

Bubble Sort

<https://www.hackerearth.com/practice/algorithms/sorting/bubble-sort/visualize/>

or

<https://visualgo.net/bn/sorting>

Sorting

- **Sorting takes an unordered collection and makes it an ordered one.**

1	2	3	4	5	6
77	42	35	12	10	5



1	2	3	4	5	6
5	1	3	4	7	10
	2	5	2	7	1

"Bubbling Up" the Largest Element

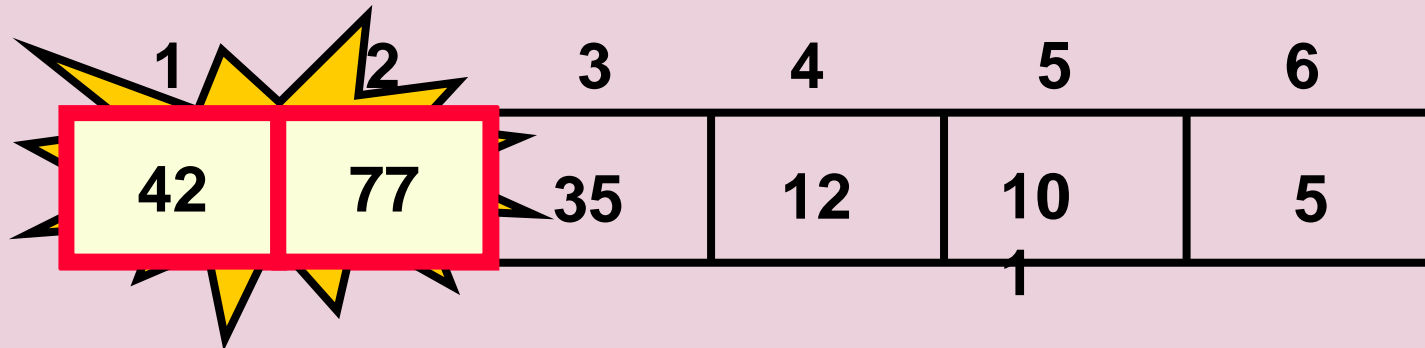
- Traverse a collection of elements
 - Move from the front to the end
 - “Bubble” the **largest value** to the end using **pair-wise comparisons and swapping**

1	2	3	4	5	6
77	42	35	12	10	5

↑

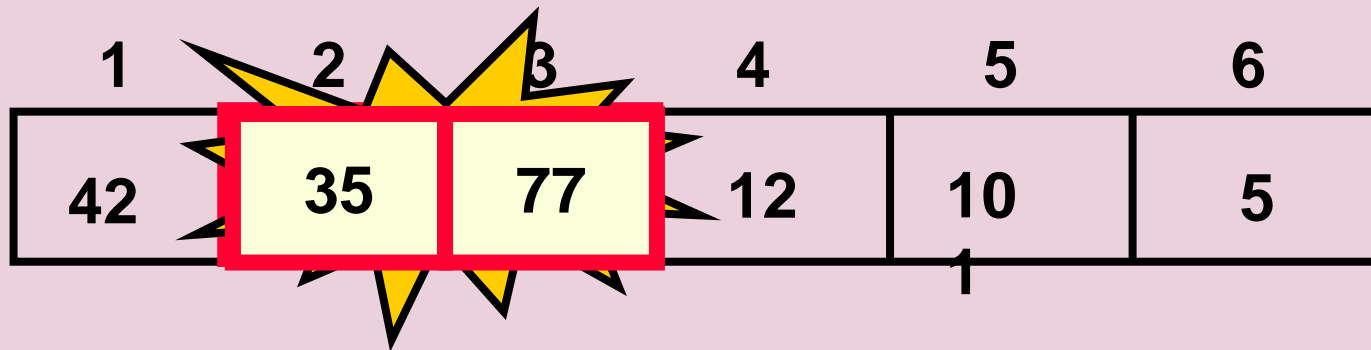
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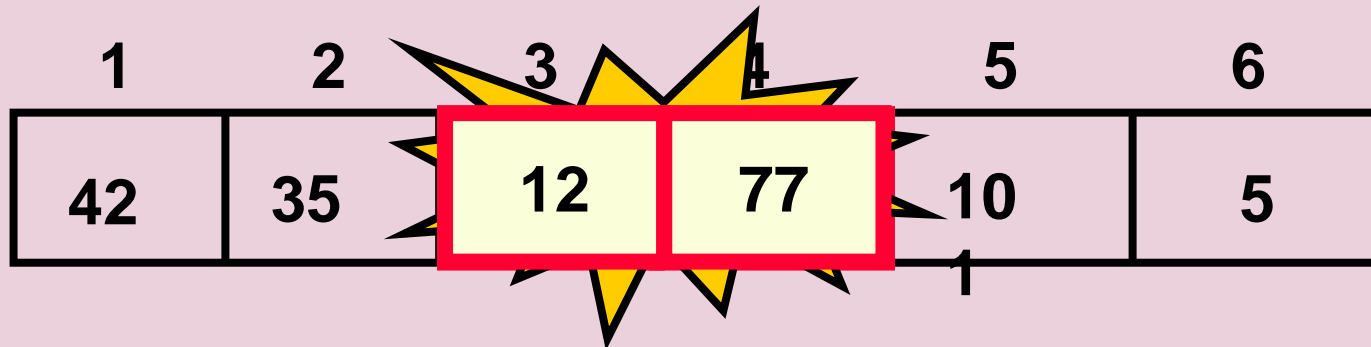
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"Bubbling Up" the Largest Element

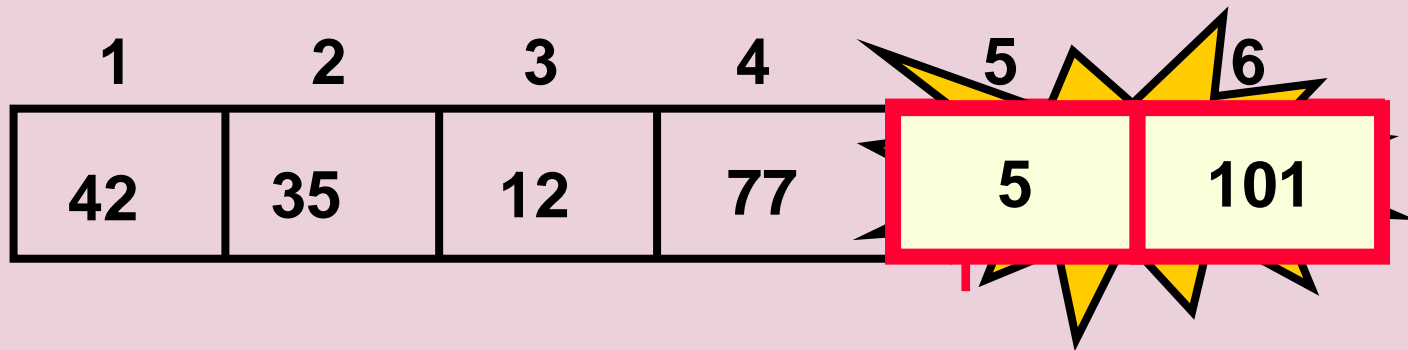
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1	2	3	4	5	6
42	35	12	77	10	5

No need to swap

"Bubbling Up" the Largest Element

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- Traverse a collection of elements
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1	2	3	4	5	6
42	35	12	77	5	101

Largest value correctly placed

Items of Interest

- Notice that only the largest value is correctly placed
- All other values are still out of order
- So we need to **repeat this process**

1	2	3	4	5	6
42	35	12	77	5	101

Largest value correctly placed

Repeat “Bubble Up” How Many Times?

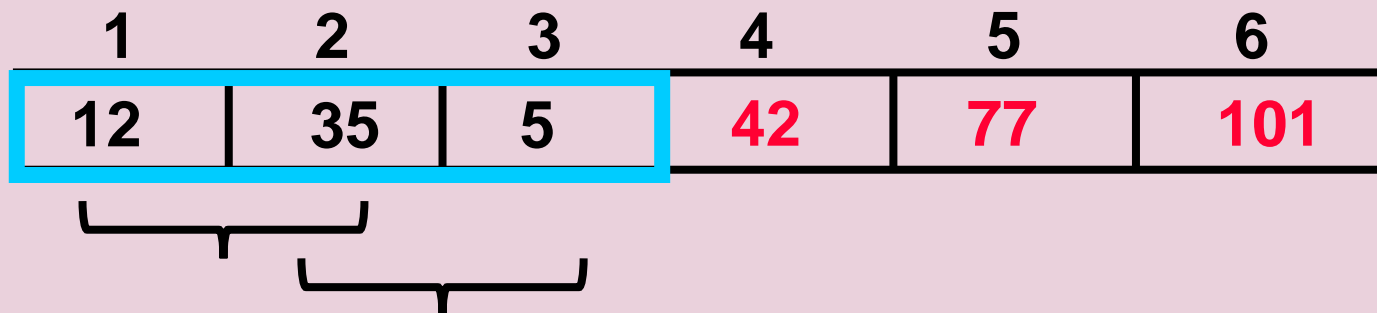
- If we have N elements...
- And if each time we bubble an element, we place it in its correct location...
- Then we repeat the “bubble up” process $N - 1$ times.
- This guarantees we'll correctly place all N elements.

“Bubbling” All the Elements

1	2	3	4	5	6
42	35	12	77	5	101
1	2	3	4	5	6
35	12	42	5	77	101
1	2	3	4	5	6
12	35	5	42	77	101
1	2	3	4	5	6
12	5	35	42	77	101
1	2	3	4	5	6
5	12	35	42	77	101

Reducing the Number of Comparisons

- On the N^{th} “bubble up”, we only need to do **MAX-N comparisons**.
- For example:
 - This is the 4th “bubble up”
 - MAX is 6
 - Thus we have **2 comparisons** to do



Already Sorted Collections?

- What if the collection was already sorted?
- What if only a few elements were out of place and after a couple of “bubble ups,” the collection was sorted?
- We want to be able to **detect this** and “**stop early**”!

1	2	3	4	5	6
5	12	35	42	77	101

Using a Boolean “Flag”

- We can use a boolean variable to determine if any swapping occurred during the “bubble up.”
- If no swapping occurred, then we know that the collection is already sorted!
- This boolean “flag” needs to be reset after each “bubble up.”

Bubble Sort Algorithm

```
bubbleSort(list,n) {
```

```
  for i (1 to n-1) {
```

```
    swapped = False
```

```
    for j (0 to n - i - 1) {
```

```
      if list[j] > list[j+1] {
```

```
        t = list[j]
```

```
        list[j] = list[j+1]
```

```
        list[j+1] = t
```

```
        swapped = True } }
```

```
  if not swapped:
```

```
    break } }
```

Inner loop

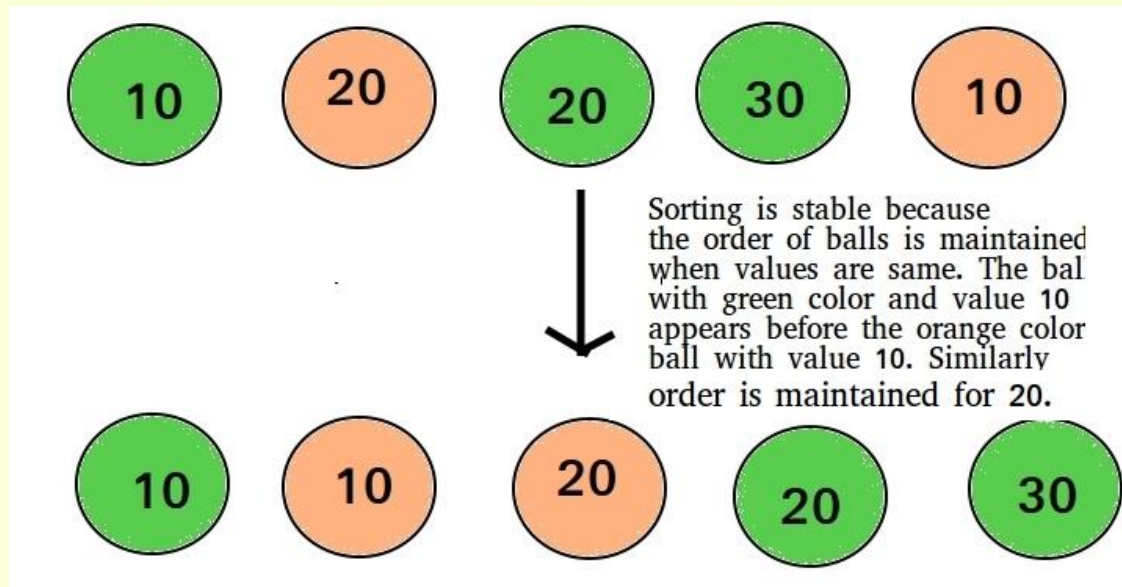
Outer loop

Time Complexity

Worst Case: $O(n^2)$

PROPERTIES

- Bubble sort is stable and in-place algorithm.
- **In-place** means that the input and output occupy the same memory
- **Stable** means the order of input elements is unchanged except where change is required to satisfy the requirements.



Summary

- “Bubble Up” algorithm will **move largest value to its correct location** (to the right)
- Repeat “Bubble Up” until all elements are correctly placed:
 - **Maximum of $N-1$ times**
 - Can finish early if **no swapping** occurs
- We reduce the number of elements we compare each time one is correctly placed

An Animated Example

N

8

 did_swap

true

to_do

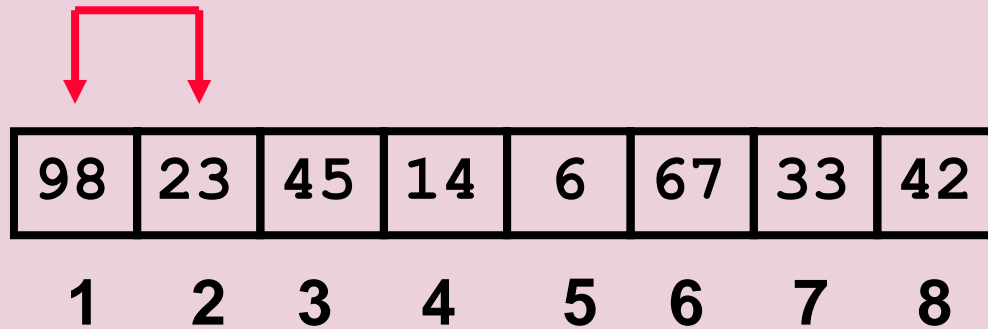
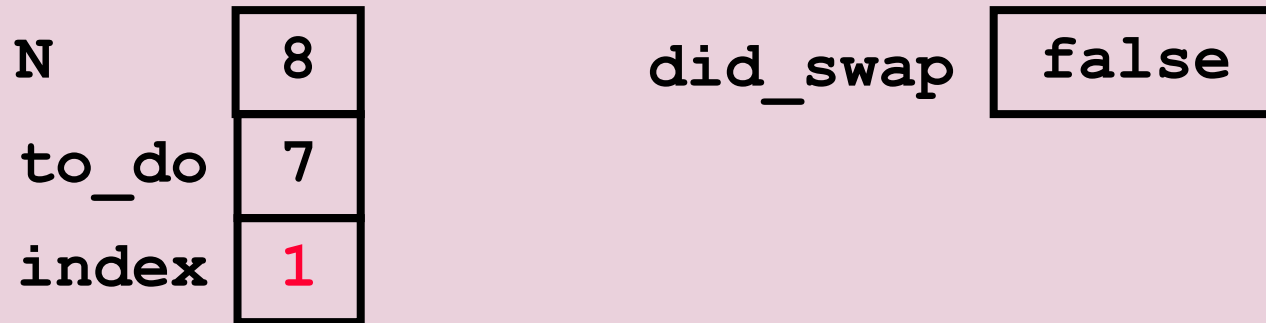
7

index

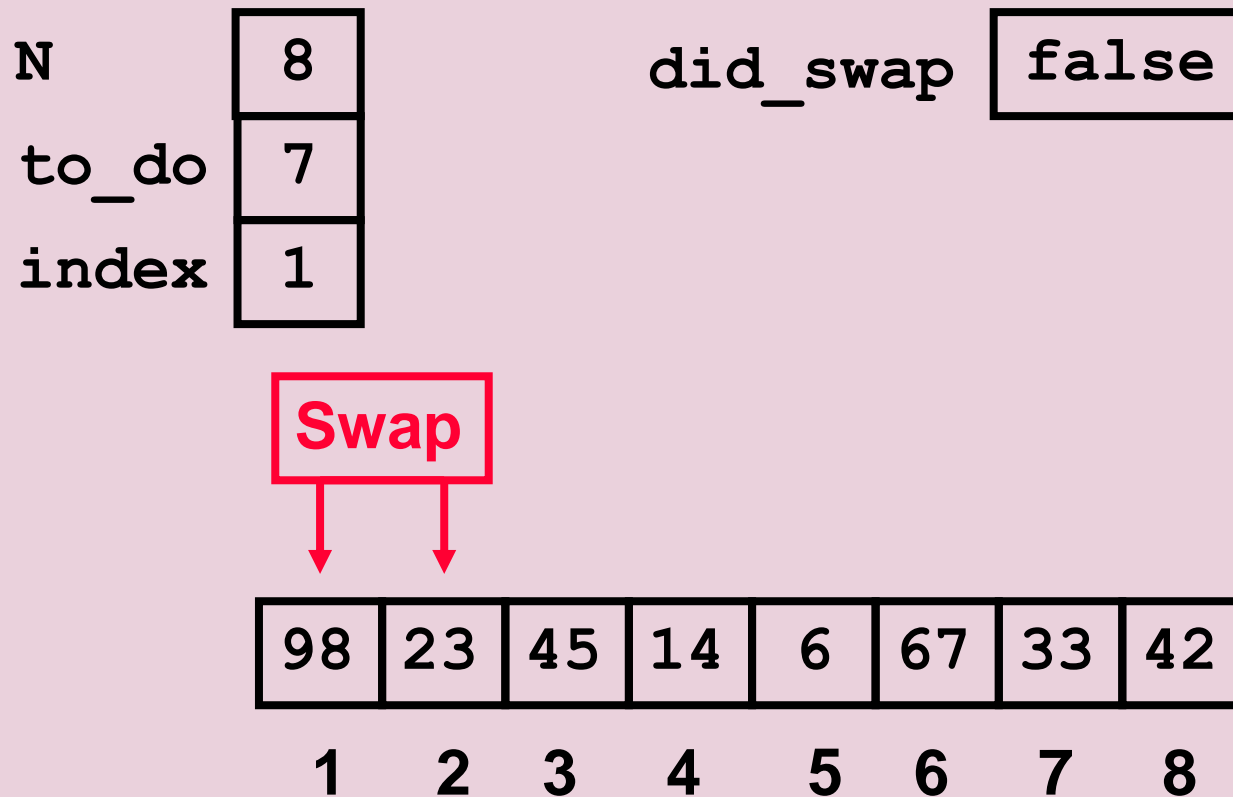
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98	23	45	14	6	67	33	42
1	2	3	4	5	6	7	8

