20MA1005

Mathematical Foundations of Computing

Course Objectives:

- To formulate physical phenomena using matrices.
- To apply differentiation and integration techniques.
- To analyse periodic signals using Fourier series.

Course Outcomes:

- The student will be able to
- Solve linear systems of equations using matrices.
- Find the Eigen values, Eigen vectors of matrices and diagonalize the matrices.
- Apply differentiation techniques to find extreme values of functions.
- Demonstrate knowledge in integration.
- Evaluate area and volume using definite integral.
- Express periodic functions as a series of sine and cosine functions.

Module 1: Linear Algebra: Matrices, Determinants, Linear Systems

Controlling Traffic Networks using Linear Algebra, Matrices: Linear Systems of Equations, Row Echelon Form, Rank of a Matrix, Determinants, Cramer’s Rule, Inverse of a Matrix, Gauss-Jordan Elimination method- Leontief input-output model.

Module 2: Linear Algebra: Matrix Eigen value Problems

Gould Index - use of Matrix to Geography, Eigen values, Eigen vectors, Cayley Hamilton Theorem, Diagonalization of a matrix, Hermitian, Unitary and Normal Matrices, bilinear and quadratic forms, orthogonal transformation to reduce quadratic form to canonical form.

Module 3: Differential Calculus

Financial Optimization using Calculus, Linear And Nonlinear Functions, Limit continuity, differentiation (definition and simple problems), Linearity of differentiation, partial derivatives, critical points, extreme points in nonlinear function, Jacobians, Maxima Minima of single variable.

Module 4: Integral Calculus

Blood Flow monitoring based on Poiseuille's Law, Integration, definite integral, Integration by parts, Integration by substitution, Integration using differentiation.

Module 5: Multiple Integration

Volume under a Surface for Remote Sensing using Double integrals (Cartesian), change of order of integration in double integrals, Area. Triple Integrals, volume. Beta and Gamma functions and their properties.

Module 6: Fourier series

Audio and Video Compression using Fourier Series, Full range, Half range Fourier sine and cosine series, Parseval's theorem, Harmonic analysis.

Text Books:

1. B.S. Grewal, “Higher Engineering Mathematics”, Khanna Publishers, 44thEdition, 2017.

Reference books:

1. R. Bronson, “Matrix methods: An introduction”, Gulf Professional Publishing, 1991.
2. David C. Lay, Steven R. Lay and Judi J. McDonald “Linear Algebra and its Applications”, Fifth Edition. Pearson, 2006.
3. C. D. Meyer, “Matrix analysis and applied linear algebra”, Vol. 71, Siam, 2000.
4. G.B. Thomas and R.L. Finney, “Calculus and Analytic geometry”, 9thEdition, Pearson, Reprint, 2002.
5. Erwin kreyszig, “Advanced Engineering Mathematics”, 9thEdition, John Wiley & Sons, 2006.
6. Veerarajan T., “Engineering Mathematics for first year”, Tata McGraw-Hill, New Delhi, 2008.
7. Ramana B.V., “Higher Engineering Mathematics”, Tata McGraw Hill New Delhi, 11th Reprint, 2010.
8. D. Poole, “Linear Algebra: A Modern Introduction”, 2nd Edition, Brooks/Cole, 2005.
9. N.P. Bali and Manish Goyal, “A text book of Engineering Mathematics”, Laxmi Publications, Reprint, 2008.
10. Dean G. Duffy. Advanced Engineering Mathematics with MATLAB, 2ndEdn. Chapman & Hall / CRC Press. New York, 2003 (Taylor and Francis, e-library, 2009).

Consider the following set of equations:

$$\begin{cases} x+y=7, \\ 3x-y=5. \end{cases}$$

It is easy to show that $x=3$ and $y=4$.

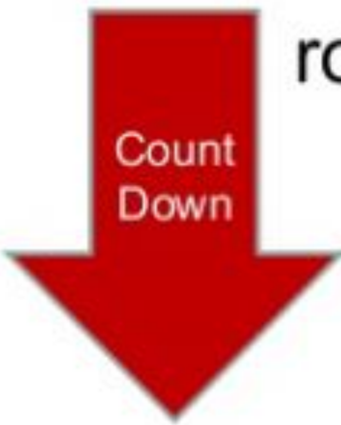
HOW ABOUT SOLVING
$$\begin{cases} x+y-2z=7, \\ 2x-y-4z=2, \\ -5x+4y+10z=1, \\ 3x-y-6z=5. \end{cases}$$

Matrices

A matrix is a rectangular array of numbers arranged in rows and columns.

The dimensions of a matrix are written as

rows x columns.



Example

$$\begin{bmatrix} -7 & 4 & 2 \\ 8 & -1 & 0 \end{bmatrix}$$

This is a 2 x 3 matrix

$$\begin{bmatrix} 5 & 3 \\ -4 & 4 \\ 2 & 0 \end{bmatrix}$$

This is a 3 x 2 matrix

In the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$$

- numbers a_{ij} are called *elements*. First subscript indicates the row; second subscript indicates the column. The matrix consists of mn elements
- It is called "the $m \times n$ matrix $A = [a_{ij}]$" or simply "the matrix A" if number of rows and columns are understood.

