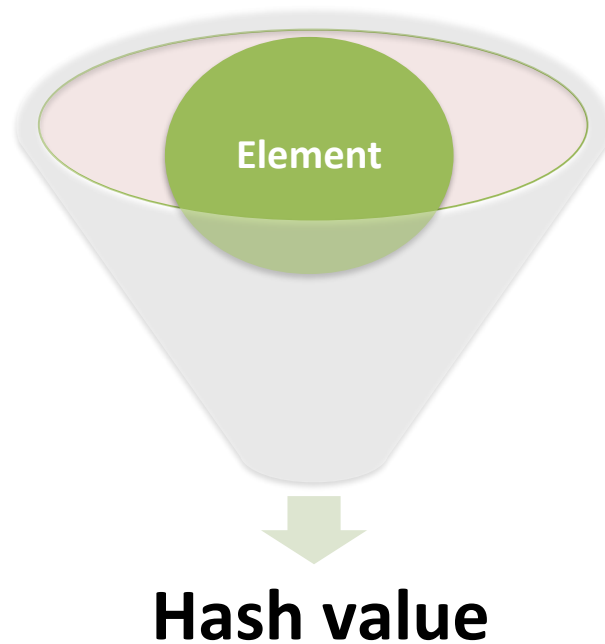


Hashing



Hashing allows user to insert/delete/find records in constant average time.

Linear Search in **arrays and linked lists**

Search : 70

20	40	50	70	80	90	10
----	----	----	----	----	----	----

0	1	2	3	4	5	6
---	---	---	---	---	---	---

Linear Search

- We need to search in a **linear fashion**, which is costly in practice.



Time complexity of linear search is **$O(N)$**

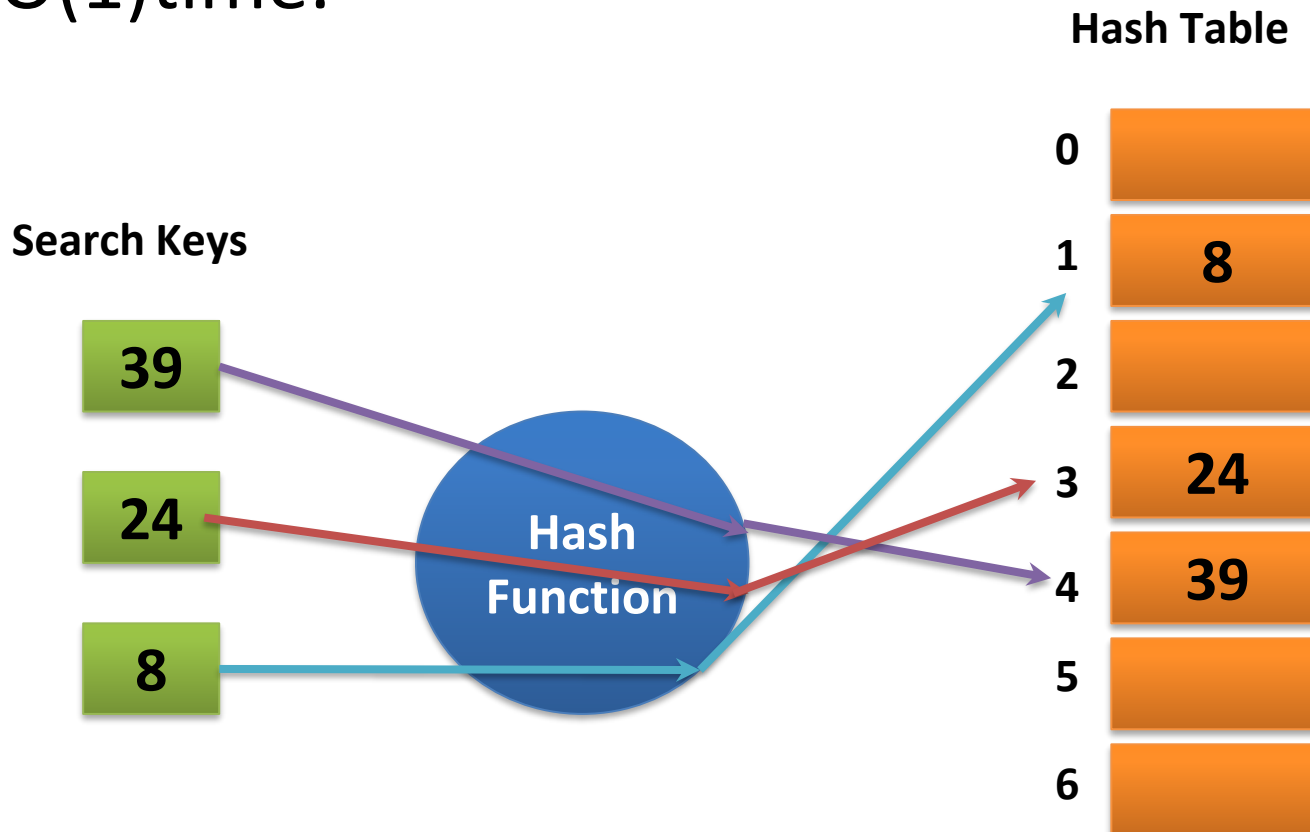
Binary Search

If we keep the data sorted, then it can be searched in **$O(\log n)$** time using Binary Search, but **insert and delete operations become costly** as we have to maintain sorted order.

Hashing



- Efficient data structure that can be searched in $O(1)$ time.



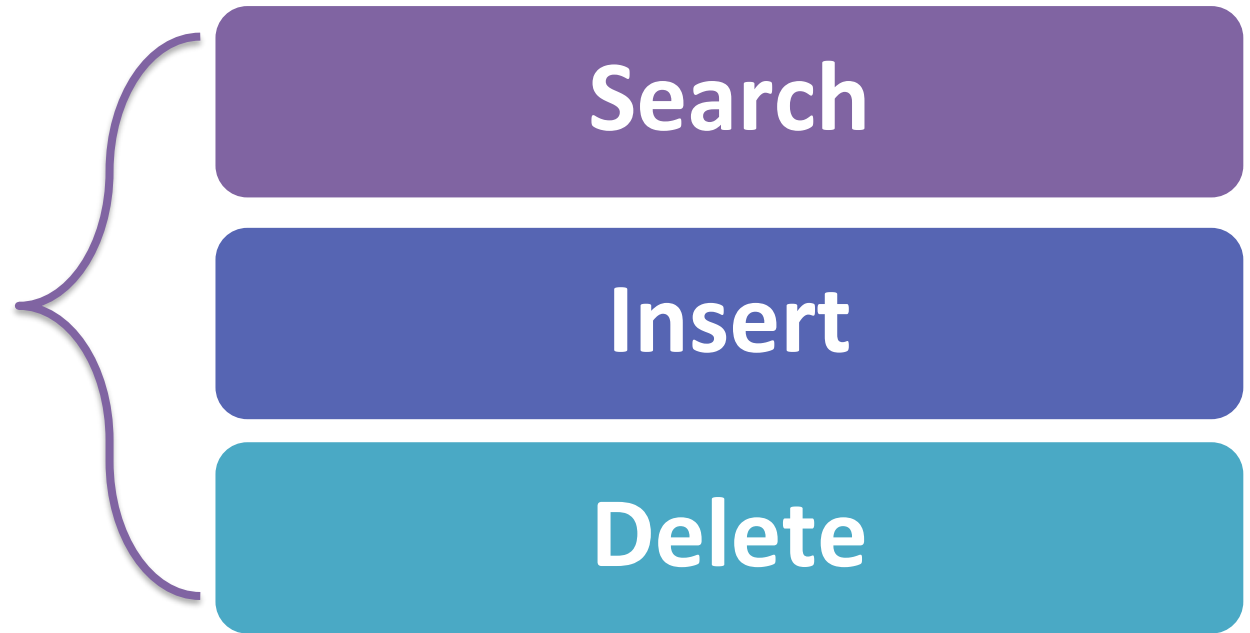


Hashing

- **Hashing** is the process of mapping the data item to a **hash table** with the help of a **hashing function**.
- A **hash table** is a collection of items which are stored in such a way as to make it easy to find them later.
- **hash function** to compute an index so that a data can be stored at a specific location in a table such that it can easily be found.
- With hashing we get **$O(1)$ search time** on average (under reasonable assumptions) and $O(n)$ in worst case.

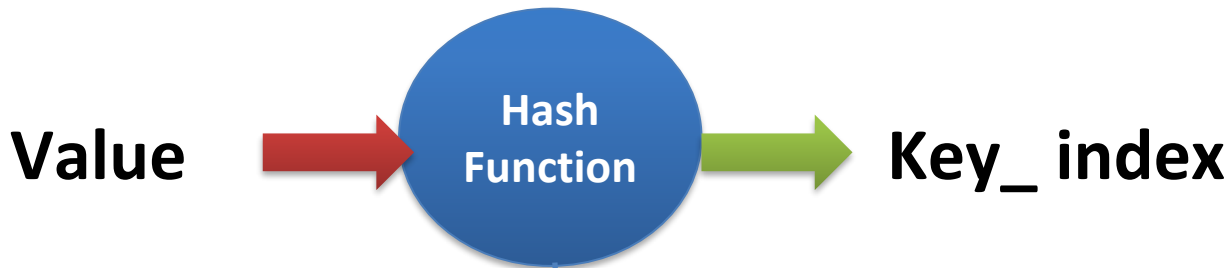
Hashing - Operations

$O(1)$
on average case



Hash Function

Hash Table size : 7



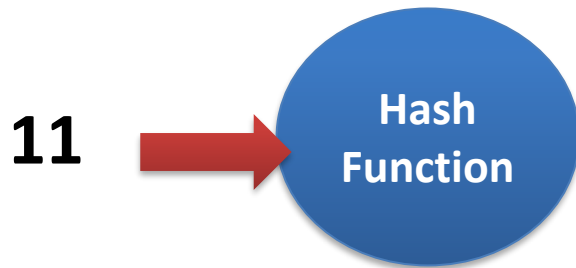
$\text{Key_index} = \text{Value} \% \text{Table size}$

Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

Hash Table

Search : 11

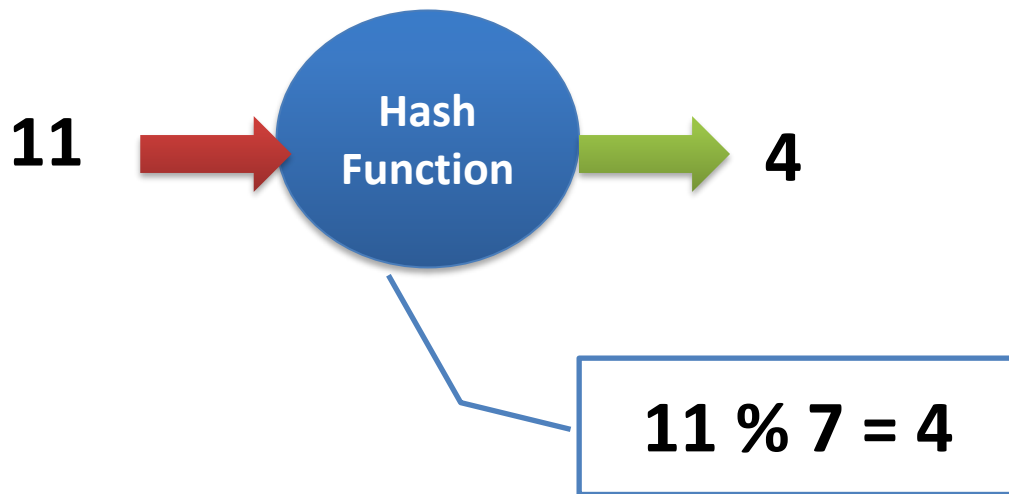


Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

Hash Table

Search : 11

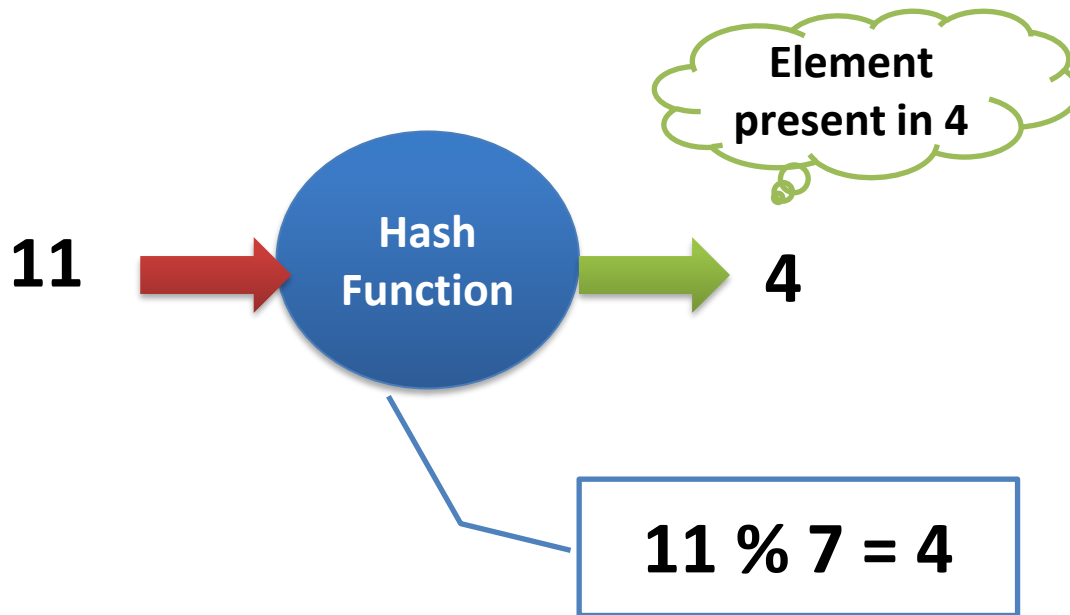


Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

Hash Table

Search : 11

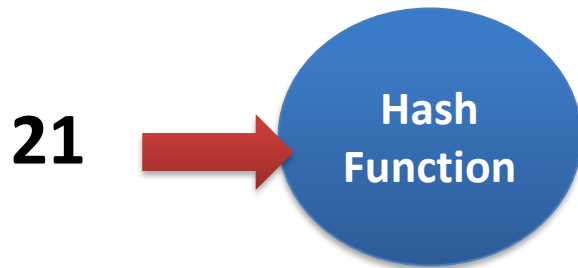


Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

Hash Table

Insert : 21

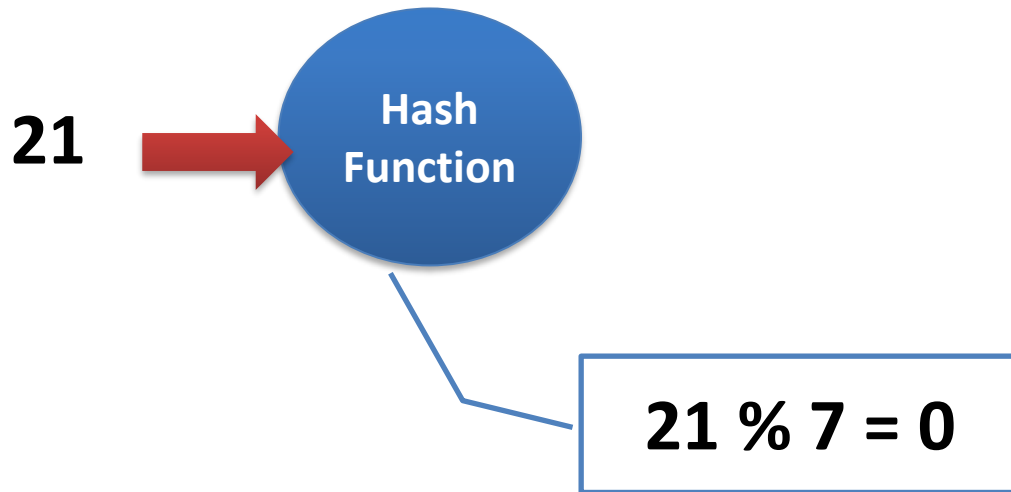


Hash Table

0	-
1	-
2	-
3	-
4	-
5	-
6	-

Hash Table

Insert : 21

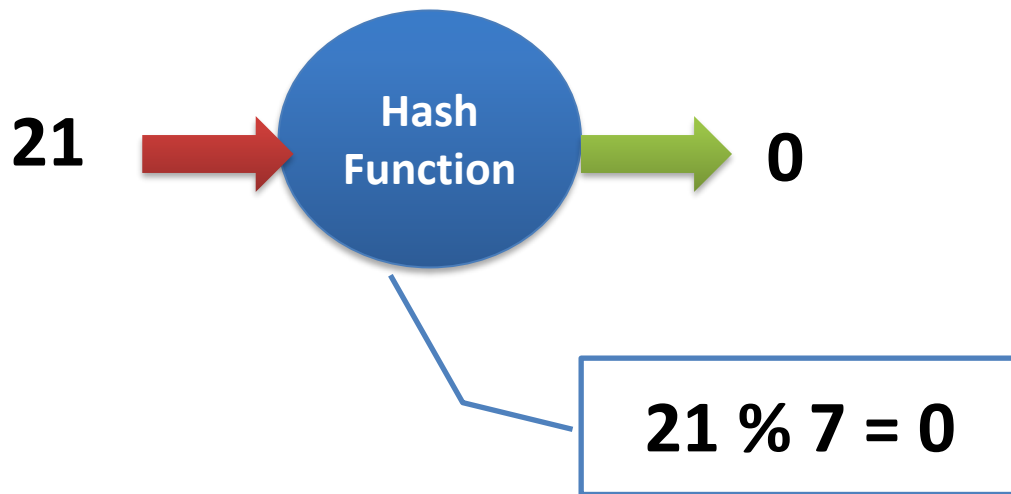


Hash Table

0	-
1	-
2	-
3	-
4	-
5	-
6	-

Hash Table

Insert : 21

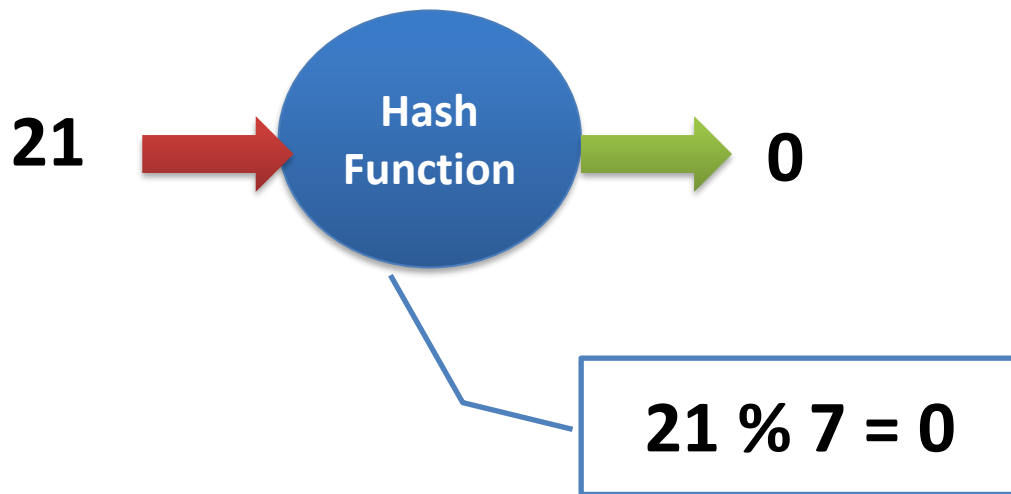


Hash Table

0	-	Free
1	-	
2	-	
3	-	
4	-	
5	-	
6	-	

Hash Table

Insert : 21

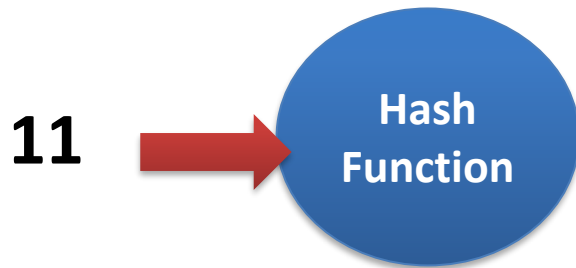


Hash Table

0	21
1	-
2	-
3	-
4	-
5	-
6	-

Hash Table

Insert : 11

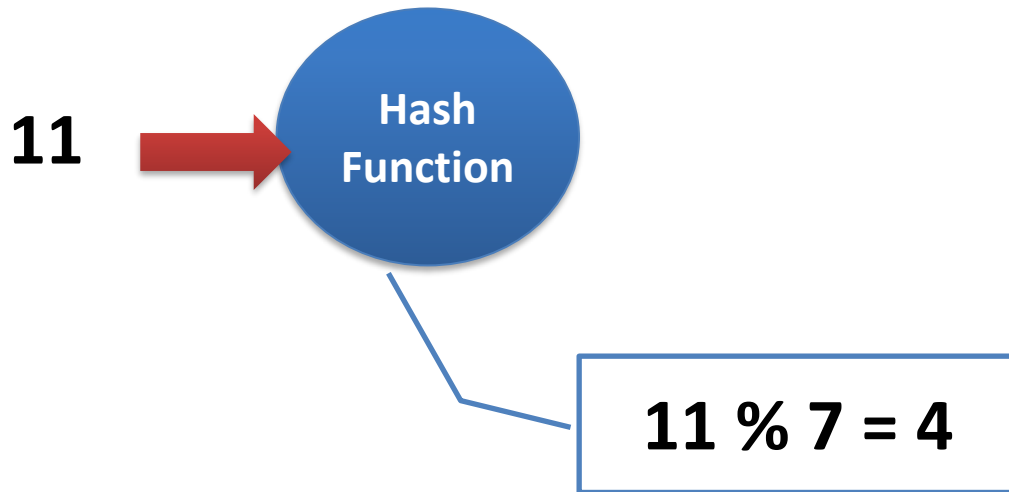


Hash Table

0	21
1	-
2	-
3	-
4	-
5	-
6	-

Hash Table

Insert : 11

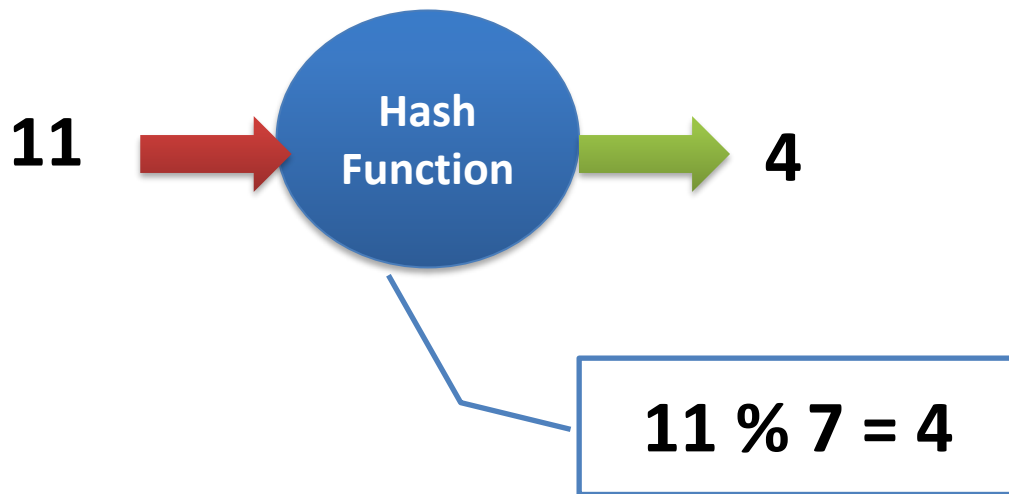


Hash Table

0	21
1	-
2	-
3	-
4	-
5	-
6	-

Hash Table

Insert : 11



Hash Table

0	21
1	-
2	-
3	-
4	11
5	-
6	-

Hash Table

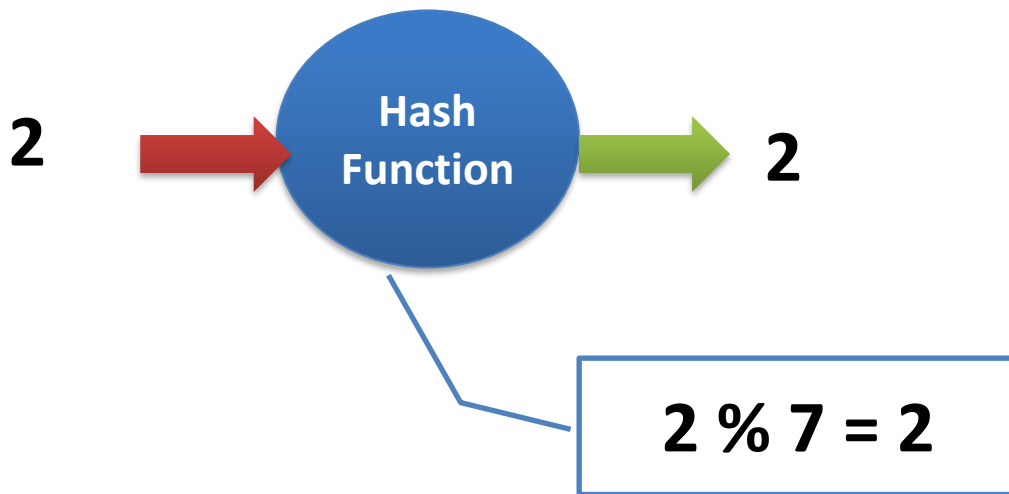
Insert : 2

Hash Table

0	21
1	-
2	-
3	-
4	11
5	-
6	-

Hash Table

Insert : 2



Hash Table

0	21
1	-
2	2
3	-
4	11
5	-
6	-

Hash Table

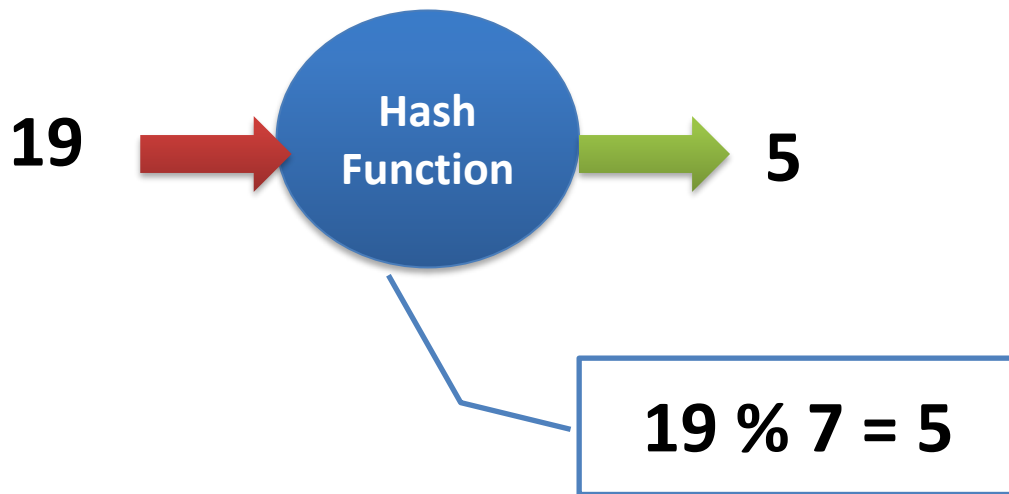
Insert : 19

Hash Table

0	21
1	-
2	2
3	-
4	11
5	-
6	-

Hash Table

Insert : 19



Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	-

Hash Table

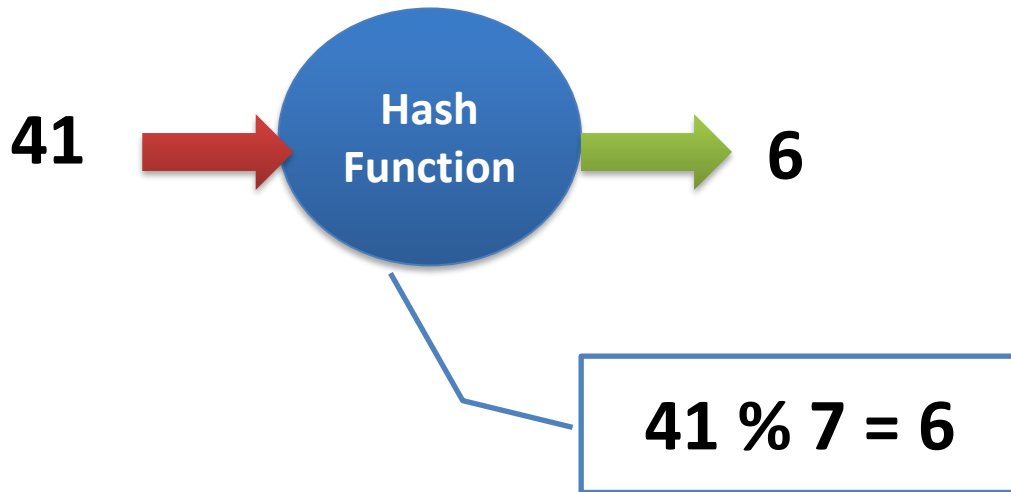
Insert : 41

Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	-

Hash Table

Insert : 41



Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

Hash Table

Load factor = $\frac{\text{Number of slots are now occupied}}{\text{Table size}}$

