What is a Graph?

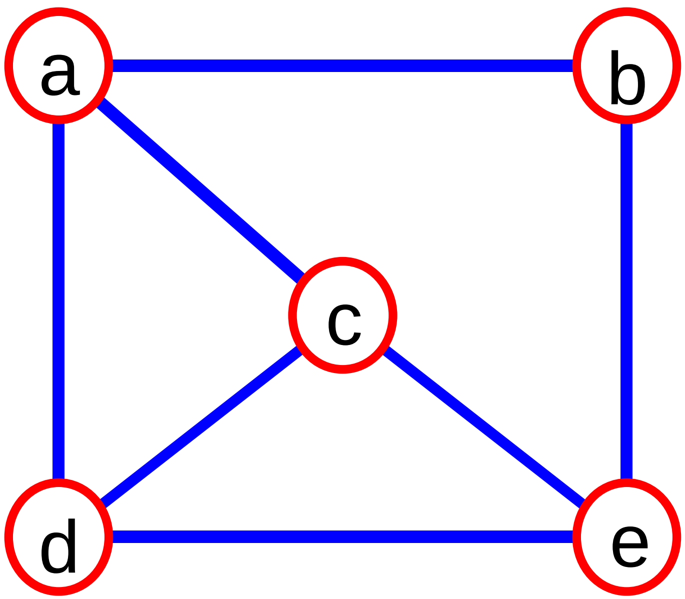
A graph G = (V,E) is composed of:

V: set of vertices

E: set of edges connecting the vertices in V

An edge e = (u,v) is a pair of vertices

Example:



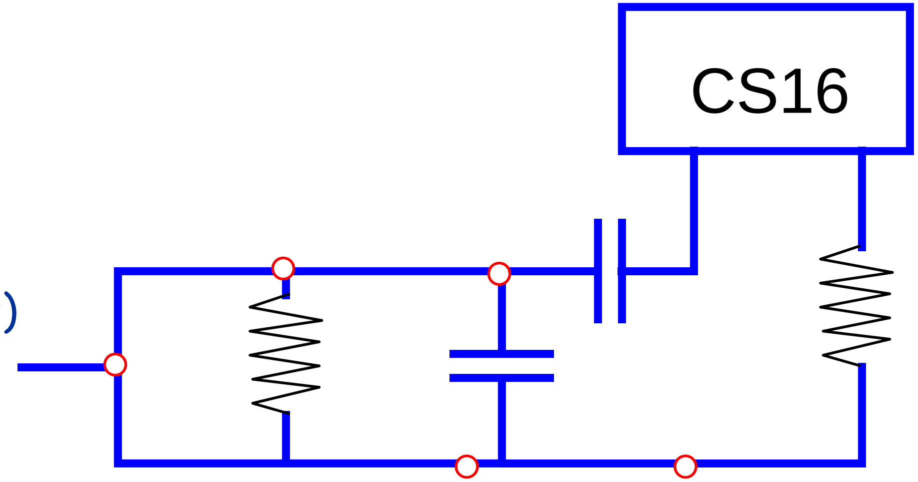
V= {a,b,c,d,e}

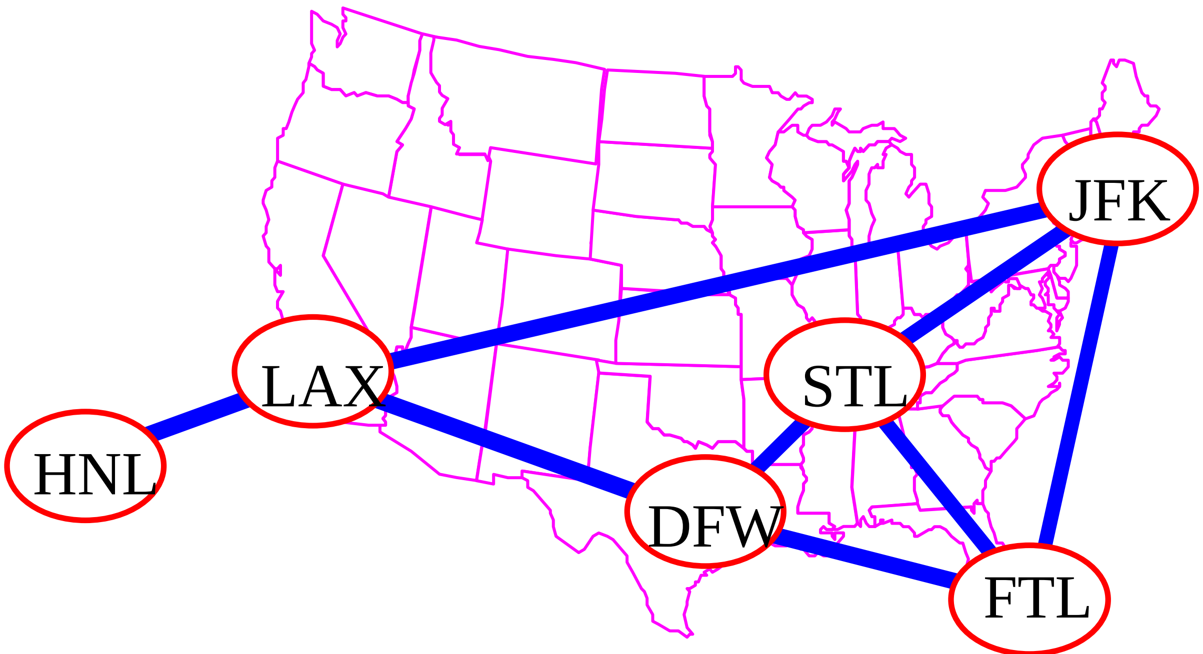
E= {(a,b),(a,c),(a,d),
(b,e),(c,d),(c,e),
(d,e)}

