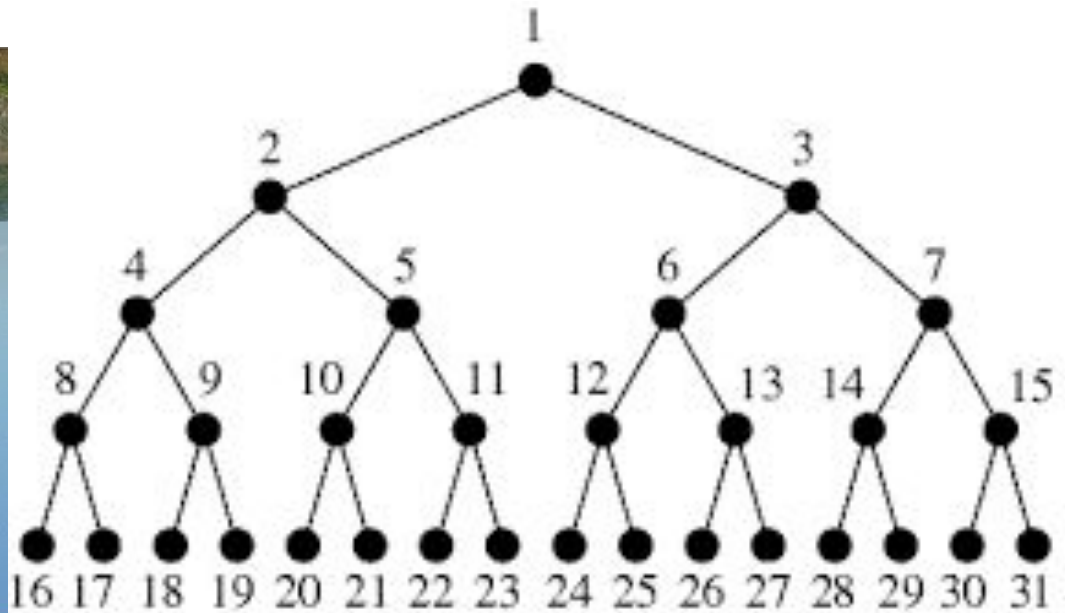


# Trees

Hierarchical (or)  
non-linear  
data structure

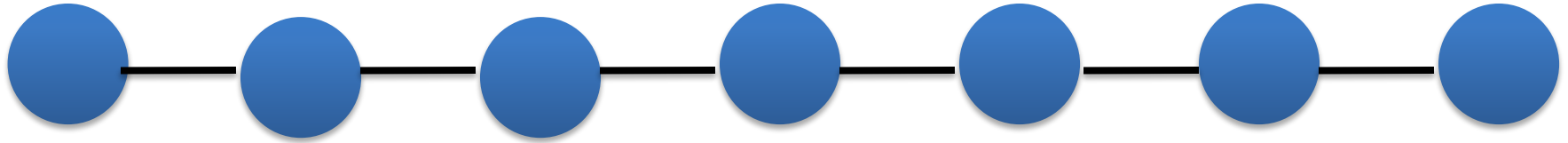


# Trees

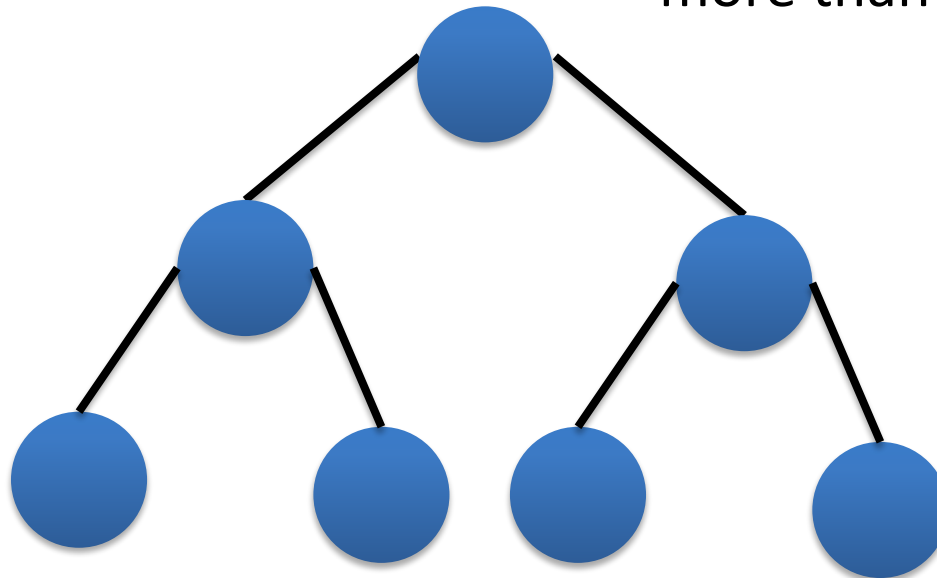


**Tree is a collection of nodes connected by edges.**

# Linear vs Non linear

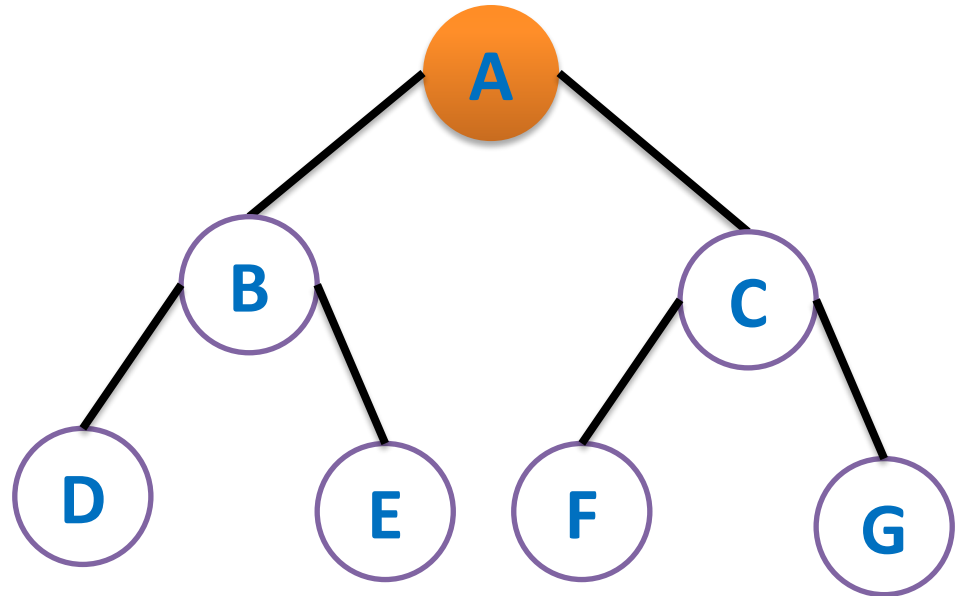


In non-linear data structure, a data item is connected to more than one data items.



# Terminologies used in Trees

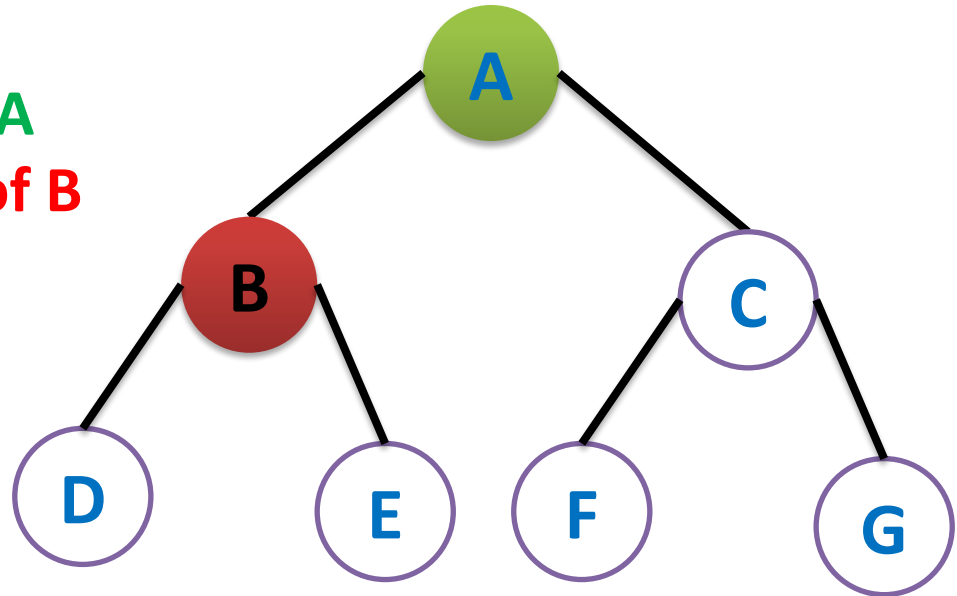
- **Root**
- **Child**
- **Parent**
- **Siblings**
- **Descendant**
- **Ancestor**
- **Leaf**
- **Degree**
- **Edge**
- **Height of node**
- **Height of tree**
- **Depth**



**The top node in a tree.**

# Terminologies used in Trees

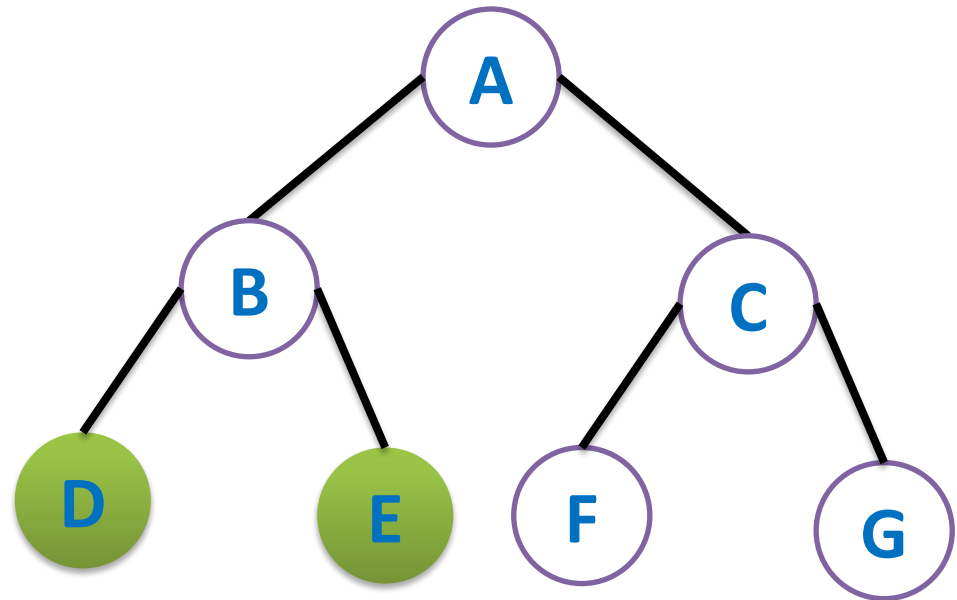
- Root
- **Child**    B is a child of A
- **Parent**    A is a parent of B
- Siblings
- Descendant
- Ancestor
- Leaf
- Degree
- Edge
- Height of node
- Height of tree
- Depth



A node directly connected to another node when moving away from the Root.

# Terminologies used in Trees

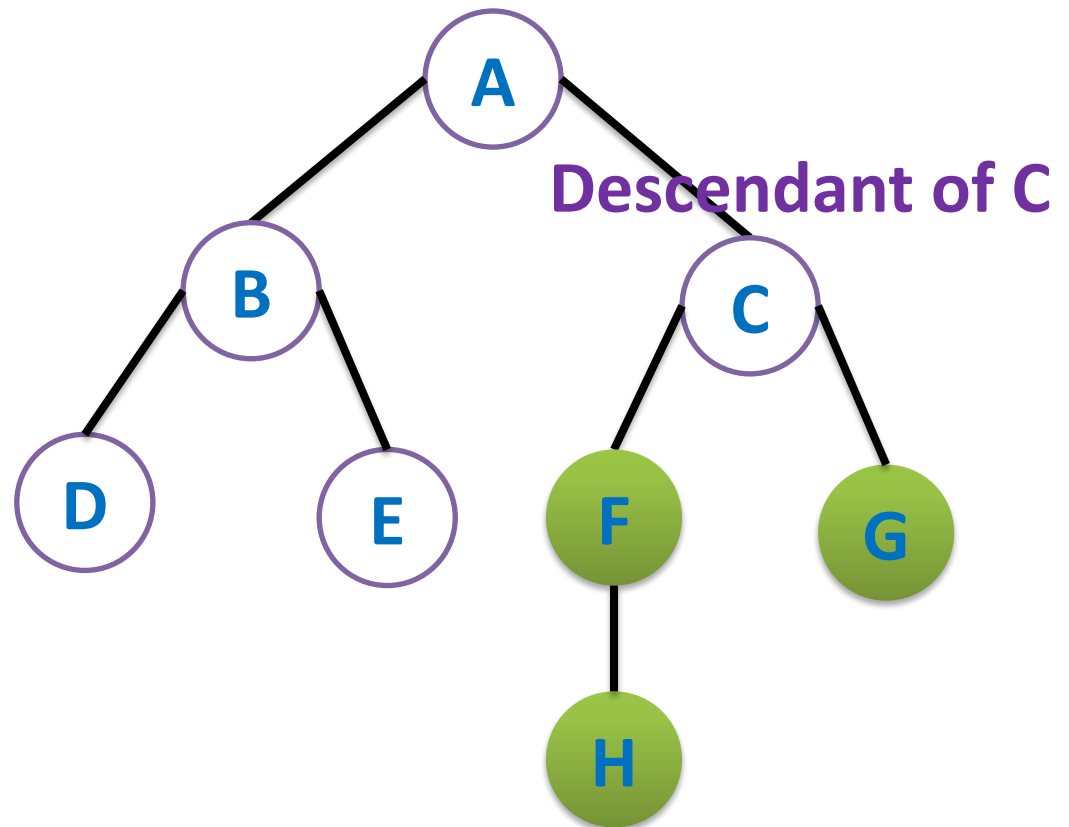
- Root
- Child
- Parent
- **Siblings**
- Descendant
- Ancestor
- Leaf
- Degree
- Edge
- Height of node
- Height of tree
- Depth



A group of nodes with the same parent.

# Terminologies used in Trees

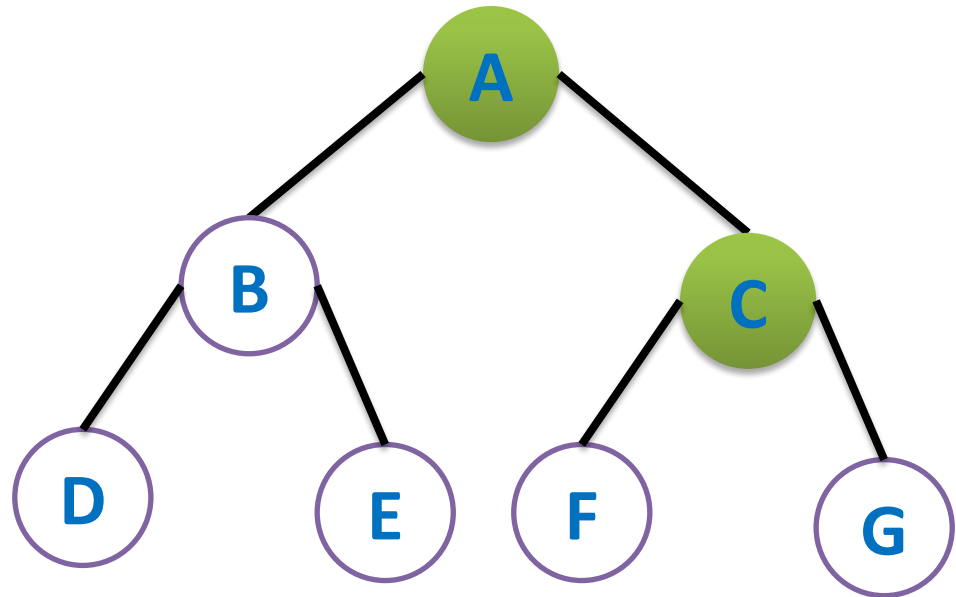
- Root
- Child
- Parent
- Siblings
- **Descendant**
- Ancestor
- Leaf
- Degree
- Edge
- Height of node
- Height of tree
- Depth



A node reachable by repeated proceeding from parent to child.

# Terminologies used in Trees

- Root
- Child
- Parent
- Siblings
- Descendant
- **Ancestor**
- Leaf
- Degree
- Edge
- Height of node
- Height of tree
- Depth



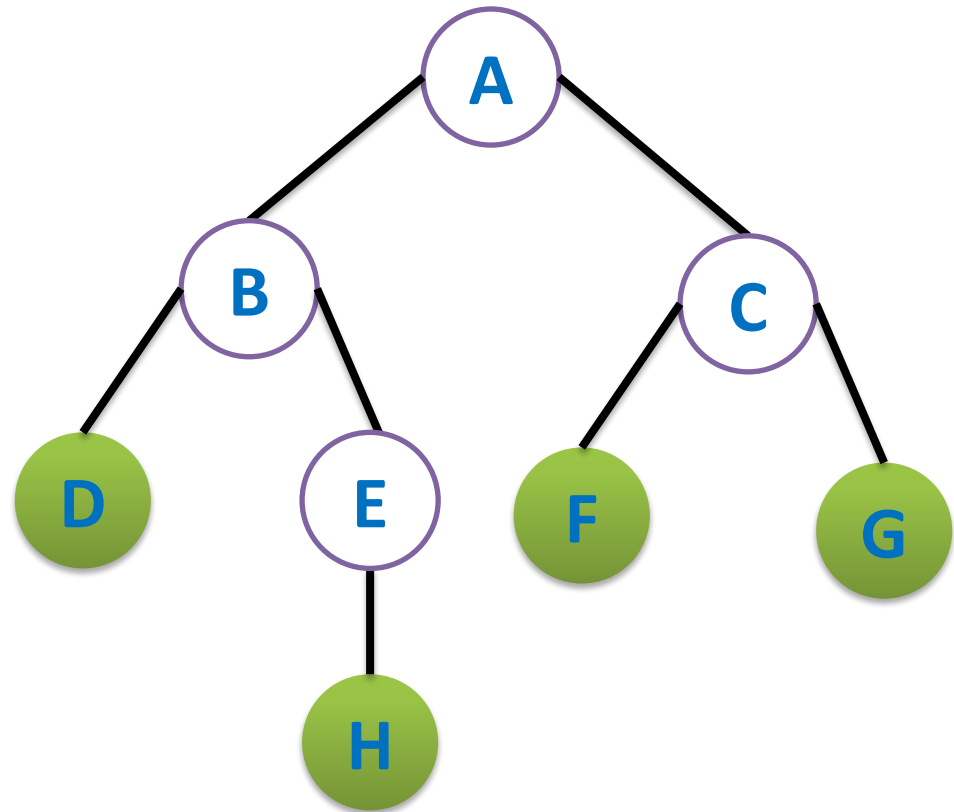
**Ancestor of F**

A node reachable by repeated proceeding from child to parent.



# Terminologies used in Trees

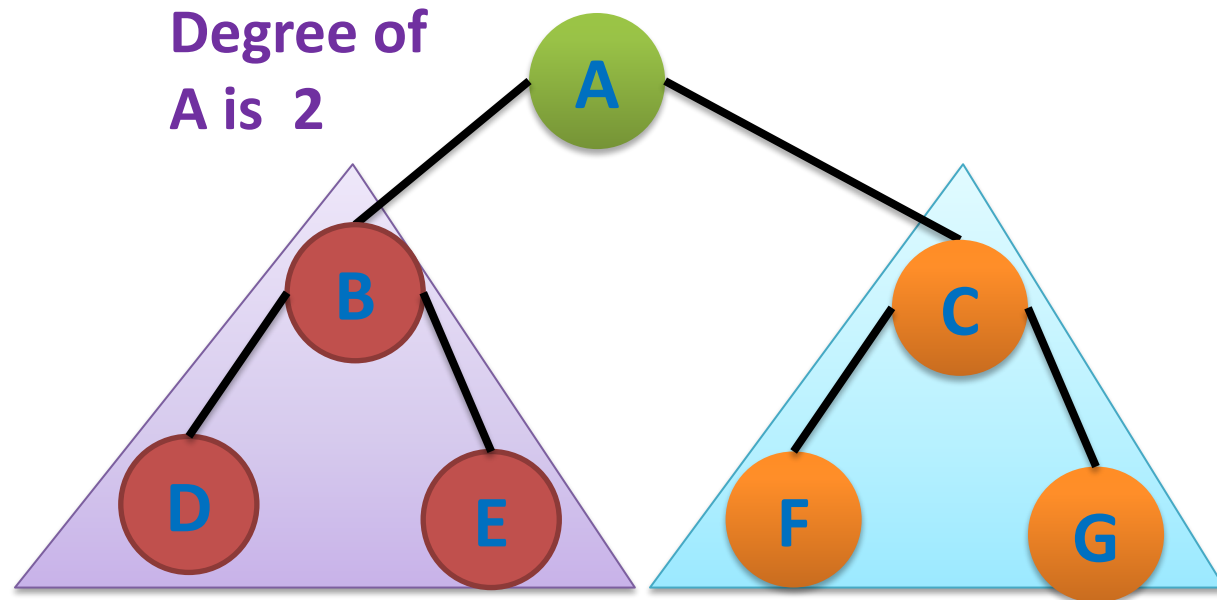
- Root
- Child
- Parent
- Siblings
- Descendant
- Ancestor
- **Leaf**
- Degree
- Edge
- Height of node
- Height of tree
- Depth



A node with no children.

