2. Vectors and Matrices

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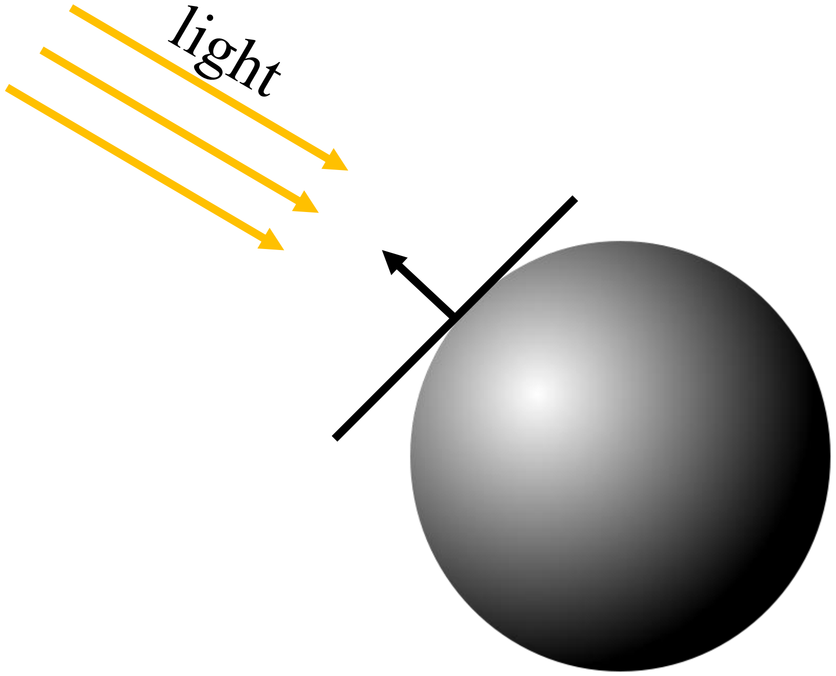
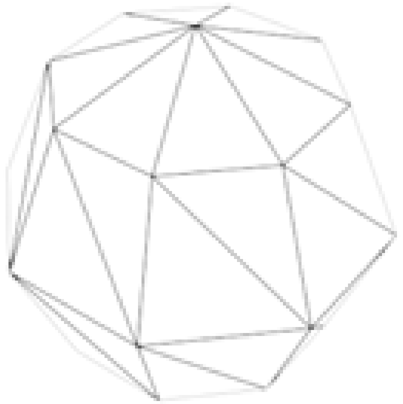
Vectors and Matrices

In 3D graphics, there are a lot of vector and matrix operations.





vertex array

	position	normal
0	(0.000, 1.000, 0.000)	(0.000, 1.000, 0.000)
1	(0.000, 0.707, 0.707)	(0.000, 0.663, 0.748)
2	(0.500, 0.707, 0.500)	(0.529, 0.663, 0.529)
3	(0.000, 0.000, 1.000)	(0.000, 0.000, 1.000)
4	(0.707, 0.000, 0.707)	(0.707, 0.000, 0.707)
	⋮	⋮
25	(0.000, -1.000, 0.000)	(0.000, -1.000, 0.000)

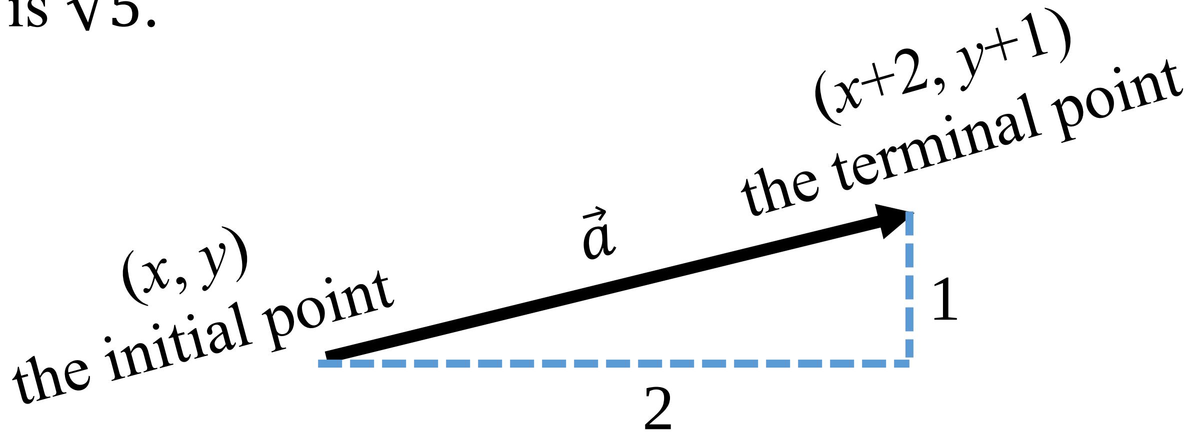
index array

0	0
1	1
2	2
3	1
4	3
5	4
	⋮
143	16

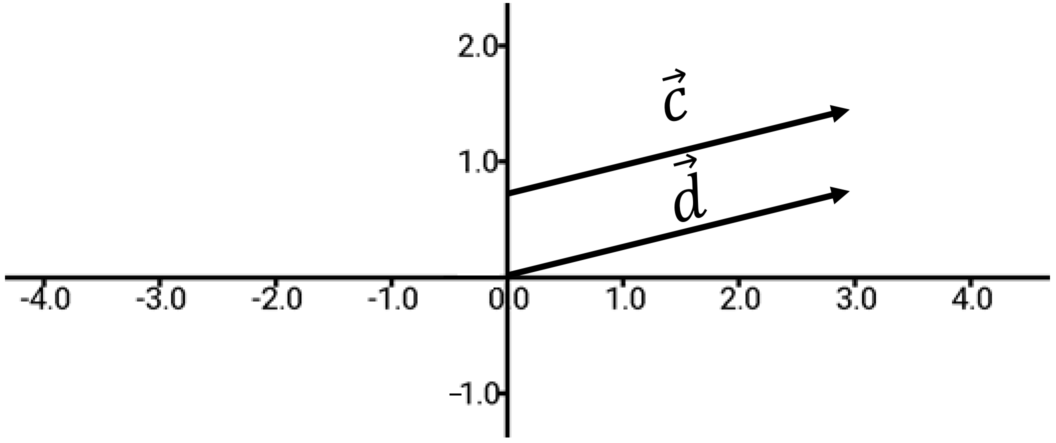
Vector

A vector is an object (or representation) that has both a magnitude and a direction.

- A vector $\vec{a} = (2, 1)$ in R^2 is shown below.
- It's magnitude is $\sqrt{5}$.

