Denoising Images using Wiener Filter in Directionalet Domain

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Abstract

An algorithm for denoising image based on directionalet is proposed, and in each high-frequency subbands, adaptive Wiener filtering (window size 1×25 *) is performed. Then, the denoised image is obtained by reconstruction from the filtered coefficients. At last, all directional sub-images are averaged.. The experiments results showed that this method outperforms Wiener filtering in standard 2-D wavelet domain. The SNR is improved 1~3dB.*

1. Introduction

In nature, most of images are noisy images, so image denoising is the key of subsequent process of image. For the multiresolution and time-frequency localization properties and extracting the local singular characteristics of the signals, Standard wavelet transform become one of the optimal algorithms for signals denoising. The most common approach in image processing is to use 2-D separable filter-banks, which consist of the direct product of two independent 1-D filter-banks in the horizontal and vertical directions. Although its computation is very simple, the standard WT uses only horizontal and vertical directions and not capture efficiently these anisotropic geometrical structures characterized by many more than the horizontal and vertical directions. So it is not perfect to use the standard WT in complex texture image denoising. Directionalet[3] is based on the one-dimensional (1-D) WT and extend the standard WT in many more than the horizontal and vertical directions. Directionalet retains the separable filtering and subsampling and the simplicity of computation and constructs the multi-directional frames and base functions.

In this paper, an algorithm based on directionalet and Wiener filtering for denoising image is proposed. Apply Mutli-directional frames of directionalet divide the image into many sets of pixels. And then 1-D WT is used along these sets of pixels. In each high-frequency subbands produced by 1-D WT Wiener filtering is used to filtering the coefficients. Experimental results on several images with different level noise shows this proposed algorithm performs better than the wiener filtering in standard 2-D wavelet[6] domain, visually and quantitatively. The SNR is improved 1~3dB.

The outline of the paper is as follows. In Section 2, at first we introduce the multi-directional frames of directionalet; the second we amend the mutli-directional frames. The method of image segmenting is corrected. At last an algorithm of Weiner filtering in dircetionalet domain is proposed. In section 3 shows the experimental results. And the conclusion is on the last section.

2. Denoising Images Using Wiener Filtering In Directionalet Domain

The multi-directional frames is proposed in [1,2], which are based on the digital line is defined in computer graphics[4,5] in 1960's, partition an image space on many directions by some given digital lines.

A set of pixels $A(x,y)\in R^2$ represents a continuous line with the slope r and intercept d if the following equality is satisfied:

$$y = rx + d\,,\ -1 \le r \le 1\,,\ \ d \in Z \qquad (1)$$

The discrete approximation of (1) is called digital line $L(r,d)$. To preserve critical sample in the transform, given a rational slope r , every pixel belongs to one and only one digital line $L(r,d)$. Thus given a slope r , the set of digital lines $\{L(r,d), d\in Z)\}$, partitions the discrete image space. Different slope will produce different digital line, which will partitions the image in different directions. Moreover, if r is not belongs to the range $-1\le r\le 1$, the lines with slopes r out of the range may be obtained by symmetry, rotating and flipping vertically the space.

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This gives access to a wealth of directions in image partition and more directional information in image.

In multi-frames of directionalet, definition of digital line requires intercept d is integer, however if intercept d is integer the image partition is not complete. The analysis in detail as follows:

Given an image A, according to (1) the pixel $A(x,y)$ can be represented:

$$y = rx + d \ \forall x \in Z \ d \in Z \ \ |r| \le 1 \qquad (2)$$

Moreover, the slope of the two vectors of lattice generator matrix is just about the digital line slope $r_i = a_i / b_i$, thus, equation (2) can be represented:

$$y = \frac{a_i}{b_i} x + d, \quad \forall x \in Z,\ d, a_i, b_i \in Z,\ |r| \le 1 \quad (3)$$

If d also is integer, they requires that x must be multiple of b_i in terms of $y \in Z$. Then, the image will be not partitioned completely with digital line and some pixels will be omitted. We think the intercept should be real number so as to partition image completely. Thus, in multi-directional frames the representation of digital line is applied as follows:

$$y = \frac{a_i}{b_i} x + d,$$

$$\forall x \in Z, \quad d \in R \quad a_i, b_i \in Z, \quad |r| \le 1 \quad (4)$$

The proposed algorithm apply the multi-directional frames of directionalet to divide an image into many sets of pixels sequences along many direction, and then use 1-D wavelet transform along these pixel sequences.

The overall denoising image algorithm proceeds as follows:

1.Choose some directions according to the geometrical characteristic of an image and then Perform the multi-directional frames of directionalet partition the image along these directions.

2.Perform 1-D WT along pixel sequences.

