PHY 114

Quantum Physics

Lecture 1- C

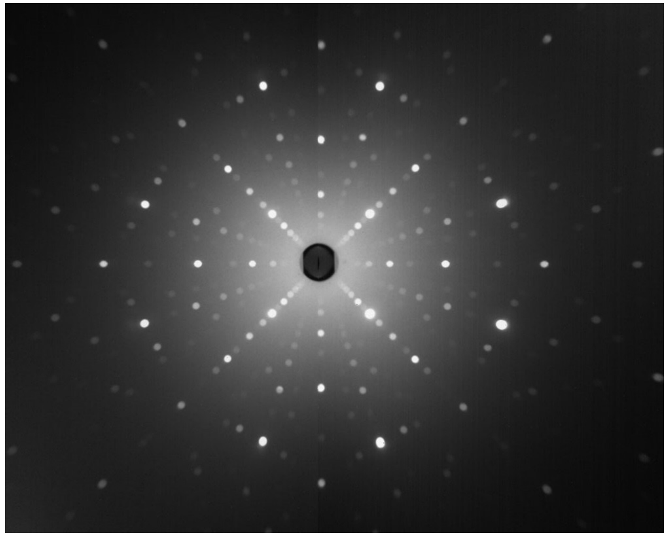
Matter Waves

Y N Mohapatra

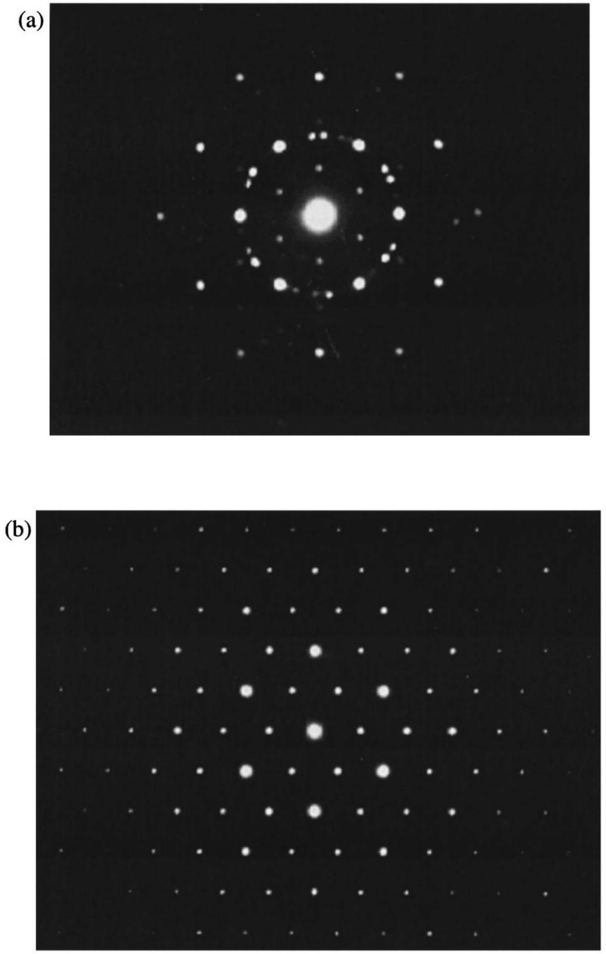
Deptt. of Physics
Materials Science Programme
National Centre for Flexible Electronics
Samtel Centre for Display Technologies

