3D Data Processing

Image processing-2

Hyoseok Hwang

contents

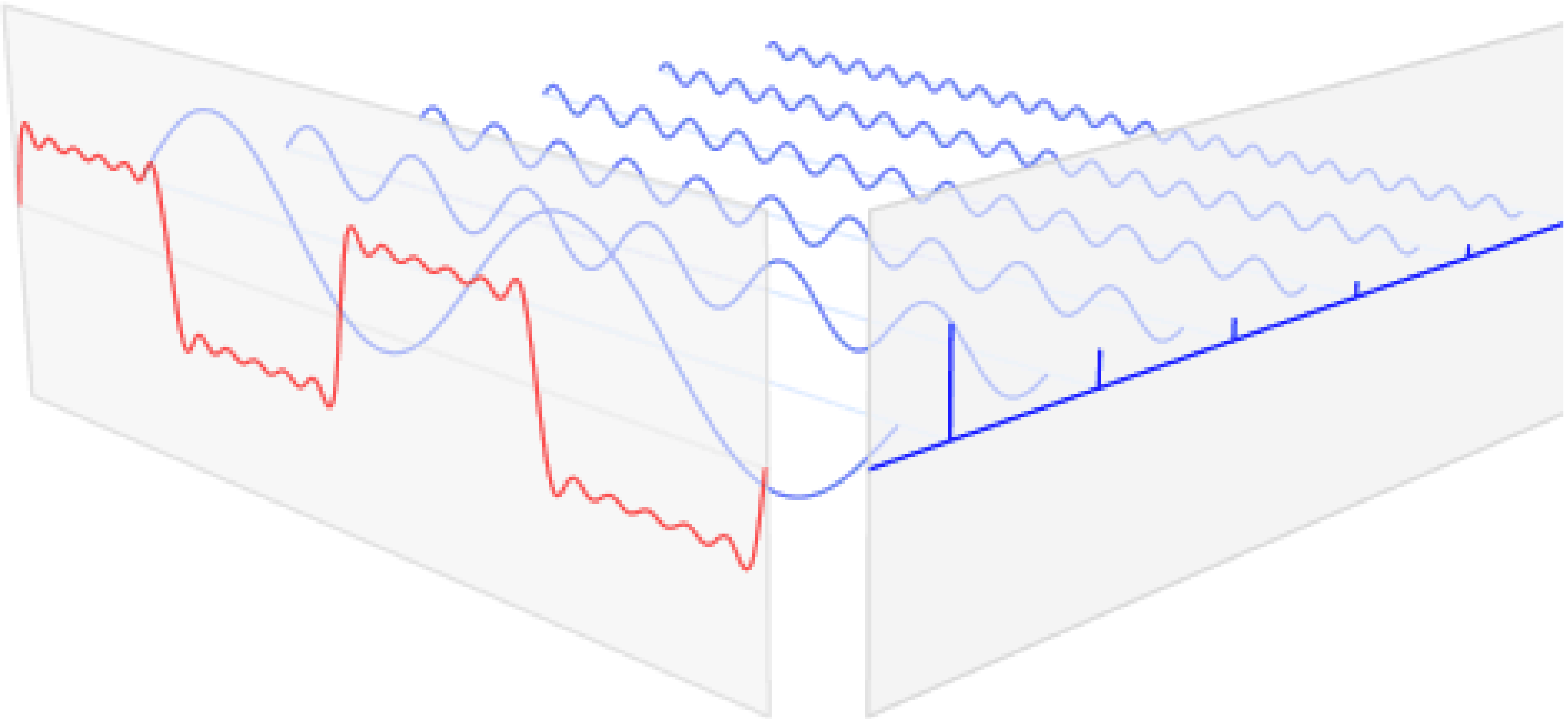
- Introduction of image processing
- Pixel point processing
- Geometric transform
- Domain transform
- Spatial filtering

Image transform

- Hough Transform
- Fourier Transform

Fourier Transform

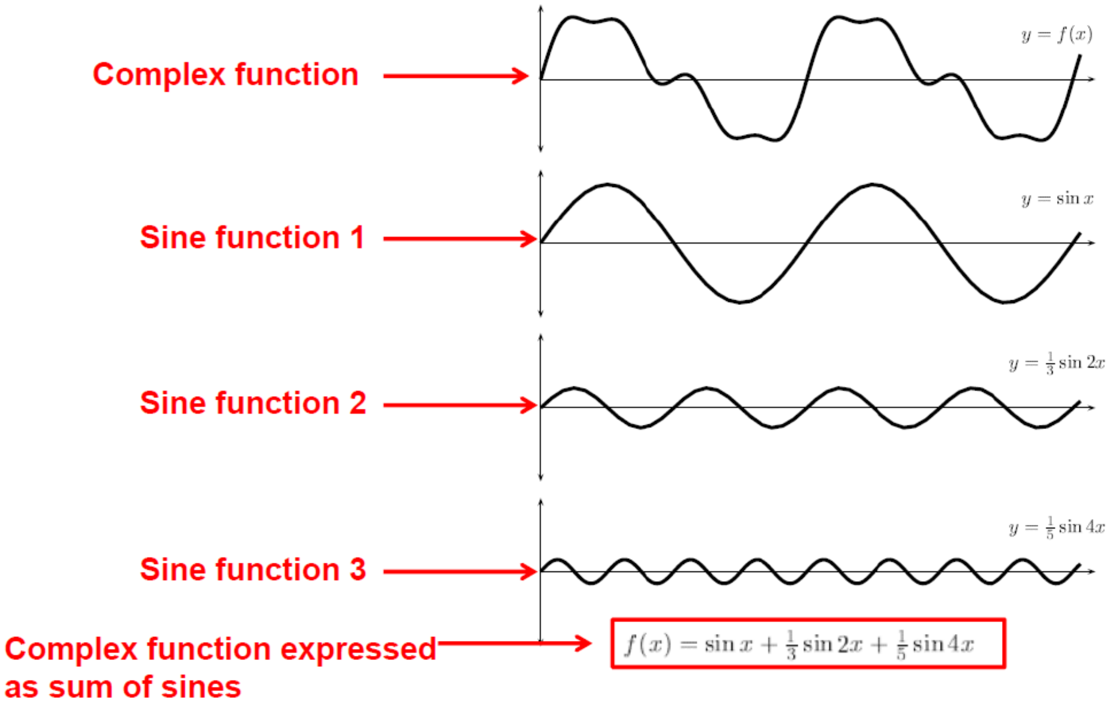
- Main idea: Any periodic function can be decomposed into a summation of sines and cosines



https://www.thefouriertransform.com/

Fourier Transform

- Main idea: Any periodic function can be decomposed into a summation of sines and cosines
- Mathematially easier to analyze effects of transmission medium, noise, etc. on simple sine functions, then add to get effect on complex signal



Fourier transform

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) \cdot \Big[\cos(\omega x) - \mathrm{i} \cdot \sin(\omega x)\Big] \, \mathrm{d}x$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) \cdot e^{-\mathrm{i}\omega x} \, \mathrm{d}x.$$

