<u>Modelling the evolution of predators and prey taking into account pollution and ecological factors</u>

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INTRODUCTION

The Lotka-Voltera equations are widely used to describe the dynamics of biological systems. It is mainly to describe the interaction of a set of prey with their predators in a biological system. This phenomenon is then modelled by a pair of ordinary nonlinear first-order differential equations. These equations were introduced independently by Alfred Lotka in 1925 [1] and by Vito Volterra in 1926 [2]. Voltera wanted to interpret the periodicity of the number of individuals of certain species of fish caught in the Adriatic Sea.

He then separated these species into two categories: prey and predators. Under certain assumptions, he was able to establish a model that links the variation of prey to that of predators, taking into account birth and mortality rates, and the initial number of individuals. This model is simplified by assuming that the prey have an unlimited source of food, that there is no competition between them. This model in its most simplified form is then written:

$$(S1)\ \begin{cases} x'(t) = x(t)(a - by(t)) \\ y'(t) = y(t)(dx(t) - c) \\ x(0) = x_0, y(0) = y_0 \end{cases}$$

Where,

- t is the time,
- $x(t)$ is the number of prey as a function of time,
- $y(t)$ is the number of predators as a function of time,
- $a > 0$ characterizes the intrinsic reproduction rate of prey,
- $b > 0$ characterizes the mortality rate of prey due to predators encountered,
- $c > 0$ characterizes the intrinsic mortality rate of predators, regardless of the number of prey,
- $d > 0$ characterizes the reproduction rate of predators according to the prey encountered and eaten.

The positive sign of parameters a, b, c and d is necessary for the model to have a biological significance.

The aim is to apply this model to study the evolution of lizards (prey) and a bird species (predators) in an ecological system. First, the simplified model is studied. Then a modified model is introduced taking into account the influence of pesticide uses in the environment of growth of these species. Finally, a further modification is made to the model considering the limitation of resources that could result from deforestation.

PART - A

A.1. Explanation for the equation: $-\boldsymbol{bx(t)y(t)}$

The Lotka-Volterra model for predator-prey interactions is given by:

$$\begin{cases} \boldsymbol{x'(t) = x(t)(a - by(t))} \\ \boldsymbol{y'(t) = y(t)(dx(t) - c)} \end{cases}$$

Were,

$\boldsymbol{x(t)}$: number of preys at time t,
$\boldsymbol{y(t)}$: number of predators at time t,
$\boldsymbol{a > 0}$: intrinsic reproduction rate of the prey,
$\boldsymbol{b > 0}$: mortality rate of the prey due to encounters with predators,
$\boldsymbol{c > 0}$: intrinsic mortality rate of the predators,
$\boldsymbol{d > 0}$: reproduction rate of predators based on prey encounters.

As we can see the equation $-\boldsymbol{bx(t)y(t)}$ is present in the parent equation $\boldsymbol{x'(t) = x(t)(a -}$ **by** $\boldsymbol{(t))}$. Let us breakdown the equation to understand,

Insights we can deduct from the Equation:

1. Product of $x(t)$ and $y(t)$:

   - The prey mortality depends on the interaction between prey and predators. The more predators and prey there are, the more encounters occur, leading to increased predation.
   - **This product term $x(t)y(t)$ models the likelihood of encounters between prey and predators, which increases the mortality rate of the prey.**

2. Coefficient b :

   - The parameter b represents how lethal each encounter is for the prey population. **A higher value of $\boldsymbol{b}$ means each encounter has a greater impact on prey mortality.**

3. Negative Sign:

   - **The negative sign in $-bx(t)y(t)$ indicates that this term reduces the prey population.** As predators encounter and consume prey, the prey count decreases, which is reflected by this negative influence on $x'(t)$.

Hence, $-\boldsymbol{bx(t)y(t)}$ represents the rate at which prey is removed from the population due to encounters with predators.

A.2. Explanation of the Term $-\boldsymbol{cy(t)}$

As we can see the equation $-\boldsymbol{bx(t)y(t)}$ is present in the parent equation $\boldsymbol{y'(t) = y(t)(dx(t) - c)}$. Let us see more into the equation,

Insights we can deduct from the Equation:

1. Coefficient c :

   - **The parameter $\boldsymbol{c}$ represents the rate at which predators die due to natural causes** unrelated to prey availability, such as age, disease, or other environmental factors.

2. Negative Sign:

    - **The negative sign indicates a reduction in the predator population.** In the absence of sufficient prey, predators may not find enough food to sustain themselves, leading to a natural decrease in their numbers over time.

3. Dependency on $y(t)$:

    - Since this term is proportional to $y(t)$, it models that **the number of predators that die in each time interval is directly related to the size of the predator population itself**. More predators mean more deaths due to natural causes.

Hence, $-cy(t)$ is the equation that represents the natural decrease in the predator population due to factors independent of prey availability.

A.3 Proofs

i) Prey population increases exponentially in the absence of predators:

In the absence of predators, we assume $\boldsymbol{y(t) = 0}$. Thus,

$$\boldsymbol{x'(t) = x(t) \cdot a}$$

This is a differential equation with respect **to $\boldsymbol{x(t)}$ only**, and it can be rewritten as:

$$\boldsymbol{\frac{dx}{dt} = a \cdot x(t)}$$

by separating variables:

$$\boldsymbol{\int \frac{1}{x(t)} dx = \int \ adt}$$

Integration on both sides:

$$\boldsymbol{\ln\ |x(t)| = at + C}$$

Exponentiating both sides:

$$\boldsymbol{x(t) = e^{C} \cdot e^{at} = x_0 e^{at}}$$

Where, $\boldsymbol{x_0 = e^{C}}$ **represents the initial population of the prey at $\boldsymbol{t = 0}$.**

Hence Proved,

$\boldsymbol{x(t) = x_0 e^{at}}$**,** shows that the prey population grows exponentially over time in the absence of predators. The growth rate is determined by a, the intrinsic reproduction rate of the prey.

ii) Predator Population Decline in the Absence of Prey Population:

In the absence of prey, we assume $\boldsymbol{x(t) = 0}$**:**

$$\boldsymbol{y'(t) = -c \cdot y(t)}$$

This is a differential equation for $\boldsymbol{y(t)}$, and it can be rewritten as:

$$\boldsymbol{\frac{dy}{dt} = -c \cdot y(t)}$$

This represents an exponential decay model.