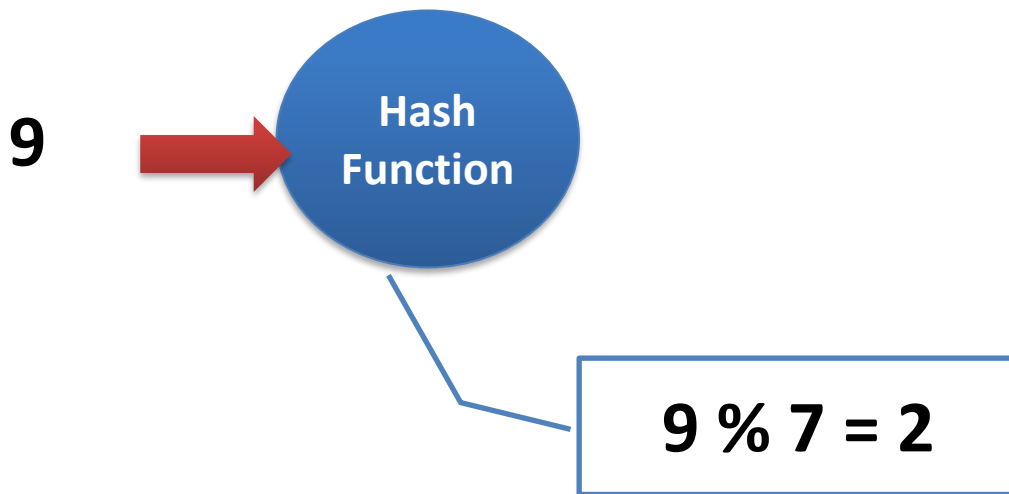
= 5 / 7

Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

linear probing

Insert : 9

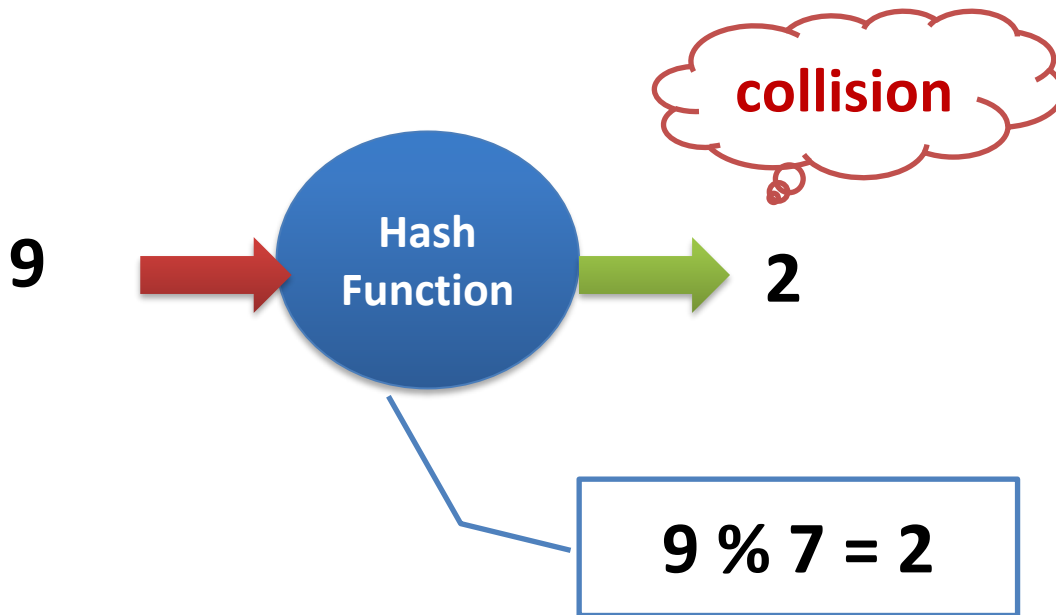


Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41

linear probing

Insert : 9



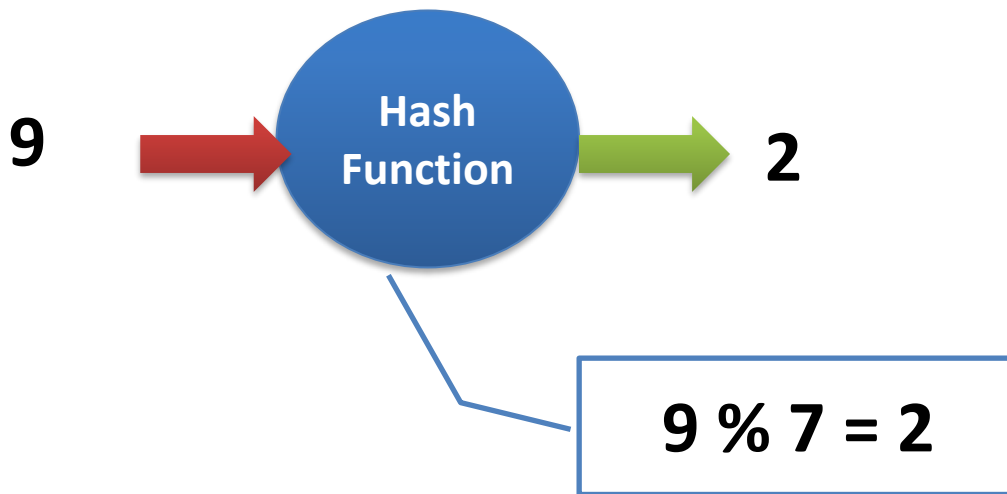
Hash Table

0	21
1	-
2	2
3	-
4	11
5	19
6	41



linear probing

Insert : 9



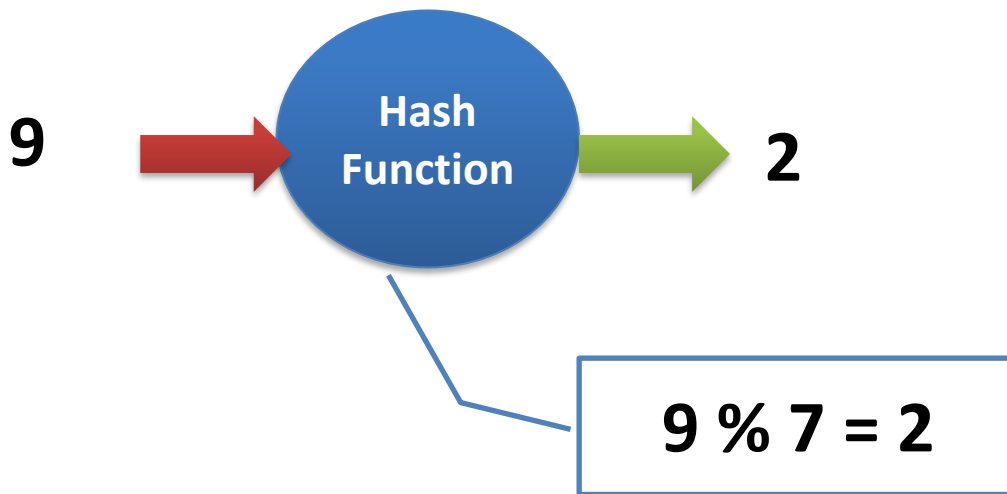
Hash Table

0	21	
1	-	
2	2	
3	-	free
4	11	
5	19	
6	41	

looking for the next available location $i + 1 =$
 $2 + 1$

linear probing

Insert : 9



Hash Table

0	21
1	-
2	2
3	9
4	11
5	19
6	41

next location $2+1=3$ is free. Insert 9 at location 3.

Collision Resolution Techniques

- Collision: $\text{hash}(x) = \text{hash}(y)$ for two different keys x and y .

- Collision Resolution techniques:

- Closed Hashing or Open Addressing

- Linear Probing
- Quadratic Probing
- Double Hashing

- Open Hashing:

- Separate Chaining

Open Addressing

- Linear Probing:
 - $(\text{hash}(\text{key}) + 1) \% \text{hashTableSize}, (\text{hash}(\text{key}) + 2) \% \text{hashTableSize}, \text{etc}$
- Quadratic Probing:
 - $(\text{hash}(\text{key}) + 1^2) \% \text{hashTableSize}, (\text{hash}(\text{key}) + 2^2) \% \text{hashTableSize}, \text{etc}$
- Double Hashing:
 - $(\text{hash1}(\text{key}) + i * \text{hash2}(\text{key})) \% \text{hashTableSize}$

Search - Procedure

1. Find the hash value of the element to be found.

position = **Element** % table_size

2. If the **hash_table[**position**] == **Element**** then,
element is found in the position.

Else

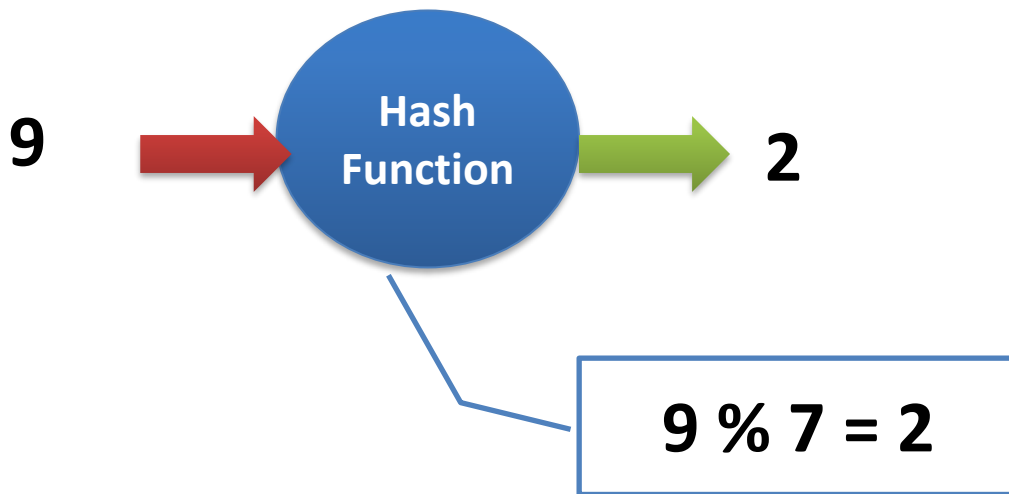
Step through an array one item at a time looking for a desired item.

The search **stops** when the item **is found** or when it **find any null value** or when the search has **examined each item without success.**

Hashing

search : 9

1. Find the hash value of the element to be found.



Hash Table

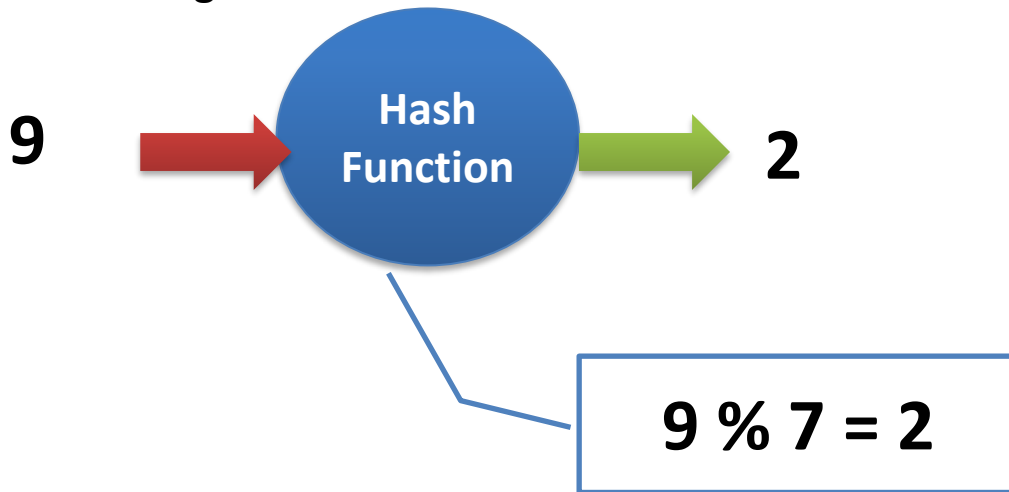
0	21
1	-
2	2
3	9
4	11
5	19
6	41

Hashing

search : 9

If (**hash_table[position] != Element**) then

Step through an array one item at a time
looking for a desired item.



Hash Table

0	21
1	-
2	2
3	9
4	11
5	19
6	41

\neq 9

Hashing

search : 9

If (hash_table[position] != Element) then

Step through an array one item at a time
looking for a desired item.

9



Hash
Function



2

Element
present in 3

$$9 \% 7 = 2$$

Hash Table

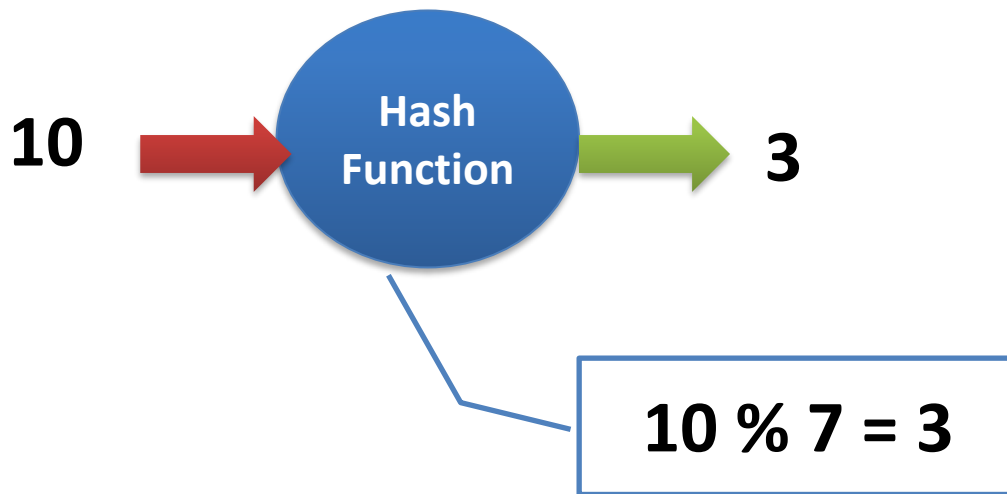
0	21
1	-
2	2
3	9
4	11
5	19
6	41

= 9

Hashing

search : 10

1. Find the hash value of the element to be found.



Hash Table	
0	21
1	-
2	2
3	9
4	11
5	19
6	41

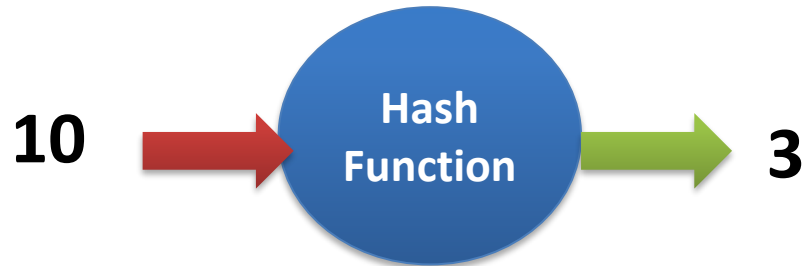
≠ 10

2. If (`hash_table[position] != Element`)then

Step through an array one item at a time looking for a desired item.

