3D Data Processing

Image stitching using Homography and RANSAC

Hyoseok Hwang

Today

- Image stitching
  - Feature extraction
  - Homography
  - Ransac

  - Least-Square method

How do we create panorama image?

- Panorama image
  - An image of (near) 360° field of view

How do we create panorama image?

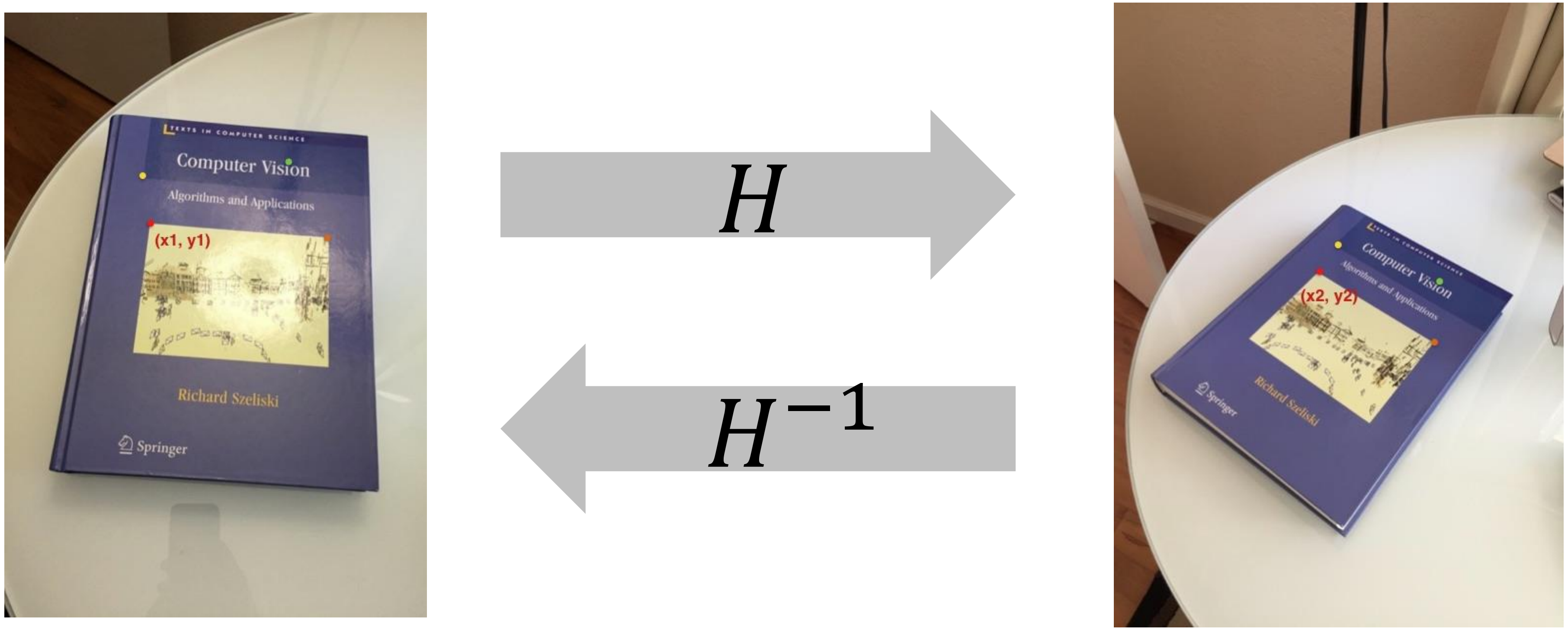
- Panorama image
  - An image of (near) 360° field of view
- Method 1 : Use a very wide-angle lens
  - Pros: Everything is done optically, single capture (up to ~200°)
  - Cons: Lens is super Expensive and bulky, lots of distortion

How do we create panorama image?

- Panorama image:
  - An image of (near) 360° field of view
- Method 2: capture multiple images and combine them
  - Capture multiple images from different viewpoints
  - Stitch them together into a virtual wide-angle image

Homography

- An isomorphism of projective spaces in projective geometry
- Transform matrix between two images

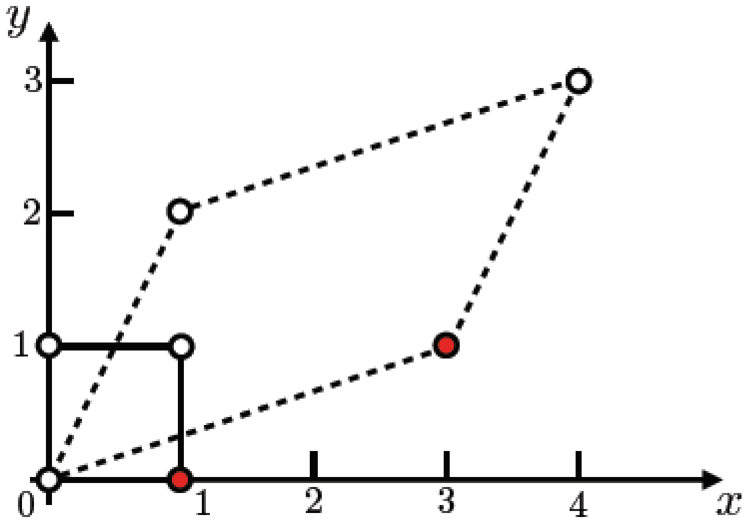


projective geometry → homogeneous coordinate

Homography

- A simple method
  - Consider the action of the unit square under, sample H

$$H = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\underbrace{\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Transformation}} \underbrace{\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{\text{Points}} = \underbrace{\begin{bmatrix} 0 & 1 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{\text{Transformed Points}}$$

Homography

- Inverse problem
  - If we know points correspondences, we can calculate H

$$\mathrm{H}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

  - We simplified this equation to $\mathrm{H}A = B$
  - How can we get H?

