Module - 4

Propositional Logic– Basic Connectives and Truth Tables

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Proposition

- A proposition is a declarative sentence that is either true or false but not both
- Example: Delhi is capital of India [True]
- 2+5=6 [False]
- America is neighboring country of India [False]
- 10+12 = 22 [True]

why are you not responding? X

Atomic statement

- Declarative sentences which cannot be further split into simpler sentences are called atomic statements (also called primary statements or primitive statements)

- Discrete Mathematics is one of the subject for Computer Science students [True]

Logical Connectives – Compound propositions – Conditional and Biconditional propositions – Truth Tables

S.No	English Language Usage	Logical Connectives	Type of Operator	Symbols
1.	and	Conjunction	binary	$\wedge$
2.	Or	Disjunction	Binary	$\vee$
3.	Not	Negation	Unary	$\neg$
4.	If ….then	Implication (or) Conditional	Binary	$\rightarrow$
5.	If and only if	Biconditional	Binary	<==> $\Leftrightarrow$

Compound Proposition

- Many mathematical statements are constructed by combining one or more propositions new propositions called compound propositions are formed from existing proposition using logical operators.

- **Truth Table:** A table, giving the truth values of a compound statement interms of its component parts is called a Truth table.

Negation ~ ⌉

The negation of a statement is generally formed by introducing the word not at a proper place in the statement

The Truth table for the negation of a proposition	
P	⌉P
T	F
F	T

P: Delhi is capital of India T
⌉P: Delhi is not " " " F

P: America is neighbour of India F
⌉P: America is not " " " T

Eg: P: Today is Wednesday[True], ⌉P: Today is not ~~Tuesday~~ Wednesday [False]
P: 3< 2 [False], ⌉P: 3>2 [True]

Conjunction [∧] [AND]

The conjunction of two statements P and Q is the statement P ∧ Q which is read as “P and Q”

The Truth table for the conjunction of two propositions		
P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

Eg: P: It is sunny[True], Q: I feel very warm[True], P ∧ Q : It is sunny and I feel very warm [True]

P: 3+4 < 5[False], Q: -3 > -5[True], P ∧ Q : 3+4 < 5 and -3 > -5[False]

P: 2 < 6 [True], Q: 2+6=9 [False], P ∧ Q : 2 < 6 and 2+6 = 9 [False]

P: 3+5 = 6 [False], Q: 3-5=4[False], P ∧ Q : 3+5=6 and 3-5=4 [False]

60
I - 50 20
E - 40
T - I + E - 45 70

Disjunction [∨] [OR]

The disjunction of two statements P ~~and~~ or Q is the statement P ∨ Q which is read as “P or Q”

The Truth table for the conjunction of two propositions		
P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Eg: P: It is sunny[True], Q: I feel very warm[True], P ∨ Q : It is sunny or I feel very warm [True]

P: 3+4 < 5[False], Q: -3 > -5[True], P ∨ Q : 3+4 < 5 or -3 > -5[True]

P: 2 < 6 [True], Q: 2+6=9 [False], P ∨ Q : 2 < 6 or 2+6 = 9 [True]

P: 3+5 = 6 [False], Q: 3-5=4[False], P ∨ Q : 3+5=6 or 3-5=4 [False]

T 80% 95

Conditional Statement [→] [If, … then]

If P and Q are any two statements then the statement P → Q which is read as "if P, then Q" is called a conditional statement.

The Truth table for the implication P → Q		
P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Eg: P: I am hungry[True], Q: I will eat [True], P → Q : If I am hungry then I will eat[True]

P: I studied B.Tech [False], Q: I will teach B.Tech [True], P → Q : If I studied B.Tech then I will teach B.Tech [True]

P: The sun is shining today [True], Q: 2+8=6 [False], P→ QIf the sun is shining today then 2+8 =6 [False]

P: 2+5=9 [False], Q: 5= 9 - 2[False], P → Q: If 2+5=9 then 5=9-2 [True]

P - Premise
- condition
- hypothesis

Q - conclusion
- Implication result

Biconditional [equivalence] Statement [⇔] [If and only if]

If P and Q are any two statements then the statement P ⇔ Q which is read as " P if and only if Q" and abbreviated as "P iff Q" is called a biconditional statement. Is has the same truth value as (P→ Q) ∧ (Q→P).