IIT Kanpur

Single Crystal : X-ray & Electron Diffraction

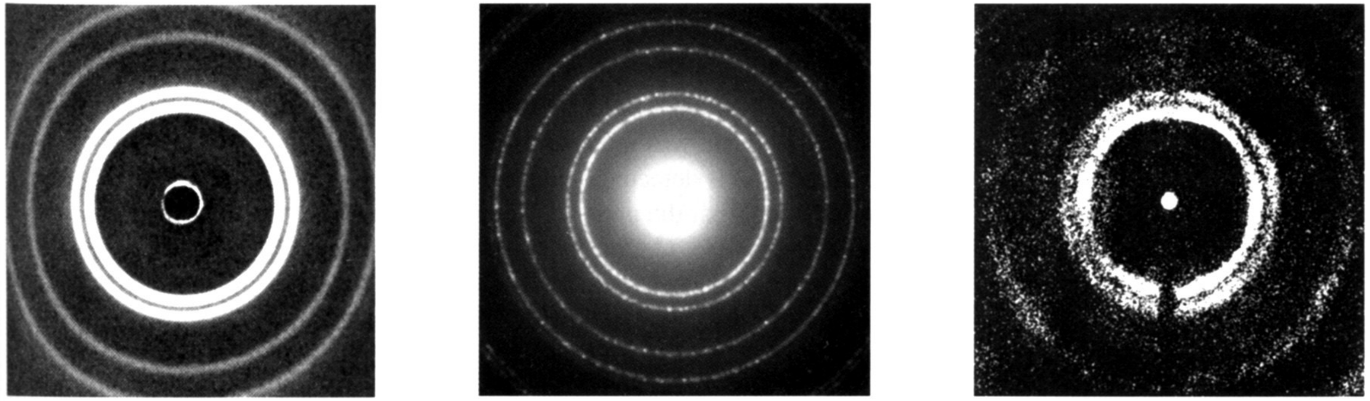


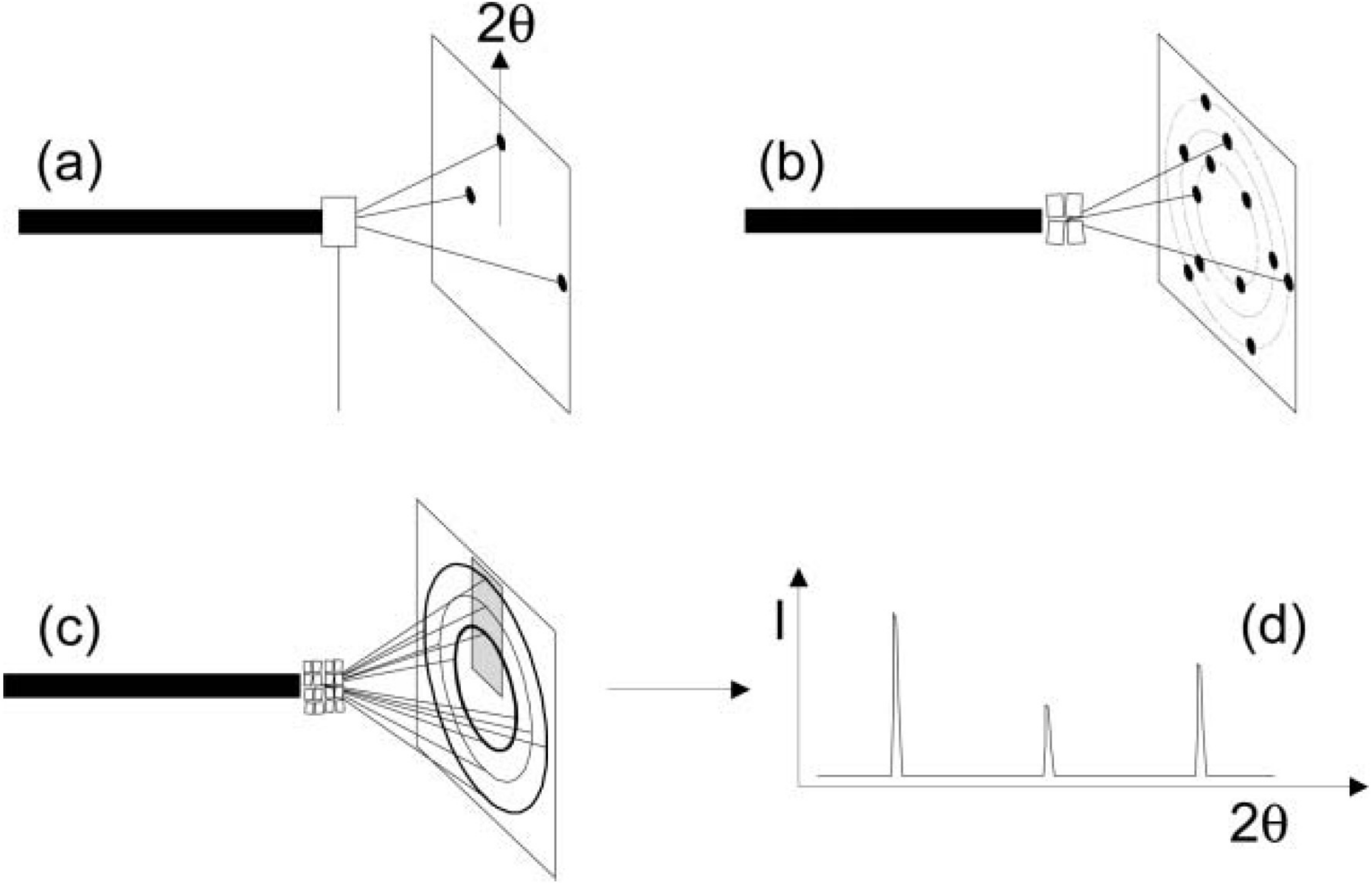
- X-ray Laue pattern



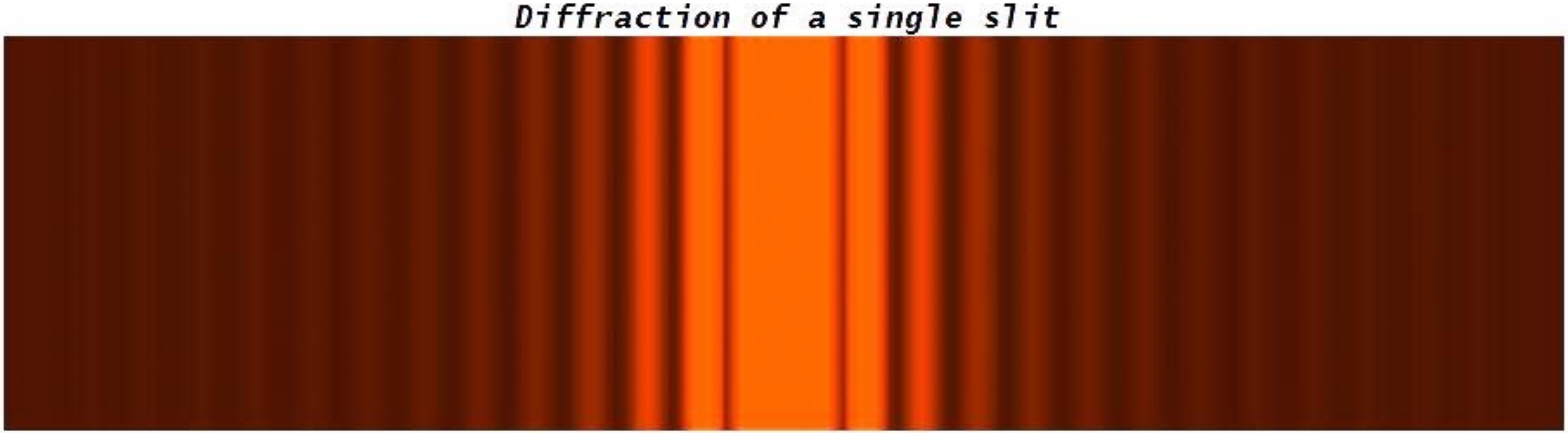
TEM micro diffraction pattern from a the subsequently deposited film, and b from a single crystal GaSe standard. Zurong Dai

X-ray & Electron Diffraction (Al thin film)