Homography

- Pseudo inverse
  - $A^{+} = A^{T}(AA^{T})^{-1}$

$$\mathrm{H}A = B$$

$$\mathrm{H}AA^{T} = BA^{T}$$

$$\mathrm{H}AA^{T}(AA^{T})^{-1} = BA^{T}(AA^{T})^{-1}$$

$$\mathrm{H} = BA^{T}(AA^{T})^{-1}$$

$$\mathrm{H} = BA^{+}$$

Homography

- Pseudo inverse
  - $A^{+} = (A^T A)^{-1} A^T$

$$AH = B$$

$$A^T AH = A^T B$$

$$(A^T A)^{-1} A^T AH = (A^T A)^{-1} A^T \mathrm{B}$$

$$H = (A^T A)^{-1} A^T \mathrm{B}$$

$$H = A^{+} B$$

Solving $\boldsymbol{Ah = b}$

- Why Pseudo inverse?
  - Result of pseudo inverse multiplication is the same with the minimization of least-square

$$E(h) = (Ah - b)^T(Ah - b) = (h^T A^T - b^T)(Ah - b)$$

$$= h^T A^T Ah - h^T A^T b - b^T Ah + b^T b$$

$$= h^T A^T Ah - 2b^T Ah + b^T b$$

$$\nabla E(h) = \frac{dE}{dh} = 2h^T A^T A - 2b^T Ah$$

$$h^T A^T A - b^T A = 0$$

$$h^T A^T A(A^T A)^{-1} = b^T A(A^T A)^{-1}$$

$$h^T = b^T A(A^T A)^{-1}$$

$$h = (b^T A(A^T A)^{-1})^T = (A^T A)^{-1} A^T b$$

Homography

- Better method

$$\begin{bmatrix} x_b \\ y_b \\ 1 \end{bmatrix} = \begin{bmatrix} kx_b \\ ky_b \\ k \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ 1 \end{bmatrix}$$

- Expand matrix multiplication

$$kx_b = h_1 x_a + h_2 y_a + h_3$$

$$ky_b = h_4 x_a + h_5 y_a + h_6$$

$$k1 = h_7 x_a + h_8 y_a + h_9$$

$$\Rightarrow$$

$$x_b = \frac{kx_b}{k} = \frac{h_1 x_a + h_2 y_a + h_3}{h_7 x_a + h_8 y_a + h_9}$$

$$y_b = \frac{ky_b}{k} = \frac{h_4 x_a + h_5 y_a + h_6}{h_7 x_a + h_8 y_a + h_9}$$

Homography

- To linear equation

$$x_b(h_7x_a + h_8y_a + h_9) = h_1x_a + h_2y_a + h_3$$

$$y_b(h_7x_a + h_8y_a + h_9) = h_4x_a + h_5y_a + h_6$$

- Rearrange

$$h_7x_ax_b + h_8y_ax_b + h_9x_b - h_1x_a - h_2y_a - h_3 = 0$$

$$h_7x_ax_b + h_8y_ax_b + h_9x_b - h_4x_a - h_5y_a - h_6 = 0$$

$$\begin{bmatrix} -x_a & -y_a & -1 & 0 & 0 & 0 & x_ax_b & y_ax_b & x_b \\ 0 & 0 & 0 & -x_a & -y_a & -1 & x_ay_b & y_ay_b & y_b \end{bmatrix} [h_1 \quad h_2 \quad h_3 \quad h_4 \quad h_5 \quad h_6 \quad h_7 \quad h_8 \quad h_9]^T$$

Homography

- One point correspondence make 2 equation

$$2N \times 9 \quad \begin{bmatrix} -x_a & -y_a & -1 & 0 & 0 & 0 & x_a x_b & y_a x_b & x_b \\ 0 & 0 & 0 & -x_a & -y_a & -1 & x_a y_b & y_a y_b & y_b \end{bmatrix} \begin{bmatrix} -x_a & -y_a & -1 & 0 & 0 & 0 & x_a x_b & y_a x_b & x_b \\ 0 & 0 & 0 & -x_a & -y_a & -1 & x_a y_b & y_a y_b & y_b \end{bmatrix} \begin{bmatrix} -x_a & -y_a & -1 & 0 & 0 & 0 & x_a x_b & y_a x_b & x_b \\ 0 & 0 & 0 & -x_a & -y_a & -1 & x_a y_b & y_a y_b & y_b \end{bmatrix} \begin{bmatrix} -x_a & -y_a & -1 & 0 & 0 & 0 & x_a x_b & y_a x_b & x_b \\ 0 & 0 & 0 & -x_a & -y_a & -1 & x_a y_b & y_a y_b & y_b \end{bmatrix} \begin{bmatrix} -x_a & -y_a & -1 & 0 & 0 & 0 & x_a x_b & y_a x_b & x_b \\ 0 & 0 & 0 & -x_a & -y_a & -1 & x_a y_b & y_a y_b & y_b \end{bmatrix} \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix}}_{9 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad 2N \times 1$$

Homography

- How to solve $Ah = 0$
    - Use Singular Value Decomposition
    - $A = U\Sigma V^T$
- If dimension of A is 18x9

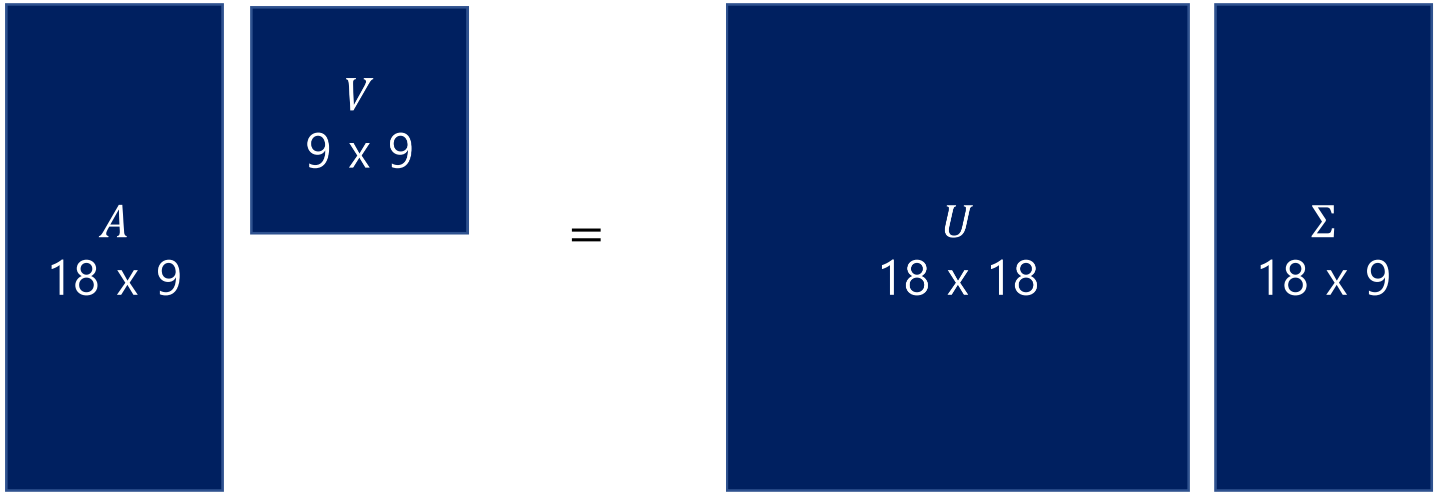
M X N matrix: U(M X M) Σ(M X N) V^T(N X N)

