Lecture - 5

12/01/2024

Motivation : Symmetry

$$p = \frac{h}{\lambda}$$

Symmetry of Nature

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h c/\lambda}{c}$$

$$E = mc^2 = \sqrt{(m_0 c^2)^2 + p^2 c^2}$$

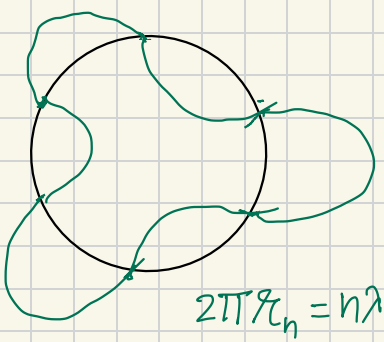
$$E = \frac{p^2}{2m}$$

$$2\pi r_n = n\lambda$$

$$L = mvr_n = pr_n$$

$$= p\left(\frac{\lambda}{2\pi}\right) \cdot n$$

$$= \frac{h}{2\pi} \cdot n = \hbar n$$



* de Broglie proposal provides sound basis to Bohr's stable orbit theory of hydrogren atom.

What about experimental evidence?

$\vec{p} = \hbar\vec{k}$: $\hbar$ is the link

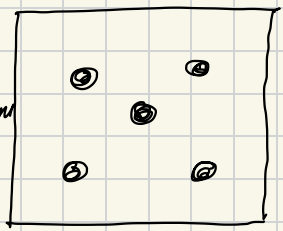
$\vec{p}$ is particle momentum

To Test for Wave nature "Diffract"

Elastic Scattering of waves from Periodical lattice

X-rays diffract from X-tal structure.

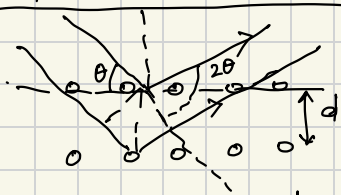
How do we know lattice X-tal structure.? Photograph of Momentum space



Laue Pattern.

Experimental Evidence:

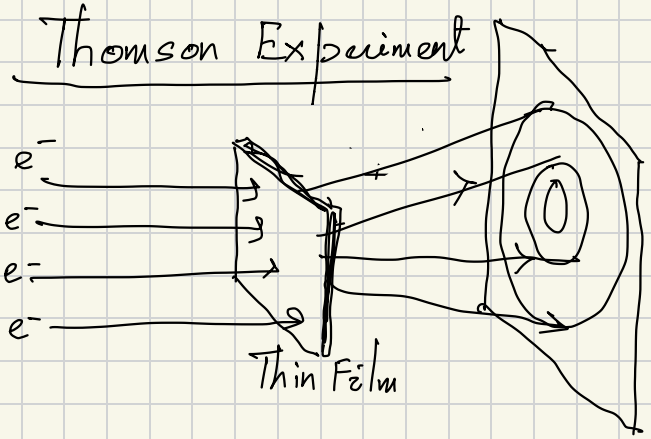
Davisson-Germer Expt. Ni (111) planes
54 eV electrons



$n\lambda = 2d\sin\theta$

Even for single electrons ⟶ the diffraction peak builds up.

Thomson Experiment



(son of J.J. Thomson)

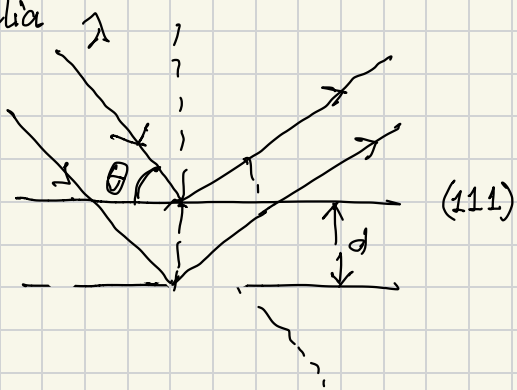
↑ for discovery of electrons

son for discovery of e⁻ as waves.

Debye-Scherrer Diffraction rings.