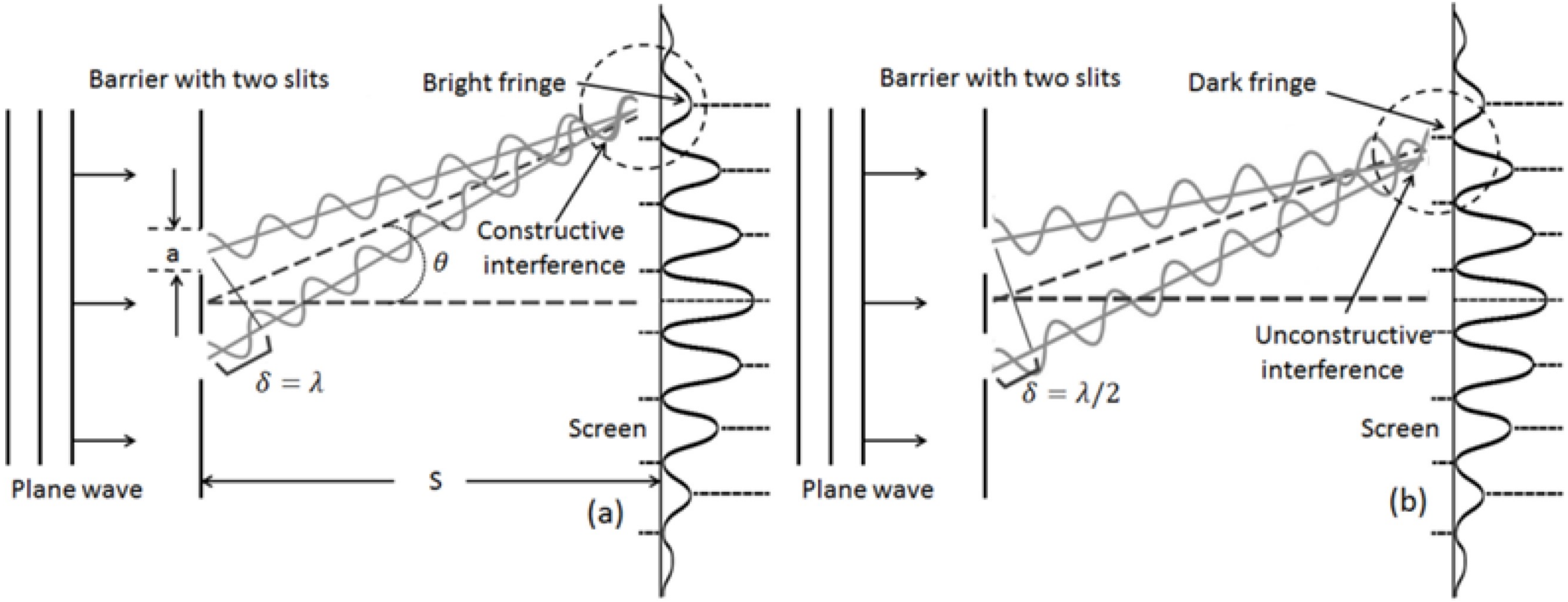


2θ
(a)
(b)
(c)
I
(d)
2θ



- The image above is from Wikipedia, https://commons.wikimedia.org/wiki/Category:Single-slit_diffraction#/media/File:Diffraction_of_a_single_slit.jpg made by Gisling CCBY3.0 June 2014

Double Slit Interference Pattern



Single electron events interference pattern in the double-slit experiment

Fig. 2 Single electron events build up to from an interference pattern in the double-slit experiments.

- The number of electron accumulated on the screen.