U, V : orthonormal matrix (norm=1)

A 18 x 9 = U 18 x 18 Σ 18 x 9 V^T 9 x 9

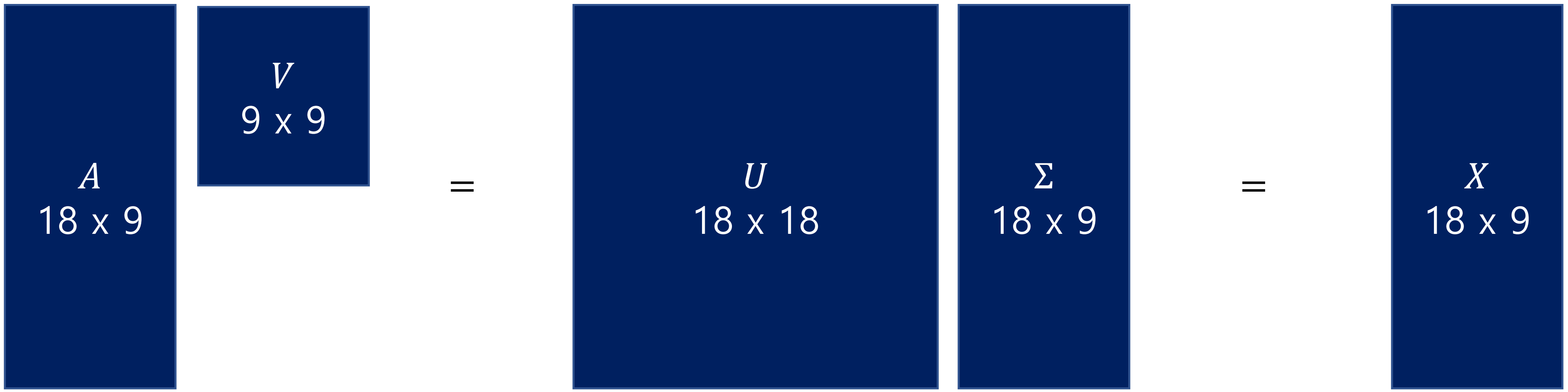
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV = U\Sigma$ (V is orthonormal matrix: transpose = inverse)



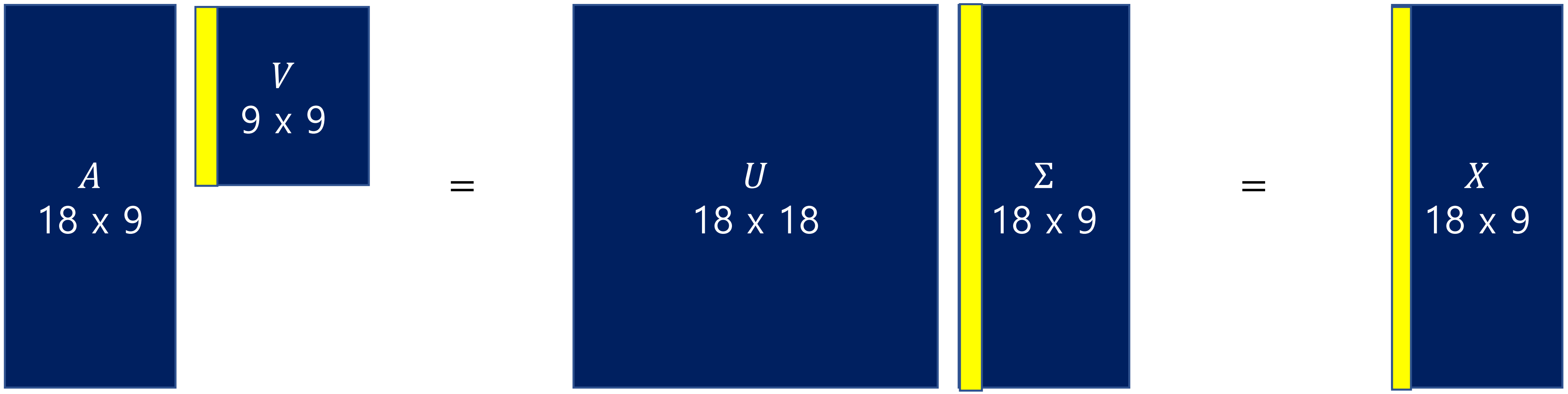
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV = U\Sigma = X$ (M X N size matrix)



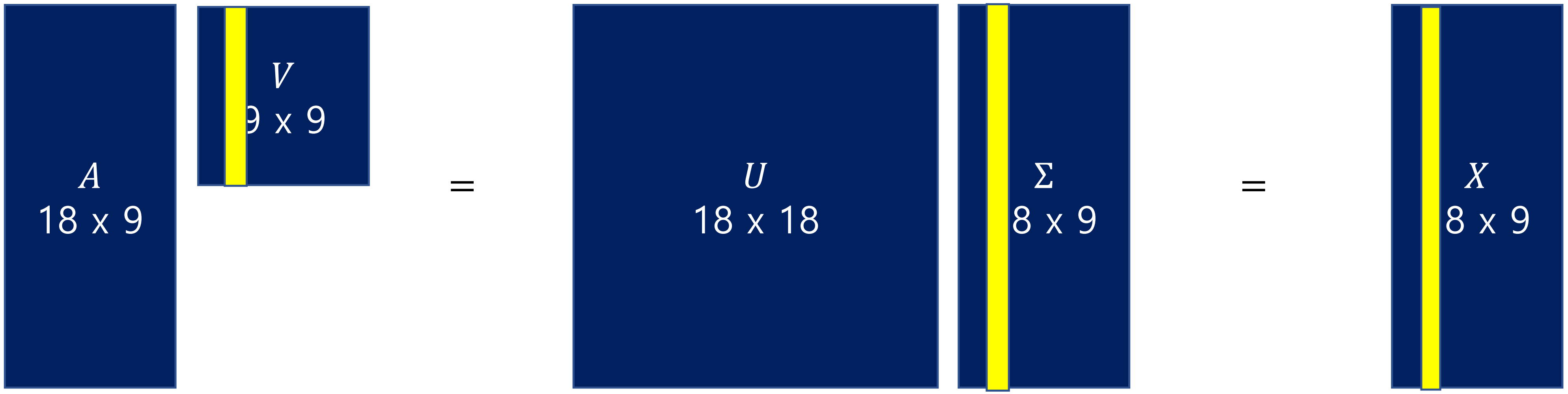
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_1 = U\Sigma_1 = \mathrm{X}_1$ (A_{x} means x-th column vector of matrix A)