The Truth table for the Biconditional P ⇔ Q		
P	Q	P ⇔ Q
T	T	T
T	F	F
F	T	F
F	F	T

Eg: P: You can take the flight[True], Q: You buy a ticket [True], P ⇔ Q : You can take the flight if and only if you buy a ticket.[True]

P: 2>3[False], Q: 4<5 [True], P ⇔ Q : 2>3 iff 4< 5[False]

P: 5<6 [True], Q: 7>8[False], P⇔ Q: 5<6 iff 7<8 [False]

P: 3 = 8-4 [False], Q: 3+4=8[False], P ⇔ Q: 3=8-4 iff 3+4=8 [True]

$p \leftrightarrow q$

$\Rightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

If I am hungry then I will eat

$P \to q$ true

$\equiv$ true

Converse.

$P \to Q$ T

$Q \to P$

Exclusive or of P and Q [P ⊕ Q]

If P and Q are any two statements . The exclusive or of P and Q, denoted by P ⊕ Q, is the proposition that is true when exactly one of P and Q is true and is false otherwise.

$P \to q$

If I get good marks then I will get placement.

P: I get good marks

Q: I will get placement

The Truth table for the Exclusive or of two proposition P ⊕ Q		
P	Q	P ⊕ Q
T	T	F
T	F	T
F	T	T
F	F	F

P: I am hungry

Q: I will eat

If I eat, then I will hungry

If I am not eating then I will not be hungry

If I am not hungry then I will not eat

Converse, Contrapositive, Inverse

1. The proposition Q → P is called the converse of P → Q
2. The proposition ⌉Q → ⌉P is called the contrapositive of P → Q
3. The proposition ⌉P → ⌉Q is called the inverse P → Q

Converse → If I get place then I will get good marks

Contrapositive → If I am not placed then I will not get good marks

Inverse → If I am not getting good marks then I will not get placed

Contrapositive

If P → Q is an implication, then the converse of P → Q is the implication Q → P, and the contrapositive of P → Q is the implication ⌉Q → ⌉P.

Example1: Give the converse and contrapositive of the implication " If it is raining, then I get wet".

Solution : P: It is raining, Q: I get wet, Q→ P(Converse): If I get wet then it is raining ⌉ Q → ⌉P (Contrapositive): If I do not get wet, then it is not raining.

Example 2: State the converse, contrapositive and inverse of the following: " A Positive integer is a prime only if it has no divisors other than 1 and itself"

Solution: Converse: A positive integer is a prime if it has no divisors other than 1 and itself

Contrapositive: If a positive integer has a divisor other than 1 and it self then it is not prime.

Inverse: If a positive integer is not prime then it has a divisor other than 1 and itself

$P \to Q$

P: A +ve integer has no divisors other than 1 + itself

Q: It is a prime

$Q \to P$

If a +ve integer is prime then it has no divisor other than 1 + itself

Contra +ve $\neg Q \to \neg P$ If a ~~+ve~~ integer is no a prime then it has divisors other 1+ itself

inverse $\neg P \to \neg Q$ If a +ve integer has divisors other than 1+ itself then it is not a prime

Contrapositive

Example: State the converse, contrapositive and inverse of " If it is very cold the I will wear sweater"

Solution:

1. Converse: I will wear sweater only if it is very cold
2. Contrapositive: If I do not wear sweater then it is not very cold.
3. Inverse: If it is not very cold, then I will not wear sweater.