# Terminologies used in Trees

- Root
- Child
- Parent
- Siblings
- Descendant
- Ancestor
- Leaf
- **Degree**
- Edge
- Height of node
- Height of tree
- Depth

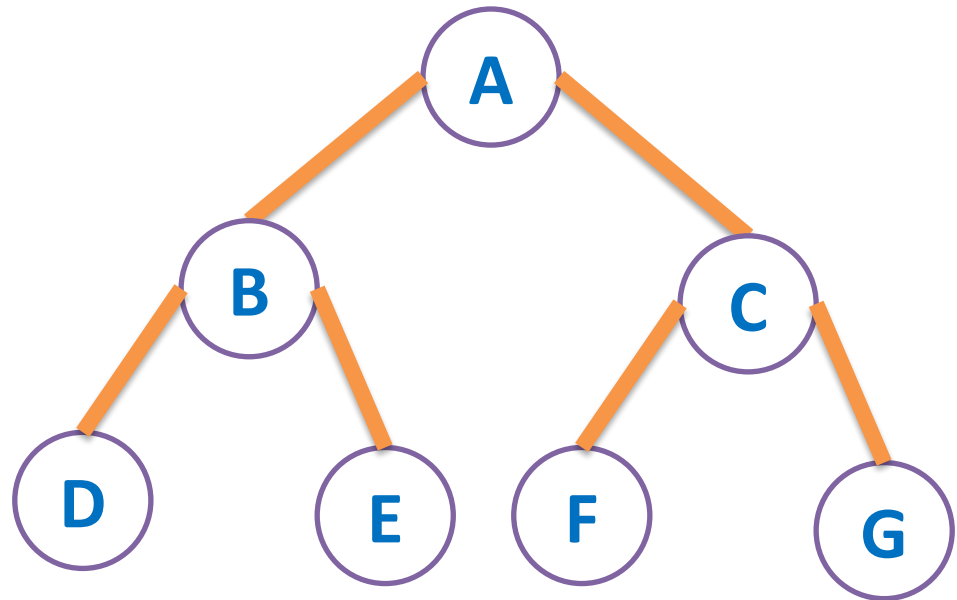


The number of sub trees of a node.

The **degree** of a **tree** is the **maximum degree** of any of its nodes.

# Terminologies used in Trees

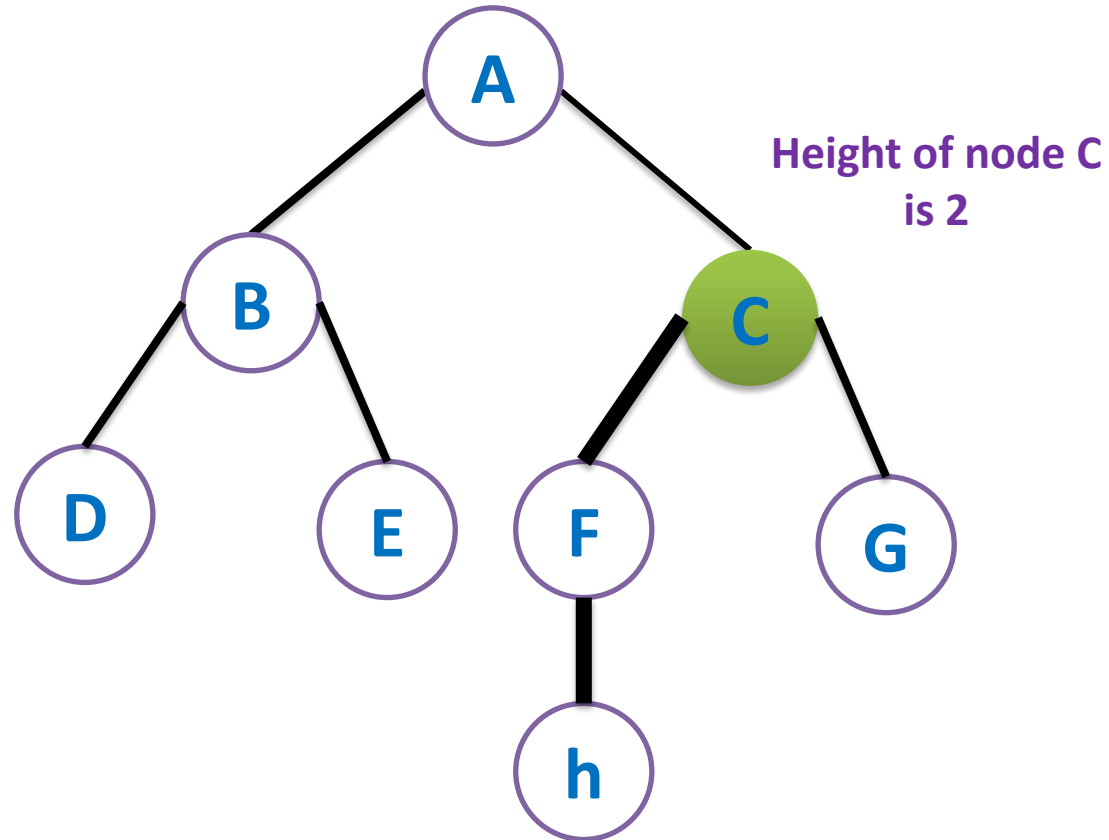
- Root
- Child
- Parent
- Siblings
- Descendant
- Ancestor
- Leaf
- Degree
- **Edge**
- Height of node
- Height of tree
- Depth



The connection between one node and another.

# Terminologies used in Trees

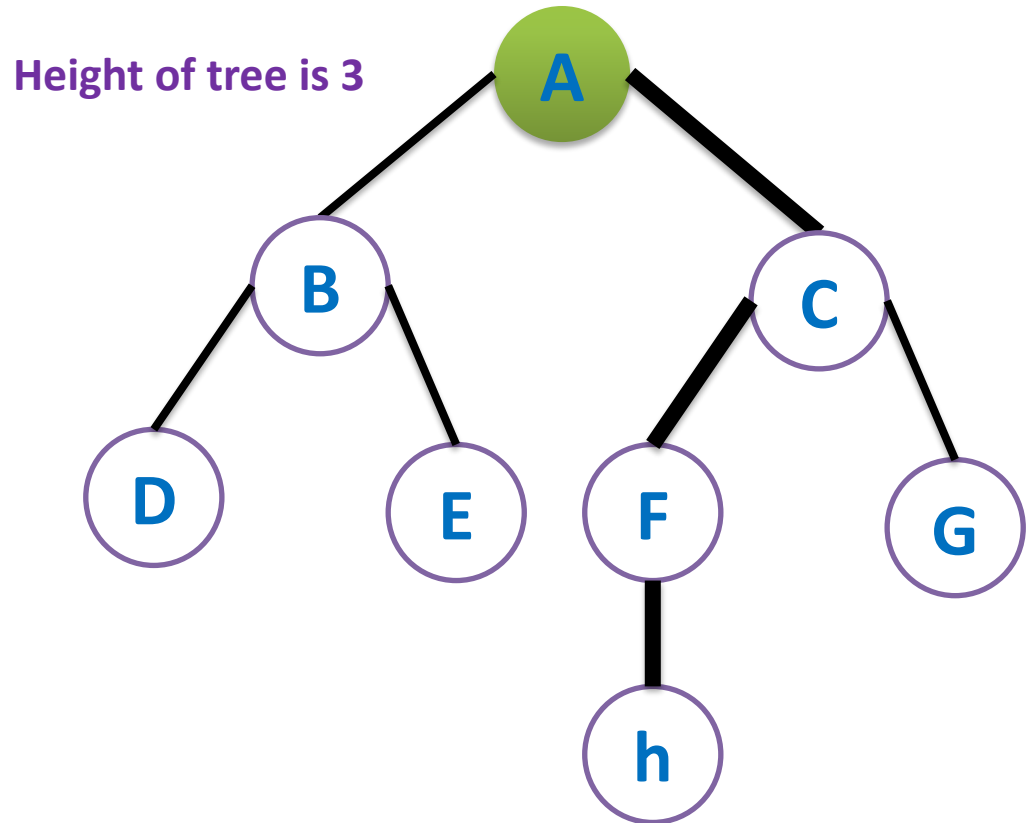
- Root
- Child
- Parent
- Siblings
- Descendant
- Ancestor
- Leaf
- Degree
- Edge
- **Height of node**
- Height of tree
- Depth



number of edges on the longest path between that node and a leaf.

# Terminologies used in Trees

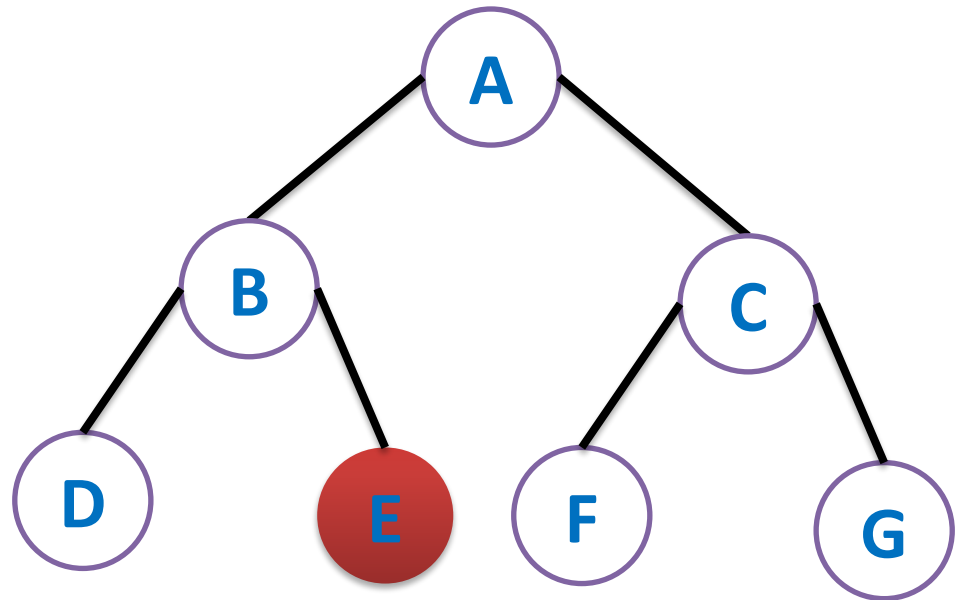
- Root
- Child
- Parent
- Siblings
- Descendant
- Ancestor
- Leaf
- Degree
- Edge
- Height of node
- **Height of tree**
- Depth



The height of a tree is the height of its root node.

# Terminologies used in Trees

- Root
- Child
- Parent
- Siblings
- Descendant
- Ancestor
- Leaf
- Degree
- Edge
- Height of node
- Height of tree
- **Depth**

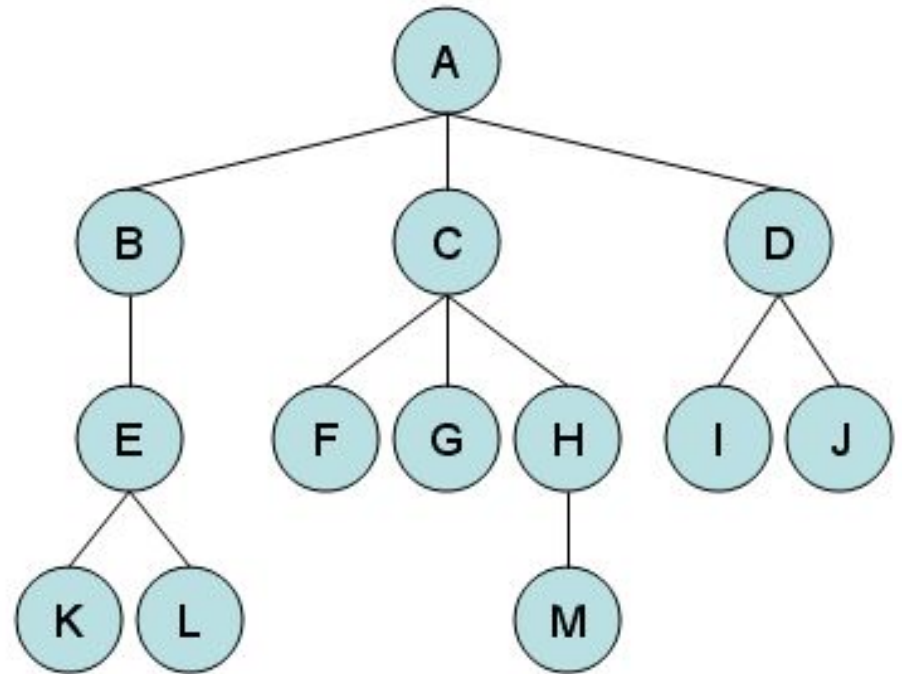


depth of a node E is 2

The depth of a node is the number of edges from the node to the tree's root node.

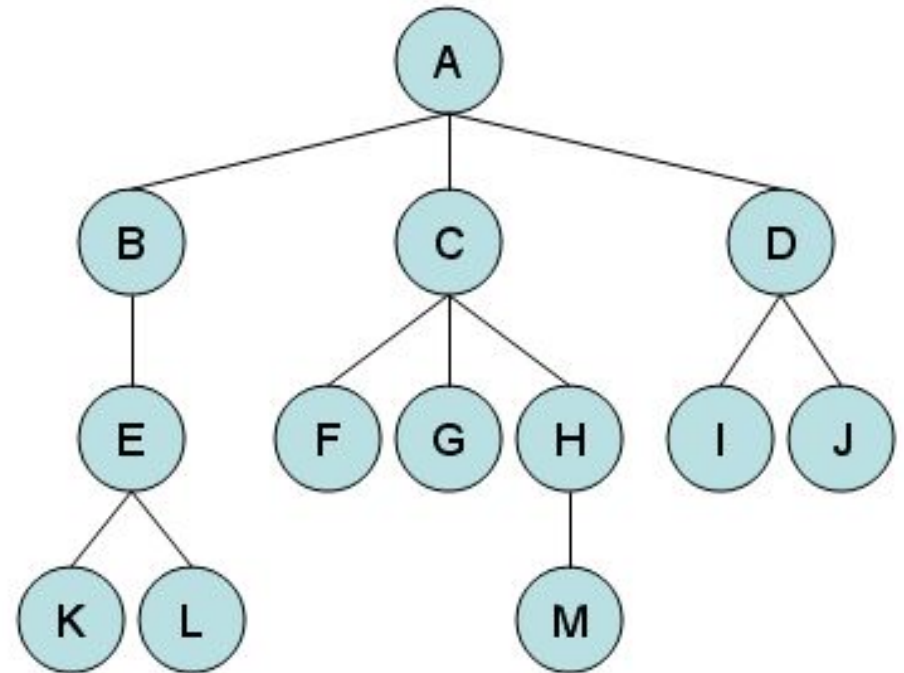
# Find the following..

- **Root**
- **Leaf**
- **Degree**
- **Height of tree**



# Answer

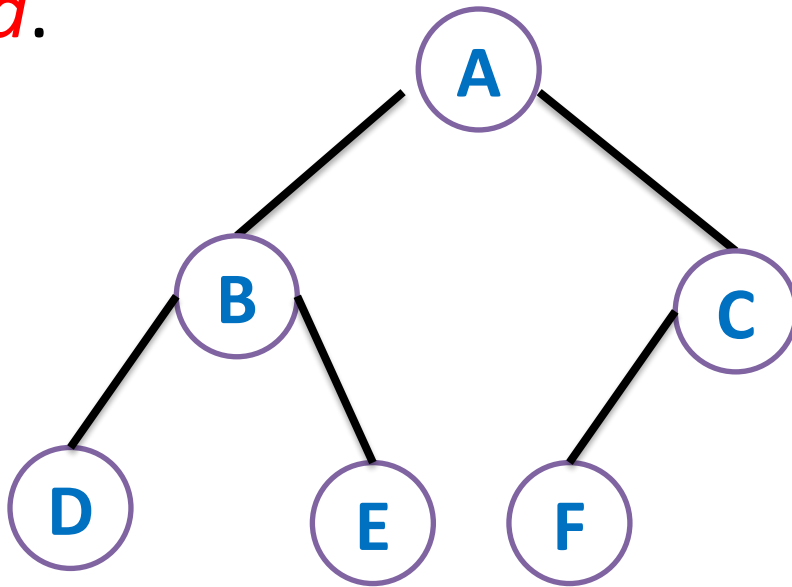
- **Root = A**
- **Leaf = K, L, F, G, M, I, J**
- **Degree = 3**
- **Height of tree= 3**





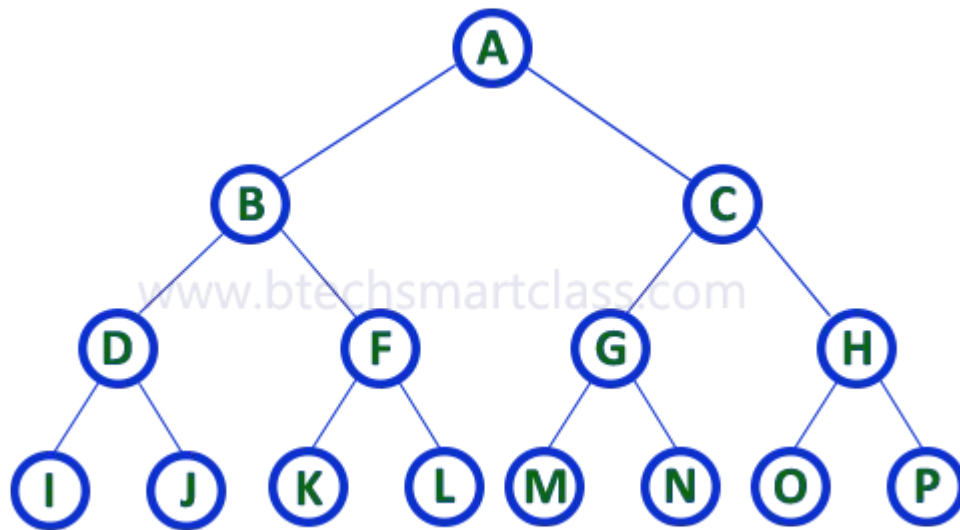
# Binary tree

**Binary tree** is a tree data structure in which each node has **at most two children**, which are referred to as the *left child* and the *right child*.



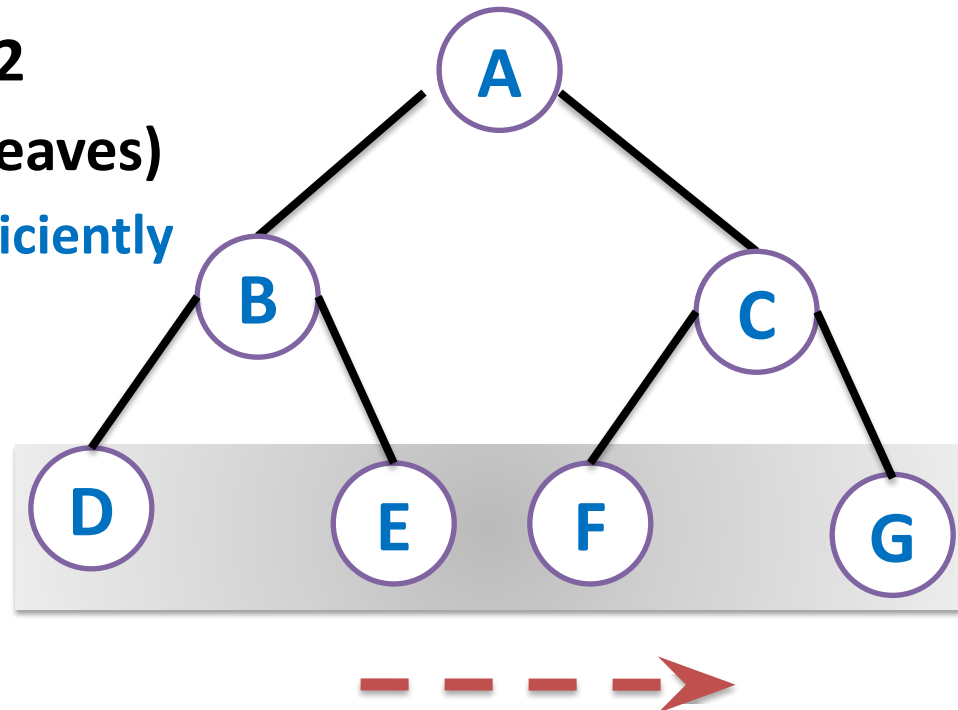
# Perfect or Full binary tree

A **perfect** binary tree is a binary tree in which all interior nodes have two children *and* all leaves have the same ***depth*** or same ***level***.



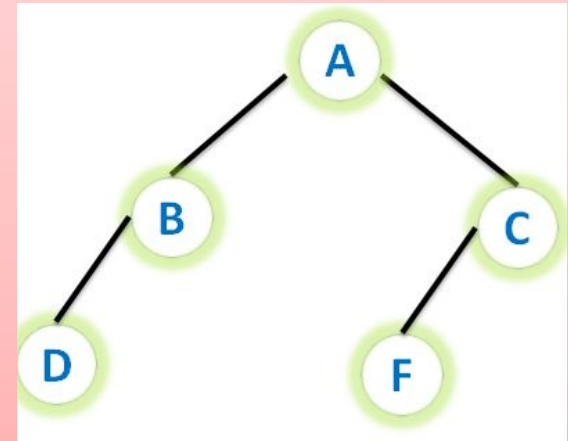
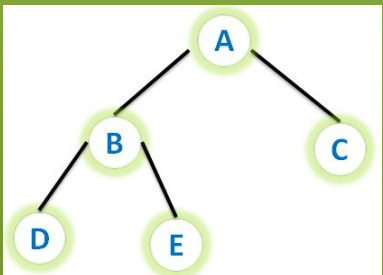
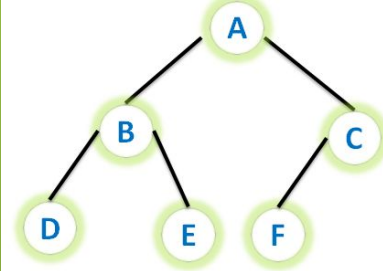
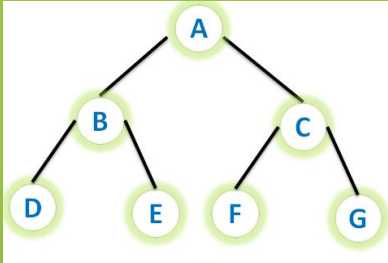
# Full Binary Tree

- In a tree of height  $h$ 
  - All the leaves are at level  $h$
  - Number of leaves 1 to  $2^h$
  - Number of internal nodes =  $2^h - 1$
- In a full binary tree of  $n$  nodes
  - Number of leaves is  $(n+1) / 2$
  - Height =  $\log_2$  (number of leaves)
- A complete binary tree can be efficiently represented using an array.