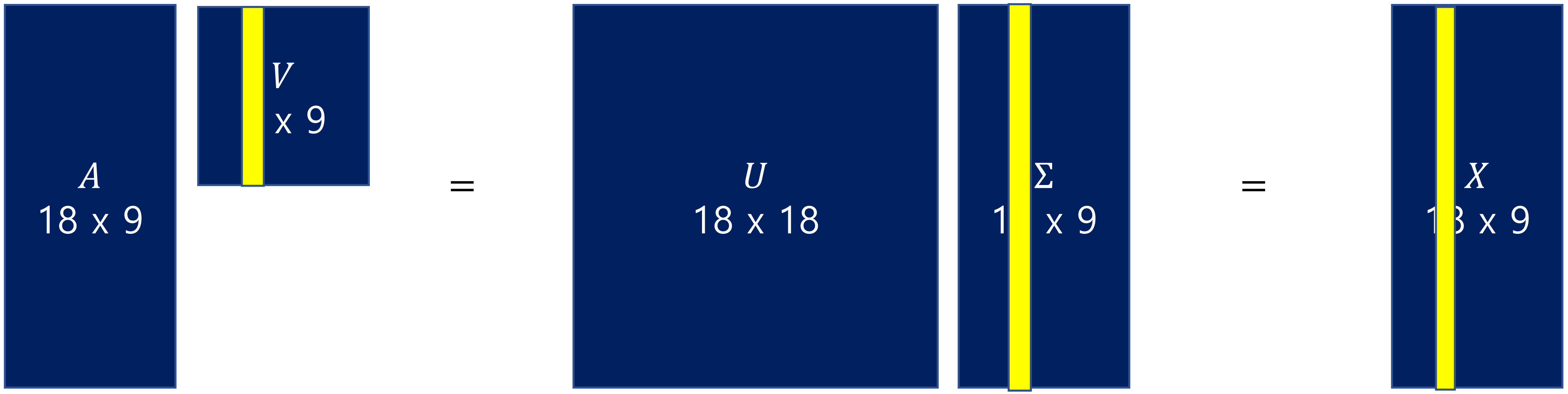
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_2 = U\Sigma_2 = X_2$



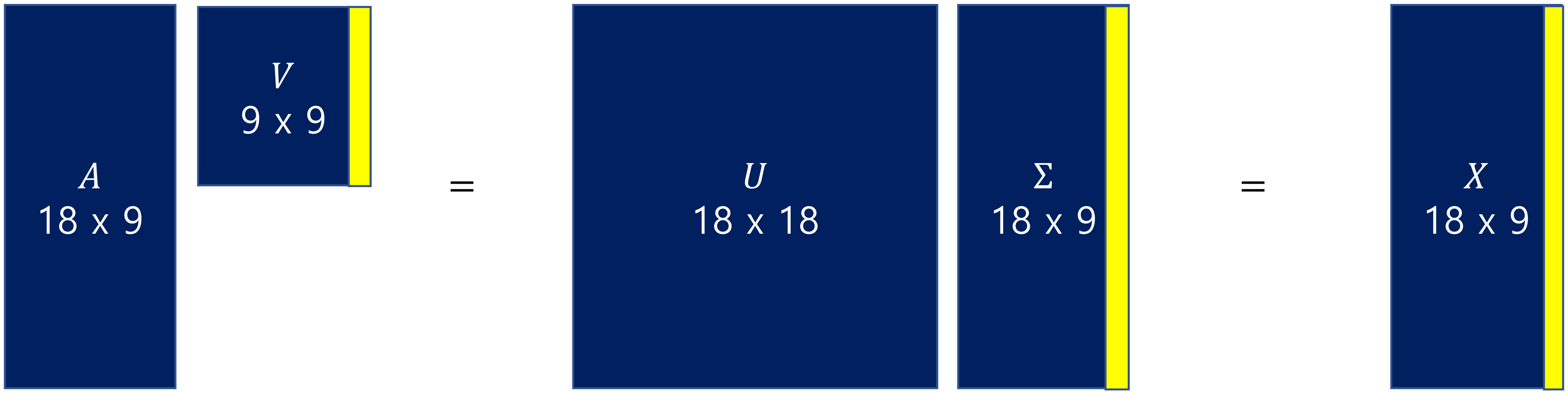
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_3 = U\Sigma_3 = \mathrm{X}_3$



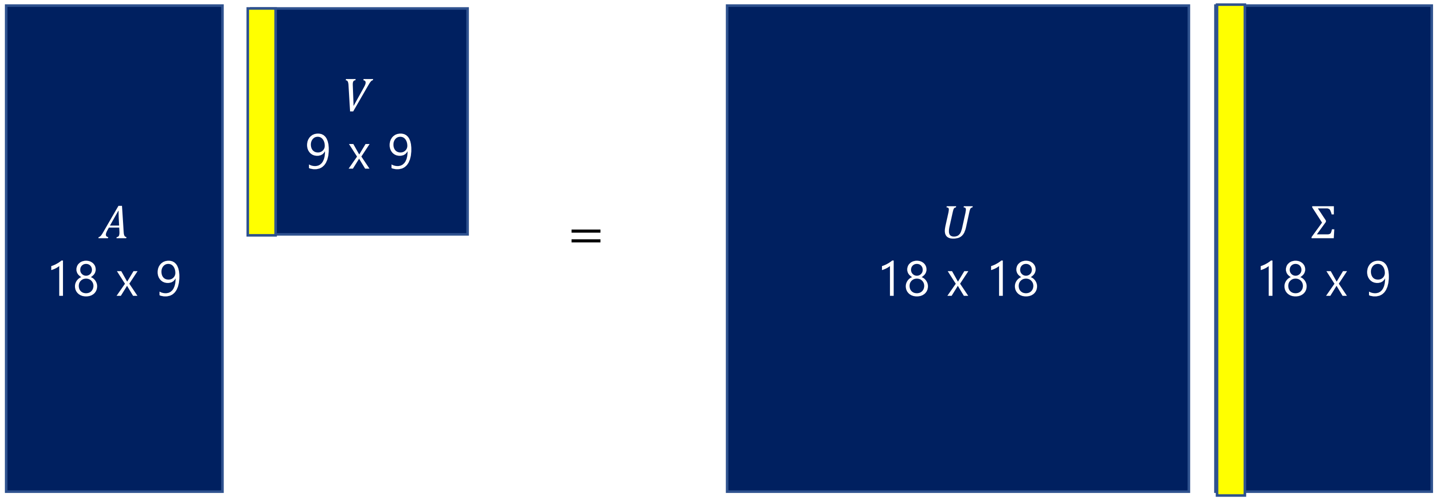
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_9 = U\Sigma_9 = \mathrm{X}_9$