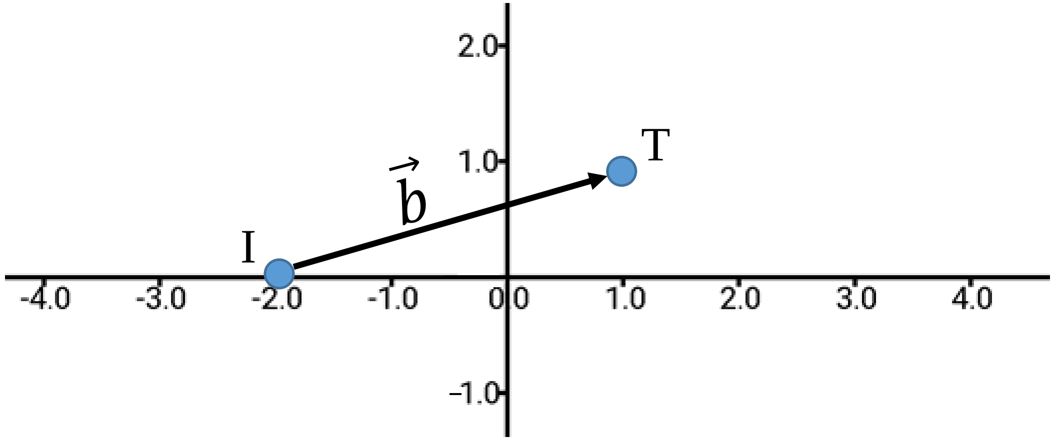


Two vectors, $\vec{c}$ and $\vec{d}$, are the same
as they have the same magnitude and direction.



Vector

Consider the vector $\vec{b}$ which has the initial point I (-2, 0) and the terminal point T (1, 1).

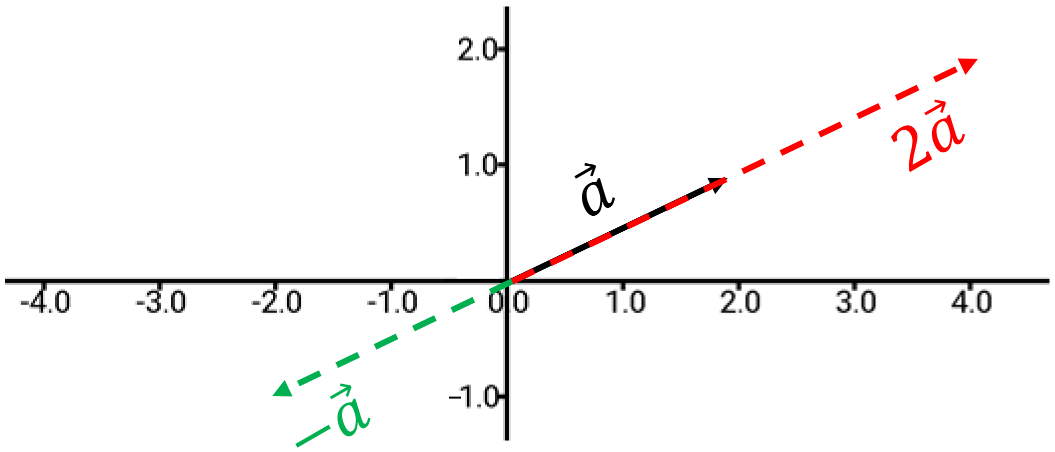


The vector can be represented as $\vec{b}$ = (1-(-2), 1-0) = (3, 1).

The magnitude of this vector is just the distance between two points, $\sqrt{3^2 + 1^2} = \sqrt{10}$

Scalar Multiplication of Vectors

Let's say we have vector $\vec{a} = (2, 1)$ in R^2.



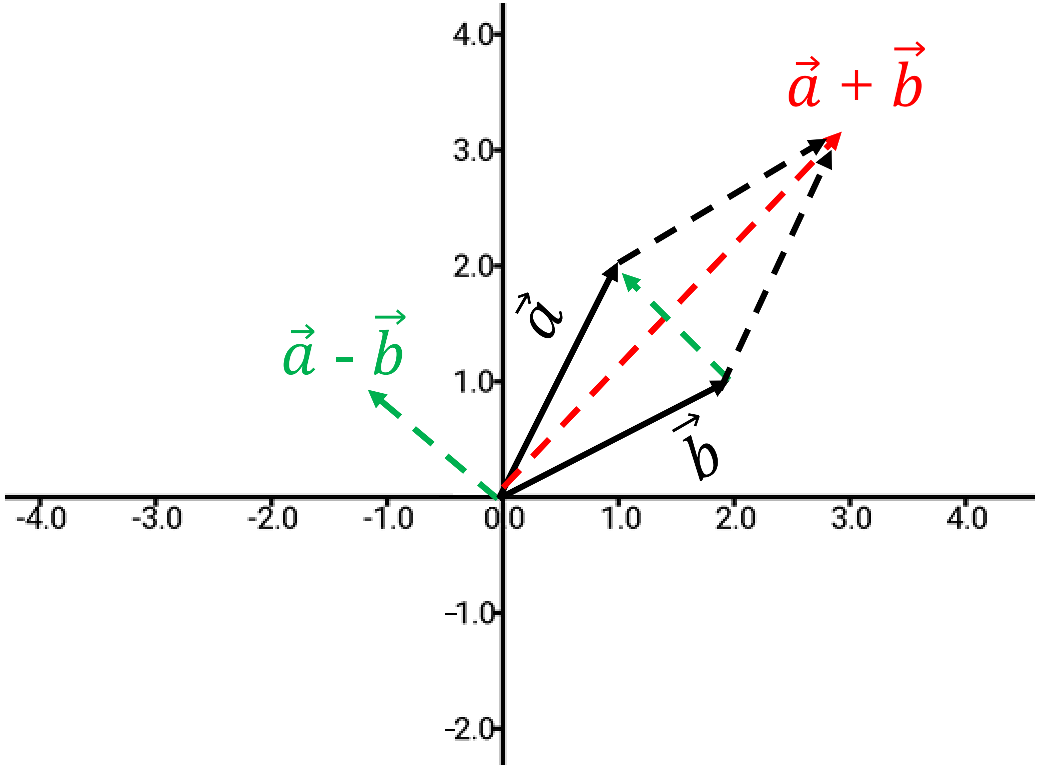
Then, $2\vec{a}$ can be computed by multiplying each of the components times 2.

- Therefore, it is going to be equal to (4, 2).
- In the same manner, we can obtain $-\vec{a} = (-2, -1)$.

Scalar Addition and Subtraction

Let's say that we have vector $\vec{a}$ = (1, 2) and $\vec{b}$ = (2, 1).

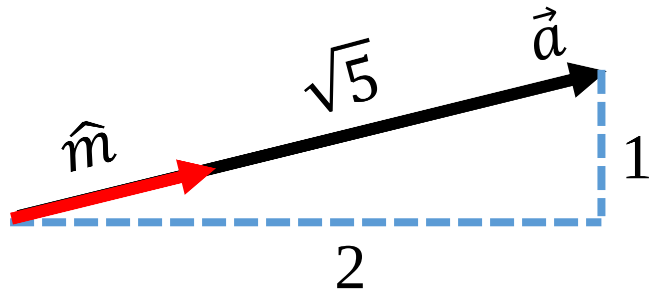
- Then $\vec{a} + \vec{b}$ is equal to (1+2, 2+1) = (3, 3).
- $\vec{a} - \vec{b}$ is equal to (1-2, 2-1) = (-1,1).



Unit Vector

A unit vector is a vector that has a magnitude of one.

- Let's say we have vector $\vec{a} = (1, 2)$. It's magnitude is $||\vec{a}|| = \sqrt{5}$.
- Then a unit vector $\hat{m}$ can be obtained by $\frac{\vec{a}}{||\vec{a}||} = (\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$.

