# Module - 4

# **Propositional Logic- Basic Connectives** and Truth Tables

8:28:09 AM

# **Proposition**

- A proposition is a declarative sentence that is either true or false but not both
- Example: Delhi is capital of India True
- 2+5=6 [False]
- America is neighboring country of India [False]
- 10+12 = 22 [7rue]

Why are you not responding?

### **Atomic statement**

- Declarative sentences which cannot be further split into simpler sentences are called atomic statements (also called primary statements or primitive statements)
- Discrete Mathematics is one of the subject for Computer Science students

### Logical Connectives - Compound propositions -Conditional and Biconditional propositions - Truth **Tables**

S.No	English Language Usage	Logical Connectives	Type of Operator	Symbols
1.	and	Conjunction	binary	^
2.	Or	Disjunction	Binary	V
3.	Not	Negation	Unary	Γ
4.	Ifthen	Implication (or) Conditional	Binary	<b>→</b>
5.	If and only if	Biconditional	Binary	<==>

### **Compound Proposition**

- Many mathematical statements are constructed by combining one or more propositions new propositions called compound propositions are formed from existing proposition using logical operators.
- **Truth Table:** A table, giving the truth values of a compound statement interms of its component parts is called a Truth table.

# Negation ~ ☐

The negation of a statement is generally formed by introducing the word not at a proper place in the statement

The Truth table for the negation of a proposition							
P	P						
T	F						
F	Т						

P. Delhi is capital of India F

1P. Delhi is not

P. America is neighbour of India F

TP. America is not

TP. America is not

Eg: P: Today is Wednesday[True], P: Today is not Tuesday [False]
P: 3 < 2 [False], P: 3 > 2 [True]

Conjunction [A] [AND]

The conjunction of two statements P and Q is the statement  $P \wedge Q$  which is



Eg: P: It is sunny[True], Q: I feel very warm[True],  $P \wedge Q$ : It is sunny and I feel very warm [True]

P: 3+4 < 5[False], Q: -3 > -5[True], P  $\land$  Q: 3+4 < 5 and -3 > -5[False]

P: 2 < 6 [True], Q: 2+6=9 [False], P  $\land$  Q: 2 < 6 and 2+6=9 [False]

P: 3+5=6 [False], Q: 3-5=4[False], P  $\land$  Q: 3+5=6 and 3-5=4 [False]

### Disjunction [V] [OR]

The disjunction of two statements P  $\stackrel{\text{OY}}{\text{and}}$  Q is the statement P  $\vee$  Q which is

read as "P or Q"

The Truth table for the conjunction of two propositions							
P	Q	$P \vee Q$					
T	Т	T					
T	F	T					
F	Т	T					
F	F	F					

Eg: P: It is sunny[True], Q: I feel very warm[True],  $P \lor Q$ : It is sunny or I feel very warm [True]

P: 3+4 < 5[False], Q: -3 > -5[True], P  $\vee$  Q: 3+4 < 5 or -3 > -5[True]

P: 2 < 6 [True], Q: 2+6=9 [False],  $P \lor Q: 2 < 6$  or 2+6=9 [True]

P: 3+5=6 [False], Q: 3-5=4[False], P  $\vee$  Q: 3+5=6 or 3-5=4 [False]

98

Conditional Statement [→] [If, ... then]

If P and Q are any two statements then the statement  $P \rightarrow Q$  which is read as "if P, then Q" is

called a conditional statement.

The Truth table for the implication $P \rightarrow Q$						
P	Q	$P \rightarrow Q$				
T	T	T				
T	F	F				
F	T	T				
F	F	T				

Eg: P: I am hungry[True], Q: I will eat [True],  $P \rightarrow Q$ : If I am hungry then I will eat [True]

P: I studied B.Tech [False], Q: I will teach B.Tech [True],  $P \rightarrow Q$ : If I studied B.Tech then I will teach B.Tech [True]

P: The sun is shining today [True], Q: 2+8=6 [False], P  $\rightarrow$  QIf the sun is shining today then 2+8=6 [False]

### Biconditional [equivalence] Statement [\$] [If and only if]

If P and Q are any two statements then the statement P  $\Leftrightarrow$  Q which is read as "P if and only if Q" and abbreviated as "P iff Q" is called a biconditional statement. Is has the same truth value as  $(P \to Q) \land (Q \to P)$ .

