Looking at polarization of light one photon at a time

Reference: P.A.M. Dirac, Principles of quantum mechanics

Background: In lectures 4 and 5, we showed and discussed about single electron or single photon interference. The conclusion arrived at were as follows:

i. When a particle is passing through two slits (say S1 and S2), there is a probability p1 that it will pass through S1 and p2 that it will pass through S2.

ii. The probabilities are described by a probability amplitude $\sqrt{p1}$ or $\sqrt{p2}$ and a phase associated with it.

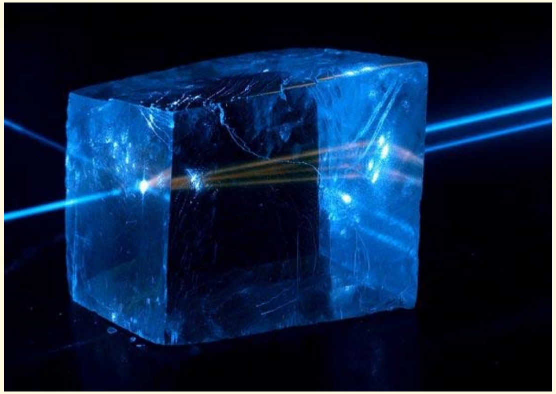
iii. In any region after the slits, the total probability amplitude is given by adding the individual probability amplitudes along with their phases.

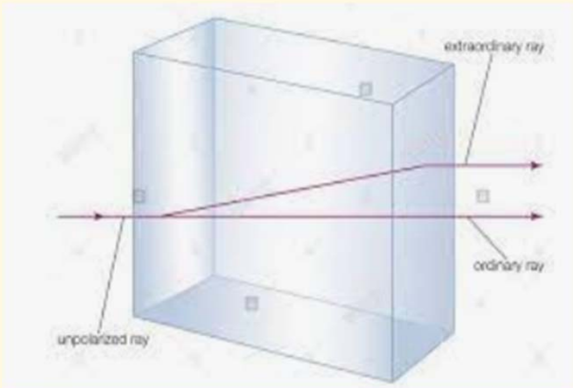
iv. Interference arises from the cross terms when total probability amplitude is squared to get the probability.

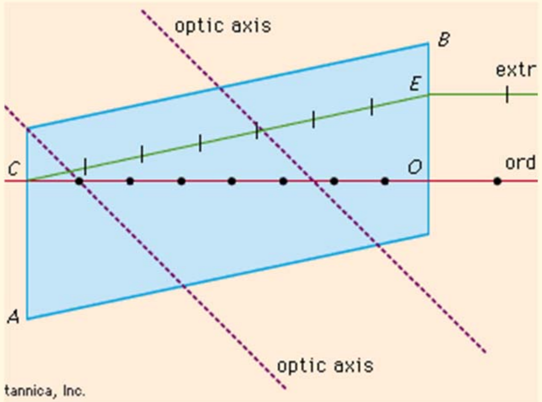
It is sometimes challenging to think in terms of probability of a particle passing through one of the slits. We will therefore look at another system which can exist in two different states and see the associated probability with each state.

For this we will consider the polarization of light and see what happens when we let light pass through polarizers with such low intensity that only photon passes through the polarizers at one time.

Consider unpolarized light falling on a polarizing crystal (it is a tourmaline crystal) that splits the light beam into two beams. The polarization of light in the two beams is perpendicular to each other. Let the orientation of the crystal be such that upper beam of light has vertical polarization and the lower one has horizontal polarization.







- For incoming light beam with polarization at an angle θ from the horizontal and having intensity I, the intensity of outgoing beams will be $I\cos^2\theta$ for the horizontally polarized light and $I\sin^2\theta$ for the vertically polarized light assuming no absorption of light.
- Now imagine the intensity of light is made so low that only one photon is incident of the crystal at any time. Question is what will be the polarization of photon after it has passed through the crystal.

Question: After passing through a tourmaline crystal, what will be the polarization of a photon incident on a with polarization at an angle θ from the horizontal?

The answer to this question is that each photon has a probability $\cos^2\theta$ of passing through the crystal with horizontal polarization and $\sin^2\theta$ with vertical polarization. This preserves the individuality of a photon (like the individuality of a particle in the double-slit experiment is preserved) and is consistent with the experimental result when probability is measure with a large number of photons (intense beam passing through the crystal).

Quantum mechanics tells us that each individual particle has this probability. Classically, on the other hand, we would say that probable number of photons with horizontal polarization is $N\cos^2\theta$ if N photons are incident on the crystal. Notice the difference between the two ways of thinking.

Exercise: Do a similar analysis if light of one polarization is absorbed completely in the crystal and only the other polarization comes out.