Problem:

2) Evaluate $\underset{x \to 0}{Lt} \dfrac{e^x + \sin x - 1}{\log(1+x)}$

$$\underset{x \to 0}{Lt} \left(\frac{e^x + \sin x - 1}{\log(1+x)} \right) = \frac{\left(\cancel{1} + \frac{x}{1!} + \frac{x^2}{2!} + \cancel{\frac{x^3}{3!}} + \frac{x^4}{4!} + \cdots\right) + \left(x - \cancel{\frac{x^3}{3!}} + \frac{x^5}{5!} - \cdots\right) - \cancel{1}}{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots}$$

$$= \frac{2x + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots}{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots}$$

$\div x$ both Nr + Dr

$$\underset{x \to 0}{Lt} \left(\frac{e^x + \sin x - 1}{\log(1+x)} \right) = \underset{x \to 0}{Lt} \left(\frac{2 + \frac{x}{2!} + \frac{x^3}{4!} + o(x^4)}{1 - \frac{x}{2} + \frac{x^2}{3} - o(x^4)} \right)$$

$$= \underline{\underline{2}}$$

3) $\underset{x\to 0}{Lt}\left(\dfrac{e^x - e^{-x} - 2x}{x - \sin x}\right)$

$$\underset{x\to 0}{Lt}\left(\frac{e^x - e^{-x} - 2x}{x - \sin x}\right) = \frac{\left(\cancel{1}+\cancel{\frac{x}{1!}}+\cancel{\frac{x^2}{2!}}+\frac{x^3}{3!}+\cancel{\frac{x^4}{4!}}+\cdots\right)-\left(\cancel{1}-\cancel{\frac{x}{1!}}+\cancel{\frac{x^2}{2!}}-\frac{x^3}{3!}+\cancel{\frac{x^4}{4!}}-\cdots\right)-\cancel{2x}}{\cancel{x}-\left\{\cancel{x}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\right\}}$$

$$= \frac{\frac{2x^3}{3!}+\frac{2x^5}{5!}+\cdots\cdots}{\frac{x^3}{3!}-\frac{x^5}{5!}+\cdots\cdots}$$

$\div$ Nr & Dr by x^3

$$\underset{x\to 0}{Lt}\left(\frac{e^x - e^{-x} - 2x}{x - \sin x}\right) = \underset{x\to 0}{Lt}\left(\frac{\frac{2}{3!}+\frac{2x^2}{5!}+O(x^4)}{\frac{1}{3!}-\frac{x^2}{5!}+O(x^3)}\right) = \frac{2/3!}{1/3!}$$

$$= 2$$

4) $\lim_{x\to 0}\left(\dfrac{e^{x}+2\sin x-e^{-x}-4x}{x^5}\right)$

$$\lim_{x\to 0}\left(\frac{e^{x}+2\sin x-e^{-x}-4x}{x^5}\right)=\lim_{x\to 0}\frac{\left\{\cancel{1}\left(\cancel{1}+\cancel{\frac{x}{1!}}+\cancel{\frac{x^2}{2!}}+\cancel{\frac{x^3}{3!}}+\cancel{\frac{x^4}{4!}}+\frac{x^5}{5!}+\cdots\right)+2\left(\cancel{x}-\cancel{\frac{x^3}{3!}}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\right)-\left(\cancel{1}-\cancel{\frac{x}{1!}}+\cancel{\frac{x^2}{2!}}-\cancel{\frac{x^3}{3!}}+\cancel{\frac{x^4}{4!}}-\frac{x^5}{5!}\cdots\right)-\cancel{4x}\right.}{x^5}$$

$$=\lim_{x\to 0}\left(\frac{4\frac{x^5}{5!}+o(x^6)}{x^5}\right)$$

$\cancel{\frac{?}{0}}\, x^5$

$$=\lim_{x\to 0}\left(\frac{4}{5!}+o(x^6)\right)$$

$$=\frac{4}{5!}=\frac{4}{120}=\frac{1}{30}$$

Binomial expansions

1) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \frac{n(n-1)(n-2)(n-3)}{4!} x^4 + \cdots \infty$

2) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \cdots$

3) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \cdots$

4) $(1+x)^{-2} = 1 - 2x - 3x^2 + 4x^3 - 5x^4 + \cdots$

5) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$

6) $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 - 15x^4 + \cdots$

7) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \cdots$

8) $(1+x)^{-4} = 1 - 4x + 10x^2 - 20x^3 + 35x^4 - \cdots$

9) $(1-x)^{-4} = 1 + 4x + 10x^2 + 20x^3 + 35x^4 + \cdots$

$\frac{-3\times -\overset{2}{\cancel{4}} \times -5}{6\cancel{x}} \cdot x^3$