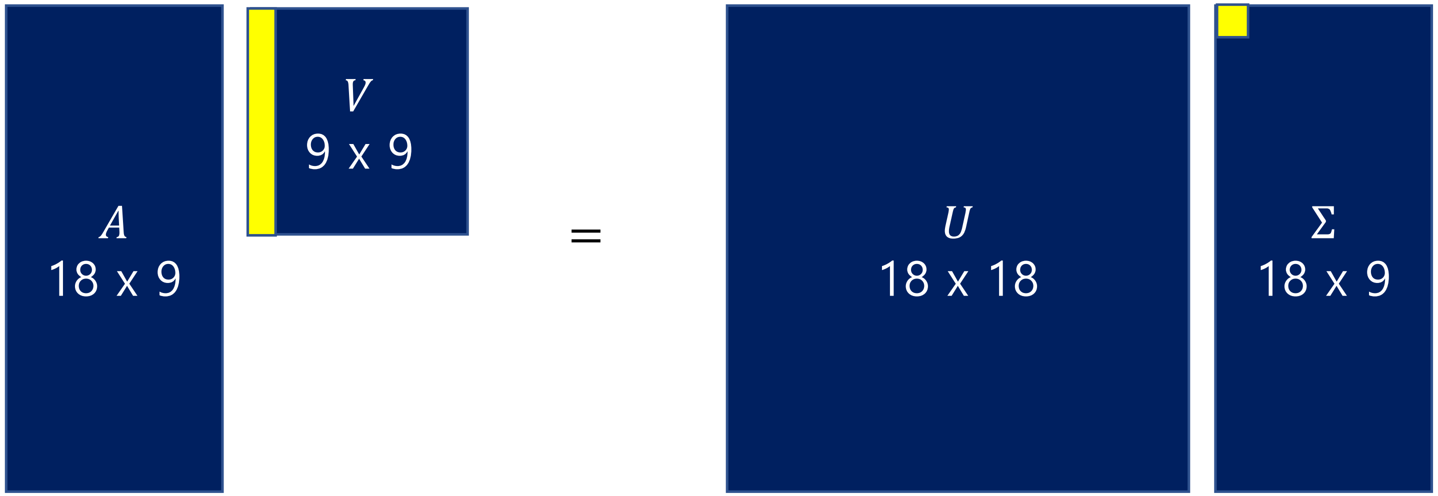
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_n = U\Sigma_\mathrm{n}$



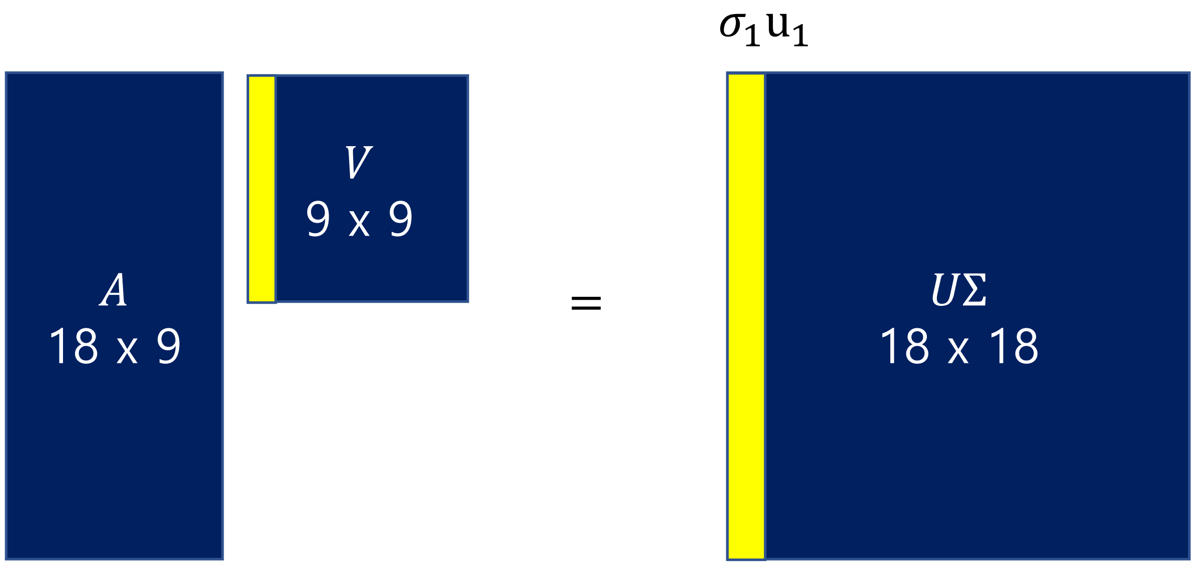
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_1 = \sigma_1 U_1$



