20MA1006	Calculus, Vector Spaces and Laplace Transform

Module 1: Calculus

Performance evaluation of Computer Systems - Evolutes and involutes; Evaluation of definite and improper integrals; Applications of definite integrals to evaluate surface areas and volumes of revolutions

Module 2: Sequences and series

Design a Calculator Software based on Convergence of sequence and series, tests for convergence; Power series, Taylor's series, Applications of Taylor series - sum of a series, evaluate limits and approximate functions, series for exponential, trigonometric and logarithm functions.

Module 3: Vector spaces

Digital image enhancement using transformations, Vector Space, linear dependence of vectors, basis, dimension; Linear transformations (maps), range and kernel of a linear map, Inverse of a linear transformation, rank- nullity theorem, composition of linear maps, Matrix associated with a linear map.

Module 4: Vector Differentiation

Decision Review System in Cricket, Path of thrown basketball, hit distance using Differentiation of vectors–Curves in space-Velocity and acceleration - Scalar and Vector point functions–Gradient–Divergence-Curl–Physical interpretations- Solenoidal and irrotational fields-Laplacian operator.

Module 5: Inner product spaces

Designing the movement of Robotic arms, Norm definition- properties -Inner product spaces, orthogonal vectors – orthonormal vectors- orthonormal basis- Gram-Schmidt orthogonalization process.

Module 6: Laplace transforms

Building integrated circuits and chips for computers using Laplace transform-Properties-Laplace transform of periodic functions-Laplace transform of unit step function, Impulse function-Inverse Laplace transform – Convolution.

Text Books:

1. B.S. Grewal, “Higher Engineering Mathematics”, Khanna Publishers, 44th Edition, 2017.

Reference Books:

1. V. Krishnamurthy, V.P. Mainra and J.L. Arora, “An introduction to Linear Algebra”, Affiliated East–West press, Reprint2005.
2. David C. Lay, Steven R. Lay and Judi J. McDonald “Linear Algebra and its Applications”, Fifth Edition. Pearson, 2006.
3. G.B. Thomas and R.L. Finney, “Calculus and Analytic geometry”, 9th Edition, Pearson, Reprint, 2002.
4. Erwin Kreyszig, “Advanced Engineering Mathematics”, 9th Edition, John Wiley & Sons, 2006.
5. D. Poole, “Linear Algebra: A Modern Introduction”, 2ndEdition, Brooks/Cole,2005.
6. Veerarajan T., “Engineering Mathematics for first year”, Tata McGraw-Hill, New Delhi, 2008.
7. Ramana B.V., “Higher Engineering Mathematics”, Tata McGraw Hill New Delhi, 11th Reprint, 2010.
8. N.P. Bali and Manish Goyal, “A text book of Engineering Mathematics”, Laxmi

Table of Derivatives.

1. $\frac{d}{dx} c = 0$, c is a constant

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(cx) = c$

4. $\frac{d}{dx}(x^n) = nx^{n-1}$

5. $\frac{d}{dx}(e^x) = e^x$

6. $\frac{d}{dx} \ln x = \frac{1}{x}$ ($\log x$)

7. $\frac{d}{dx} a^x = a^x \ln a$

8. $\frac{d}{dx} \log_a x = \frac{1}{x} \cdot \frac{1}{\ln a}$

9. $\frac{d}{dx} \sin x = \cos x$

10. $\frac{d}{dx} \cos x = -\sin x$

11. $\frac{d}{dx} \tan x = \sec^2 x$

12. $\frac{d}{dx} \cot x = -\csc^2 x$ ($-\operatorname{cosec}^2 x$)

13. $\frac{d}{dx} \sec x = \sec x \tan x$

14. $\frac{d}{dx} \csc x = -\csc x \cot x$ ($\operatorname{cosec} x$)

15. $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$ ($\sin^{-1} x$)

16. $\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$ ($\cos^{-1} x$)

17. $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$ ($\tan^{-1} x$)

18. $\frac{d}{dx} \operatorname{arccot} x = \frac{-1}{1+x^2}$ ($\cot^{-1} x$)

19. $\frac{d}{dx} \operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}} = \frac{1}{x\sqrt{x^2-1}}$ ($\sec^{-1} x$)

20. $\frac{d}{dx} \operatorname{arccsc} x = \frac{-1}{|x|\sqrt{x^2-1}} = \frac{-1}{x\sqrt{x^2-1}}$ ($\operatorname{cosec}^{-1} x$)

Differentiation Rules	
Constant Rule	$\frac{d}{dx}[c] = 0$
Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
Product Rule	$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$ $= f(x).g'(x) + g(x)f'(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
Chain Rule function of function rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

$\frac{d}{dx}(u.v) = uv' + vu'$

or $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

Calculus is used in every branch of the physical sciences, actuarial science, computer science, statistics, **engineering**, economics, business, **medicine**, demography, and in other fields wherever a problem can be mathematically modeled and an optimal solution is desired.