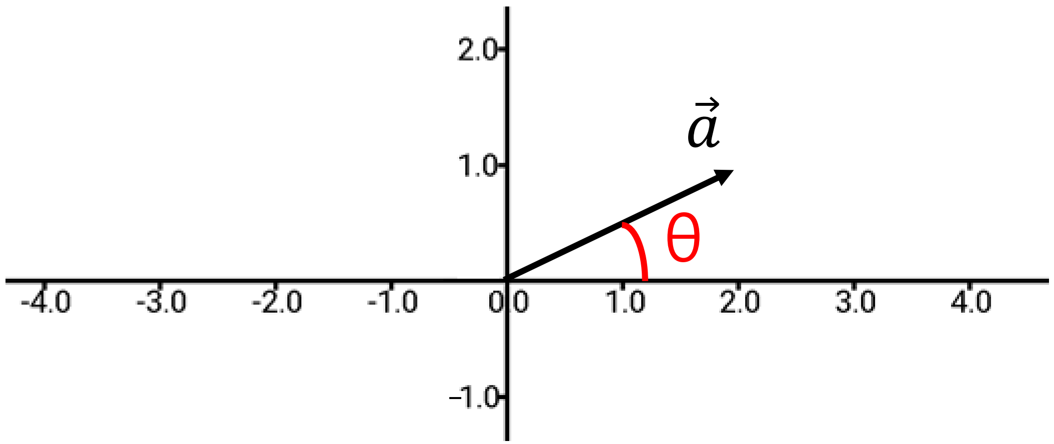


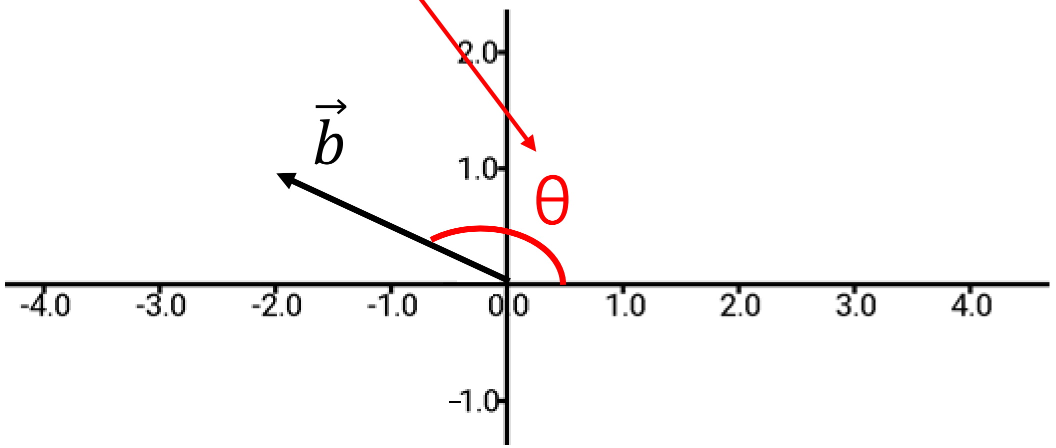
In the same manner, a unit vector $\hat{n}$ which has the same direction as $\vec{b} = (1, 1, 2)$ can be obtained by $\frac{\vec{b}}{||\vec{b}||} = (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$.

Vector Direction

The direction of a vector is the measure of the angle it makes with a horizontal line.

- Vector $\vec{a}$ = (2, 1) has the direction of $\tan^{-1}(\frac{1}{2}) \approx 26.565^\circ$.
- Vector $\vec{b}$ = (-2, 1) has the direction of $\tan^{-1}(\frac{1}{-2}) \approx -26.565^\circ$.





If we notice the vector is in Quadrant II, we must add 180° to obtain a proper angle.

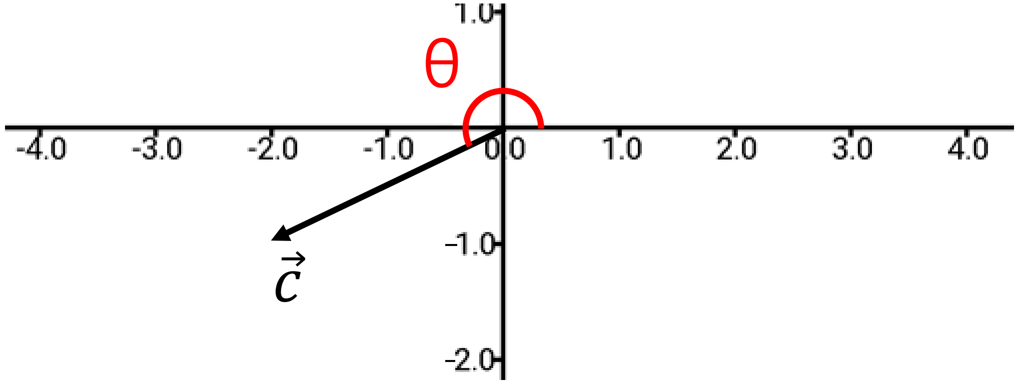
- Therefore, the direction of $\vec{b}$ is equal to $180 + -26.565 \approx 153.435^\circ$.

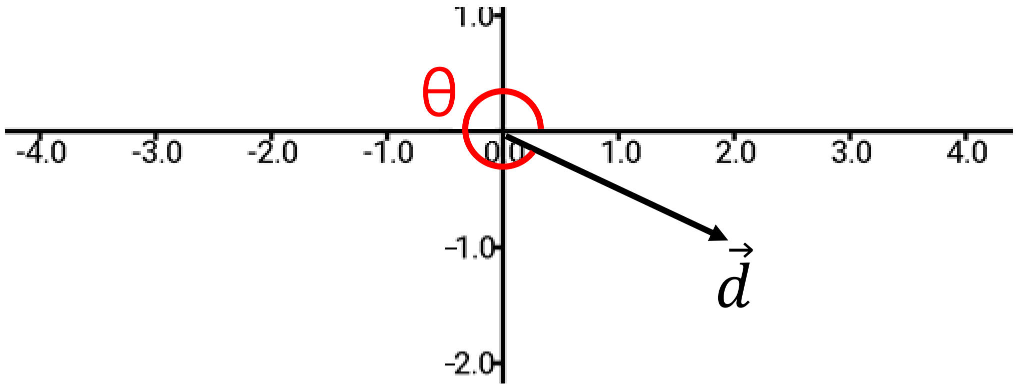
Vector Direction

The direction of a vector is the measure of the angle it makes with a horizontal line.

Vector $\vec{c}$ = (-2, -1) has the direction of $\tan^{-1}\left(\frac{-1}{-2}\right) \approx$ 26.565°.

Vector $\vec{d}$ = (2, -1) has the direction of $\tan^{-1}\left(\frac{-1}{2}\right) \approx$ -26.565°.





