Convergence / divergence of Sequences.

An ordered set of real numbers $a_1, a_2 \dots\dots a_n$ is called a sequence, denoted by (a_n)

If the no. of terms is unlimited, then the sequence is said to be an <u>infinite sequence</u> & its nth term is a_n.

<u>Limit</u>, A sequence is said to tend to a limit 'l' if for every $\varepsilon > 0$, a value N of n can be found, such that,

$$|a_n - l| < \varepsilon \quad \forall\, n \geq N$$

<u>Convergence, divergence, oscillating</u>

(i) If $\lim_{n \to \infty} (a_n) = l$ is finite & unique then the sequence is said to be cgt.

(II) If $\lim_{n\to\infty} (a_n) =$ infinite (i.e. $\pm\infty$), then the sequence is said to be divergent (dgt)

(III) If $\lim_{n\to\infty} (a_n)$ is not unique, then the sequence is oscillating.

Problems: Examine the following sequences for convergence.

1) $a_n = \dfrac{n^2 - 2n}{3n^2 + n}$.

Soln $a_n = \dfrac{\cancel{n^2}\left(1 - \frac{2}{n}\right)}{\cancel{n^2}\left(3 + \frac{1}{n}\right)}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\dfrac{1 - \frac{2}{n}}{3 + \frac{1}{n}}\right)$

$\dfrac{n}{n^2} = \dfrac{1}{n}$

$\lim_{n\to\infty} \dfrac{1}{n} = 0$

$= \dfrac{1}{3}$, finite & unique.

$\therefore$ The sequence is cgt.

2) $a_n = 2^n$

$\underset{n\to\infty}{Lt}\ a_n = \underset{n\to\infty}{Lt}\ 2^n = 2^\infty = \infty$.

$\therefore\ a_n = 2^n$ is divergent.

3) $a_n = 3 + (-1)^n$.

$(-1)^n$ ~~even~~ $= 1$ if n is even

$(-1)^n = -1$ if n is odd.

$\underset{n\to\infty}{Lt}\ a_n = \underset{n\to\infty}{Lt}\ \left(3 + (-1)^n\right)$

$= 3 + (-1)^\infty$

$= 3 + 1$ if n is even

$= 3 - 1$ if n is odd

$\underset{n\to\infty}{Lt}\ a_n = \begin{cases} 4 & \text{if } n \text{ is even} \\ 2 & \text{if } n \text{ is odd} \end{cases}$, not unique.

$\therefore$ The given sequence is oscillating.

Test for Convergence

1) $a_n = \dfrac{3n-1}{1+2n}$.

2) $a_n = 1 + \dfrac{2}{n}$

3) $a_n = \left(n + (-1)^n\right)^{-1}$ or $a_n = \dfrac{1}{n+(-1)^n}$.