Applications

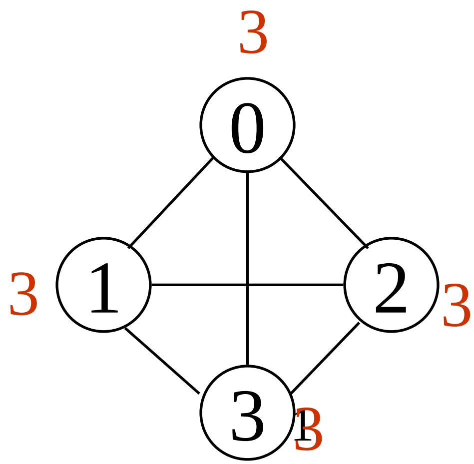
electronic circuits

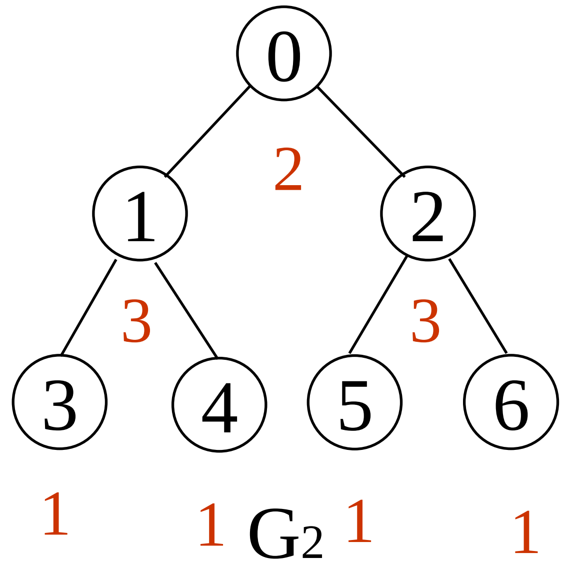
networks (roads, flights, communications)





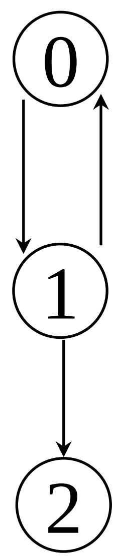
Examples





G_2

directed graph
in-degree
out-degree



in:1, out: 1

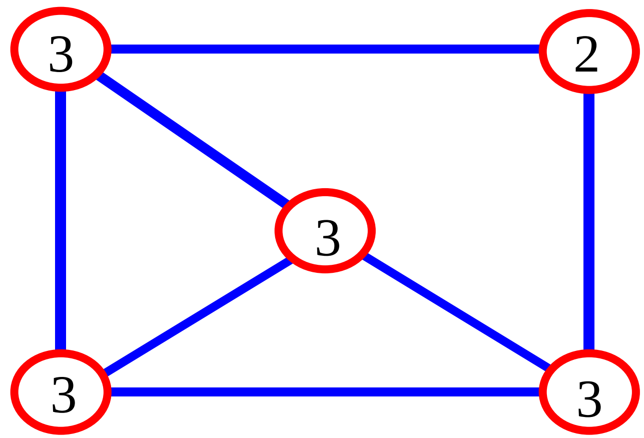
in: 1, out: 2

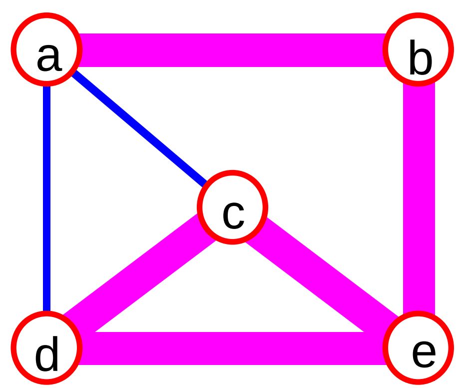
in: 1, out: 0

G_3

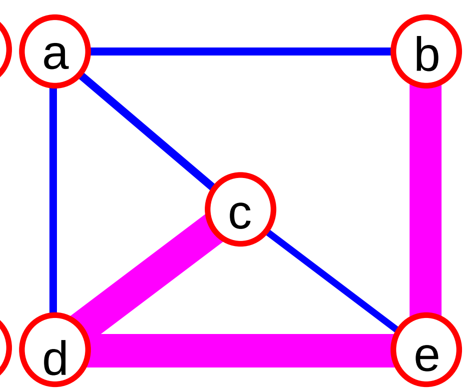
Terminology: Path

path: sequence of vertices $v_1, v_2, \ldots v_k$ such that consecutive vertices v_i and v_{i+1} are adjacent.