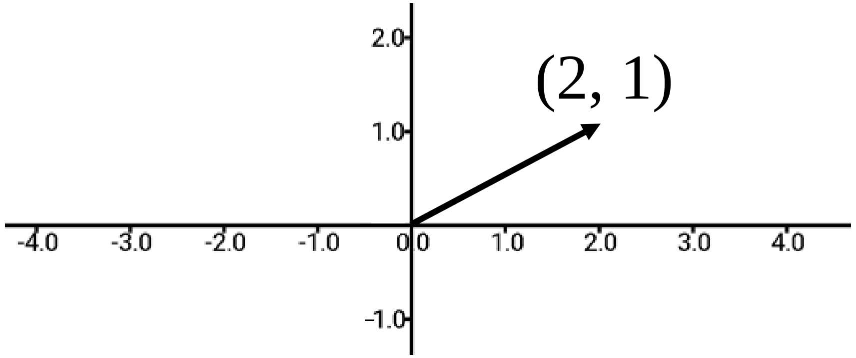
If we notice the vector is in Quadrant III, we must add 180° to obtain a proper angle.
Therefore, the direction of $\vec{c}$ is equal to 180 + 26.565 ≈ 206.565°.

If we notice the vector is in Quadrant IV, we must add 360° to obtain a proper angle.
Therefore, the direction of $\vec{d}$ is equal to 360 - 26.565 ≈ 333.435°.

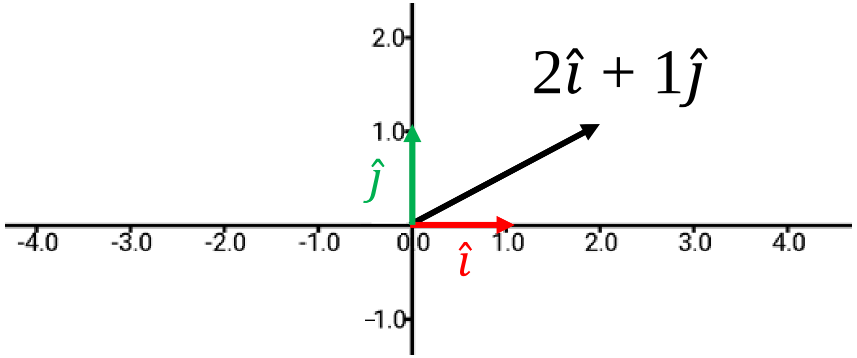
Vector Representation

There are three types of vector representations.

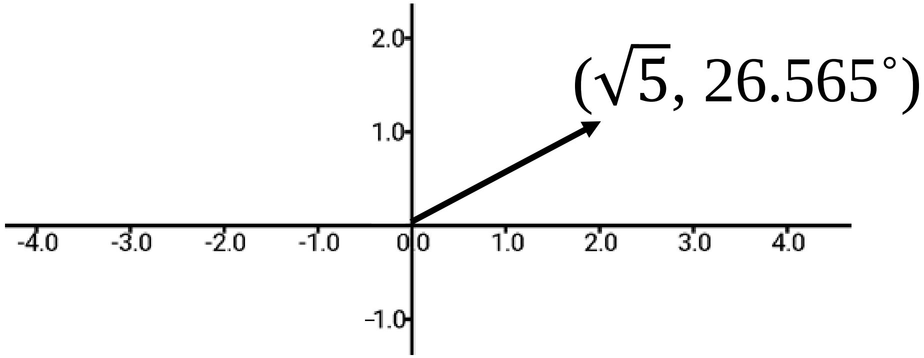
- Component form: (a, b)



- Unit vector form: $a\hat{\imath} + b\hat{\jmath}$, where $\hat{\imath} = (1, 0)$, $\hat{\jmath} = (0, 1)$



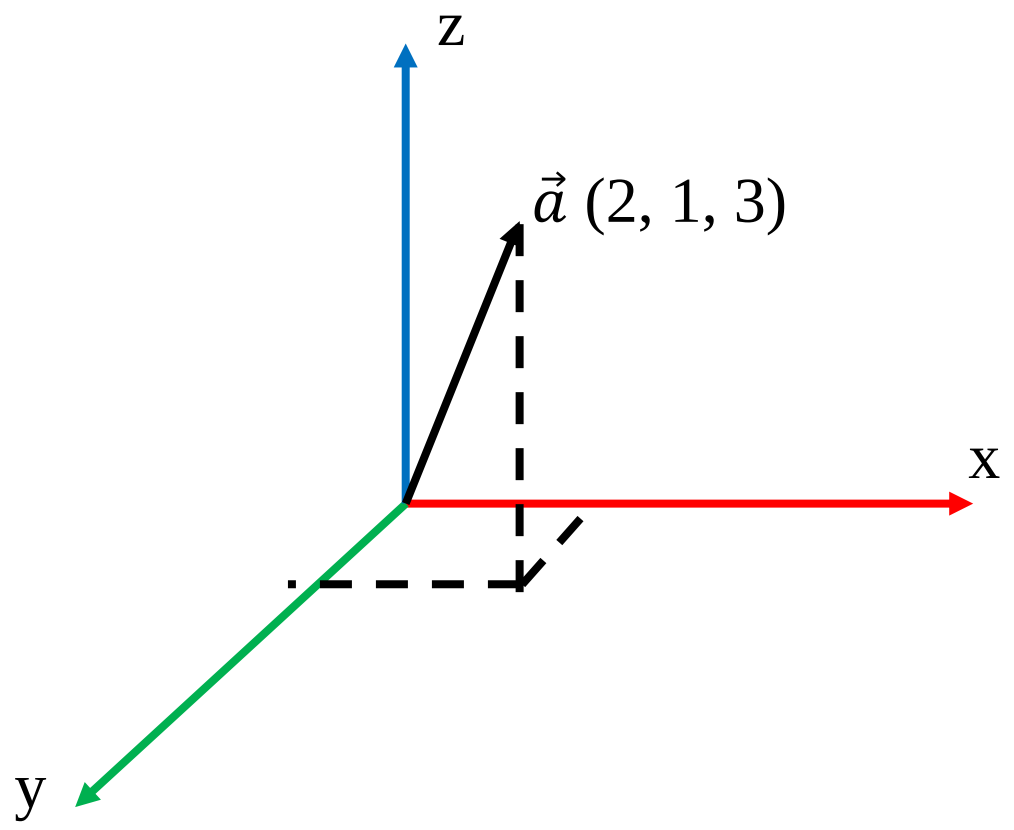
- Magnitude and direction form: $(\|\vec{a}\|, \theta)$



Vector in 3D

Let's extend the idea to (x, y, z) axes.

