3D Data Processing

Point Clouds Registration

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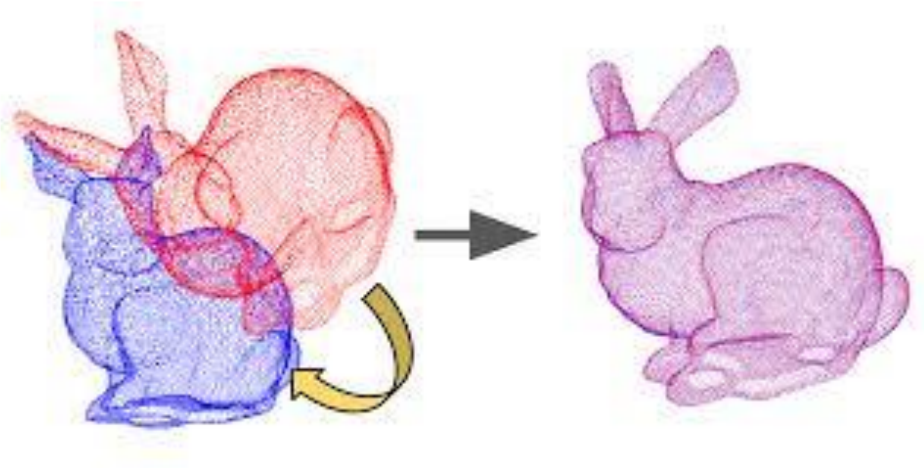
Lectures are based on Open3D functions

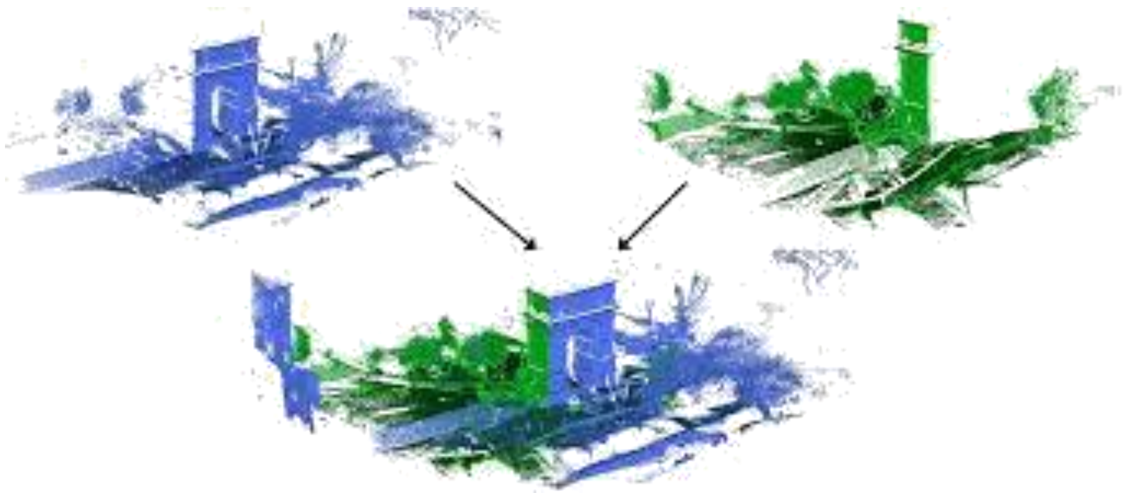
Today

- Registration
- ICP

Registration (Association)

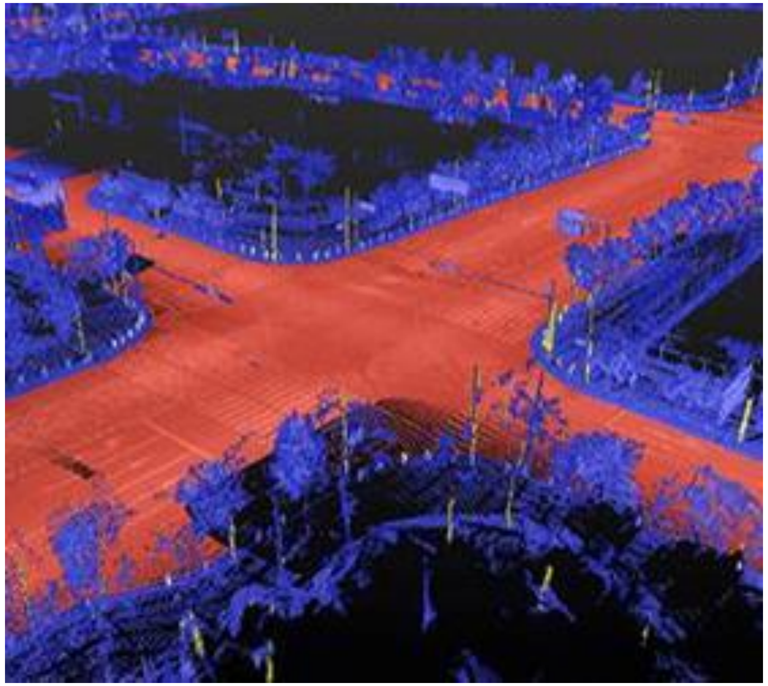
- Registration
  - Connection! ←→ What do we need to connect?
  - Process of finding a spatial transformation (e.g., scaling, rotation and translation) that aligns two point clouds.
  - Merging multiple data sets into a globally consistent model (or coordinate frame)
  - Fundamental problem in geometry analysis





