Harmonic Analysis.

(circular form of Fourier Series)

When a function $f(x)$ is given by the numerical values at 'q' equally spaced points, the coefficients in the Fourier Series representing $f(x)$ can be obtained by numerical integration.

The reqd Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{(or)}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Expanding we get,

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + (a_3 \cos 3x + b_3 \sin 3x) + \dots\dots$$

Here $a_1 \cos x + b_1 \sin x$ is called fundamental harmonic or first harmonic.

$a_2 \cos 2x + b_2 \sin 2x$ is called second harmonic.

$a_3 \cos 3x + b_3 \sin 3x$ is called 3rd harmonic & so on

where $a_0 = \frac{2}{q} \sum f(x)$, q is the no. of ordinates gn in the pblm.

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In otherwords

$a_0 = 2$ (mean value of $f(x)$)

$$a_1 = \frac{2}{q} \Sigma f(x).\cos x.$$

$$a_2 = \frac{2}{q} \Sigma f(x) \cos 2x$$

$$a_3 = \frac{2}{q} \Sigma f(x) \cos 3x$$

$$\vdots$$

$$b_1 = \frac{2}{q} \Sigma f(x) \sin x$$

$$b_2 = \frac{2}{q} \Sigma f(x) \sin 2x$$

$$b_3 = \frac{2}{q} \Sigma f(x) \sin 3x$$

$$\vdots$$

Problem

Calculate the first three harmonics of the Fourier series of $f(x)$ given by the following table.

x:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
$f(x)$:	1	1.4	1.9	1.7	1.5	1.2

S.No x	$f(x)$	$\cos x$	$\cos 2x$	$\cos 3x$	$\sin x$	$\sin 2x$	$\sin 3x$
0	1	1	1	0	0	0	0
$\frac{\pi}{3} = 60^\circ$	1.4	$1/2$	$-1/2$	-1	0.866	$.866$	0
$\frac{2\pi}{3} = 120^\circ$	1.9	$-1/2$	$-1/2$	1	$-.866$	-0.866	0
$\pi = 180^\circ$	1.7	-1	1	-1	0	0	0
$\frac{4\pi}{3} = 240^\circ$	1.5	$-1/2$	$-\frac{1}{2}$	1	$-.866$	$.866$	0
$5\pi/3 = 300^\circ$	1.2	$1/2$	$-1/2$	-1	$-.866$	$-.866$	0

Now, The reqd F.S is

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + (a_3 \cos 3x + b_3 \sin 3x)$$

$q = 6$

$$a_0 = \frac{2}{q} \sum f(x)$$

$$= \frac{2}{6}(1 + 1.4 + 1.9 + 1.7 + 1.5) = 2.9$$

$$a_1 = \frac{2}{q} \sum f(x).\cos x = \frac{2}{6}(1\times1 + 1.4\times.5 + 1.9(-.5) + 1.7\times1 + 1.5\times(-.5) + 1.2\times.5)$$

$$= -0.37$$

$$a_2 = \frac{2}{q} \sum f(x) \cos 2x = \frac{2}{6}(1\times1 + 1.4\times-.5 + 1.9\times-.5 + 1.7\times1 + 1.5(-.5) + 1.2(-.5))$$

$$a_2 = -0.1$$

$$a_3 = \frac{2}{9} \sum f(x)\cos 3x$$

$$= \frac{2}{6}\left\{1(1) + 1.4(-1) + 1.9\times 1 + 1.7\times(-1) + 1.5(1) + 1.2(-1)\right\}$$

$$= 0.03.$$

$$b_1 = \frac{2}{9} \sum f(x)\sin x$$

$$= \frac{2}{6}\left\{1\times 0 + 1.4\times .866 + 1.9\times .866 + 1.7\times 0 + 1.5\times -.866 + 1.2\times -.866\right\}$$

$$= 0.17$$

$$b_2 = \frac{2}{9} \sum f(x)\sin 2x$$

$$= \frac{2}{6}\left\{1\times 0 + 1.4\times .866 + 1.9(-.866) + 1.7\times 0 + 1.5\times .866 + 1.2\times -.866\right\}$$

$$= -0.06.$$

$b_3 = \frac{2}{9} \sum f(x) \sin 3x$

$= 0 \quad (\because \sin 3x = 0 \ \forall x.$

$\therefore$ The reqd F.S is:

$f(x) = \frac{2.9}{2} + (-.3\cancel{7} \cos x + 0.17 \sin x) + (-.1 \cos 2x - 0.06 \sin 2x) + 0.03 \cos 3x \quad (\because b_3 = 0)$

$a_0 \rightarrow 2(\text{mean of } f(x))$

$a_1 \rightarrow 2(\text{mean of } f(x) \cos x)$

$a_2 \rightarrow 2(\text{mean of } f(x) \cos 2x)$

$a_3 = 2(\text{mean of } f(x) \cancel{\cos} 3x$

$b_1 = 2$ (mean of $f(x)\sin x$)

$b_2 = 2$ (mean of $f(x)\sin 2x$)

$b_3 = 2$ (mean of $f(x)\sin 3x$).

$a_1 \cos x + b_1 \sin x \rightarrow$ first / fundamental harmonic

$a_2 \cos 2x + b_2 \sin 2x \rightarrow$ 2nd harmonic

$a_3 \cos 3x + b_3 \sin 3x \rightarrow$ 3rd harmonic.