The Truth table for the Biconditional P⇔Q						
P	Q	P ⇔ Q				
T	T	T	,			
T	F	F ,				
F	T	F /				
F	F	T	-			

Eg: P:You can take the flight[True], Q:You buy a ticket [True],  $P \Leftrightarrow Q$ :You can take the flight if and only if you buy a ticket.[True]

P: 2>3[False], Q: 4<5 [True], P \( \infty \) Q: 2>3 iff 4< 5[False]

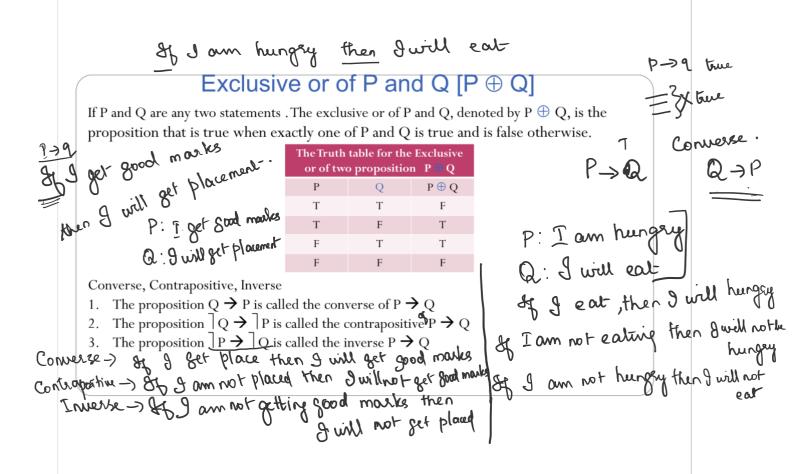
P: 5<6 [True], Q: 7>8 [False], P⇔ Q: 5<6 iff 7<8 [False]

P: 3 = 8-4 [False], Q: 3+4=8[False], P \(\Diffineq\) Q: 3=8-4 iff 3+4=8 [True]

 $P \iff Q$   $\Rightarrow (P \Rightarrow Q) \land$   $(Q \hookrightarrow P)$ 

P- Premise
- condution
- hypothesis

- conclusion
- implication
- implication



### Contrapositive

If  $P \rightarrow Q$  is an implication, then the converse of  $P \rightarrow Q$  is the implication  $Q \rightarrow P$ , and the contrapositive of  $P \rightarrow Q$  is the implication  $|Q \rightarrow P|$ .

Example 1: Give the converse and contrapositive of the implication "If it is raining, then I get wet".

Solution: P: It is raining, Q: I get wet,  $Q \rightarrow P(Converse)$ : If I get wet then it is raining  $Q \rightarrow |P(Contrapositive)|$ : If I do not get wet, then it is not raining.

Example 2: State the converse, contrapositive and inverse of the following: "A Positive integer is a prime only if it has no divisors other than 1 and itself"

Solution: Converse: A positive integer is a prime if it has no divisors other than 1 and itself

Contrapositive: If a positive integer has a divisor other than 1 and it self then it is not

Inverse: If a positive integer is not prime then it has a divisor other than 1 and itself

is prime then it has TP > 7 a the integer is no a prime then it has is prime the TP > 7 a divisors other 14 itself no divisors other than 14 itself involve & a prime then it is priot a prime no divisor other tham 1 4 itself

Pal

has nodivisors Other Than It itself

Q 9+ is a prime

# Contrapositive

Example: State the converse, contrapositive and inverse of " P: 9 - in well and inverse of " P Q: I will weak sweater If it is very cold the I will wear sweater" Q > P Solution:

1. Converse: I will wear sweater only if it is very cold

Off I we ar sweeter then it will be nery cold 2. Contrapositive: If I do not wear sweater then it is not very @7Q->7P cold.

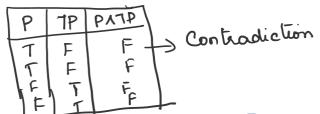
3. Inverse: If it is not very cold, then I will not wear sweater.