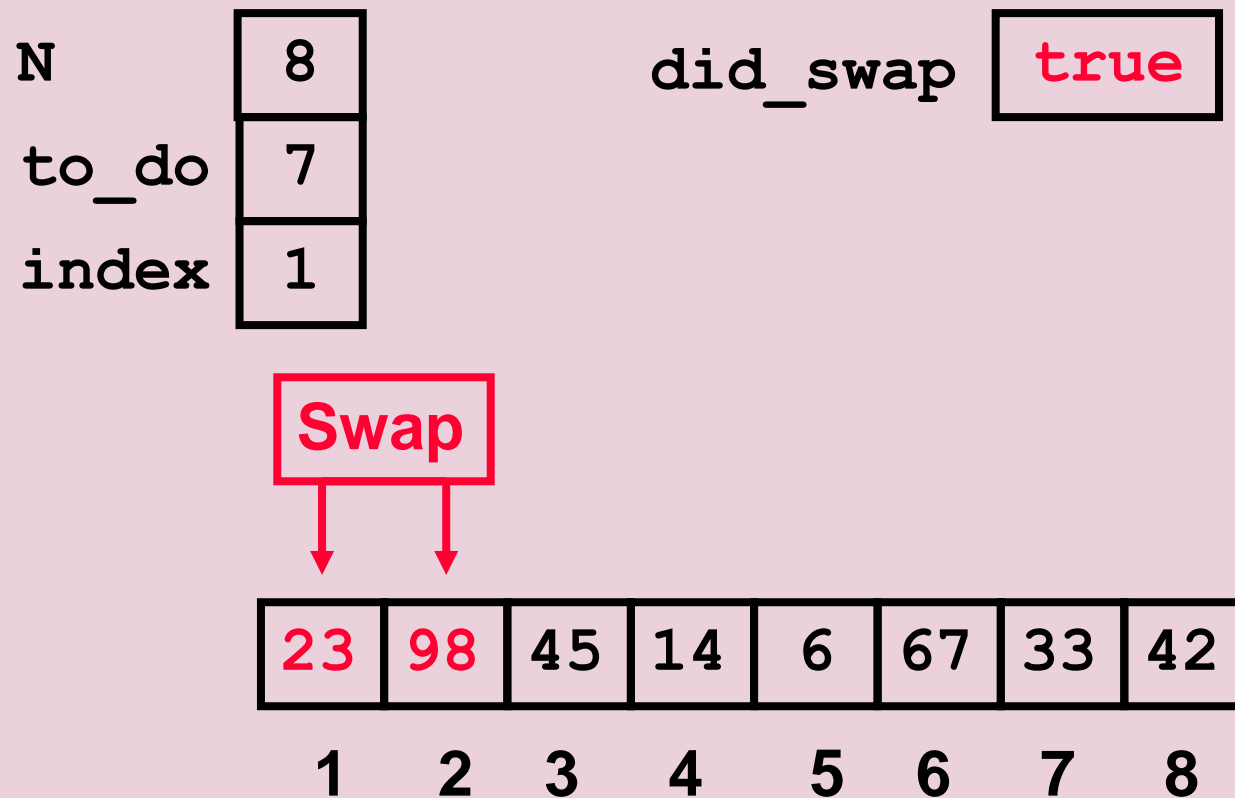
An Animated Example



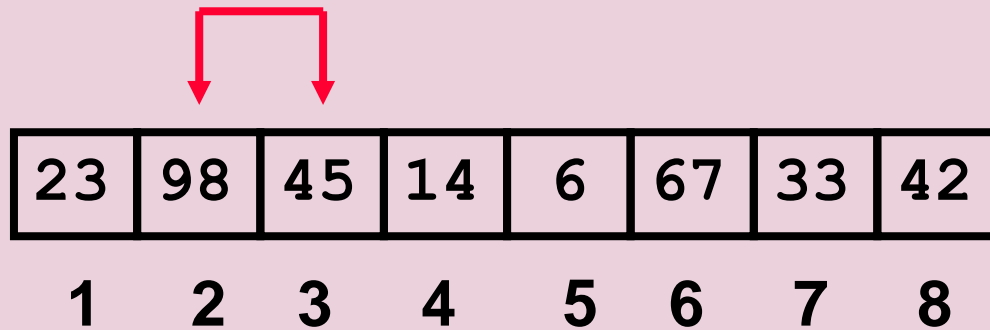
An Animated Example



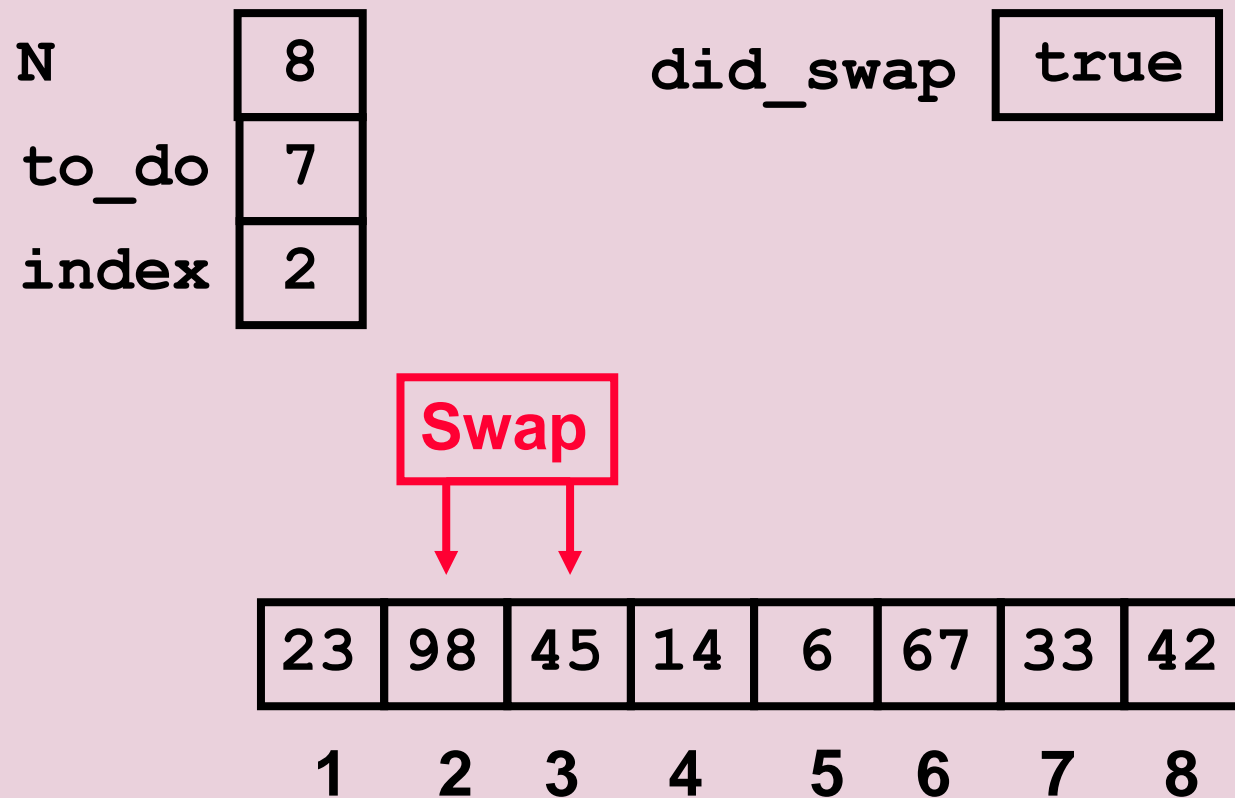
An Animated Example



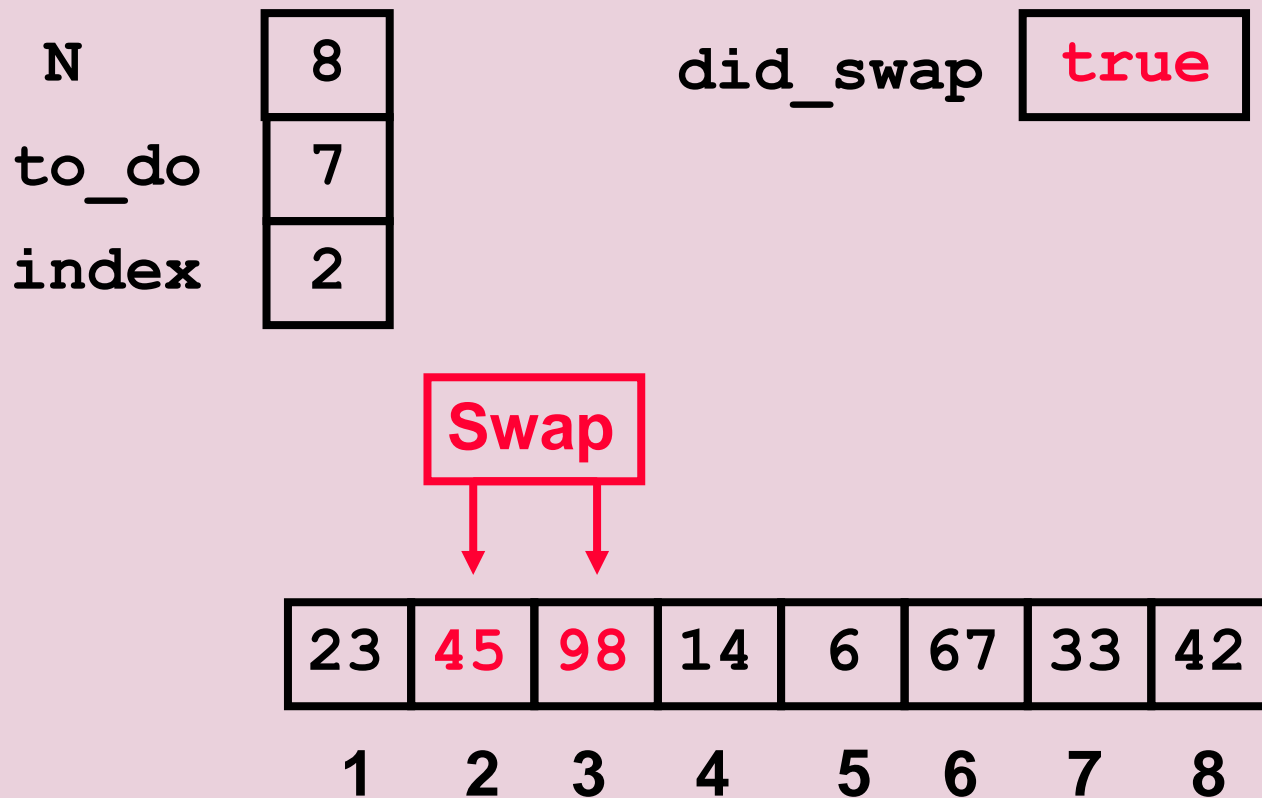
An Animated Example



An Animated Example



An Animated Example



An Animated Example

N

8

to_do

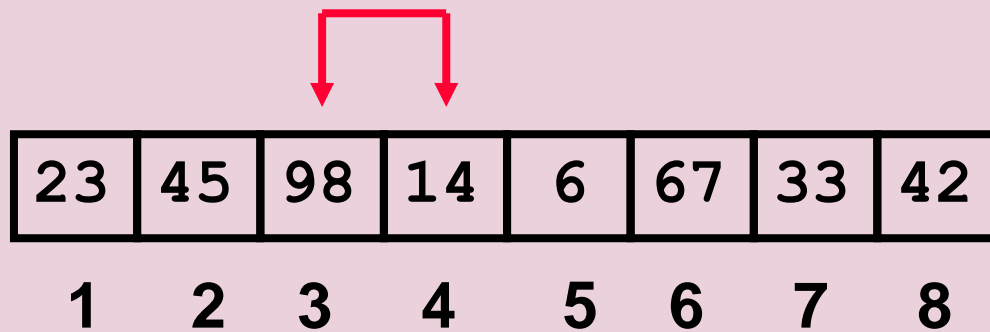
7

index

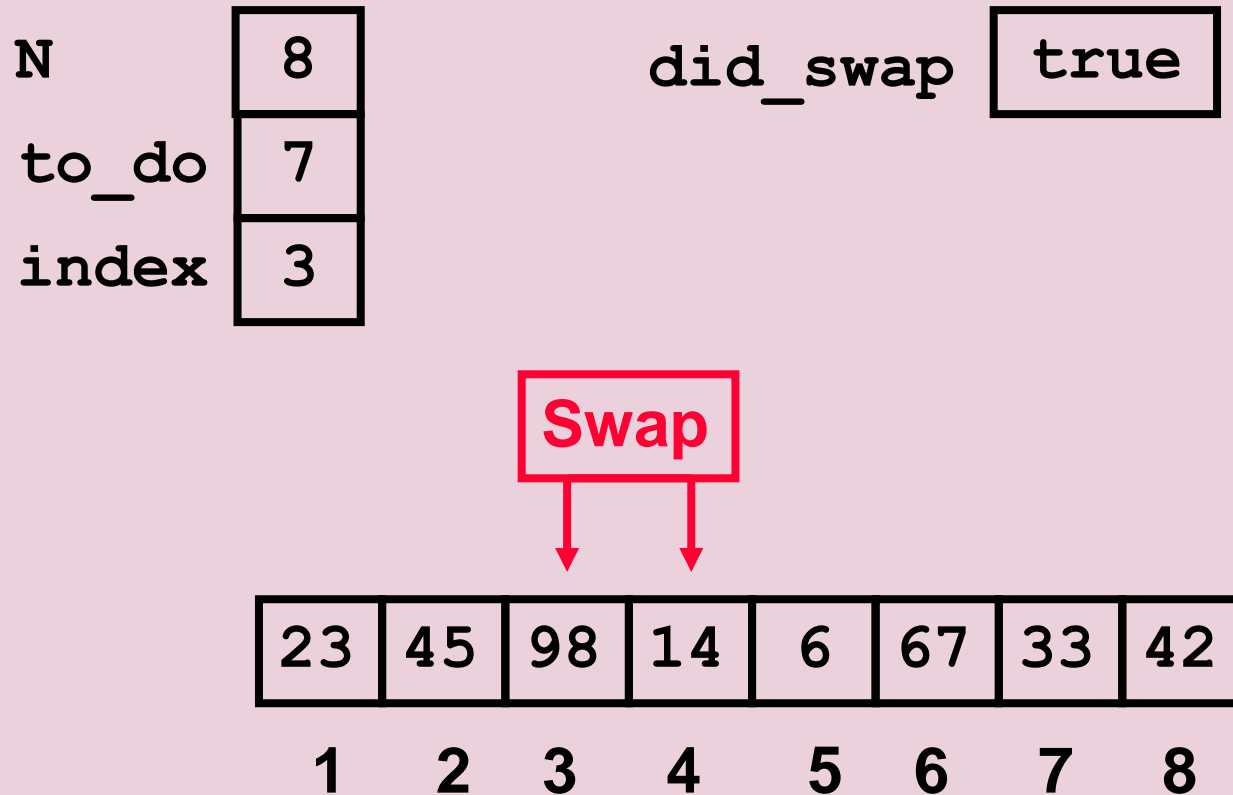
3

did_swap

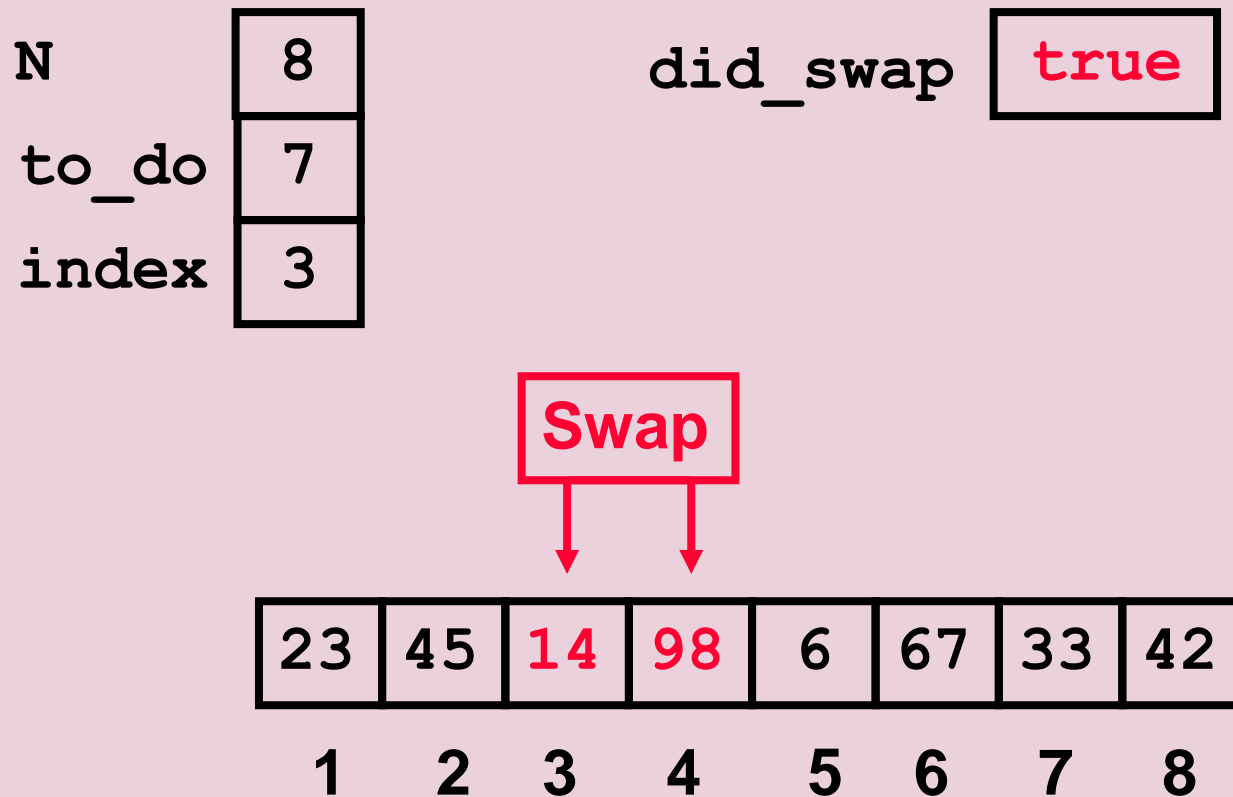
true



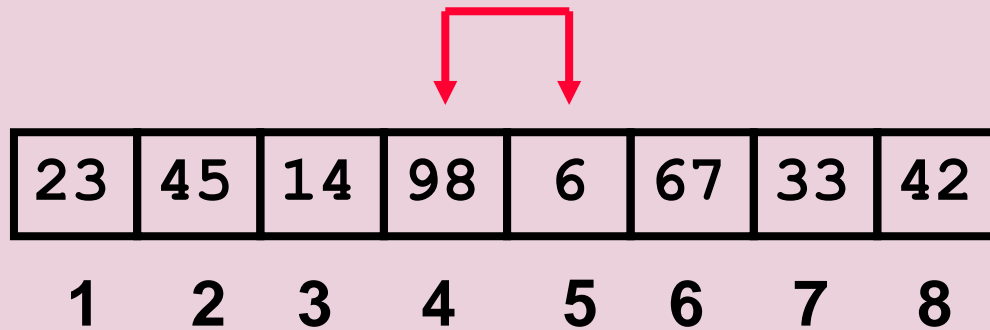
An Animated Example



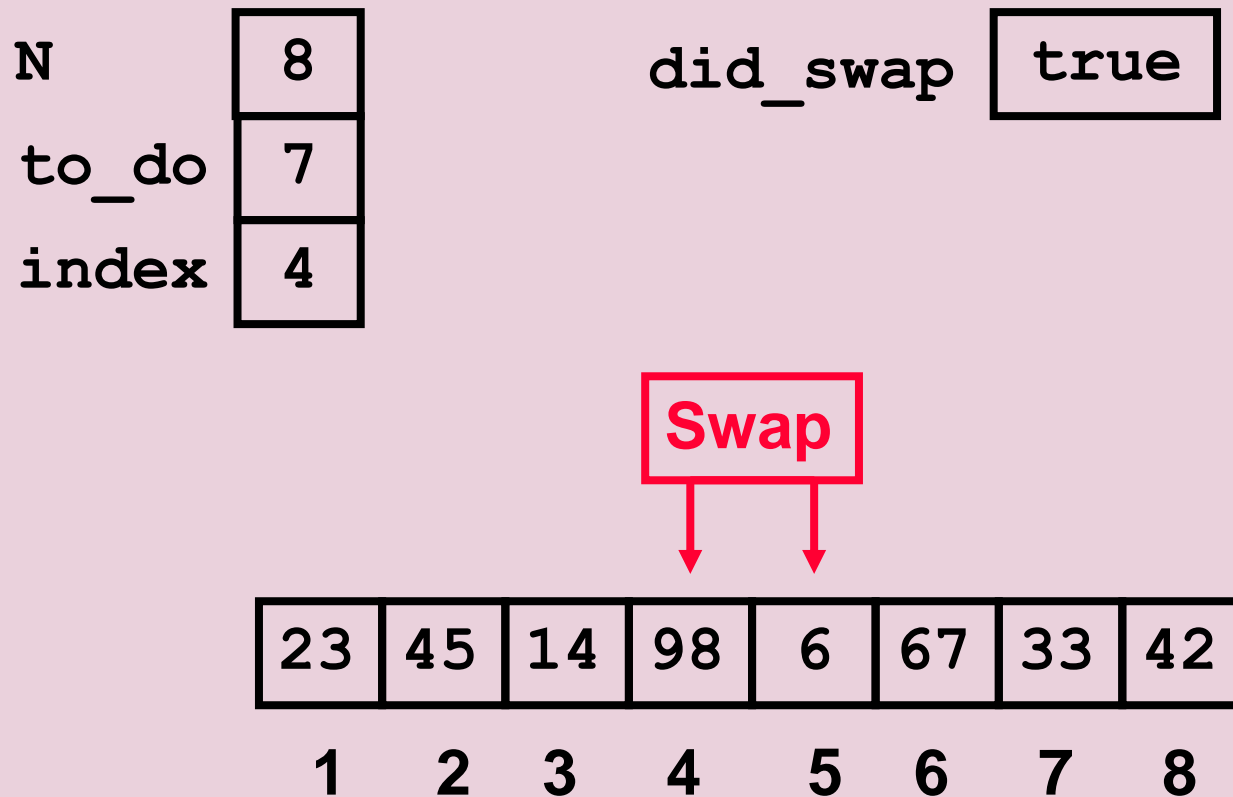
An Animated Example



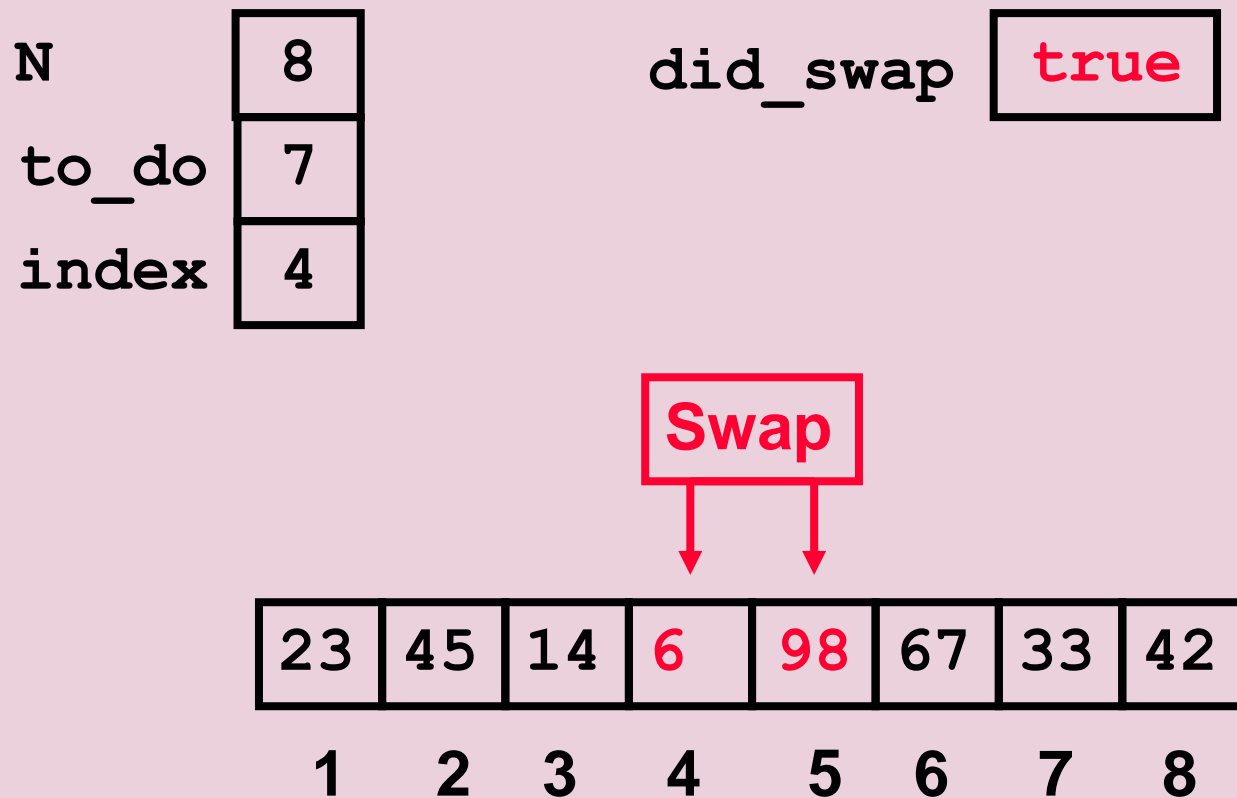
An Animated Example



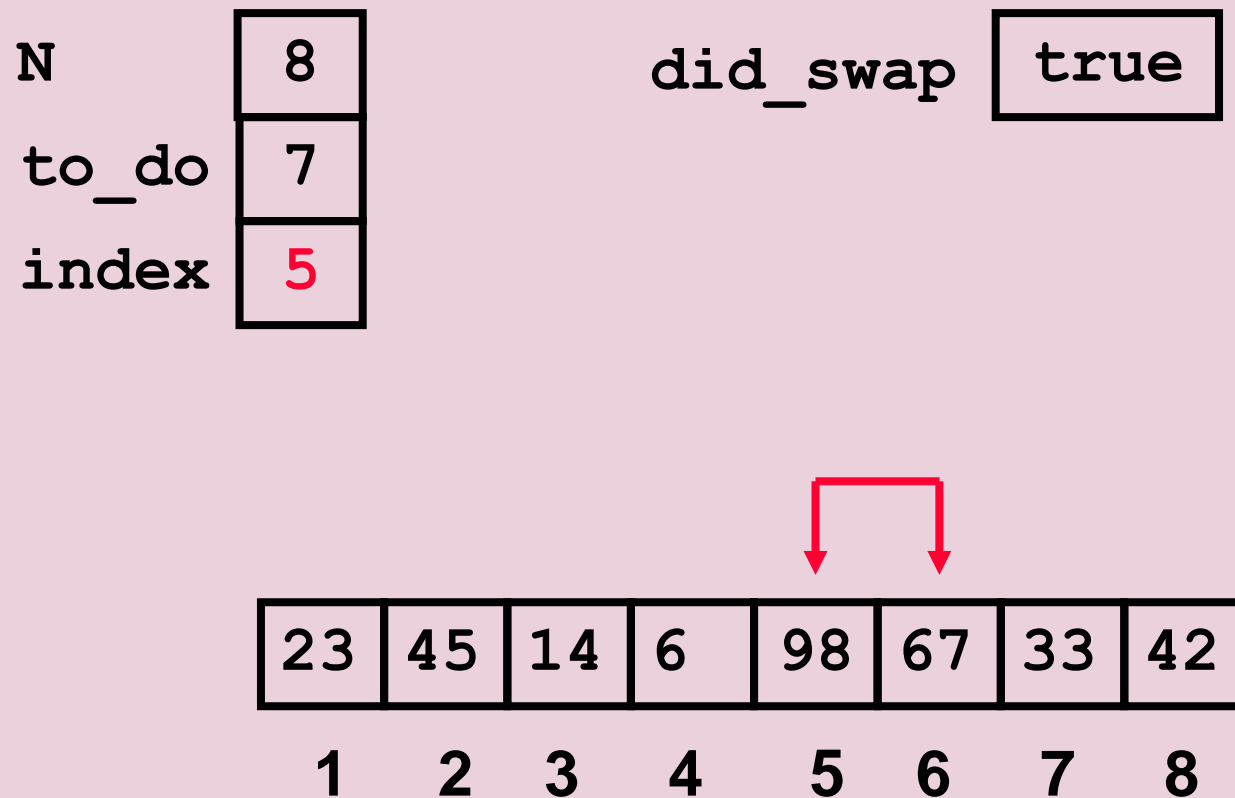
An Animated Example