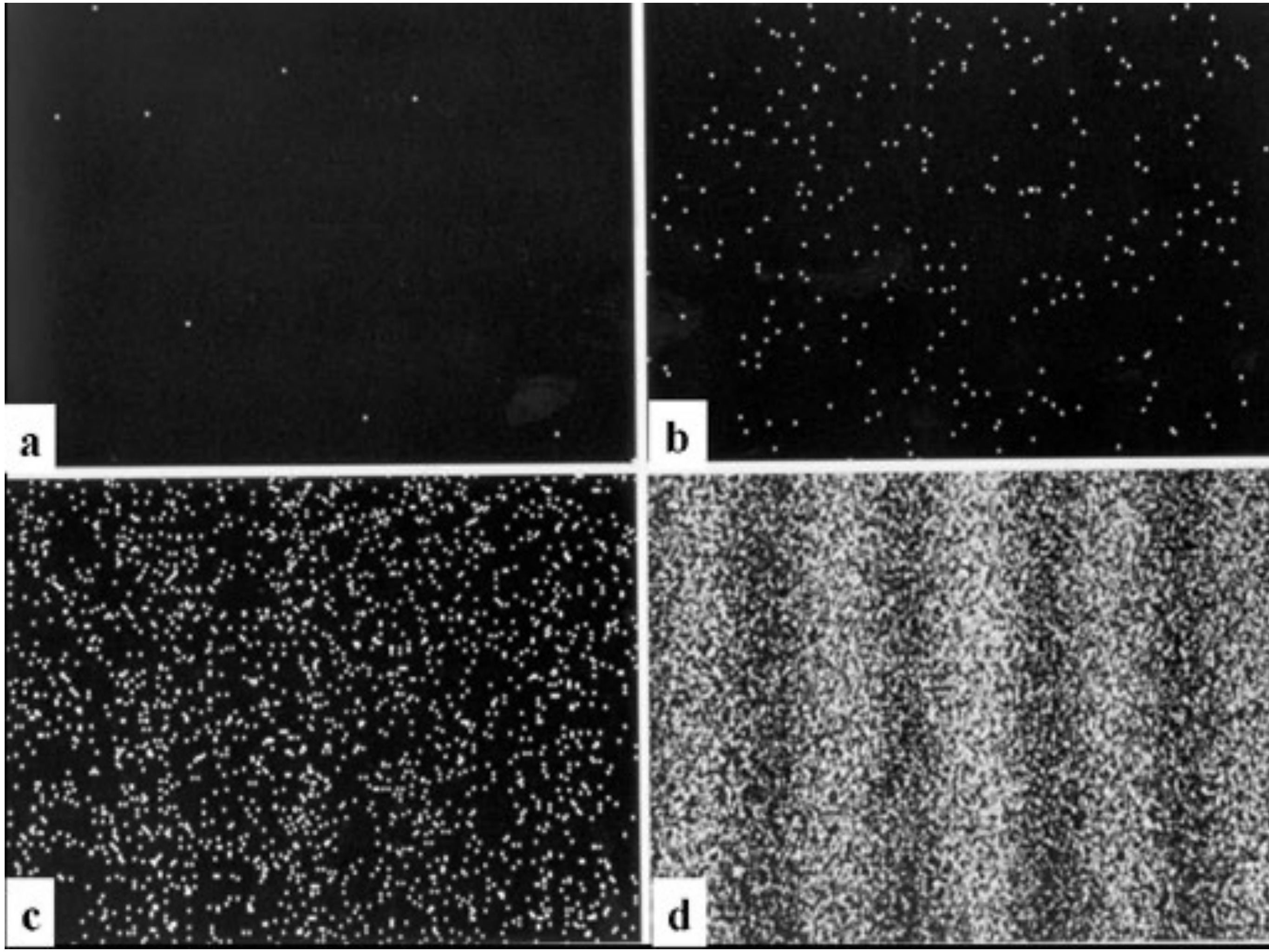
(a) 8 electrons;

(b) 270 electrons;

(c) 2000 electrons;

(d) 160,000.

The total exposure time from the beginning to the stage (d) is 20 min.



Hitachi Ltd. 1994, 2024

How to make a ‘particle’ out of ‘waves’ ?

For a particle, waves must be **LOCALIZED** in the region of space where probability of finding them is high.

How ?

Linear Superposition of Plane waves.

$$\Psi(x,t) = \sum_k A_k \, e^{i(kx-\omega t)}$$

Wavepacket

Wave Packets : Superposition

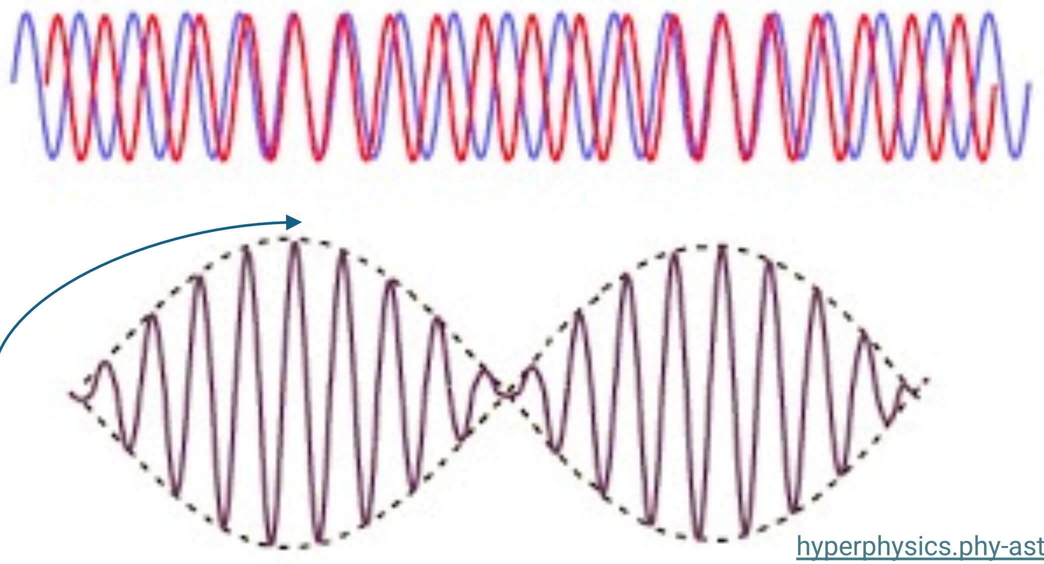
Beat as an elementary example

Heterodyne:
Radio + music

$$\Phi_1(x,t) = A\cos\left[\left(k + \frac{1}{2}\Delta k\right)x - \left(\omega + \frac{1}{2}\Delta\omega\right)t\right]$$