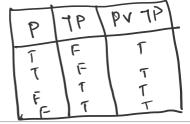
Sweath then it is not very cold then I will not very cold then I will not very cold then I will not ween sweater

### **Truth Table**

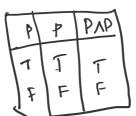
Conditio ion ional positive Q P→Q P⇔Q  $P \rightarrow Q$  $Q \rightarrow P$ P∧Q P∨Q T / F Т F

Problem 1: Construct the truth table P ∧ P





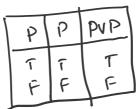
Problem 3: Construct the truth table P ^ P



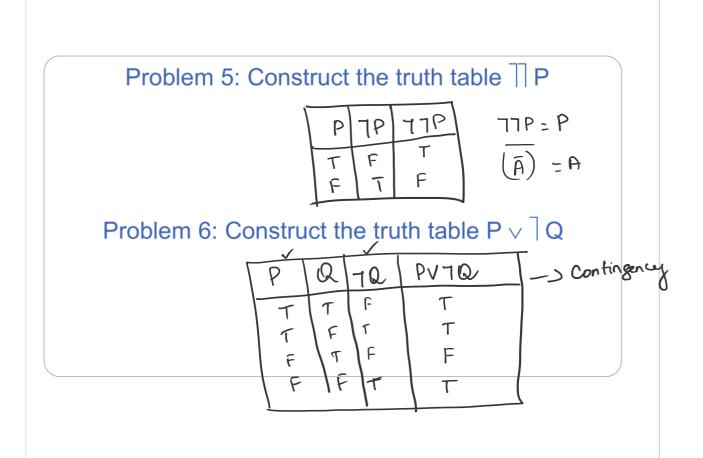
PAP=P

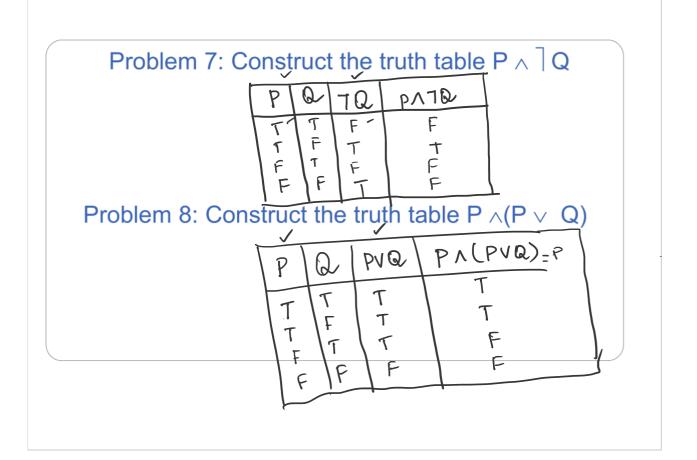
ACA:A

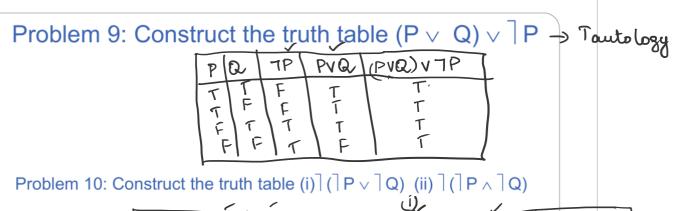
Problem 4: Construct the truth table P v P



PVP=P AUA=A







	_						
LP.	Ø	72	72	7PV7Q	7 (7PV7Q)	GENGE	1(7PATQ)
TTF	T F T	F F	FTF	FFF	T	F	T
1 +	IF	T	7	T	-	<u> </u>	F

Problem	11: Co	onstru	ct the	truth ta	able ( P	<sup>2</sup> ∧(	R)∫∨((Q∧R)∨(F	P ^ R))	- (1)		
	0	0	0,	×	<u> </u>	x	0			69	
3	P	0	R	7P	70	7Q 12	TPA (TQAR	(QAR	PAF		3
	TTTFFFF	TTFFTTF	THTHTH	中、中、中、下、丁、丁丁丁	に F T T E F T T	かけ トード ままして よ	11 11 11 11 11 11 11 11 11 11 11 11 11	FFFFFFF	TETFFFF	T F T F T F F F	T F T F T F T F