Hashing

search : 10



Step through an array one item at a time looking for a desired item.

looking for the next slot $3 + 1 = 4$

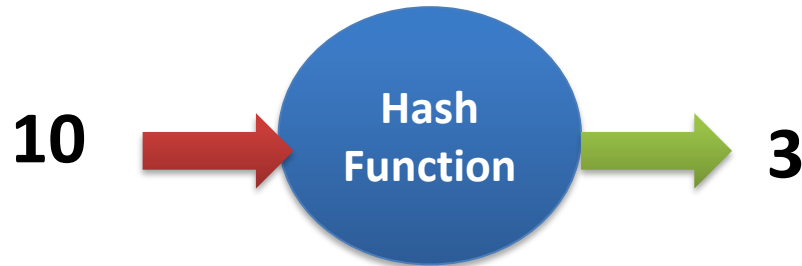
The search **stops** when the item **is found** or when it **find any null value** or when the search has **examined each item without success**.

Hash Table

0	21
1	-
2	2
3	9
4	11 \neq 10
5	19
6	41

Hashing

search : 10



Step through an array one item at a time looking for a desired item.

looking for the next slot $4 + 1 = 5$

The search **stops** when the item **is found** or when it **find any null value** or when the search has **examined each item without success**.

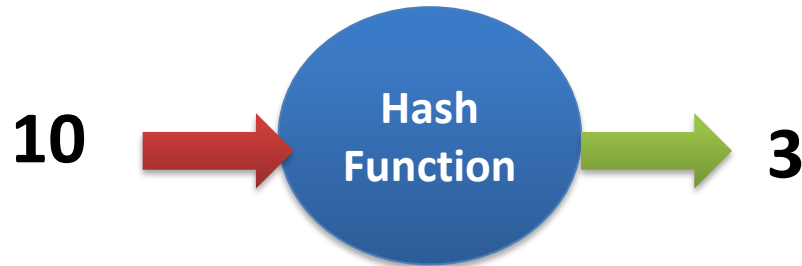
Hash Table

0	21
1	-
2	2
3	9
4	11
5	19
6	41

\neq 10

Hashing

search : 10



Step through an array one item at a time looking for a desired item.

looking for the next slot $5 + 1 = 6$

The search **stops** when the item **is found** or when it **find any null value** or when the search has **examined each item without success**.

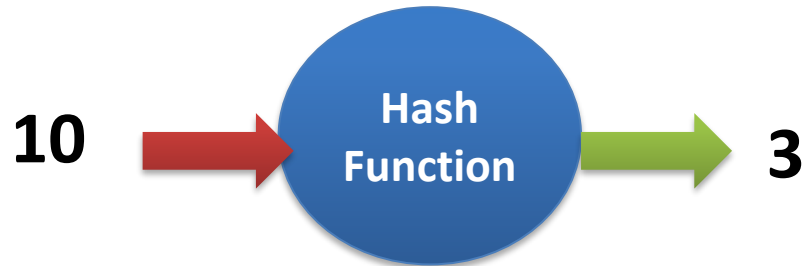
Hash Table

0	21
1	-
2	2
3	9
4	11
5	19
6	41

≠ 10

Hashing

search : 10



Step through an array one item at a time looking for a desired item.

looking for the next slot $6 + 1 = 7$ ✗
 $(6+1) \bmod 7 = 0$

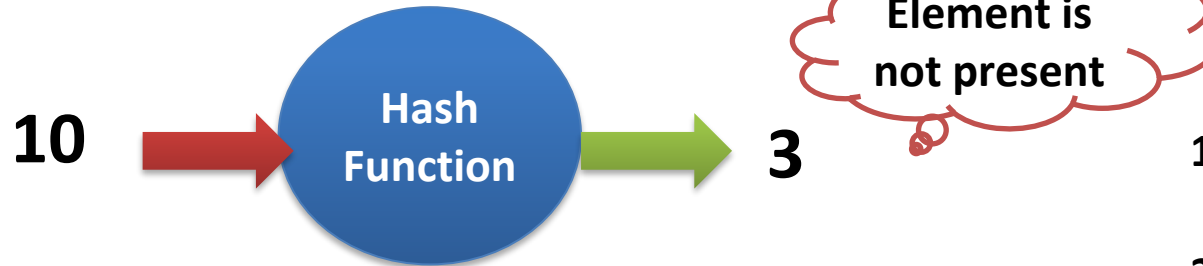
The search **stops** when the item **is found**
or when it **find any null value** or when the
search has **examined each item without
success.**

Hash Table

0	21	≠ 10
1	-	
2	2	
3	9	
4	11	
5	19	
6	41	

Hashing – unsuccessful search

search : 10



Step through an array one item at a time looking for a desired item.

The search **stops** when the item **is found** or when it **find any null value** or when the search has **examined each item without success.**

Hash Table

1	21	\neq 10 $=$ -
1	-	
2	2	Search process stops
3	9	
4	11	
5	19	
6	41	

Quadratic Probing

Let $\text{hash}(x)$ be the slot index computed using the hash function.

- If the slot $\text{hash}(x) \% S$ is full, then we try $(\text{hash}(x) + 1*1) \% S$.
- If $(\text{hash}(x) + 1*1) \% S$ is also full, then we try $(\text{hash}(x) + 2*2) \% S$.
- If $(\text{hash}(x) + 2*2) \% S$ is also full, then we try $(\text{hash}(x) + 3*3) \% S$.
- This process is repeated for all the values of i until an empty slot is found.

Separate chaining

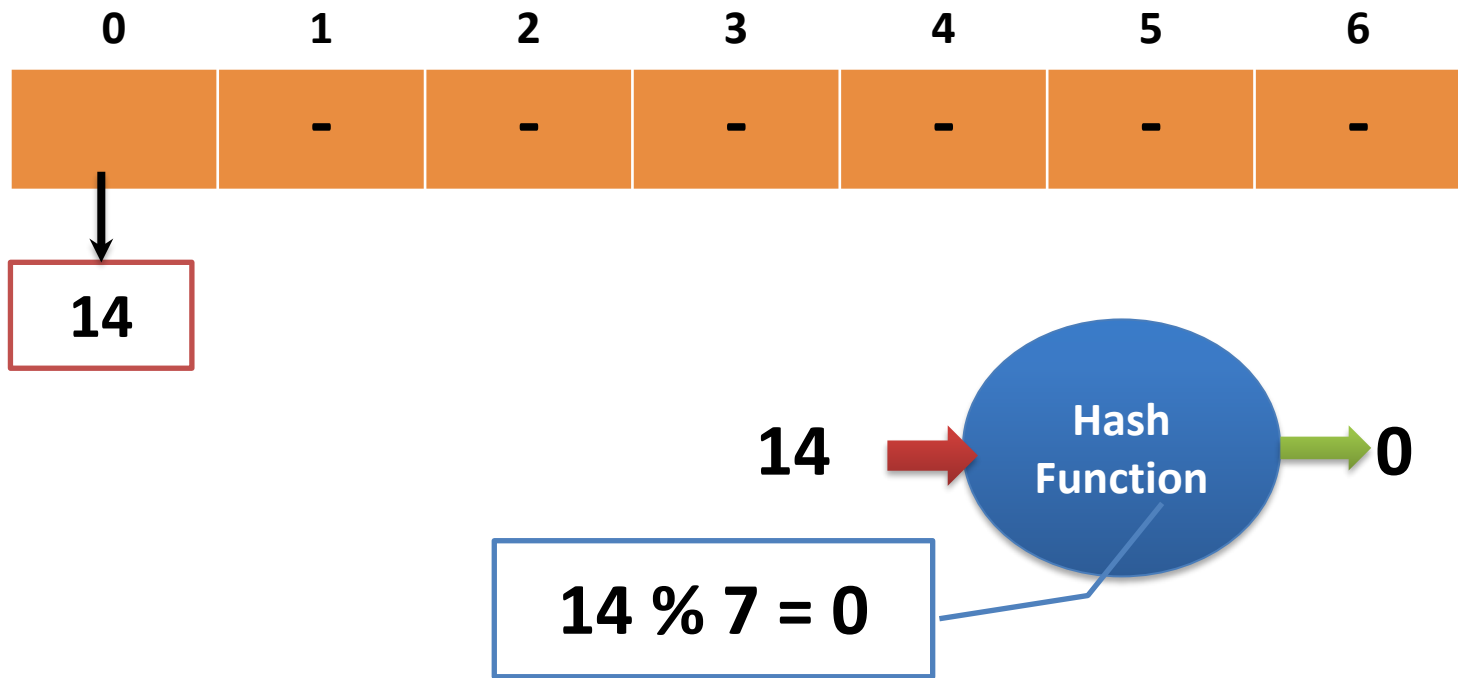
The process of creating a **linked list of values** if they hashed into the same location.

In **open addressing**, each array element can **hold just one entry**. When the array is full, no more records can be added to the table.

But in **Chaining** each component of the hash table's array **can hold more than one entry**.