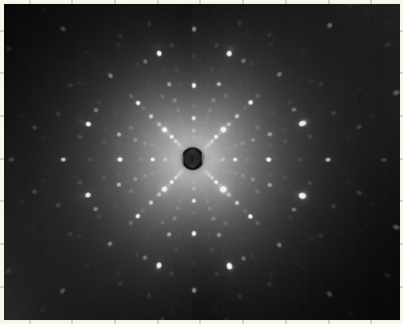
Electron Diffraction : Davisson-Germer

Single Crystal Diffraction



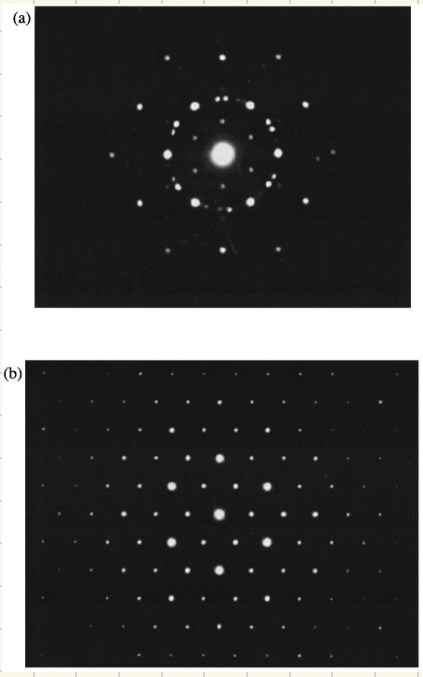
$$2d\sin\theta = n\lambda$$

Ni crystals electrons



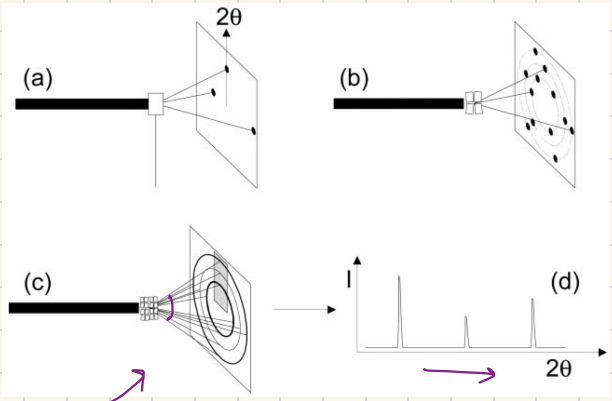
Exactly like Laue X-ray photographs.

Each spot corresponds to a set of parallel planes. (momentum-space photo)

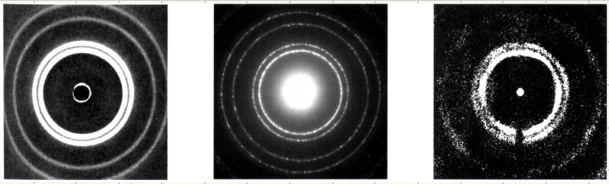


Transmission Electron microscope (TEM) in local area diffraction mode.

Multiple Crystal (polycrystal) diffraction.



Debye-Scherrer



X-ray

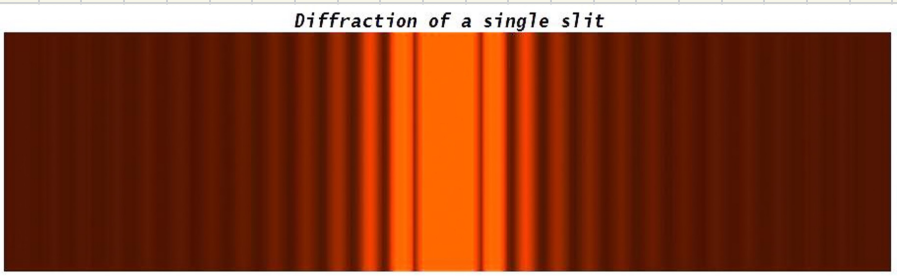
Electron Diffraction of Al thin-films.

Thomson experiment proves X-ray and electron diffraction equivalence.