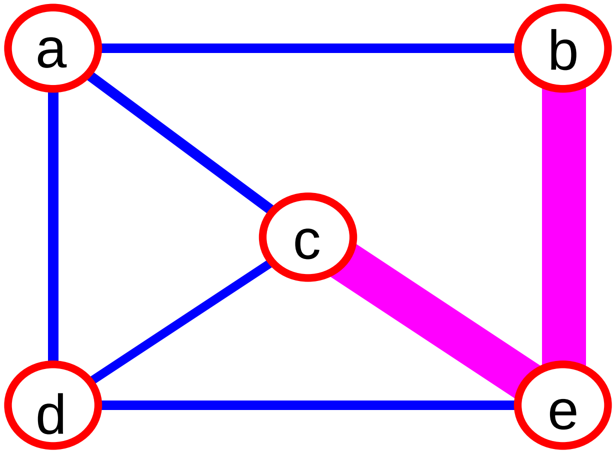
a b e d c



b e d c

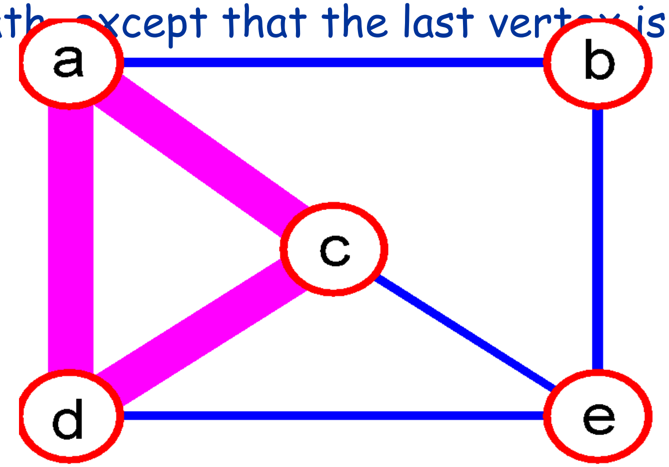
More Terminology

simple path: no repeated vertices

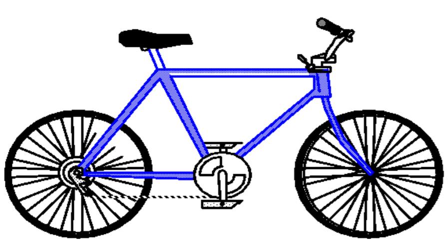


b e c

cycle: simple path, except that the last vertex is the same as the first vertex

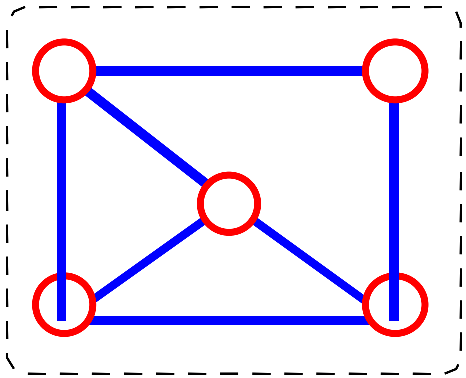


a c d a

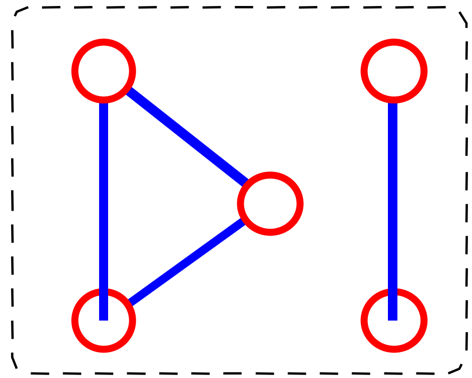


Even More Terminology

- connected graph: any two vertices are connected by some path