- Vector $\vec{a} = (2, 1, 3)$ in R^3 is shown below.
- It's magnitude can be computed by $\sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$



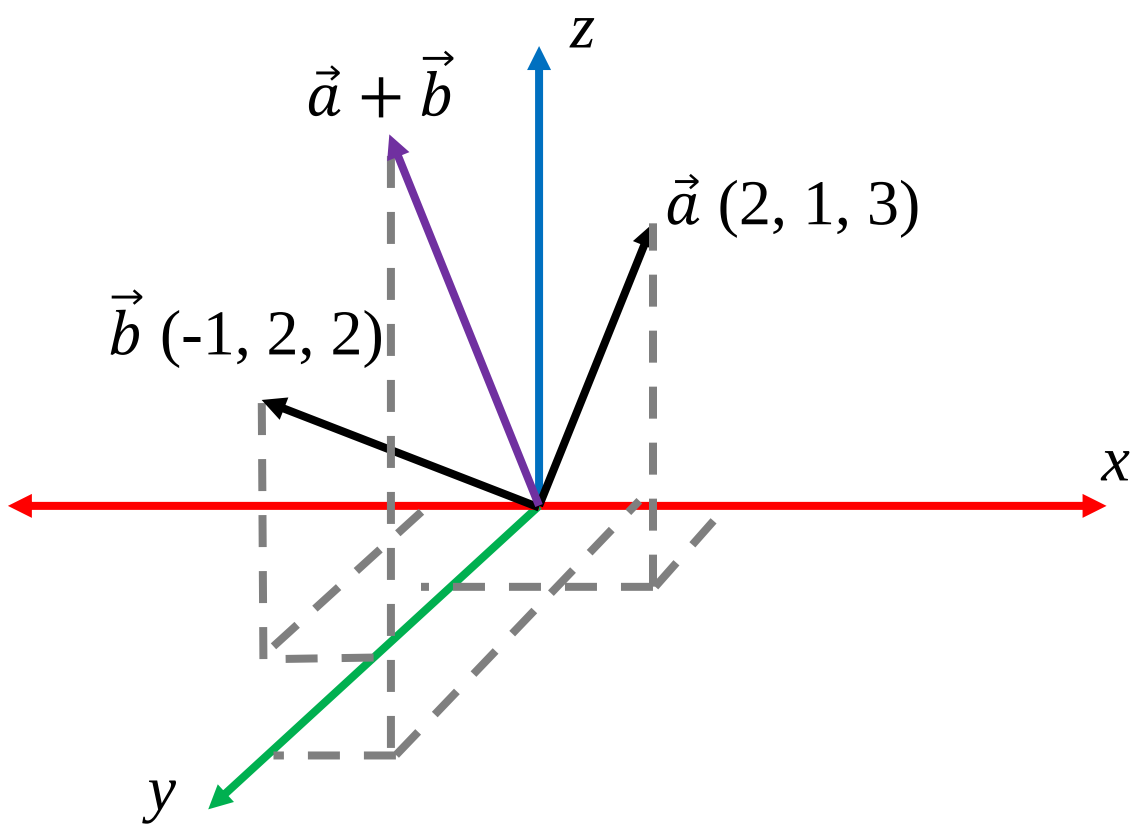
Vector in 3D

Vector $\vec{a}$ = (2, 1, 3) and $\vec{b}$ = (-1, 2, 2) can be represented by sum of three unit vectors of i (1, 0, 0), j (0, 1, 0) and k (0, 0, 1).

- $\vec{a} = 2i + j + 3k$ and $\vec{b} = -i + 2j + 2k$ are shown in the figure.

Sum of two vectors can be computed by:

- $\vec{a} + \vec{b} =$ (2-1, 1+2, 3+2) = (1, 3, 5).



Dot Product

Dot product (or scalar product) is an algebraic operation that takes two equal-length sequences of numbers (usually coordinate vectors) and returns a single number.

Algebraically, the dot product is the sum of the products of corresponding entries of the two sequences of numbers.

- The dot product of two vectors of $\vec{a}$ = (2, 1, 3) and $\vec{b}$ = (-1, 2, 2) is defined as:
  $\vec{a} \cdot \vec{b}$ = 2*(-1) + 1*2 + 3*2 = 6

- The dot product of two vectors of $\vec{c}$ = (0, -1, 1) and $\vec{d}$ = (2, 1, 3) is defined as:
  $\vec{c} \cdot \vec{d}$ = 0*(2) + -1*1 + 1*3 = 2

Dot Product

Geometrically, dot product is the product of the magnitude of two vectors and the cosine of the angle between them.

- The dot product of two vectors of $\vec{a}$ = (2, 1, 3) and $\vec{b}$ = (-1, 2, 2) is defined as:
  $\vec{a} \cdot \vec{b} = ||\vec{a}||||\vec{b}||\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

- The angle between two vectors can be obtained by using dot product. We learned that $\vec{a} \cdot \vec{b} =$ 2*(-1) + 1*2 + 3*2 = 6.

- We can derive $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||||\vec{b}||}$ and $\arccos\frac{\vec{a} \cdot \vec{b}}{||\vec{a}||||\vec{b}||} = \theta$.

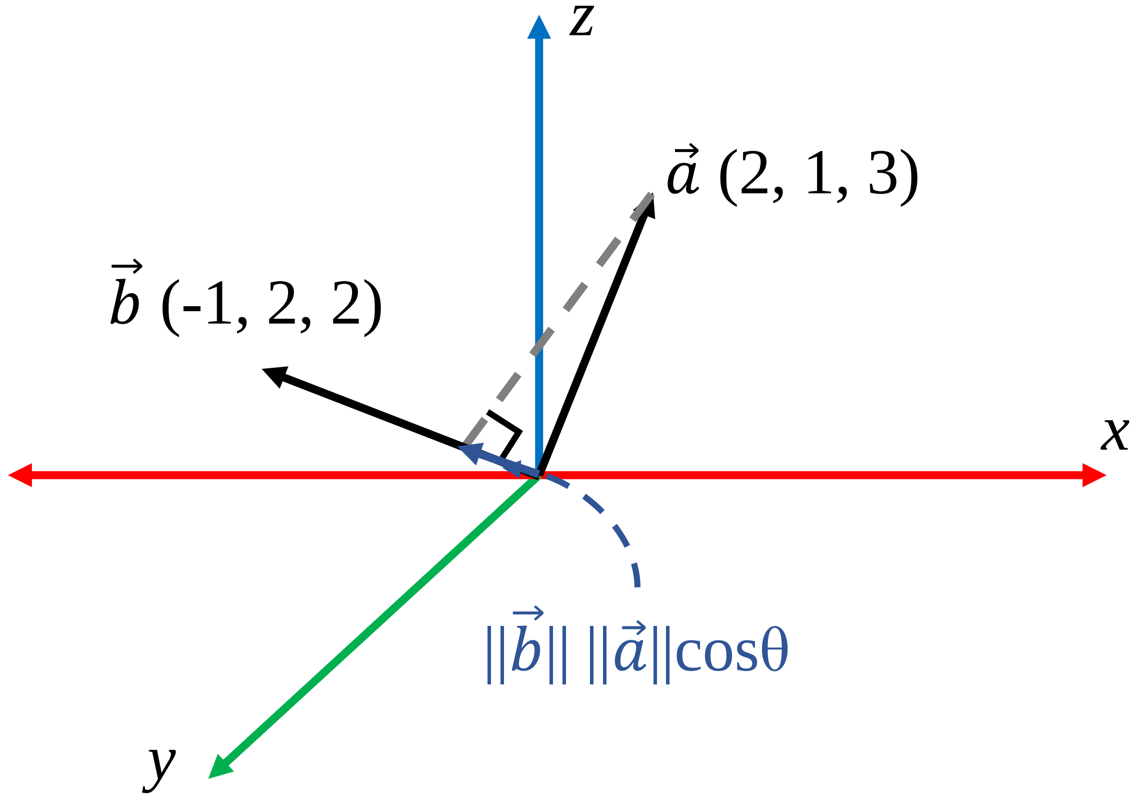
- Therefore, we can compute θ by $\arccos\frac{6}{\sqrt{14}\sqrt{9}} \approx 57.69^\circ$

Dot Product

The dot product of two vectors implies the scalar projection of one vector in the direction of the other vector.