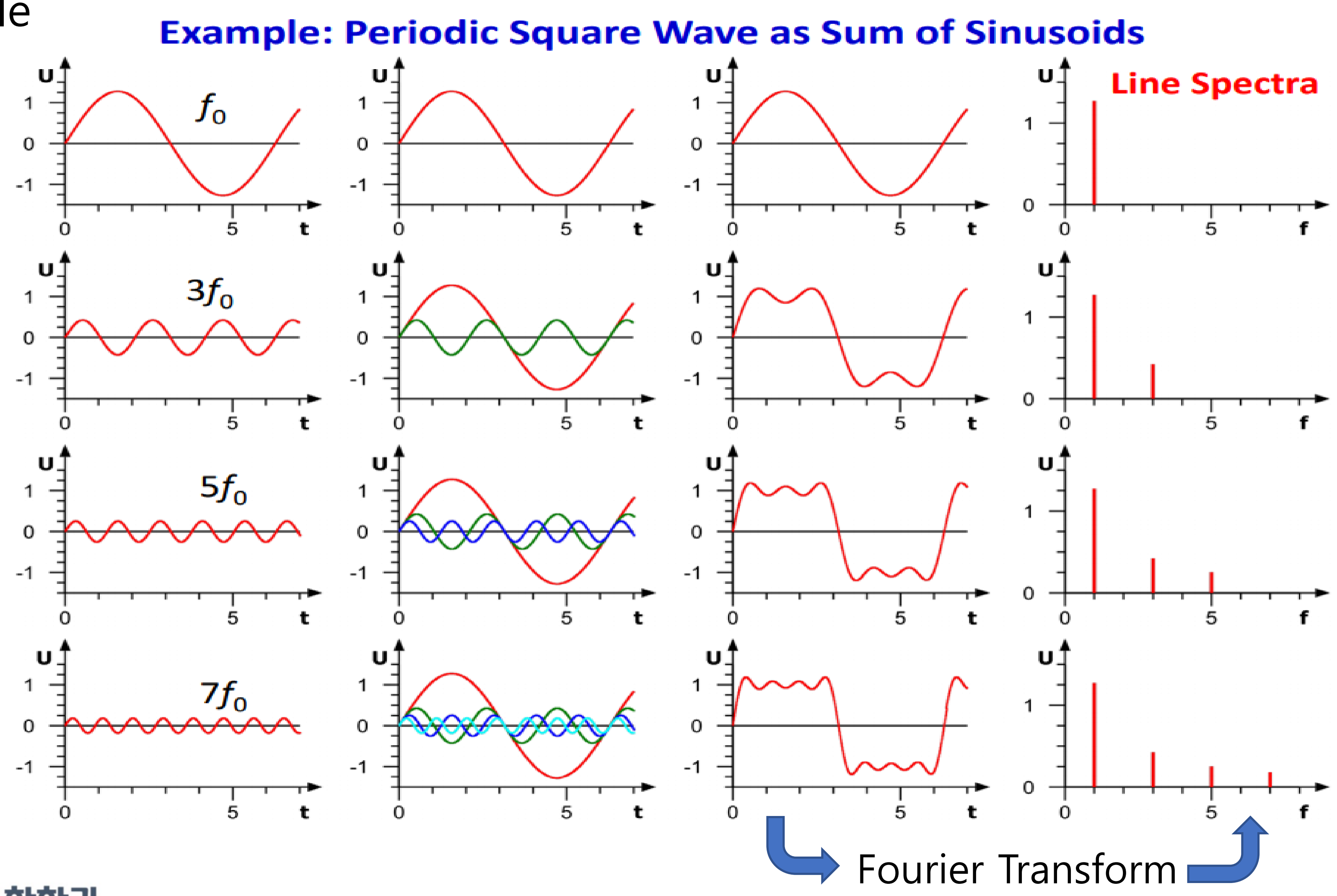
Inverse Fourier transform

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) \cdot \Big[\cos(\omega x) + \mathrm{i} \cdot \sin(\omega x)\Big] \, \mathrm{d}\omega$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) \cdot e^{\mathrm{i}\omega x} \, \mathrm{d}\omega.$$

https://www.thefouriertransform.com/

Fourier Transform

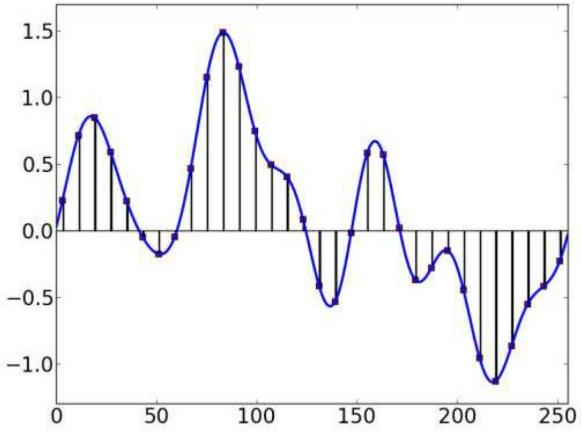
- Example



https://www.thefouriertransform.com/

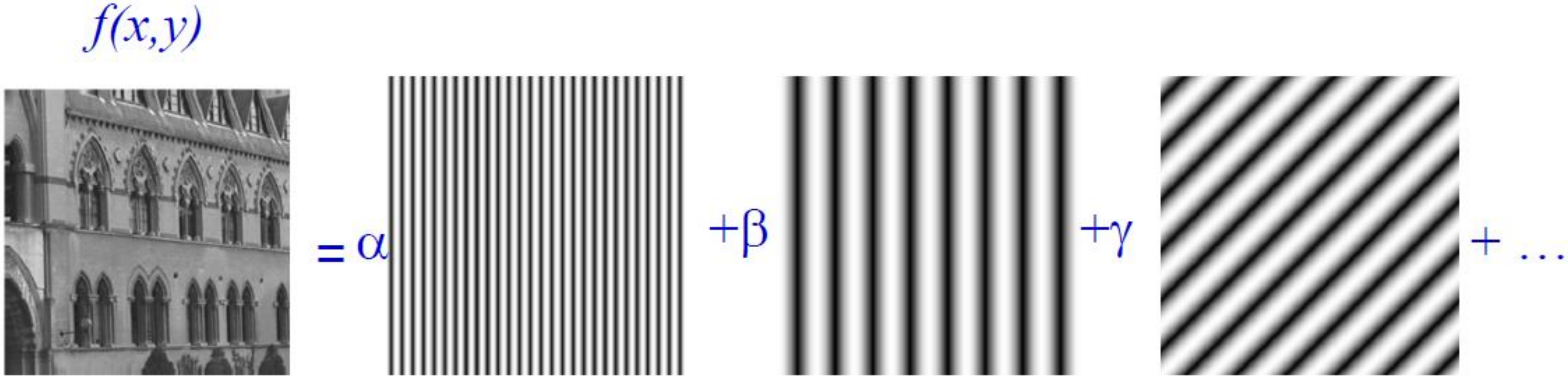
Fourier Transform

- Fourier transform
  - Convert **continuous** signals of time domain into frequency domain
- Inverse Fourier transform
  - Convert **continuous** signals of frequency domain into time domain
- Discrete Fourier transform (DFT)
  - The equivalent of the continuous Fourier Transform for **discrete(sampling)** signals.
  - Discrete signals = Quantization(Sampling(continuous signals))
- Fast Fourier transform (FFT)
  - A fast algorithm for computing the Discrete Fourier Transform
  - In image processing, FFT (iFFT) is mainly used.



Fourier Transform

- 2D Fourier Transform
  - Fourier transform can be generalized to higher dimensions
- Images as functions
  - Gray scale images: 2D functions – Domain of the functions: set of (x,y) values for which f(x,y) is defined : 2D lattice [i,j] defining the pixel locations – Set of values taken by the function : gray levels

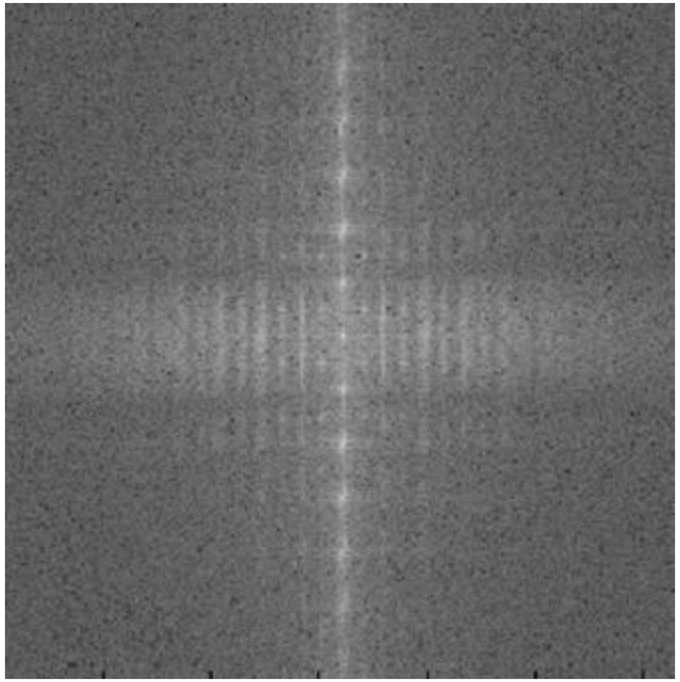


Fourier Transform

- 2D Fourier Transform
  - Fourier transform is invertible.