Calculus is **used** for calculating efficiency of algorithms, which is kind of important. You may not need it for many modern **applications**, but it's vital for understanding programs on a deeper level.

Calculus. **Calculus** is another **important** part of **programming**. **Calculus** problems show up practically all the time in machine learning. In any machine learning problem, the ultimate goal is to optimize the cost function

$y = f(x)$ $\frac{dy}{dx}$ ~~partial~~ ordinary derivative

The Definition of **Differentiation**

The **derivative** is the instantaneous rate of change of a function with respect to one of its variables. This is equivalent to finding the slope of the tangent line to the function at a point.

$x = f(y)$ $\frac{dx}{dy}$ $\frac{d}{dx}$ or $\frac{d}{dy}$

$z = f(x, y)$ $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

Differentiate the following.

1) $y = x^5$ (2) $y = x$ (3) $y = \frac{1}{7x^2}$ (4) $y = 4x^2 - 3\cos x + e^x + 2\sin x$

(5) $y = \cos 6x + \sin 4x$ (6) $y = (x^2+5)^2$ (7) $y = (3x^2+1)^3$ 8) $y = \frac{3}{\sqrt{x}}$

9) $y = 3x^4 e^x$ (10) $y = (x^2+1)(x^2+2)$ (11) $y = \cos 2x \, e^{3x}$

$\frac{dy}{dx} = y_1$ or y'

$\frac{d}{dx}(x^n) = n x^{n-1}$

12) $y = \frac{x^3}{x-2}$ (13) $y = \frac{2x-3}{4x+5}$

Answer

1) $y = x^5$

$\frac{dy}{dx} = 5x^4$

2) $y = x = x^1$

$\frac{dy}{dx} = 1$

3) $y = \frac{1}{7x^2} = \frac{1}{7}x^{-2}$

$\frac{dy}{dx} = \frac{1}{7} \times (-2) x^{-2-1}$

$= -\frac{2}{7} x^{-3}$

$= -\frac{2}{7x^3}$

4) $\frac{dy}{dx} = 8x + 3\sin x + e^x + 2\cos x$

5) $\frac{dy}{dx} = -\sin 6x \times 6 + 4\cos 4x = -6\sin 6x + 4\cos 4x$

6) $y = (x^2+5)^2$

$\frac{dy}{dx} = 2(x^2+5)^{2-1}.(2x+0)$

$= 4x.(x^2+5)^1$

$4 \times 2x^{2-1}$

$8x^1$

$8x$

7) $y = (3x^2+1)^3$

$\frac{dy}{dx} = 3(x^2+1)^2 \cdot (6x+0)$

$= 18x(x^2+1)^2$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$n = \frac{-1}{2}$

8) $y = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}}$ Note $\sqrt{x} = x^{1/2}$

$y = 3x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 3 \times -\frac{1}{2} (x)^{-\frac{1}{2}-1} \cdot 1$

$= -\frac{3}{2} \cdot x^{-3/2}$

9) $y = 3x^4 e^x$

$$\frac{dy}{dx} = 3\left(x^4 . e^x + e^x . 4x^3\right)$$

$$= 3x^4 e^x + 12 e^x x^3$$

$$= 3x^3 e^x (x+4)$$

10) $y = \underbrace{(x^2+1)}_{u} \underbrace{(x^2+2)}_{v}$

$$\frac{dy}{dx} = (x^2+1) 2x + (x^2+2).2x$$

$$= 2x(x^2+1+x^2+2)$$

$$= 2x(2x^2+3)$$

11) $y = \underbrace{\cos 2x}_{u} \underbrace{e^{3x}}_{v}$

$$\frac{dy}{dx} = \cos 2x.(e^{3x}.3) + e^{3x}(-\sin 2x).2$$

$$= e^{3x}(3\cos 2x - 2\sin 2x)$$

12) $y = \frac{x^3}{x-2}$

$$\frac{dy}{dx} = \frac{(x-2).3x^2 - x^3.1}{(x-2)^2}$$

$$= \frac{3x^3 - 6x^2 - x^3}{(x-2)^2}$$

$$= \frac{2x^3 - 6x^2}{(x-2)^2} = \frac{2x^2(x-3)}{(x-2)^2}.$$

13) $y = \dfrac{2x-3}{4x+5}$

$$\frac{dy}{dx} = \frac{(4x+5).2 - (2x-3).4}{(4x+5)^2}$$

$$= \frac{\cancel{8x}+10 - \cancel{8x}+12}{(4x+5)^2}$$

$$= \frac{22}{(4x+5)^2}.$$

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