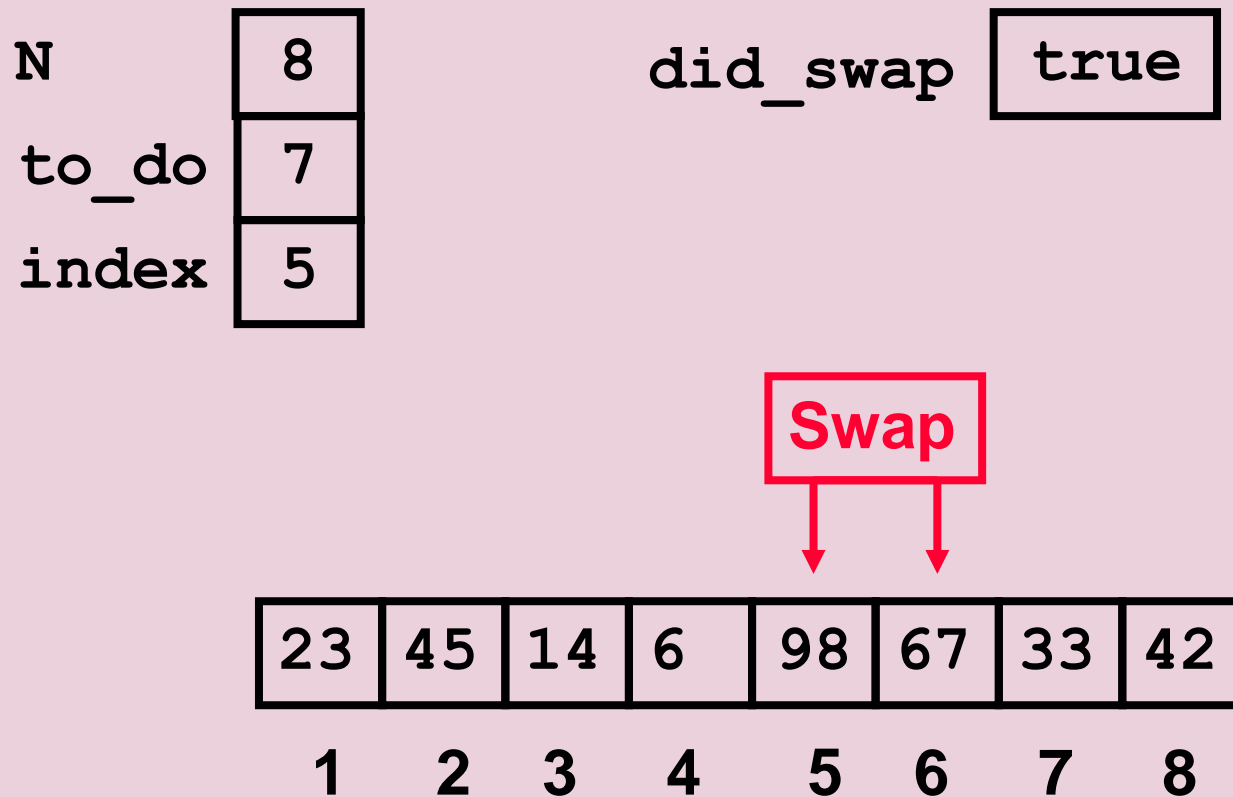
An Animated Example



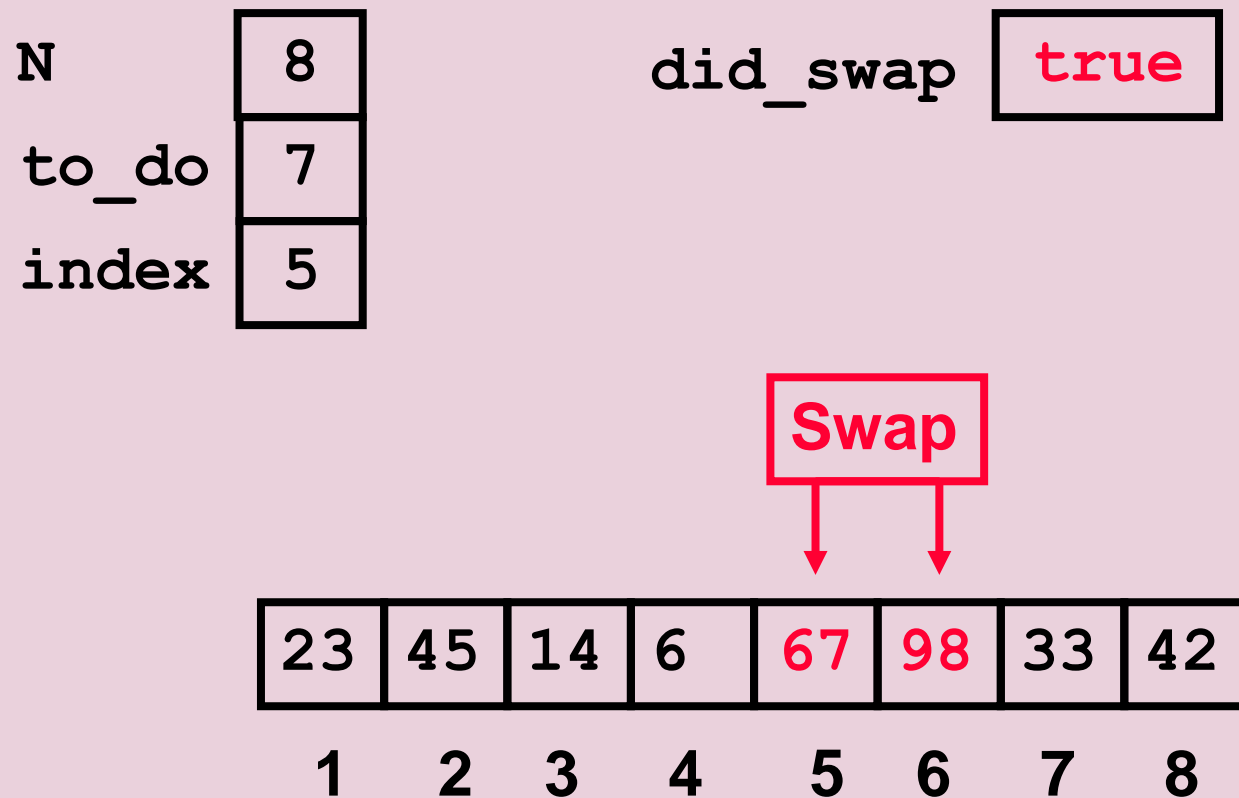
An Animated Example



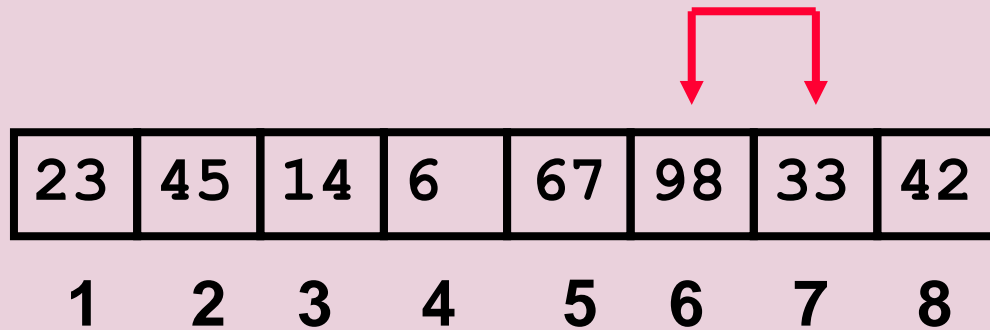
An Animated Example



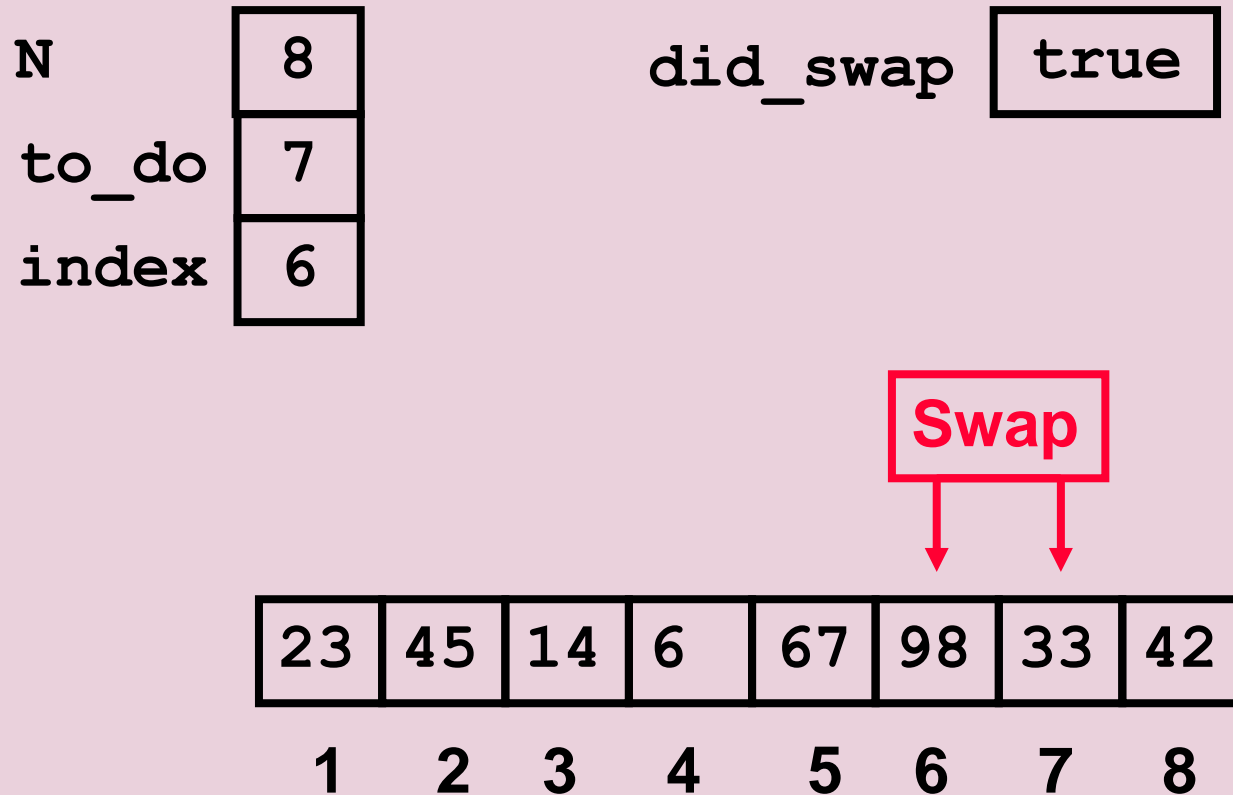
An Animated Example



An Animated Example



An Animated Example



An Animated Example

N
to_do
index

8
7
6

did_swap

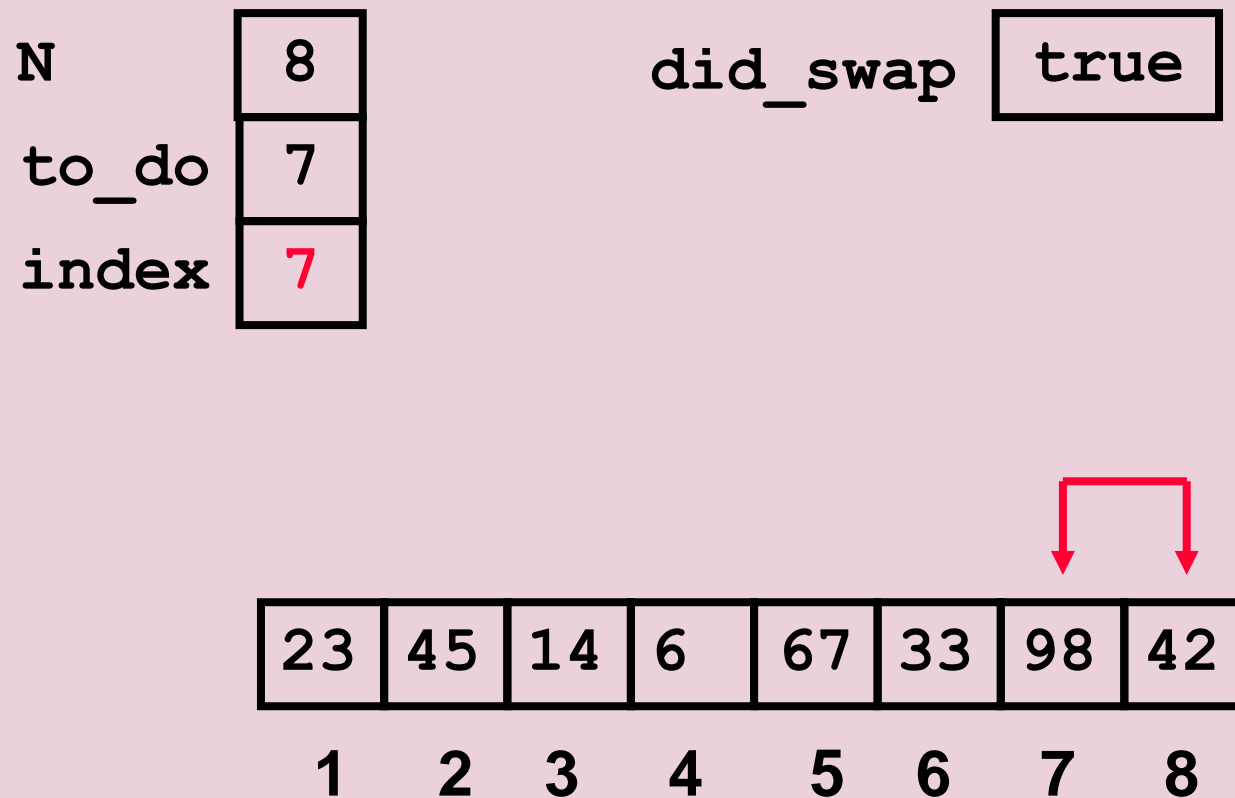
true

Swap

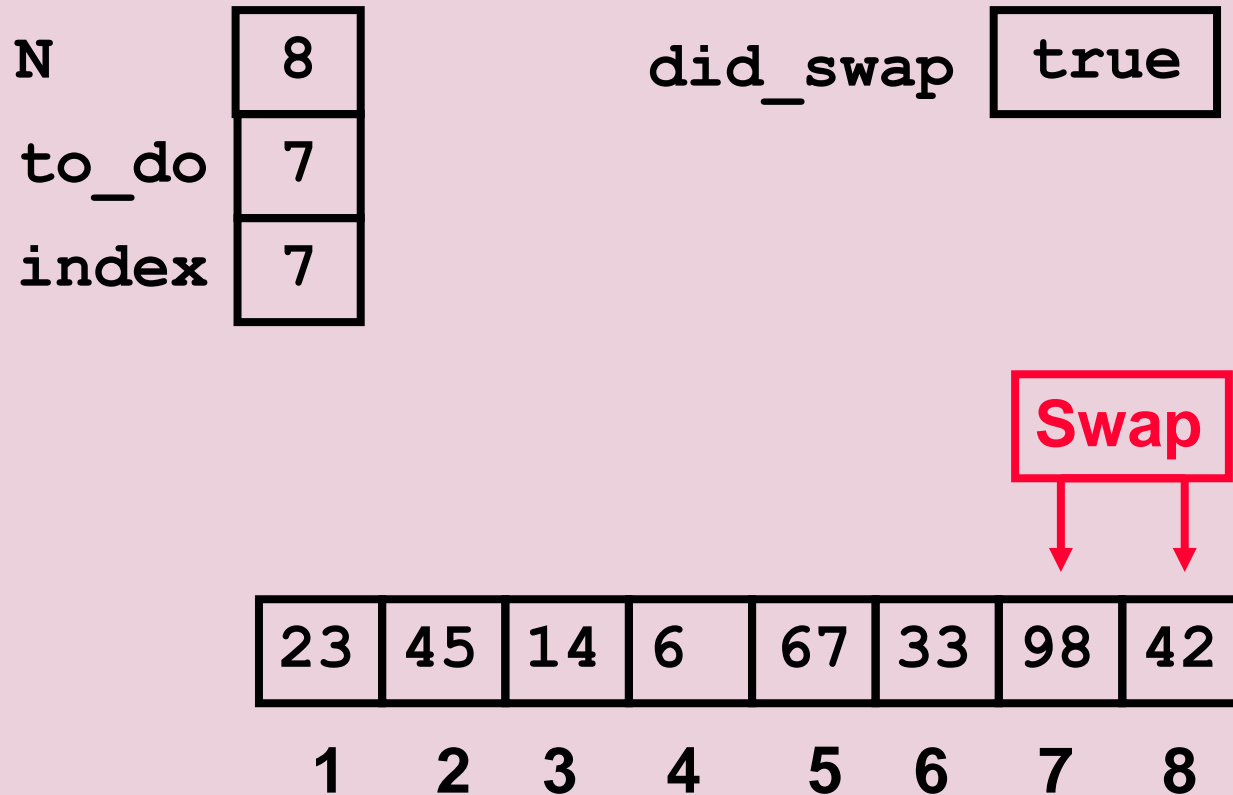


23	45	14	6	67	33	98	42
1	2	3	4	5	6	7	8

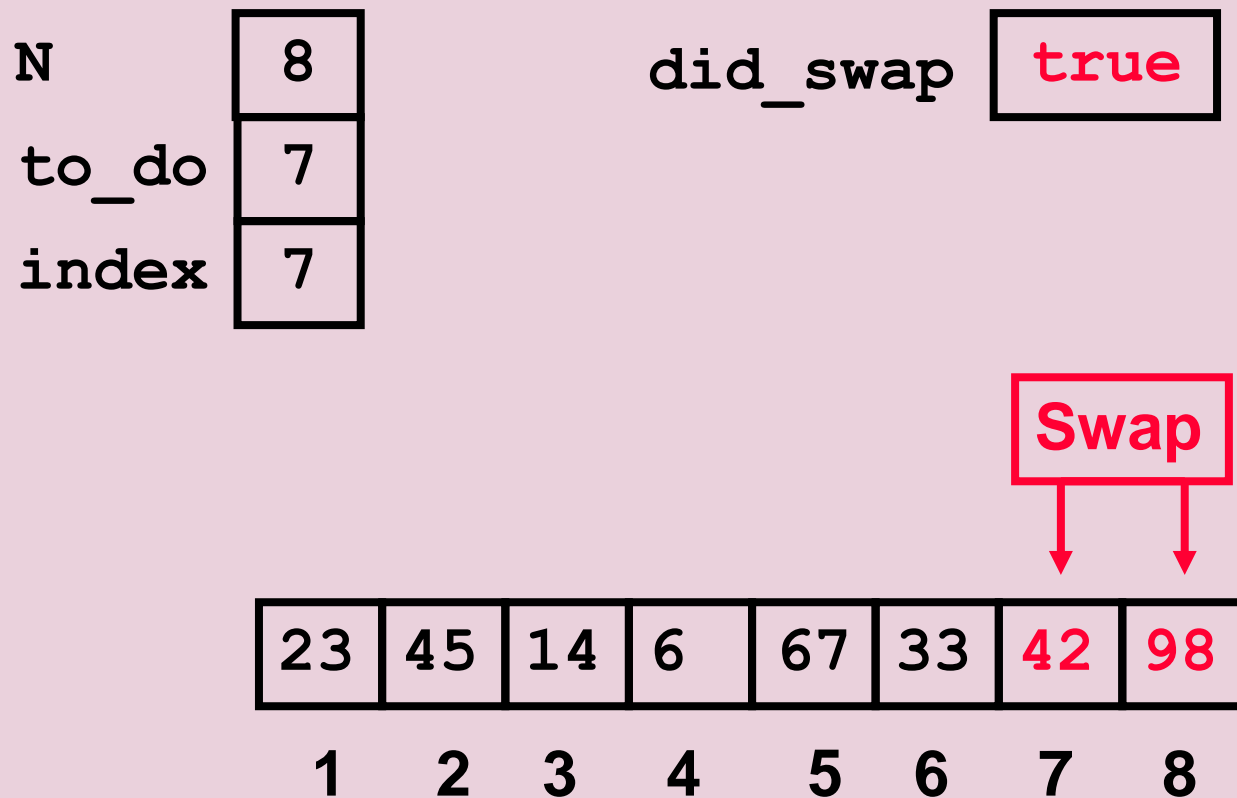
An Animated Example



An Animated Example



An Animated Example



After First Pass of Outer Loop

N
to_do
index

8
7
8

did_swap

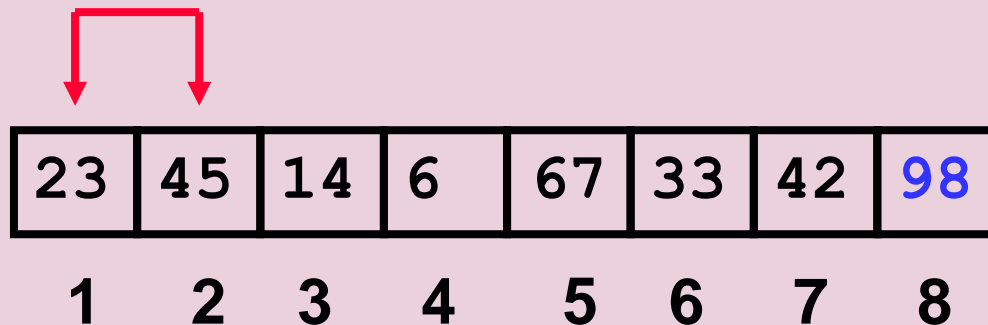
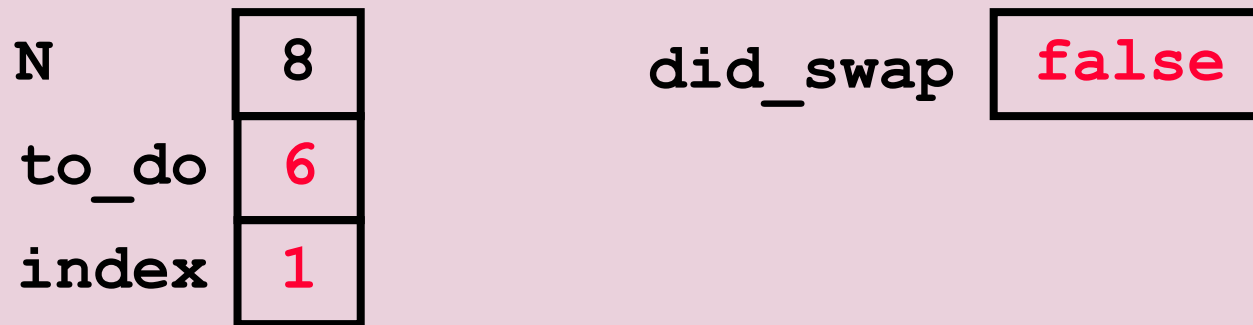
true