P: It is very cold
Q: I will wear sweater

$Q \to P$

① If I wear sweater then it will be very cold

② $\neg Q \to \neg P$
If I am not wearing sweater then it is not very cold

③ $\neg P \to \neg Q$
If it is not very cold then I will not wear sweater

Truth Table

		Conjunction	Disjunction	Negation	Conditional	Biconditional	Converse	Inverse	Contrapositive
P	Q	$P \wedge Q$	$P \vee Q$	$\neg P$	$P \to Q$	$P \Leftrightarrow Q$	$Q \to P$	$\neg P \to \neg Q$	$\neg Q \to \neg P$
T	T	T	T	F	T	T	T	T	T
T	F	F	T	F	F	F	T	T	F
F	T	F	T	T	T	F	F	F	T
F	F	F	F	T	T	T	T	T	T

Problem 1: Construct the truth table $P \wedge \neg P$

P	$\neg P$	$P \wedge \neg P$
T	F	F
T	F	F
F	T	F
F	T	F

$\to$ Contradiction

Problem 2: Construct the truth table $P \vee \neg P$

$\to$ Tautology

P	$\neg P$	$P \vee \neg P$
T	F	T
T	F	T
F	T	T
F	T	T

Problem 3: Construct the truth table $P \wedge P$

P	P	$P \wedge P$
T	T	T
F	F	F

$P \wedge P = P$

$A \cap A = A$

Problem 4: Construct the truth table $P \vee P$

P	P	$P \vee P$
T	T	T
F	F	F

$P \vee P = P$

$A \cup A = A$

Problem 5: Construct the truth table ⌉⌉P

P	⌉P	⌉⌉P
T	F	T
F	T	F

⌉⌉P = P

$\overline{(\overline{A})} = A$

Problem 6: Construct the truth table P ∨ ⌉Q

P	Q	⌉Q	P∨⌉Q
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

→ Contingency

Problem 7: Construct the truth table P ∧ ⌉Q

P	Q	⌉Q	P∧⌉Q
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Problem 8: Construct the truth table P ∧(P ∨ Q)

P	Q	P∨Q	P∧(P∨Q) = P
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

Problem 9: Construct the truth table $(P \vee Q) \vee \neg P$ → Tautology

P	Q	¬P	P∨Q	(P∨Q)∨¬P
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Problem 10: Construct the truth table (i) $\neg(\neg P \vee \neg Q)$ (ii) $\neg(\neg P \wedge \neg Q)$

P	Q	¬P	¬Q	¬P∨¬Q	¬(¬P∨¬Q)	¬P∧¬Q	¬(¬P∧¬Q)
T	T	F	F	F	T	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	T	F	T	F

Problem 11: Construct the truth table $(\neg P \wedge (\neg Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))$ – 1

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P	Q	R	¬P	¬Q	¬Q∧R	¬P∧(¬Q∧R)	(Q∧R)	P∧R	(Q∧R)∨(P∧R)	3
T	T	T	F	F	F	F	T	T	T	T
T	T	F	F	F	F	F	F	F	F	F
T	F	T	F	T	T	F	F	T	T	T
T	F	F	F	T	F	F	F	F	F	F
F	T	T	T	F	F	F	T	F	T	T
F	T	F	T	F	F	F	F	F	F	F
F	F	T	T	T	T	T	F	F	F	T
F	F	F	T	T	F	F	F	F	F	F

tautology

Problem 12: Construct the truth table ¬(P ∧Q) ⇔ ¬P ∨ ¬Q

2 P – T F

2 Q – T F

$2^2 = 4$

P	Q	P∧Q	¬(P∧Q)	¬P	¬Q	¬P∨¬Q	¬(P∧Q) ⟺ ¬P ∨ ¬Q
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Problem 13: Construct the truth table (P→Q) → P

P	Q	P→Q	(P→Q)→P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

Problem 14: Construct the truth table (P → Q) ∧ (Q→ P)

P	Q	P→Q	Q→P	(P→Q) ∧(Q→P)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Problem 15: Construct the truth table (P ∨ ¬Q) → Q

P	Q	¬Q	P∨¬Q	(P∨¬Q)→Q
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F