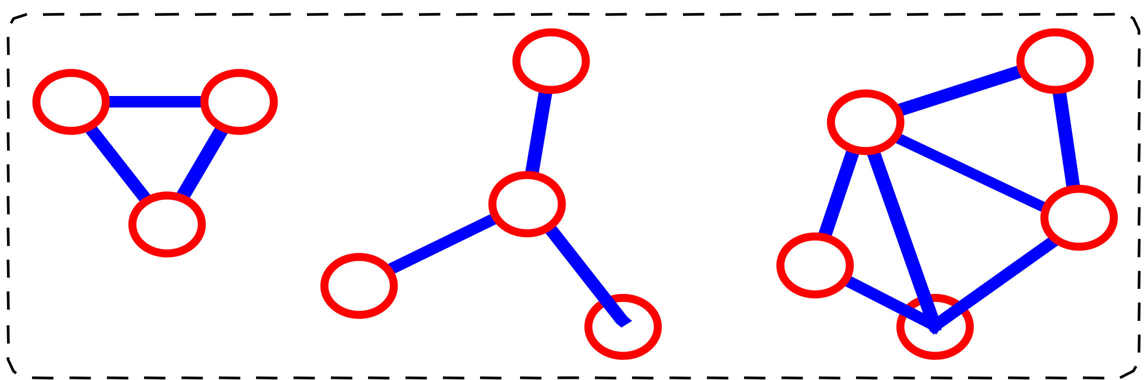
connected



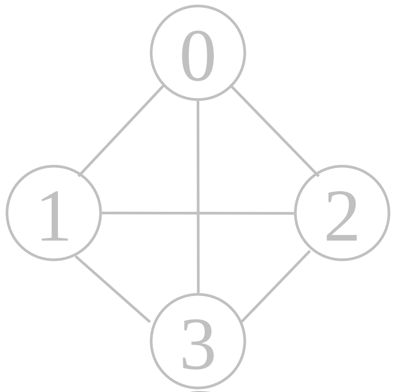
not connected

subgraph: subset of vertices and edges forming a graph

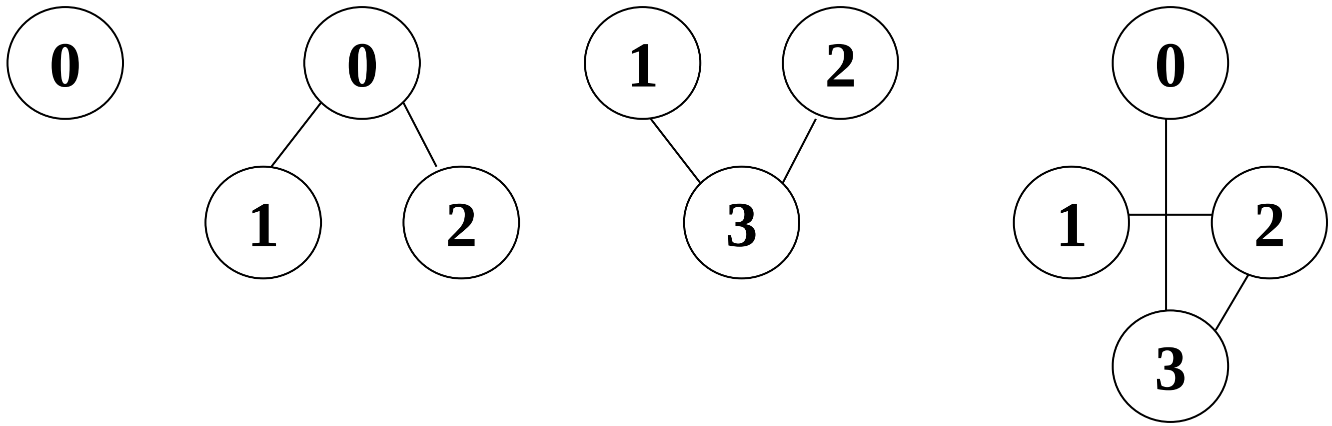
connected component: maximal connected subgraph. E.g., the graph below has 3 connected components.



Subgraphs Examples

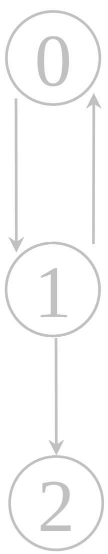


G1

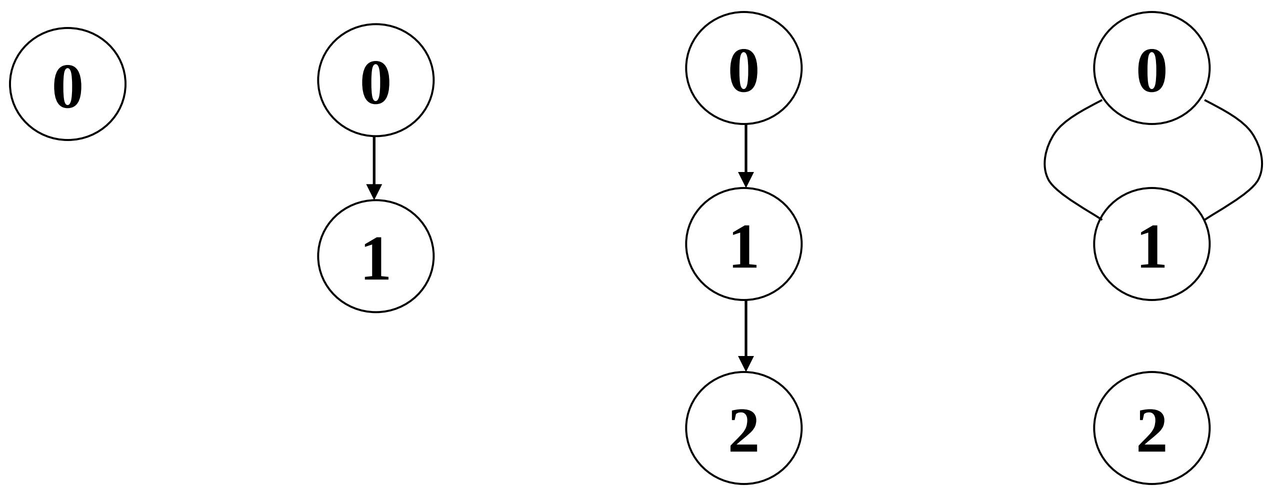


(i) (ii) (iii) (iv)