# Complete Binary Tree

Completely filled, with the possible exception of the bottom level, which is filled from left to right.

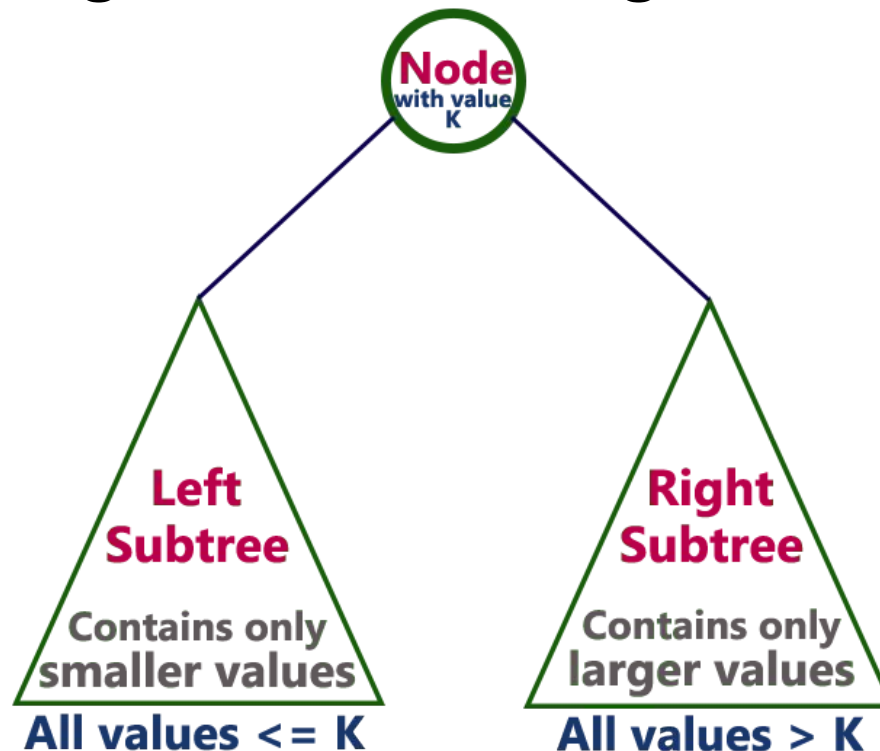


**A complete binary tree can be efficiently represented using an array.**

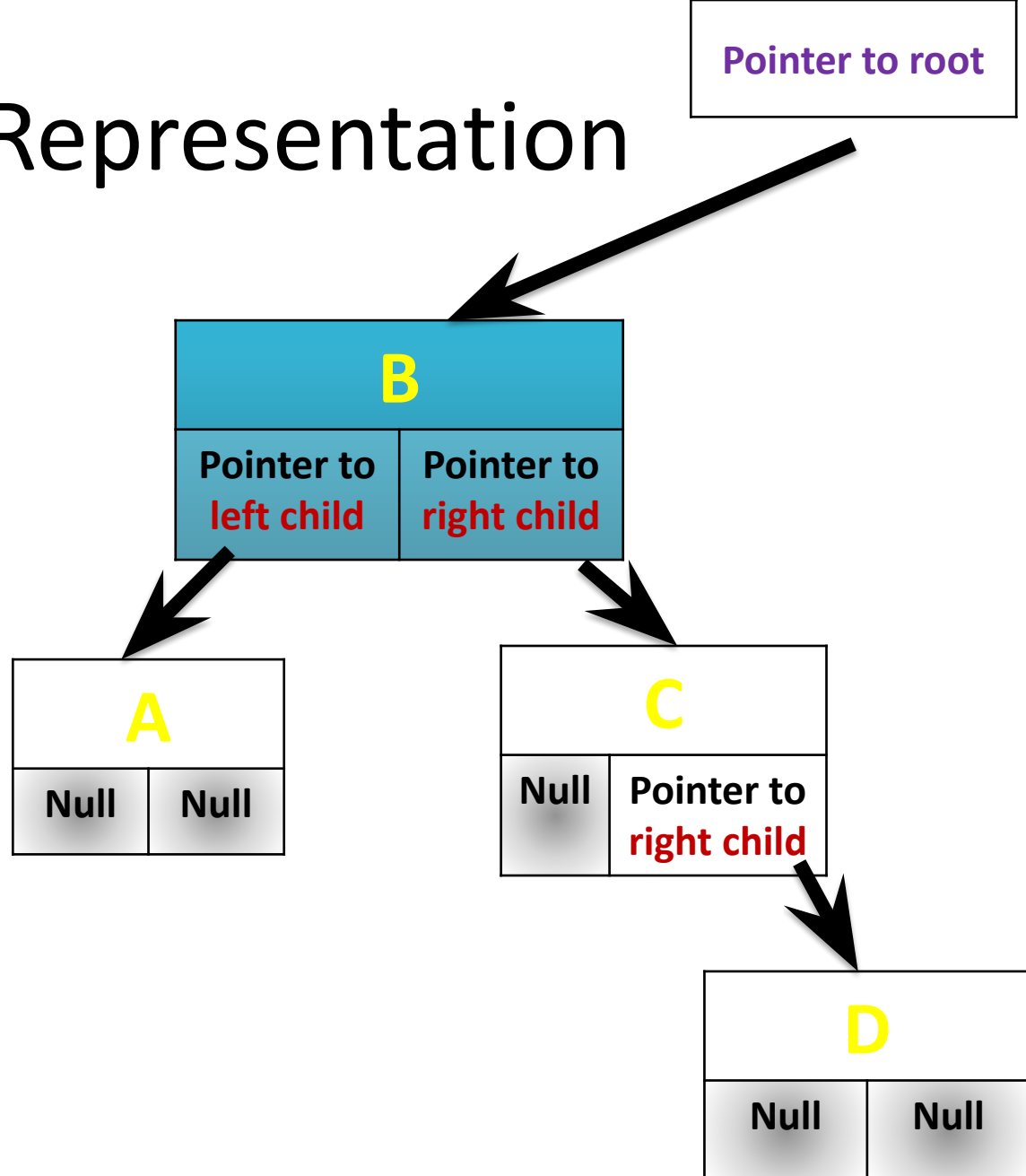
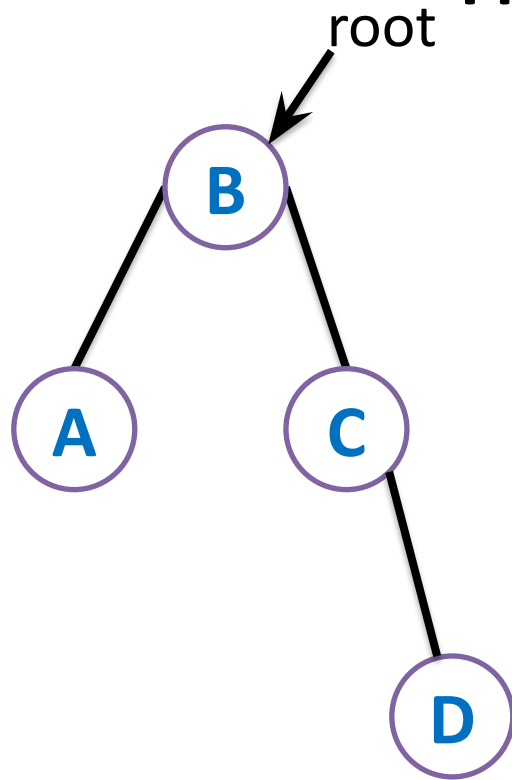
# Binary search trees (BST)

ordered or sorted binary trees,

- Binary Search Tree is a binary tree in which every node contains only **smaller values in its left** subtree and only **larger values in its right** subtree.

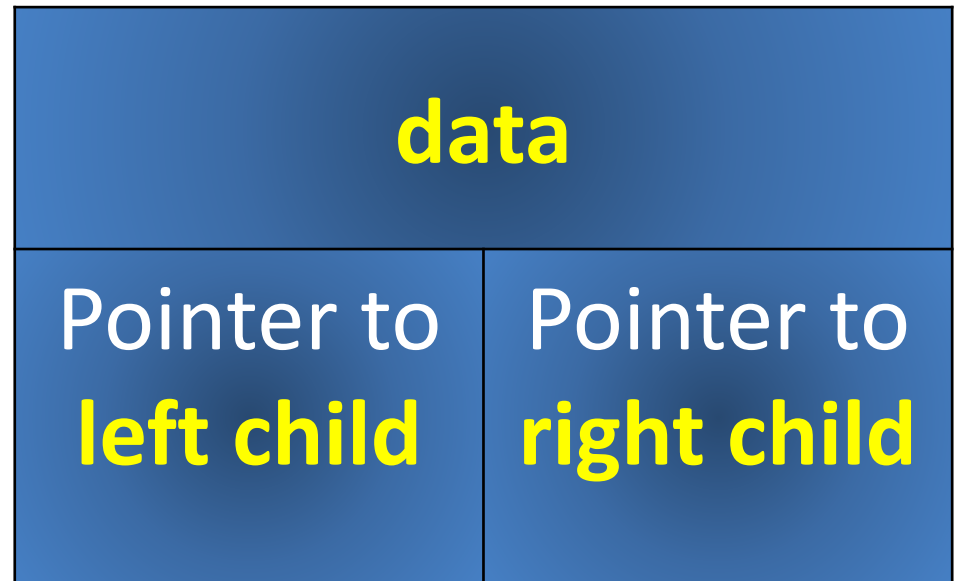


# Tree Representation



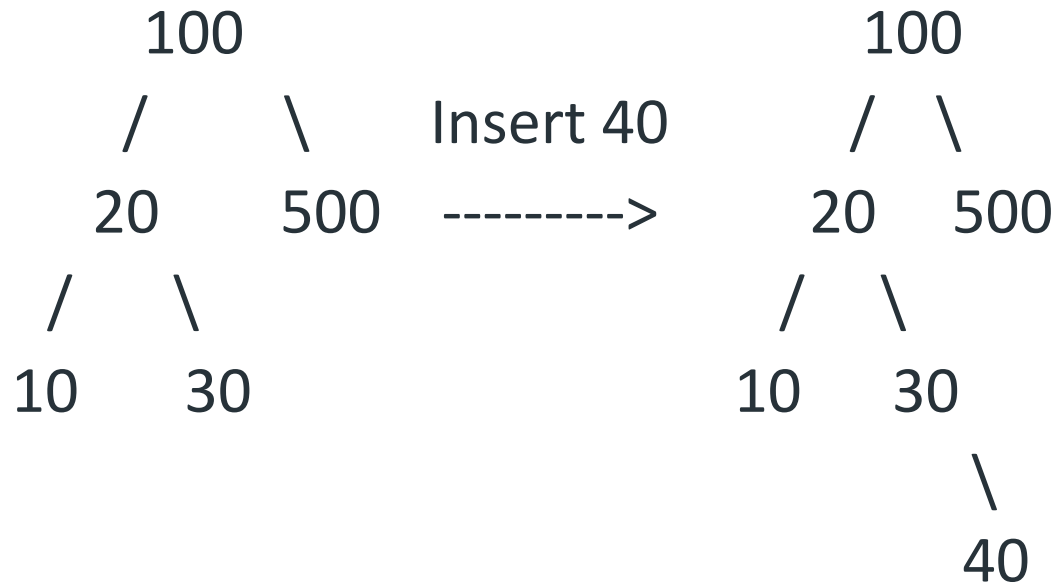
# Node Structure

```
struct Treenode{  
    int data;  
    Treenode *left ;  
    Treenode *right ;  
}
```



# BST Insertion

- A new key is always inserted at the leaf.
- Start searching a key from the root until a leaf node is hit;  
The new node is added as a child of the leaf node.





# BST Insertion

```
def insert(root, key):  
    if root is None:  
        return Node(key)  
    else:  
        if root.val == key:  
            return root  
        elif root.val < key:  
            root.right = insert(root.right, key)  
        else:  
            root.left = insert(root.left, key)  
    return root
```

# Tree Traversal

- Process of **checking** and/or **updating** each node in the tree data structure, **exactly once**.
- Such **traversals** are classified by the order in which the nodes are visited.
  - Inorder
  - Preorder
  - Postorder

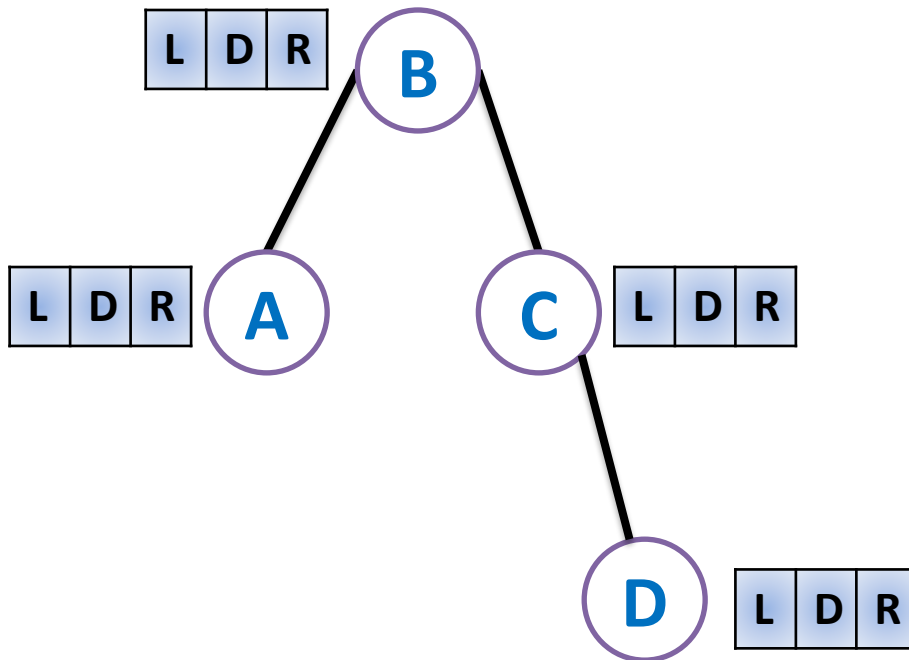
# Inorder traversal

## Algorithm **Inorder**(tree)

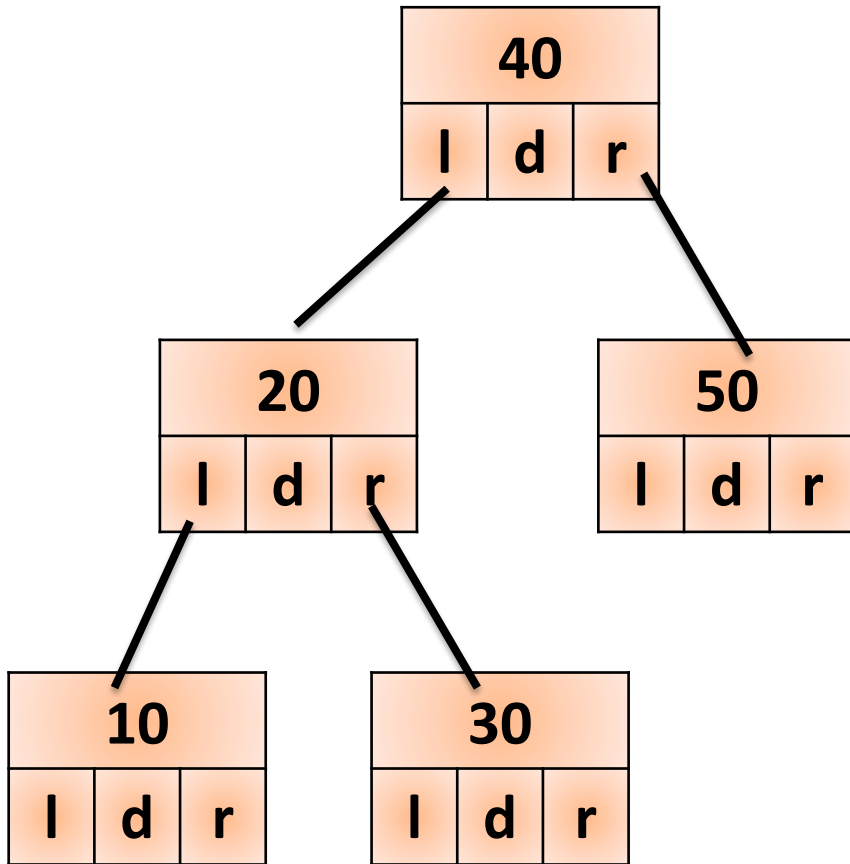
Traverse the left subtree, i.e., call **Inorder**(left-subtree)

Print Data.

Traverse the right subtree, i.e., call **Inorder**(right-subtree)