- The illustration of $||\vec{b}||\ ||\vec{a}||\cos\theta$ is shown below.

The dot product is zero when two vectors are at right angles to each other.



Dot Product

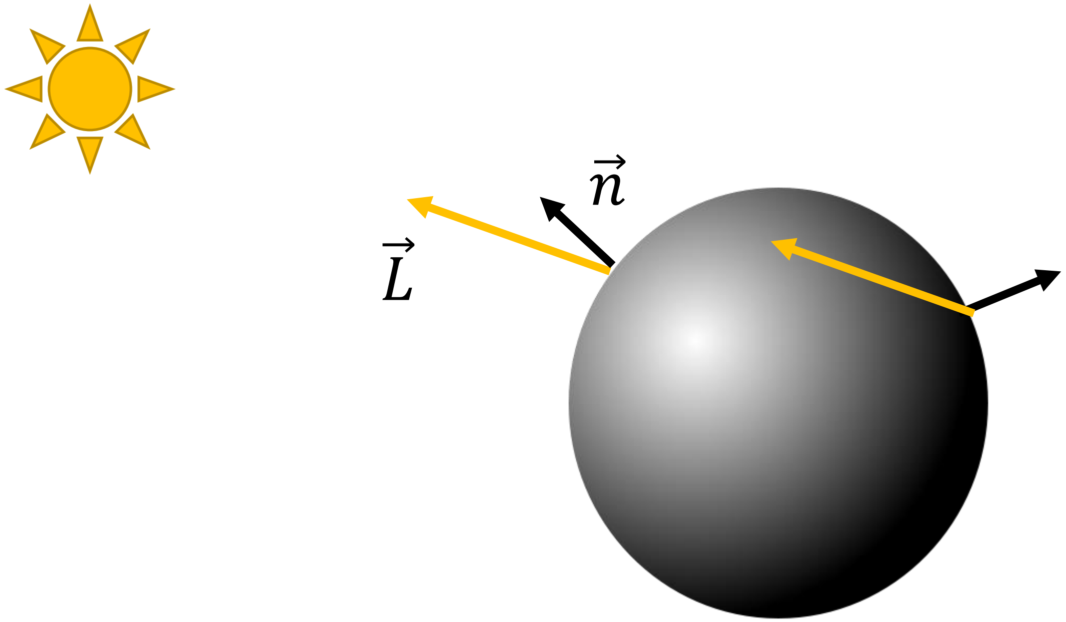
Dot product fulfills the following properties.

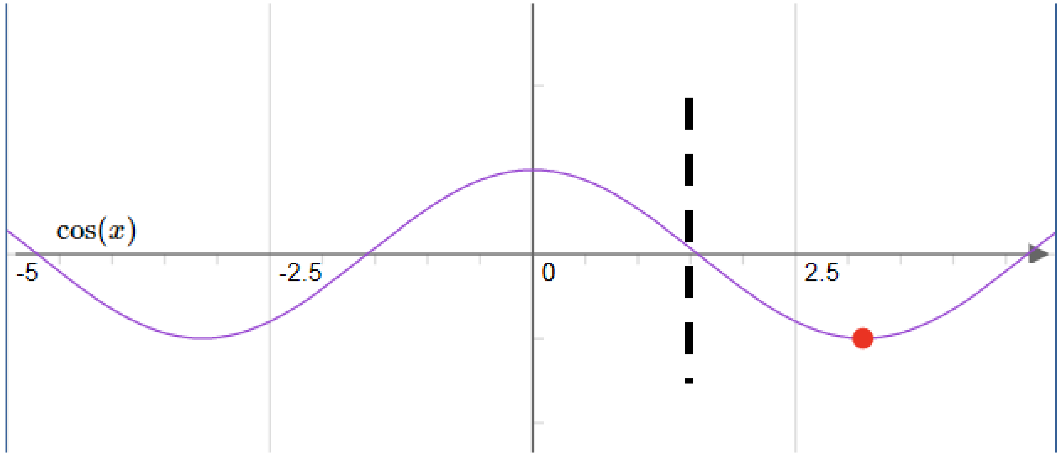
- Commutative: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- Distributive: $\vec{a} \cdot (\vec{b} + \vec{c}\,) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- Orthogonal: Two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$.
- No cancellation: even though $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a}$ is non-zero vector, $\vec{b} \neq \vec{c}$.

Dot Product

The dot product can be used for calculating the lighting result.

$$\vec{L} \cdot \vec{n} = ||\vec{L}||||\vec{n}||\cos\theta$$



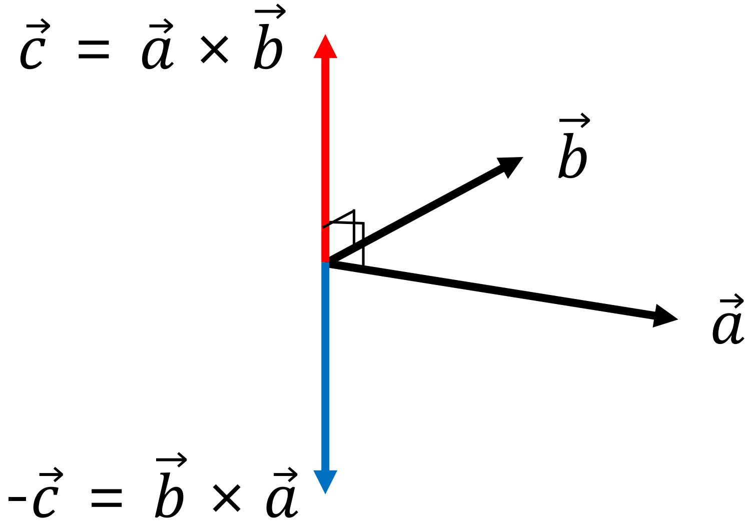


Cross Product

The cross product[1] (or vector product) is a binary operation on two vectors in R^3 and is denoted by the symbol $\times$.

- Note that dot product is defined in any dimension.

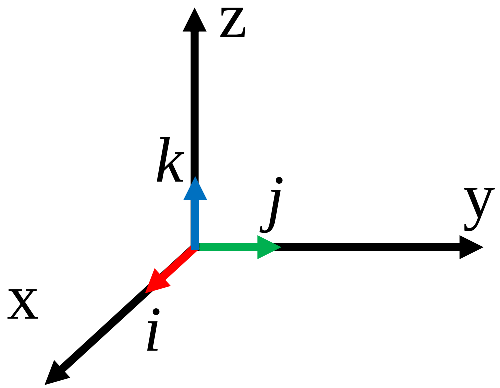
The cross product of two vectors, $\vec{a} \times \vec{b}$, is defined as a vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$, with a direction given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span. As the cross product is anti-commutative, $\vec{b} \times \vec{a} = -\vec{c}$



Cross Product

The standard basis vectors *i*, *j*, and *k* satisfy the following equalities:

- $i \times j = k$ and $j \times i = -k$
- $j \times k = i$ and $k \times j = -i$
- $k \times i = j$ and $i \times k = -j$
- $i \times i = j \times j = k \times k = 0$



Two vectors of $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ can be defined as the sum of standard basis vectors: $\vec{u} = u_1 i + u_2 j + u_3 k$ and $\vec{v} = v_1 i + v_2 j + v_3 k$.