(a) Some of the subgraph of G_1



G3



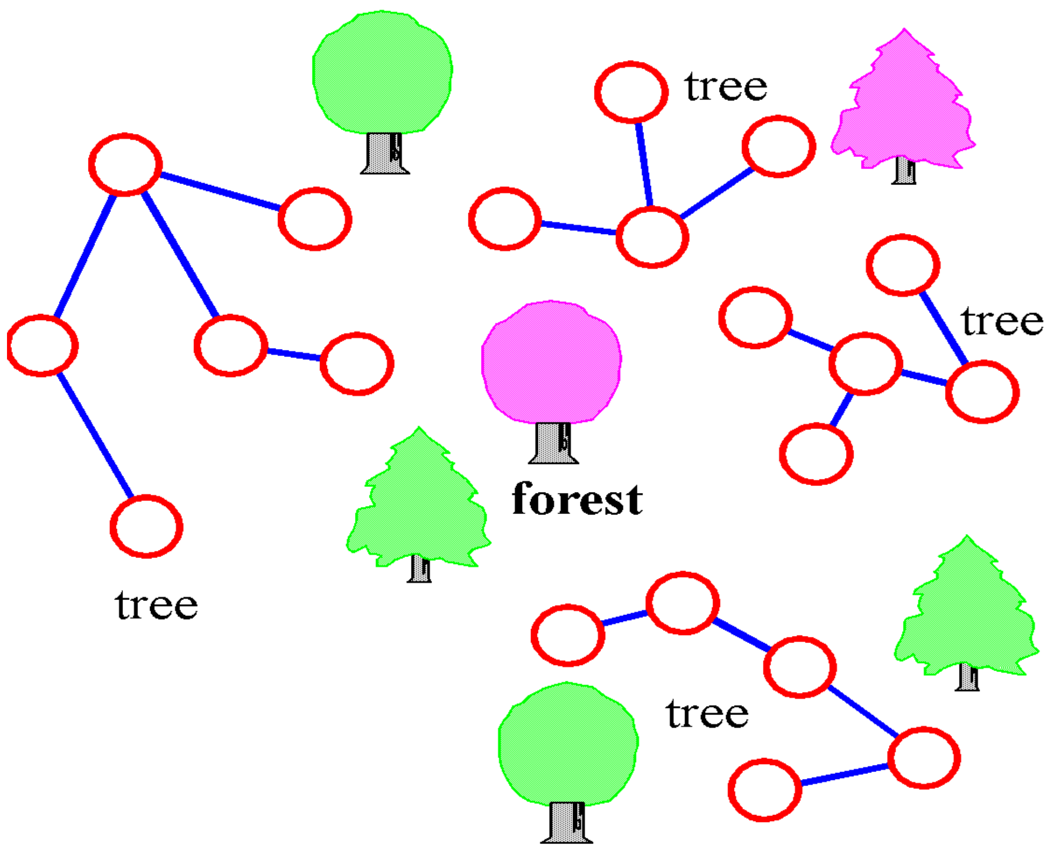
(i) (ii) (iii) (iv)

(b) Some of the subgraph of G_3

More…

tree - connected graph without cycles

forest - collection of trees



Connectivity

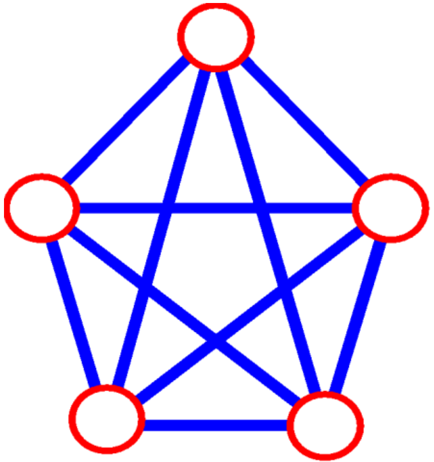
Let **n** = #vertices, and **m** = #edges

A complete graph: one in which all pairs of vertices are adjacent

How many total edges in a complete graph?

- Each of the n vertices is incident to n-1 edges, however, we would have counted each edge twice! Therefore, intuitively, m = **n**(**n** -1)/2.