Finished first “Bubble Up”

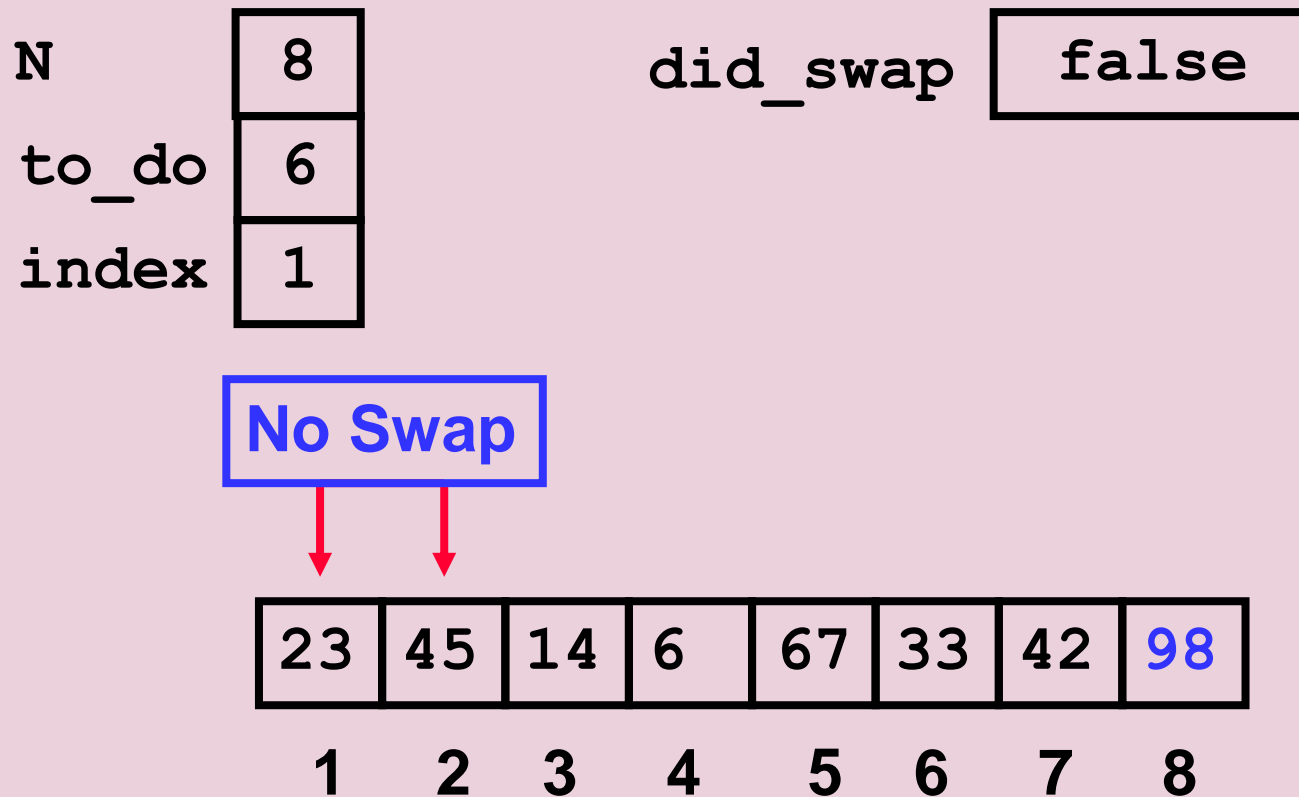
23	45	14	6	67	33	42	98
1	2	3	4	5	6	7	8



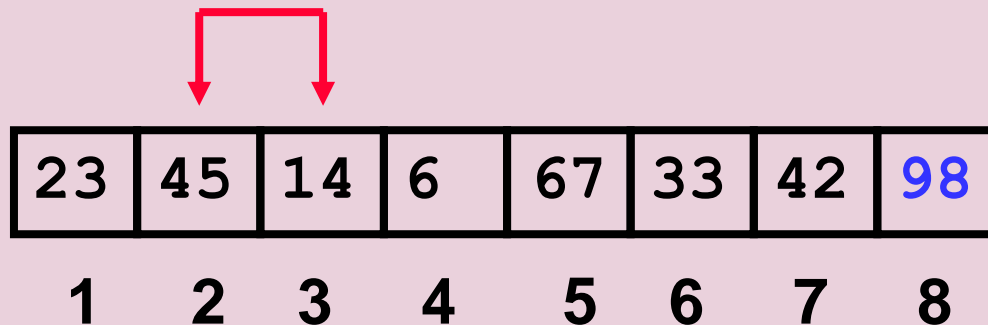
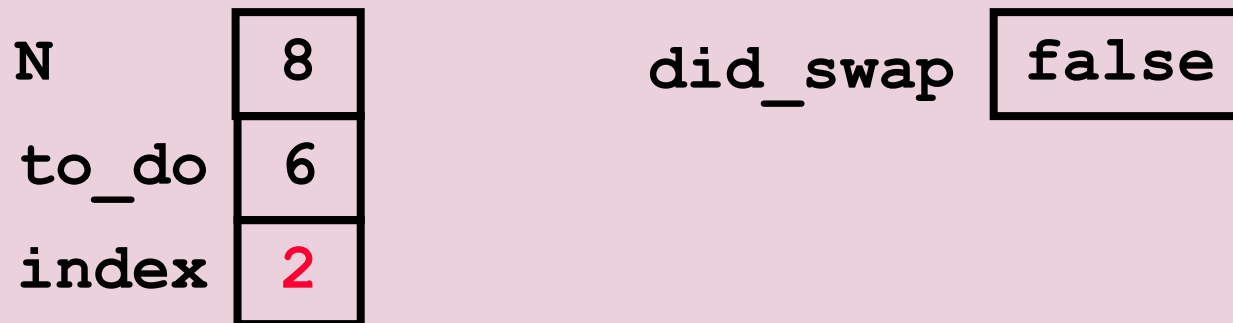
The Second “Bubble Up”



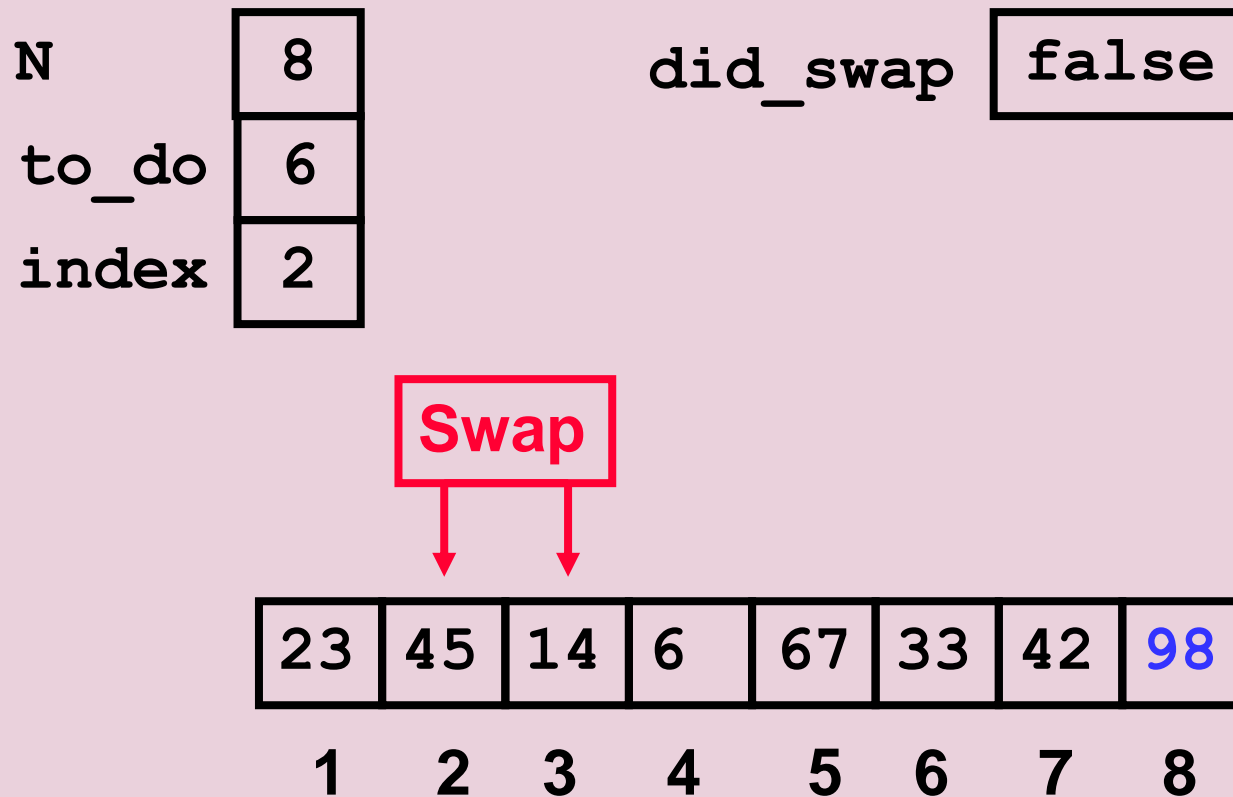
The Second “Bubble Up”



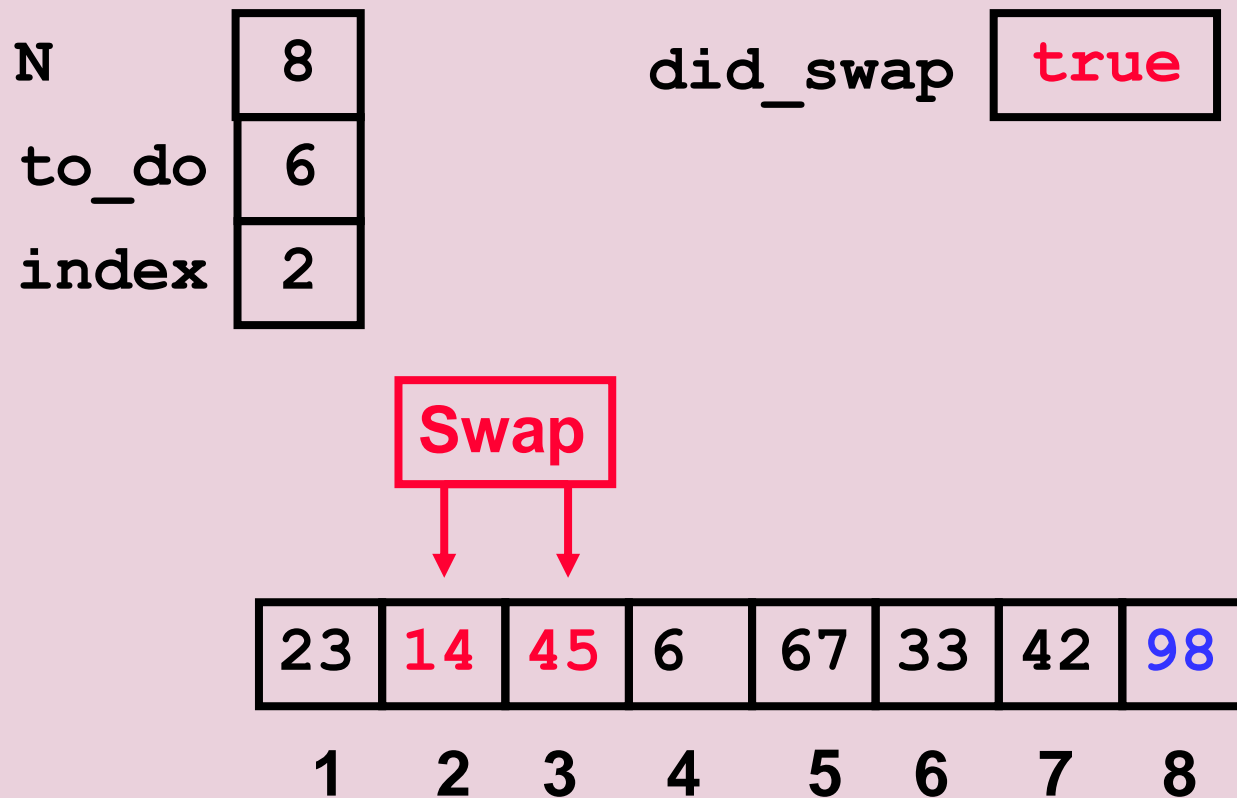
The Second “Bubble Up”



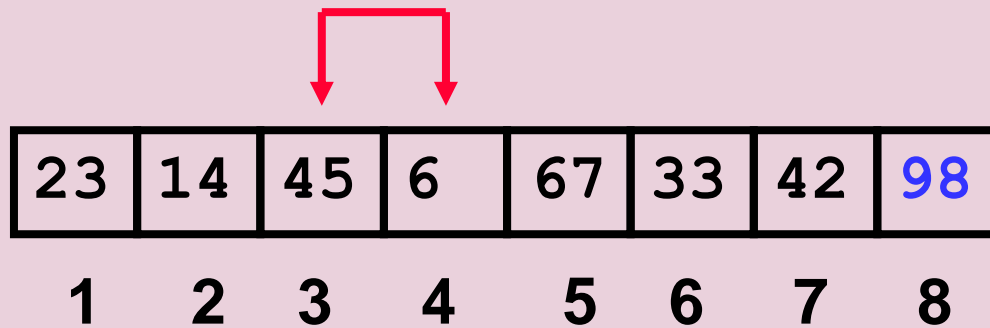
The Second “Bubble Up”



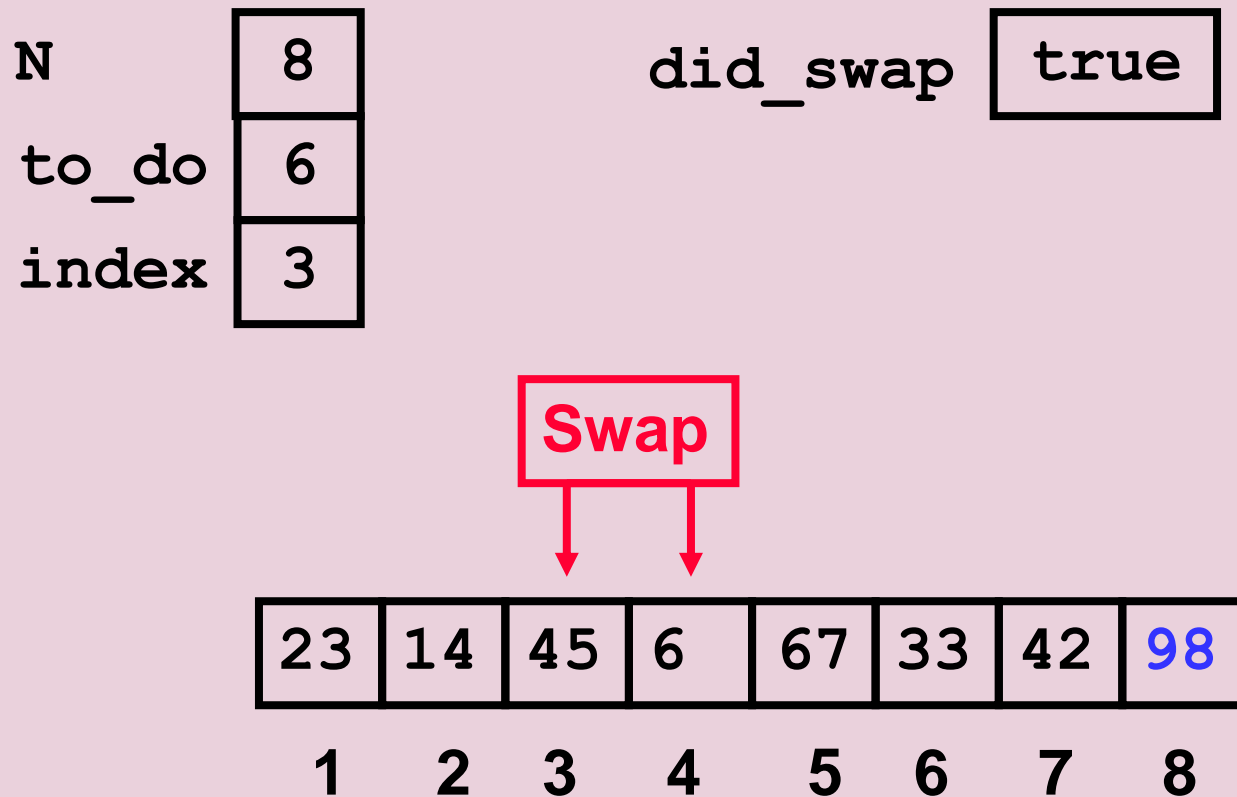
The Second “Bubble Up”



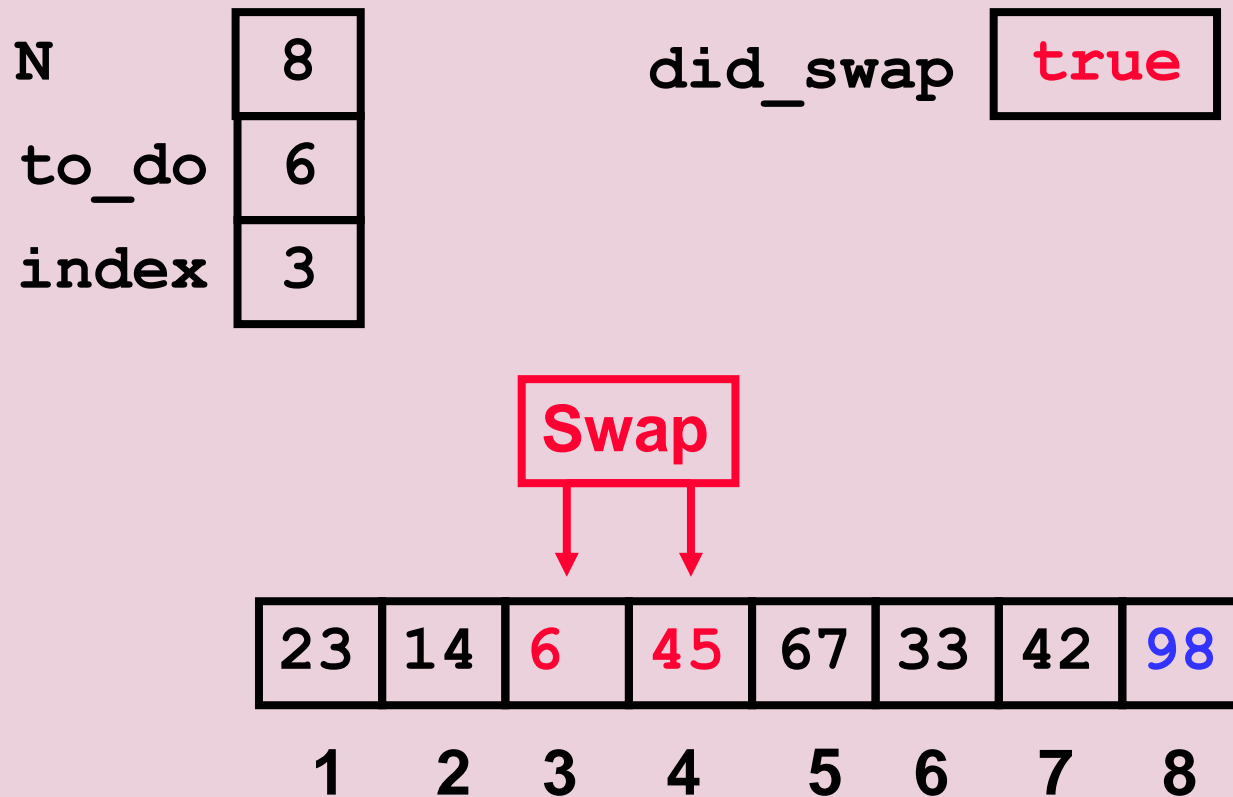
The Second “Bubble Up”



The Second “Bubble Up”



The Second “Bubble Up”



The Second “Bubble Up”

