

## Week-5 (Time Domain Analysis) –ZSR

### Finding Zero – State Response:

- **METHOD 1:** Use the “ode23” function in MATLAB to directly solve the Differential Equation (DE), obtaining the solution in array form.

Here, **only the forcing function  $x(t)$  is considered by setting the initial conditions to zero** and the procedure is carried out as explained for the Zero-Input Response (ZIR).

For the given example differential equation,  $(D^2+4D+3)y(t) = (3D+5)x(t)$  and the input  $x(t) = 2t+3$ , manually compute  $Q(D)x(t)$  as:  $(3D+5)\{x(t)\} = 3D\{(0.2t+0.2)\} + 5\{(0.2t+0.2)\} = t+1.6$ .

Then, include these terms in the set of differential equation (1) and (2a) (See-Week 4-ZIR) as:

$$Dy_1(t) = y_2(t) \quad \text{---- (1)} \quad \text{and} \quad Dy_2(t) = -4y_2(t) - 3y_1(t) + t+1.6 \quad \text{---- (2)}$$

- **METHOD 2 : Using Impulse Response in symbolic form of the differential equation – by convolution method. (i.e. using Graphical Convolution Method)**

Manually compute the impulse response for the DE, and use it as convolution of two signals:

Convolve the two signals  $h(t) = (e^{-3t} + e^{-t})u(t)$  and  $x(t) = (0.2t+0.2)u(t)$

### MATLAB code:

```
x=inline('(0.2*t+0.2).*(t>=0)','t')
h=inline('(exp(-t)+2*exp(-3*t)).*(t>=0)','t')
tvec = -0.25:0.01:5; %defines time range for the output presentation (lower and upper value depend
on width property of convolution, accounting the causality of both the signals)
dtau = 0.05; %defines an incremental value for the convolution
tau= -3.5:dtau:5; %defines the integrating ranges for the convolution (tau axis, lower value selected
for better visibility)
%maximum allowed lower limit value of tau is depend on the causality of x(t) OR may be same as tvec
%minimum allowed upper limit value of tau is depend on the causality of h(t) OR everlasting nature.
%If the signal h(t) has the non-zero value for  $t \geq -m$ , then the minimum value chosen depends on the
value chosen for the upper value of tvec. That is at least tvec+m.
ti= 0; %initialising the index counter, i.e. pointer to the solution to DE for y(t)
y=NaN*zeros(1,length(tvec)); %NAN is not a number (no value is defined as o/p)
for t = tvec, % Start of loop for convolution, here t is scalar value varies in loop
    ti=ti+1; % incrementing the counter pointer for the solution to DE in y
    xh= x(tau).* h(t-tau); % Multiplication of two signals
    y(ti) = sum(xh.*dtau); % Integration part, i.e. computes output y(t) and stores in an index “ti”
    figure; subplot(2,1,1), plot(tau,x(tau),'r_',tau,h(t-tau),'m-',t,0,'ro');
    % This window is for the display of the overlap ranges of two signals in “tau” axis.
    %axis([tau(1) tau(end) -0.2 5.1]); % optional axis range for “tau” axis
    subplot(2,1,2), plot(tvec,y,'b_',tvec(ti),y(ti),'ro'); % This window is for the display of output in tvec axis
    %axis([tvec(1) tvec(end) -0.2 5.1]); % optional axis range for “tvec” axis
    drawnow; pause;
end
figure; plot(tvec,y) % output is sketched in new window
```

### Finding Total Response:

Here, consider the given boundary conditions (initial conditions) along with input  $x(t)$  by manually computing  $Q(D)x(t)$  and solve using “ode23” to get the total response in array form.

### Exercises

1. Following are the Differential Equations (DE) of the LTIC system,
  - a)  $D(D+1)y(t) = (D+2)x(t)$  and the input  $x(t) = u(t)$  OR  $x(t) = e^{-2t}u(t)$ .
  - b)  $(D^2+9)y(t) = (3D+2)x(t)$  and the input  $x(t) = u(t)$  OR  $\sin(3t)u(t)$  and  $\sin(2.9t)u(t)$ , .
  - c)  $(D^2+D+13)y(t) = 4(D+2)x(t)$  and the input  $x(t) = e^{-0.5t} \sin(3.5707t)u(t)$  OR  $e^{-0.5t} \sin(3.5t)u(t)$
  - d)  $D^2+1.5D+0.5625)y(t) = 4(D+2)x(t)$  and the input  $x(t) = e^{-0.75t}u(t)$  OR  $e^{-t}u(t)$
  - i. Find manually the **zero state response**  $\{y_s(t)\}$ .
  - ii. Using “**ODE23**” function in MATLAB get  $y_s(t)$  in array form and sketch the same.
  - iii. Using “graphical convolution procedure”, in MATLAB, get  $y_s(t)$  in array form and sketch the same.
  - iv. Identify the, characteristic modes in the total response of the system.
  - v. Give comments on which class of signal model the output response belongs to.
  - vi. Find the steady state values.
  - vii. Find the approximate time taken by the output response to reach steady value.
  - viii. Find the period of oscillation of output response (if applicable).
2. Find manually the **total response** of a systems specified by the DEs, of exercise 1) given initial conditions:  $y(0^-) = 2$ ,  $\dot{y}(0^-) = -3$ . Use “**ODE23**” function in MATLAB to solve the unit impulse response for these DEs.
3. Try using “**dsolve**” function to solve the Differential Equations in symbolic form.

**NOTE:** Solve all of the above Differential Equations manually (i.e. hand calculation) before entering the lab. Weightage is mainly based on the how well you prepare and come.