Therefore, if a graph is not complete, m < **n**(**n** -1)/2



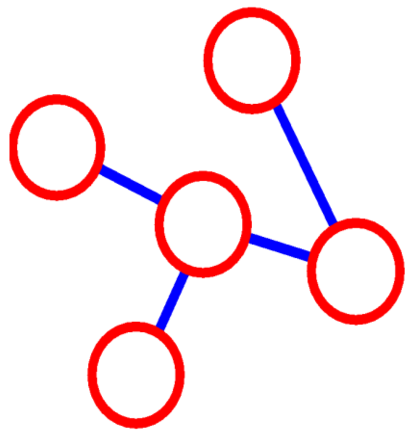
$\mathbf{n} = 5$
$\mathbf{m} = (5 * 4)/2 = 10$

More Connectivity

n = #vertices

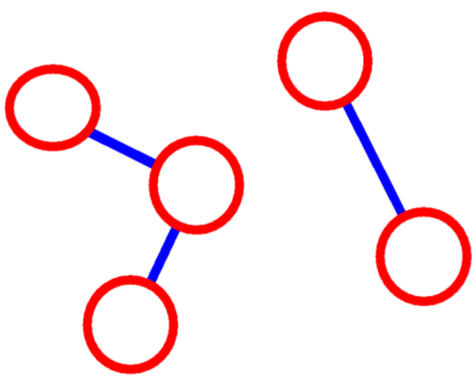
m = #edges

For a tree $m = n - 1$



$n = 5$
$m = 4$

If $m < n - 1$, G is not connected



$n = 5$
$m = 3$

Directed vs. Undirected Graph

An undirected graph is one in which the pair of vertices in a edge is unordered, $(v_0, v_1) = (v_1, v_0)$

A directed graph is one in which each edge is a directed pair of vertices, $\langle v_0, v_1 \rangle$!= $\langle v_1, v_0 \rangle$

Graph Representations

Adjacency Matrix

Adjacency Lists

Adjacency Matrix

Let G=(V,E) be a graph with n vertices.

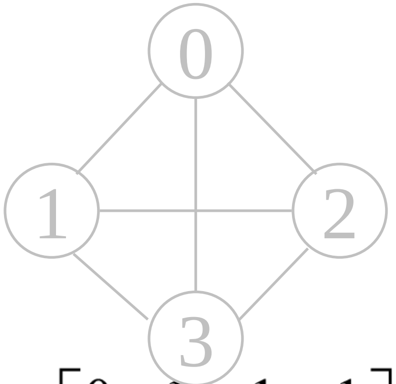
The adjacency matrix of G is a two-dimensional n by n array, say adj_mat

If the edge (v_i, v_j) is in E(G), adj_mat[i][j]=1

If there is no such edge in E(G), adj_mat[i][j]=0

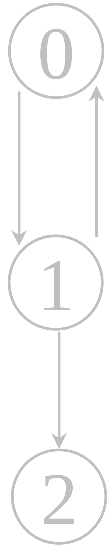
The adjacency matrix for an undirected graph is symmetric; the adjacency matrix for a digraph need not be symmetric

Examples for Adjacency Matrix



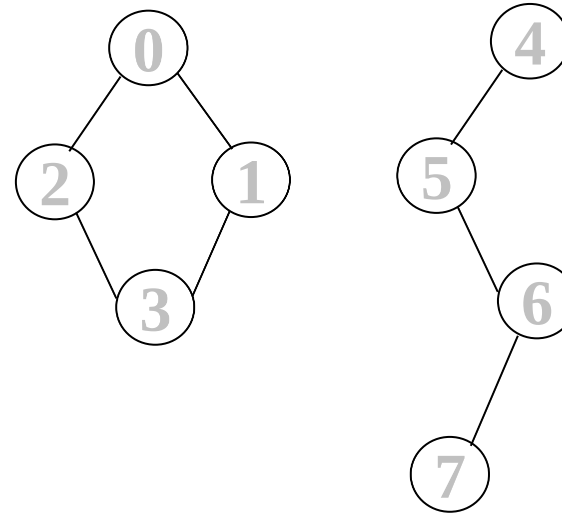
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