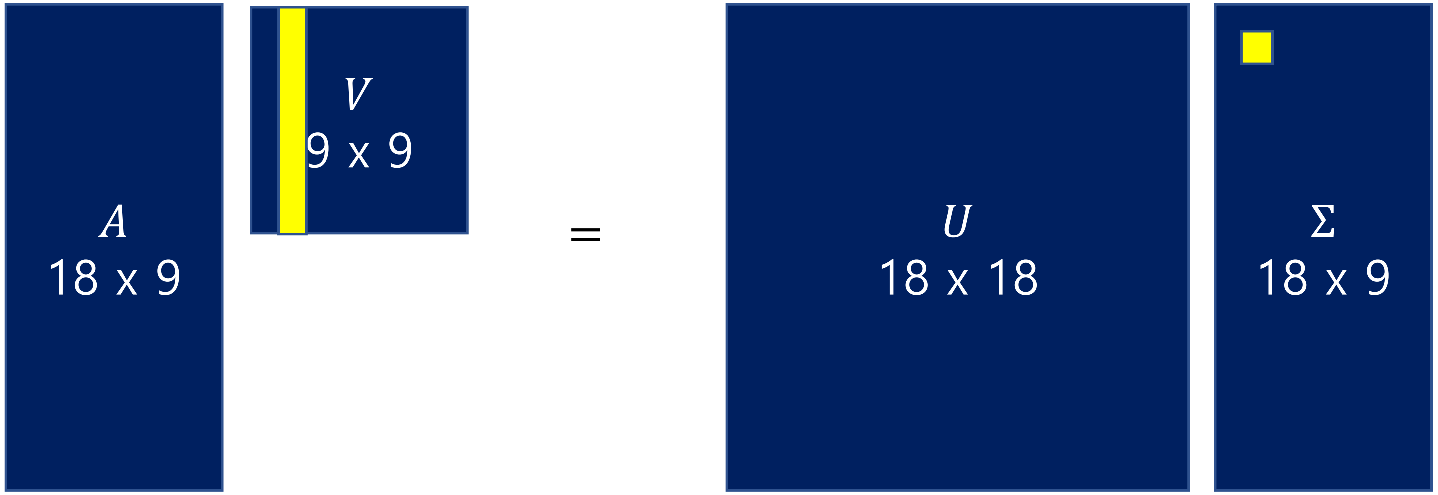
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_1 = \sigma_1 U_1$



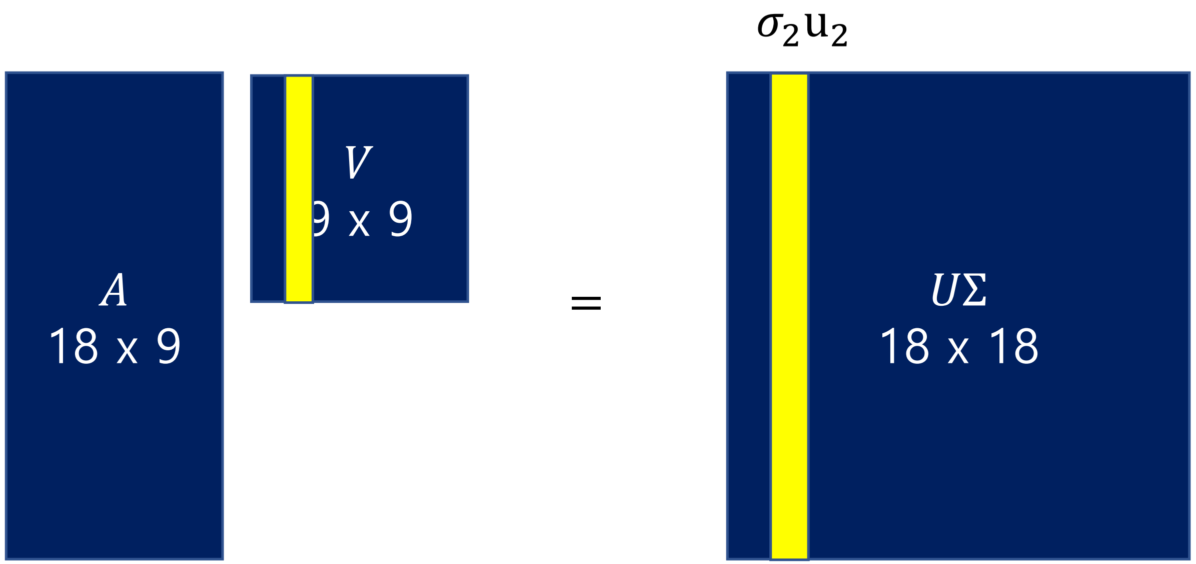
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_2 = \sigma_2 U_2$