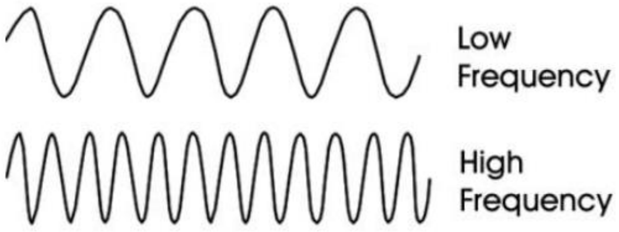
2D FFT

2D iFFT

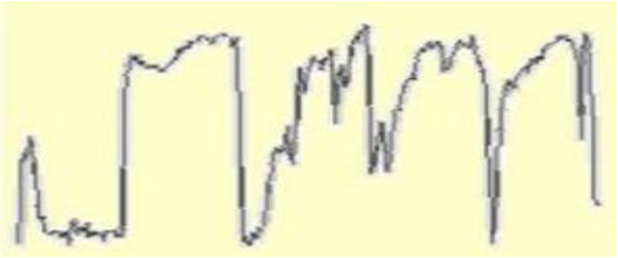


Fourier Transform

- Frequency on images
  - High frequency: edges, points
  - Low frequency: All the rest



Check the region of Low/High frequency



Intensity of N-th row

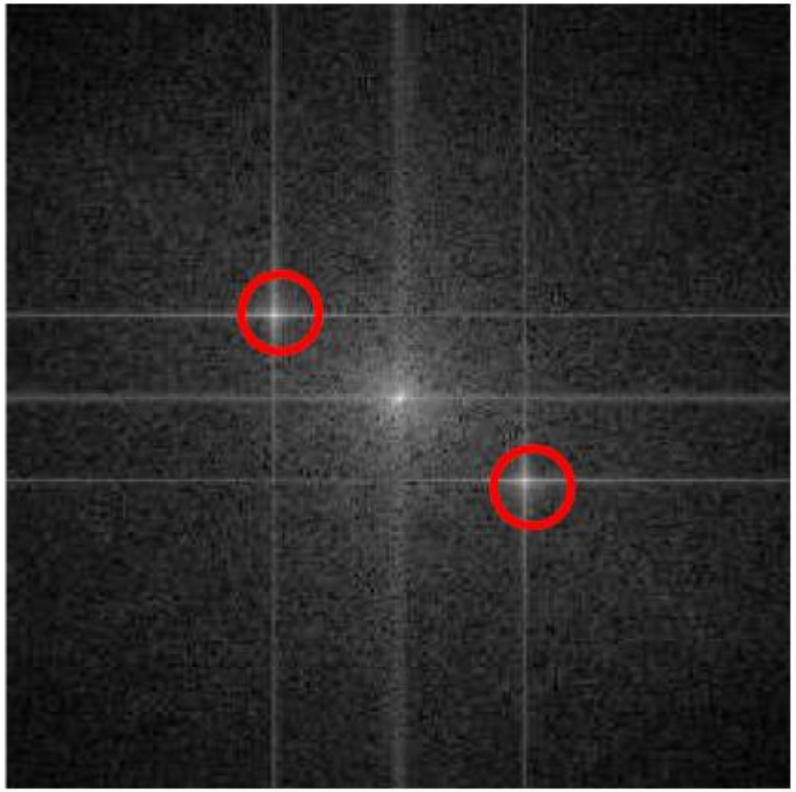
Image restoration

- Removing artifacts by frequency analysis
- Types of artifacts
  - Noise
    - Noise is a large difference from the nearby values → high frequency
  - Unwanted pattern
    - The periodic pattern is distributed throughout the image(spatial domain), but is displayed as a single dot in the frequency domain.
- Methodology
  - Convert image to frequency domain using FFT
  - Remove artifacts
  - Convert frequency values to image using iFFT

Image restoration

- Removing unwanted pattern (periodic noise)

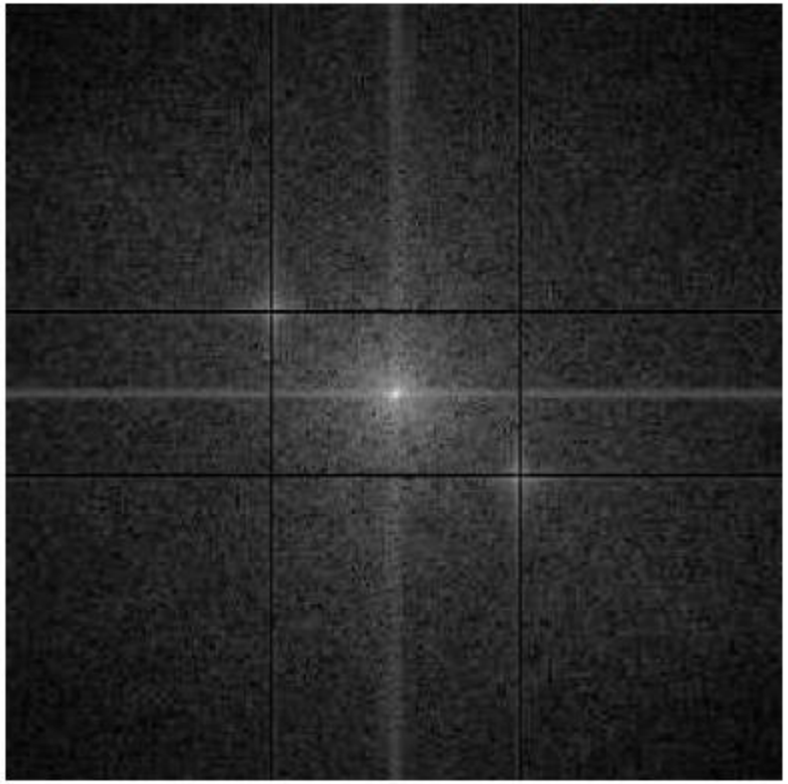
Image with periodic Noise



DFT of Image

Image restoration

- Removing unwanted pattern (periodic noise)



Notch Filter

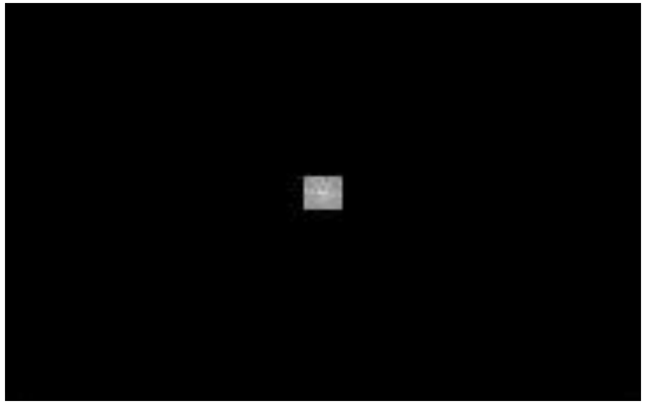
Result after notch filter applied then inverted

Image restoration

- Removing random noise

2D FFT

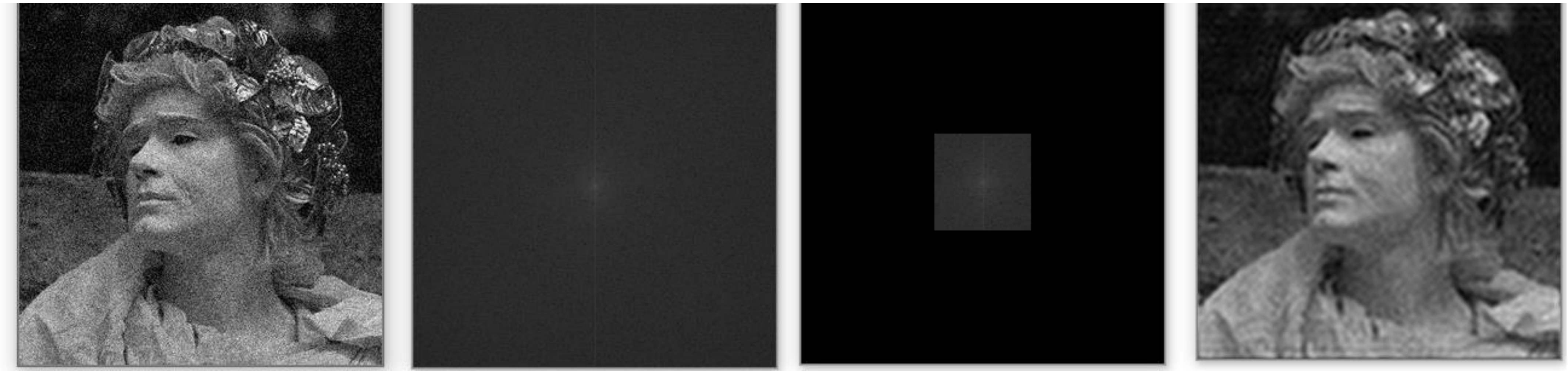
Remove high frequency (low-pass filter)



