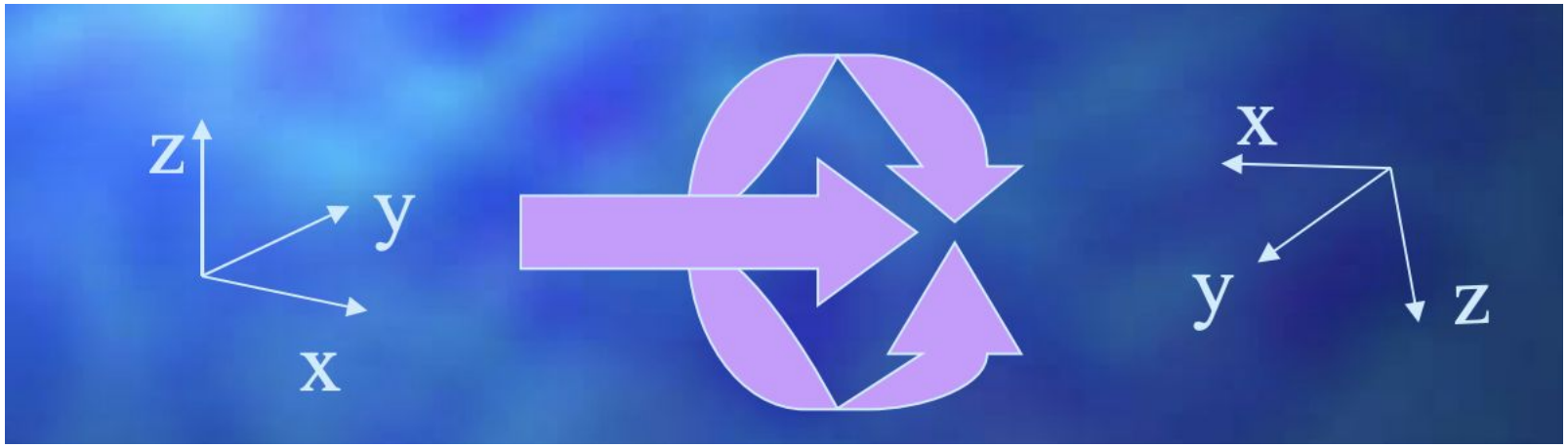
KINEMATICS

- KINEMATICS – the analytical study of the geometry of motion of a mechanism:
  - with respect to a fixed reference co-ordinate system
  - without regard to the forces or moments that cause the motion.
- In order to control and programme a robot we must have knowledge of both its spatial arrangement and a means of reference to the environment.

Kinematic Relationship

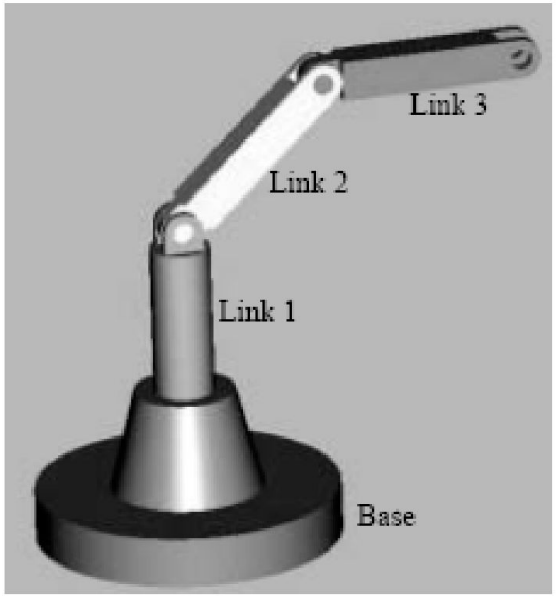
- Between two frames we have a kinematic relationship - basically a translation and a rotation.



- This relationship is mathematically represented by a 4 × 4 Homogeneous Transformation Matrix

OPEN CHAIN MANIPULATOR KINEMATICS

- Mechanics of a manipulator can be represented as a kinematic chain of rigid bodies (links) connected by revolute or prismatic joints.
- One end of the chain is constrained to a base, while an end effector is mounted to the other end of the chain.
- The resulting motion is obtained by composition of the elementary motions of each link with respect to the previous one.