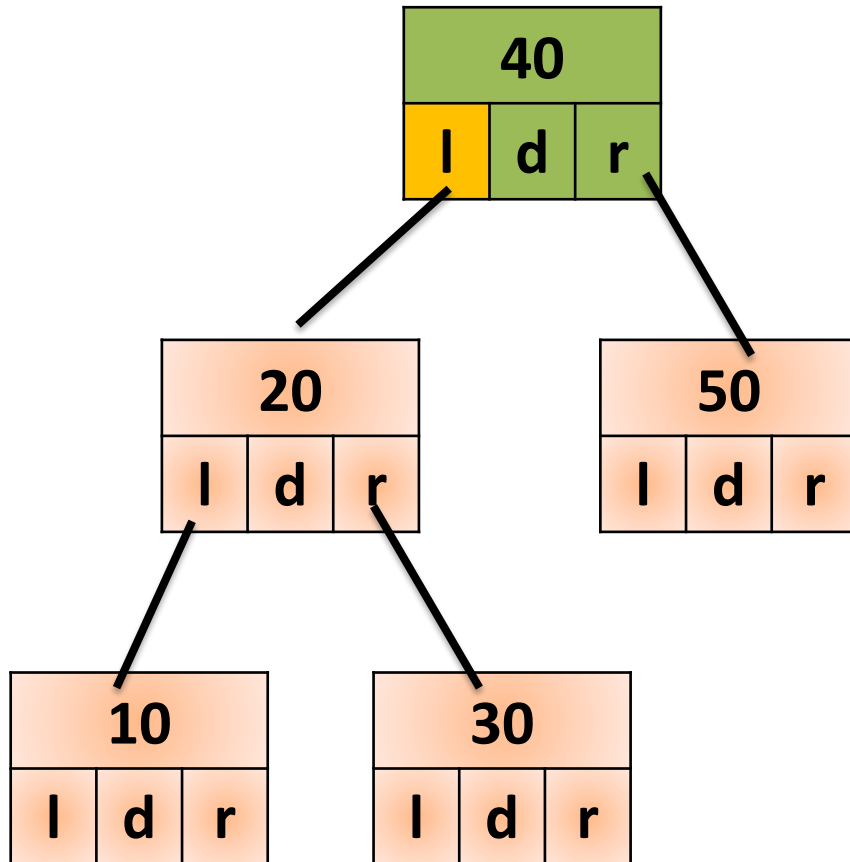
# Inorder



OUTPUT



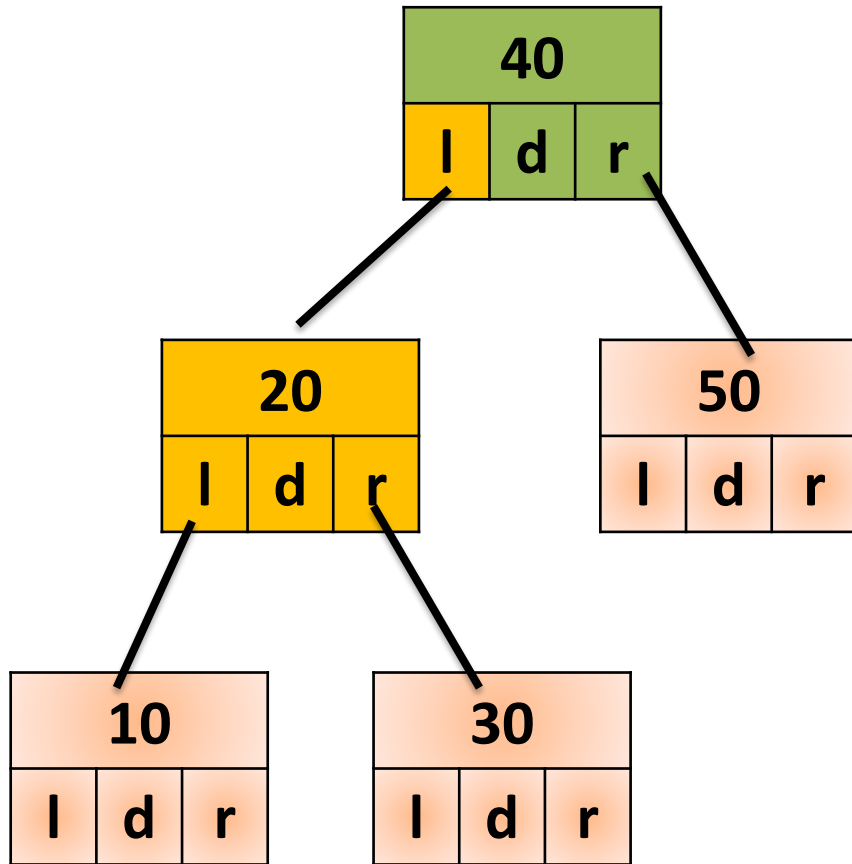
# Inorder



OUTPUT



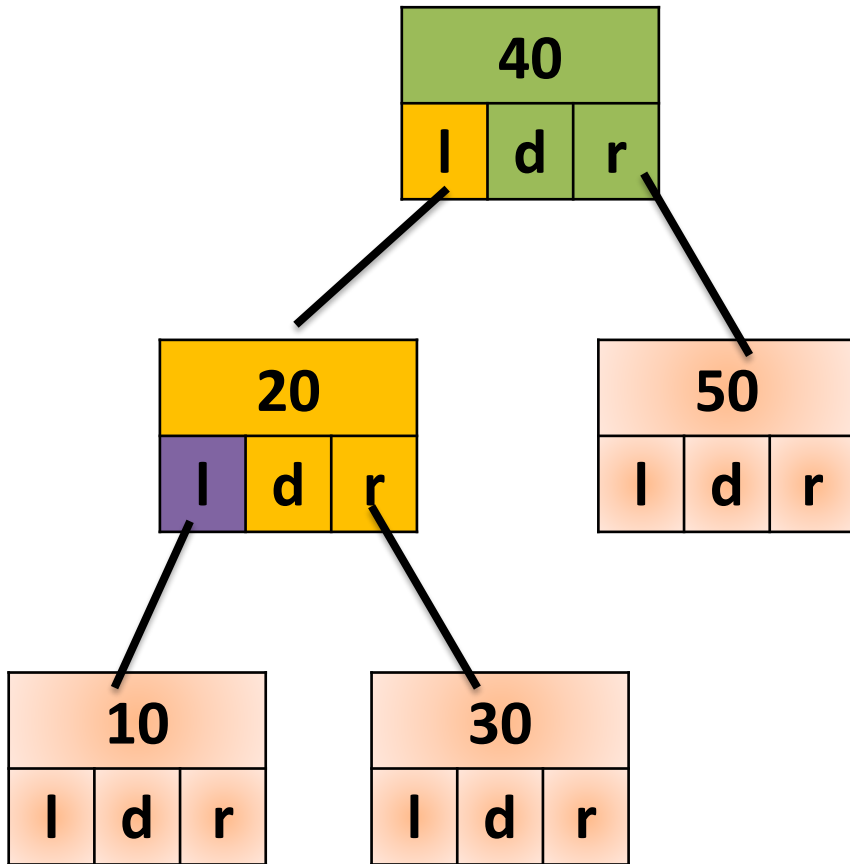
# Inorder



OUTPUT



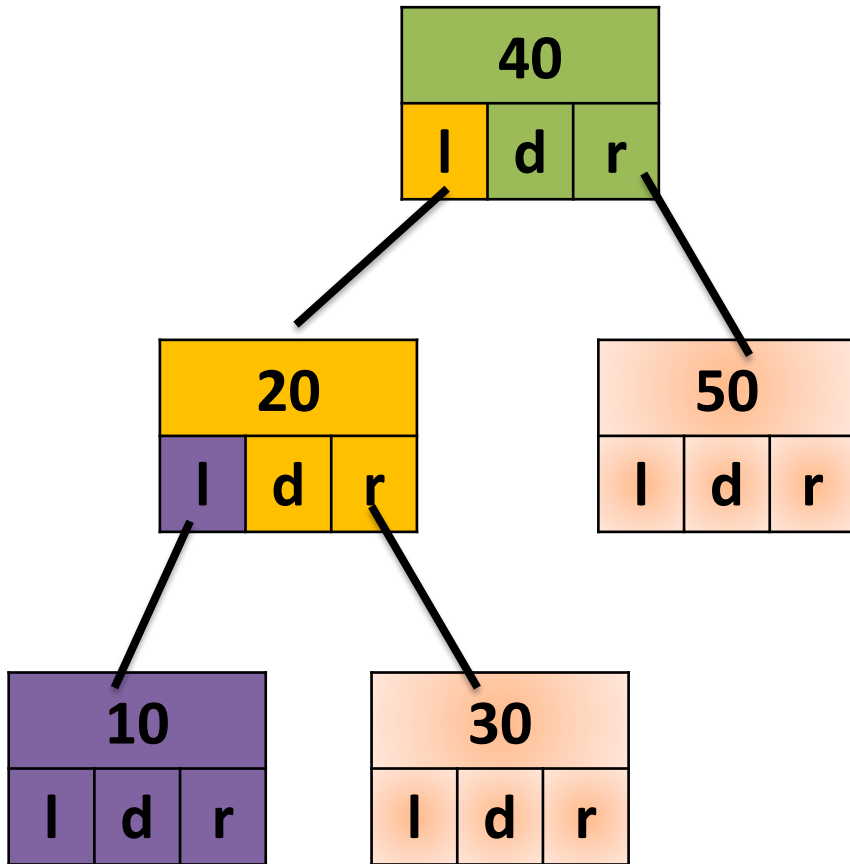
# Inorder



OUTPUT



# Inorder

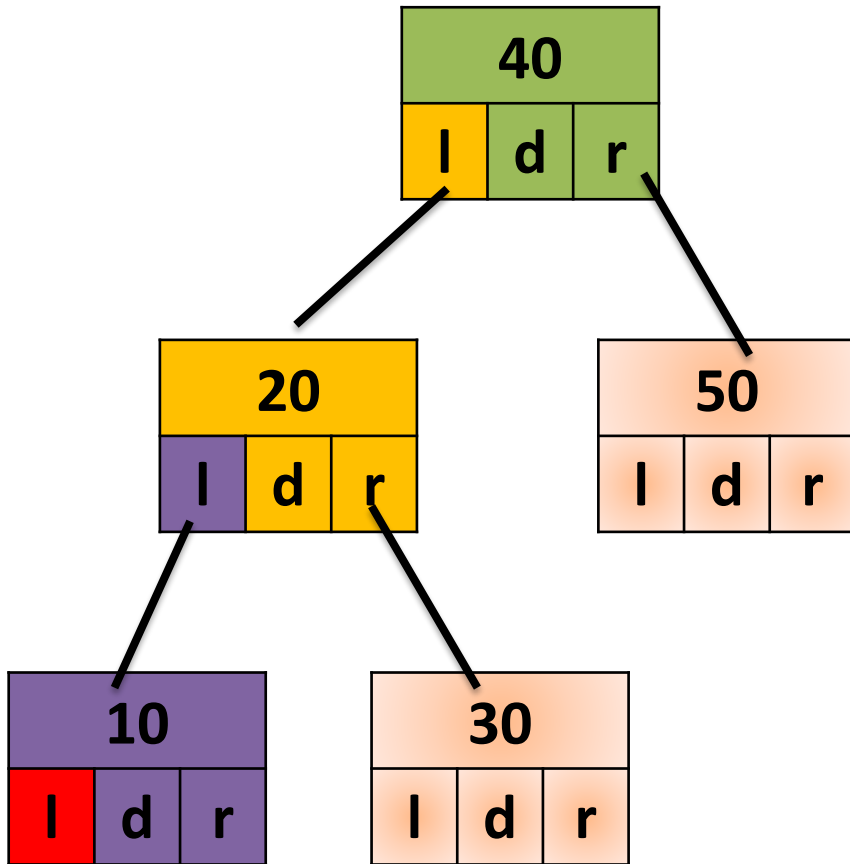


OUTPUT





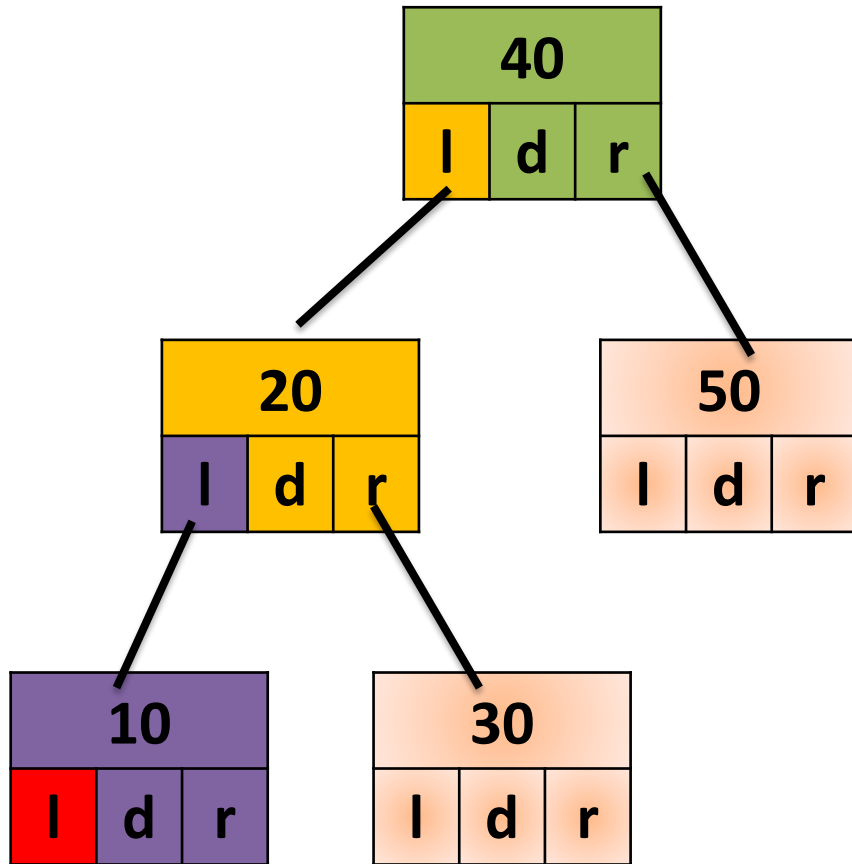
# Inorder



OUTPUT



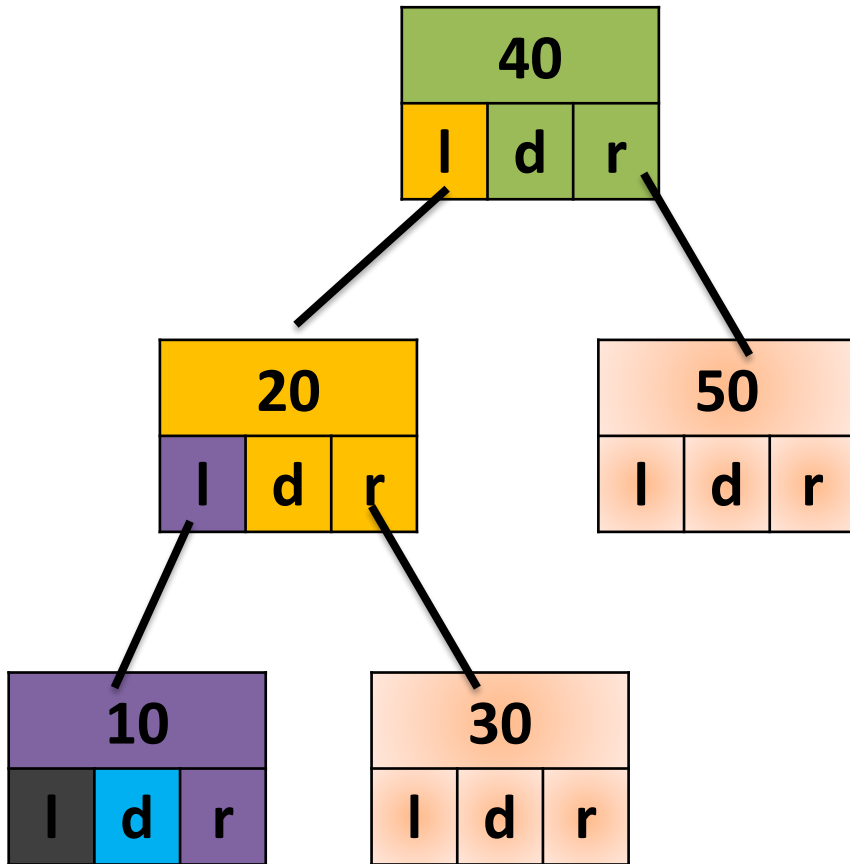
# Inorder



OUTPUT



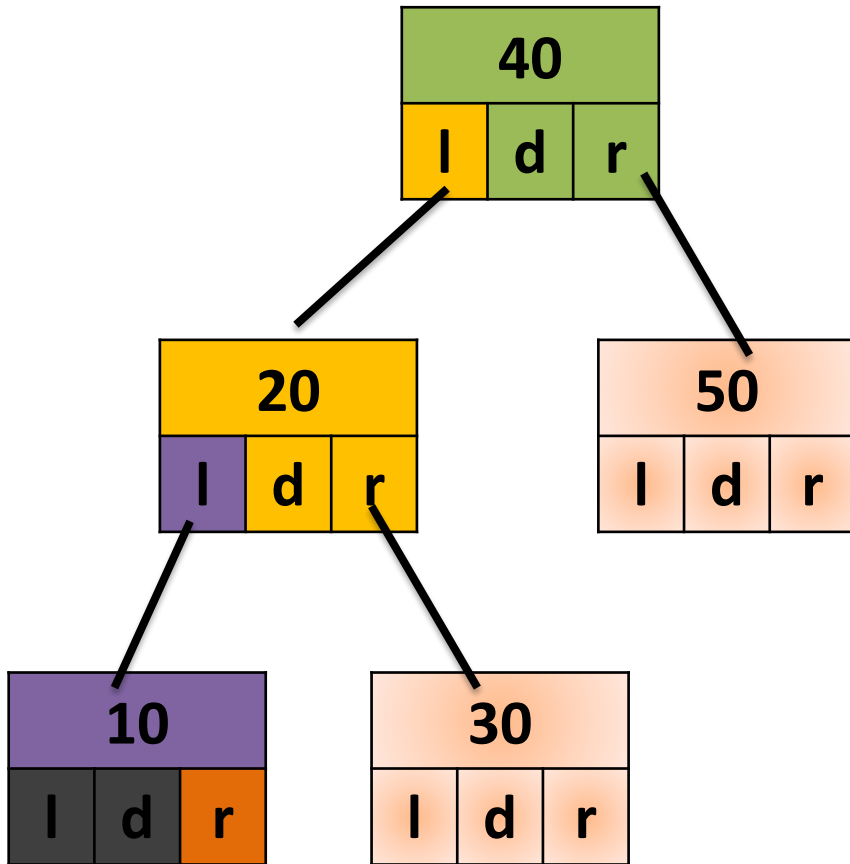
# Inorder



OUTPUT



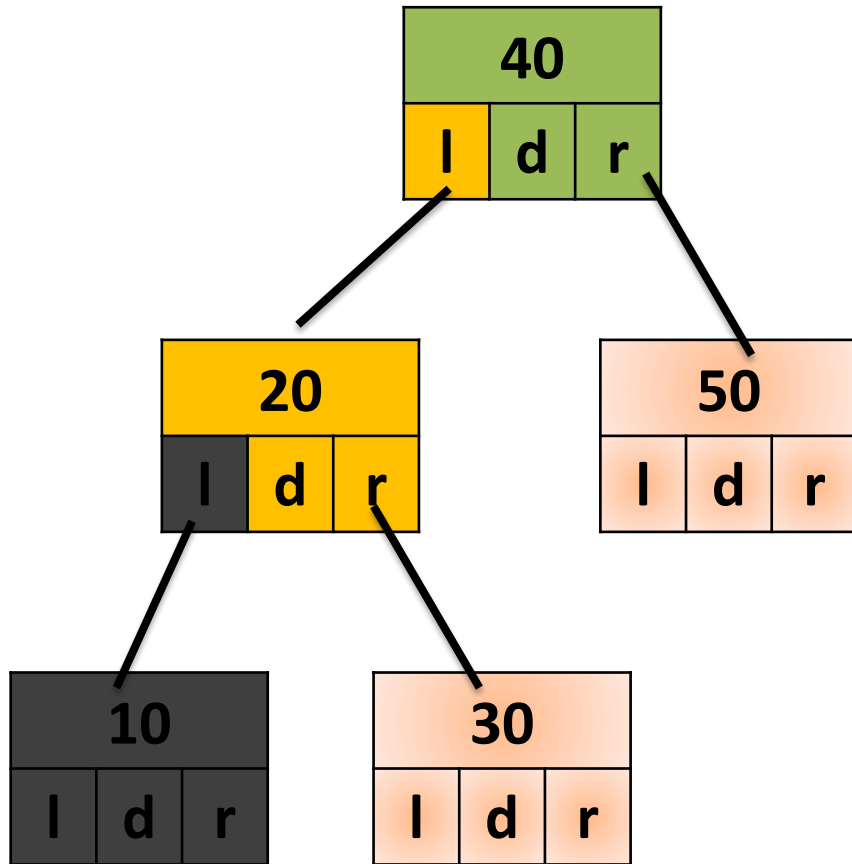
# Inorder



OUTPUT



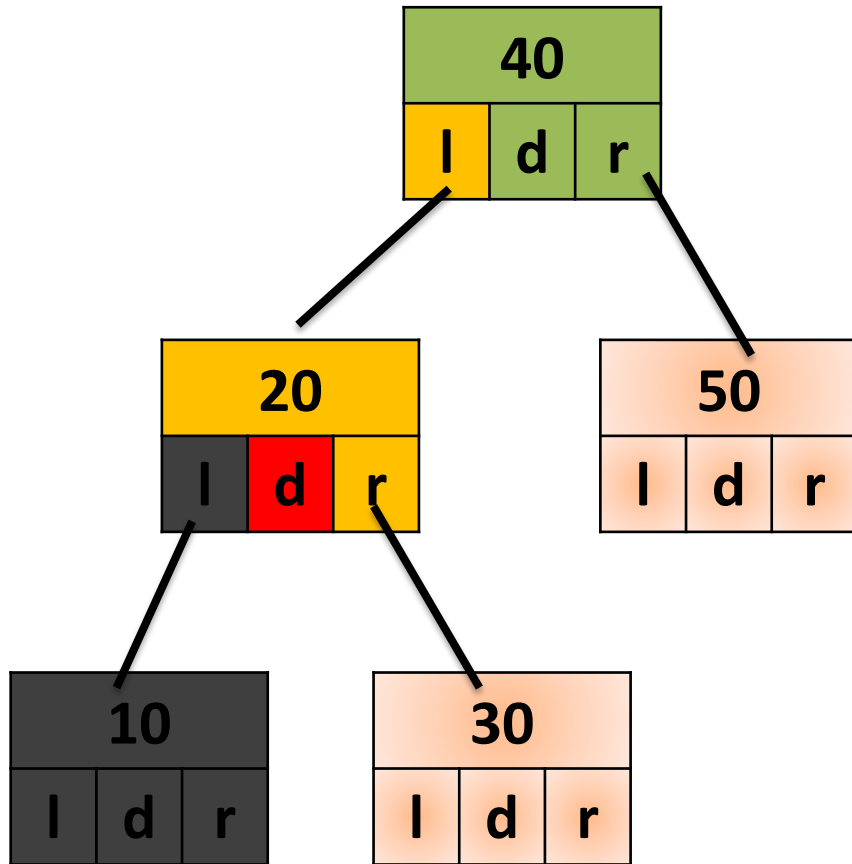
# Inorder



OUTPUT



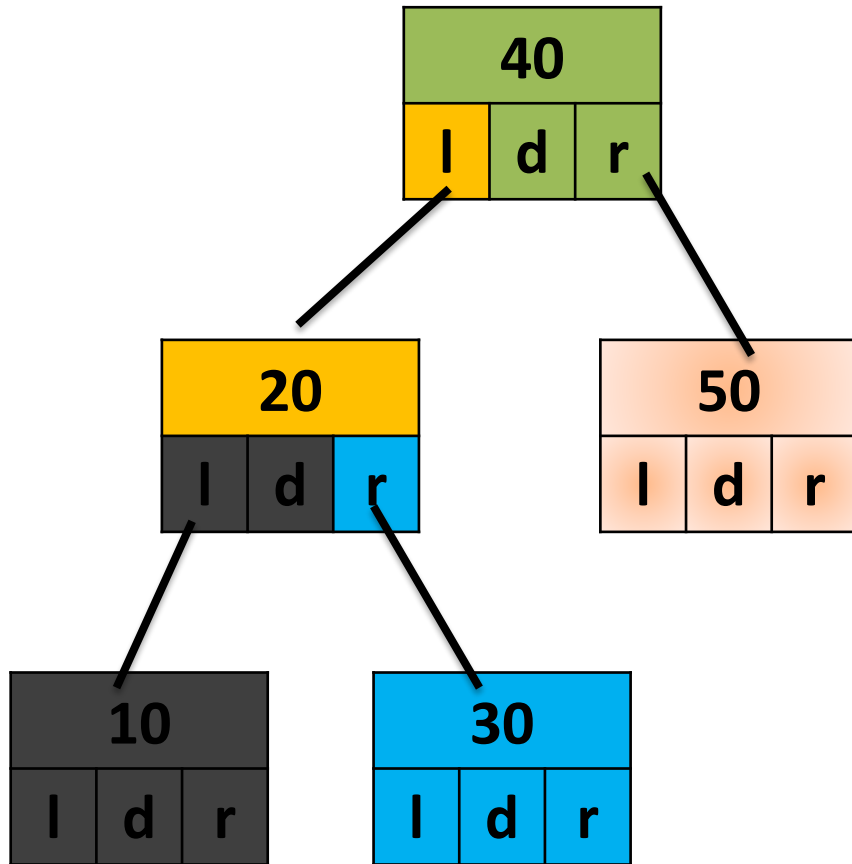