2D iFFT

Image restoration

- Exercise
  - File name: 3_2D_FFT_denoising.py



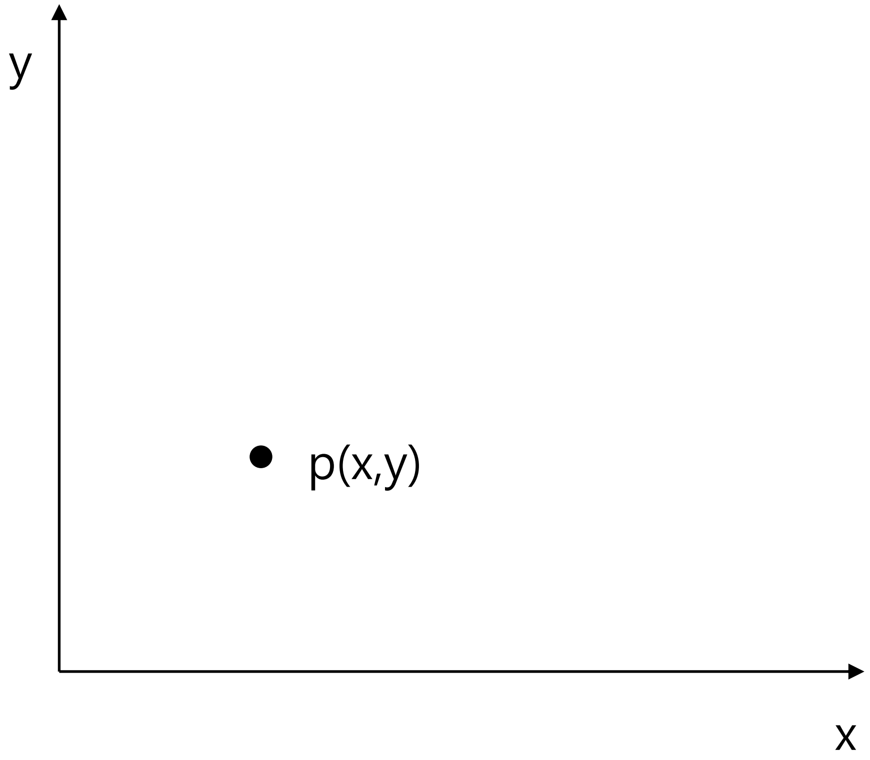
2D FFT   low-pass filter   2D iFFT

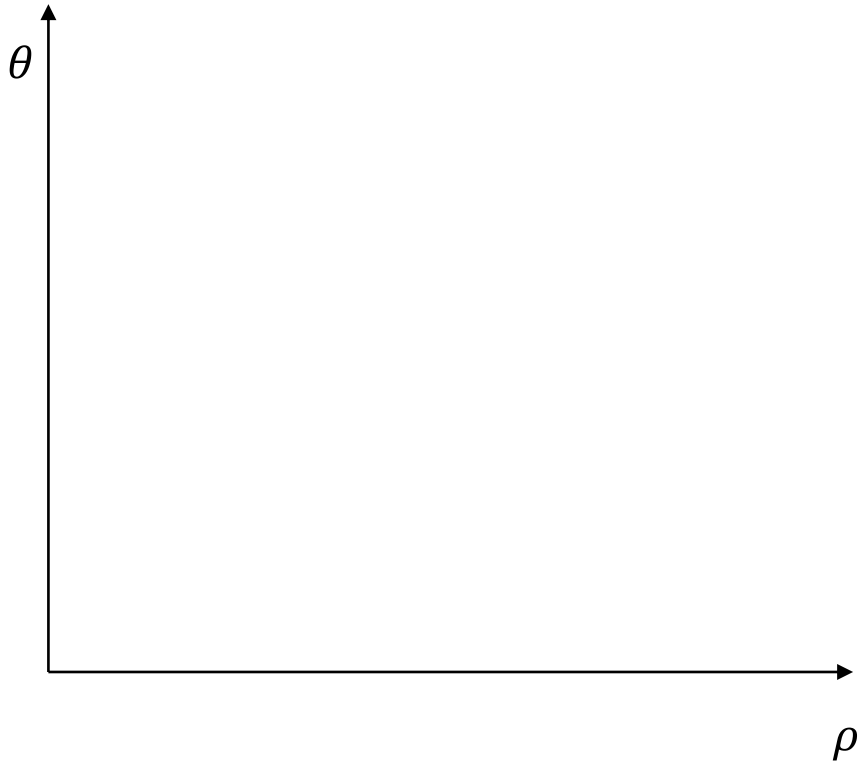
Hough transform

- Convert (x,y) domain to (ρ, θ)
  - Represent a line on (x,y) coordinate to a point on the (ρ, θ) coordinate.
- Hough transform is used for detection certain shapes (especially lines)
  - Suppose that there is a point p(x,y)
  - Infinitely many straight lines pass through that point.
  - Each line can be represented to a point in (ρ, θ) coordinate
  - Lines passing through p(x,y) are represented in the form of a curve.
  - The straight line we are looking for appears as a superposition(such as a point) of the curves in A.

Hough transform

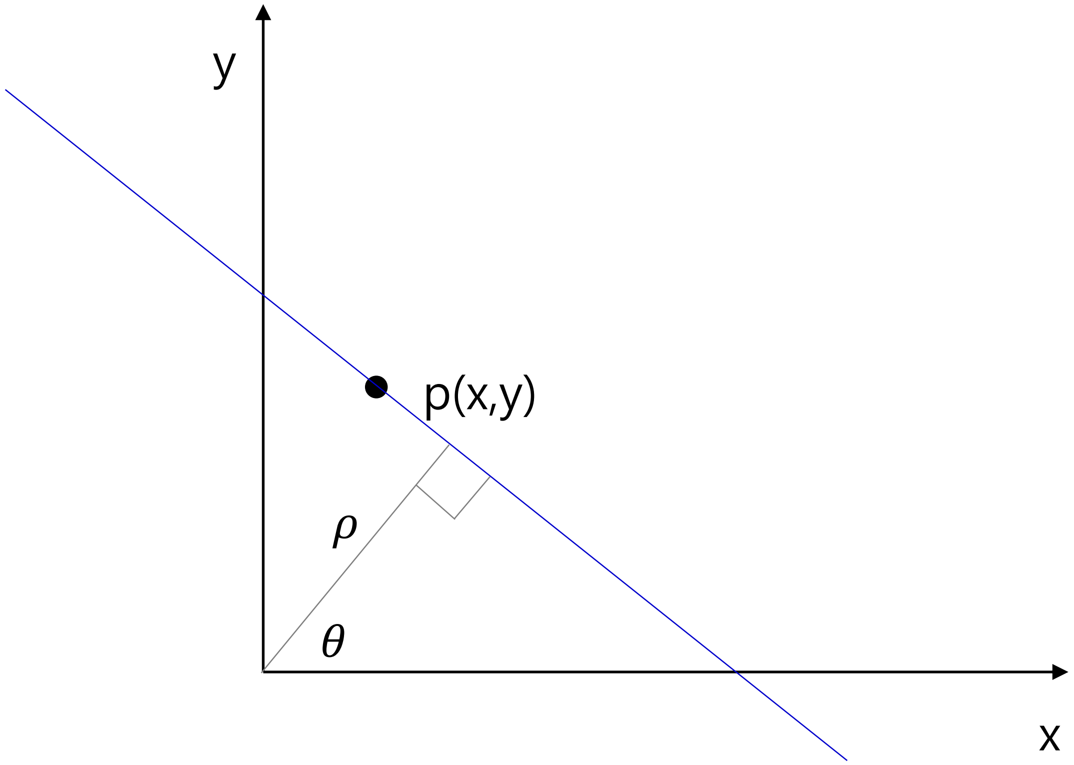
- Transform of a line on the point p(x,y)