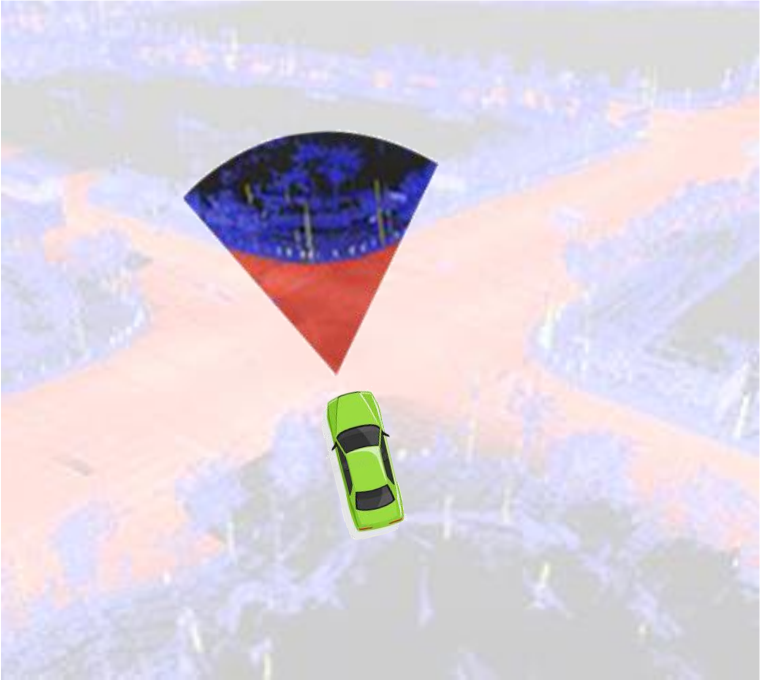
Registration

- Applications
  - Autonomous Vehicle Localization
  - Visual Odometry for UAVs and Unmanned Ground Vehicles



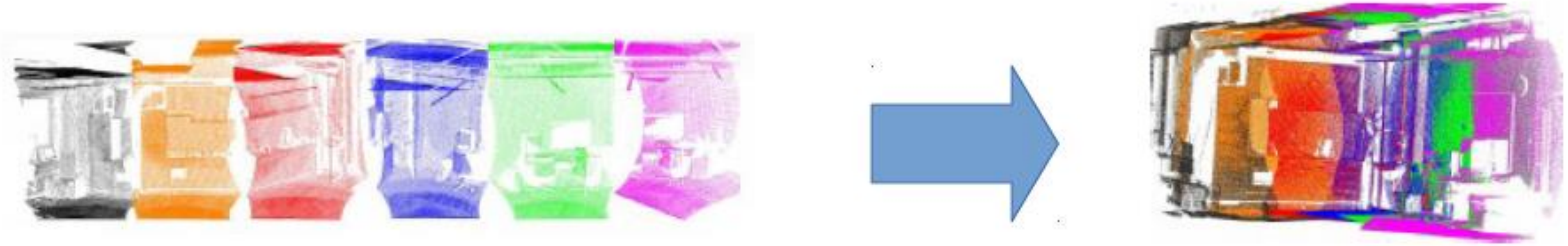
map

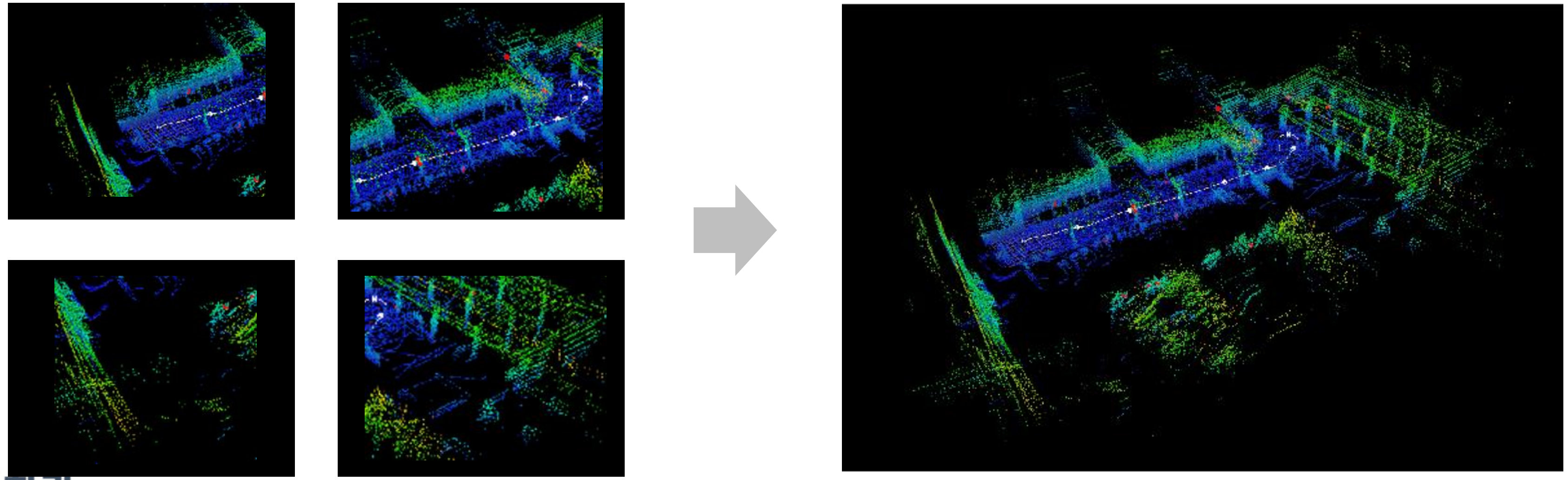
Where am I?