$$\Phi_2(x,t) = A\cos\left[\left(k - \frac{1}{2}\Delta k\right)x - \left(\omega - \frac{1}{2}\Delta\omega\right)t\right]$$

$$\Phi_1 + \Phi_2 = 2A\cos\left[\frac{1}{2}(\Delta kx - \Delta\omega t)\right]\cos(kx - \omega t)$$



hyperphysics.phy-astr.gsu.edu

What is the velocity of this peak ?

The modulating envelope (dashed line), the position of the peak is given by

$$\Delta kx - \Delta\omega t = 0$$

Therefore , the increments in x and t at the position of the peak is given by

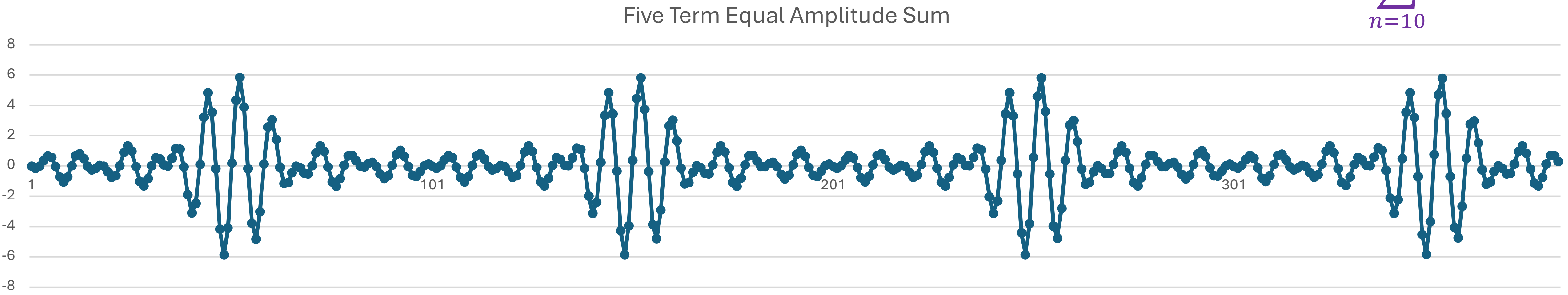
$$\Delta k\, dx - \Delta\omega\, dt = 0 \longrightarrow v_g = \frac{dx}{dt} = \frac{\Delta\omega}{\Delta k} \longrightarrow \boxed{v_g \equiv \frac{dx}{dt} = \frac{d\omega}{dk}}$$

Phase velocity : $\boxed{v_p \equiv \frac{\omega}{k}}$

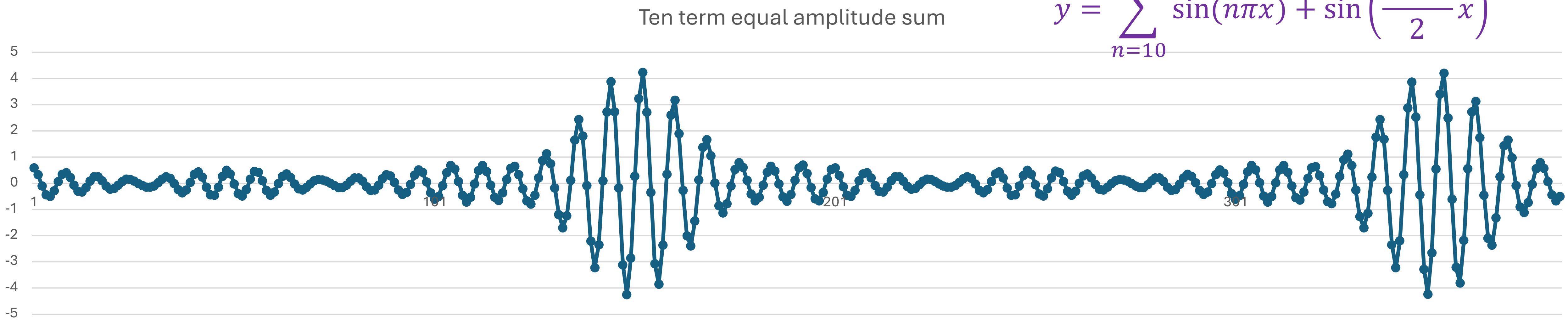
Group velocity :