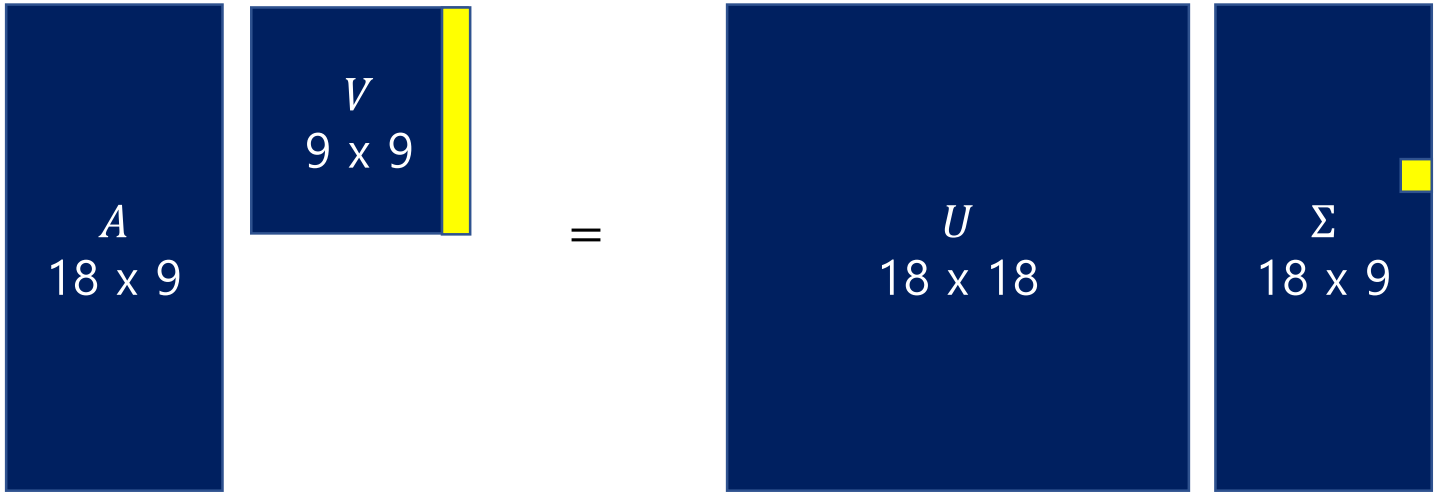
Homography

- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_2 = \sigma_2 U_2$



Homography

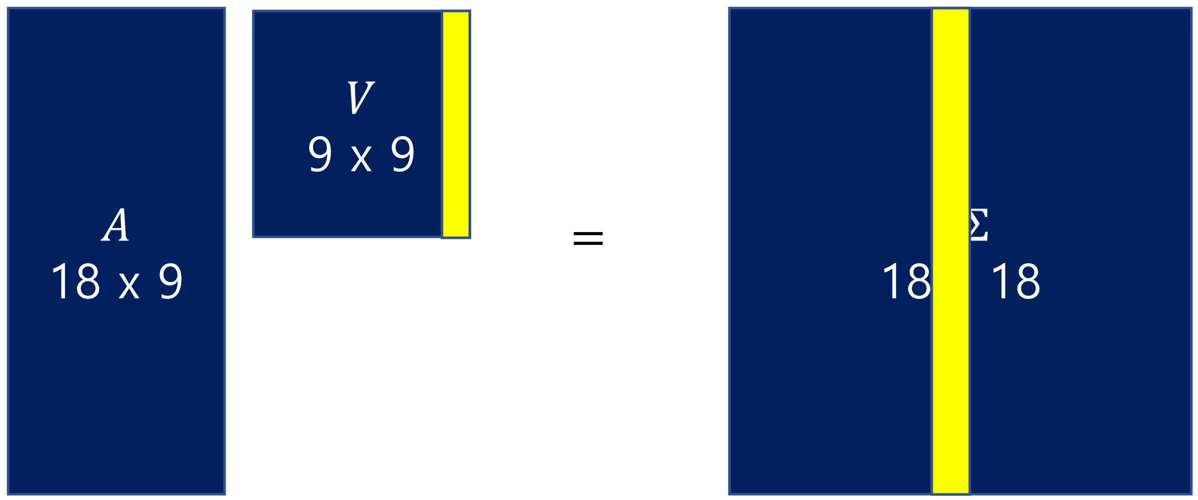
- How to solve $Ah = 0$
  - Use Singular Value Decomposition
  - $A = U\Sigma V^T$
- $AV_9 = \sigma_9 U_9$



Homography

- How to solve $Ah = 0$
    - Use Singular Value Decomposition
    - $A = U\Sigma V^T$
- $AV_9 = \sigma_9 U_9$

Almost zero vector, because $\sigma_9 \approx 0$



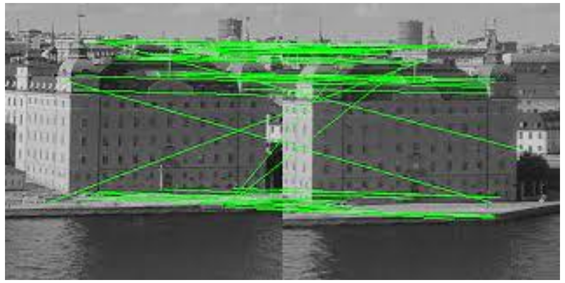
Homography

- Therefore, the last column of V is solution
- It is the same as the last row of V^T

Homography

- Does least-square method always guarantee the "optimal" H ?
- Are all corresponding point pairs important?

- No, because
  - A homography is a transform on a relation in the plane only.
  - Not all correspondence pairs are correct.
    - Wrong matches → outliers
    - Feature position errors



- It is important to use features that are not outliers.

Strategies to match feature points robustly

- Working with individual features
  - For each feature point, find most similar point in other images
  - Reject ambiguous matches where there are too many similar points

- Working with all the features
  - Reject homographies that don't have many feature matches
  - Reject features that are not matches by a majority-satisfying homography.

- This is why RANSAC is used.
  - Assumption: Outliers are a minority.

RANSAC (RANdom Sample Concensus)

- An Algorithm, a strategy or a methodology
  - To get H using inliers as much as possible.

- Algorithm
  - During iteration
    - Select random pair-points (more than 4-pairs inHomography)
    - Compute H
    - Transform X_a, and Get X_b' by $X_b' = HX_a$
    - Counting inliers that satisfies: $|X_{bi}' - X_{bi}| < \theta$
    - Update H which have the maximum inliers
    - (Optional) Compute new H using inliers
    - (Optional) Inliers or errors satisfy condition, stop loop

Limitation of RANSAC

- Non-determinisitic algorithm
  - RANSAC uses random choice, which induces different results
- Uncertainty
  - It does not guarantee optimal results.
- Depend on distribution and density
  - Having a lot of features in a particular area increases its influence.
  - If there are many outliers, you won't get the correct value.

Image mosaicing

- For given images
  - Find feature points

Image mosaicing

- For given images
  - Establish point-correspondences by matching descriptors
  - Several wrong correspondences