Registration

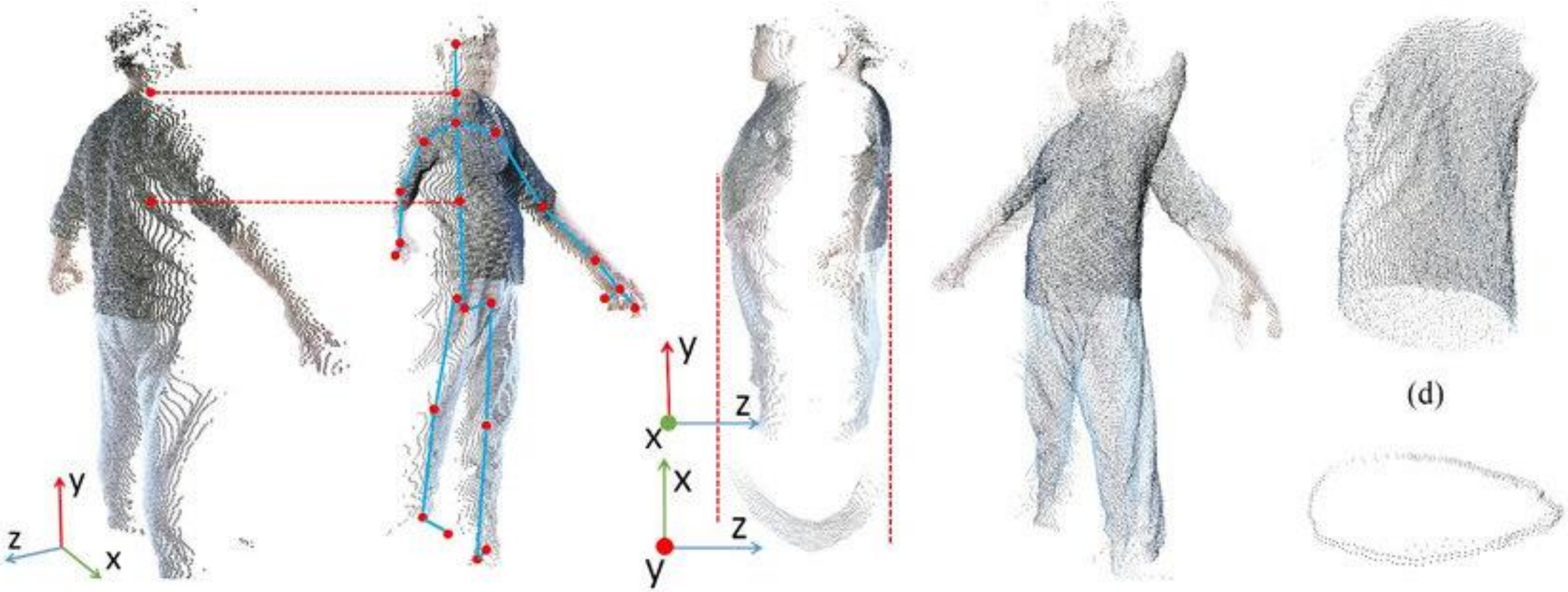
- Applications
  - 3D Terrain Mapping





Registration

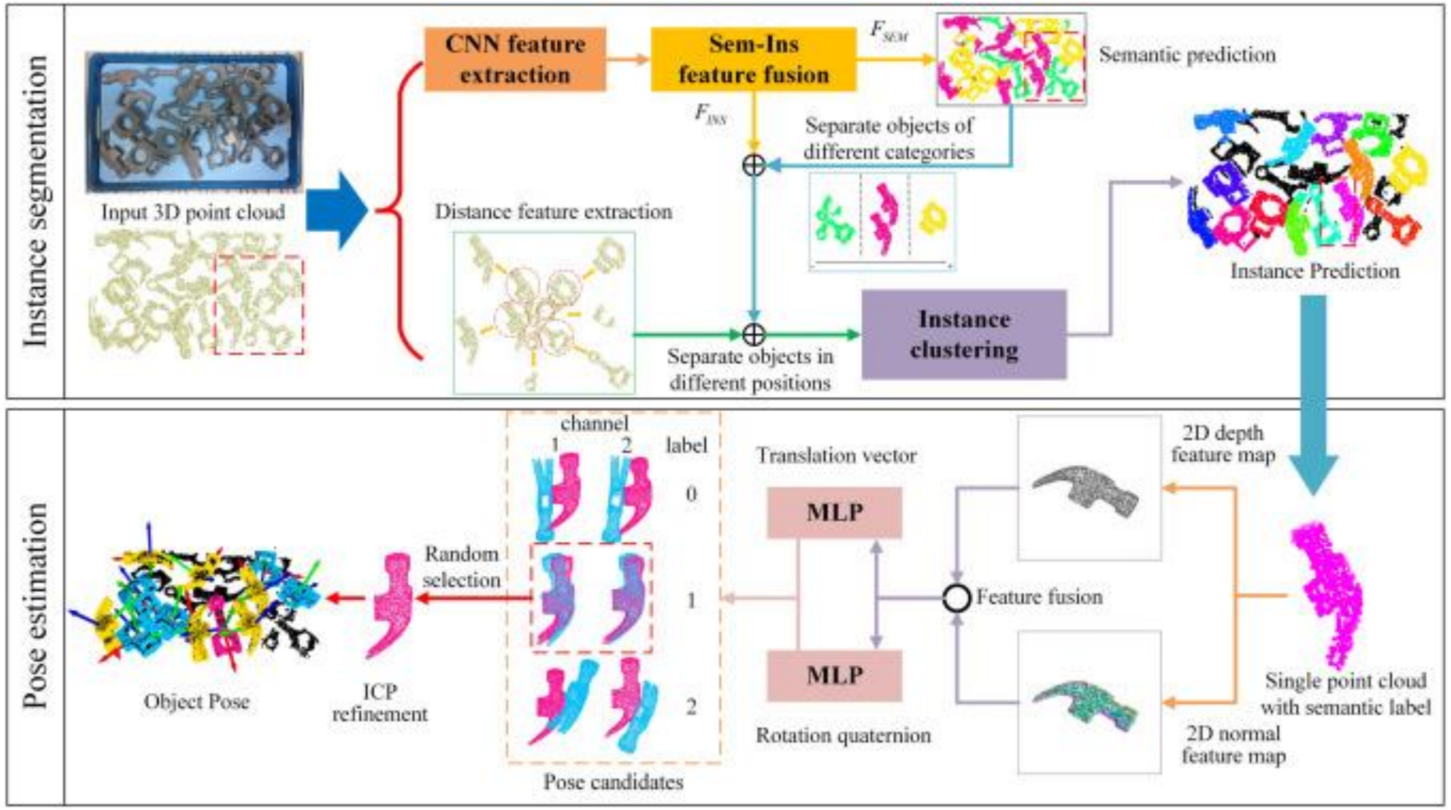
- Applications
  - Object 3D reconstruction



(a) (b) (c)

Registration

- Applications
  - 6DOF Pose estimation (refinement)

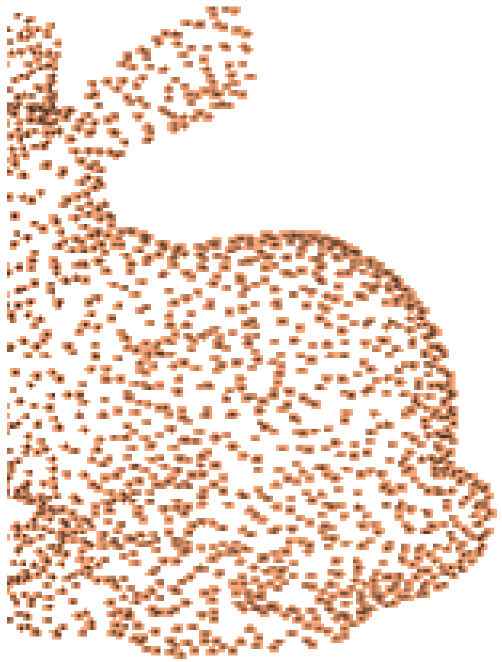


Instance segmentation based 6D pose estimation of industrial objects using point clouds for robotic bin-picking

Registration

- Principles
  - How can we register two partial point clouds?

P1



P1

Registration

- Principles
  - You may overlap similar areas.

P1 P2

RIGHT!

Registration

- Principles
  - How can we express this mathematically?
  - How can we write this code?