Then, $\vec{u} \times \vec{v} = (u_1 i + u_2 j + u_3 k) \times (v_1 i + v_2 j + v_3 k) = u_1v_1(i \times i) + u_1v_2(i \times j) + u_1v_3(i \times k) +$

$$u_2v_1(j \times i) + u_2v_2(j \times j) + u_2v_3(j \times k) +$$

$$u_3v_1(k \times i) + u_2v_2(k \times j) + u_3v_3(k \times k)$$

$$= (u_2v_3 - u_2v_2)i + (u_3v_1 - u_1v_3)j + (u_1v_2 - u_2v_1)k$$

Cross Product

Cross product can also be computed in the matrix form.

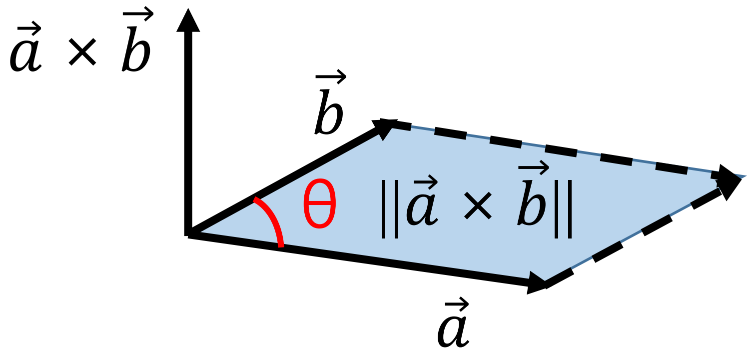
$$\vec{u} \times \vec{v} = (u_1 i + u_2 j + u_3 k\ ) \times (v_1 i + v_2 j + v_3 k) = (u_2 v_3 \text{ - } u_2 v_2) i + (u_3 v_1 + u_1 v_3) j + (u_1 v_2 \text{ - } u_2 v_1) k$$

$$\begin{bmatrix} \boxed{i} & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix} \qquad \begin{bmatrix} i & \boxed{j} & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix} \qquad \begin{bmatrix} i & j & \boxed{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$$

$$(u_2 v_3 - u_3 v_2) i \qquad (u_3 v_1 - u_1 v_3) j \qquad (u_1 v_2 - u_2 v_1) k$$

Cross Product

The magnitude of cross product ($||\vec{a} \times \vec{b}||$)can be interpret as the positive area of the parallelogram made by $\vec{a}$ and $\vec{b}$. Therefore, $||\vec{a} \times \vec{b}||$ equals to $||\vec{a}||\ ||\vec{b}||$ sinθ.



Dot product and cross product are related by the properties as follows:

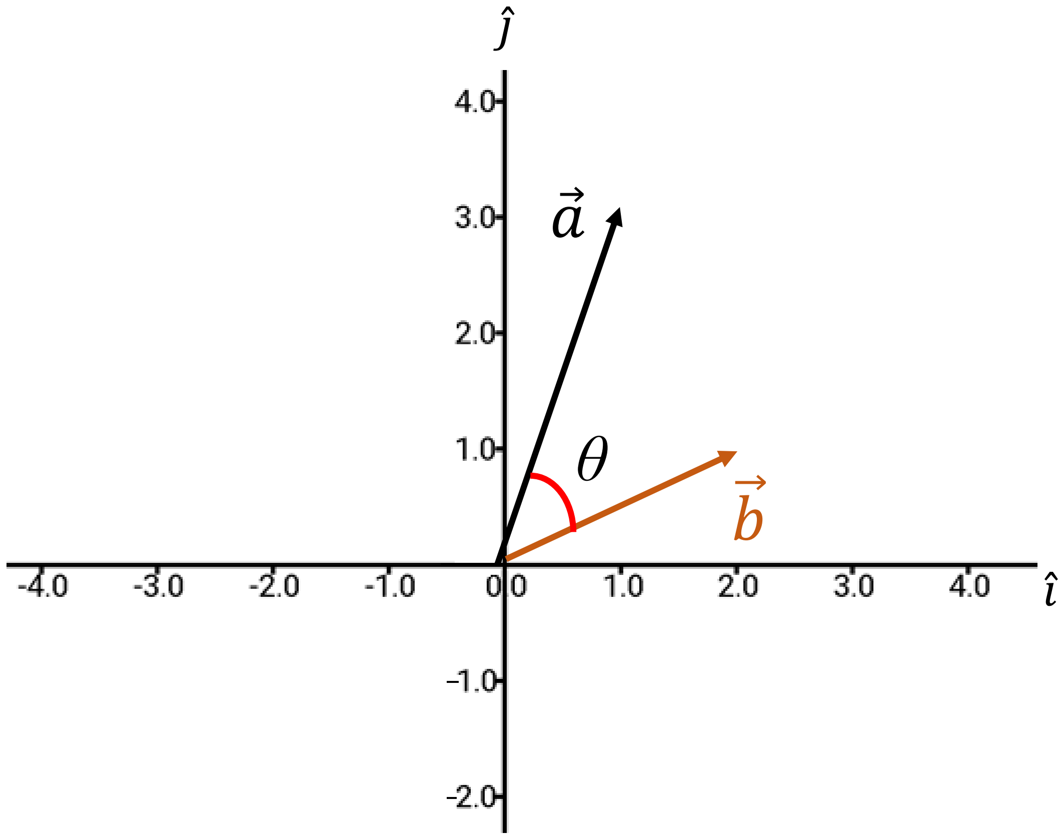
- $(\vec{a}\times\vec{b}) \cdot \vec{a} = 0$
- $||\vec{a}\times\vec{b}||^2 = ||\vec{a}||^2\ ||\vec{b}||^2\ ||\sin\theta||^2 = ||\vec{a}||^2\ ||\vec{b}||^2\ ||1 - \cos\theta||^2 = ||\vec{a}||^2\ ||\vec{b}||^2 - ||\vec{a}||^2\ ||\vec{b}||^2\ ||\cos\theta||^2$
  $= ||\vec{a}||^2\ ||\vec{b}||^2 - (\vec{a} \cdot \vec{b})^2$

Practice

1. Calculate the cosθ where θ is the angle between the two vectors of $\vec{a} = \hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath}$.
   ($\hat{\imath} = (1, 0)$, $\hat{\jmath} = (0, 1)$)

Practice - Solution

1. Calculate the $\cos\theta$ where θ is the angle between the two vectors of $\vec{a} = \hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath}$.
   ($\hat{\imath} = (1, 0)$, $\hat{\jmath} = (0, 1)$)



$$\sqrt{10} \quad \sqrt{5}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$1 * 2 + 3 * 1 = 5$$

$$\Rightarrow \cos\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Matrix

A matrix is a rectangular arrangement of numbers.
Shown below is a matrix P with *m* rows and *n* columns:

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{pmatrix}$$

Matrix A which has two rows and three columns and matrix B which has three rows and two columns are as follows:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \end{pmatrix}$$

Matrix Elements

A matrix element (simply matrix entry) is identified by row and column.
Let's consider matrix A.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$$

The element in row i and column j of matrix is denoted as $a_{i,j}$.
In the example, the element $a_{1,2}$ is the entry in the first row and the second column. It is 2.