N

8

 did_swap

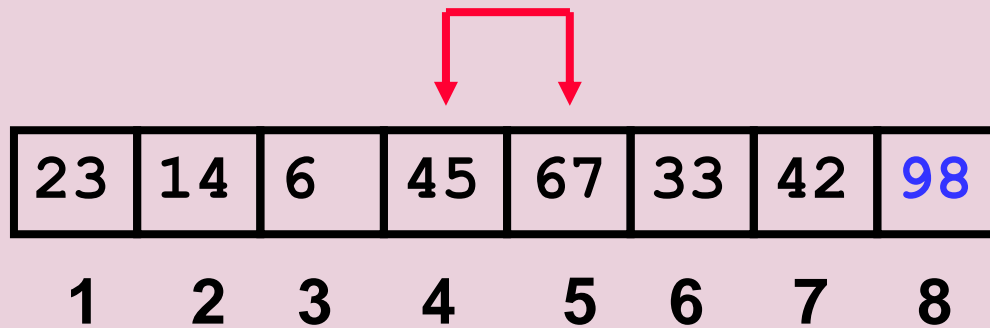
true

to_do

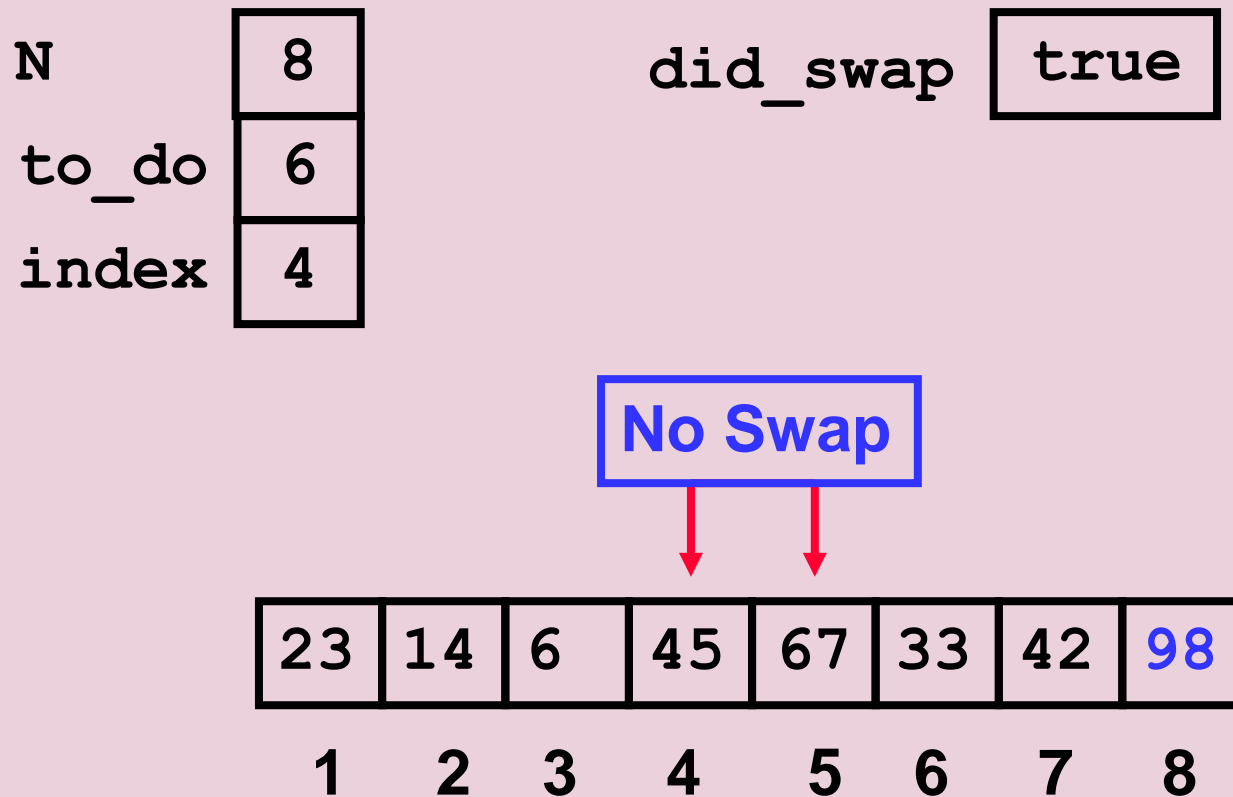
6

index

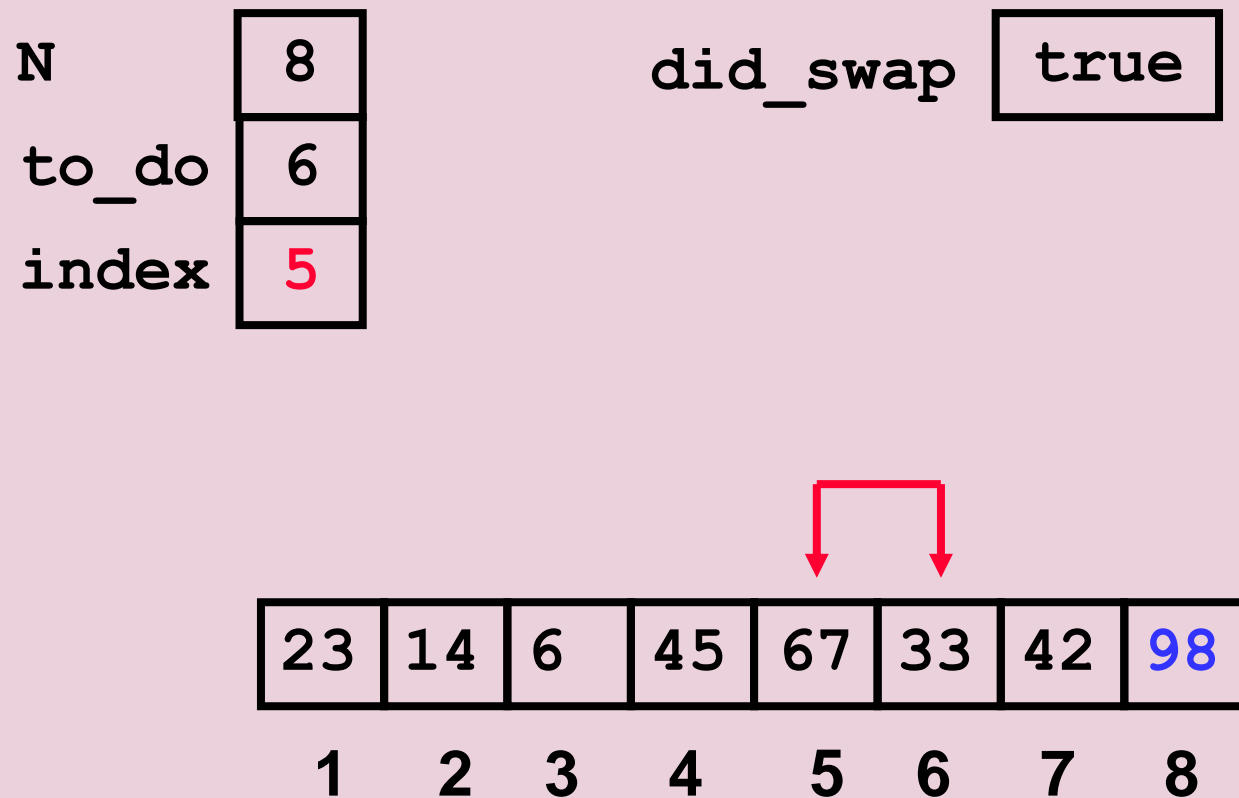
4



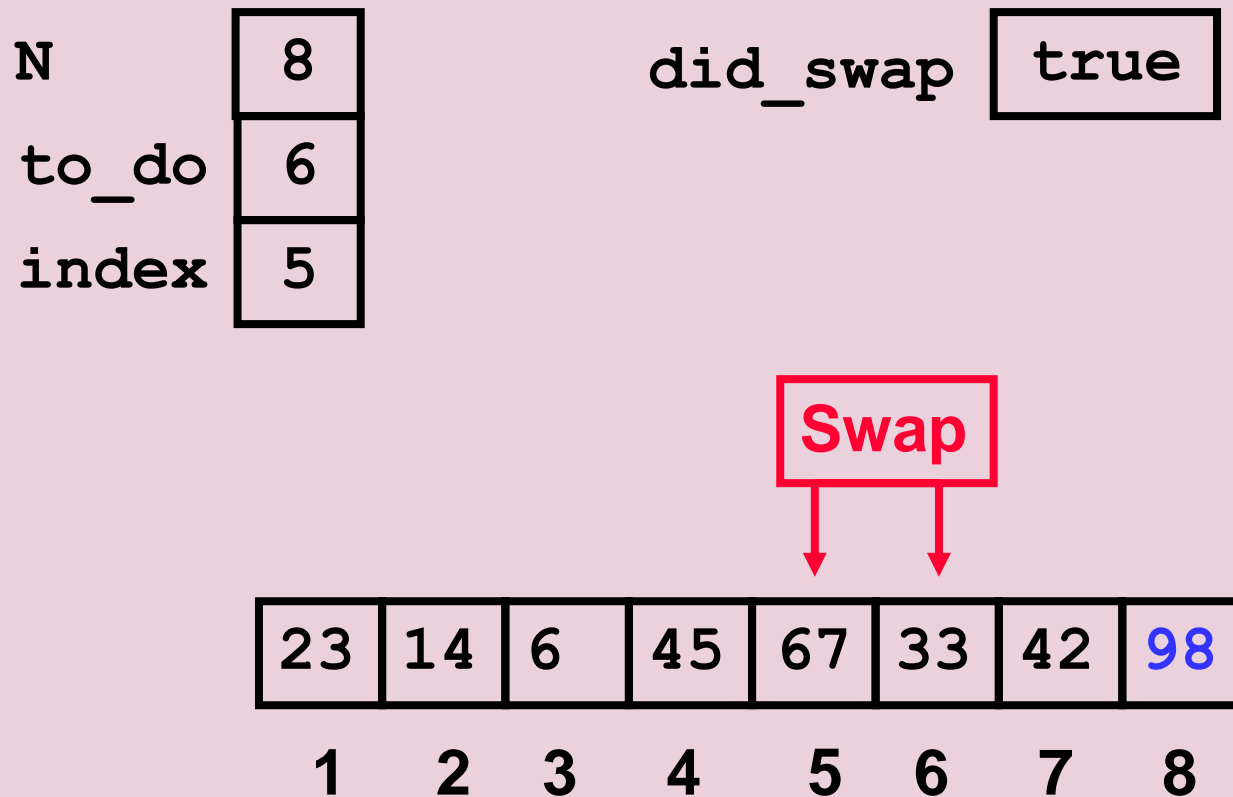
The Second “Bubble Up”



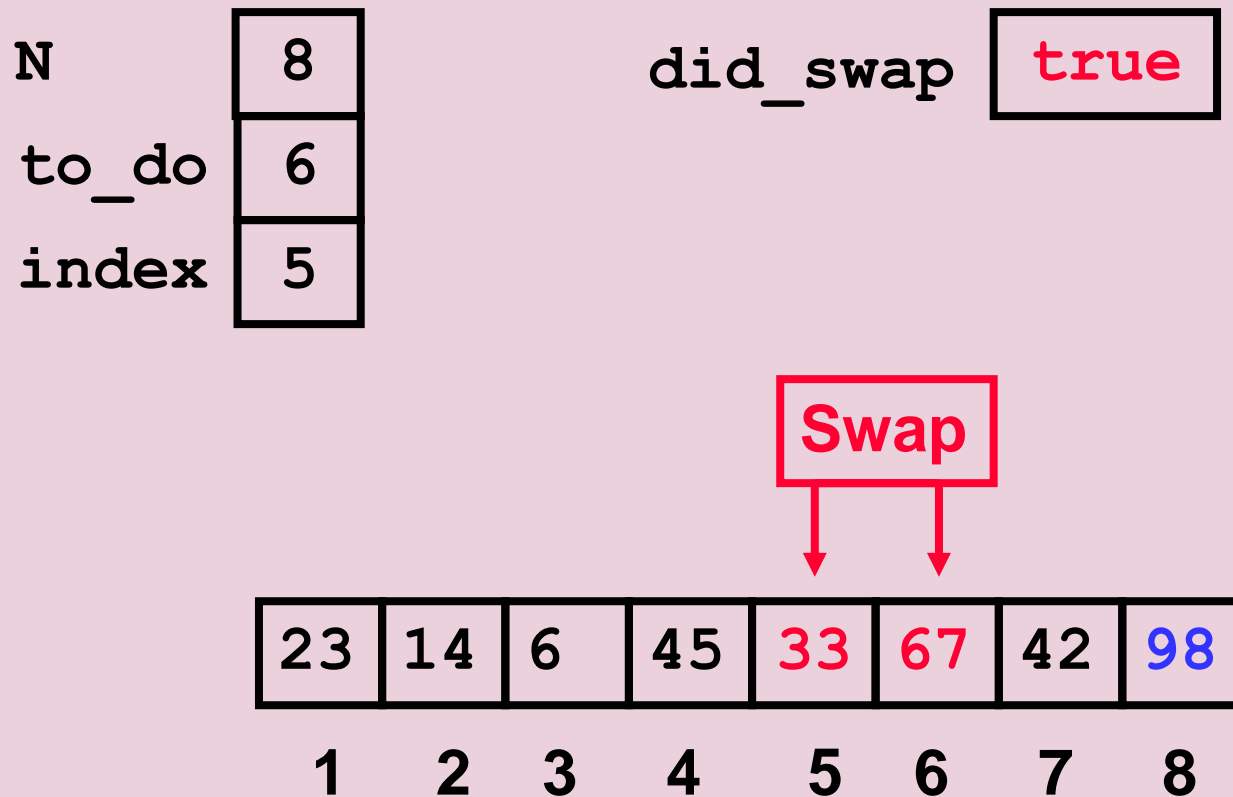
The Second “Bubble Up”



The Second “Bubble Up”



The Second “Bubble Up”



The Second “Bubble Up”

N

8

to_do

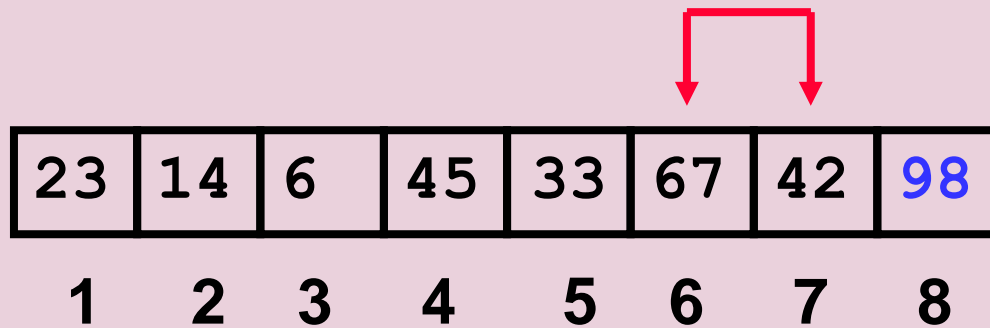
6

index

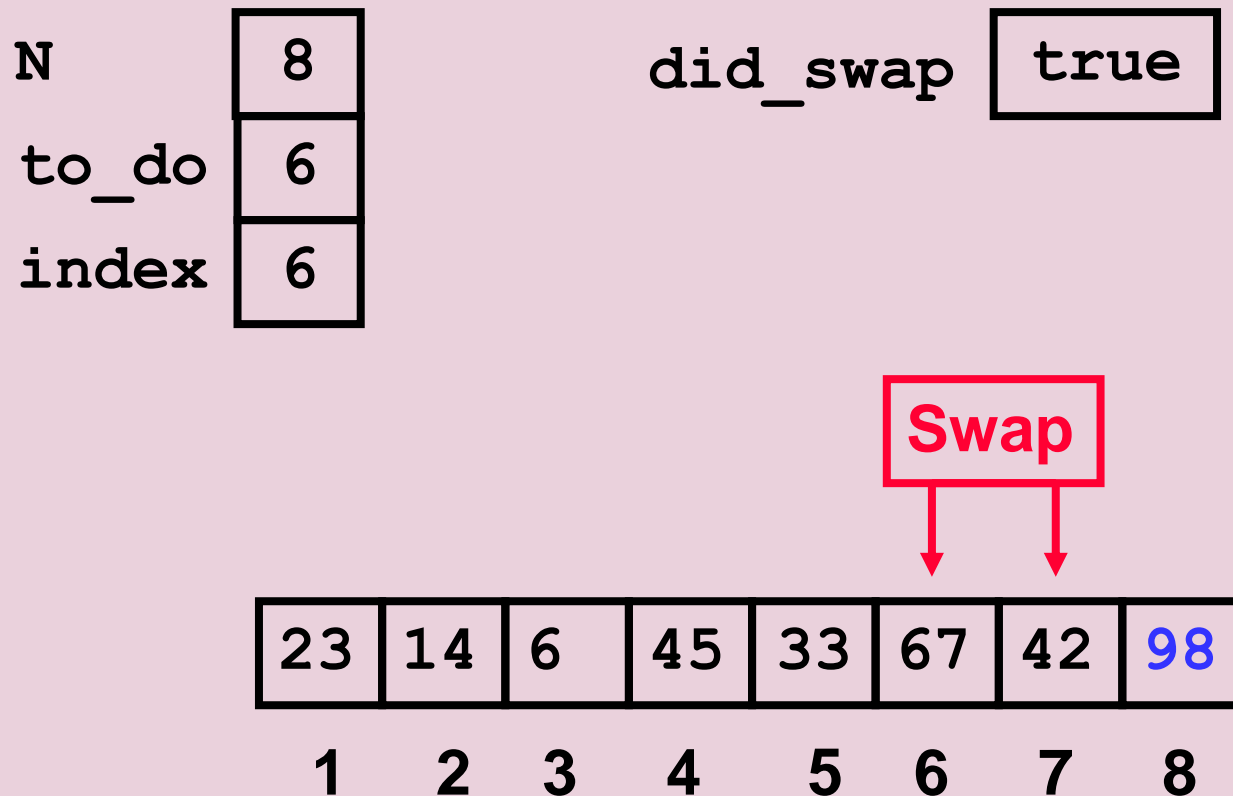
6

did_swap

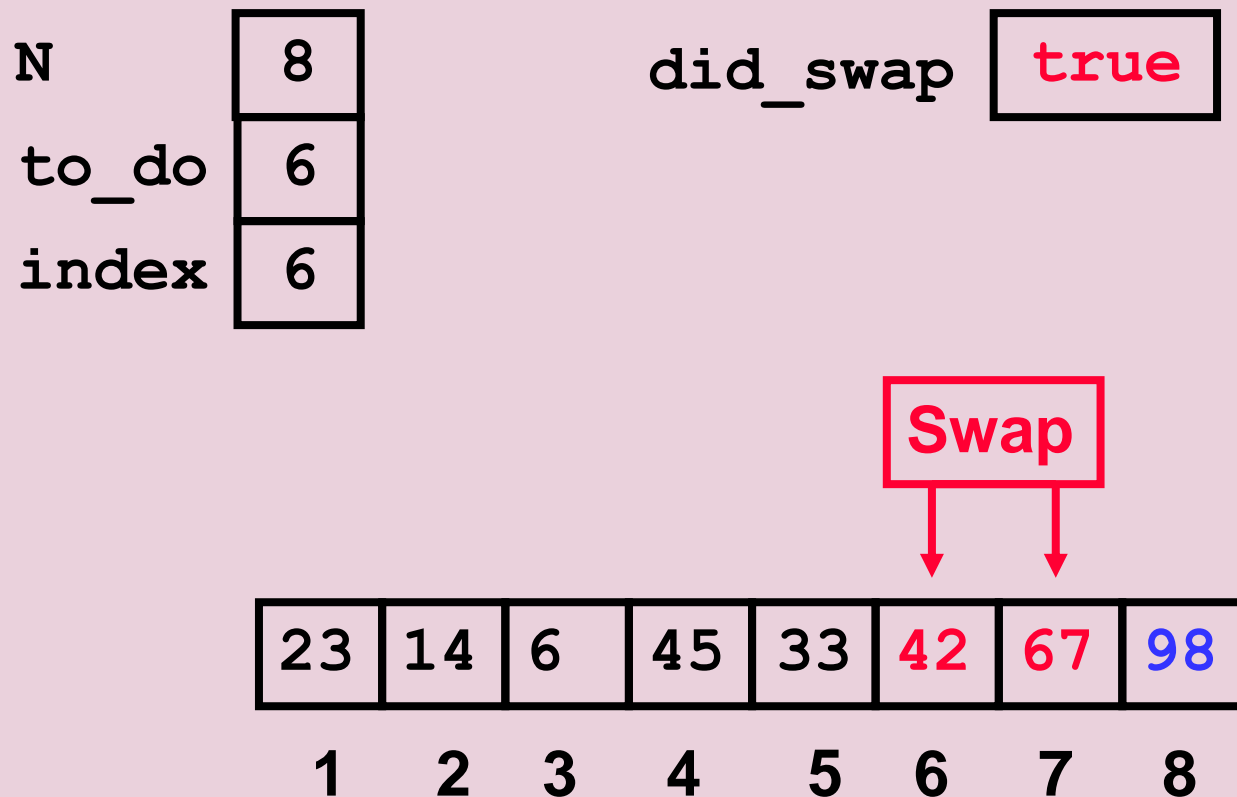
true



The Second “Bubble Up”



The Second “Bubble Up”



After Second Pass of Outer Loop


N
to_do
index

8
6
7

did_swap

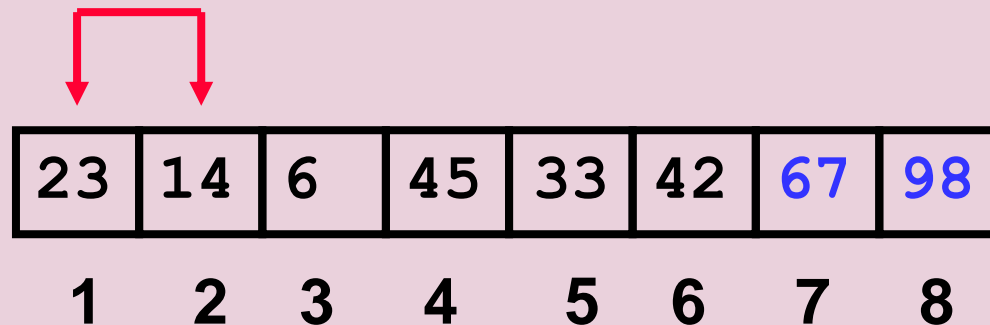
true

Finished second "Bubble Up"