P1 P2

Registration

- Principles
  - Hint: Overlap, Superposition
    - They are actually the same point

$X_1 = [x_1, y_1, z_1]^\top$

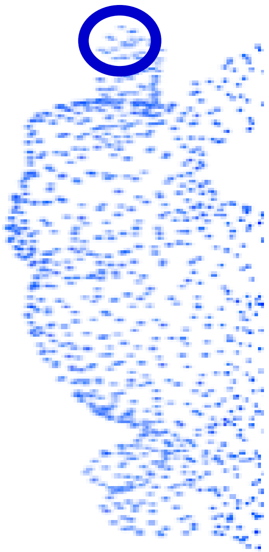
P1

$X'_1 = [x'_1, y'_1, z'_1]^\top$

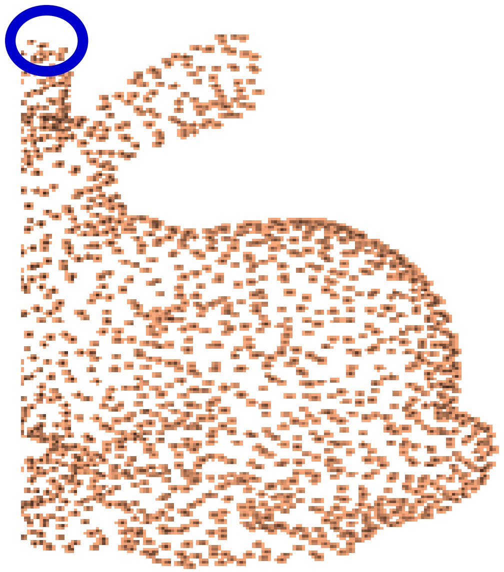
P2

Registration

- Principles
  - Hint: Overlap, Superposition
    - They are actually the same point
      - $X_1 = X'_1$?



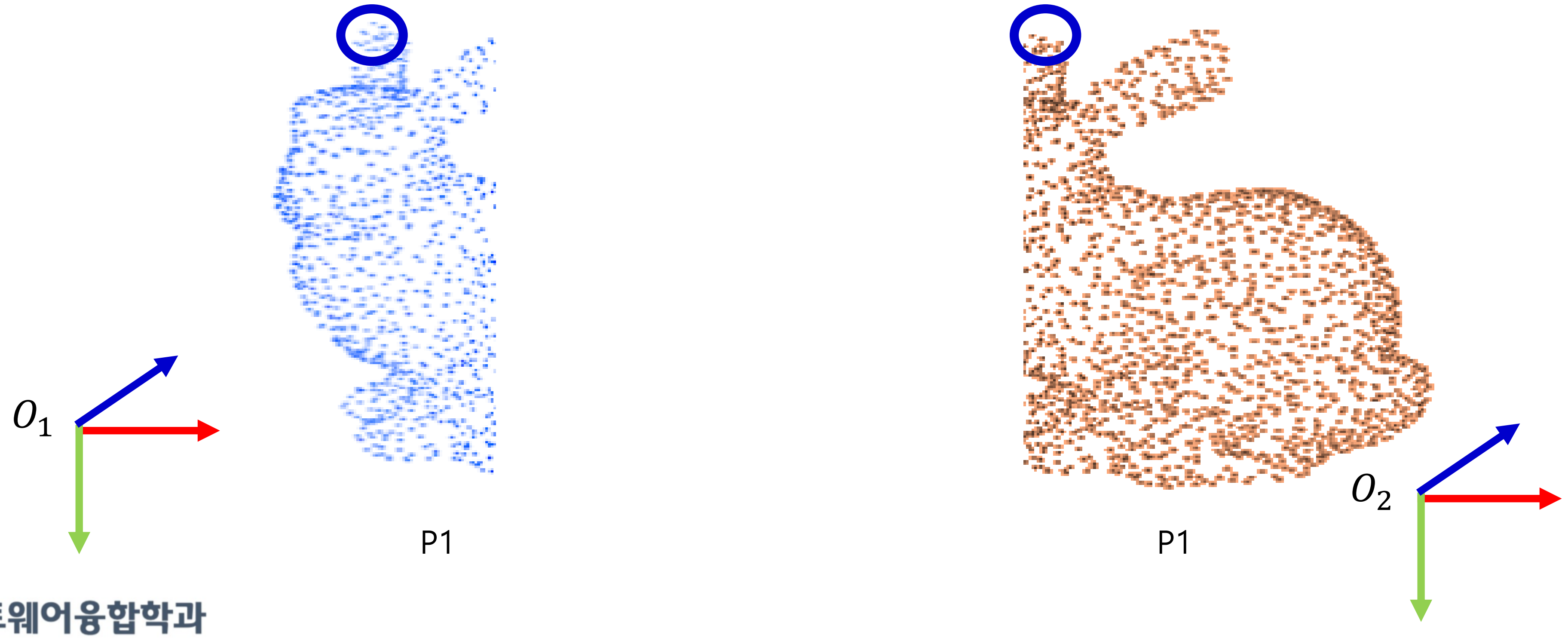
P1



P1

Registration

- Principles
  - Hint: Overlap, Superposition
    - They are actually the same point
      - All positions are coefficients in the base coordinates.



Registration

- Principles
  - Hint: Overlap, Superposition
    - They are actually the same point

$$X_1 = RX'_1 + t$$

    - R, t : Rotation matrix (3x3), translation vector of Coordinate system 2 from Coordinate system 1.

  - Homogeneous coordinate

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{31} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'_1 \\ y'_1 \\ z'_1 \\ 1 \end{bmatrix} \qquad X_1 = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} X'_1 \qquad X_1 = TX'_1$$

Registration

- Registration is finding the relative rotation and translation of the coordinate axes between the two datasets.