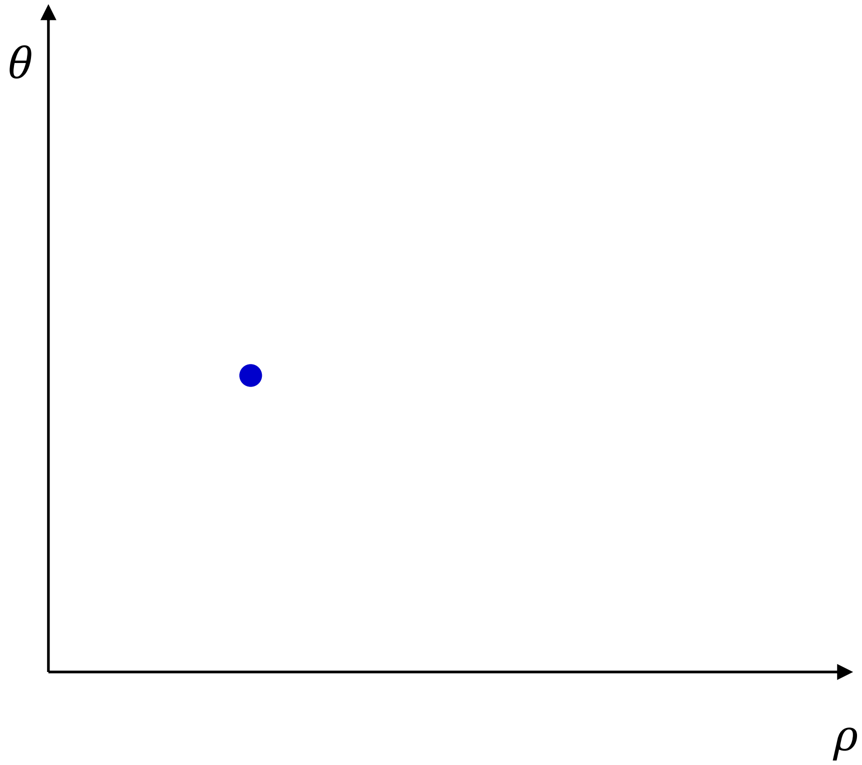




Hough transform

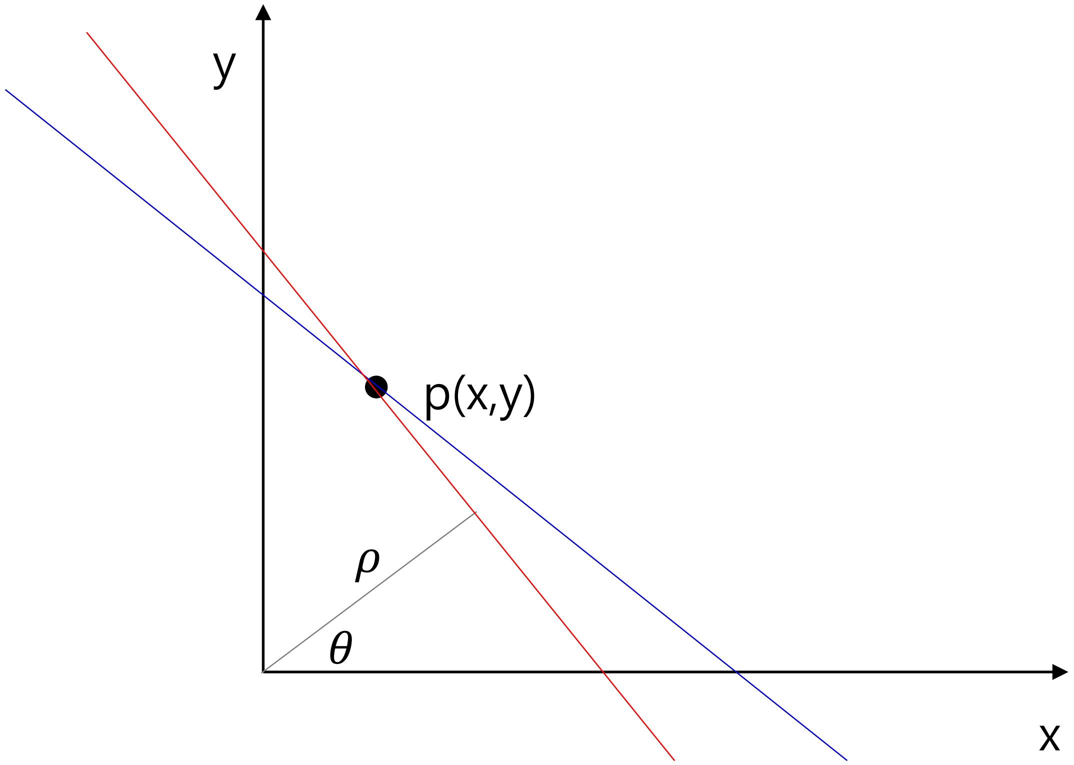
- Transform of a line on the point p(x,y)
  - ρ: The orthogonal distance between the origin and a straight line.
  - θ: angle of orthogonal line

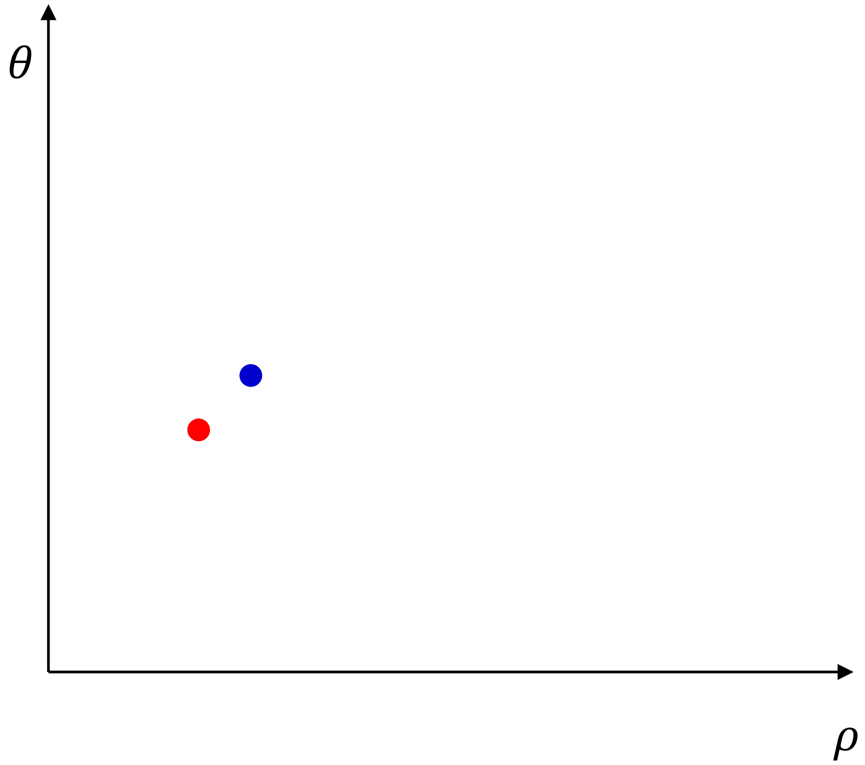




Hough transform

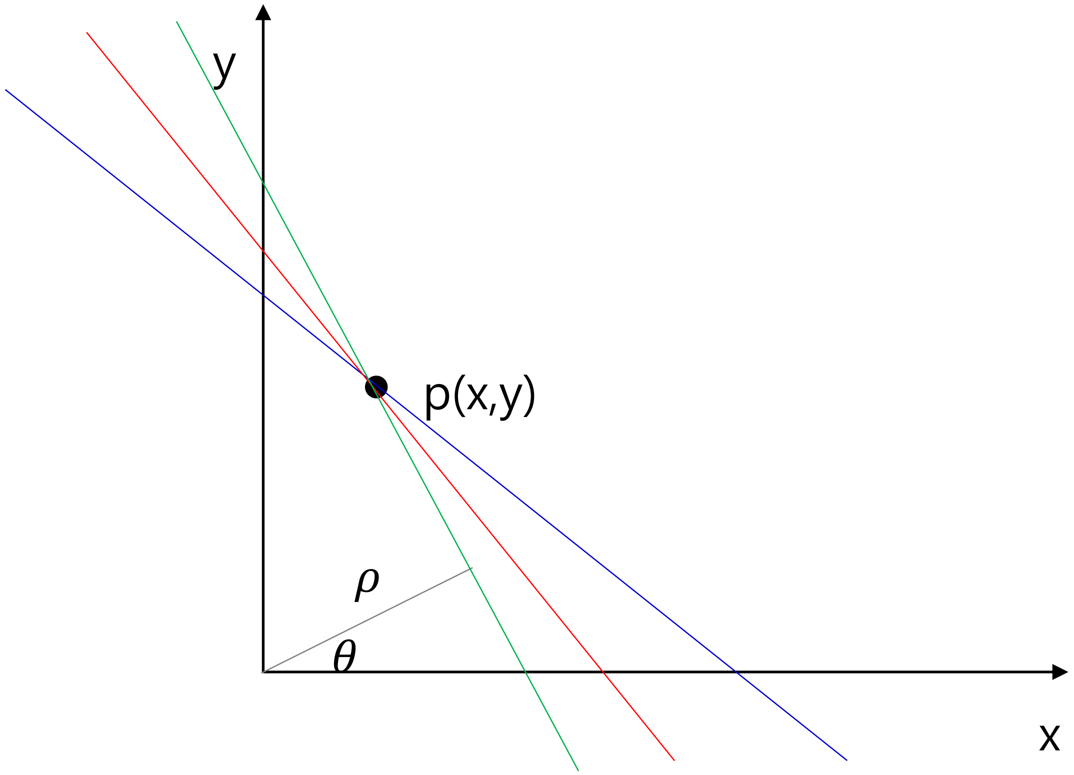
- Transform of a line on the point p(x,y)
  - Repeat getting parameters of other lines passing through p(x,y)