# Inorder



OUTPUT



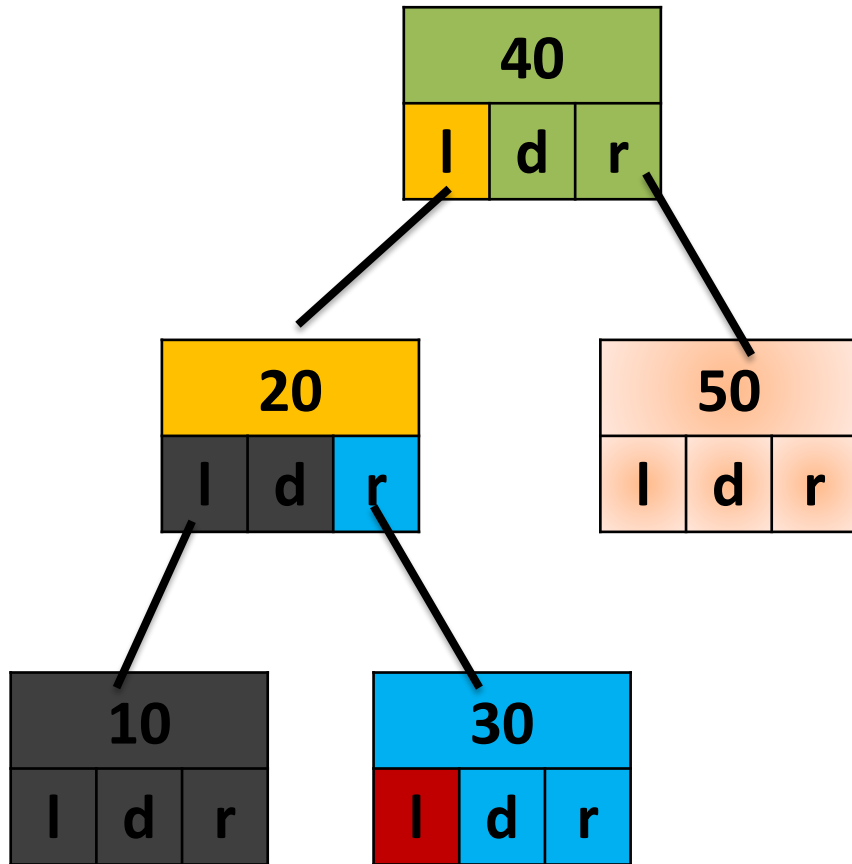
# Inorder



OUTPUT



# Inorder

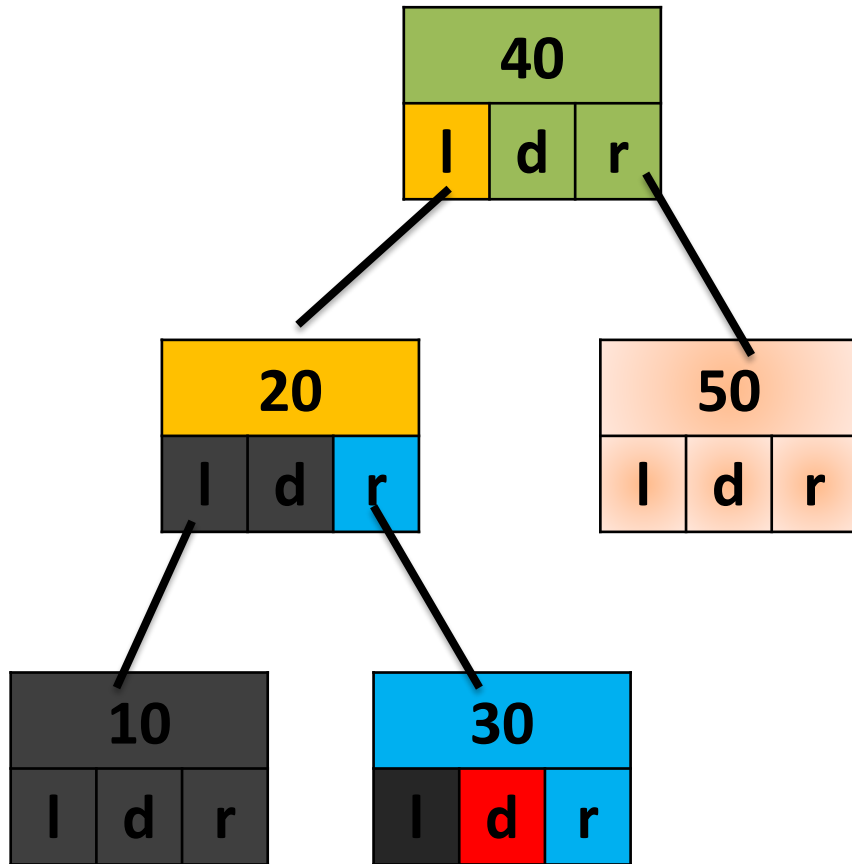


OUTPUT





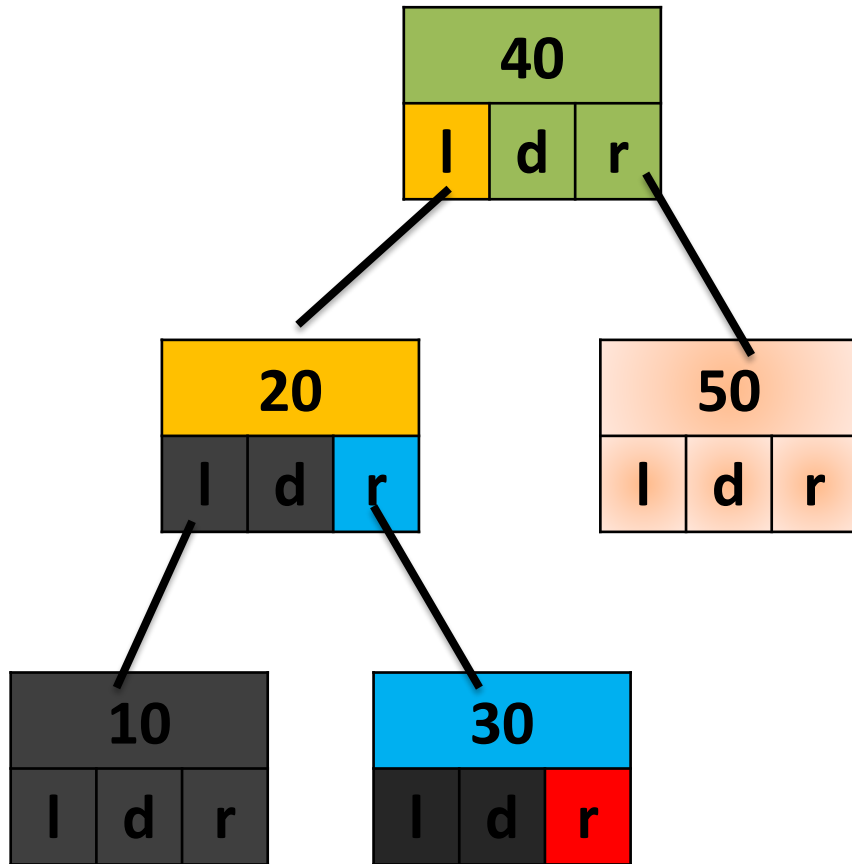
# Inorder



OUTPUT



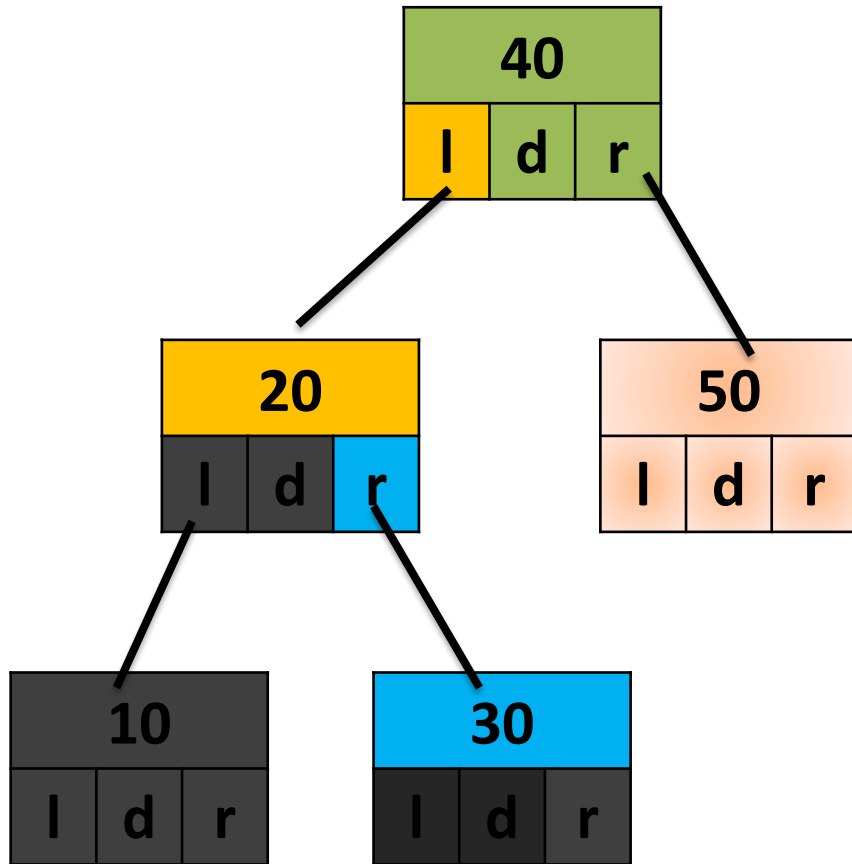
# Inorder



OUTPUT



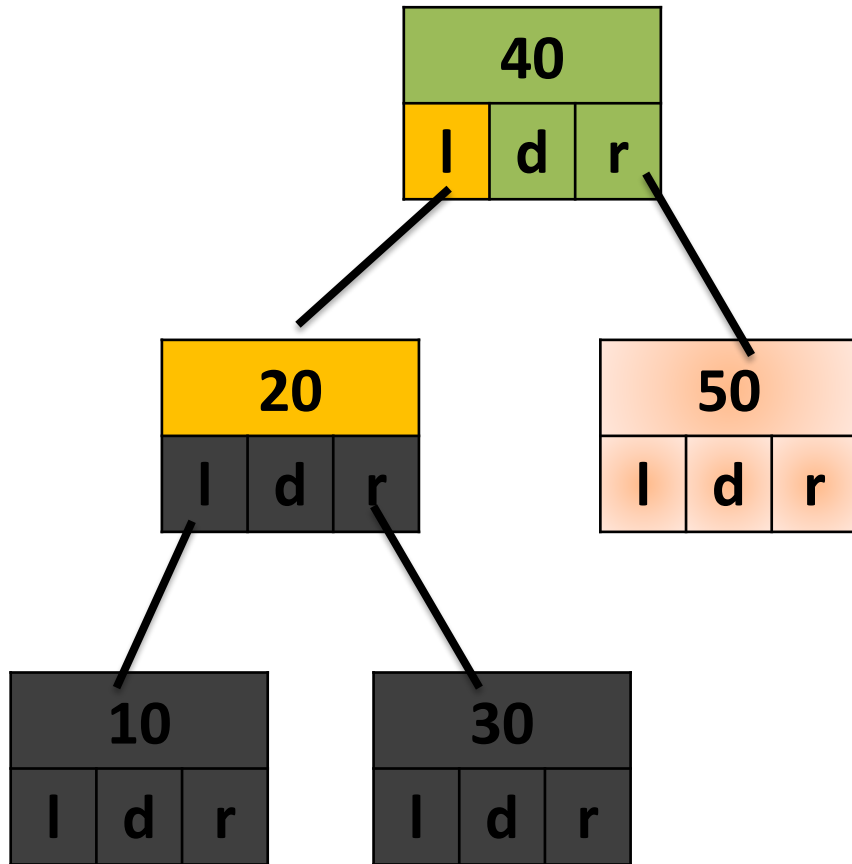
# Inorder



OUTPUT



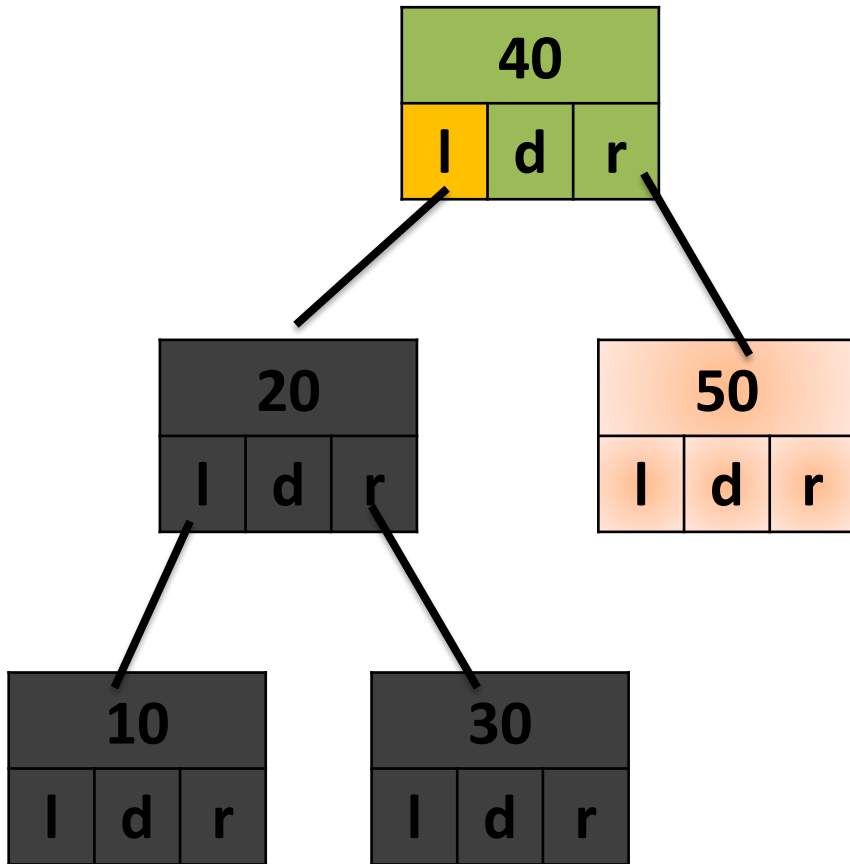
# Inorder



OUTPUT



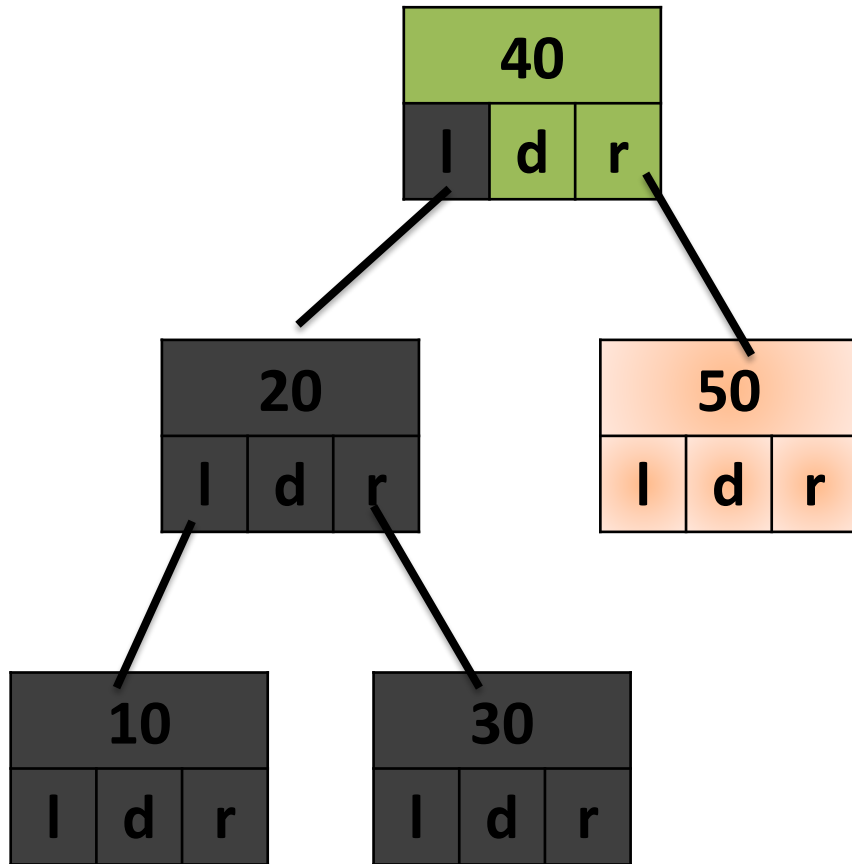
# Inorder



OUTPUT



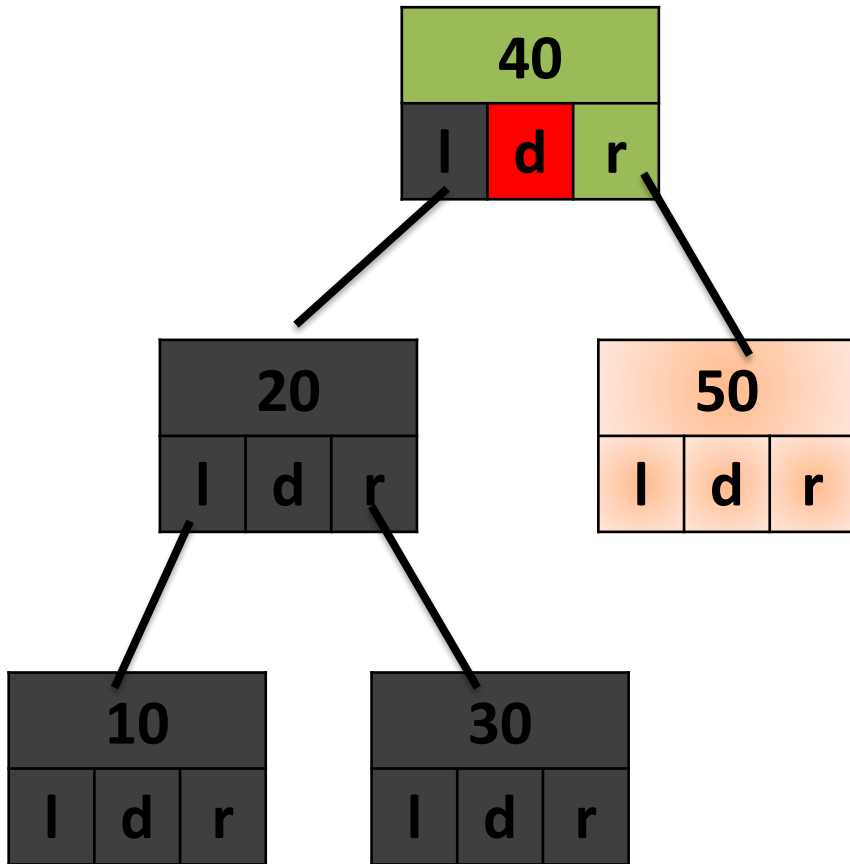
# Inorder



OUTPUT



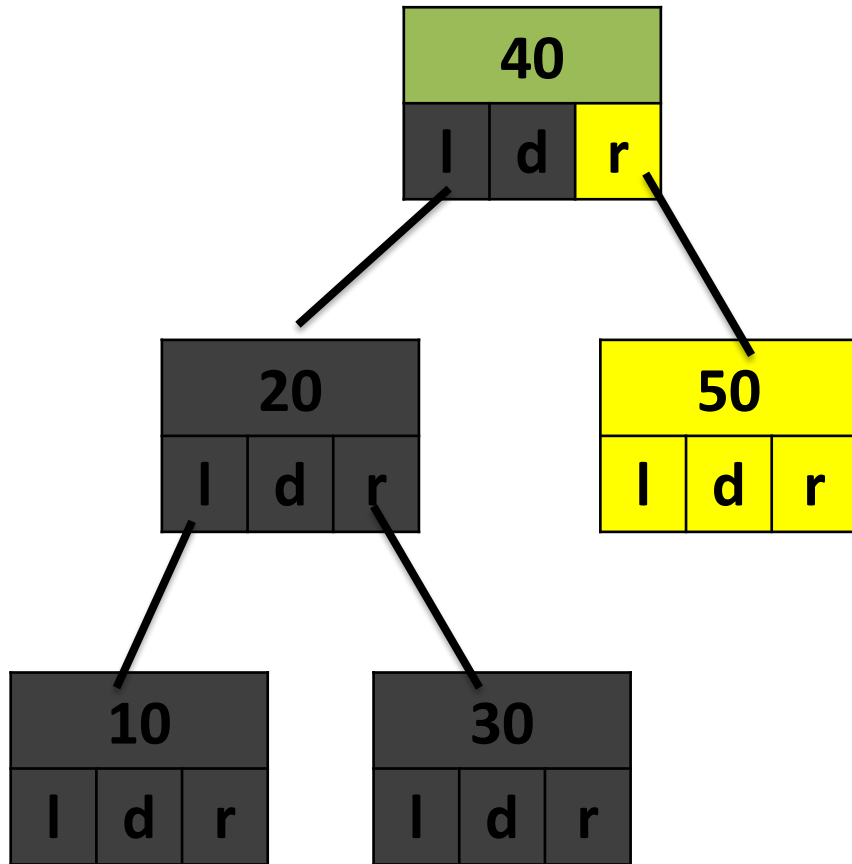
# Inorder



OUTPUT



# Inorder

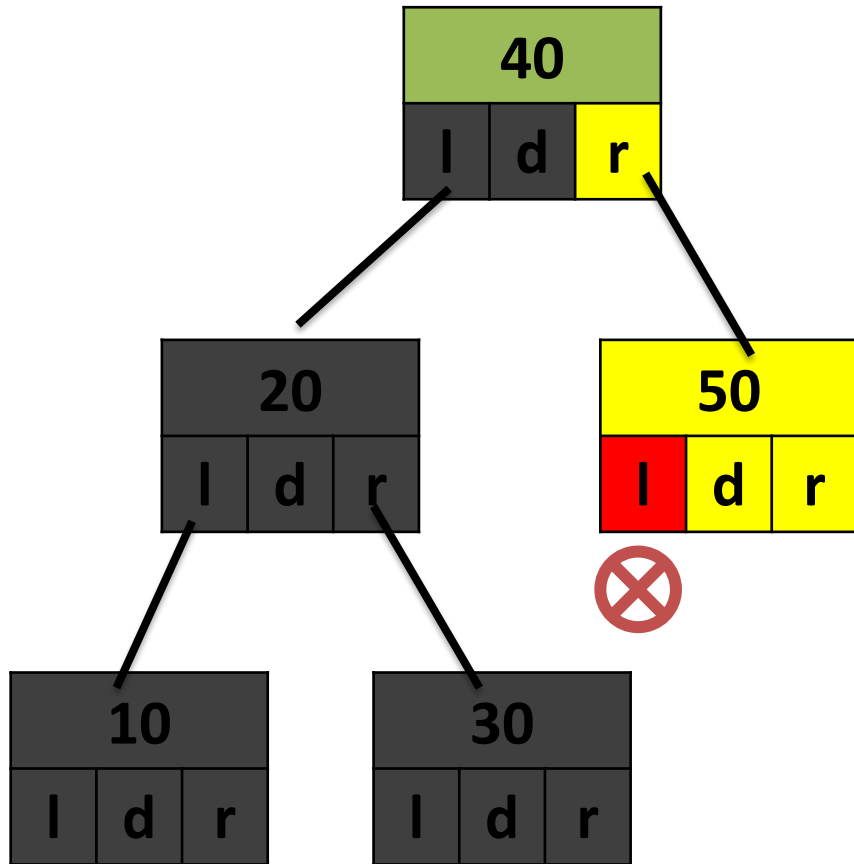


OUTPUT