- How can we find rotation and translation?
  - Method 1
    - using corresponding points

$$X_1 = TX'_1 \Rightarrow [X_1 \quad X_2 \quad \ldots] = T[X'_1 \quad X'_2 \quad \ldots] \Rightarrow \mathbb{X} = T\ \mathbb{X}'$$

    - Solve using Least-Square Method (or SVD)

$$\mathbb{X}\mathbb{X}'^{\mathsf{T}}(\mathbb{X}'\mathbb{X}'^{\mathsf{T}})^{-1} = T$$

    - What if we don't know the corresponding points? → ICP

ICP

- ICP: Iterative Closest Points algorithm
- ICP is one of the widely used algorithms in aligning 2D/3D data
  - Simply said it is a widely used registration method
- The algorithm iteratively revises the transformation
  - Combination of translation and rotation needed to minimize an error metric.
- Usually the error metric is a distance from the source to the reference point cloud, such as the sum of squared differences between the coordinates of the matched pairs.

$$Error = \min \Sigma_{j=1}^{m} \left\| X_j - T{X'}_j \right\|^2$$

ICP

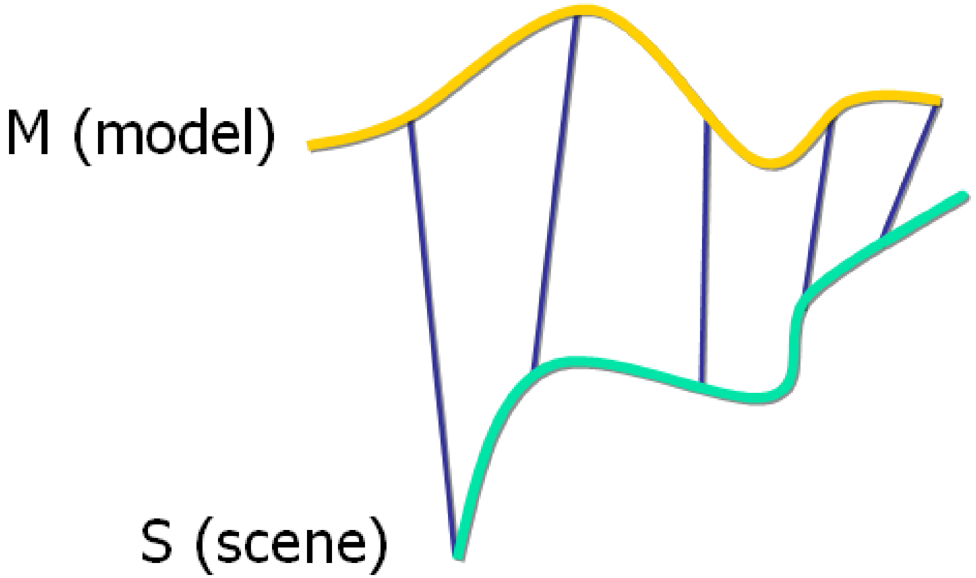
- Algorithm
  - Find correspondence
  - Calculate alignment
  - Apply alignment
  - Update error

```
ICP (point set P (Source scan), point set Q (Target Scan))

    Compute (Centre of mass for P and Q){ }
    Translate (Centre of mass for Q into P){ }

    while ( d(T)> E_max || Number of iteration>  for example 20) {
        for each  p_i  in P {
            s_i  = nearest point( p_i ,Q)
        }
        transformation T =  min_T E(T) = min_T Σ_i ||q_i − T(p_i)||^2
        P = Transform point set(P,T)
        Number of iteration++
    }
```

$$\text{transformation T} = \min_T E(T) = \min_T \sum_i \left\| q_i - T(p_i) \right\|^2$$

ICP

- Find correspondence
  - Let M be a model point set.
  - Let S be a scene point set.
  - N_x: number of point set x
  - X_i: i -th point of point set x



- Assumption
  - $N_m = N_s$ (not strict)
  - Each point S_i correspond to M_i

- Given a points M_i , S_i has the minimal distance from M_i

ICP

- Calculate alignment
  - Center of Mass
    - Suppose that two point clouds are P, Q
    - The centers of mass of the correspond. points in both sets

$$\boldsymbol{\mu}_Q = \frac{1}{|C|} \sum_{(i,j)\in C} \boldsymbol{q}_i \quad \boldsymbol{\mu}_P = \frac{1}{|C|} \sum_{(i,j)\in C} \boldsymbol{p}_j$$

    - Subtract the corresponding center of mass from every point

$$\begin{aligned} Q' &= \{\boldsymbol{q}_i - \boldsymbol{\mu}_Q\} = \{\boldsymbol{q}'_i\} \\ P' &= \{\boldsymbol{p}_j - \boldsymbol{\mu}_P\} = \{\boldsymbol{p}'_j\} \end{aligned}$$