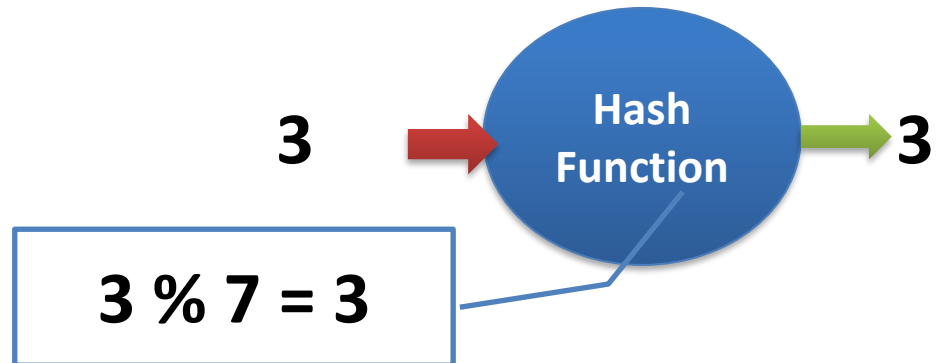
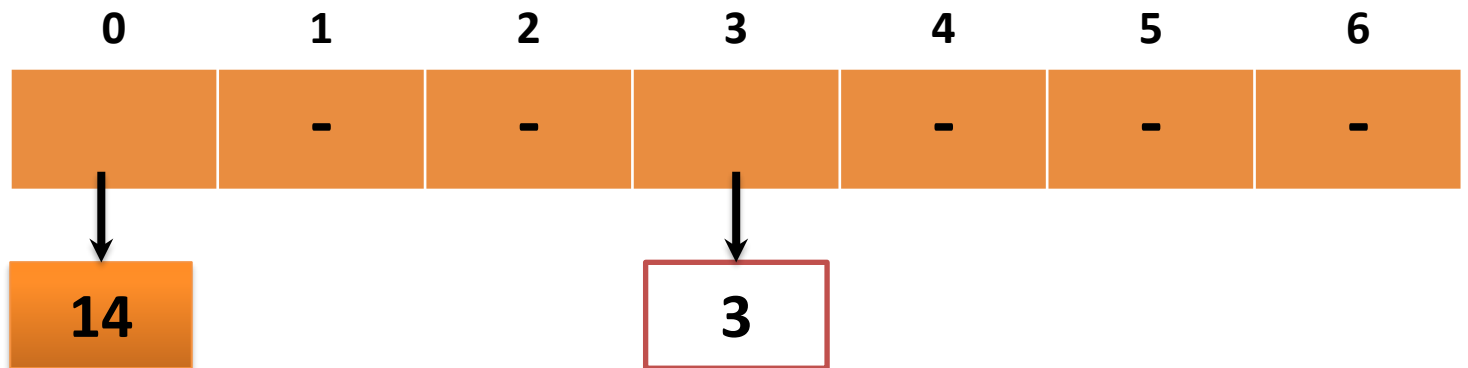
Separate chaining