Separate the variables:

$$\boldsymbol{\int \frac{1}{y(t)} dy = -\int \, cdt}$$

Integration both sides, we get:

$$\boldsymbol{\ln\,|y(t)| = -ct + C}$$

Exponentiating both sides:

$$\boldsymbol{y(t) = e^{C} \cdot e^{-ct} = y_0 e^{-ct}}$$

Where, $\boldsymbol{y_0 = e^{C}}$ **represents the initial population of the predators at** $\boldsymbol{t = 0}$**.**

Hence Proved,
$\boldsymbol{y(t) = y_0 e^{-ct}}$**,** shows that the predator population decreases exponentially over time in the absence of prey. The decay rate is determined by c, the intrinsic mortality rate of predators.

PART - B

B.1. Rewriting the model S1 in Matrix form:

The Lotka-Volterra equations we're working with are:

$$\begin{cases} \boldsymbol{x'(t) = x(t)(a - by(t))} \\ \boldsymbol{y'(t) = y(t)(dx(t) - c)} \end{cases}$$

system in matrix form:

$$\begin{pmatrix} \boldsymbol{x'(t)} \\ \boldsymbol{y'(t)} \end{pmatrix} = \boldsymbol{F} \begin{pmatrix} \boldsymbol{x(t)} \\ \boldsymbol{y(t)} \end{pmatrix} = \begin{pmatrix} \boldsymbol{f_1(x(t), y(t))} \\ \boldsymbol{f_2(x(t), y(t))} \end{pmatrix}$$

Here, f_1 and f_2 represent the expressions for $x'(t)$ and $y'(t)$ from the model equations. Expressions for f_1 and f_2:

1. $\boldsymbol{f_1(x(t), y(t)) = x(t)(a - by(t))}$
2. $\boldsymbol{f_2(x(t), y(t)) = y(t)(dx(t) - c)}$

So, the equations can be rewritten as:

$$\begin{pmatrix} \boldsymbol{x'(t)} \\ \boldsymbol{y'(t)} \end{pmatrix} = \begin{pmatrix} \boldsymbol{x(t)(a - by(t))} \\ \boldsymbol{y(t)(dx(t) - c)} \end{pmatrix}$$

The matrix form is created by using the formulas of Lotka-Volterra given.

B.2. The theorem and assumptions are:

The Cauchy-Lipschitz Theorem (also known as the Picard-Lindelöf Theorem) guarantees the existence and uniqueness of solutions to an ordinary differential equation (ODE) for a given initial condition, under certain assumptions about the function F in the equation.

Assumptions:

1. Continuity: The function $F(t, y)$ must be continuous in both variables t and y.
2. Lipschitz Continuity: The function $F(t, y)$ must satisfy a Lipschitz condition in y on some domain, meaning there exists a constant L such that:

$$|F(t, y_1) - F(t, y_2)| \leq L|y_1 - y_2|$$

for all y_1 and y_2 in the domain.

These conditions ensure that for an initial condition $y(t_0) = y_0$, the matrix equation has a unique solution over a specific interval around t_0.

B.3. Finding the equilibrium points:

An equilibrium point is a point where **both $x'(t)$ and $y'(t)$ are zero**, meaning that the populations of prey and predators do not change over time.

From the equations:

1. $x'(t) = x(t)(a - by(t)) = 0$
2. $y'(t) = y(t)(dx(t) - c) = 0$

For both derivatives to be zero, there are two cases to consider:

Case 1:
If $x(t) = 0$ and $y(t) = 0$, then $x'(t) = 0$ and $y'(t) = 0$.

So, the first equilibrium point is:

$$(x, y) = (0, 0)$$

This represents the extinction of both populations.

Case 2:
For non-zero values, we need each equation to be zero independently:

1. **From $x(t)(a - by(t)) = 0$,** we get.

$$a - by(t) = 0 \Rightarrow y(t) = \frac{a}{b}.$$

2. **From $y(t)(dx(t) - c) = 0$,** we get.

$$dx(t) - c = 0 \Rightarrow x(t) = \frac{c}{d}.$$

So, the second equilibrium point is:

$$(x, y) = \left(\frac{c}{d}, \frac{a}{b}\right)$$

B.4. Euler explicit scheme for the system (S1):

The Lotka-Volterra equation is:

$$\begin{cases} x'(t) = x(t)(a - by(t)) \\ y'(t) = y(t)(dx(t) - c) \end{cases}$$

We'll apply the Euler method to these equations to derive the approximate,

Euler Method Setup
Given:

1. Initial conditions $x(0) = x_0$ **and** $y(0) = y_0$.
2. Step size h, which we calculate based on the total time T and number of subdivisions N as $h = \frac{T - t_0}{N}$.

3. Recursive formulas for $\boldsymbol{x_{n+1}}$ **and** $\boldsymbol{y_{n+1}}$ based on the Euler scheme.

Deriving Recursive Formulas for x and y
Using the Euler method, the update formulas for x and y become:

1. For the prey x :

$$\boldsymbol{x_{n+1} = x_n + h \cdot x_n \cdot (a - b y_n)}$$

2. For the predators y :

$$\boldsymbol{y_{n+1} = y_n + h \cdot y_n \cdot (d x_n - c)}$$

Derivation of the values:

Step 1: Calculating x_1 and y_1
Using the initial conditions x_0 and y_0 :

$$\boldsymbol{x_1 = x_0 + h \cdot x_0 \cdot (a - b y_0)}$$
$$\boldsymbol{y_1 = y_0 + h \cdot y_0 \cdot (d x_0 - c)}$$

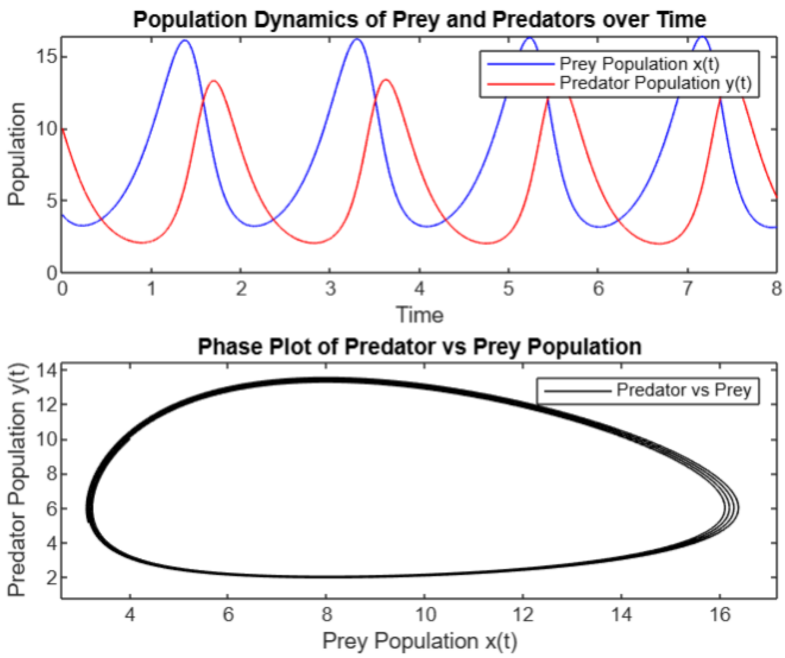
Step 2: Calculating x_2 and y_2
For the next time step, we use x_1 and y_1 :

$$\boldsymbol{x_2 = x_1 + h \cdot x_1 \cdot (a - b y_1)}$$
$$\boldsymbol{y_2 = y_1 + h \cdot y_1 \cdot (d x_1 - c)}$$