ICP

- Calculate alignment
  - Orthogonal Procrustes Problem
  - Minimizing $$E(R, \boldsymbol{t}) = \sum_{(i,j) \in \mathcal{C}} \|\boldsymbol{q}_i - R\boldsymbol{p}_j - \boldsymbol{t}\|^2$$
  - Is equivalent to minimizing

$$E'(R) = \|[\boldsymbol{q}'_1 \dots \boldsymbol{q}'_n] - R[\boldsymbol{p}'_1 \dots \boldsymbol{p}'_n]\|_F^2$$

  - Can be solved through SVD

ICP

- Singular Value Decomposition
  - Compute the cross-covariance matrix

$$W = \sum_{(i,j)\in C} \boldsymbol{q}'_i \boldsymbol{p}'^{\top}_j$$

  - Use the SVD to decompose

$$W = UDV^{\top}$$

  - The matrices are 3 by 3 matrices
  - U, V are rotation matrices
  - Diagonal matrix $D = \mathrm{Diag}(\sigma_1, \sigma_2, \sigma_3)$

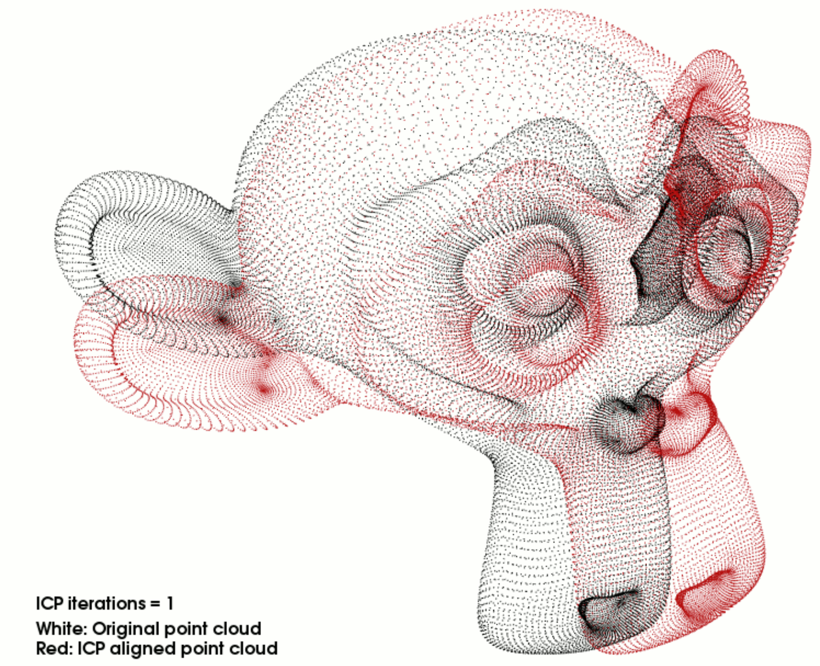
$$\begin{aligned} R &= UV^{\top} \\ \boldsymbol{t} &= \boldsymbol{\mu}_Q - R\boldsymbol{\mu}_P \end{aligned}$$

https://ocw.mit.edu/courses/mathematics/18-065-matrix-methods-in-data-analysis-signal-processing-and-machine-learning-spring-2018/video-lectures/lecture-34-distance-matrices-procrustes-problem-first-project/

ICP

- Apply derived rotation and translation to S
- Calculate distance errors again
- If the error is larger than threshold, calculate alignment

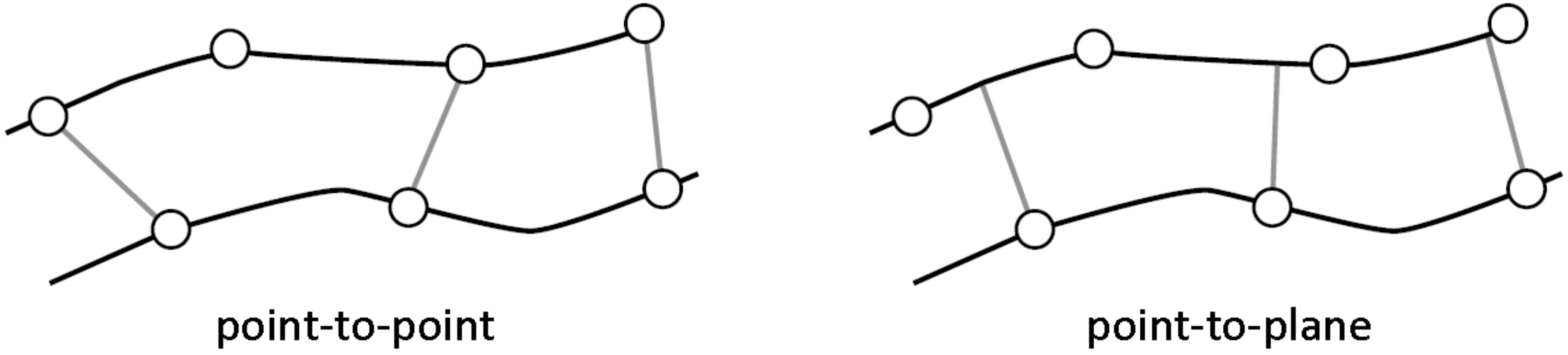
Init the error to ∞
↓
Calculate correspondence
↓
Calculate alignment
↓
Apply alignment
↓
Update error
↓
If error > threshold