Matrix Addition and Subtraction

Given A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, let's find A + B.

- A + B = $\begin{pmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{pmatrix}$ = $\begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$

Similarly, subtraction can be computed by subtracting the corresponding entries.

- A - B = $\begin{pmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{pmatrix}$ = $\begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$

Matrix Addition and Subtraction

If we have A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, B = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, and A + C = B. Then, we can obtain C.

- A + C = B
- C = B – A = $\begin{pmatrix} 5-1 & 6-2 \\ 7-3 & 8-4 \end{pmatrix}$ = $\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

Matrix Multiplication

Scalar multiplication refers to the product of a real number and a matrix.

Given that A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, let’s find -3A.

- -3A = $\begin{pmatrix} -3*1 & -3*2 \\ -3*3 & -3*4 \end{pmatrix}$ = $\begin{pmatrix} -3 & -6 \\ -9 & -12 \end{pmatrix}$

Let’s solve a matrix equation. Let 2M = $\begin{pmatrix} 2 & 6 \\ -4 & -2 \end{pmatrix}$. Then, what is M?

- M = $\frac{1}{2}\begin{pmatrix} 2 & 6 \\ -4 & -2 \end{pmatrix}$ = $\begin{pmatrix} 1 & 3 \\ -2 & -1 \end{pmatrix}$

Matrix Multiplication

Matrix multiplication refers to the product of two matrices.

Given that A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and B = $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$. Then, A · B can be obtained by

- A · B = $\begin{pmatrix} a_{11} * b_{11} + a_{12} * b_{21} & a_{11} * b_{12} + a_{12} * b_{22} \\ a_{21} * b_{11} + a_{22} * b_{21} & a_{21} * b_{12} + a_{22} * b_{22} \end{pmatrix}$

In the linear system, this can be represented by dot product.

$$\begin{array}{cc} & \begin{array}{cc} & \begin{array}{cc} \overrightarrow{b_1} & \overrightarrow{b_2} \\ \Downarrow & \Downarrow \end{array} \end{array} \\ \begin{array}{c} \overrightarrow{a_1} \Rightarrow \\ \overrightarrow{a_2} \Rightarrow \end{array} & \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} * \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} \overrightarrow{a_1} \cdot \overrightarrow{b_1} & \overrightarrow{a_1} \cdot \overrightarrow{b_2} \\ \overrightarrow{a_2} \cdot \overrightarrow{b_1} & \overrightarrow{a_2} \cdot \overrightarrow{b_2} \end{pmatrix} \end{array}$$

Matrix Multiplication Dimensions

In matrix multiplication, the number of entries in each row of the first matrix must be the same as the number of entries in each column of the second matrix.

Let's say A = $\begin{pmatrix} 0 & 2 & 4 \\ -4 & -2 & 0 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & -5 \\ 3 & -3 \\ 5 & -1 \end{pmatrix}$.

- The number of entries in each row of the A is 3.
- The number of entries in each column of the B is 3.

On the other hand, A = $\begin{pmatrix} 0 & 2 & 4 \\ -4 & -2 & 0 \end{pmatrix}$ and C = $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$.

- The number of entries in each row of the A is 3.
- The number of entries in each column of the C is 1.
- Therefore, we cannot compute AC.

Matrix Multiplication Dimensions

The dimensions of matrix can be referred as rows × columns.

For example, A = $\begin{pmatrix} 0 & 2 & 4 \\ -4 & -2 & 0 \end{pmatrix}$ is a 2×3 matrix and B = $\begin{pmatrix} 1 & -5 \\ 3 & -3 \\ 5 & -1 \end{pmatrix}$ is a 3×2 matrix.

The multiplication of an $x \times y$ matrix and an $y \times z$ is an $x \times z$ matrix.

- A is a 2×3 matrix and B is a 3×2 matrix.
- Therefore, AB is a 2×2 matrix. AB = $\begin{pmatrix} 26 & -10 \\ -10 & 26 \end{pmatrix}$

Identity Matrix

The identity matrix is a square matrix whose entries on the diagonal from the upper left to the bottom right are all 1 while other entries are all 0.

The 2×2, 3×3, and 4×4 identity matrices are as follows:

- $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

The product of any matrix of A and identity matrix of I is always A.

- $A \cdot I = I \cdot A = A$

Properties of Matrix Operation

Matrix addition and subtraction fulfill following properties:

- Commutative: $A+B = B+A$
- Associative: $(A+B)+C= A+(B+C)$
- Distributive: $A(B+C) = AB + BC$
- Additive identity: There is a unique $m \times n$ matrix O (zero matrix) with $A + O = A$
- Additive inverse : For any $m \times n$ matrix A, there is an $m \times n$ matrix B with $A+B = O$

Matrix multiplication fulfills following properties:

- Not commutative: $AB \neq BA$
- Associative: $(AB)C = A(BC)$
- Distributive: $A(B+C) = AB + BC$
- Multiplicative identity: There is a unique matrix I with $AI = IA = A$

Matrix Transpose

Transposing a matrix means flipping a matrix. We denote the transpose of matrix A by A^T.

- $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$, then $A^T = \begin{pmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \end{pmatrix}$

This can be used for converting between row vector and column vector.

- $X = (x_1 \quad x_2 \quad \cdots \quad x_n)$ and $X^T = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

Matrix Inverse

Inverse matrix can be computed by

- $A^{-1} = \frac{1}{determinant(A)} adjugate(A)$

- In this class, we only covers 2 × 2 matrix.
- So, the inverse matrix of A $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ equals to $A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. $(ad - bc \neq 0)$

Inverse matrix of A $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ equals to $\frac{1}{10 - 12} \begin{pmatrix} 5 & -3 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{-5}{2} & \frac{-3}{2} \\ 2 & -1 \end{pmatrix}$.

Linear System

A linear system can be represented by an augmented matrix.

- $\begin{cases} 3x + 2y = 10 \\ x + 4y = 10 \end{cases}$ (linear system)

- $\begin{pmatrix} 3 & 2 & 10 \\ 1 & 4 & 10 \end{pmatrix}$ (augmented matrix)

This is another example.

- $\begin{cases} w + 2y = 4 \\ 2x + 3y + z = 8 \\ w + z = 1 \end{cases}$ (linear system)

- $\begin{pmatrix} 1 & 0 & 2 & 0 & 4 \\ 0 & 2 & 3 & 1 & 8 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}$ (augmented matrix)

Practice

Let's say that $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, and $B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$.

- Determine AB.
- Determine BA.
- AB = BA? Why?
- Determine (A+B)O.
- Determine $I^2(IB + OAB)$

Practice - Solution

Let's say that $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, and $B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$.

- Determine AB. *sol*: $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & 4 \end{pmatrix}$
- Determine BA. *sol*: $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & 4 \end{pmatrix}$
- AB = BA? Why? *sol*: A = 2I. Therefore, AB = 2B, BA = 2B.
- Determine (A+B)O. *sol*: 0
- Determine $I^2(IB + OAB)$ *sol*: $B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$