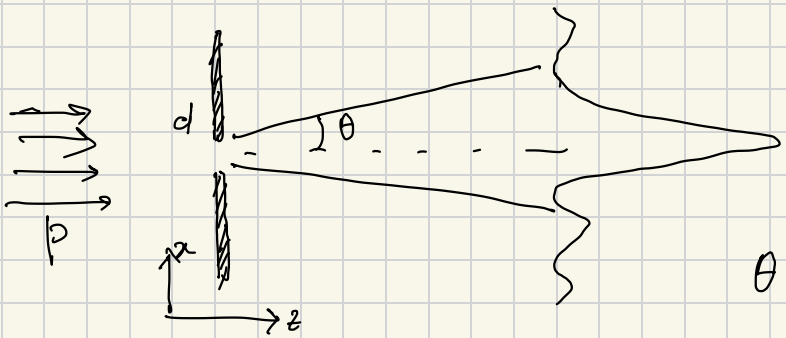
You get rings because polycrystals have the planes in random orientation.

Single Slit Diffraction:

Heuristic Introduction to Heisenberg Uncertainity Principle.



- The image above is from Wikipedia, https://commons.wikimedia.org/wiki/Category:Single-slit_diffraction#/media/File:Diffraction_of_a_single_slit.jpg made by Gisling CCBY3.0 June 2014



$$\theta \approx \frac{\lambda}{d} \sim \frac{\lambda}{\Delta x}$$

$$\frac{\lambda}{\Delta x} \sim \frac{\Delta p_x}{p} \qquad \tan\theta \sim \theta \sim \frac{\Delta p_x}{p}$$

$$\Rightarrow \boxed{\Delta x \, \Delta p_x \sim p\lambda \sim h}$$

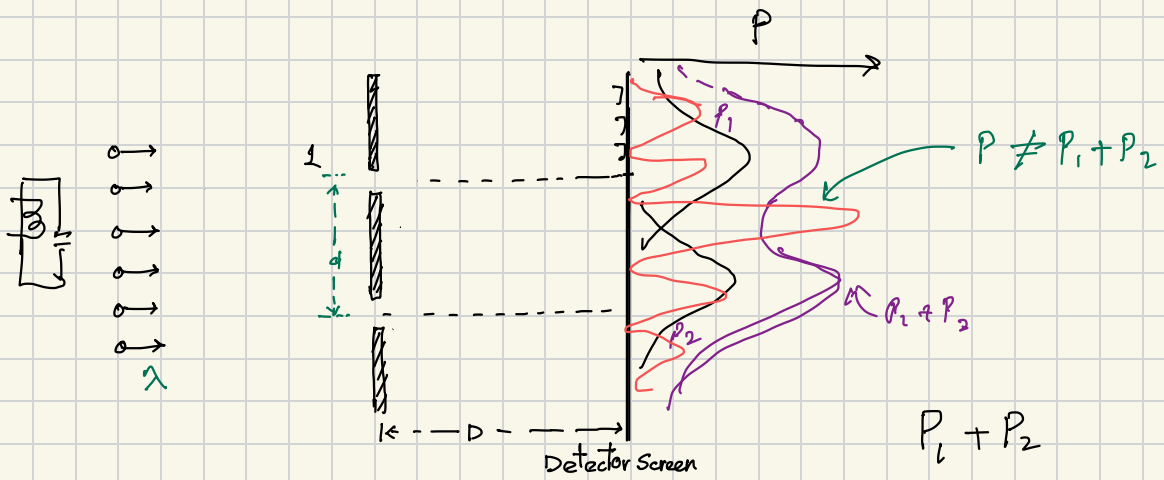
Heisenberg Uncertainty Relationship.

$$I = \underbrace{4 I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2}_{\alpha = \frac{\pi a \sin\theta}{\lambda}} \underbrace{\cos^2(\delta/2)}_{\delta = \frac{2\pi d \sin\theta}{\lambda}}$$

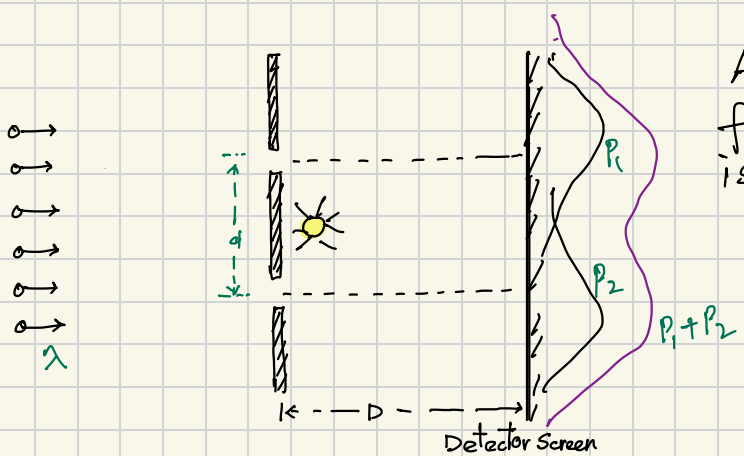
Single slit diffraction

Double slit interference

Double Slit Experiment: True Test of Waves.