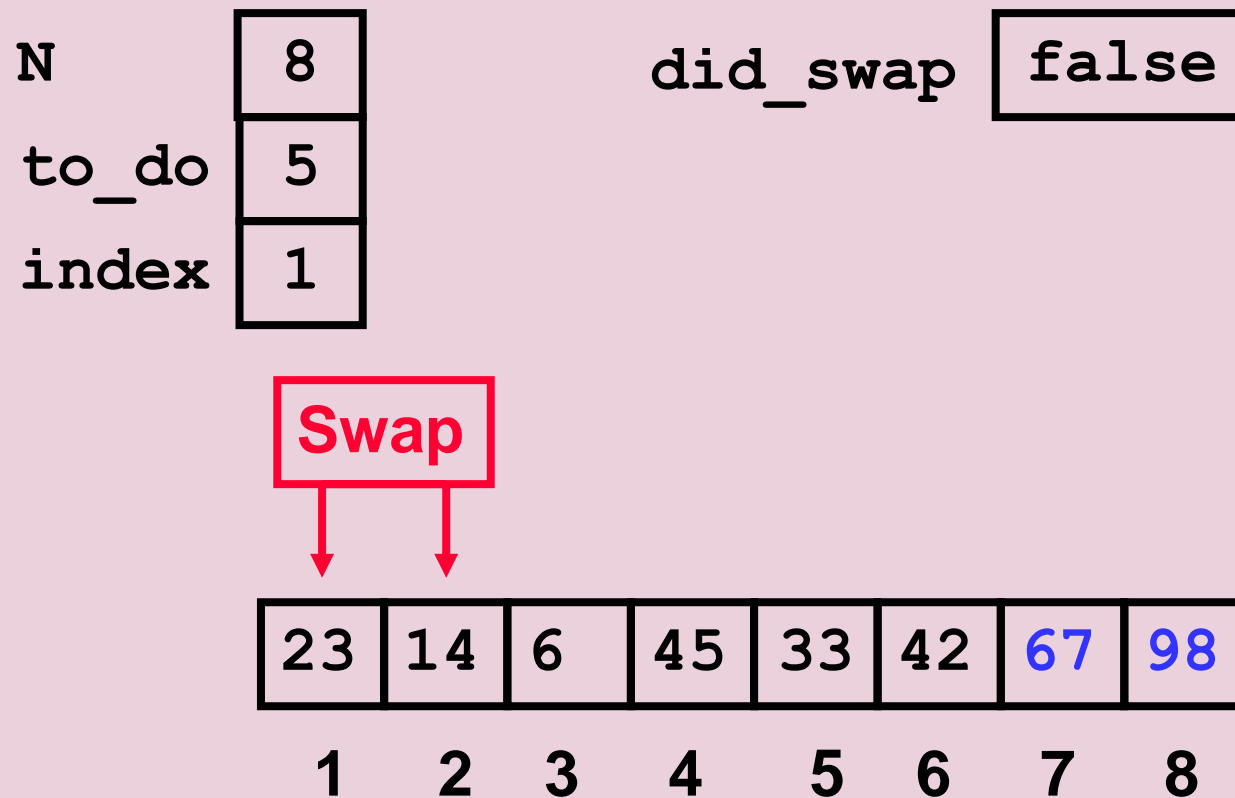


23	14	6	45	33	42	67	98
1	2	3	4	5	6	7	8

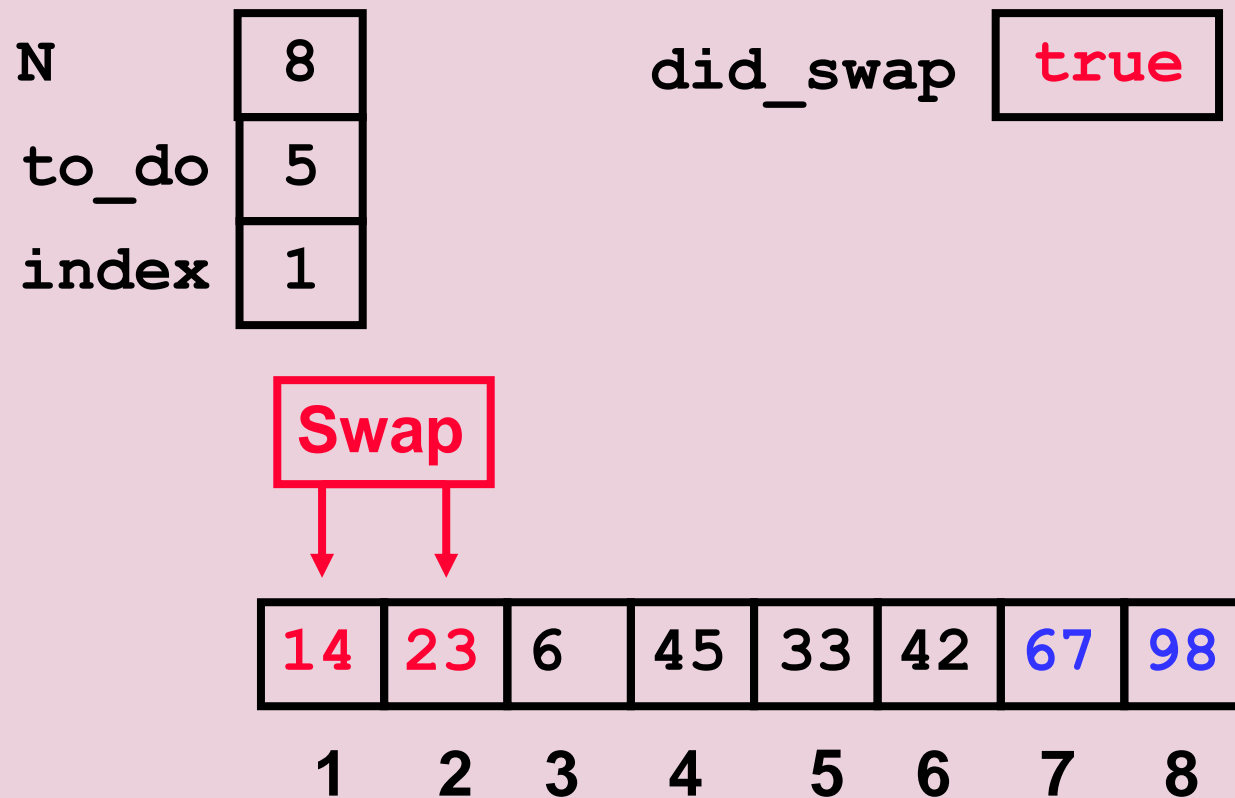
The Third “Bubble Up”



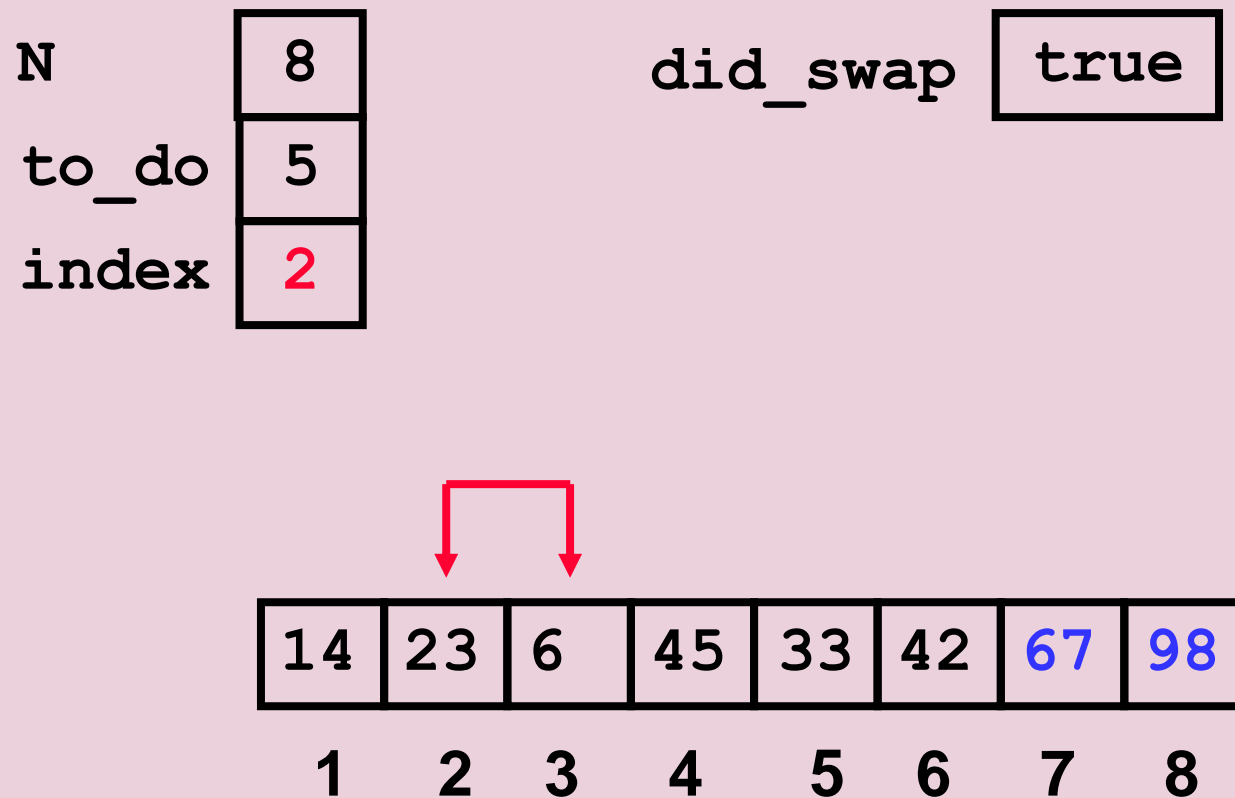
The Third “Bubble Up”



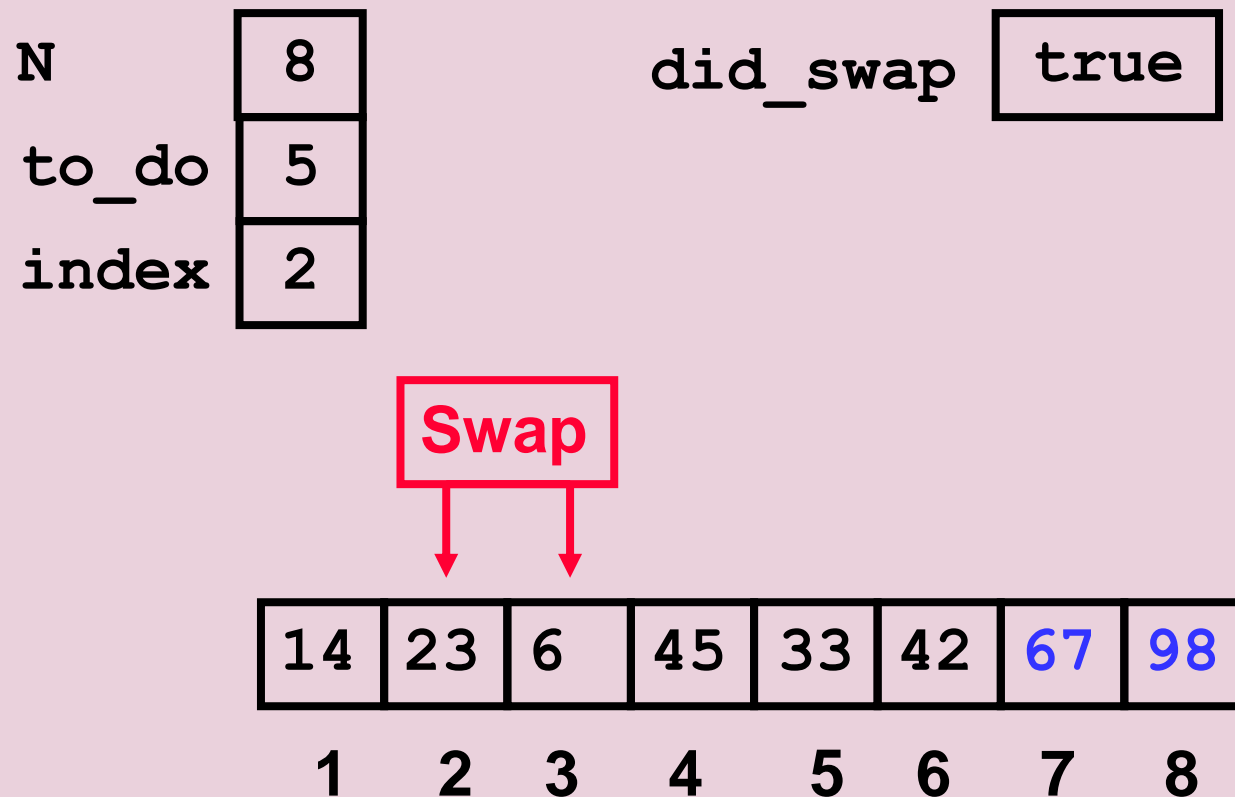
The Third “Bubble Up”



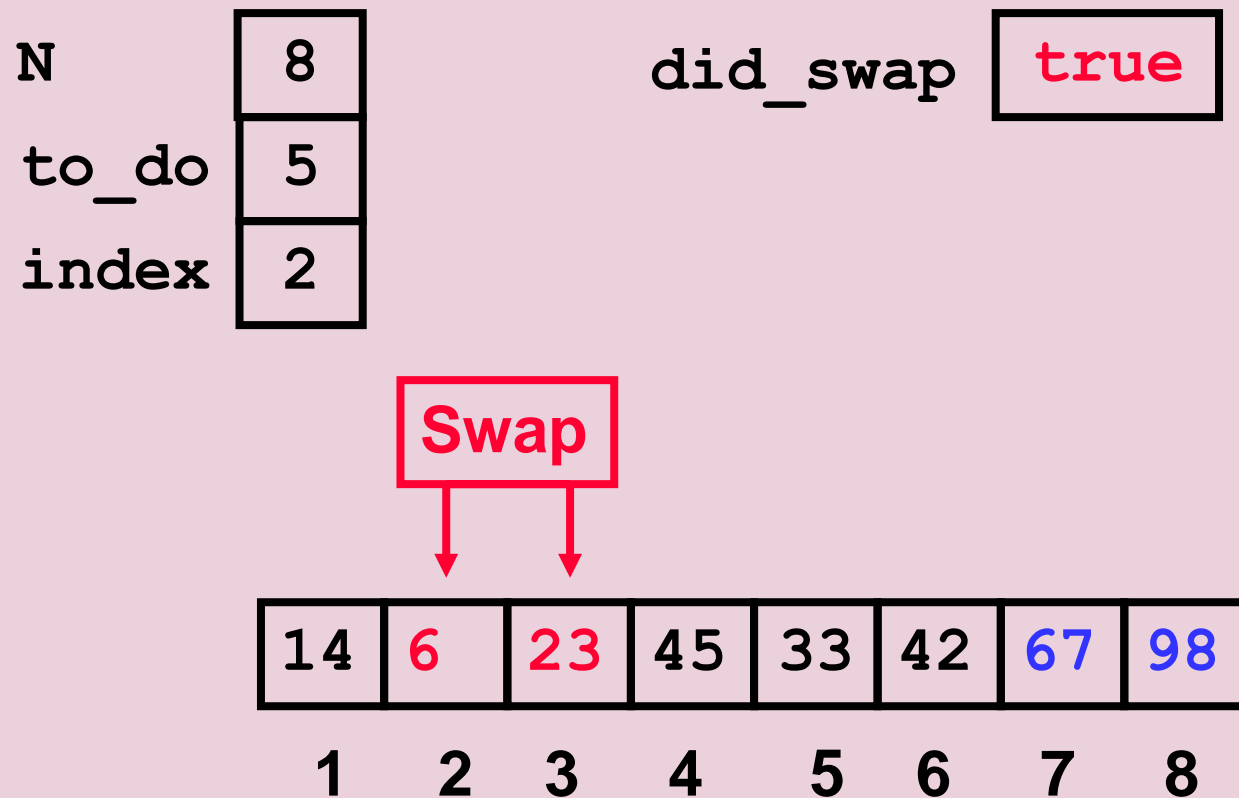
The Third “Bubble Up”



The Third “Bubble Up”



The Third “Bubble Up”



The Third “Bubble Up”

N

8

to_do

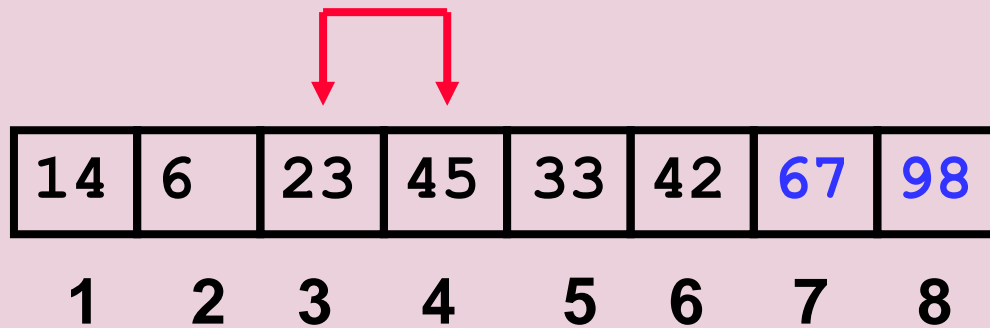
5

index

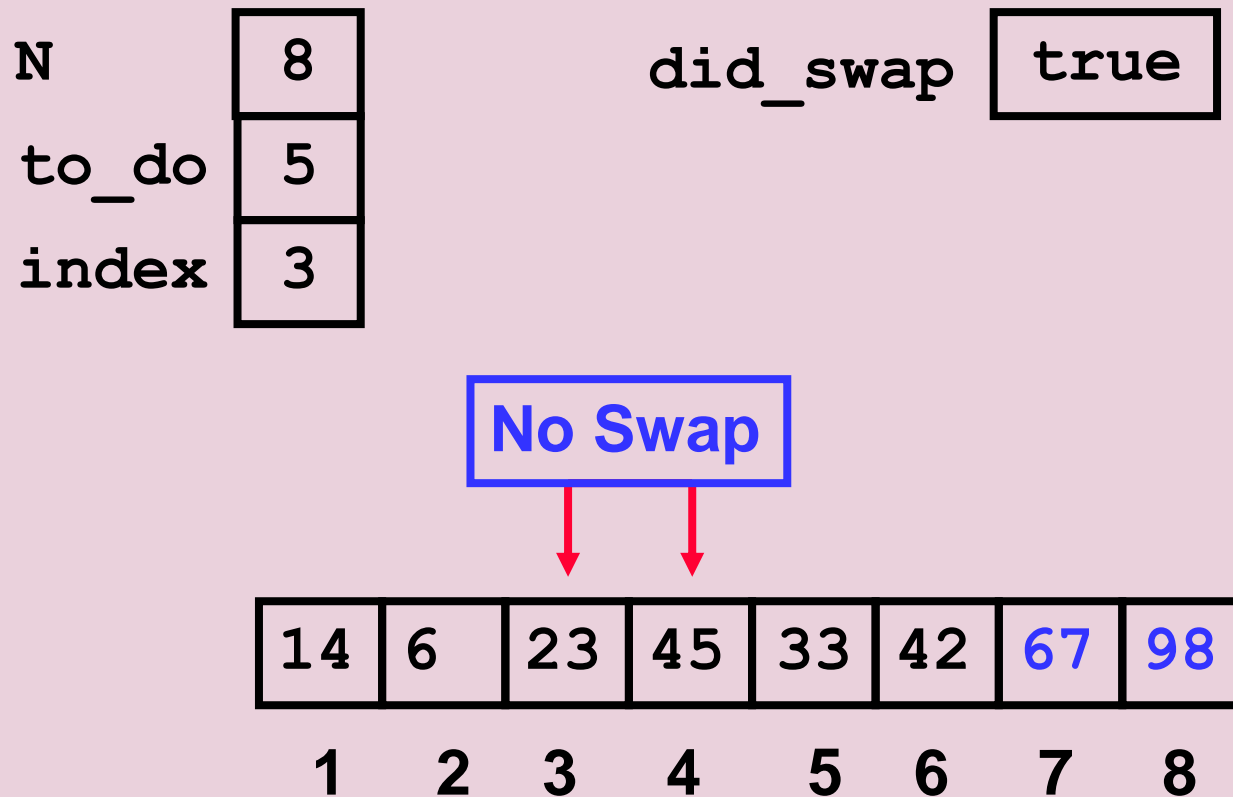
3

did_swap

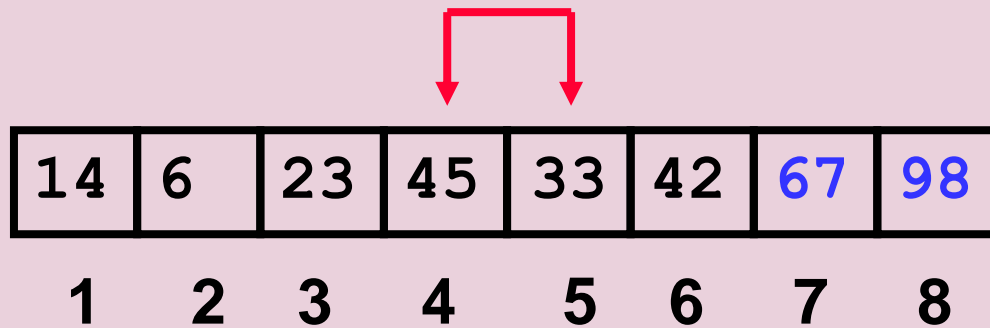
true



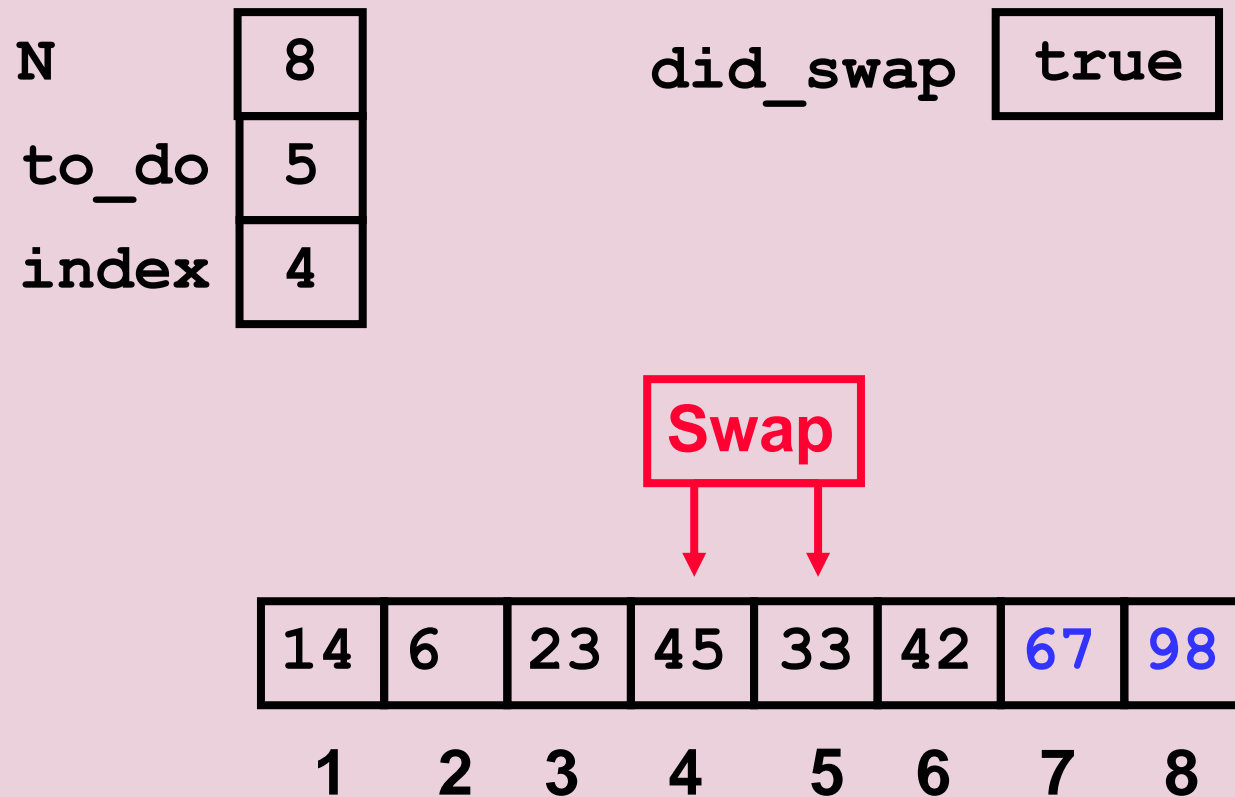
The Third “Bubble Up”