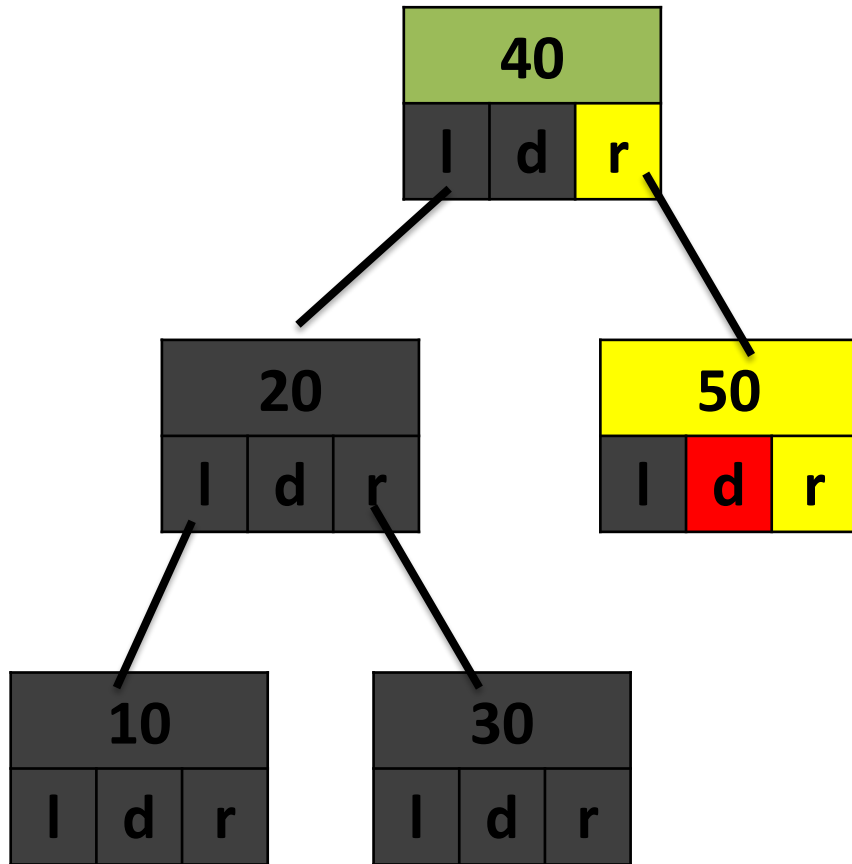
# Inorder



OUTPUT



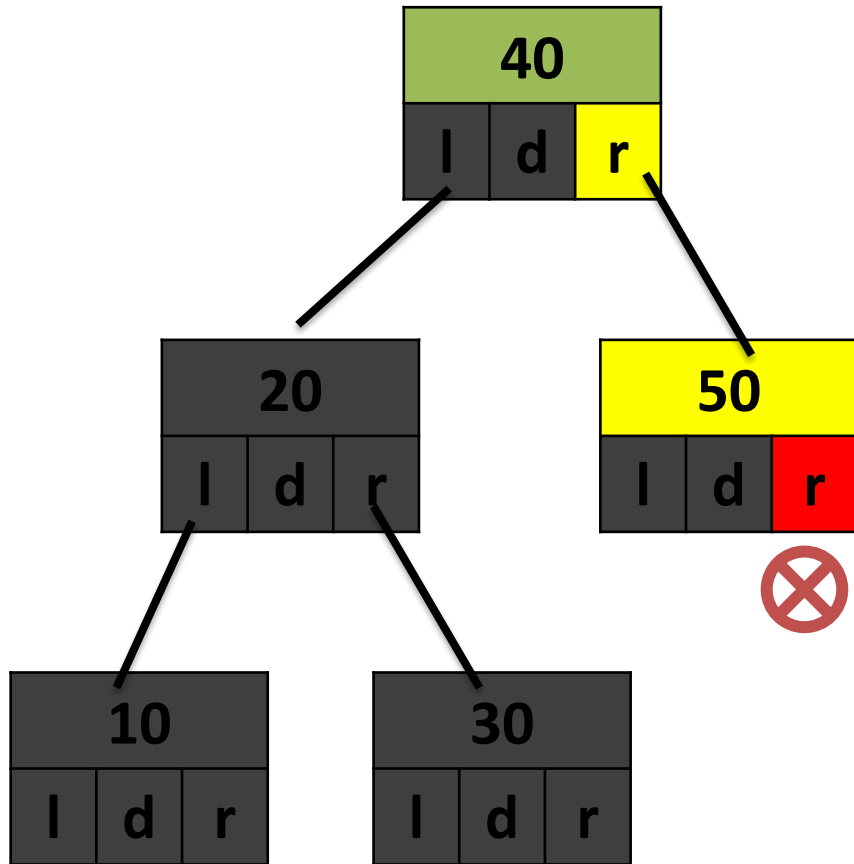
# Inorder



OUTPUT

10	20	30	40	50
----	----	----	----	----

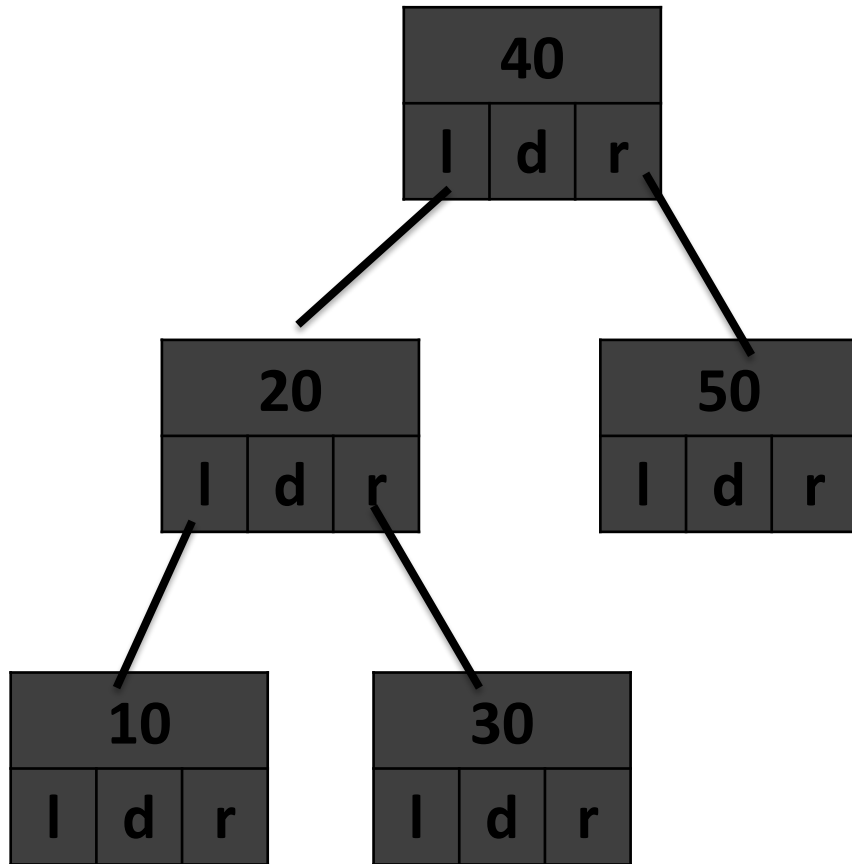
# Inorder



OUTPUT

10	20	30	40	50
----	----	----	----	----

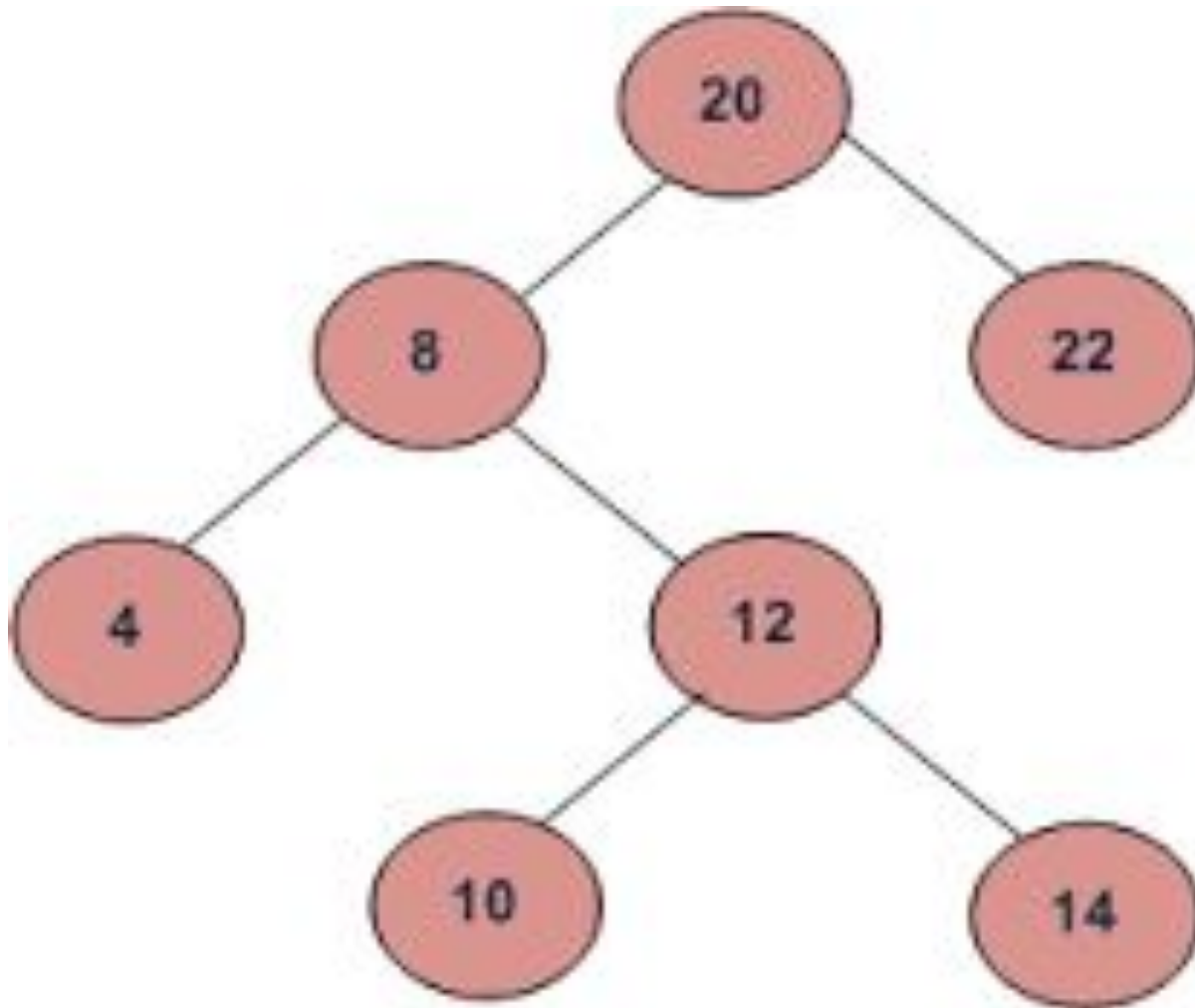
# Inorder



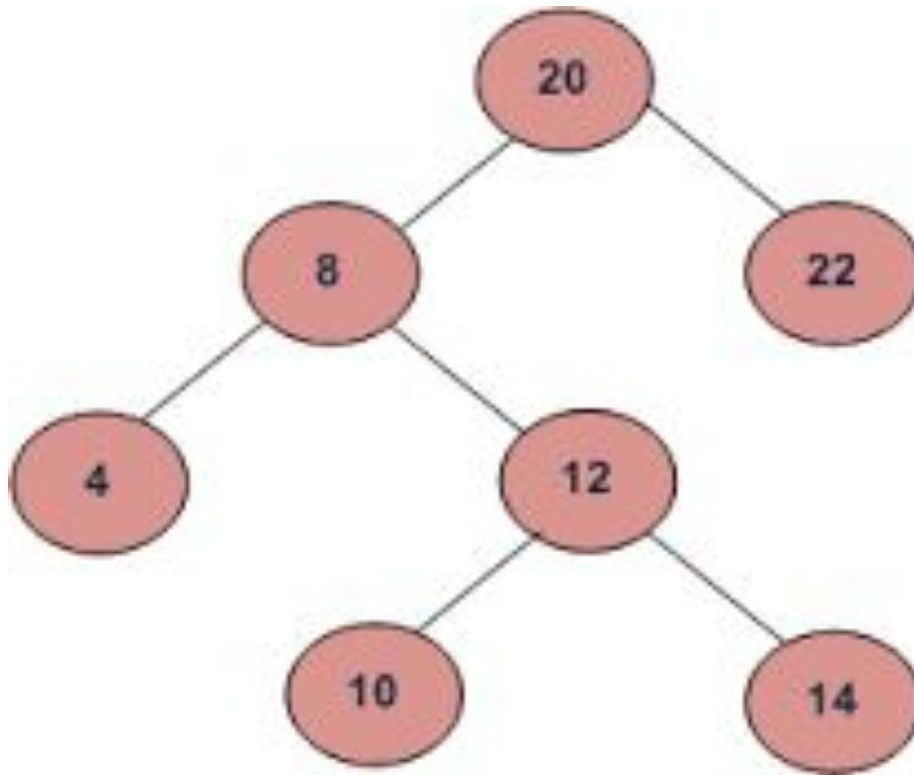
OUTPUT

10 20 30 40 50

Find inorder traversal sequence of this tree.



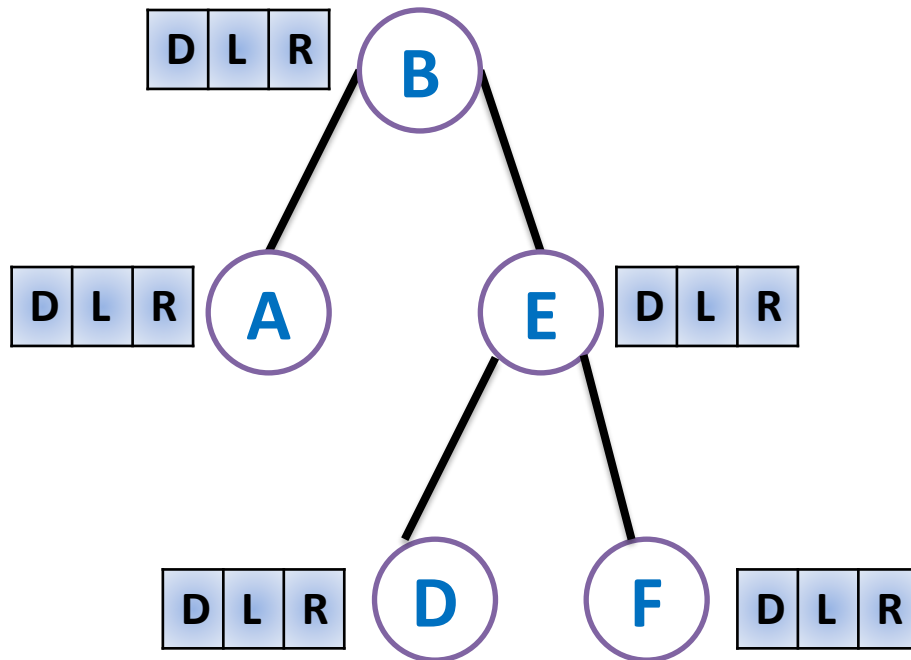
Find inorder traversal sequence of this tree.



**Answer : 4, 8,10, 12,14,20,22**

# Preorder traversal

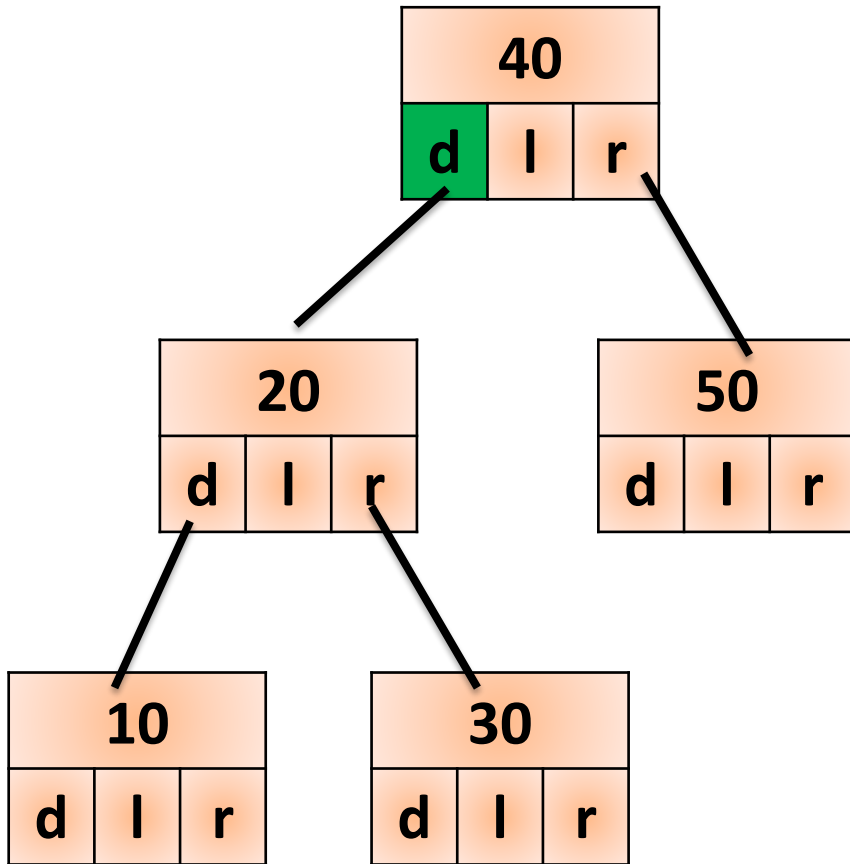
1. Print data.
2. Traverse the left subtree
3. Traverse the right subtree