Types of Matrices

- Row Matrix: An m x n matrix is called row matrix if m = 1. Ex: A = $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$
- Column Matrix: An m x n matrix is called row matrix if n = 1. Ex: A = $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$
- Square Matrix: A square matrix is a matrix that has the same number of rows and columns i.e. if $m = n$. Ex: A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Types of Matrices

- Zero Matrix: A matrix each of whose elements is zero & is called a zero matrix. It is usually denoted by "O". It is also called *"Null Matrix"*

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 0 \end{bmatrix}$$

Types of Matrices

- **Diagonal Matrix:** A square matrix with its all non *diagonal elements* as zero. i.e if $A = [a_{ij}]$ is a diagonal matrix, then $a_{ij} = 0$ whenever $i \neq j$. ***Diagonal elements*** are the a_{ij} elements of the square matrix A for which $i = j$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Types of Matrices

- Diagonal elements are said to constitute the **main diagonal** or **principal diagonal** or simply a **diagonal**.
- The diagonals which lie on a line perpendicular to the diagonal are said to constitute **secondary diagonal.**

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Here main diagonal consists of 1 & 4 and secondary diagonal consists of 2 & 3

Types of Matrices

- Scalar Matrix: It's a diagonal matrix whose all elements are equal.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- Unit Matrix: It's a scalar matrix whose all diagonal elements are equal to unity. It is also called a Unit Matrix or Identity Matrix. It is denoted by I_n.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Types of Matrices

- Triangular Matrix: If every element above or below the diagonal is zero, the matrix is said to be a triangular matrix.

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$ Upper Triangular Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 5 & -6 & 3 \end{bmatrix}$$ Lower Triangular Matrix

Equality of Matrices

- Two matrices A & B are said to be equal iff:
  1. A and B are of the same order
  2. All the elements of A are equal as that of corresponding elements of B
- Two matrices A = [aij] & B = [bij] of the same order are said to be equal if aij = bij

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

If A & B are equal, then

$x=1$, $y=2$, $z=3$, $w=4$

TRACE OF A MATRIX

- In a square matrix A, the sum of all the diagonal elements is called the trace of A. It is denoted by tr A.
  - Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 7 & 6 & 1 \end{bmatrix}$ $\quad$ $\text{tr } A = 1+4+1 = 6$
  - Ex: If $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad$ $\text{tr } B = 1+4 = 5$

Operations on Matrices

- Addition/Subtraction
- Scalar Multiplication
- Matrix Multiplication

Addition and Subtraction

- Two matrices may be added (or subtracted) iff they are the same order.

- Simply add (or subtract) the corresponding elements. So, $\mathbf{A} + \mathbf{B} = \mathbf{C}$

Scalar Multiplication

- To multiply a scalar times a matrix, simply multiply each element of the matrix by the scalar quantity

$$k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

- Ex: If $A = \begin{bmatrix} 3 & 8 & 11 \\ 6 & -3 & 8 \end{bmatrix}$, then

$$10A = \begin{bmatrix} 30 & 80 & 110 \\ 60 & -30 & 80 \end{bmatrix}$$

Matrix Multiplication

Example: $A=\begin{bmatrix}1&2&3\\0&1&4\end{bmatrix}$, $B=\begin{bmatrix}-1&2\\2&3\\5&0\end{bmatrix}$, Evaluate $C=AB$.

$$\begin{bmatrix}1&2&3\\0&1&4\end{bmatrix}\begin{bmatrix}-1&2\\2&3\\5&0\end{bmatrix}\Rightarrow\begin{cases}c_{11}=1\times(-1)+2\times2+3\times5=18\\c_{12}=1\times2+2\times3+3\times0=8\\c_{21}=0\times(-1)+1\times2+4\times5=22\\c_{22}=0\times2+1\times3+4\times0=3\end{cases}$$

$$C=AB=\begin{bmatrix}1&2&3\\0&1&4\end{bmatrix}\begin{bmatrix}-1&2\\2&3\\5&0\end{bmatrix}=\begin{bmatrix}18&8\\22&3\end{bmatrix}$$

TRANSPOSE OF A MATRIX

- The matrix obtained by interchanging the rows and columns of a matrix A is called the transpose of A (written as A^T *or* A`).

Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

The transpose of A is $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

- For a matrix $A = [a_{ij}]$, its transpose $A^T = [b_{ij}]$, where $b_{ij} = a_{ji}$.

Symmetric & Skew Symmetric Matrices

- A matrix A such that $A^T = A$ is called symmetric, i.e., $a_{ji} = a_{ij}$ for all i and j.

- A matrix A such that $A^T = -A$ is called skew-symmetric, i.e., $a_{ji} = -a_{ij}$ for all i and j.