Inferential Statistics

Estimation & Hypothesis Testing

Hypothesis Testing

- Hypothesis is a claim that should be tested.
- The null hypothesis (H_0), stated as the null, is a statement about a population parameter, such as the population mean, that is assumed to be true.
- An alternative hypothesis (H_1) is a statement that directly contradicts a null hypothesis by stating that the actual value of a population parameter is less than, greater than, or not equal to the value stated in the null hypothesis.

Suppose that children in the United States watch an average of 3 hours of tv per week with a population standard deviation of 1.5.

H_0 : children in the United States watch an average of 3 hours of TV per week.
H_1 : children watch more than (>) or less than (<) 3 hours of TV per week or not equal to (≠) 3 hours.

Example :1

- It is believed that a candy machine makes chocolate bars that are on average 5g. A worker claims that the machine after maintenance no longer makes 5g bars. Write H_0 and H_1.

$$H_0 : \mu = 5g$$

$$H_1 : \mu \neq 5g$$

H_0 and H_1 are mathematical opposites.

Four steps for hypothesis testing

- Step 1: State the hypotheses. (null and alternative)
- Step 2: Set the criteria (level of significance) for a decision.
- Step 3: Compute the test statistic.
- Step 4: Make a decision.

STATE/SET/COMPUTE/MAKE

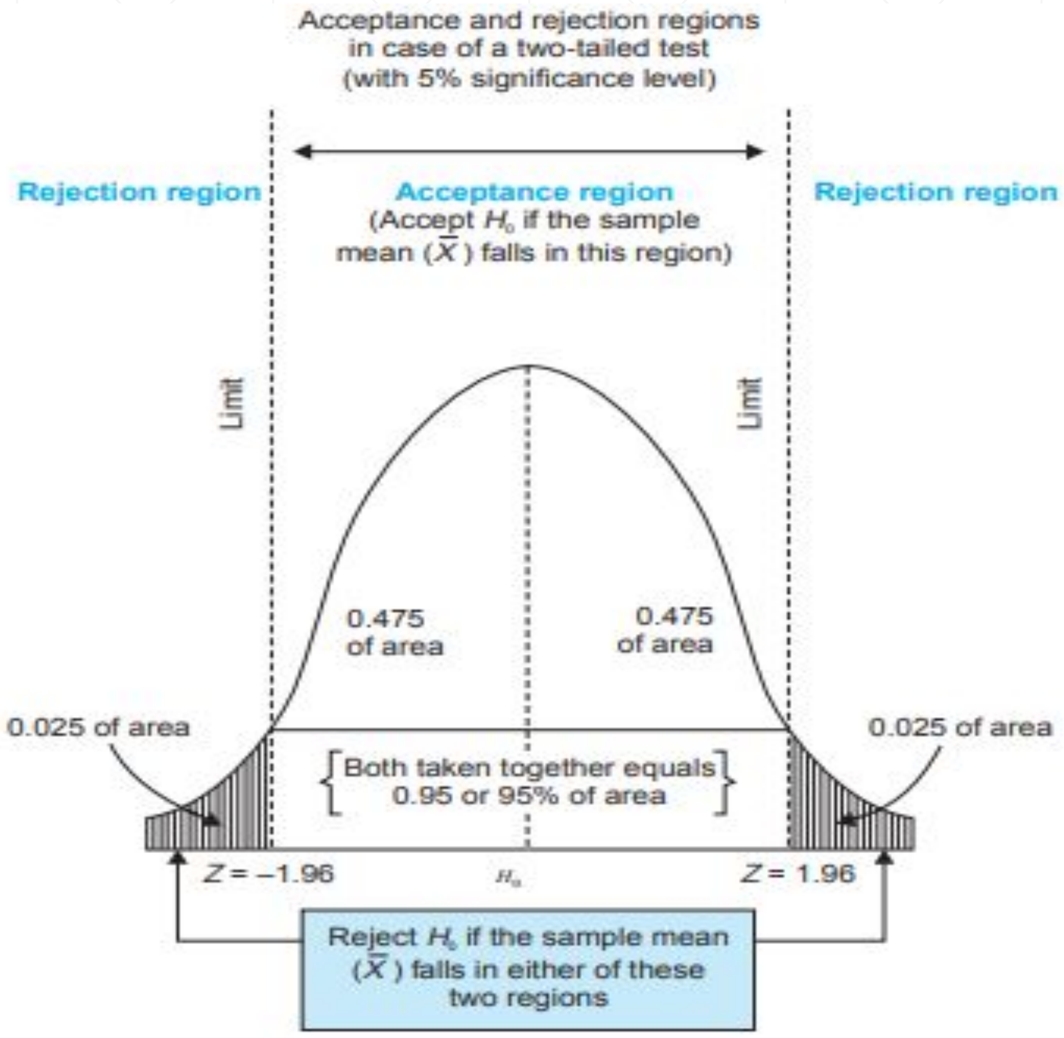
Directional & Non-Directional

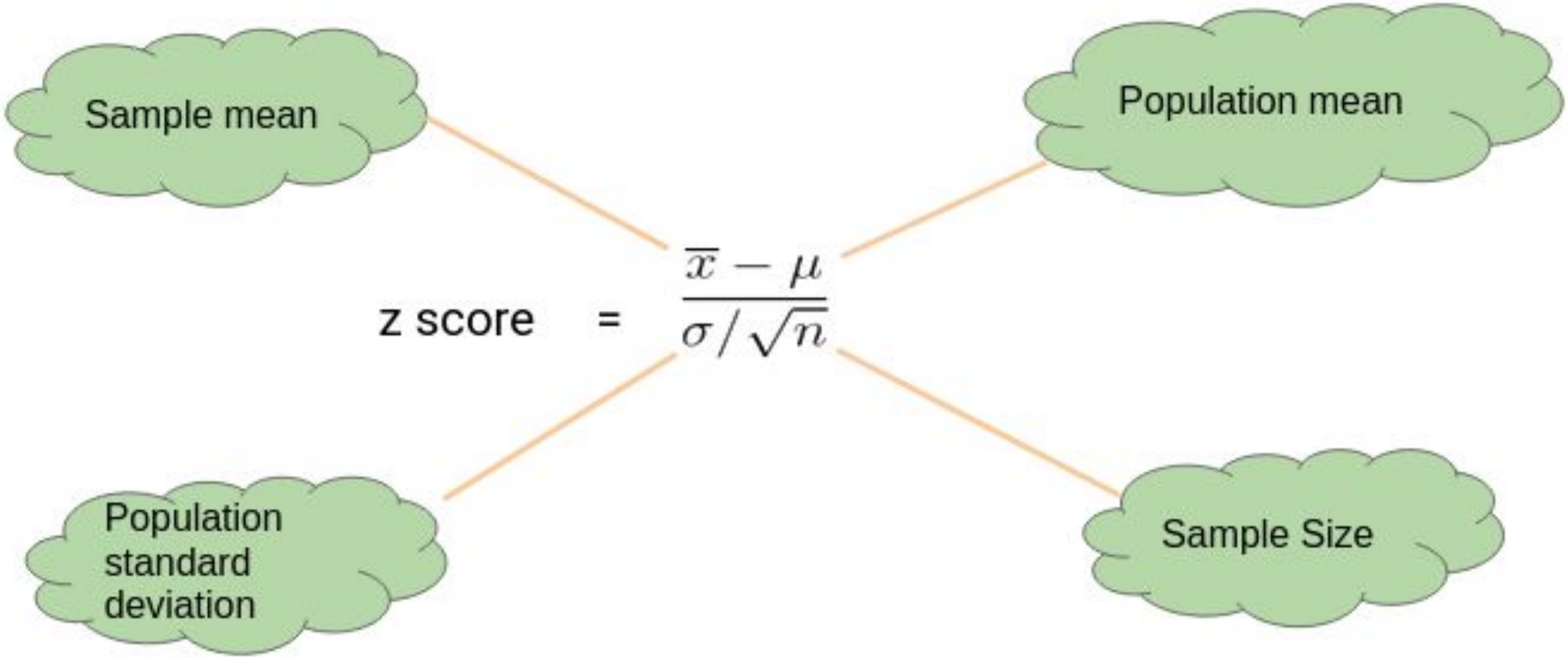
- The hypothesis is directional
  - **Null hypothesis**: *All adults sleep 7 hours a day*
  - **Alternative hypothesis**: *All adults sleep more than 7 hours a day*
  - **Null hypothesis**: *All adults sleep 7 hours a day*
  - **Alternative hypothesis**: *All adults sleep less than 7 hours a day*
- The hypothesis is non-directional
  - **Null hypothesis**: *All adults sleep 7 hours a day*
  - **Alternative hypothesis**: *All adults do not sleep 7 hours a day*

