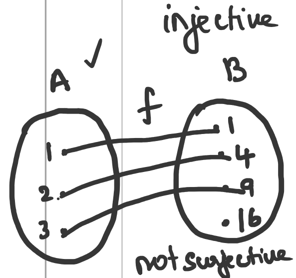
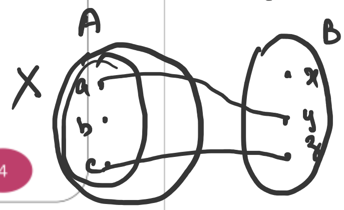
Functions

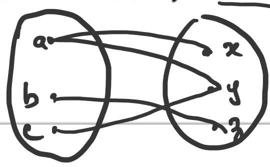
- Let f be a function from X to Y, Now we say that X is a domain of f and Y is the co-domain of f. If f(x) = y, we say that y is the image of x and x is a preimage of y.
- The range of f is the set of all images of elements of Y. It is denoted by R_f and defined by R_f = {y/there exist x such that X and y=f(x)}
- Eg: 1.Consider the two sets A={1,2,3}, B={1,4,9,16} Let f: A→B defined by f(x)= x^2; f = {(1,1), (2,4), (3,9)} ✓
- Eg.2 Let A={a,b,c}, B={x,y,z}. Then f: A→ B is defined by
  - f(a)=y, f(c)=z
  - f(a)=x, f(a)=y, f(b)=z, f(c)=y. Check whether it is a function

injective

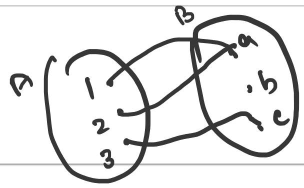


not surjective





f(a) = x, y X



neither injective not surjective

$x \neq y \quad f(x) \neq f(y)$

$f(x) = f(y) \Rightarrow x = y$

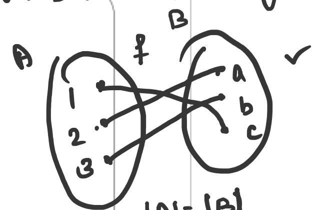
Classification of Functions

- **One-to-one Function:**

  A function f: X→Y is said to be one-one or 1-1 or (injective) if distinct elements in X have distinct images in Y under f.

  f is 1-1 then x≠y → f(x) ≠ f(y) ; x,y ∈ X or equivalently

  $\boxed{f(x) = f(y) \rightarrow x=y \; ; \; x,y \in X}$

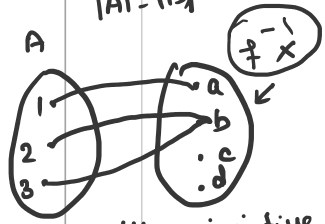


|A| = |B|

- **Onto Function:**

  A function f: X→Y is said to be onto (surjective) if every element of Y has a pre-image in X, (i.e) if the range of f=Y then f is onto

f^{-1} X

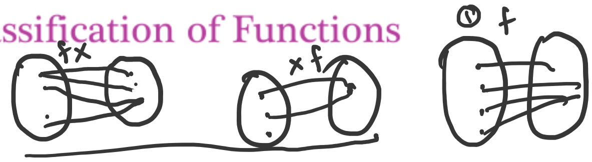


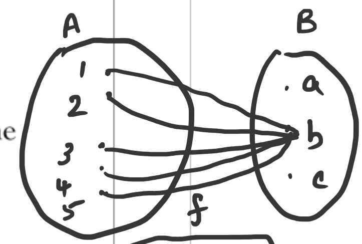
neither injective nor surjective

- **Bijective Function:**

  If f:X→Y is both 1-1 and onto(i.e both injective and surjective) then f is said to be 1-1 correspondence or bijective function.

Classification of Functions





$\boxed{f_R = \{b\}}$

❖ **Constant Function:**

A function f: A→ B is called a constant function if every element of A has the same image in B, (ie) f(x) = c where c is constant.
The range of the function is a singleton.

❖ **Inverse Function:**

If f: X →Y be a bijection. Then for each y ∈Y, there exists a unique element x ∈X such that f(x) =y. We now define $f^{-1}: Y \rightarrow X \; by f^{-1}(y) = x. \; f^{-1}$ is called the inverse of a function f. It is also called as invertible function.

$f : x \rightarrow y$

$x \quad f^{-1} : Y \rightarrow x$

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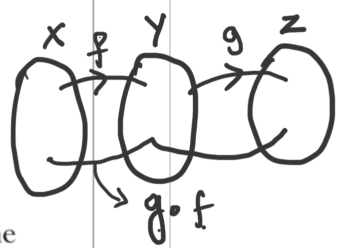
Composition of Functions



❖ Let f: X→Y and g:Y → Z be two functions. We define the composite of these functions $g \cdot f : X \rightarrow Z$ by the rule $g \cdot f$ (x) = g(f(x)) for all x ∈ X.

❖ Note : $X \rightarrow Y$

- ❖ The composition $g \cdot f$ cannot be defined unless the range of f is a subset of the domain of g .
- ❖ In general $g \cdot f \neq f \cdot g$ not commuative
- ❖ $f \cdot$g means f is acting on g

$a + b = b + a$

$2+3 = 3+2$

$\sin^2 x$

❖ Example:

- ❖ f: R→ R is given by f(x) = x^2
- ❖ g: R → R is given by g(x) = sinx
- ❖ f.g(x)=f(g(x))=f(sinx)= $(\sin x)^2$
- ❖ g.f(x) = g(f(x))=g(x^2) = sin x^2

$f \circ g(x) = f(g(x))$
$= f(\sin x) = (\sin x)^2$

$g \circ f(x) = g(f(x)) = g(x^2) = \sin(x^2)$

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$f \circ g \neq g \circ f$