3.Apply Wiener filtering[7] along sequences and the size of windows is 1×25.

4.Average all subband images are produced by above three proceeds.

3. Experimental Results

In this section, we evaluate the performance of our proposed algorithm for denoising image. Three standard test images: *Cameraman, Lena and Barbara*, ordered with increasing degree of texture content, were corrupted by Gaussian white noise $\sigma = 10 \sim 25$. For quantitative evaluation, SNR have been used:

$$SNR = 10\lg\left[\frac{\sum_{i=1}^{M}\sum_{j=1}^{N} S_0(i,j)^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}[S_0(i,j) - S(i,j)]^2}\right] \qquad (5)$$

The SNR is defined as the ratio of the variance of the noise-free signal S_0 to the mean-squared error between the noise-free signal and the denoised signal S .

Table1.SNRs for three images with different level Gaussian noise proceeded by two algorithms

Original image	σ	Noisy image	WFW	WFD
Cameraman	10	22.52	21.84	24.99
	15	19.02	20.97	23.50
	20	16.52	19.49	22.08
	25	14.56	18.38	20.66
Lena	10	22.47	26.19	27.89
	15	18.97	23.79	25.46
	20	16.45	21.71	23.32
	25	14.51	20.08	21.53
Barbara	10	22.24	20.89	23.47
	15	18.73	19.29	22.85
	20	16.24	18.31	21.76
	25	14.29	17.69	19.67

In wavelet, the choice of base function is very important. Different base function have different characteristic and adapt to different fields. According to the analysis of their performance, we use db8 wavelet, which is orthogonal and tight as 1-D base function of directionalet to decompose pixel sequences in two scales. These sequences are obtained along 6 directions partition. The experiment result shows that Wiener filtering in directionalet(WFD) can retain more textures in image than Wiener filtering in standard wavelet domain(WFW), moreover its SNR outperforms the WFW other method 1~3dB. Table1 shows the SNR that two methods are used for different noise level three images. Fig1~4 show the results of two algorithms. Fig.1 is original image. Fig.2 is image with noise $\sigma = 20$. Fig.3 is denoised image with WFW. Fig.4 is the proposed methods. We can two methods reduce the noise, but the denoising performance of the WFD method outperform the WFW method. And the WFD method preserves more the structure of the original image than the WFW method.

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Figure1. Original image

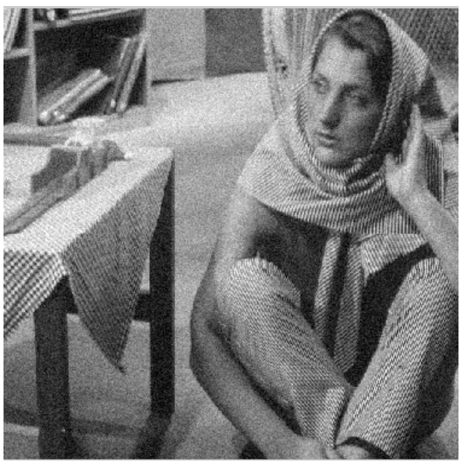

Figure2. Image with Gaussian white noise

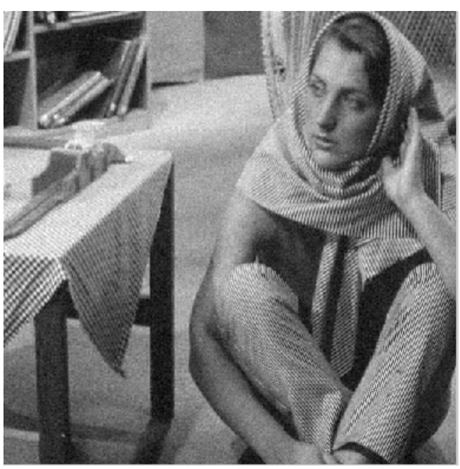

Figure3. Denoised image with WFW

Figure4. Denoised image with WFD

4. Conclusion

We have presented a new algorithm of Wiener filtering in directionalet domain for denoising image. This algorithm is based on the multi-directional frames of directionalet and apply digital line to divided the noisy image into sets of pixels sequences along many directions, thus 1-D wavelet is applied along these sequences. The decomposed coefficients of high-frequency subbands are filtered by Wiener filter, at last all directional subband images are averaged. The computation of this method is simple, moreover the denosing performance outperforms Weiner filter in wavelet domain under most noise-level conditions.

Directionalet is a new part of wavelet as contourlet[8,9] for image processing. Its filter is separable and is simpler than contourlet, but it performance is equal to contourlet. So some algorithms based on directionalet for image fusing, edges detection are very interesting.

5. References

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