Test statistic – Z-Score

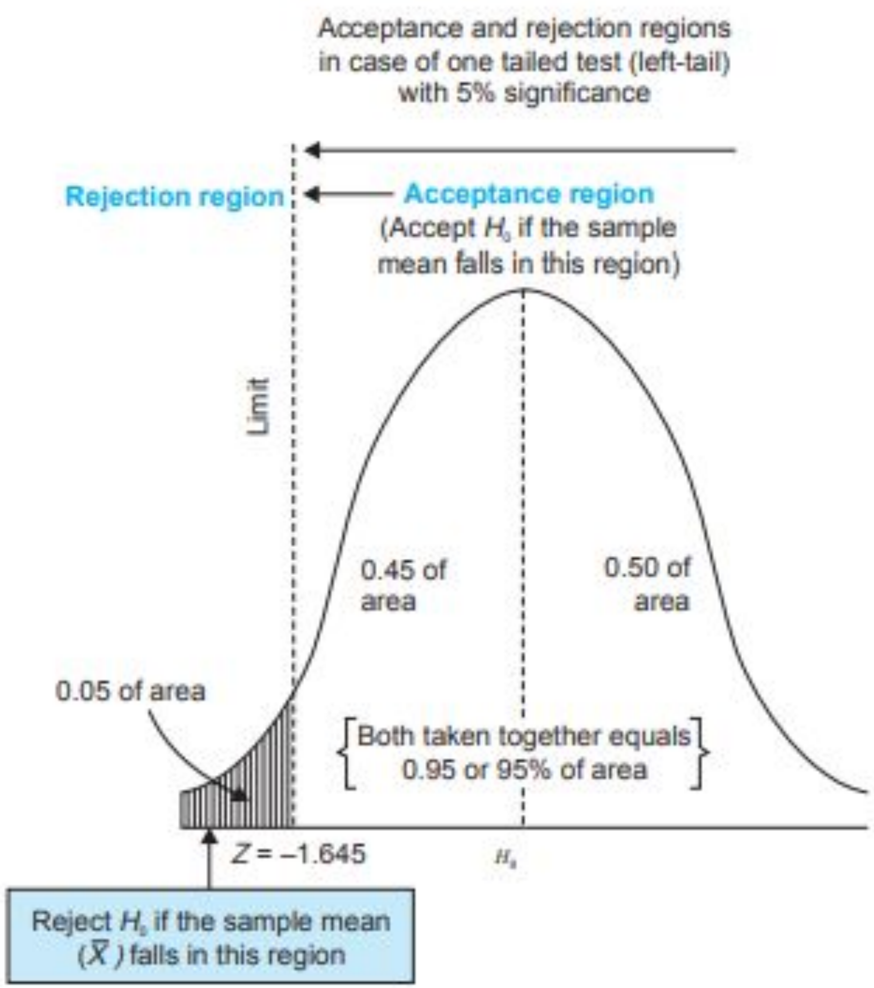
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

C	α	One tail test	Two tail test
0.90	0.10	1.28	± 1.645
0.95	0.05	1.645	± 1.96
0.98	0.02	2.05	± 2.33
0.99	0.01	2.33	± 2.575