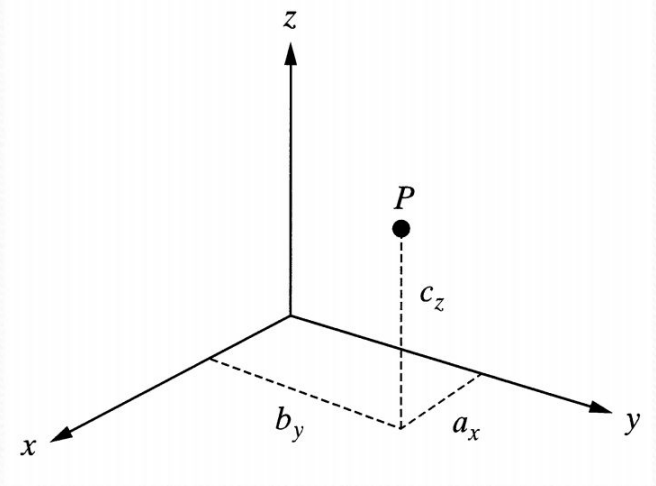
CLOSED KINEMATIC CHAIN

- Much more difficult.
- Even analysis has to take into account statics, constraints from other links, etc.
- Synthesis of closed kinematic mechanisms is very difficult

- Forward Kinematics:
  - to determine where the robot's hand is? (If all joint variables are known)
- Inverse Kinematics:
  - to calculate what each joint variable is? (If we desire that the hand be located at a particular point)
- Forward Kinematics and Inverse Kinematics equation for position analysis and three types of standard robot coordinate system are:
- (a) Cartesian (gantry, rectangular) coordinates.
- (b) Cylindrical coordinates.
- (c) Spherical coordinates.

Matrix Representation

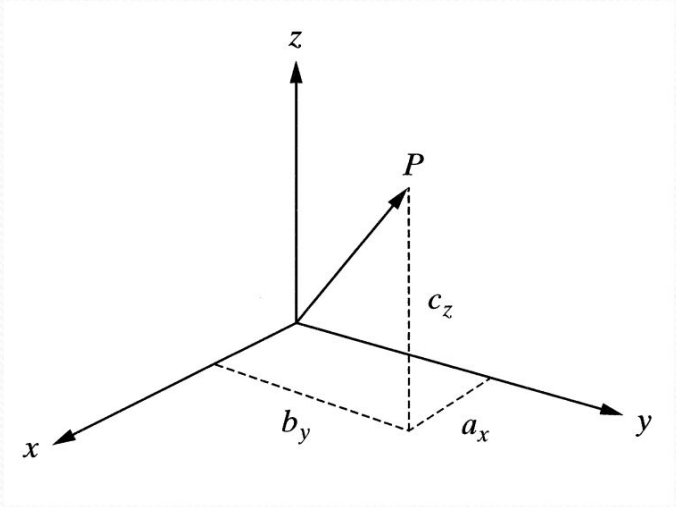
- Representation Of A Point In Space
- A point P in space :
    - 3 coordinates relative to a reference frame



Representation of a point in space

$$P = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

- -Representation of a Vector in Space
  - A Vector P in space : 3 coordinates of its tail and of its head