Insert Elements in this order **14**, 3, 9, 7, 23, 19, 35



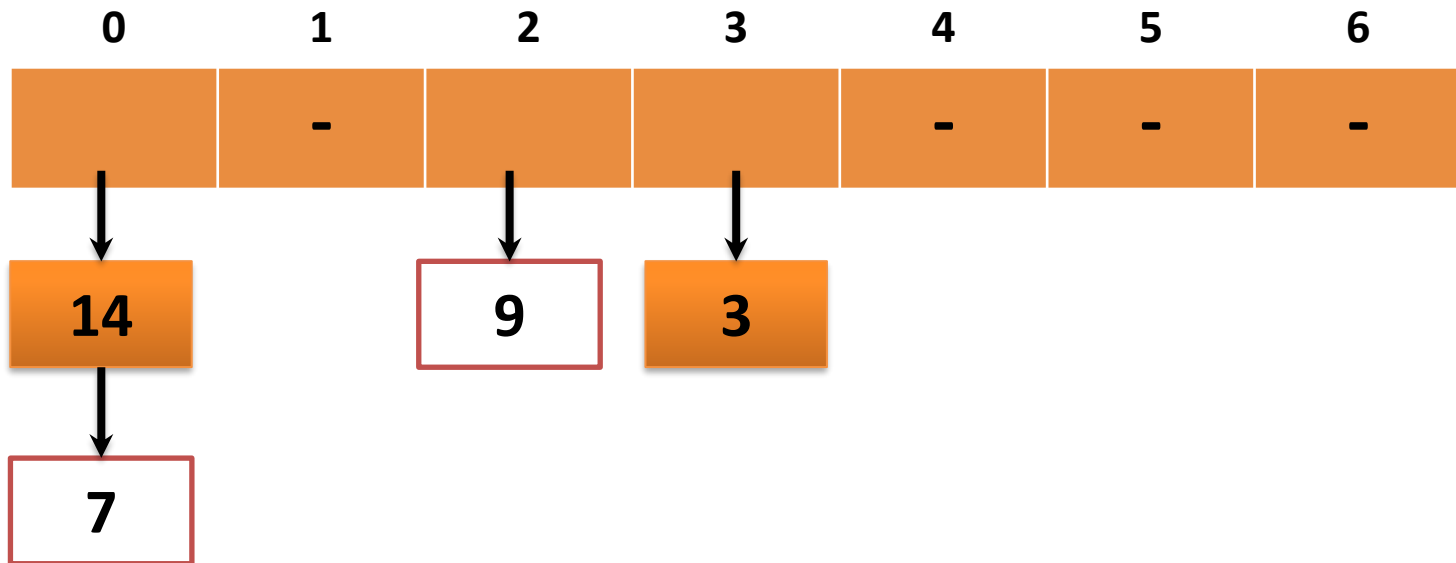
Separate chaining

Insert Elements in this order 14, **3**, 9, 7, 23, 19, 35



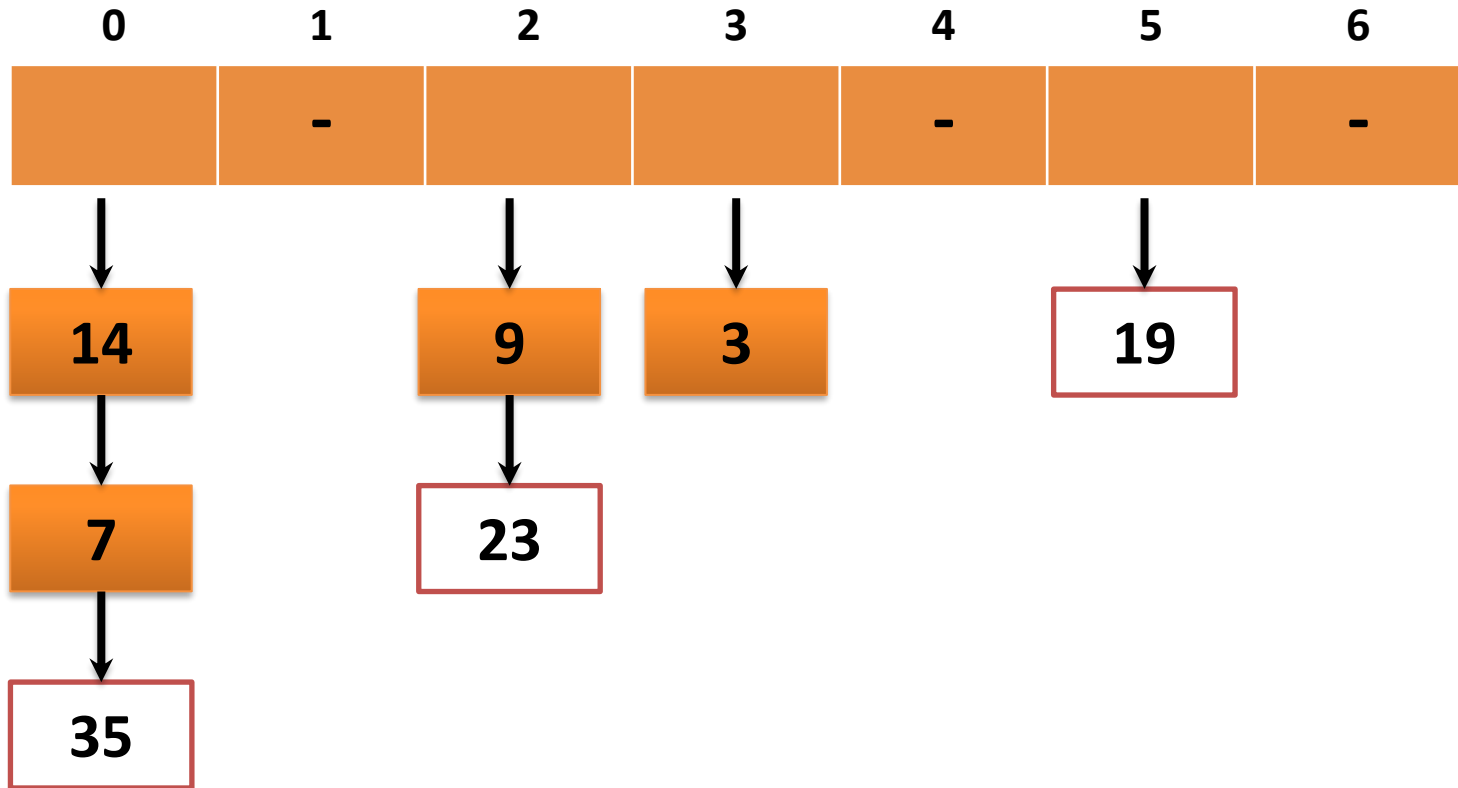
Separate chaining

Insert Elements in this order 14, 3, 9, 7, 23, 19, 35

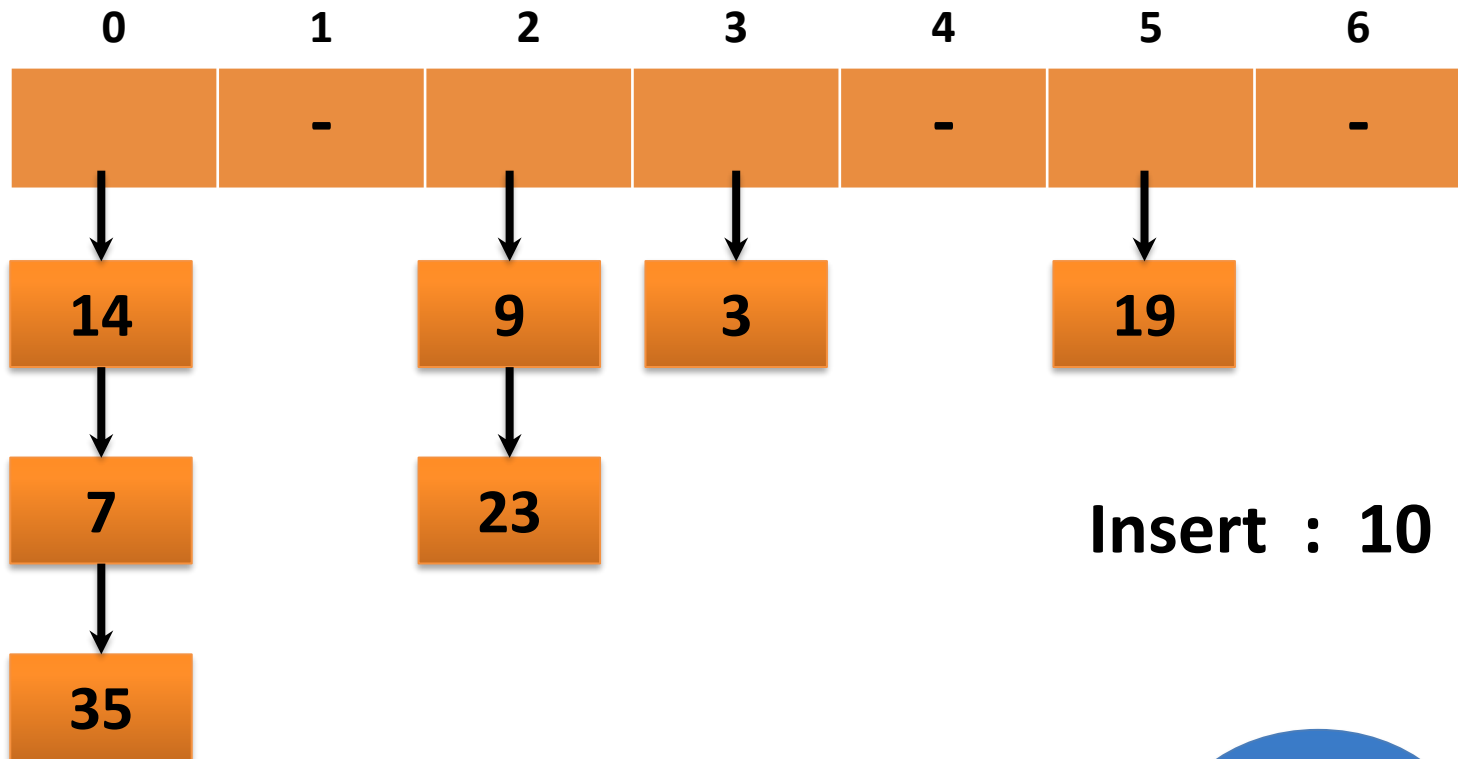


Separate chaining

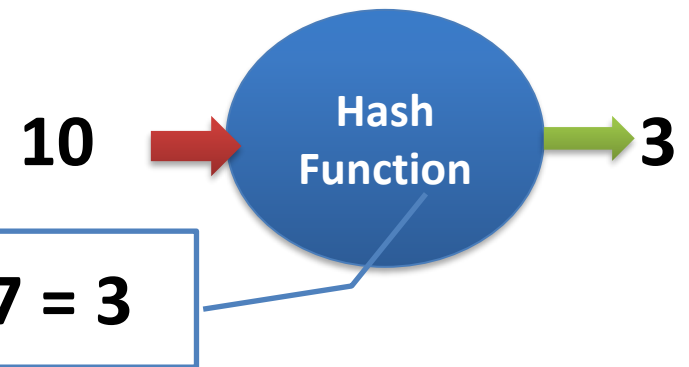
Insert Elements in this order 14, 3, 9, 7, **23**, **19**, **35**



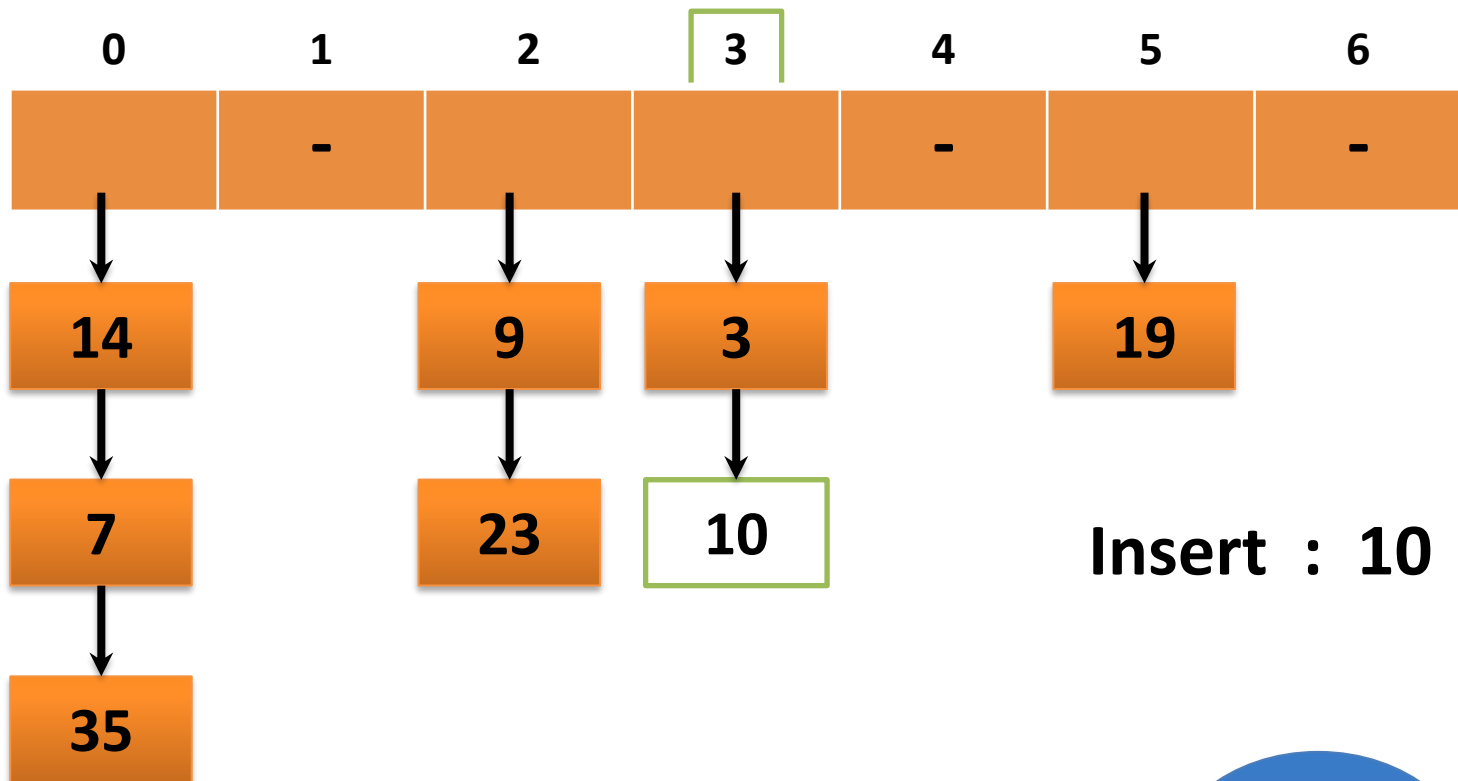