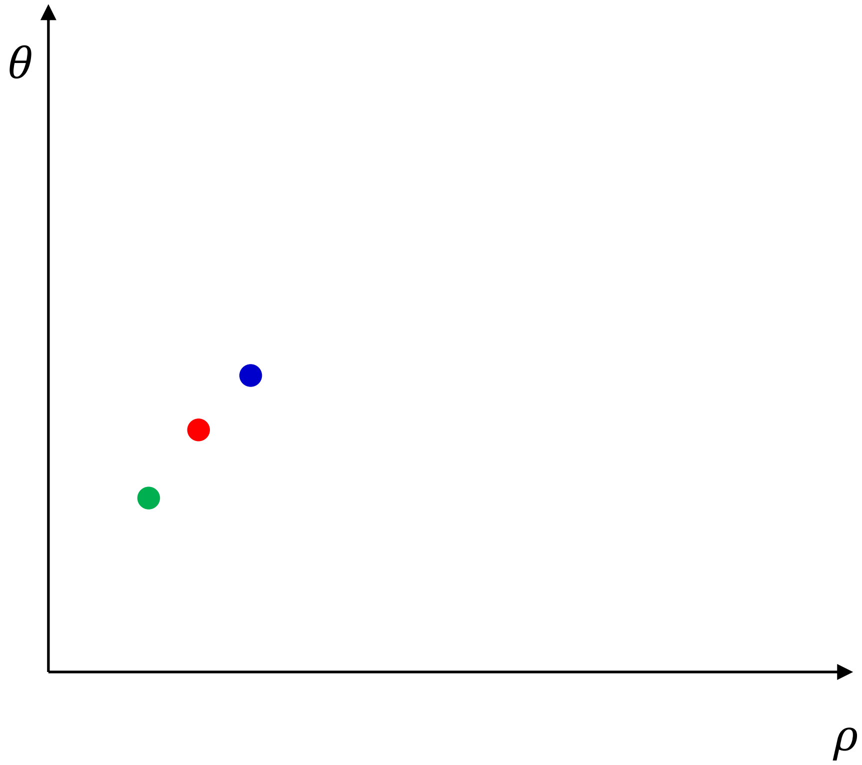




Hough transform

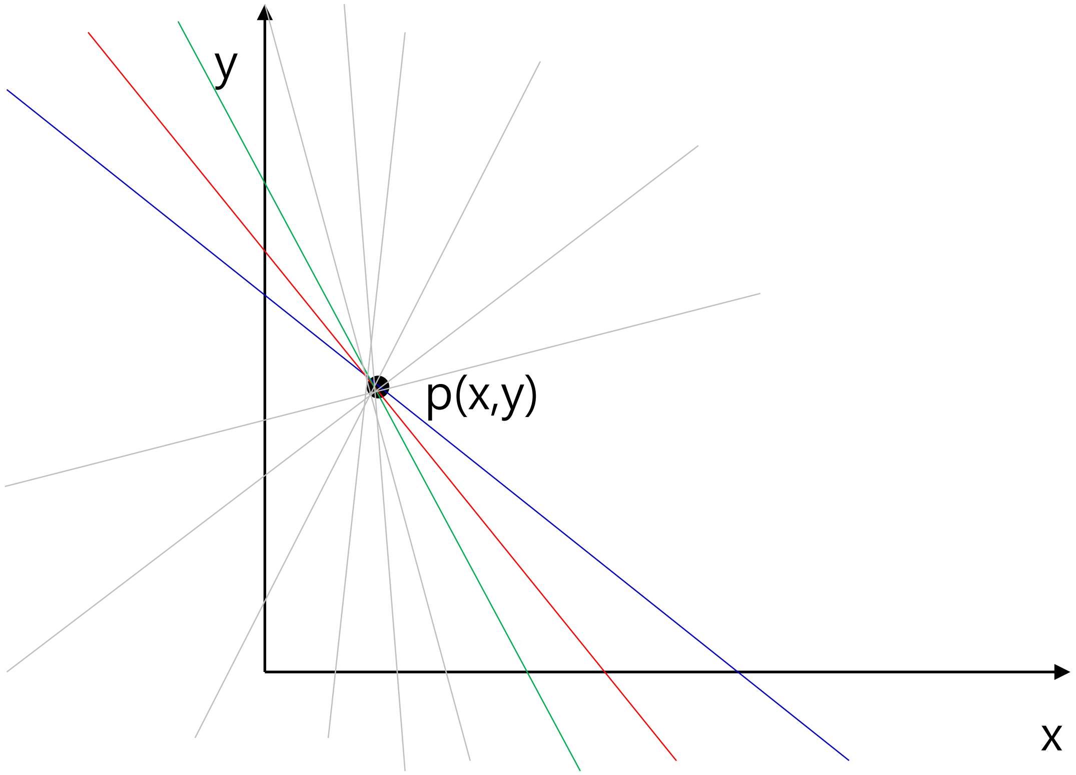
- Transform of a line on the point p(x,y)
  - Repeat getting parameters of other lines passing through p(x,y)