Demo Superposition : Equal Amplitude

$$y = \sum_{n=10}^{15} \sin n\pi x$$

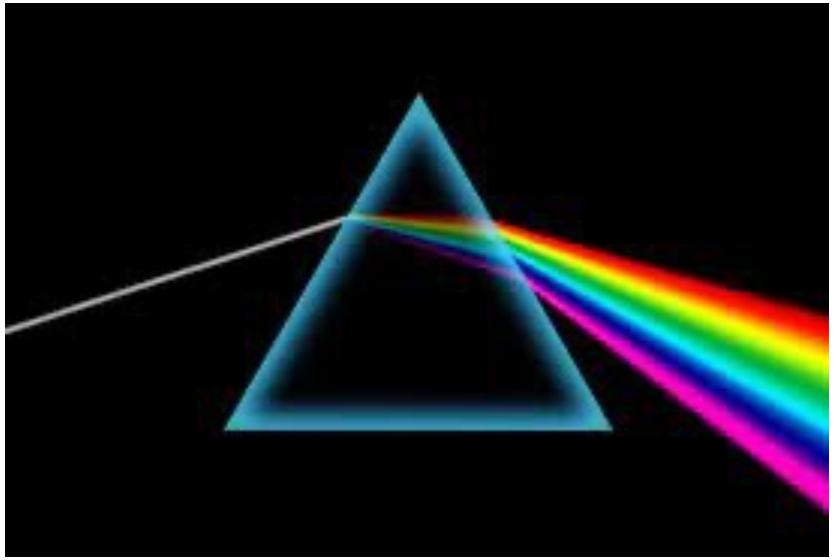


$$y = \sum_{n=10}^{15} \sin(n\pi x) + \sin\left(\frac{n+1}{2}x\right)$$

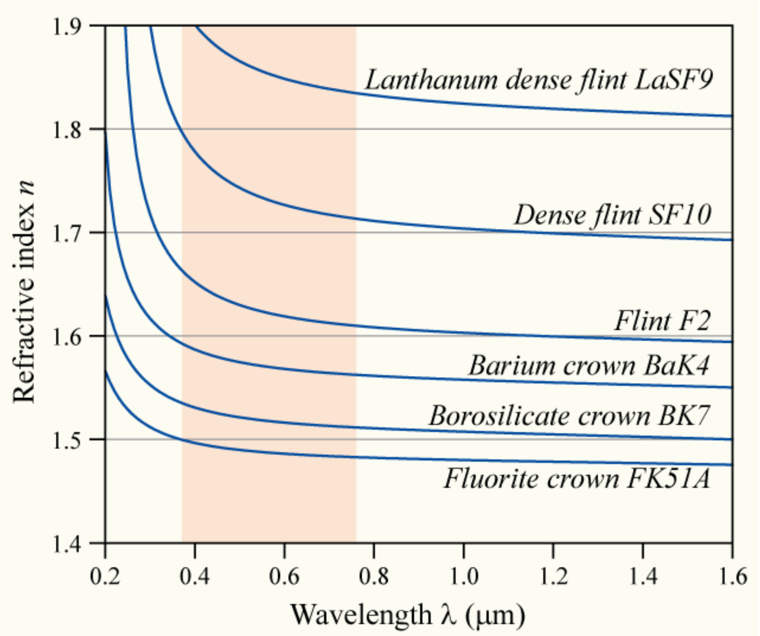


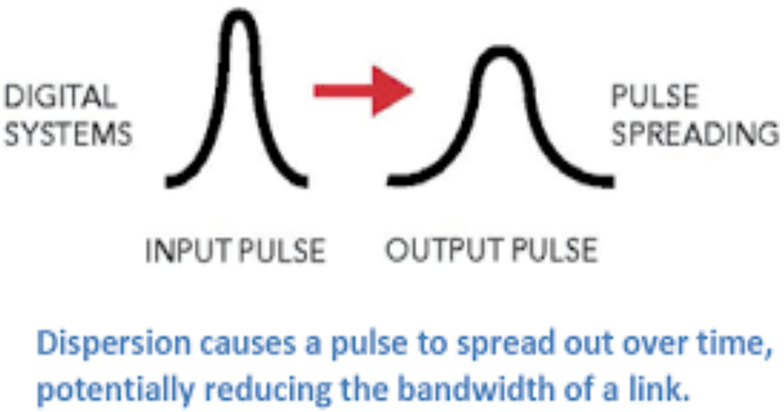
Dispersion:

frequency dependence of speed in the medium



dkfindout.com





ofsoptics.com

<u>Wave Packet</u> : A group of harmonic waves of different *k*

$$\Phi(x,t) = \sum \tilde{A}_j \, e^{i(k_j x - \omega t)}$$

$$\Phi(x,t) = \int A(k) \, e^{i(kx - \omega t)} \, dk$$

$$\boxed{\omega(k)}$$

<u>Dispersion Relationship</u> : Defines all about the medium.

Example : Dispersion of water waves in deep water.

Water waves in deep water have a frequency dependence given by

$$\omega = \sqrt{gk}$$

,where g is the acceleration due to gravity.