Separate chaining



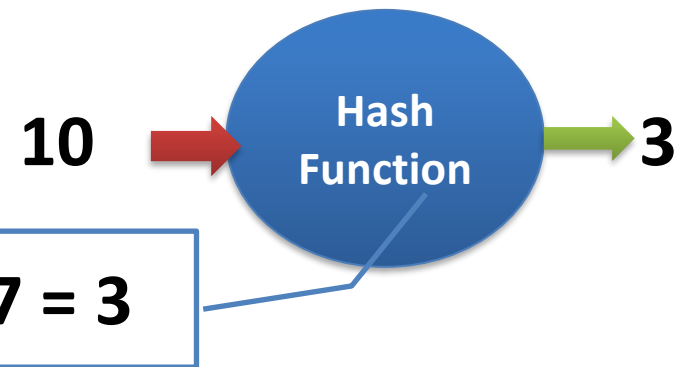
Insert : 10



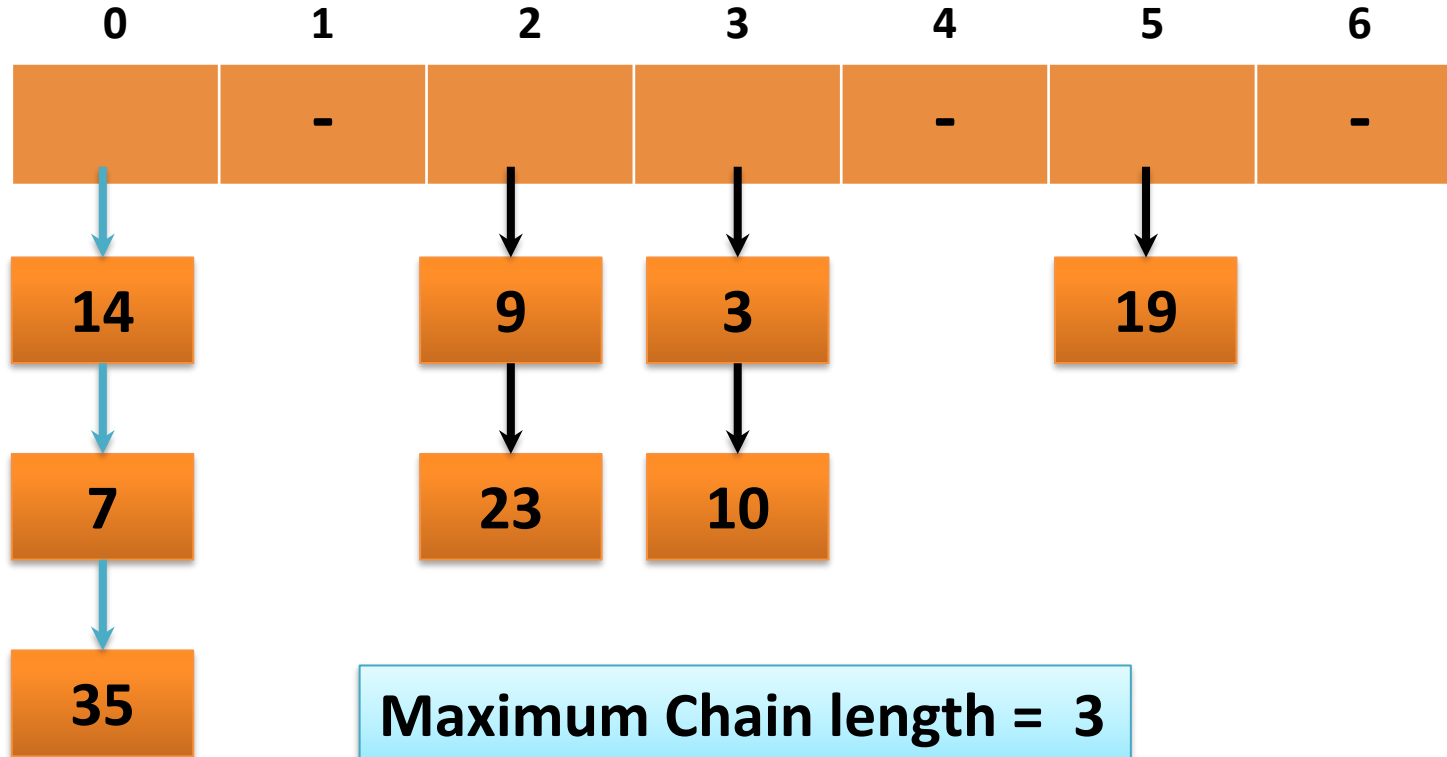
Separate chaining



Insert : 10



Separate chaining



Maximum Chain length = 3

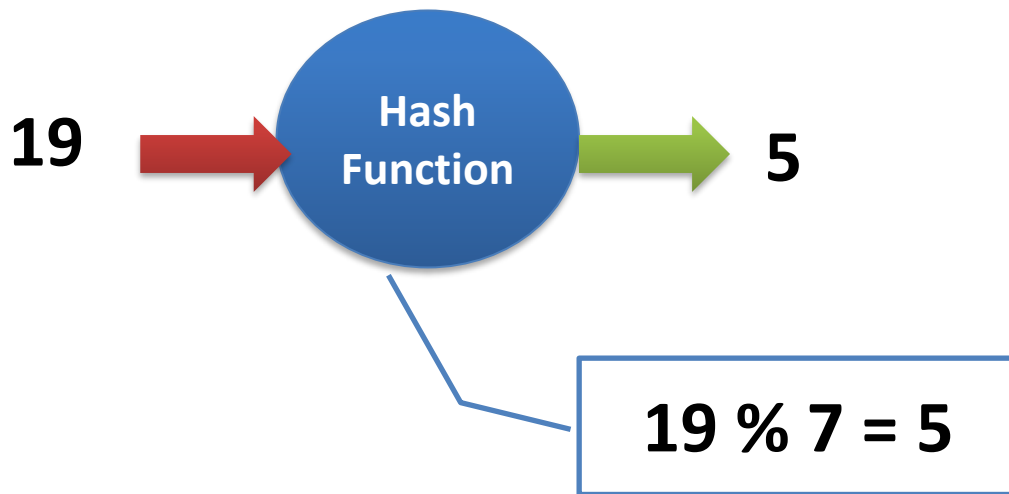
Minimum Chain length = 0

Average Chain length = $(3 + 0 + 2 + 2 + 0 + 1 + 0) / 7 = 8 / 7$

- **separate chaining:** if the number of records is not known in advance
- **open addressing:** if the number of the records can be predicted and there is enough memory available