G_1



$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

G_2



$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

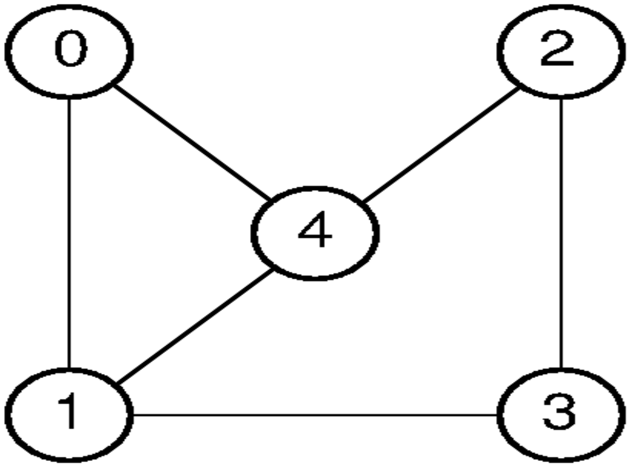
G_4

symmetric

undirected: $n^2/2$

directed: n^2

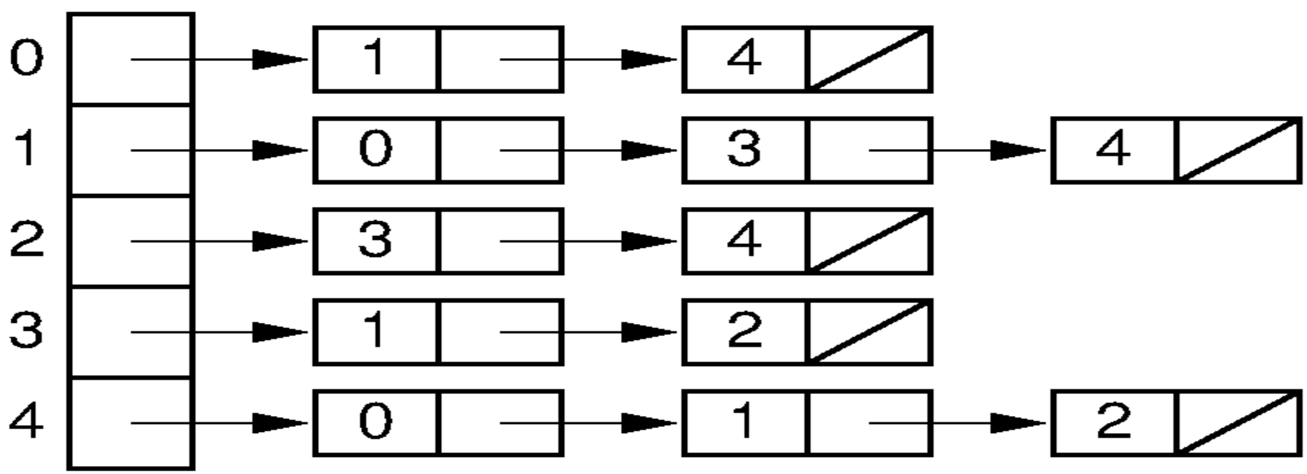
Graphs: Adjacency List



(a)

	0	1	2	3	4
0		1			1
1	1			1	1
2				1	1
3		1	1		
4	1	1	1		

(b)



(c)