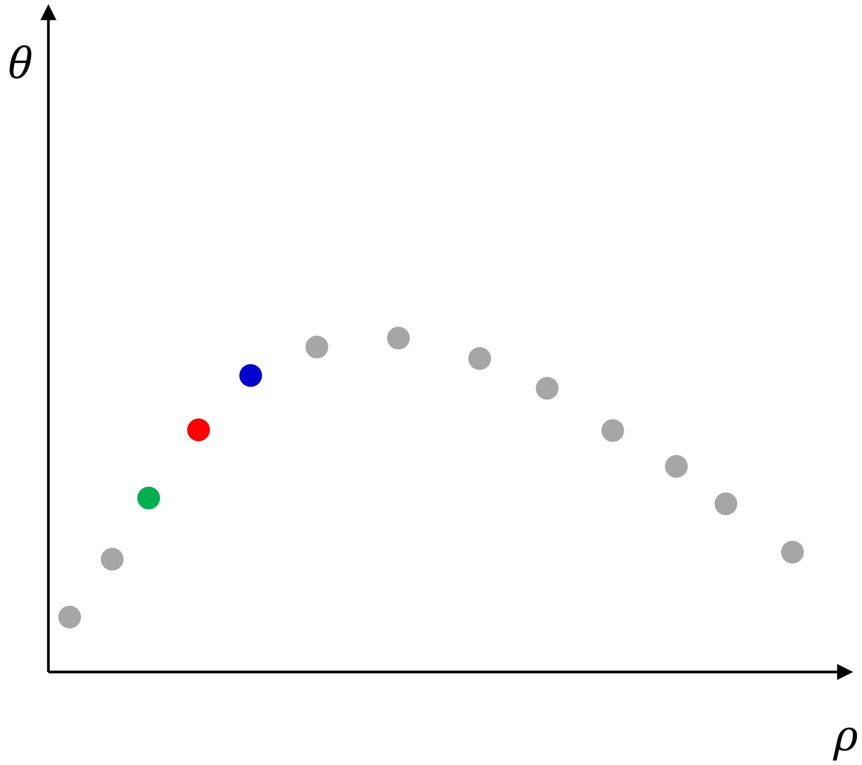




Hough transform

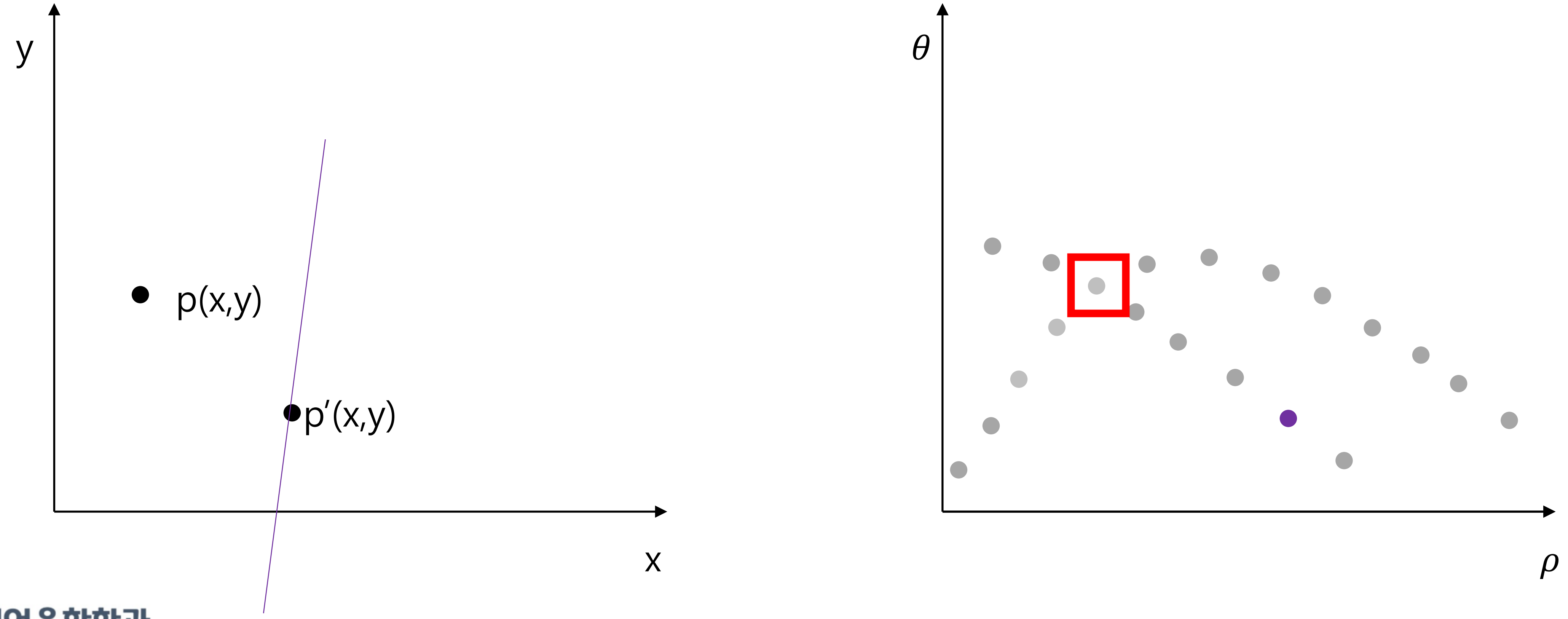
- Transform of a line on the point p(x,y)
  - Repeat getting parameters of other lines passing through p(x,y)





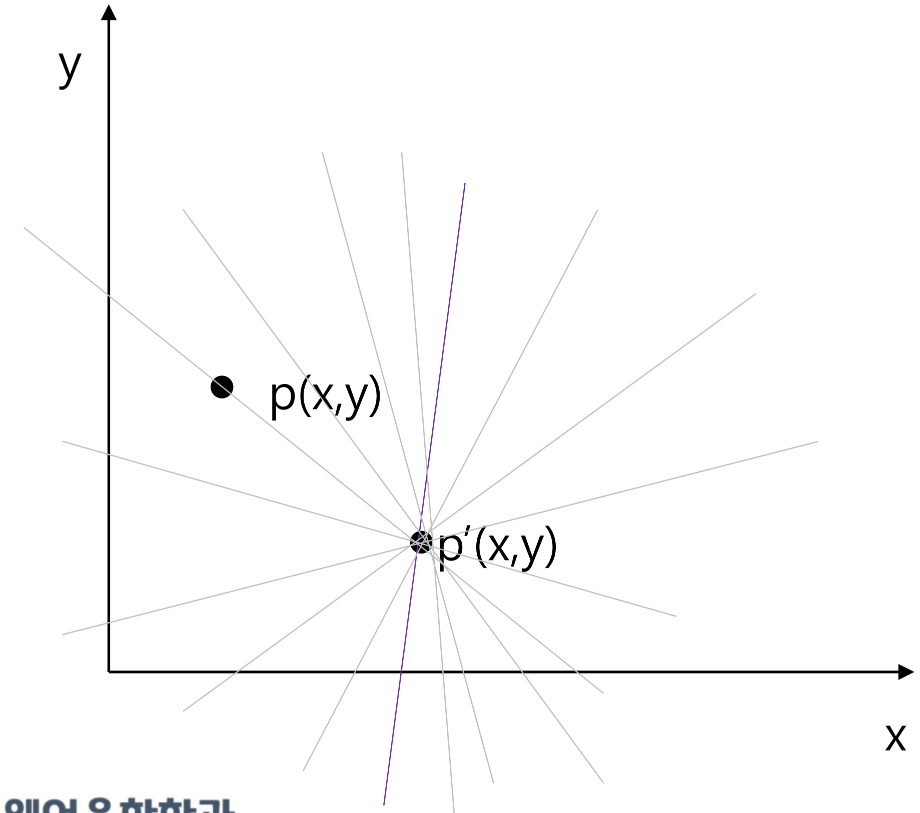
Hough transform

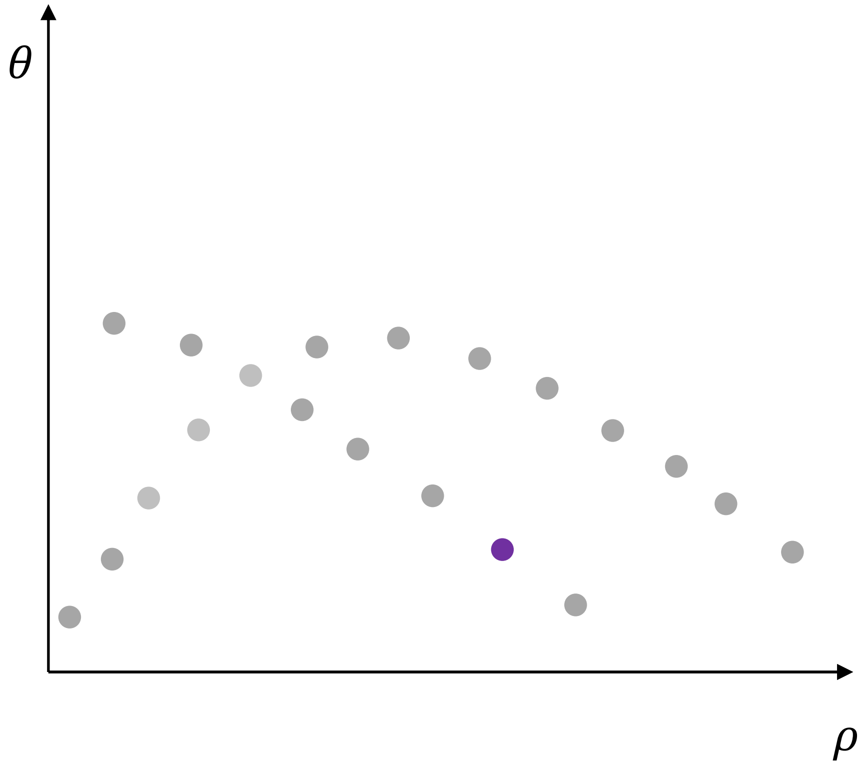
- Transform of a line on the point p(x,y)
  - Get parameters of other lines passing through p′(x,y)