OUTPUT

**B A E D F**

# preorder

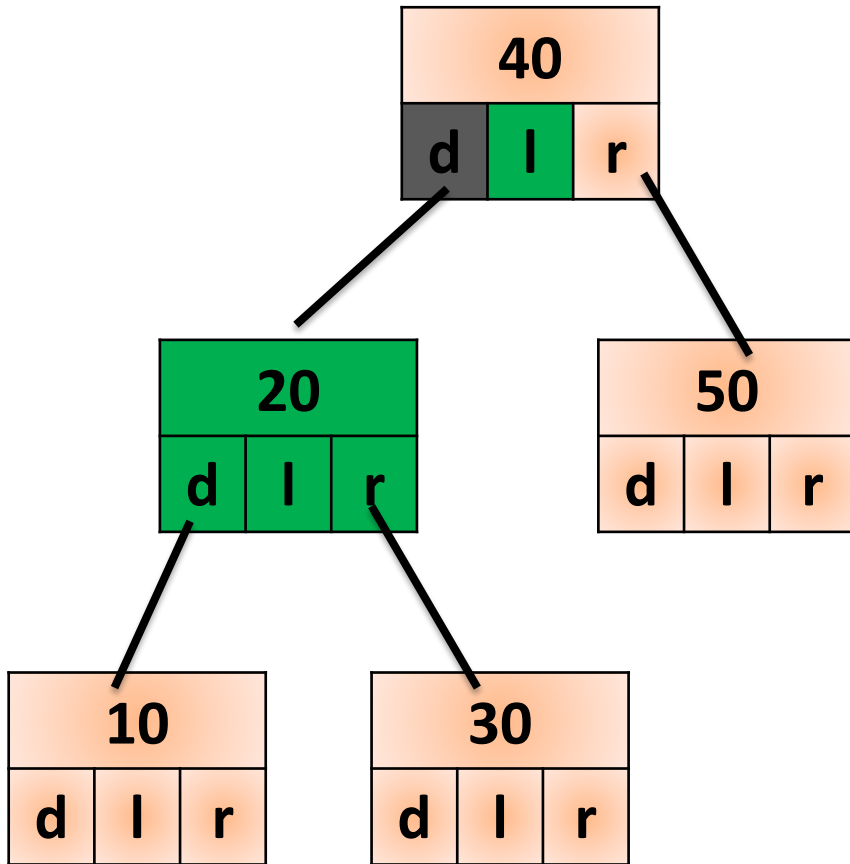


OUTPUT





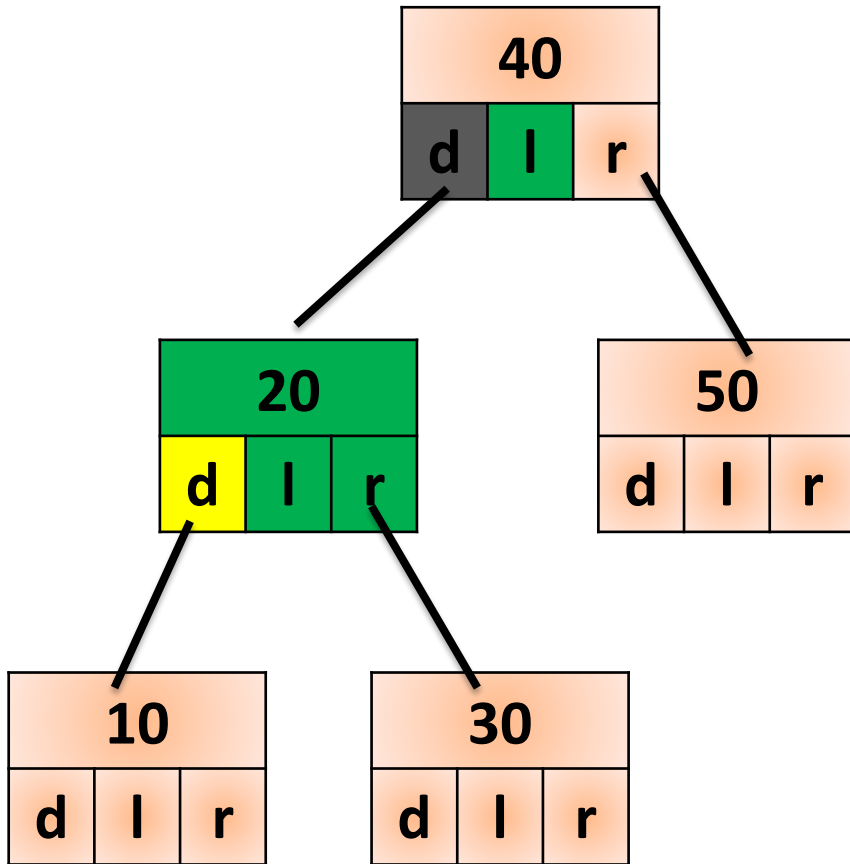
# preorder



OUTPUT



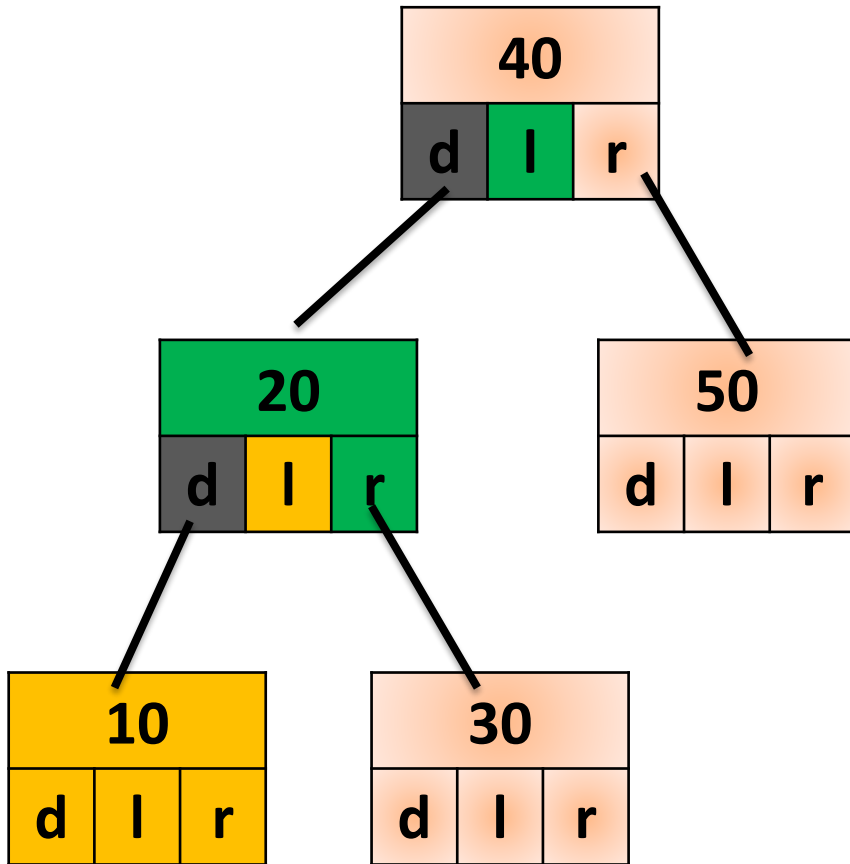
# preorder



OUTPUT



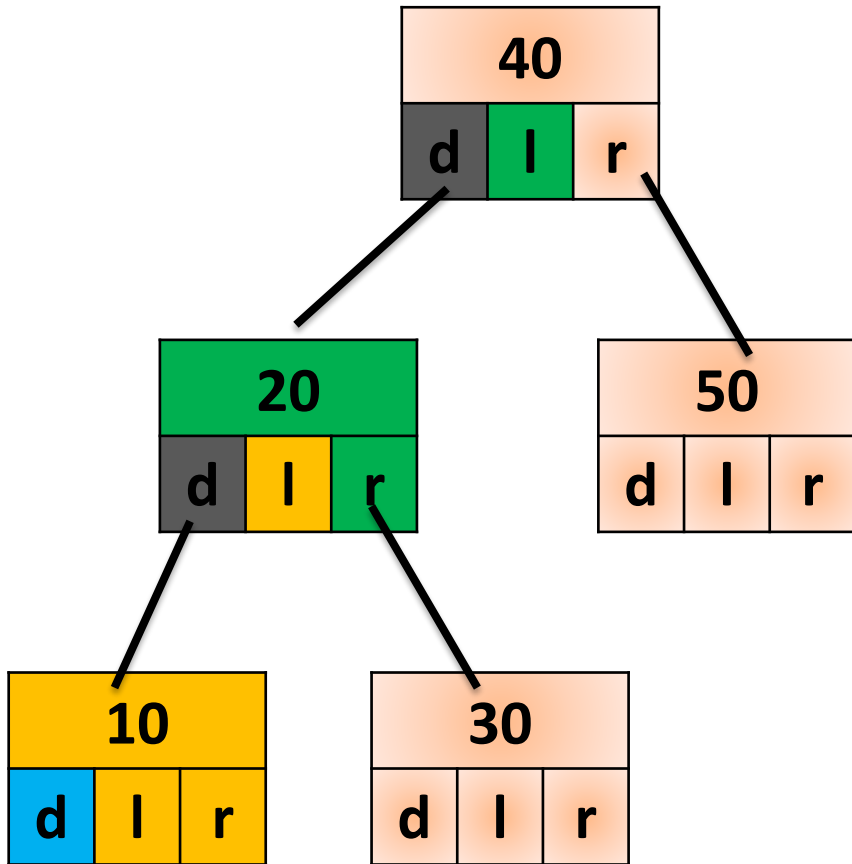
# preorder



OUTPUT



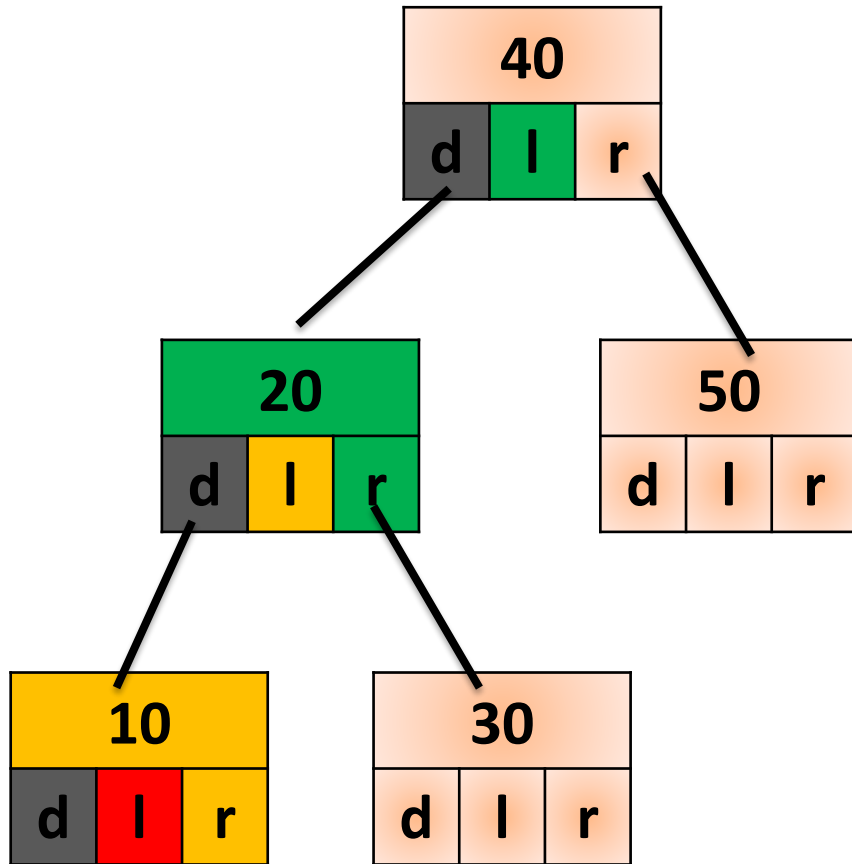
# preorder



OUTPUT



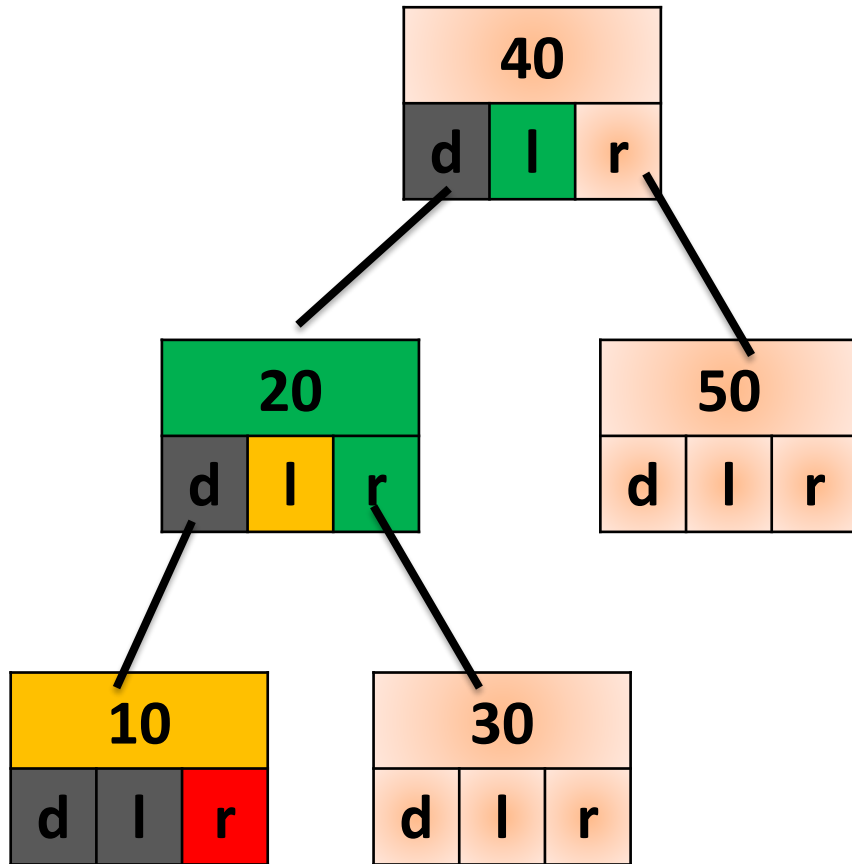
# preorder



OUTPUT



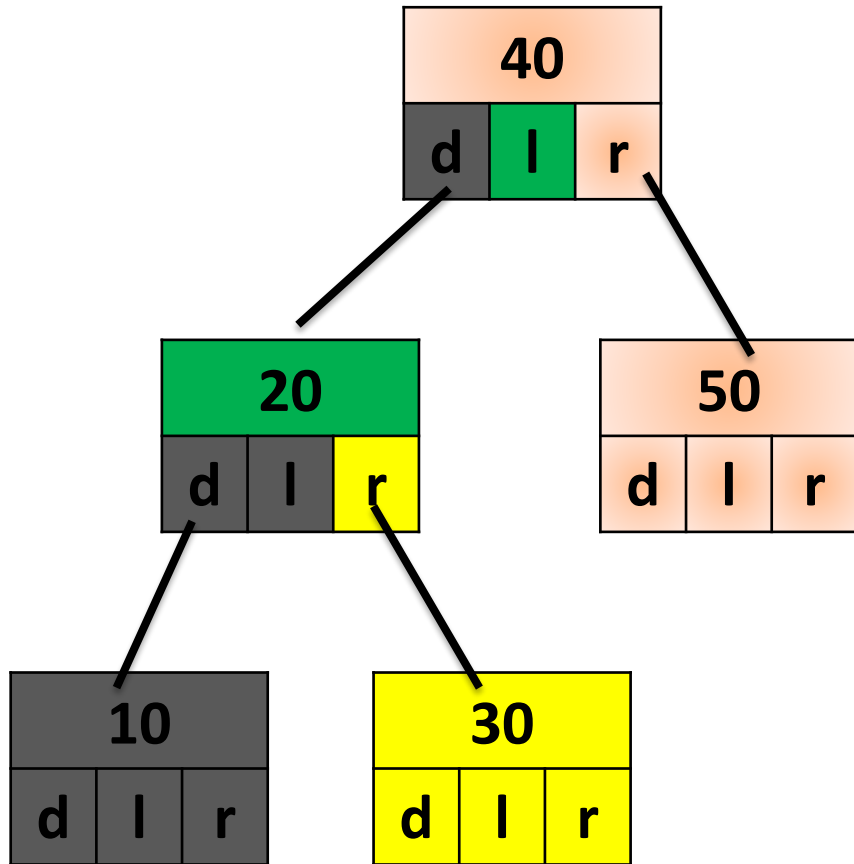
# preorder



OUTPUT



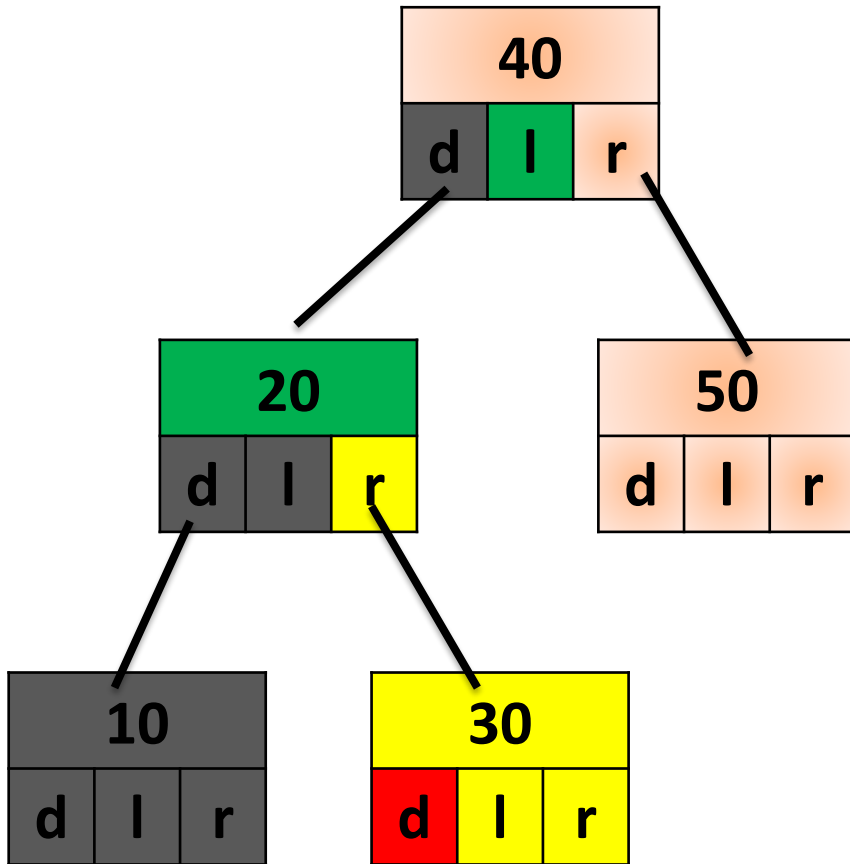
# preorder



OUTPUT



# preorder

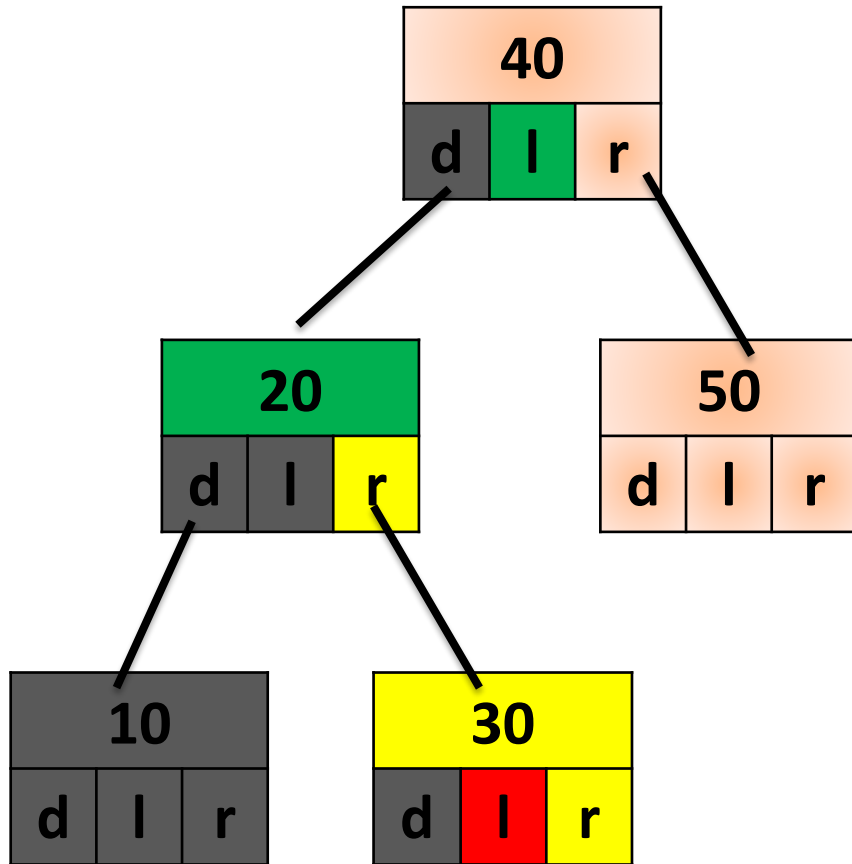


OUTPUT





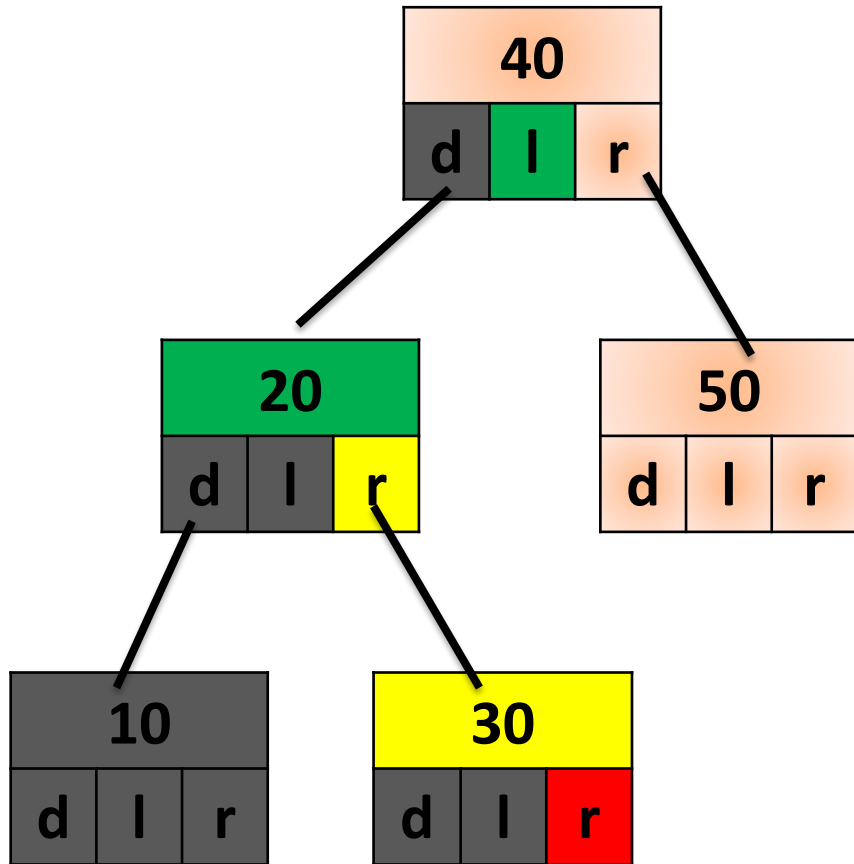
# preorder



OUTPUT



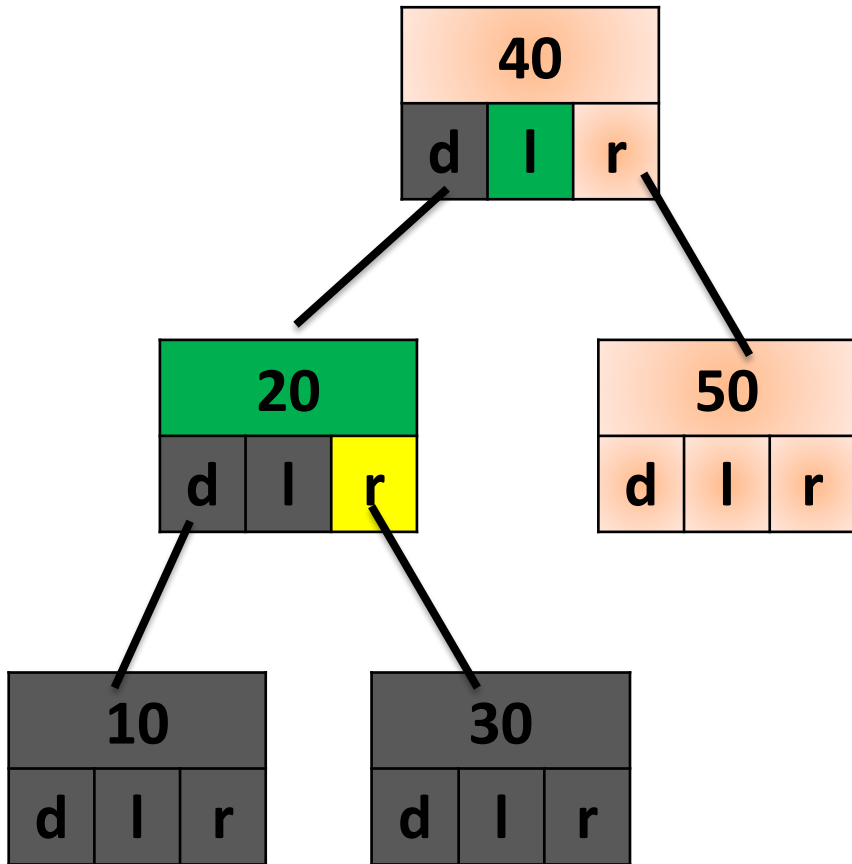
# preorder



OUTPUT

40	20	10	30	
----	----	----	----	--

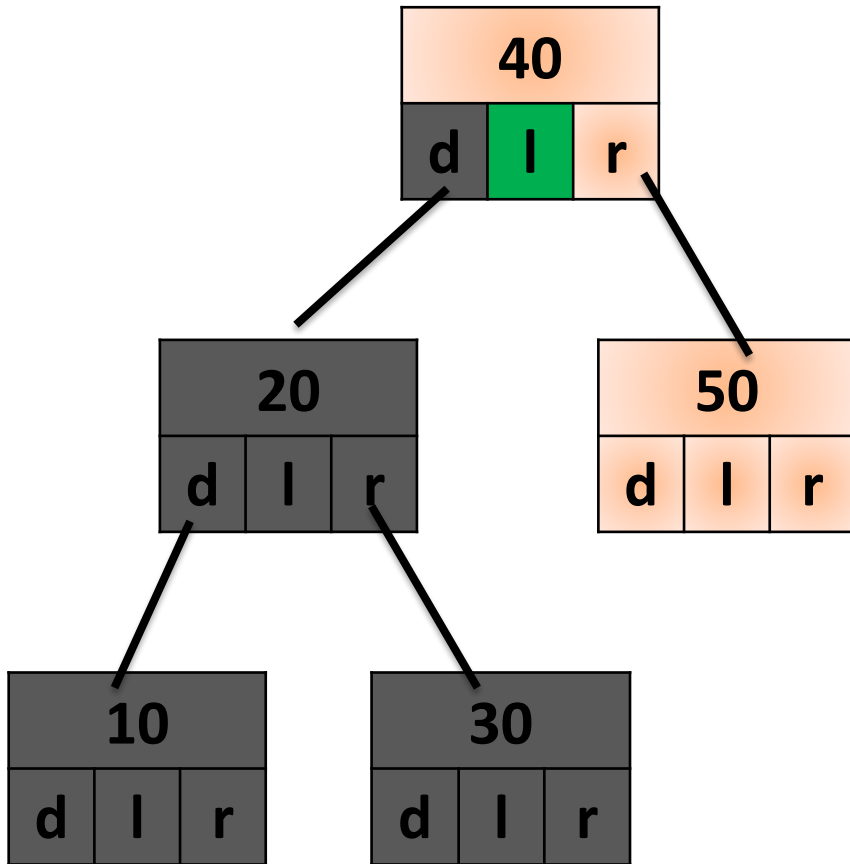