Representation of a vector in space

$$\overline{P} = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

$$\overline{P} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

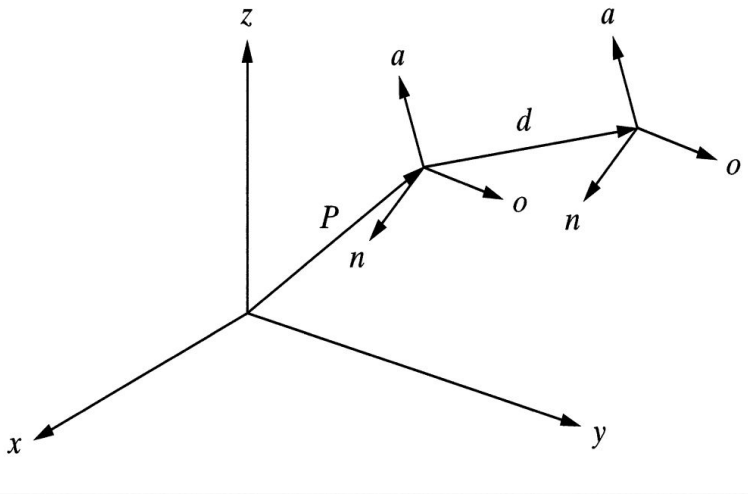
Where w is Scale factor

Transformations

- A transformation is defined as making a movement in space.
- Types of Transformation are:
  - A pure translation
  - A pure rotation
  - A combination of translation and rotation

Representation of a Pure Translation

- If a frame moves in space without any change in its orientation

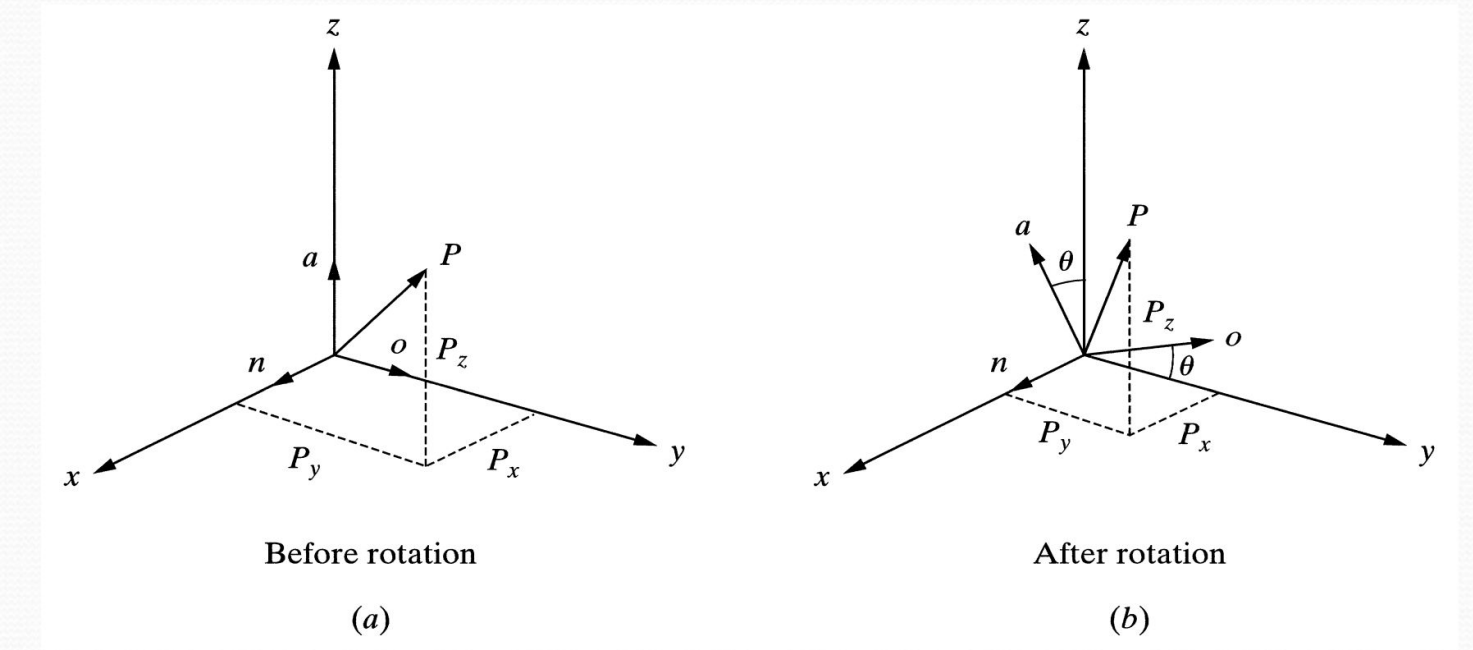


Representation of an pure translation in space

$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Pure Rotation about an Axis

- Assumption : The frame is at the origin of the reference frame and parallel to it



Coordinates of a point in a rotating frame before and after rotation.

COORDINATE FRAMES