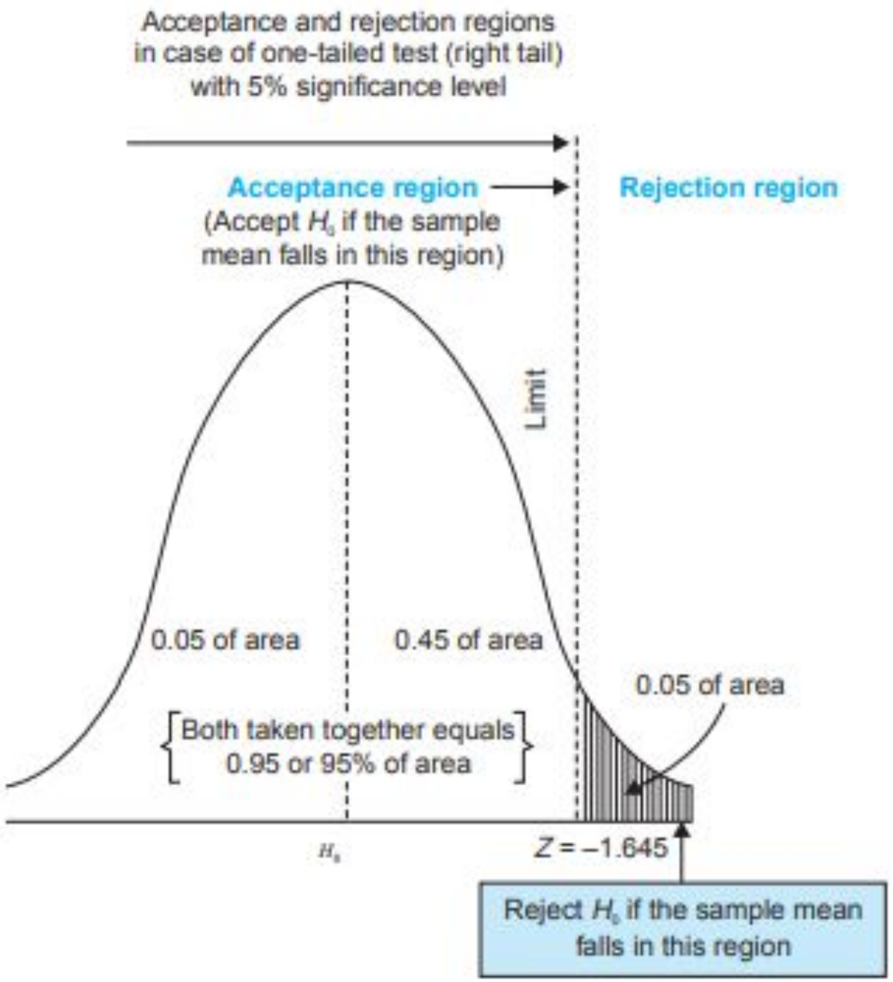



Sample mean
Population mean
z score = $\frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$
Population standard deviation
Sample Size