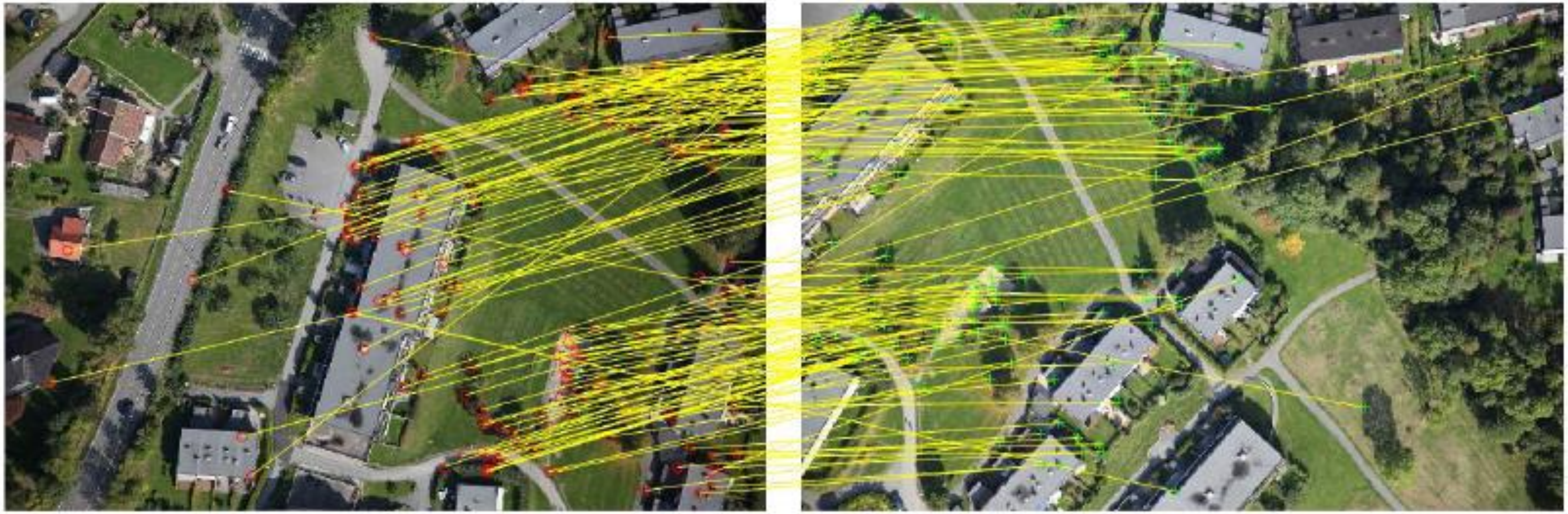


Image mosaicing

- For given images
  - Estimate homography with RANSAC

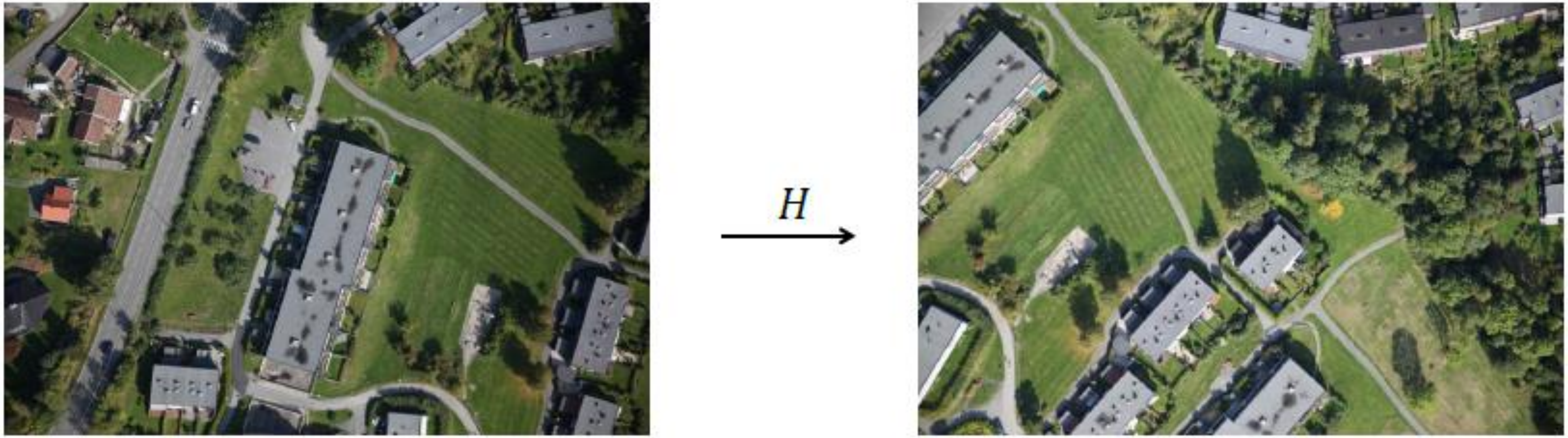


Image mosaicing

- For given images
  - Image warping : Move all pixel points using H

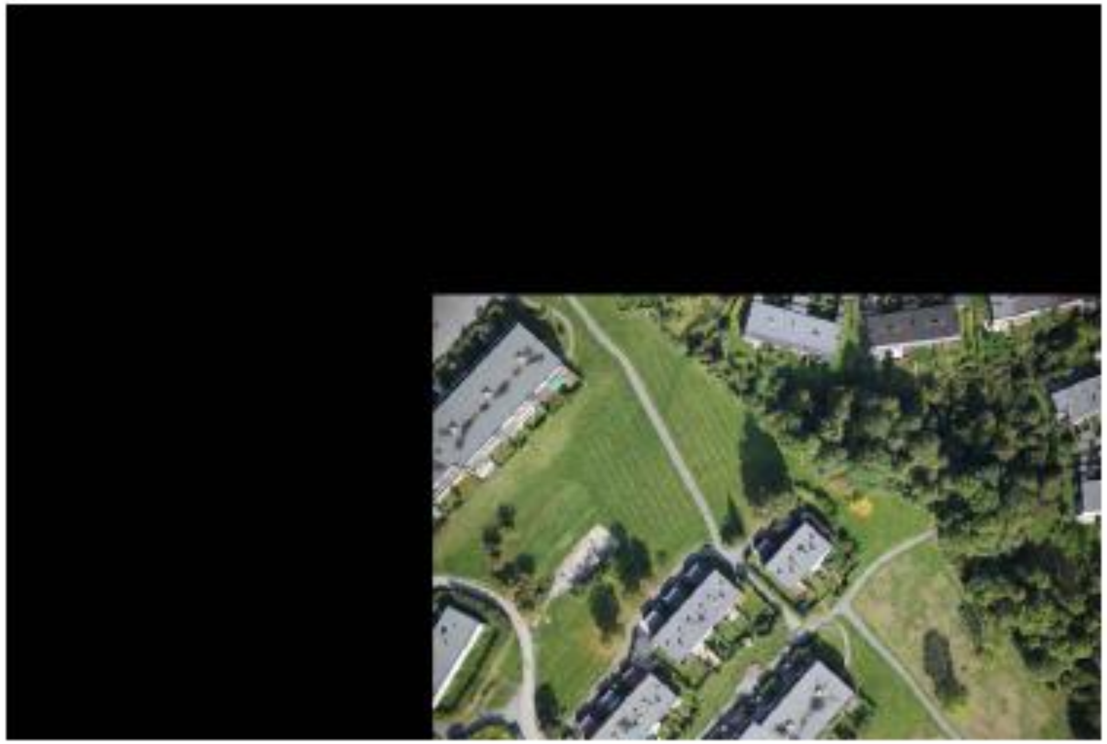


Image mosaicing

- For given images
  - Image warping : Move all pixel points using H

Thank you