$$v_p = \frac{\omega}{k}$$

$$v_p = \sqrt{\frac{g\lambda}{2\pi}}$$

,i.e. water waves of longer wavelength have have a greater velocity.

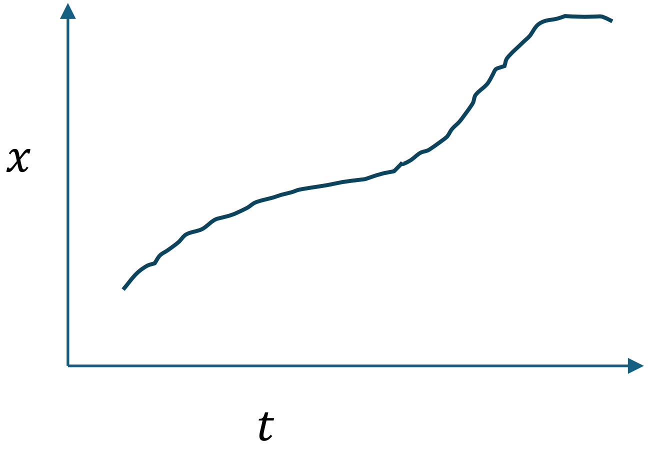
,i.e. surface of water waves is a dispersive medium.

$$\boxed{v_g = \frac{d\omega}{dk}}$$

$$\boxed{v_g = \frac{d}{dk}\sqrt{gk} = \frac{1}{2}\sqrt{\frac{g}{k}}}$$

Note here : $v_g = \frac{1}{2} v_p$

Characterizing Waves in a medium



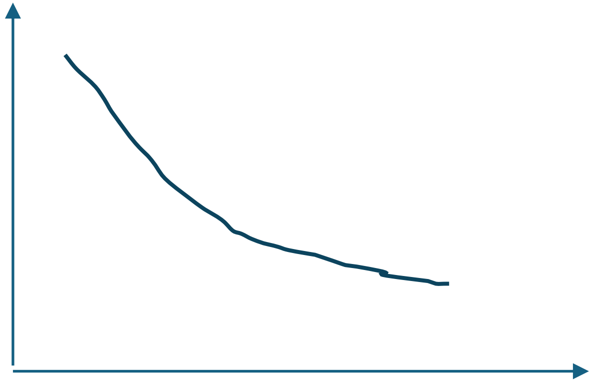
For a particle, we need to know $x(t)$:

we can then find : $\frac{dx}{dt}, \frac{d^2x}{dt^2}$

$$\omega = \frac{2\pi}{T}$$

$\sim E$

Energy



$$k = \frac{2\pi}{\lambda} \quad \sim p$$

Momentum

$$\boxed{v_p \equiv \frac{\omega}{k}}$$

$$\boxed{v_g \equiv \frac{d\omega}{dk}}$$

What about a wave ?

It is characterized by λ *and* T.