Assignment 5: Double Integrals

1. Evaluate the following iterated integrals:

   (a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx,$

   (b) $\displaystyle\int_0^\pi \int_x^\pi \frac{\sin y}{y}\,dydx,$

   (c) $\displaystyle\int_0^1 \int_y^1 x^2 \exp^{xy}\,dxdy.$

2. Evaluate $\displaystyle\iint_R xdxdy$ where R is the region $1 \leq x(1-y) \leq 2$ and $1 \leq xy \leq 2$.

3. Using double integral, find the area enclosed by the curve $r = \sin 3\theta$ given in polar cordinates.

4. Compute $\displaystyle\lim_{a \longrightarrow \infty} \iint_{D(a)} \exp^{-(x^2+y^2)}\,dxdy$, where

   (a) $D(a) = \{(x, y) : x^2 + y^2 \leq a^2\}$

   (b) $D(a) = \{(x, y) : 0 \leq x \leq a,\ 0 \leq y \leq a\}$.

   Hence prove that (i) $\displaystyle\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2},$ (ii) $\displaystyle\int_0^\infty x^2 e^{-x^2}dx = \frac{\sqrt{\pi}}{4}.$

5. Find the volume of the solid which is common to the cylinder $x^2+y^2 = 1$ and $x^2+z^2 = 1$.

Assignment 6: Triple Integrals, Surface Integrals, Line integrals

1. Evaluate the integral $\iiint\limits_W \dfrac{dzdydx}{\sqrt{1+x^2+y^2+z^2}}$; where W is the ball $x^2+y^2+z^2 \leq 1$.

2. What is the integral of the function x^2z taken over the entire surface of a right circular cylinder of height h which stands on the circle $x^2+y^2=a^2$?

   What is the integral of the given function taken throughout the volume of the cylinder?

3. find the area of the surface $x = uv,\ y = u+v,\ z = u-v$, where $(u,v) \in D = \{(u,v)|\ u^2+v^2 \leq 1\}$

4. Find the line integral of the vector field $F(x,y,z) = y\vec{i} - x\vec{j} + \vec{k}$ along the path $\mathbf{c}(t) = (\cos t, \sin t, \frac{t}{2\pi}),\quad 0 \leq t \leq 2\pi$ joining $(1,0,0)$ to $(1,0,1)$.

5. Evaluate $\int\limits_C T \cdot dR$, where C is the circle $x^2+y^2=1$ and T is the unit tangent vector.

6. Show that the integral $\int\limits_C yzdx+(xz+1)dy+xydz$ is independent of the path C joining $(1,0,0)$ and $(2,1,4)$.

Assignment 7 : Green's /Stokes' /Gauss' Theorems

1. Use Green's Theorem to compute $\int_C (2x^2-y^2)\ dx+(x^2+y^2)\ dy$ where C is the boundary of the region $\{(x,y) : x, y \geq 0,\quad x^2+y^2 \leq 1\}$.

2. Use Stokes' Theorem to evaluate the line integral $\int_C -y^3\ dx + x^3 dy - z^3 dz$, where C is the intersection of the cylinder $x^2+y^2 = 1$ and the plane $x+y+z = 1$ and the orientation of C corresponds to counterclockwise motion in the xy-plane.

3. Let S be the unit sphere $x^2+y^2+z^2 = 1$. Evaluate the following surface integral using Divergence Theorem.
$$\iint_S [x(2x+3e^{z^2}) + y(-y-e^{x^2}) + z(2z+\cos^2 y)]d\sigma.$$

4. Let $\overrightarrow{F} = \frac{\overrightarrow{r}}{|\overrightarrow{r}|^3}$ where $\overrightarrow{r} = x\,\overrightarrow{i} + y\,\overrightarrow{j} + z\,\overrightarrow{k}$ and let S be any surface that surrounds the origin. Prove that $\iint_S \overrightarrow{F}.n\ d\sigma = 4\pi$.

5. Let D be the solid bounded by $z = 0$ and the paraboloid $z = 4-x^2-y^2$. Let S be the boundary of D. If $F(x,y,z) = (x^3+\cos(yz),\ y^3,\ x+\sin(xy))$, use divergence theorem to evaluate $\iint_S F \cdot n\ d\sigma$ where n is the outward normal to the surface S.

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