Electrons start as particles
arrive at the detectors as particles
but during travel they behave as waves.



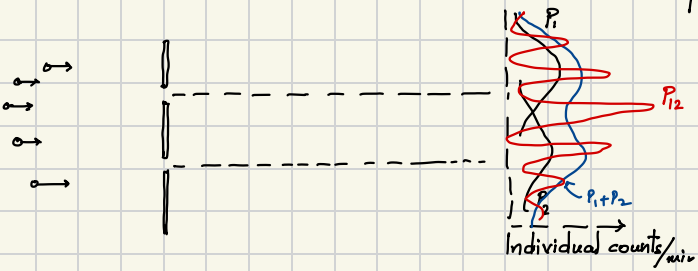
Add flasher to find out which slit e^- is going through
→ Less intensity fewer flashes. e not detected interfere
→ hν too less $\lambda \gg d$ can't find out which slit.

Wave Detection : Probabilities = $|\phi_1 + \phi_2|^2$

Particle $P_1 + P_2 = |\phi_1|^2 + |\phi_2|^2$

Double SLIT EXPT. with electrons

- First, block one of the slits, S_2
  electrons are delivered whole, no break up.



- Instead of $P_1 + P_2$ as we expect of particles,
  we get P_{12} : interference pattern : Direct Evidence of Wave Nature
- Could it be due to e-e repulsion? We can send single electrons.
  Initially it may appear random, but interference pattern is built up.

  ⇒ even a single electron somehow goes through both slits (without breaking up)

- If you try to find out through which slit it went through the distribution is particle like i.e. $P_1 + P_2$ and no interference is observed. (Finding out process makes them behave like particles.)

How do I know? From Experiment. No logical thinking or derivation can give that.

Nature behaves like that. !

Our attempt to simultaneously determine the x and p cannot succeed.

Results of Double Slit Interference with electrons in an Electron Microscope.

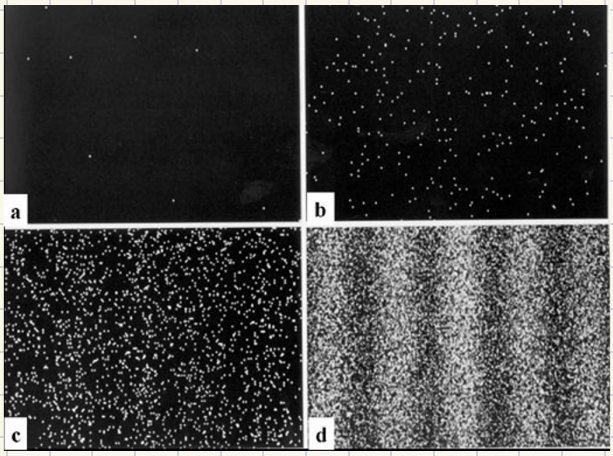


Fig. 2 Single electron events build up to from an interference pattern in the double-slit experiments.

- The number of electron accumulated on the screen.

(a) 8 electrons;

(b) 270 electrons;

(c) 2000 electrons;

(d) 160,000.

The total exposure time from the beginning to the stage (d) is 20 min.

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The rate at which electrons arrive is so low that at anytime only one electron is going through the double slit system.

yet the characteristic interference fringes appear.

Bohr's Complementarity Principle:

Experiment to detect particles will detect particles,
detect wave nature will show wave nature.

never simultaneously both!

Which slit e^- went through? $\Rightarrow$ Particle Prob.

Is there an interference pattern? $\Rightarrow$ Wave Nature.

"Measurement" forces dynamic system to one of its pure state

When you are not looking (or measuring) the system is evolving as per dynamics, but moment you measure it realizes one of its possible states.