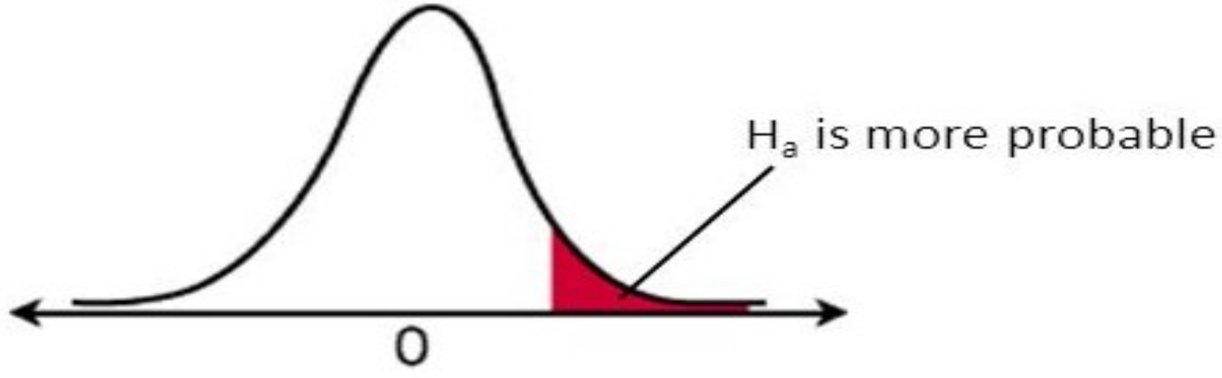
Critical Values Z_α	Level of significance (α)			
	1%	2%	5%	10%
Two-tailed test	$\lvert Z_\alpha \rvert=2.58$	$\lvert Z_\alpha \rvert=2.33$	$\lvert Z_\alpha \rvert=1.96$	$\lvert Z_\alpha \rvert=1.645$
Right tailed test	$Z_\alpha=2.33$	$Z_\alpha=2.055$	$Z_\alpha=1.645$	$Z_\alpha=1.28$
Left tailed test	$Z_\alpha= -2.33$	$Z_\alpha= -2.055$	$Z_\alpha= -1.645$	$Z_\alpha= -1.28$



Acceptance and rejection regions
in case of one tailed test (left-tail)
with 5% significance
Rejection region
Acceptance region
(Accept H_0 if the sample
mean falls in this region)
Limit
0.45 of
area
0.50 of
area
0.05 of area
Both taken together equals
0.95 or 95% of area
$Z = -1.645$
H_0
Reject H_0 if the sample mean
($\overline{X}$) falls in this region

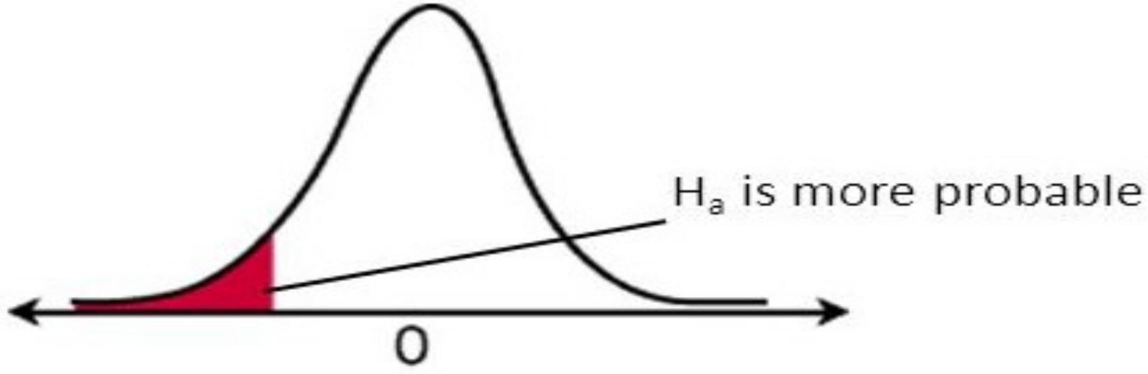


Acceptance and rejection regions
in case of one-tailed test (right tail)
with 5% significance level
Acceptance region
(Accept H_0 if the sample
mean falls in this region)
Rejection region
Limit
0.05 of area
0.45 of area
0.05 of area
Both taken together equals
0.95 or 95% of area
H_0
$Z = -1.645$
Reject H_0 if the sample mean
falls in this region