GRAPH ------- DEPTH FIRST TRAVERSAL

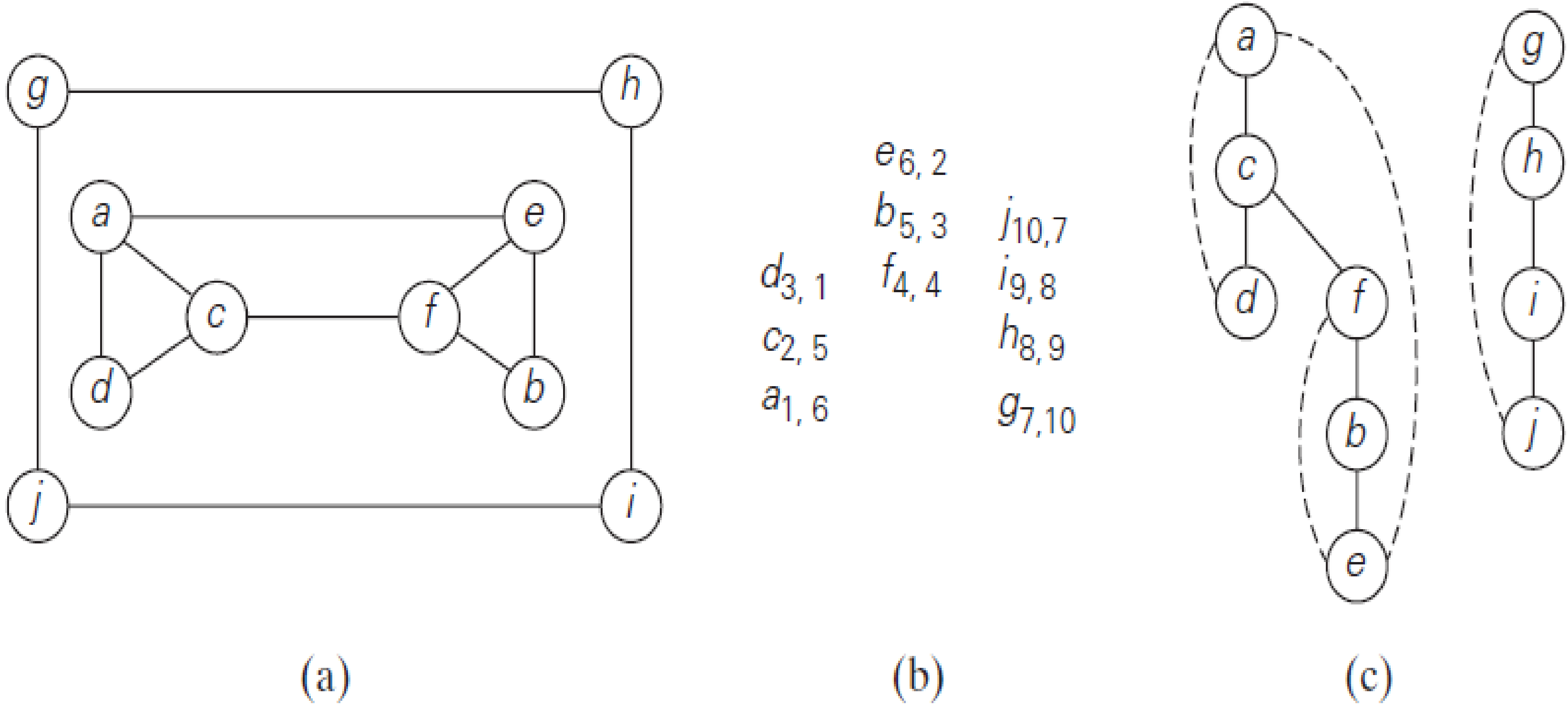


FIGURE 3.10 Example of a DFS traversal. (a) Graph. (b) Traversal's stack (the first subscript number indicates the order in which a vertex is visited, i.e., pushed onto the stack; the second one indicates the order in which it becomes a dead-end, i.e., popped off the stack). (c) DFS forest with the tree and back edges shown with solid and dashed lines, respectively.

GRAPH ------- BREADTH FIRST TRAVERSAL

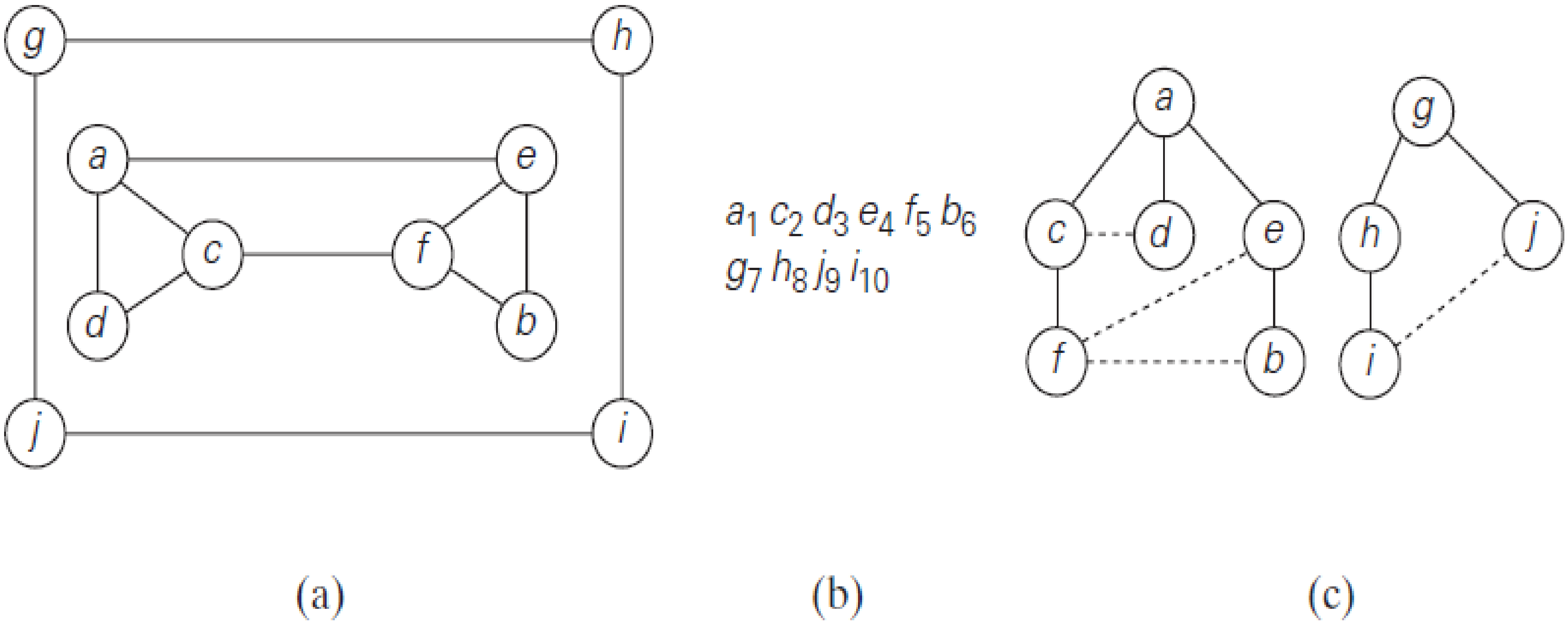


FIGURE 3.11 Example of a BFS traversal. (a) Graph. (b) Traversal queue, with the numbers indicating the order in which the vertices are visited, i.e., added to (and removed from) the queue. (c) BFS forest with the tree and cross edges shown with solid and dotted lines, respectively.