Log

Definition

$\log_b(a) = x \iff b^x = a$

- b: base (b > 0 and b ≠ 1)
- a > 0

Basic Logarithm Operations

1. **Sum of Logarithms** $\log_b(x) + \log_b(y) = \log_b(x \cdot y)$
2. **Difference of Logarithms** $\log_b(x) - \log_b(y) = \log_b(x/y)$
3. **Multiplication by a Constant** $k \cdot \log_b(x) = \log_b(x^k)$
4. **Change of Base** $\log_b(a) = \log_c(a)/\log_c(b)$ Where c is any base (e.g., c = 10 or c = e)
5. **Logarithm of the Base** $\log_b(b) = 1$
6. **Logarithm of 1** $\log_b(1) = 0$

Special Logarithms

1. **Natural Logarithm** (ln) $\ln(x) = \log_e(x)$, where $e \approx 2.718$
2. **Decimal Logarithm** (log) $\log(x) = \log_{10}(x)$

Exponent Properties

Definition

b^x where $b > 0$ and $x \in \mathbb{R}$

Basic Exponent Operations

1. **Sum of Exponents** $b^x \cdot b^y = b^{(x + y)}$
2. **Difference of Exponents** $b^x/b^y = b^{(x - y)}$, $b \neq 0$
3. **Power of a Power** $(b^x)^y = b^{(x \cdot y)}$
4. **Product of Powers with Different Bases** $a^x \cdot b^x = (a \cdot b)^x$
5. **Roots as Fractional Exponents** $b^{(1/n)} = \sqrt[n]{b}$, $n > 0$
6. **Negative Exponent** $b^{(-x)} = 1/b^x$, $b \neq 0$

Special Exponent Cases

1. **Base 1** $1^x = 1$
2. **Exponent 0** $b^0 = 1$, $b \neq 0$
3. **Base 0** $0^x = 0$, $x > 0$

Relationship Between Logarithms and Exponents

$\log_b(b^x) = x$ and $b^{(\log_b(x))} = x$