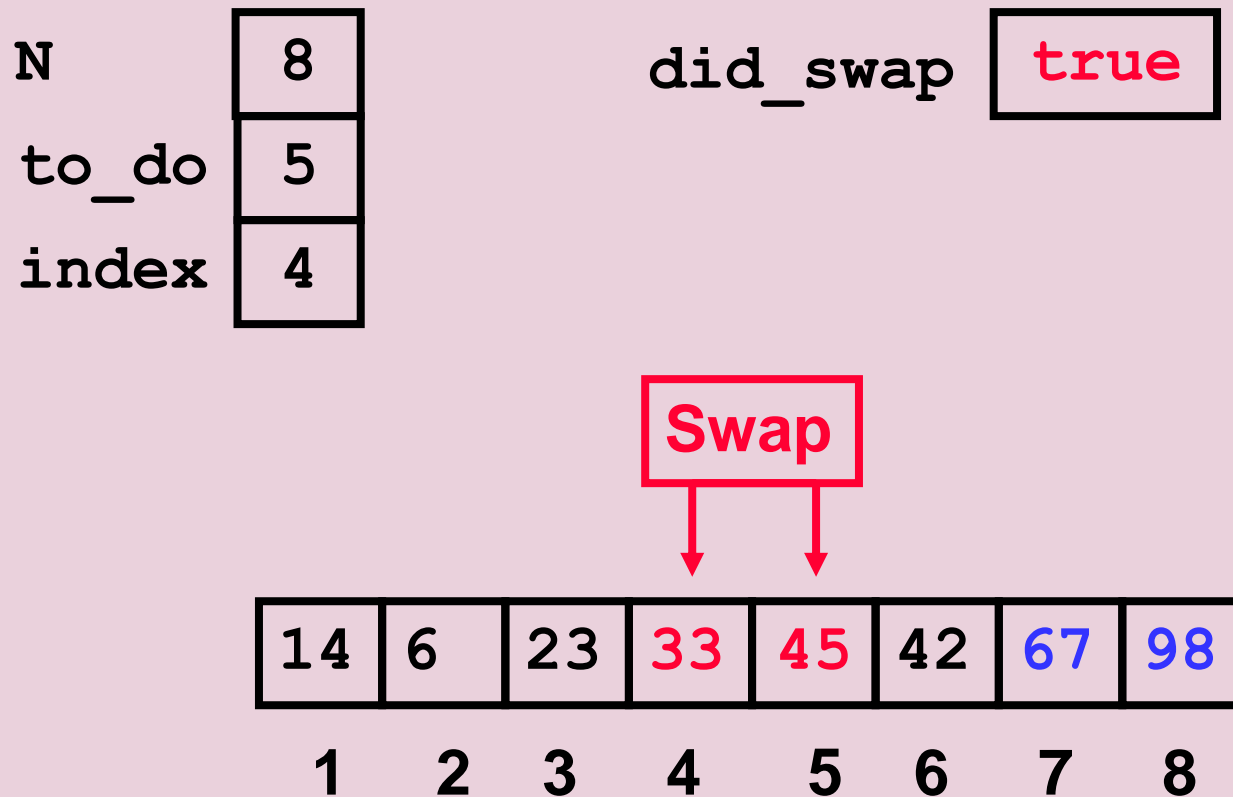
The Third “Bubble Up”



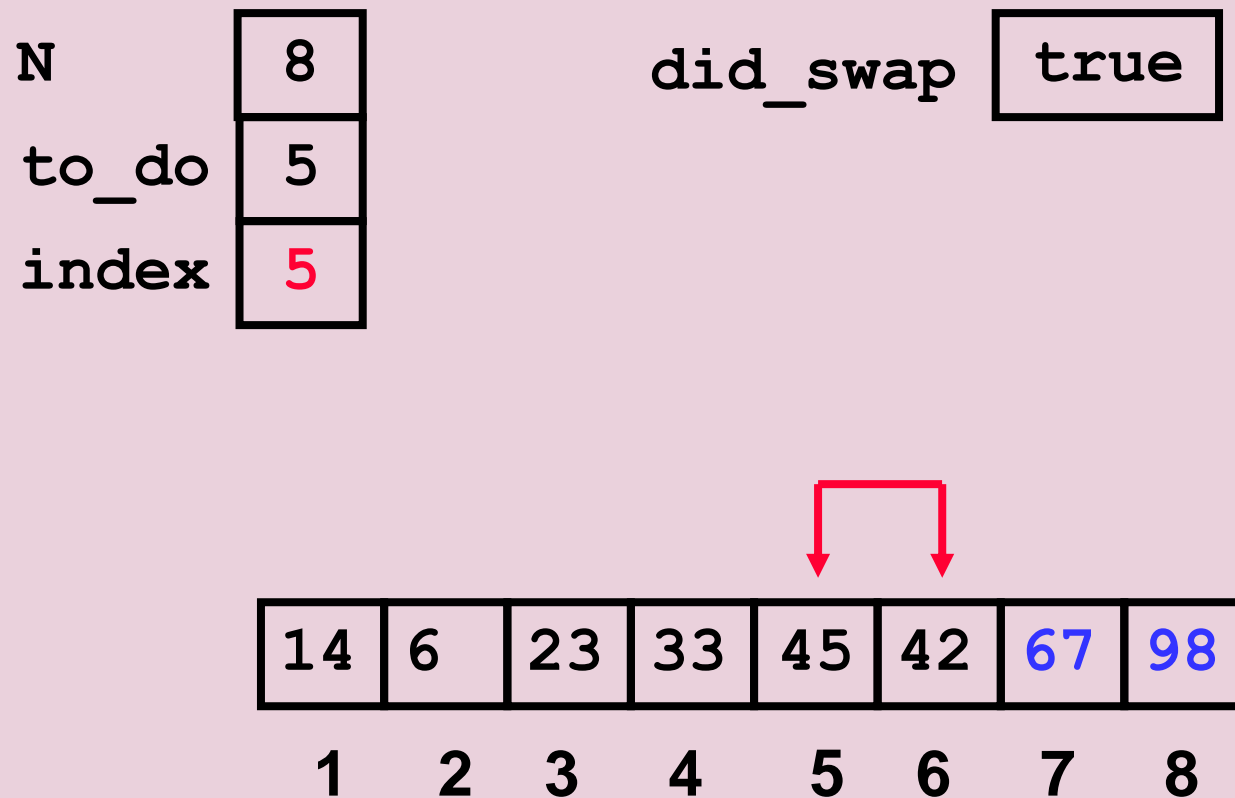
The Third “Bubble Up”



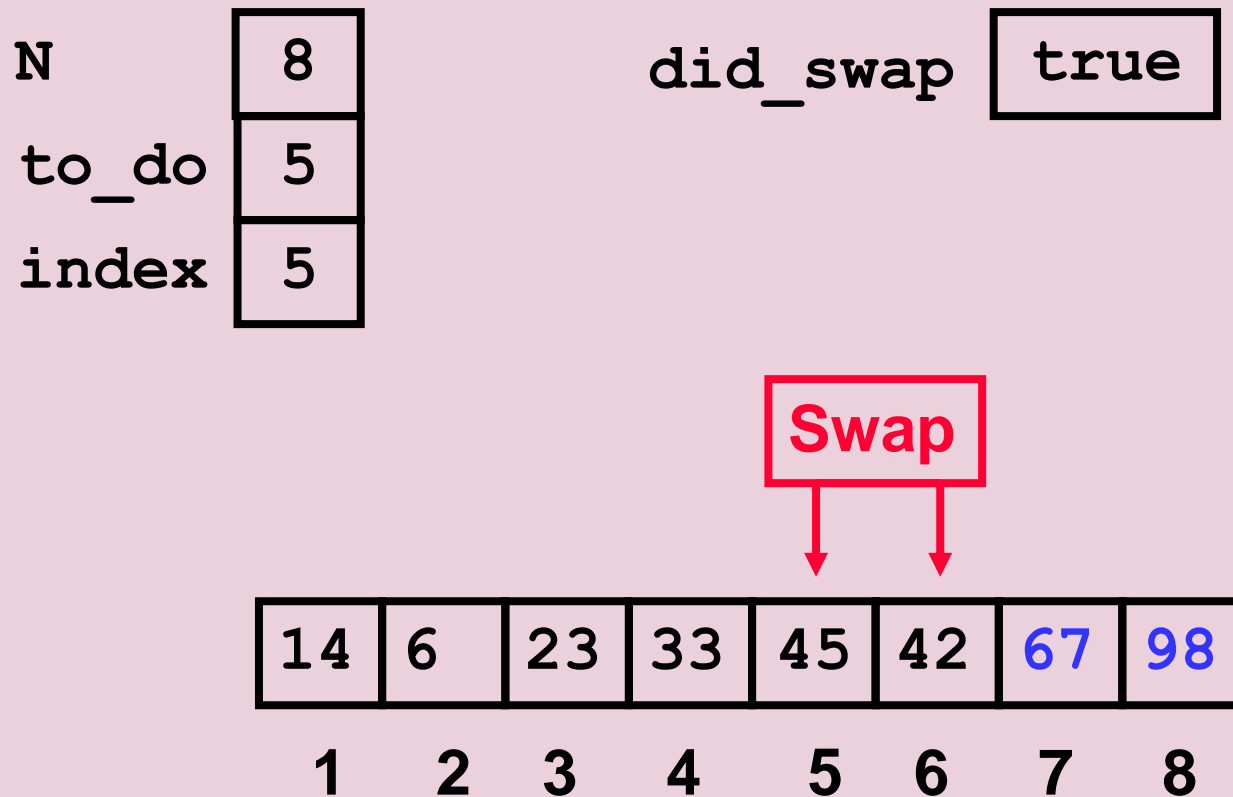
The Third “Bubble Up”



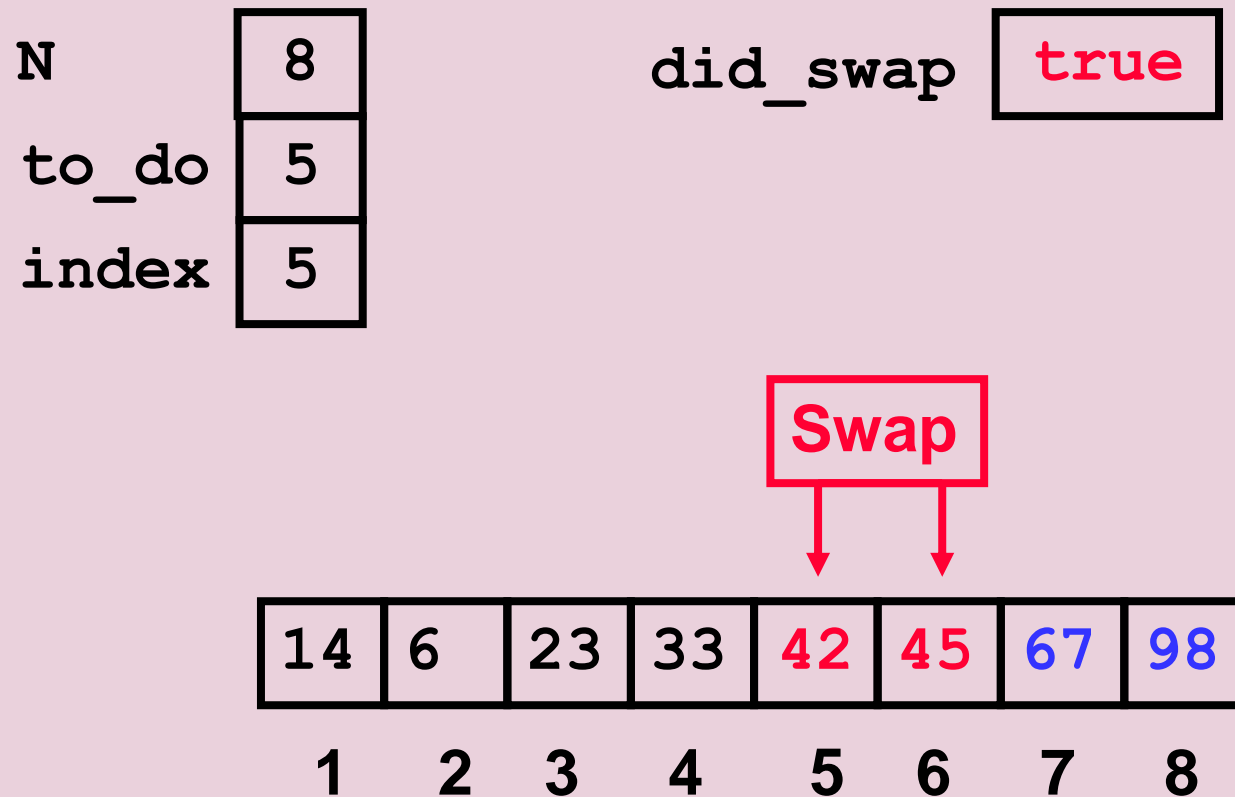
The Third “Bubble Up”



The Third “Bubble Up”



The Third “Bubble Up”



After Third Pass of Outer Loop

N

8

to_do

5


index

6

did_swap

true

Finished third “Bubble Up”



14	6	23	33	42	45	67	98
1	2	3	4	5	6	7	8

The Fourth “Bubble Up”

N

8

 did_swap

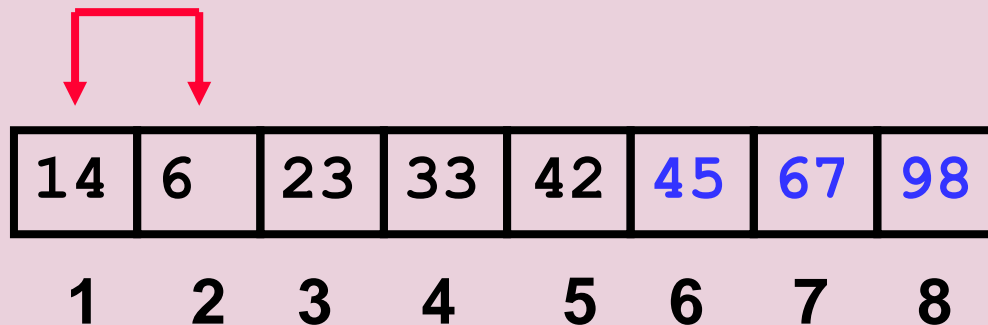
false

to_do

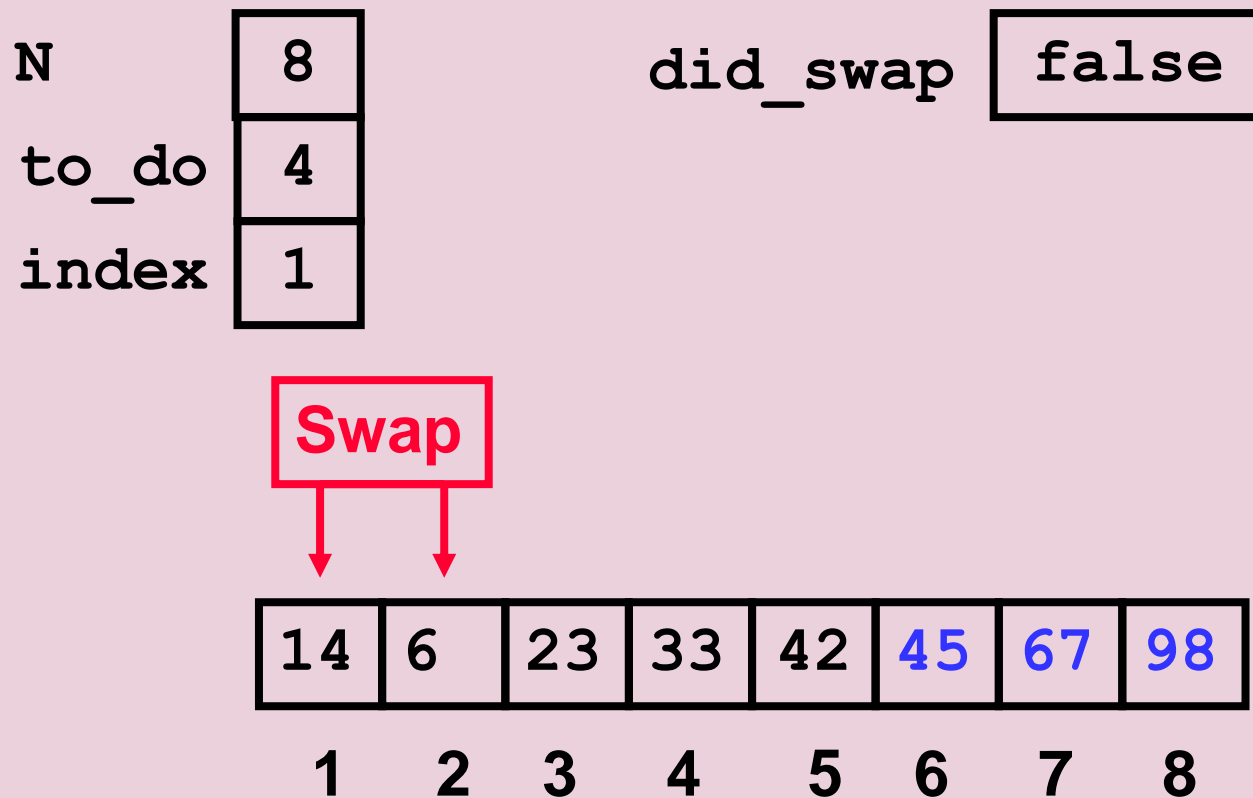
4

index

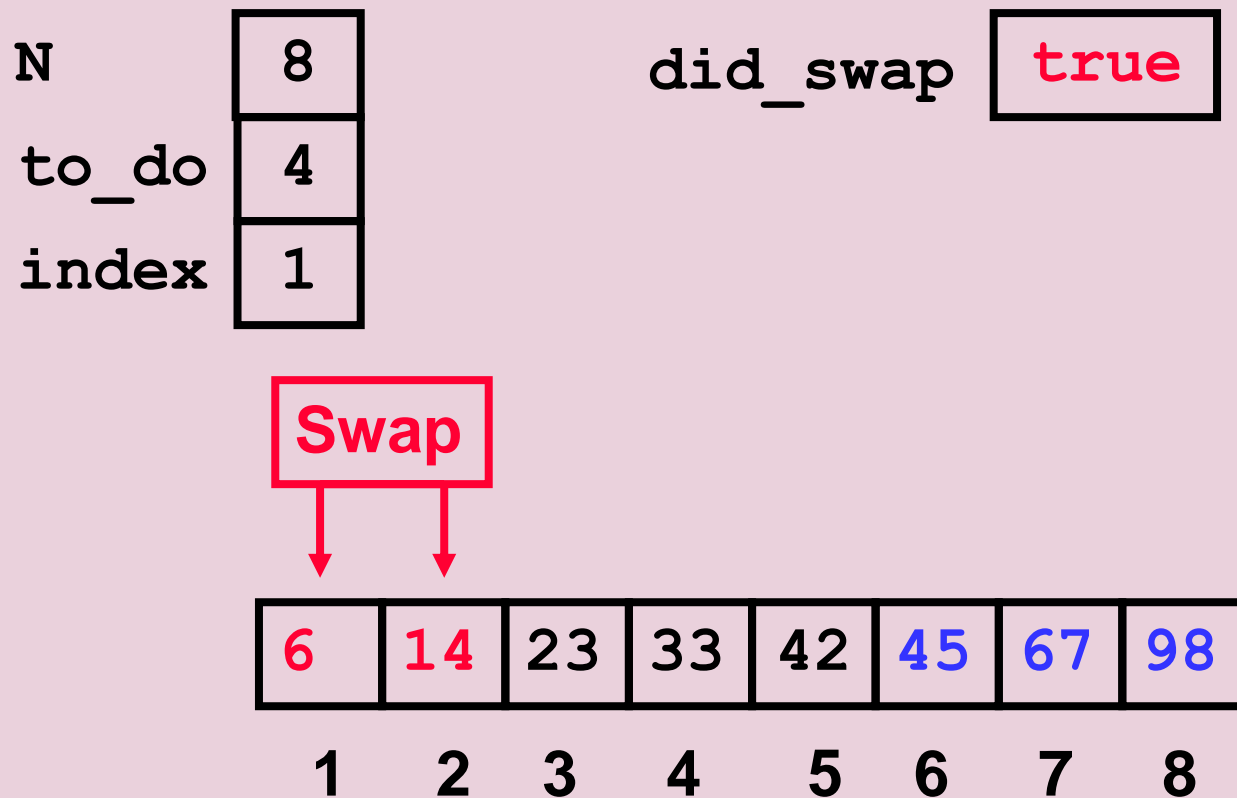
1



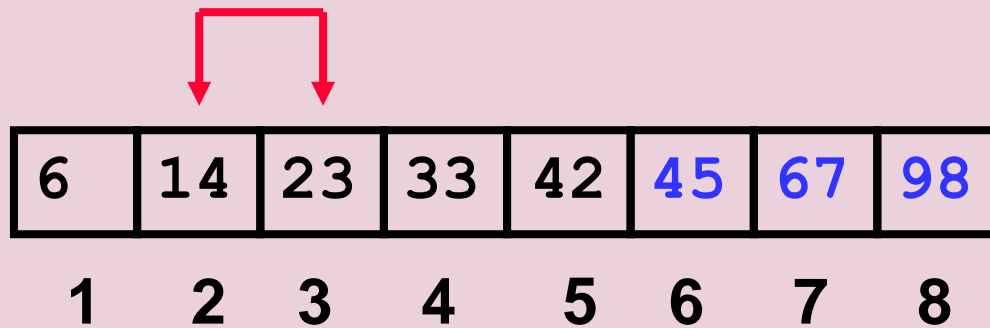
The Fourth “Bubble Up”



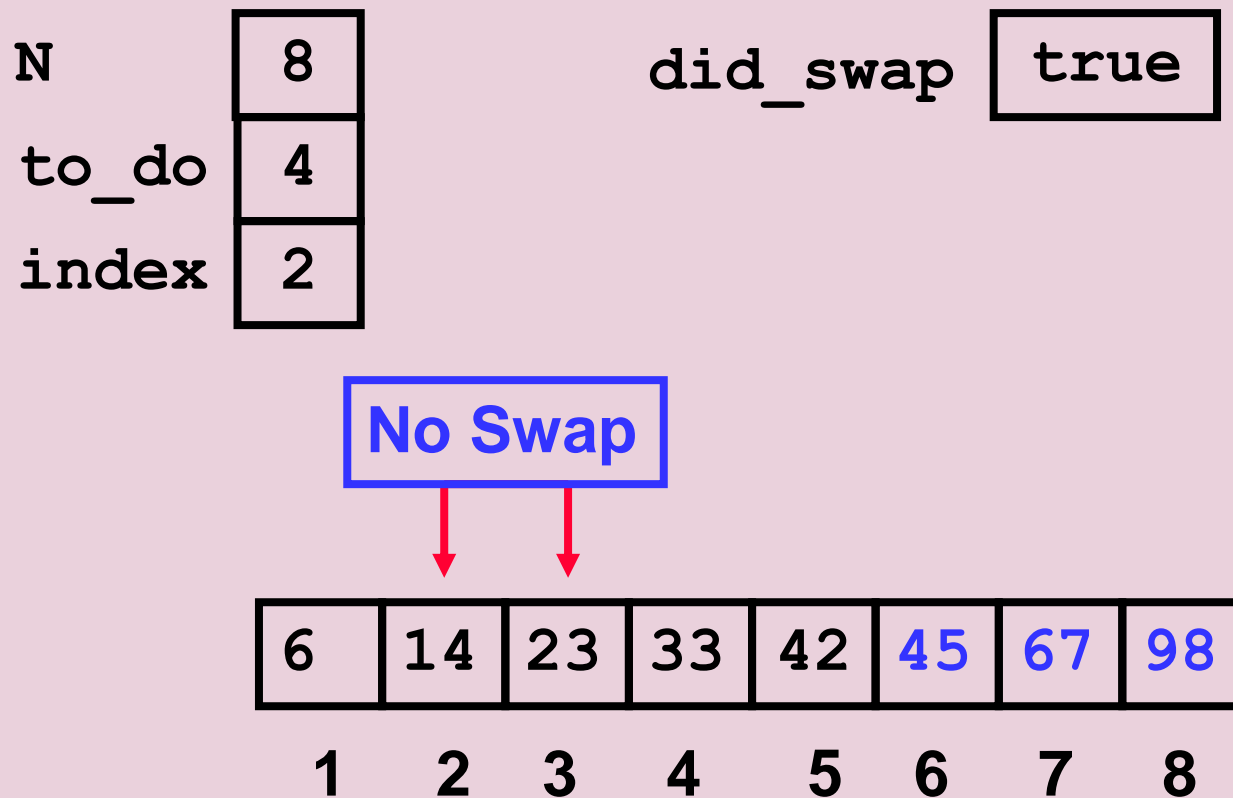
The Fourth “Bubble Up”



