

Time and Space Trade-Offs

Example: Sorting by Counting

Definition:

In computer science, a **space-time** or **time-memory tradeoff** is a way of solving a problem in :

- 1.) Less time by using more memory) or,
- 2.) By solving a problem in very little space by spending a long time.

Types of Trade Off:

1. **Compressed / Uncompressed Data**
2. **Re-Rendering / Stored Images**
3. **Smaller Code / Loop Unrolling**
4. **Lookup Table / Recalculation**

Compressed / Uncompressed Data

- A space -time trade off can be applied to the problem of data storage.
- If data is stored uncompressed, it takes more space but less time.
- If the data is stored compressed, it takes less space but more time to run the decompression algorithm.

Re-Rendering / Stored Images

- Storing only the source and rendering it as an image everytime the page is requested would be trading time for space.

More time used but less space.

- Storing the images would be trading space for time.

More space used but less time.

Smaller Code(with loop) / Larger Code (without loop)

- Smaller code occupies **less space** in memory but it requires **high computation time** which is required for jumping back to the beginning of the loop at the end of each iteration.
- Larger code or loop unrolling can be traded for higher program speed. It occupies **more space** in memory but requires **less computation time**.

(as we need not perform jumps back and forth since there is no loop.)

Lookup Table / Recalculation

1. In lookup table, an implementation can include the entire table which **reduces computing time** but **increases** the amount of **memory needed**.
1. It can recalculate i.e. compute table entries as needed, **increasing computing time** but **reducing memory requirements**.

Example

More time, Less space	More Space, Less time
<ol style="list-style-type: none">1. int a,b;2. printf("enter value of a \n");3. scanf("%d",&a);4. printf("enter value of b \n");5. scanf("%d",&b);6. b=a+b;7. printf("output is:%d",b);	<ol style="list-style-type: none">1. int a,b,c;2. printf("enter values of a,b and c \n");3. scanf("%d%d%d",&a,&b,&c);4. printf("output is:%d",c=a+b);

COUNTING SORT

- Step-1: Create a count array to store the count of each unique object.
- Step-2: Modify count array by adding the previous number.
- Step-3: Create output array by decrease count array.



Let the Array in range 0 to 5

Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---

Create an array that will hold the count of each number, with index ranges from 0 to 5

	[0]	[1]	[2]	[3]	[4]	[5]
Count	0	1	2	2	1	1



Modify count array by adding the previous number:

Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---

Sum Count

[0]	[1]	[2]	[3]	[4]	[5]
0	1	3	5	6	7

Output each object from the input sequence followed by decreasing count by 1:

Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---

[0] [1] [2] [3] [4] [5]

Sum Count

0	1	3	5	6	7
---	---	---	---	---	---

[1] [2] [3] [4] [5] [6] [7]

output

--	--	--	--	--	--	--



Output each object from the input sequence followed by decreasing count by 1:



Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---

[0] [1]  [2] [3] [4] [5]

Sum Count

0	1	3	5	1	1
---	---	---	---	---	---


[1] [2] [3] [4] [5] [6] [7]

output

1						
---	--	--	--	--	--	--




Input



1	4	3	2	3	5	2
---	---	---	---	---	---	---

[0] [1] [2] [3] [4] [5]

Sum Count



0	0	3	5	6	7
---	---	---	---	---	---


[1] [2] [3] [4] [5] [6] [7]

output

1					4	
---	--	--	--	--	---	--




Input



1	4	3	2	3	5	2
---	---	---	---	---	---	---

[0] [1] [2] [3] [4] [5]

Sum Count



0	0	3	5	5	7
---	---	---	---	---	---


[1] [2] [3] [4] [5] [6] [7]

output

1				3	4	
---	--	--	--	---	---	--



Input



1	4	3	2	3	5	2
---	---	---	---	---	---	---



[0] [1] [2] [3] [4] [5]

Sum Count

0	0	3	4	5	7
---	---	---	---	---	---

[1] [2] [3] [4] [5] [6] [7]

output

1		2		3	4	
---	--	---	--	---	---	--



Input



1	4	3	2	3	5	2
---	---	---	---	---	---	---

[0] [1] [2] [3] [4] [5]

Sum Count

0	0	2	4	5	7
---	---	---	---	---	---

[1] [2] [3] [4] [5] [6] [7]

output

1		2	3	3	4	
---	--	---	---	---	---	--



Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---



[0] [1] [2] [3] [4] [5]

Sum Count

0	0	2	3	5	7
---	---	---	---	---	---

[1] [2] [3] [4] [5] [6] [7]

output

1		2	3	3	4	5
---	--	---	---	---	---	---



Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---



[0] [1] [2] [3] [4] [5]

Sum Count

0	0	2	3	5	6
---	---	---	---	---	---

[1] [2] [3] [4] [5] [6] [7]

output

1	2	2	3	3	4	5
---	---	---	---	---	---	---



Input

1	4	3	2	3	5	2
---	---	---	---	---	---	---

[0] [1] [2] [3] [4] [5]

Sum Count

0	0	1	3	5	6
---	---	---	---	---	---

[1] [2] [3] [4] [5] [6] [7]

output

1	2	2	3	3	4	5
---	---	---	---	---	---	---



ARRAY IS NOW SORTED

[1] [2] [3] [4] [5] [6] [7]

output

1	2	2	3	3	4	5
---	---	---	---	---	---	---



Algorithm:

countingSort(array, n) // 'n' is the size of array

max = find maximum element in the given array

create count array with size maximum + 1

Initialize count array with all 0's

for i = 0 to n

find the count of every unique element and

store that count at ith position in the count array

for j = 1 to max

Now, find the cumulative sum and store it in count array

for i = n to 1

Restore the array elements

Decrease the count of every restored element by 1

end countingSort

Coding

```
// Store count of each character
for (i = 0; arr[i]; ++i)
    ++count[arr[i]];

// Change count[i] so that count[i] now contains
actual
// position of this character in output array
for (i = 1; i <= RANGE; ++i)
    count[i] += count[i - 1];

// Build the output character array
for (i = 0; arr[i]; ++i) {
    output[count[arr[i]] - 1] = arr[i];
    --count[arr[i]];
}
// Copy the output array to arr, so that arr now
// contains sorted characters
for (i = 0; arr[i]; ++i)
    arr[i] = output[i];
}
```

Time Complexity: $O(n)$