ICP variants

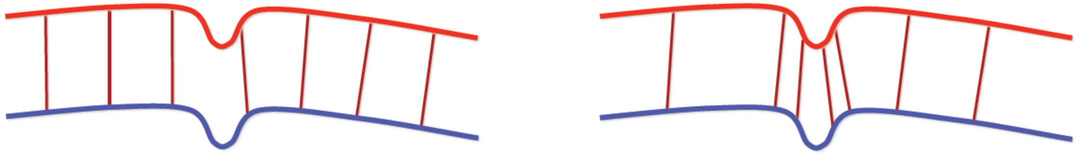
- Nearest neighborhood method for finding corresponding points are too e xpensive
  - Solution: K-D tree

- Corresponding points are not accurate enough
  - Using point-to-plane distance instead of point-to-point allows flat regions slide alon g each other



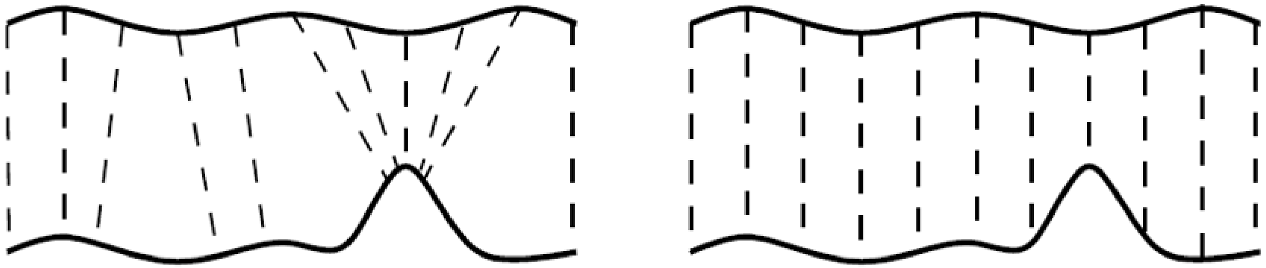
ICP variants

- Stable sampling (Normal-Space Sampling)
  - Select samples that constraint all degrees of freedom of the rigid-body transformation



Uniform Sampling　　Stable Sampling

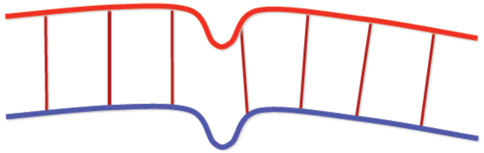
- Projection
  - Project the sample point onto the destination mesh

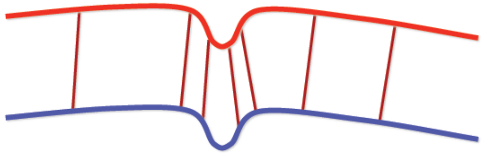


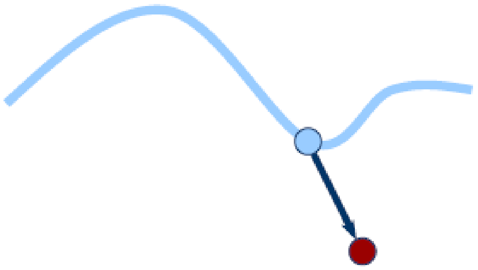
point-to-point　　projection

ICP variants

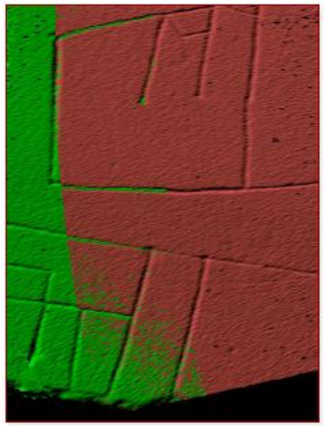
- Normal-Space Sampling
  - Select samples that constraint all degrees of freedom of the rigid-bod y transformation

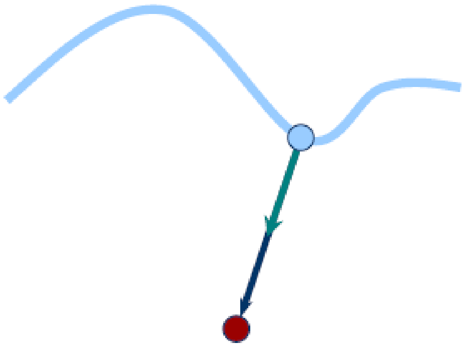

uniform sampling


normal-space sampling




Random sampling


Normal-space sampling



Advantages and Disadvantages of ICP

- Advantages:
  - Relatively easy to understand
  - Does not require local feature extraction
  - Algorithm can be generalized to n-dimensional space

- Disadvantages:
  - Converges to local minima
  - **Convergence time depends on initializations of Rotation and Translation**
  - Is sensitive to outliers
  - High "time complexity" in finding point associations
  - **Cannot handle partial point cloud registration**

Thank you