General Form for x_n and y_n
By following this iterative approach, we get a general form for x_n and y_n after n steps:

$$\boldsymbol{x_{n+1} = x_n + h \cdot x_n \cdot (a - b y_n)}$$
$$\boldsymbol{y_{n+1} = y_n + h \cdot y_n \cdot (d x_n - c)}$$

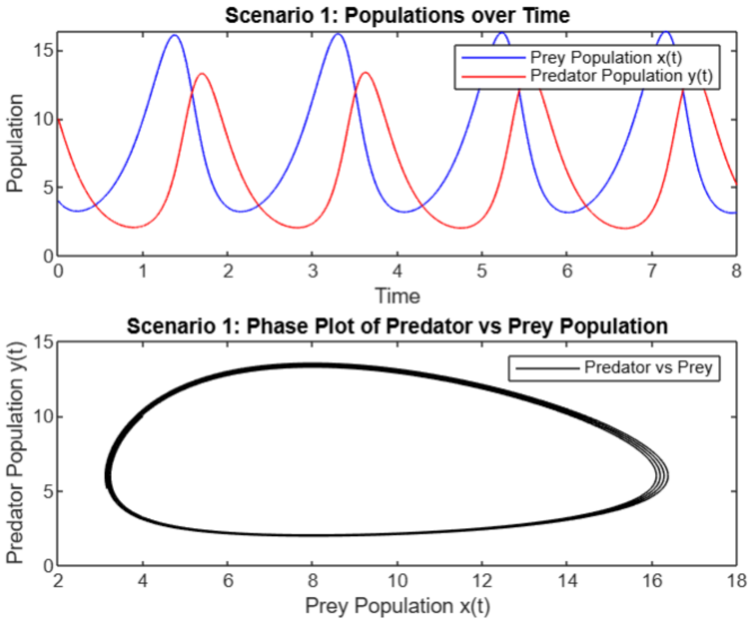
Graph Generated Using MATLAB code:



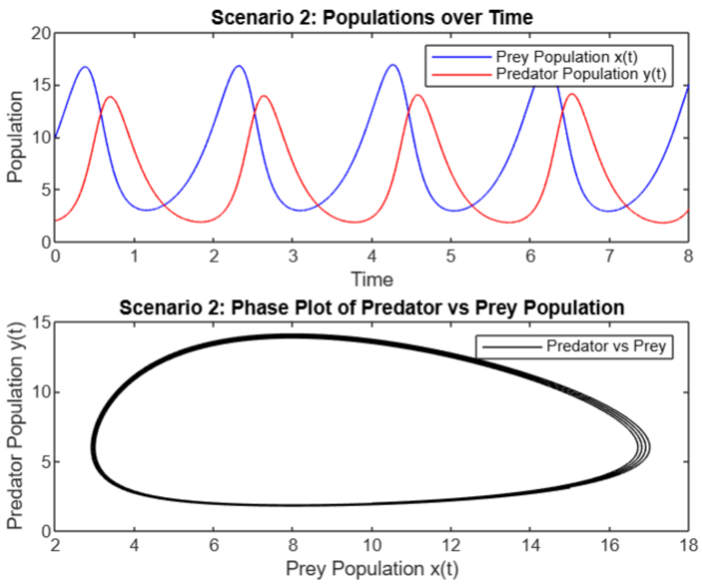
B.5. Interpretation of the graphs generated for the following below Scenarios:

1. Initial time t0 = 0, final time T = 8, subdivisions number N = 10000, x0 = 4, y0 = 10 and a =3, b=0.5, c=4, d =0.5.
2. Initial time t0 = 0, final time T = 8, subdivisions number N = 10000, x0 = 10, y0 = 2 and a =3, b=0.5, c=4, d =0.5.
3. Initial time t0 = 0, final time T = 8, subdivisions number N = 10000, x0 = 4, y0 = 10 and a =3, b=0.5, c=4, d =2.
4. Initial time t0 = 0, final time T = 8, subdivisions number N = 10000, x0 = 4, y0 = 10 and a =8, b=0.5, c=4, d =0.5.

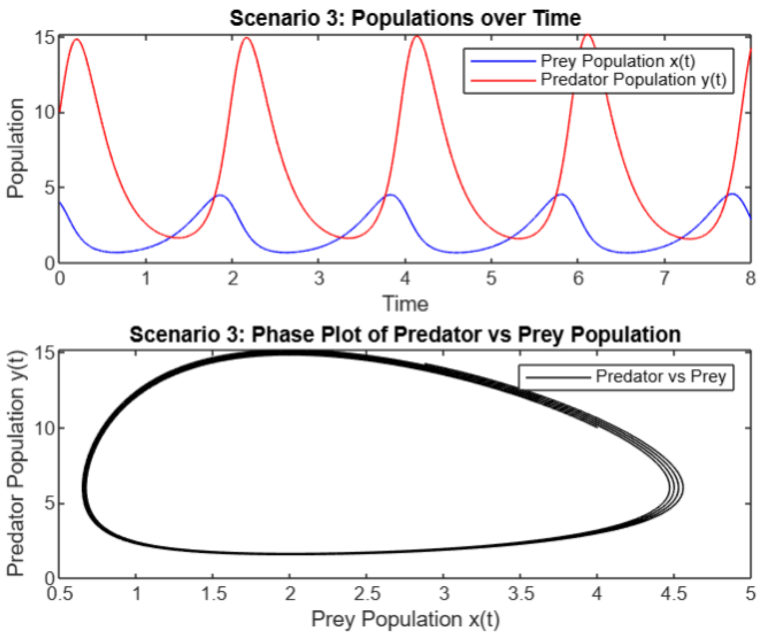
Scenario 1:



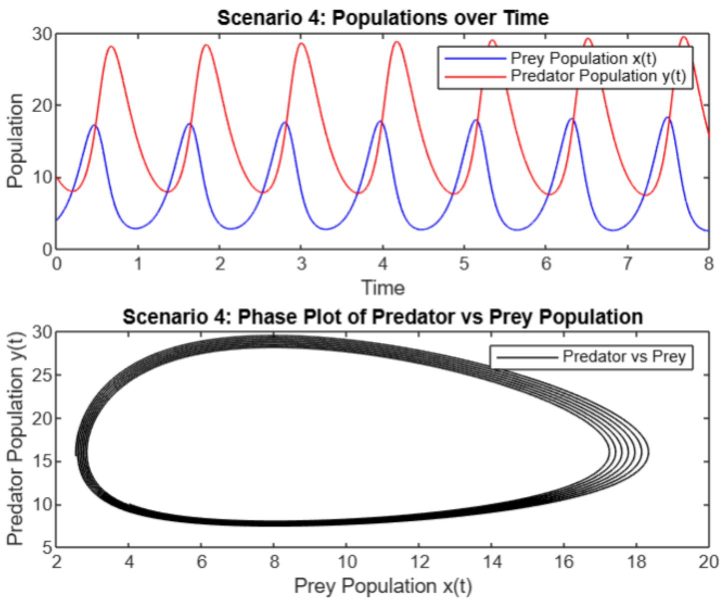
Scenario 2:



Scenario 3:



Scenario 4:



Interpretation:

I. In scenarios 1 and 2, which are mostly similar, a peak in the prey population is often accompanied by a peak in the predator population. This suggests a direct, balanced relationship between the two.

II. In scenarios 3 and 4, a high degree of imbalance is evident. In scenario 3, the lower amplitude indicates a significant imbalance, and the worst predator-prey relationship can be observed. As the predator population increases, the prey population continues to decline, leading to further imbalance.

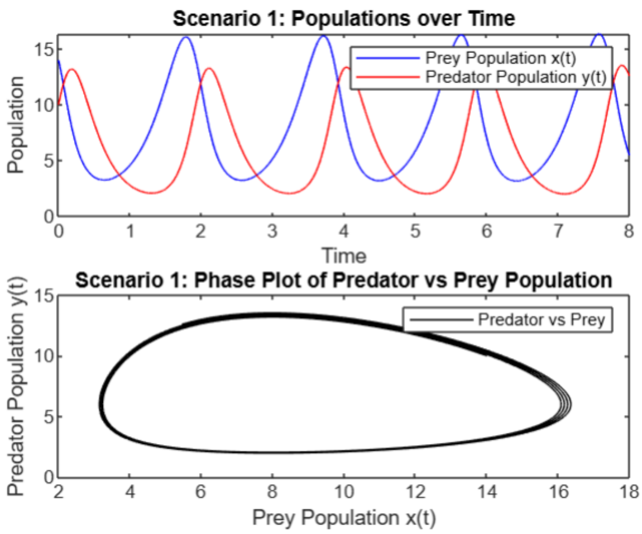
B.6. Taking as initial time t0 = 0, as final time T = 8, a subdivisions number N = 10000, and a=3, b=0.5, c=4, d=0.5, plot on the same coordinate system the evolution of predators according to prey for different initial values of x0 and y0.

Graphs Generated using MATLAB Code:

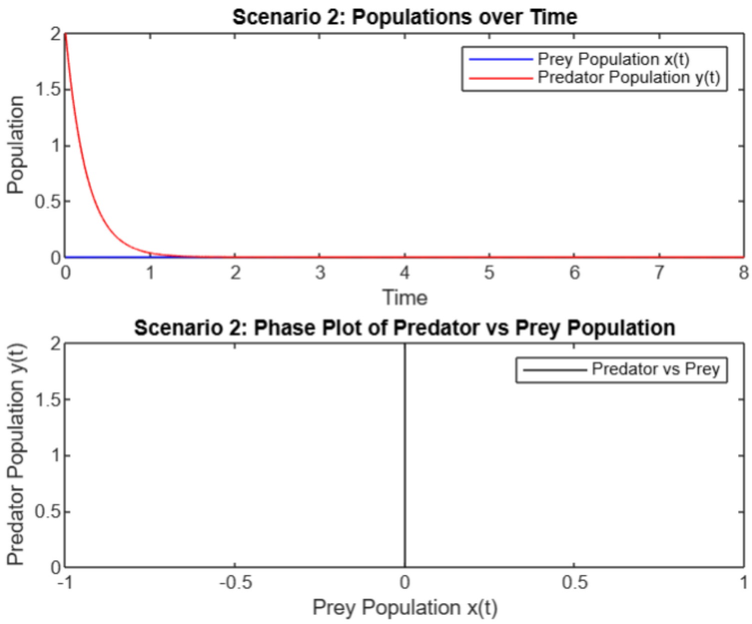
Initial Condition Scenarios:

- x0 = 14, **y0** =10
- x0 = 0, **y0** =2
- x0 = 5, **y0** =10
- x0 = 40, **y0** =10

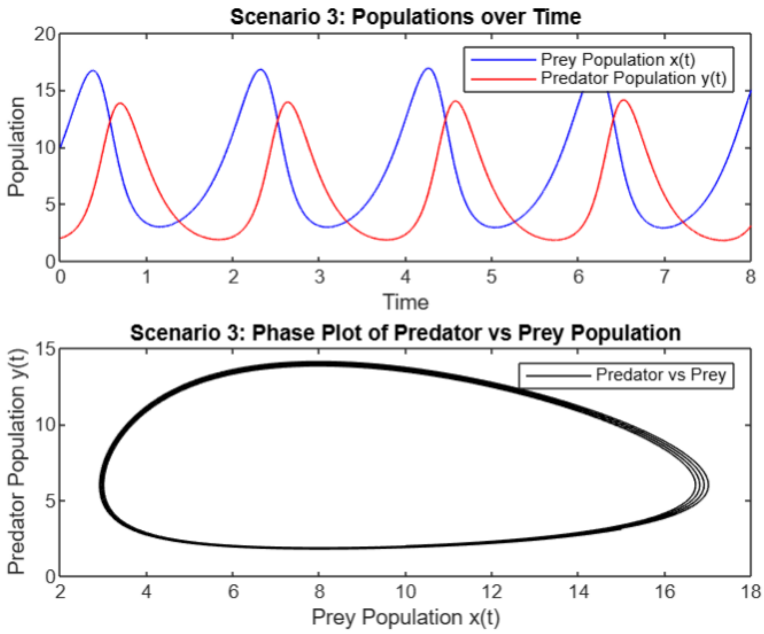
Scenario 1:



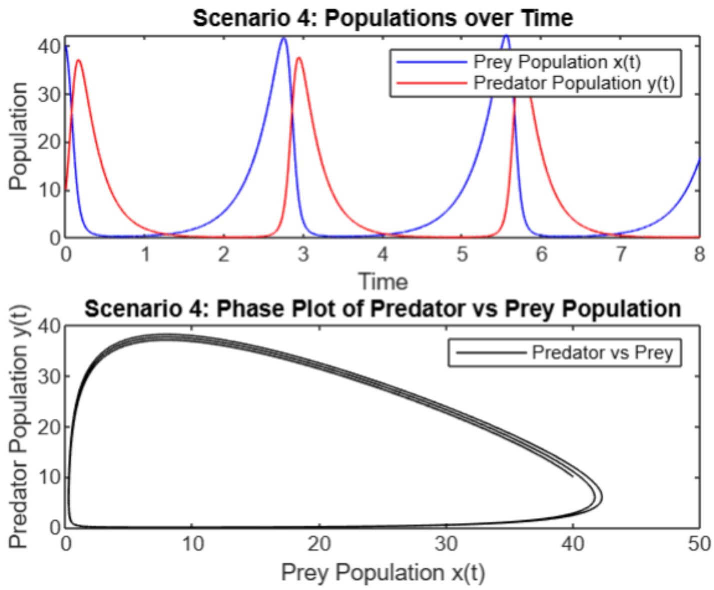
Scenario 2:



Scenario 3:



Scenario 4:



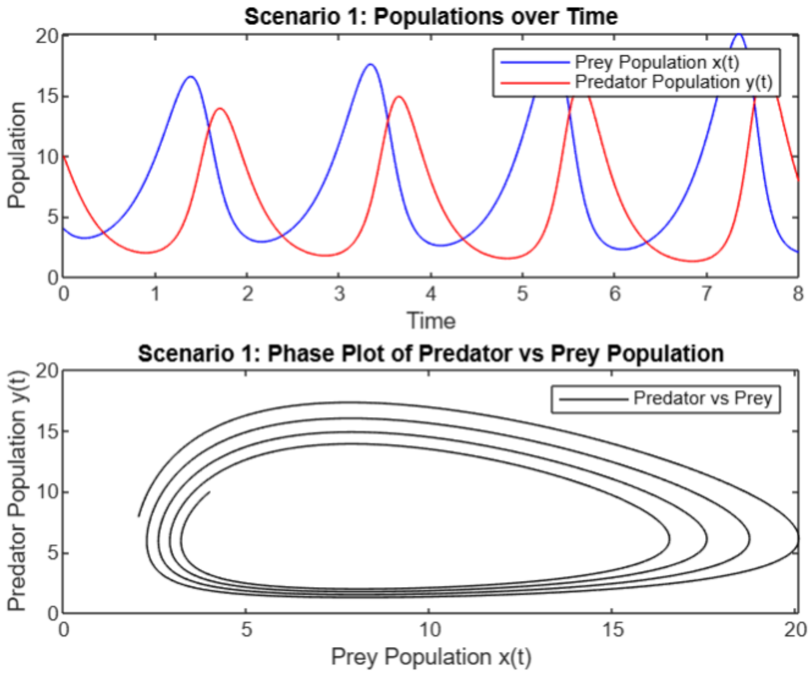
Interpretation:

I. We can interpret that the change in initial conditions did not significantly impact the overall trend in the prey and predator populations over time. The amplitude of the peaks and troughs in the graphs, which are proportional to both populations, suggests that while initial conditions can influence the population dynamics, they affect both the prey and predator populations similarly.

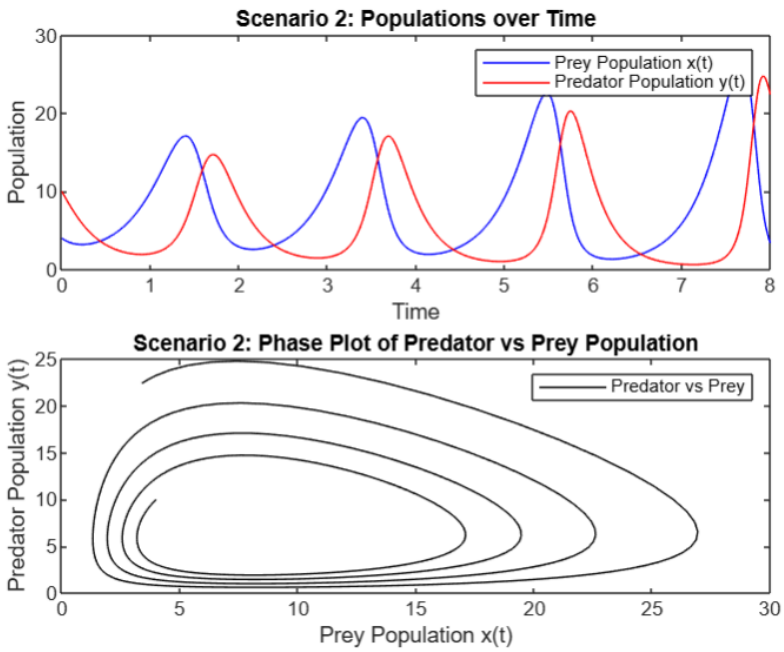
II. A special condition is observed in **scenario 2,** where the initial prey population is zero. In this case, the predator population, deprived of its food source, gradually declines to near-zero levels, demonstrating the interdependence of the two populations.

B.7. Interpreting Changes in Subdivision and Their Effects.

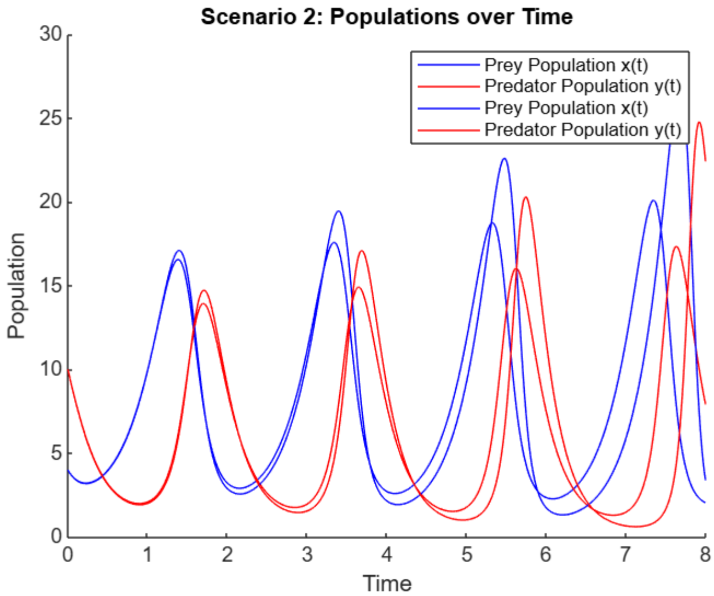
N= 10000:



N = 500:

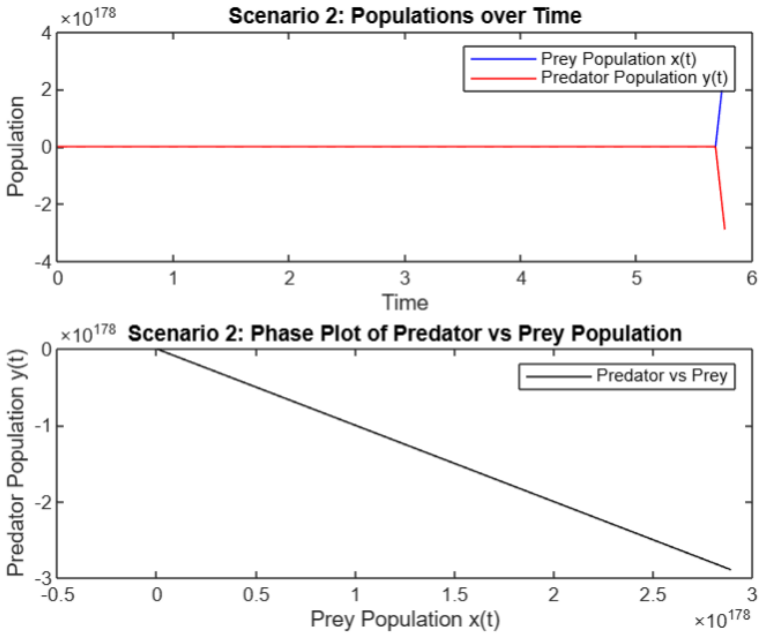


Combined graph:



We can observe that as the number of subdivisions increases, the graph becomes more accurate in representing the dynamics between the prey and predator populations. However, when the number of subdivisions is excessive, as in the case of **N = 100**, the graph becomes meaningless and fails to provide any meaningful insights into the relationship between the two populations.

N = 100:



PART – C

C.1. Populations sensitivity to pesticide effects:

Death rate assumptions:

Let $x(t)$ represent the prey population at time t and assume that the rate of deaths in the prey population due to pesticides is proportional to the size of the population at any time t.

We can express the death rate due to pesticides as:

$$\textbf{Death rate for prey } = \boldsymbol{\epsilon} \cdot \boldsymbol{x}(\boldsymbol{t})$$

where ϵ is a positive constant of proportionality (the sensitivity to pesticides).

Similarly, if $y(t)$ represents the predator population, the death rate for predators due to pesticides can be written as:

$$\textbf{Death rate for predators } = \boldsymbol{\epsilon} \cdot \boldsymbol{y}(\boldsymbol{t})$$

Thus, the change in the population due to natural growth and pesticide related deaths, we can incorporate this death term into a differential equation.

For the prey population $x(t)$, the rate of change of population including deaths due to pesticides is:

$$\frac{d\boldsymbol{x}}{d\boldsymbol{t}} = \textbf{ natural growth of } \boldsymbol{x} - \boldsymbol{\epsilon} \cdot \boldsymbol{x}$$

Similarly, for the predator population $y(t)$:

$$\frac{d\boldsymbol{y}}{d\boldsymbol{t}} = \textbf{natural growth of } \boldsymbol{y} - \boldsymbol{\epsilon} \cdot \boldsymbol{y}$$

Interpretations:

- The term $\boldsymbol{\epsilon} \cdot \boldsymbol{x}$ ***or*** $(\boldsymbol{\epsilon} \cdot \boldsymbol{y})$ implies that deaths scale linearly with the population size.
- This means that as the population grows, the number of deaths due to pesticides also increases proportionally, which is realistic because a larger population would result in more individuals being exposed to the pesticide.
- ϵ is positive because it represents a mortality factor. A positive ϵ reduces the population rather than increasing it.
- A higher ϵ corresponds to greater sensitivity to the pesticide, meaning that more individuals die for the same population size.

Proportionality:
Epsilon represents the sensitivity of each population to the effects of pesticides. This is biologically intuitive because larger populations will be more affected by pesticides, as more individuals are exposed and consequently face pesticide-related mortality.

Positive Proportionality:
Higher values of Epsilon imply a greater sensitivity to pesticides, leading to a lower growth rate for that species. This effectively reduces the population growth, simulating pesticide effects that reduce reproduction or increase mortality.

C.2. In case of ϵ >a:

If $\epsilon > a$, this means that the prey's sensitivity to pesticides (as represented by ϵ) is greater than its natural reproduction rate a. In other words, the pesticide effect is so strong that it outweighs the prey's ability to reproduce. As a result, the prey population will continually decline over time, leading to eventual extinction even in the absence of predators.

Proof:

Starting, let us take the Differential Equation for Prey Population that we have created in the C.1:

Given the prey population $x(t)$, the differential equation that accounts for intrinsic reproduction and pesticide-related deaths can be written as:

$$\frac{dx}{dt} = a \cdot x(t) - \epsilon \cdot x(t)$$

where:

- **$a \cdot x(t)$ is the natural reproduction term (positive growth]\),**
- **$\epsilon \cdot x(t)$ is the pesticide-related death term (negative growth).**

By Simplifying the Equation:

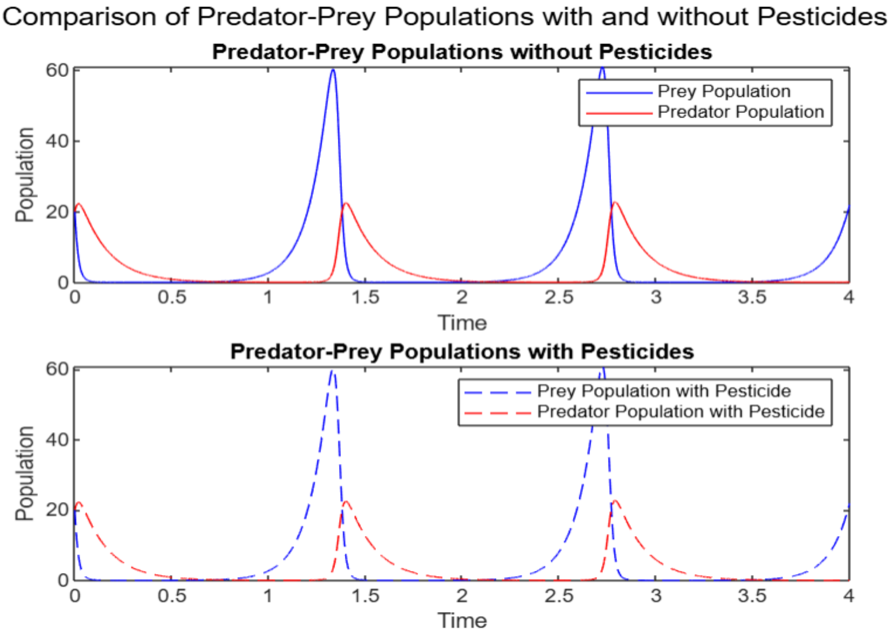
Factoring **out $x(t)$,** we get:

$$\frac{dx}{dt} = (a - \epsilon) \cdot x(t)$$

The trend of growth Rate:

- **If $\epsilon > a$, then $(a - \epsilon) < 0$, making the growth rate negative:**
$$\frac{dx}{dt} < 0$$
- **This implies that $x(t)$, the prey population will decrease over time because the term $(a - \epsilon\ )$ is negative.**

C.3. Graph if the S2 for the given initial conditions, comprising both With and without Epsilon (Pesticide sensitivity factor).

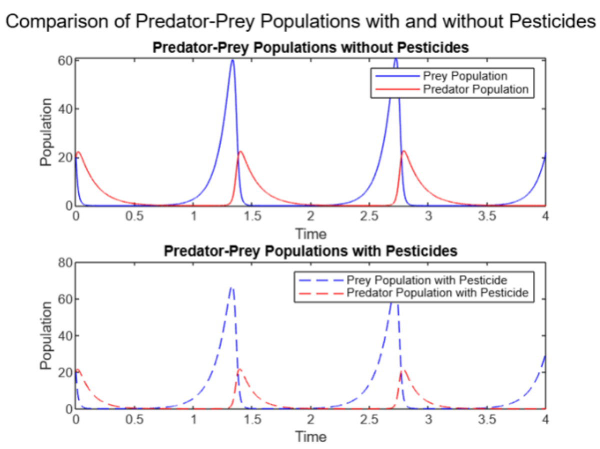


Interpretation:

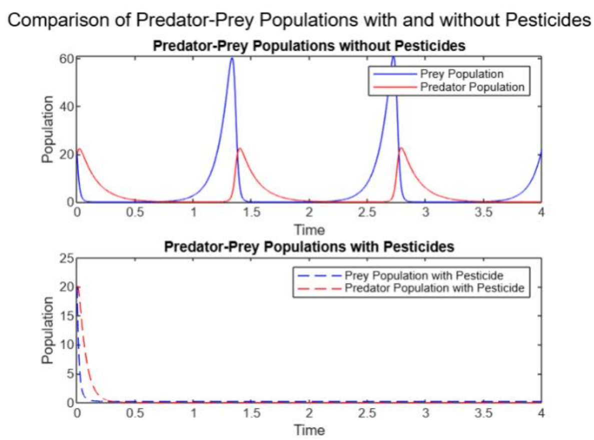
I. **Effect of Pesticides:** Pesticides reduce the amplitude of oscillations in both prey and predator populations, suggesting a damping effect that stabilizes or lowers the population sizes.

II. **Cycle Comparison:** The population peaks are lower and less frequent when pesticides are introduced, indicating that pesticides disrupt the natural predator-prey dynamics.

III. Even with a relatively small epsilon value, the graph above demonstrates the impact on the system. To further illustrate this effect, I have included examples with larger epsilon values, such as 2 and 10. Only then did I notice a significant difference in the results, such as the prey population declining first, followed by a slower decline in the predator population due to the reduced prey availability.



Epsilon = 2



Epsilon = 10

<u>PART – D</u>

D.1. Addition of the new term.

The modified model for the prey, **incorporating the added term, is given by:**

$$\boldsymbol{x'(t) = x\left(a - \frac{a \cdot x}{K} - b \cdot y\right)}$$

where:

- $x(t)$ is the prey population,
- a is the intrinsic growth rate of the prey,
- K is the carrying capacity (the maximum sustainable population for prey),
- $b \cdot y$ represents the predation term, where y is the predator population is.

Interpretation:

- The term $\frac{a \cdot x}{K}$ reduces the prey's growth rate as the prey population x approaches the carrying capacity K.
- When x is small compared to K, the effect of this term is minimal, and the prey population can grow nearly exponentially.
- As x approaches K, the term $\frac{a \cdot x}{K}$ approaches a, which reduces the overall growth rate to zero, preventing the prey population from exceeding K.

Therefore, adding this term ensures that the prey population $\boldsymbol{x(t)}$ theoretically cannot exceed the carrying capacity $\boldsymbol{K}$, as the growth rate approaches zero as $\boldsymbol{x}$ nears $\boldsymbol{K}$. This reflects the natural limitation in ecosystems, where resources are finite, and population growth is constrained.

D.2. Equilibrium points and the biological sense.

To find the equilibrium points, we set $\boldsymbol{x'(t) = 0}$ **and** $\boldsymbol{y'(t) = 0}$**.**

Case 1: Prey Equation:

$$\boldsymbol{x'(t) = x\left(a - \frac{a \cdot x}{K} - b \cdot y\right) = 0}$$

This yields two cases:

- $\boldsymbol{x = 0}$ **:** The prey population is extinct.
- $\boldsymbol{a - \frac{a \cdot x}{K} - b \cdot y = 0}$**which simplifies** to:

$$\boldsymbol{y = \frac{a}{b} - \frac{a \cdot x}{b \cdot K}}$$

Case 2: Predator Equation:

$$\boldsymbol{y'(t) = y(d \cdot x - c) = 0}$$

This gives two cases:

- $\boldsymbol{y = 0}$ **:** The predator population is extinct.
- $\boldsymbol{d \cdot x = c}$**which simplifies** to:

$$\boldsymbol{x = \frac{c}{d}}$$

Solving for Equilibrium Points:

By combining cases 1 and 2, we get:

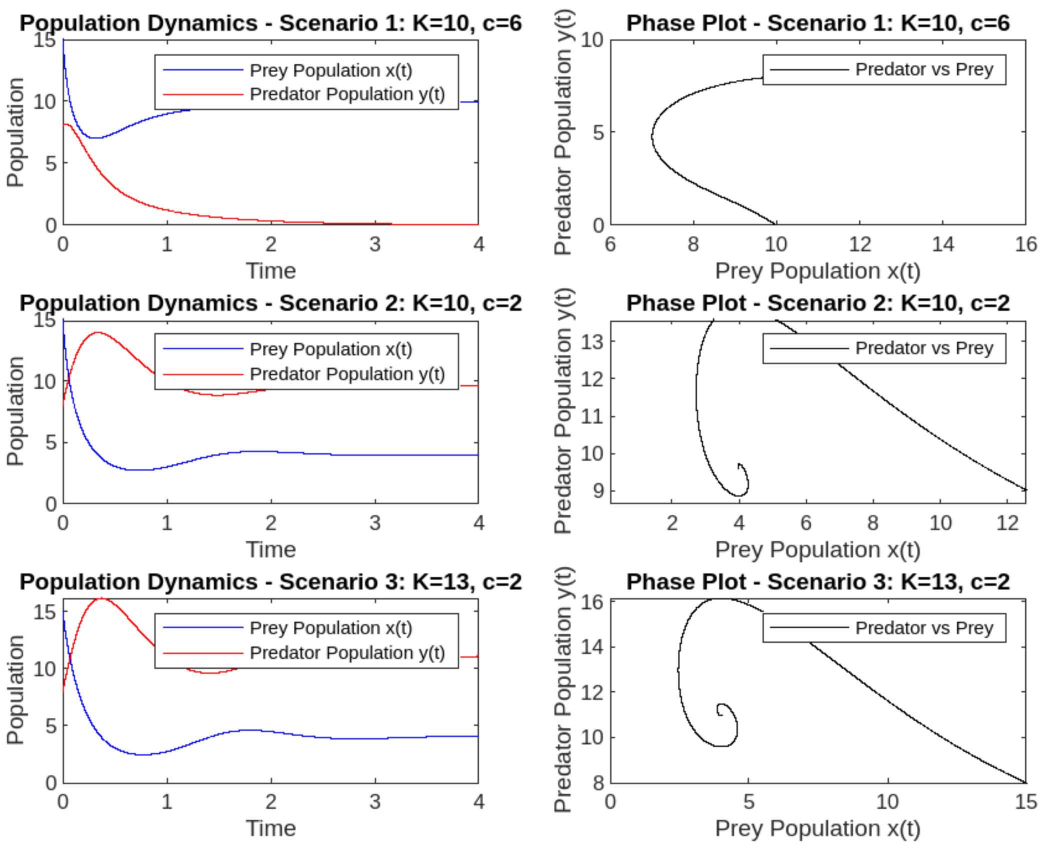
1. $\boldsymbol{(x, y) = (0, 0)}$ **:** Extinction of both prey and predators.
2. $\boldsymbol{(x, y) = (K, 0)}$ **:** Prey population reaches carrying capacity K, but there are no predators.
3. $\boldsymbol{(x, y) = \left(\frac{c}{d}, \frac{a}{b}\left(1 - \frac{c}{K \cdot d}\right)\right)}$ **:** A balance point where both prey and predator populations coexist.

Biological Interpretation:

- **Point (0,0):** Complete extinction of both populations, often an unstable equilibrium.
- **Point** $\boldsymbol{(K, 0)}$: The prey population reaches its maximum sustainable level without any predators, as the environment can support prey but not predators.
- **Point** $\boldsymbol{\left(\frac{c}{d}, \frac{d}{b}\left(1 - \frac{c}{K \cdot d}\right)\right)}$: Both populations coexist in a balance, with prey below carrying capacity and a stable predator population that the prey population can sustain.

D.3. and D.4.:

Graph generated using the S3 system equations in MATLAB:



Interpretation:

The plots show how the predator-prey dynamics evolve over time under different scenarios with varying carrying capacities (K) and predator mortality rates (c). Each scenario has time series and phase plots, indicating the interactions between the prey population (x) and predator population (y).

Scenario 1:

- The prey population rapidly decreases initially and then stabilizes at a low value. The predator population similarly declines, reaching near-zero, indicating a lack of sufficient prey for survival.
- Shows a path where the predator population rapidly declines as prey levels fall, eventually reaching an equilibrium at very low population levels for both species.
- **Equilibrium Point**: The system moves toward a state where both prey and predator populations stabilize close to zero.

Scenario 2:

- The prey population initially decreases, then stabilizes at a mid-level due to the lower predator mortality rate ($c = 2$). The predator population, after an initial increase, also stabilizes around a moderate level.
- Shows a spiral pattern, indicating an oscillatory convergence towards a stable equilibrium point.
- **Equilibrium Point**: A stable equilibrium with moderate population levels for both prey and predators, suggesting a balance between the species at the given carrying capacity.

Scenario 3:

- The prey population initially declines, followed by recovery and stabilization at a higher level than in Scenario 2 due to the higher carrying capacity ($K = 13$). The predator population mirrors this, stabilizing at a relatively higher level than Scenario 2.
- Exhibits a spiral pattern with wider oscillations than Scenario 2, settling towards a higher equilibrium point.
- **Equilibrium Point**: A stable equilibrium at higher population levels for both species, indicating that the higher carrying capacity allows for a larger sustainable predator and prey population.

Proposed Further Simulations and Effects of Deforestation:

To simulate the effects of deforestation, we can gradually reduce the carrying capacity K and observe the impacts on the predator-prey dynamics, especially when K becomes less than c. This situation models an environment where resources for prey are critically limited, potentially destabilizing the ecosystem.

Hypothesis for Simulations with Lower K (deforestation scenario):

- **If $K < c$**: The carrying capacity of the environment for prey becomes too low to sustain a viable population. This scenario would likely lead to a rapid decline in the prey population, followed by a collapse in the predator population due to lack of food resources.

- **Expected Dynamics**: We would expect both the prey and predator populations to spiral towards extinction or very low, unsustainable levels. The phase plot should show a trajectory that approaches zero for both populations.

Suggested Parameters for Simulation

1. **Moderate Deforestation**: K=5, c=2
   - This would test how the population dynamics adjust when the carrying capacity is halved but still above the predator mortality rate.

2. **Severe Deforestation**: K=2, c=2
   - Here, the carrying capacity matches the predator mortality rate, which could destabilize the ecosystem.

3. **Critical Deforestation**: K=1, c=2
   - This models an environment on the brink of collapse, with K less than c.

Discussion of Expected Results

If we observe the simulations under these conditions:

- **Scenario where K > c**: Populations may stabilize at lower levels, but some equilibrium can still be reached.
- **Scenario where K = c**: This represents a critical point where the prey population struggles to sustain the predators, possibly leading to cyclic fluctuations with increasingly smaller amplitudes or even extinction.
- **Scenario where K < c**: The environment cannot support a sufficient prey population for predator survival, potentially leading to an extinction vortex, where both species decline to unsustainable levels.

Thus, it is significant that drastic reductions in **K** from deforestation or habitat loss could destabilize predator-prey relationships, leading to population crashes or extinctions, especially if the environment cannot recover sufficiently.