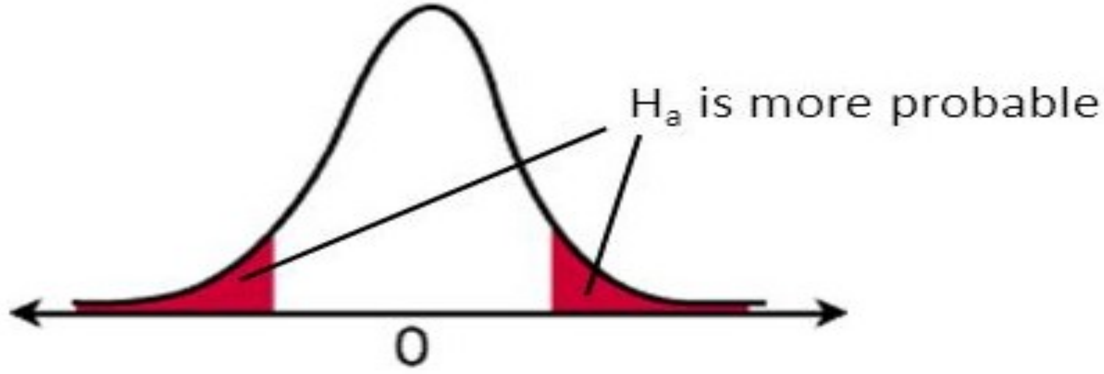
Right-tail test

H_a: $\mu >$ value



Left-tail test

H_a: $\mu <$ value



Two-tail test

H_a: $\mu \neq$ value

1. A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance

2. The mean of a certain production process is known to be 50 with a standard deviation of 2.5. The production manager may welcome any change is mean value towards higher side but would like to safeguard against decreasing values of mean. He takes a sample of 12 items that gives a mean value of 48.5. What inference should the manager take for the production process on the basis of sample results? Use 5 per cent level of significance for the purpose.

3. An alternator manufacturer must produce its alternators so that they are 95% confident that it runs at less than 71.1^{o} C under stress test in order to meet the production requirements for sale to the US government. The stress test is performed on random samples drawn from the production line on a daily basis. Today's sample of 7 alternators has a mean of 71.3^{o} C and a standard deviation of 0.214^{o} C. Is there a production quality issue?

4. Suppose that children in the United States watch an average of 3 hours of tv per week with a population standard deviation of 1.5. To test whether is true for preschool children a researcher records the number of hours a sample of preschool children (N= 36) watch tv and finds their average to be M=4.5 hours. Based on the data, what should be concluded?

5. Boys of a certain age are known to have a mean weight of $\mu = 85$ pounds. A complaint is made that the boys living in a municipal children's home are underfed. As one bit of evidence, $n = 25$ boys (of the same age) are weighed and found to have a mean weight of $\bar{x} = 80.94$ pounds. It is known that the population standard deviation σ is 11.6 pounds. Based on the available data, what should be concluded concerning the complaint?

6. A teacher wanted to know how well the gifted students in here class perform relative to her other classes. She administers a standardized test with a mean of 50 and standard deviation of 10. Her class of 31 students has an average score of 55. Based on the data, what should be concluded?

7. A rental car company claims the mean time to rent a car on their website is 60 seconds with a standard deviation of 30 seconds. A random sample of 36 customers attempted to rent a car on the website. The mean time to rent was 75 seconds. Based on the data, what should be concluded?

8. The life span of 100 W light bulbs manufactured by a particular company follows a normal distribution with a standard deviation of 120 hours and its half-life is guaranteed under warranty for a minimum of 800 hours. At random, a sample of 50 bulbs from a lot is selected and it is revealed that the half-life is 750 hours. With a significance level of 0.01, should the lot be rejected by not honoring the warranty?

9. A manufacturer of electric lamps is testing a new production method that will be considered acceptable if the lamps produced by this method result in a normal population with an average life of 2,400 hours and a standard deviation equal to 300. A sample of 100 lamps produced by this method has an average life of 2,320 hours. Can the hypothesis of validity for the new manufacturing process be accepted with a risk equal to or less than 5%?

10. The quality control division of a factory that manufactures batteries suspects defects in the production of a model of mobile phone battery which results in a lower life for the product. Until now, the time duration in phone conversation for the battery followed a normal distribution with a mean of 300 minutes and a standard deviation of 30. However, in an inspection of the last batch produced before sending it to market, it was found that the average time spent in conversation was 290 minutes in a sample of 60 batteries. Assuming that the time is still normal with the same standard deviation: Can it be concluded that the quality control suspicions are true at a significance level of 1%?