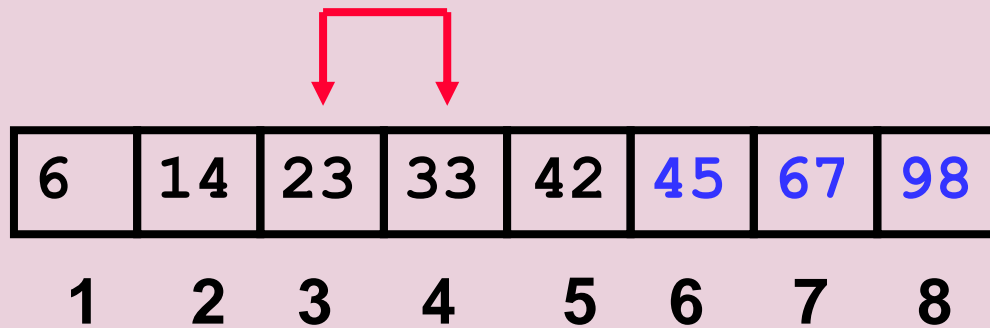
The Fourth “Bubble Up”



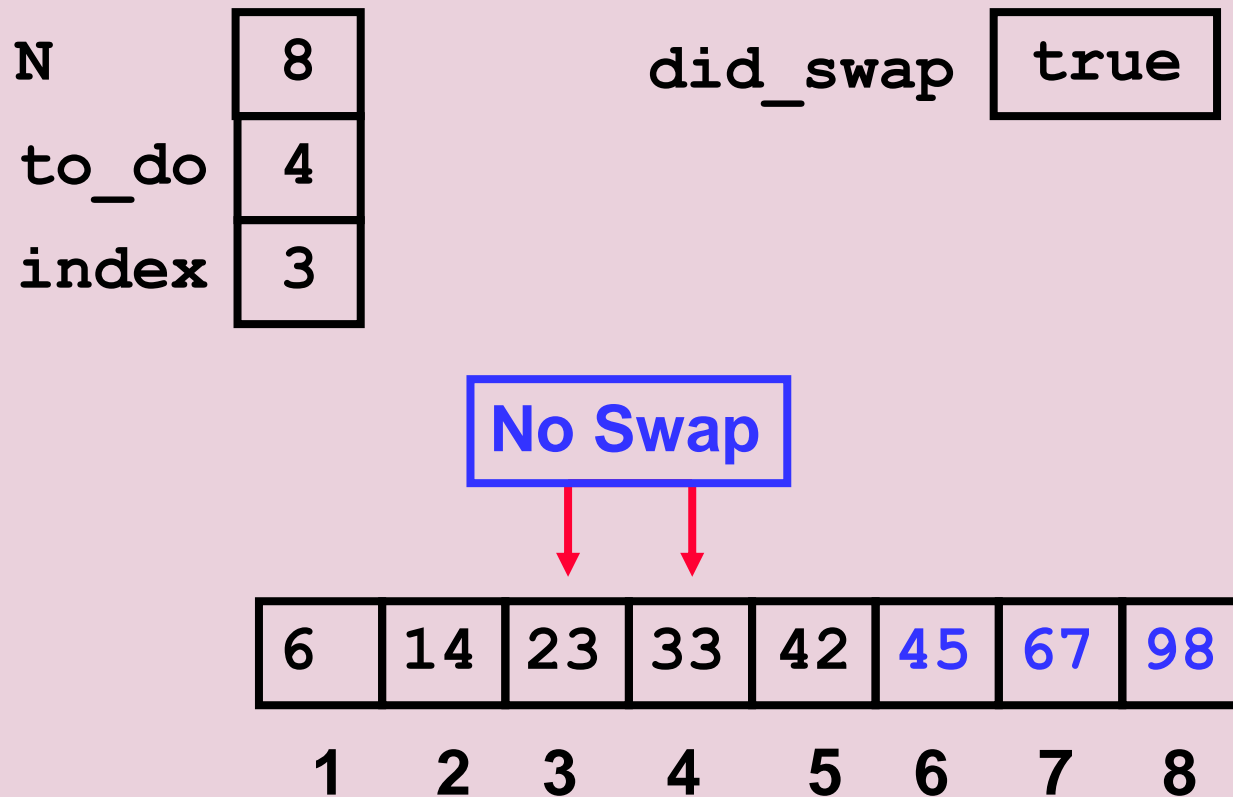
The Fourth “Bubble Up”



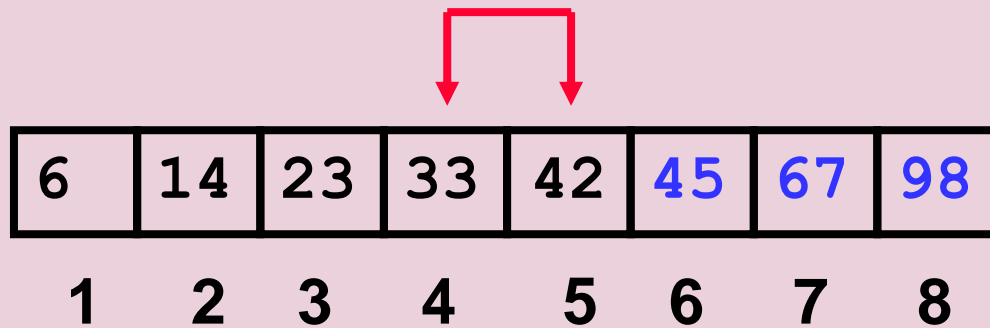
The Fourth “Bubble Up”



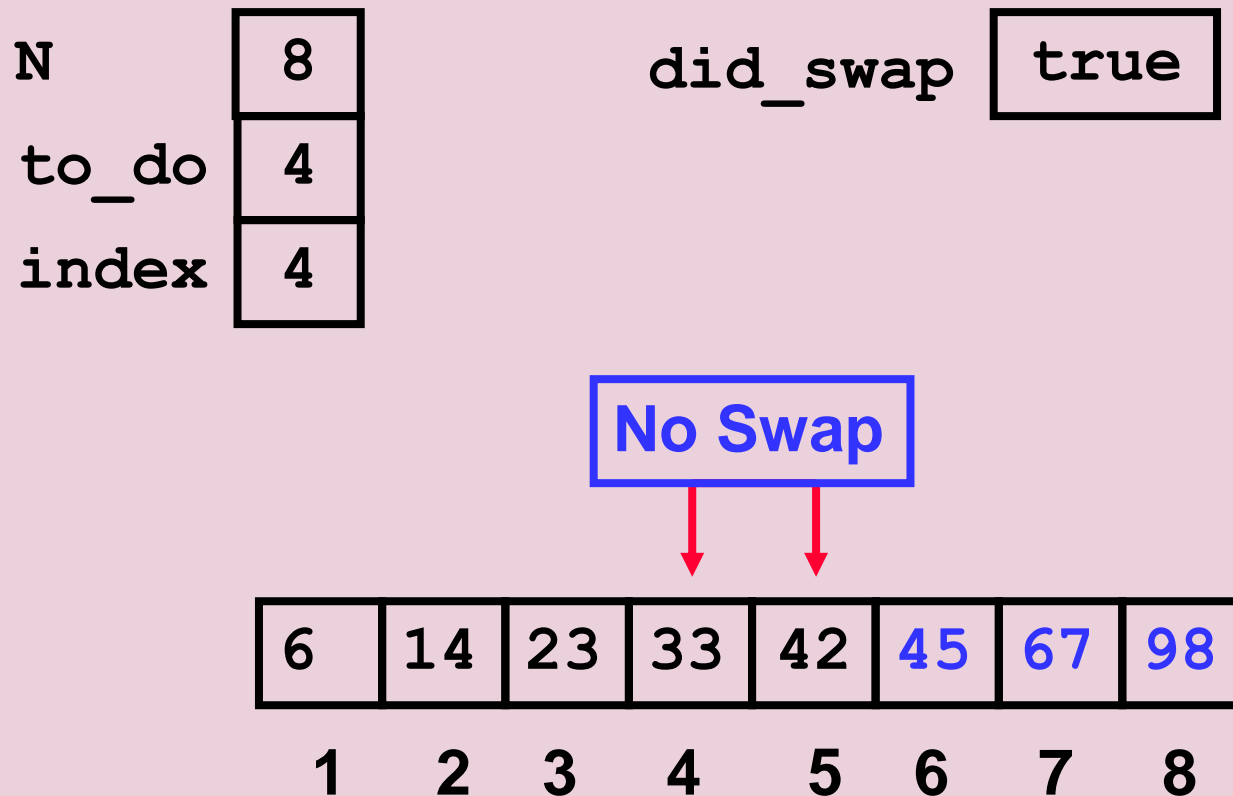
The Fourth “Bubble Up”



The Fourth “Bubble Up”



The Fourth “Bubble Up”



After Fourth Pass of Outer Loop

N

8

to_do

4

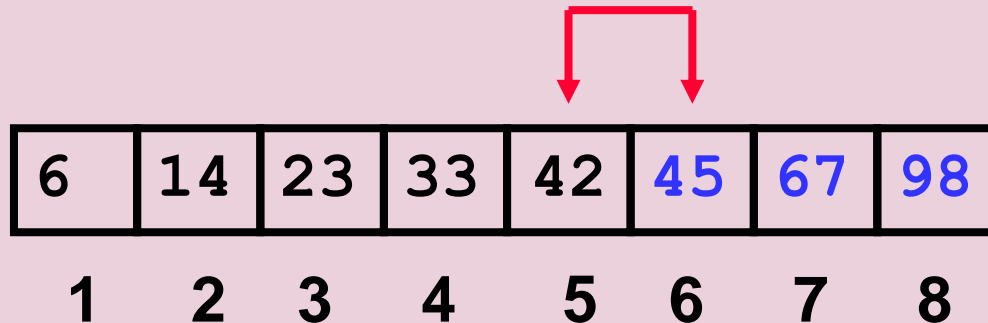
index

5

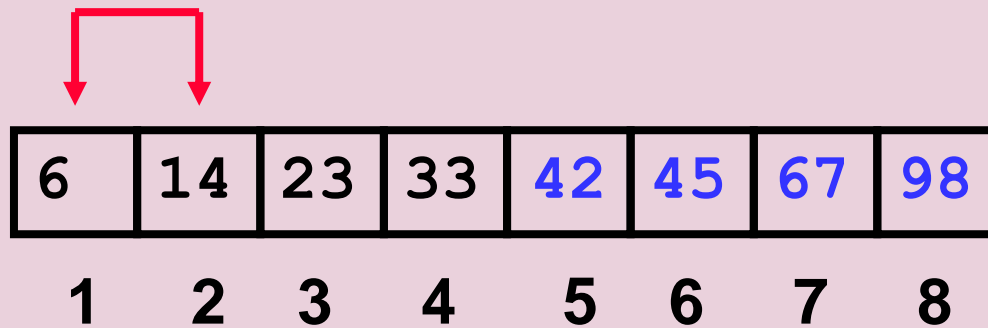
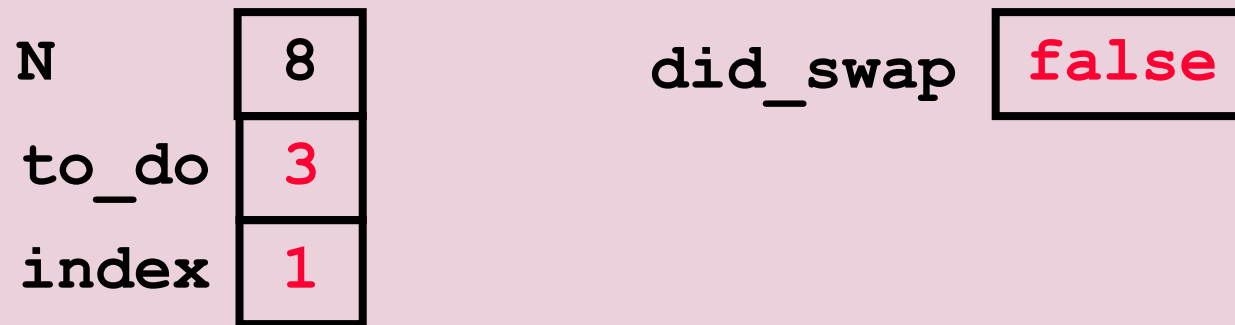
did_swap

true

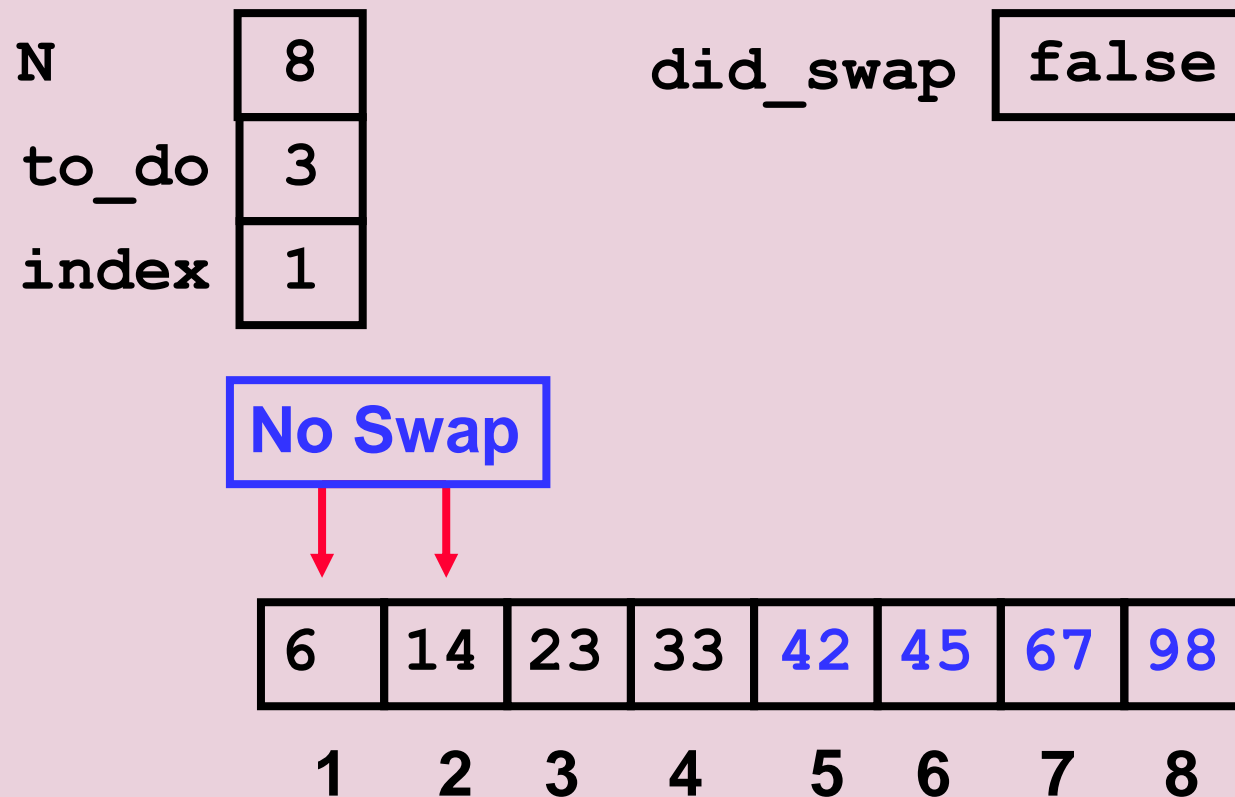
Finished fourth “Bubble Up”



The Fifth “Bubble Up”



The Fifth “Bubble Up”



The Fifth “Bubble Up”

N

8

 did_swap

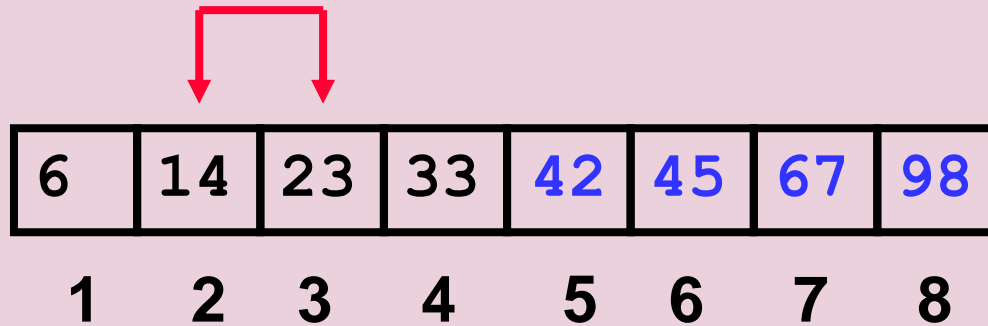
false

to_do

3

index

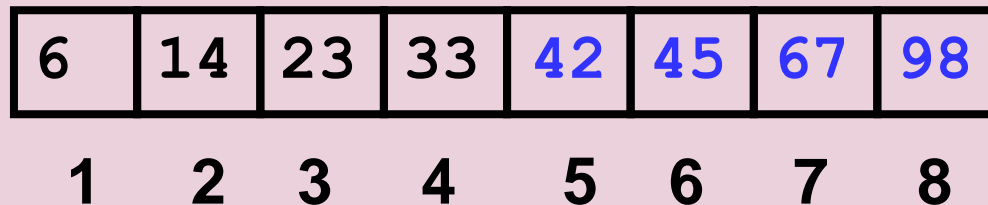
2



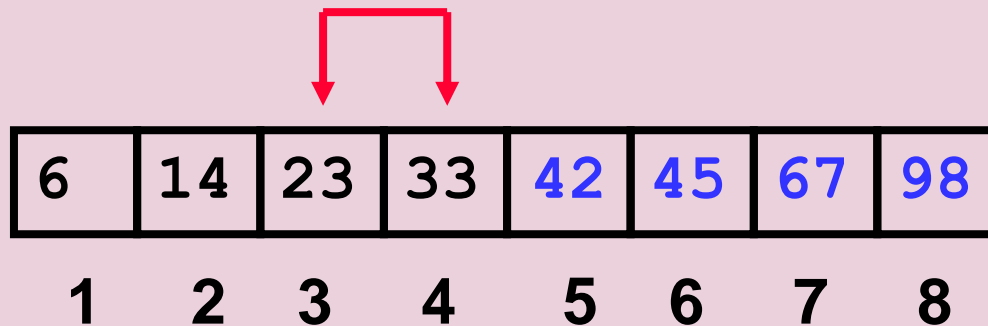
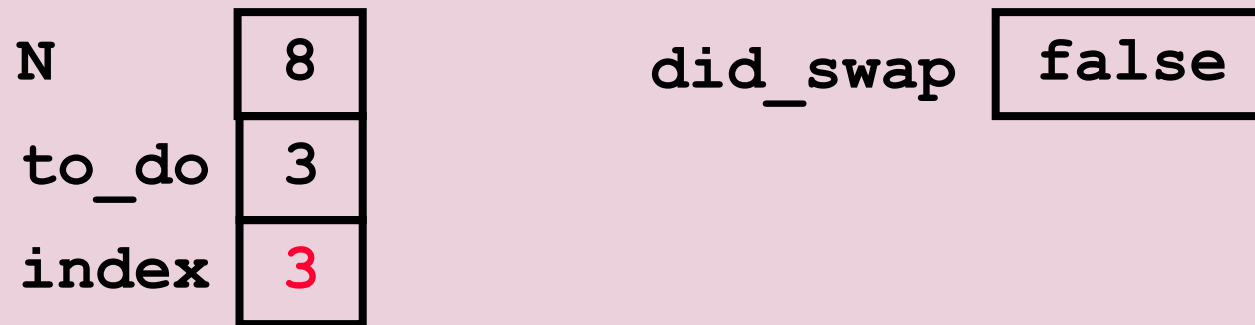
The Fifth “Bubble Up”



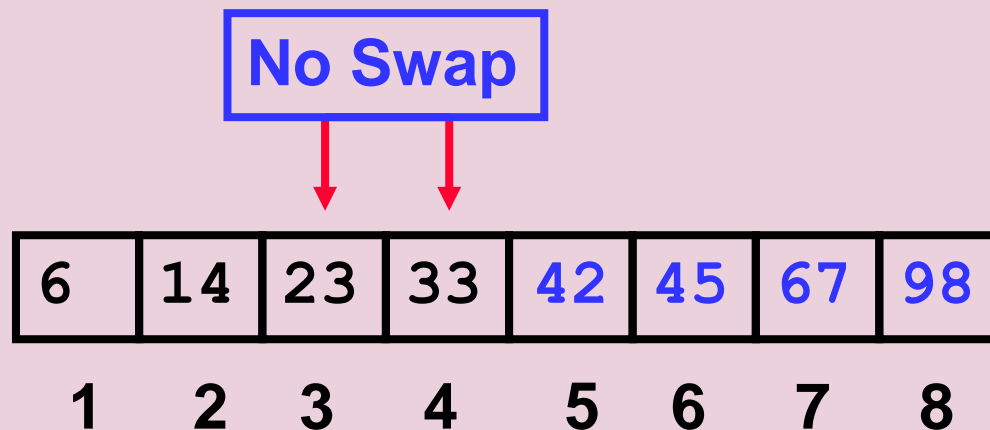
No Swap



The Fifth “Bubble Up”



The Fifth “Bubble Up”



After Fifth Pass of Outer Loop

N

8

 did_swap

false

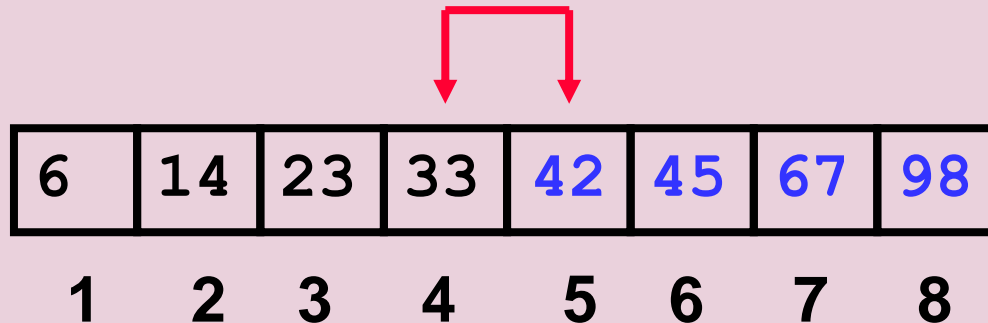
to_do

3

index

4

Finished fifth “Bubble Up”



Finished “Early”

N	8
to_do	3
index	4

did_swap **false**

We didn't do any swapping,
so all of the other elements
must be correctly placed.

We can “skip” the last two
passes of the outer loop.

6	14	23	33	42	45	67	98
1	2	3	4	5	6	7	8