Hash Table – Lazy Deletion

delete : 19

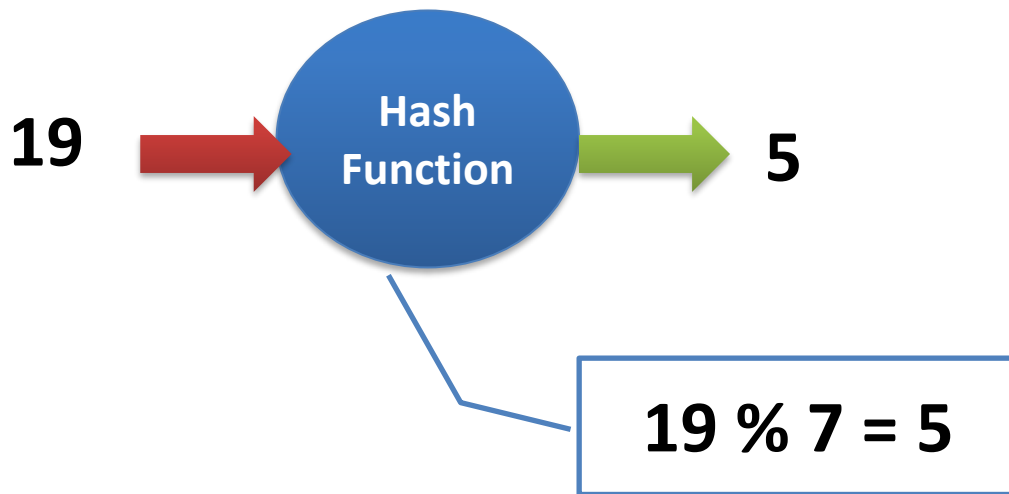


Hash Table

0	21
1	18
2	2
3	-
4	11
5	19
6	41

Hash Table- Lazy Deletion

delete : 19



Hash Table	
0	21
1	18
2	2
3	-
4	11
5	19
6	41

**= 19 ,
then
delete**

Hash Table - Lazy Deletion

Data 19 is removed and we put a flag 🚩 to indicate the element is deleted.

The flag indicates that a record once occupied the slot but does so no longer.

deletions are done by marking an element as deleted, rather than erasing it entirely .

Hash Table	
0	21
1	18
2	2
3	-
4	11
5	🚩
6	41

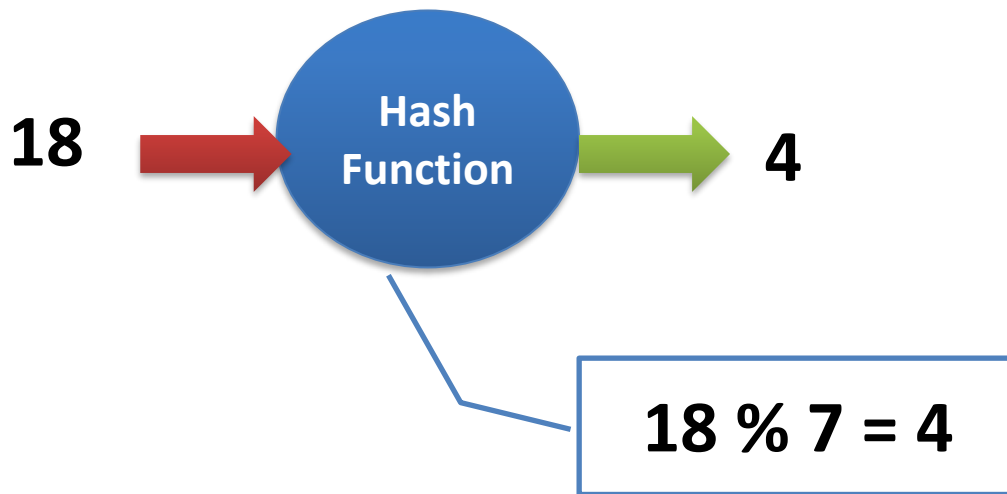
Assign a flag

Hash Table - Lazy Deletion

- Deleted locations are treated as **empty** when inserting and as **occupied** during a search.

Hash Table - Lazy Deletion

Search : 18



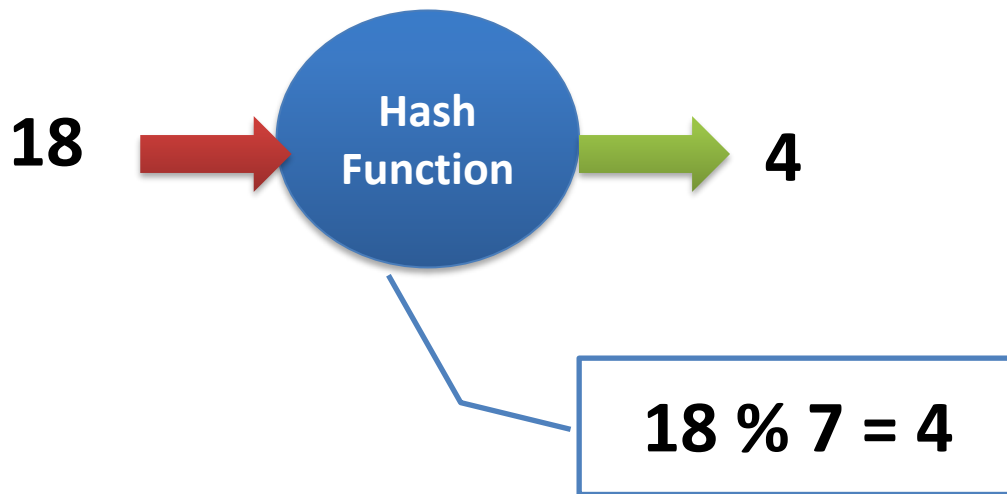
Hash Table

0	21
1	18
2	2
3	-
4	11
5	🚩
6	41

Assign
a flag


Hash Table - Lazy Deletion

Search : 18



Deleted locations are treated as **occupied** during a search.

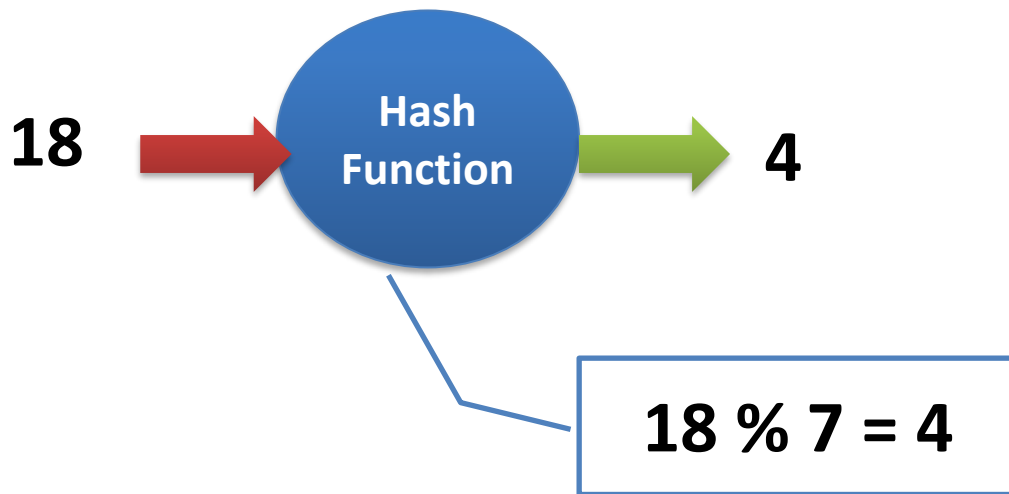
Hash Table

0	21
1	18
2	2
3	-
4	11
5	
6	41


Assign
a flag

Hash Table - Lazy Deletion

Search : 18



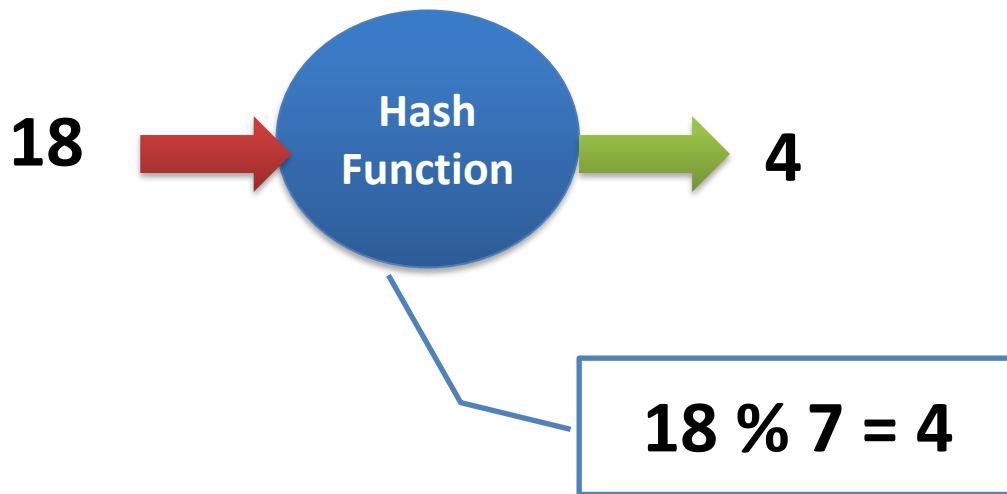
Hash Table

0	21
1	18
2	2
3	-
4	11
5	
6	41

Assign
a flag

Hash Table - Lazy Deletion

Search : 18



Hash Table

0	21
1	18
2	2
3	-
4	11
5	🚩
6	41

Assign
a flag

Hash Table - Lazy Deletion