Hough transform

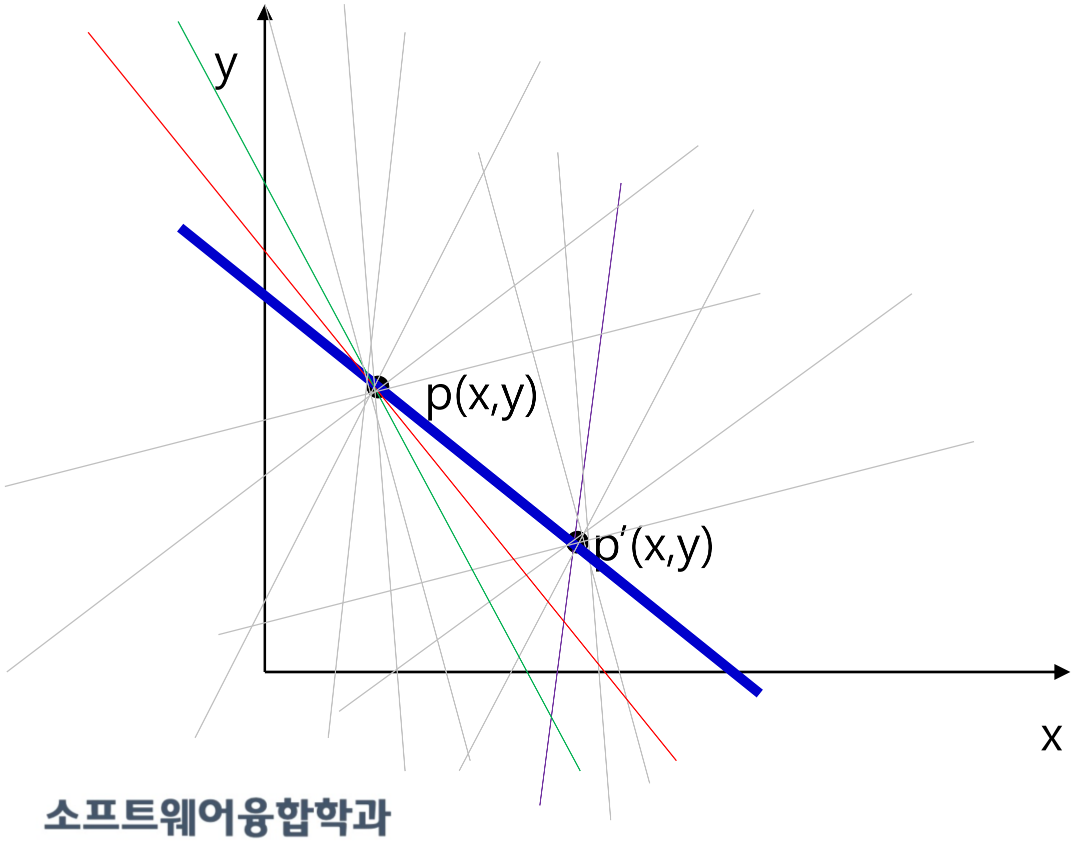
- Transform of a line on the point p(x,y)
  - Repeat getting parameters of other lines passing through p'(x,y)

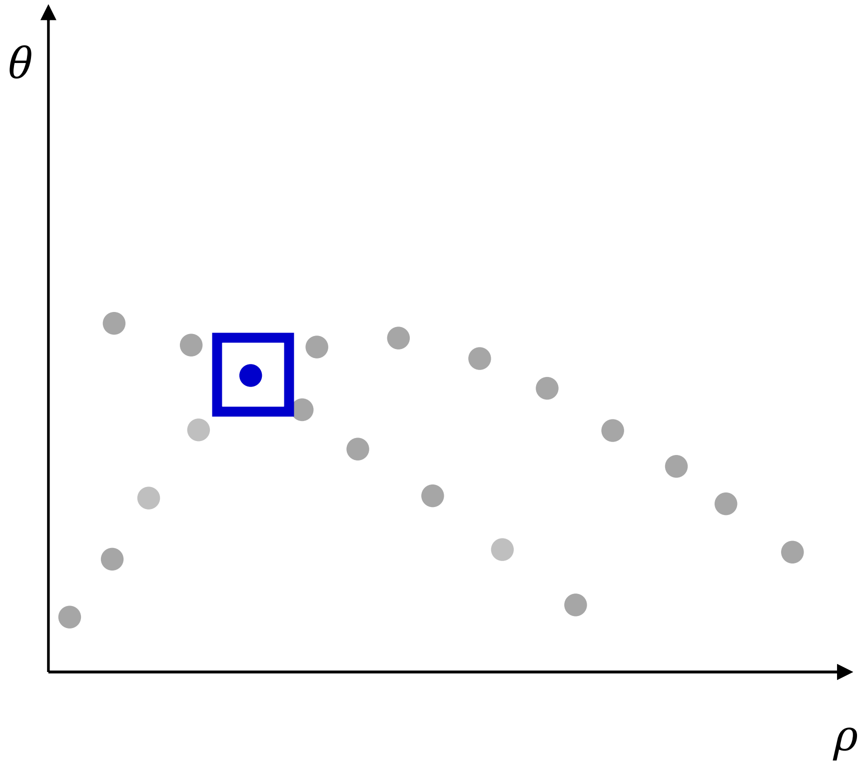




Hough transform

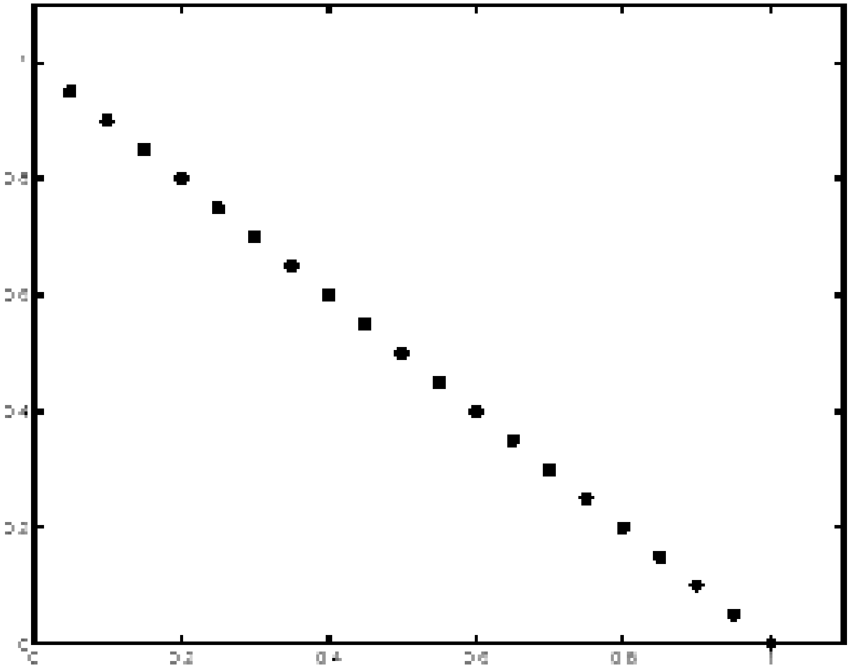
- Transform of a line on the point p(x,y)
  - They have a same point on (ρ, θ)coordinate if they are on the same line.

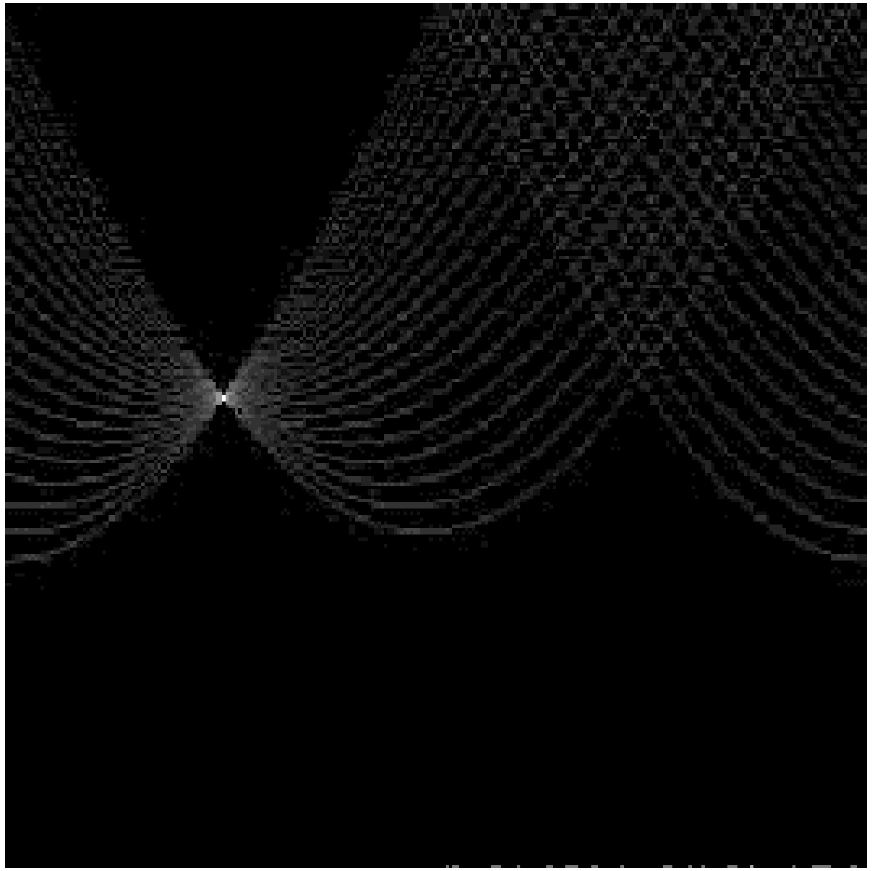




Hough transform

- An example of Hough transform





Thank you