# preorder



OUTPUT



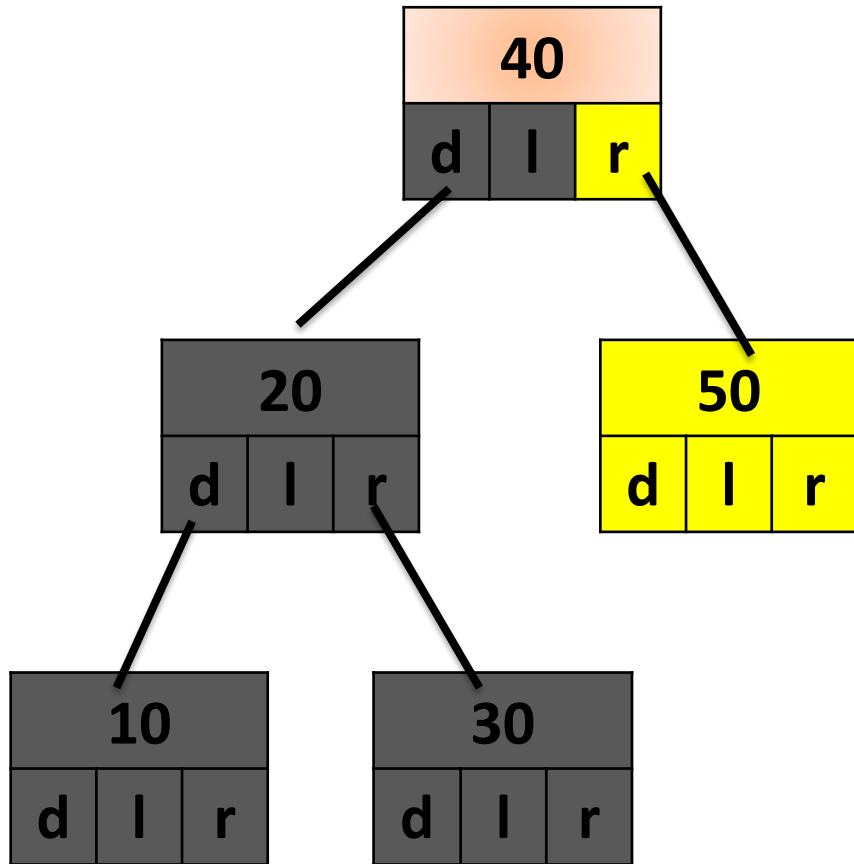
# preorder



OUTPUT



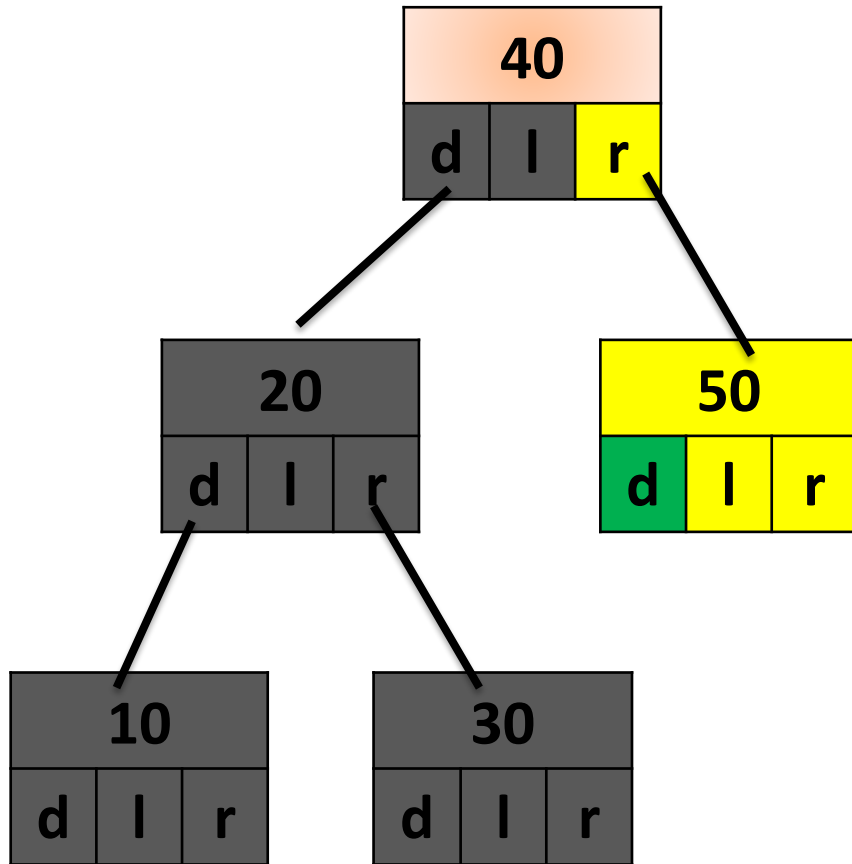
# preorder



OUTPUT



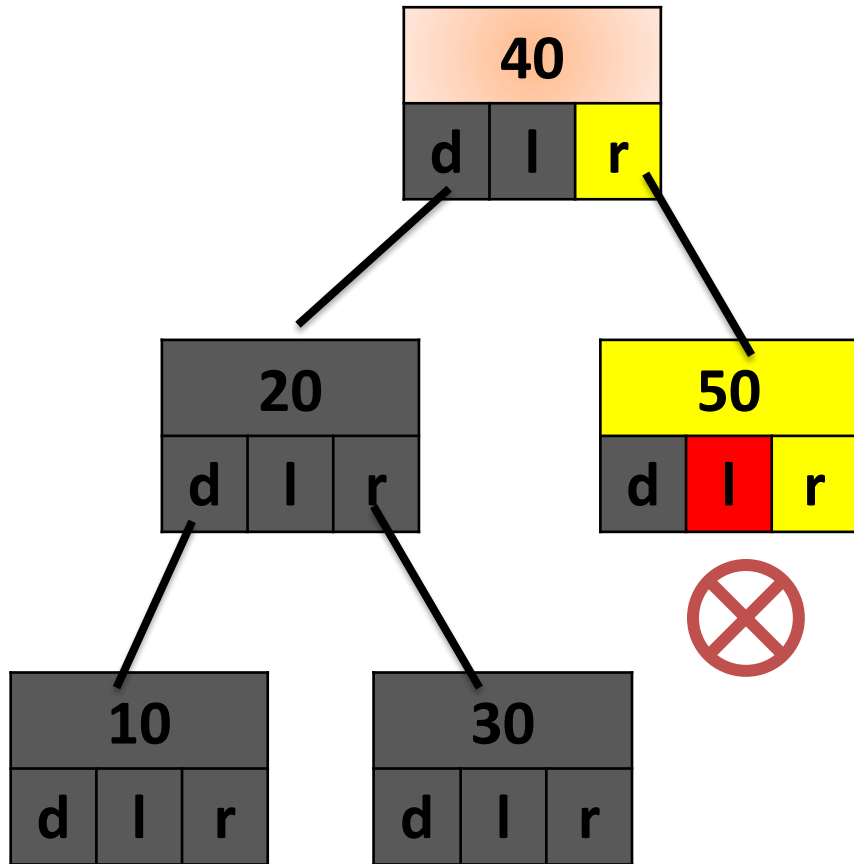
# preorder



OUTPUT

40	20	10	30	50
----	----	----	----	----

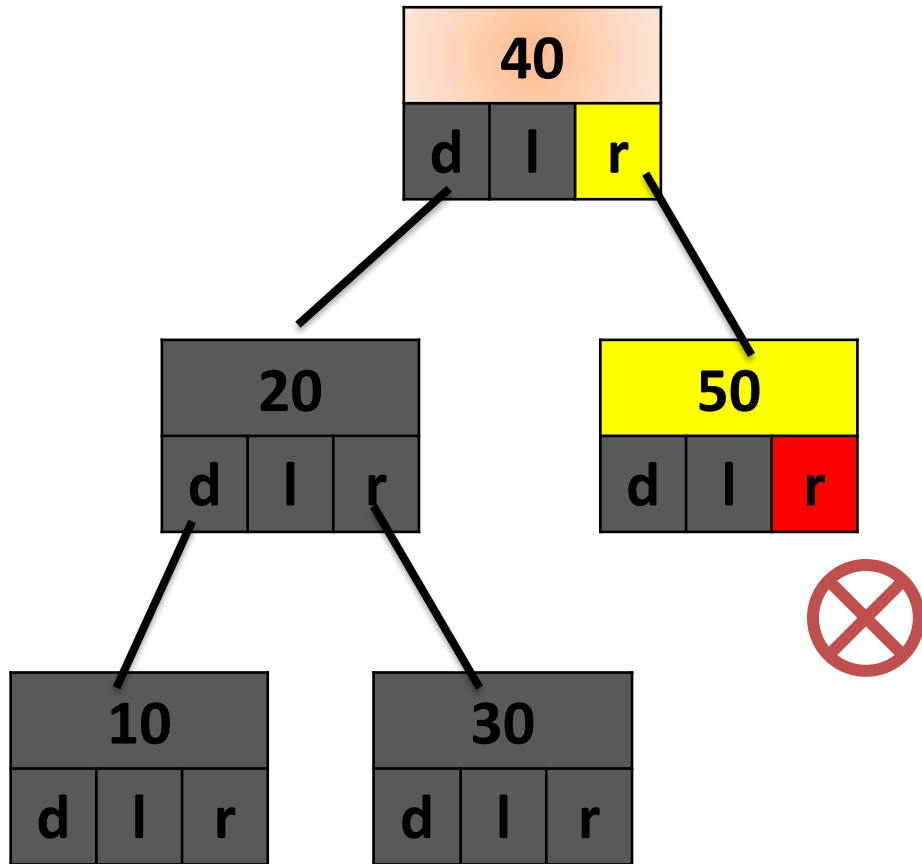
# preorder



OUTPUT

40	20	10	30	50
----	----	----	----	----

# preorder

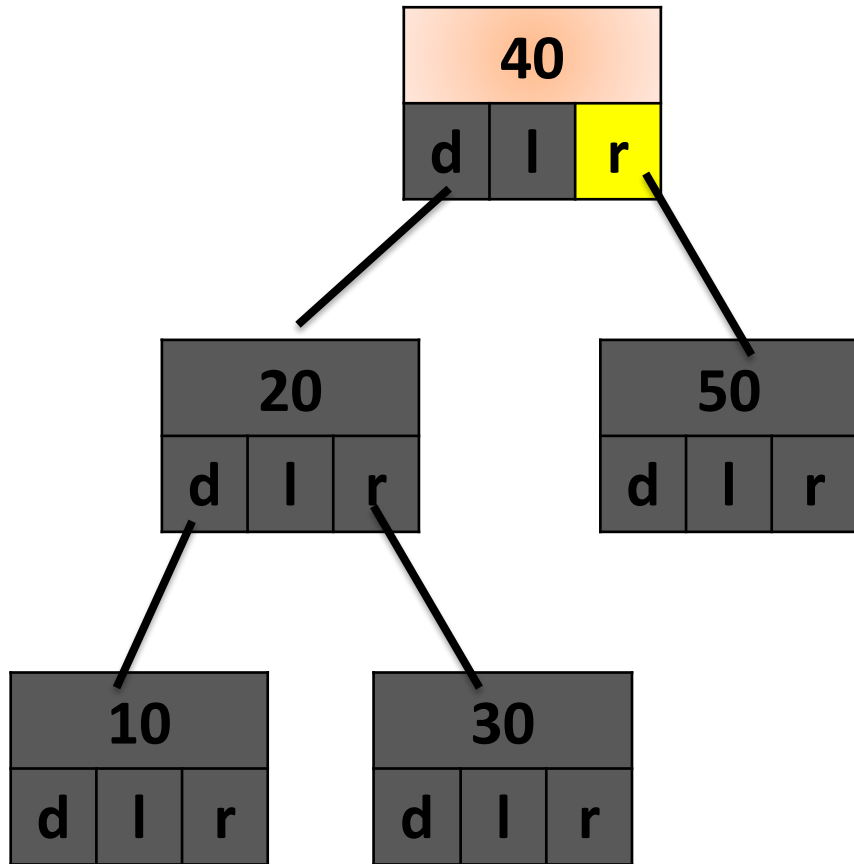


OUTPUT

40	20	10	30	50
----	----	----	----	----



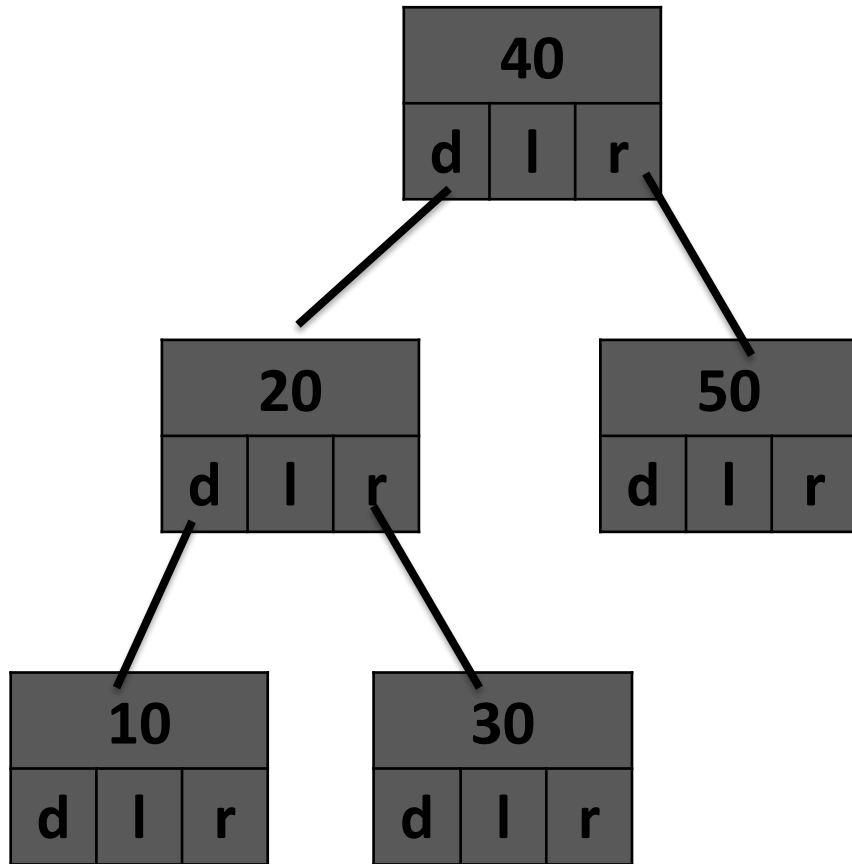
# preorder



OUTPUT

40 20 10 30 50

# preorder

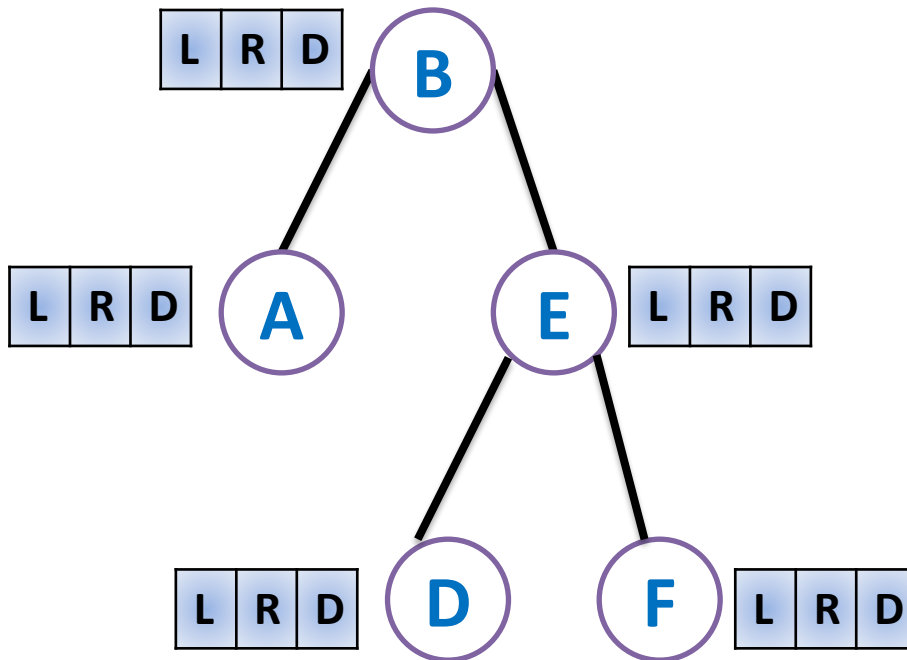


OUTPUT

40 20 10 30 50

# Postorder traversal

1. Traverse the left subtree
2. Traverse the right subtree
3. Print data.



OUTPUT

A D F E B

Find preorder & postorder traversal sequence of this tree.

