

PHY 114

Quantum Physics

Lecture 1-B-2

Waves as Particles

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Prediction of Classical vs Planck for $u(\nu)$?

We know **Maxwell's EM Theory** & **Boltzmann's Classical Thermodynamics**.

Calculate the number density of oscillators in a cavity, and multiply the average energy of each oscillator.

Classical

Density of Modes: $g(\nu) = \frac{8\pi}{c^3} \nu^2$

Average Energy: $\langle \epsilon \rangle = k_B T$

Energy density u_ν : $u_\nu = \frac{8\pi}{c^3} \nu^2 k_B T$

Planck's Quantum

$$g(\nu) = \frac{8\pi}{c^3} \nu^2$$

$$\langle \epsilon \rangle = \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

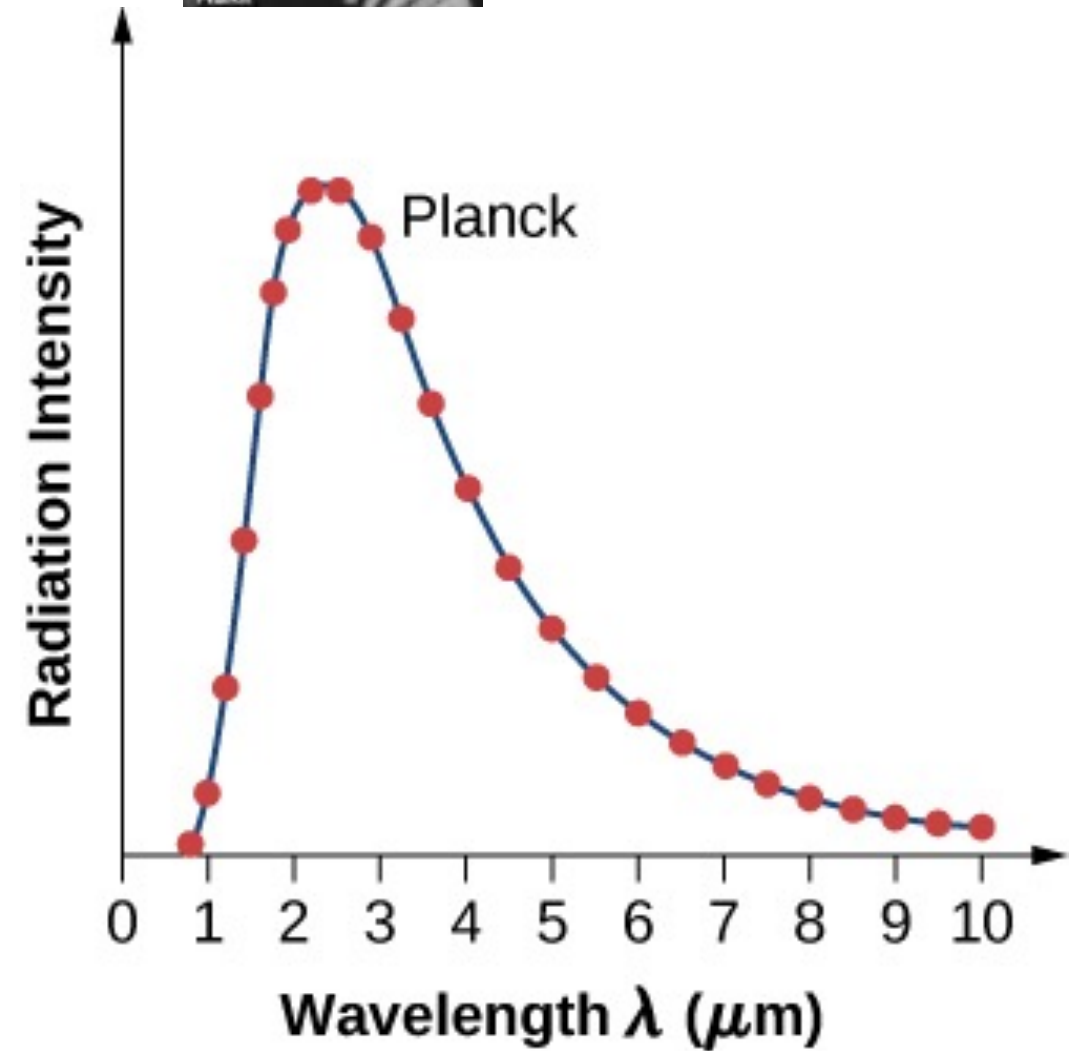
$$u_\nu = \frac{8\pi}{c^3} \nu^2 \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

Perfect Match with Experiment

- Energy of oscillator discrete $E_n = n h \nu$
- A new universal constant is born h
- Indication of a new occupation statistics.
- With h quantum age came into being !
- But, h is quantum of which Physical Quantity ?

$$h = 6.62607015 \times 10^{-34} \text{ J-s}$$

QUANTUM of
ACTION !!



An Act of Desperation.....

*“Kurz zusammengefasst, kann ich die ganze Tut nur als einen
Akt der Verzweiflung bezeichnen.”*

*“In summary, I can only characterize the entire work as an
act of desperation.”*

- Max Planck in 1931

Heat Capacity of Solids :

A simple single frequency model.

Dulong-Petit : $U = 3N_A k_B T$ $c_v = 3N_A k_B$

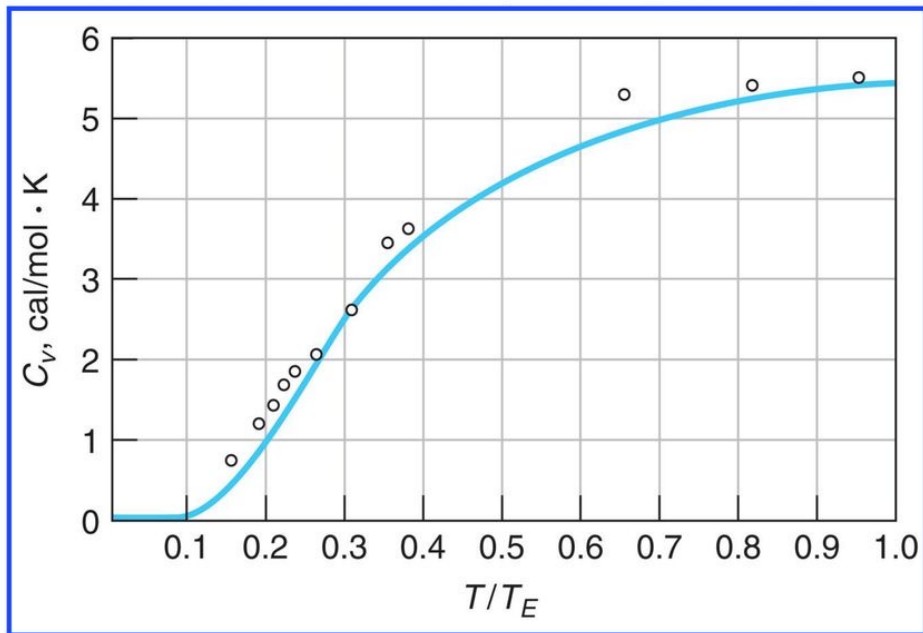
$$U = N_A \frac{3 h \nu}{e^{h\nu/k_B T} - 1}$$

$$c_v = 3N_A k_B \frac{x^2 e^x}{(e^x - 1)^2}$$

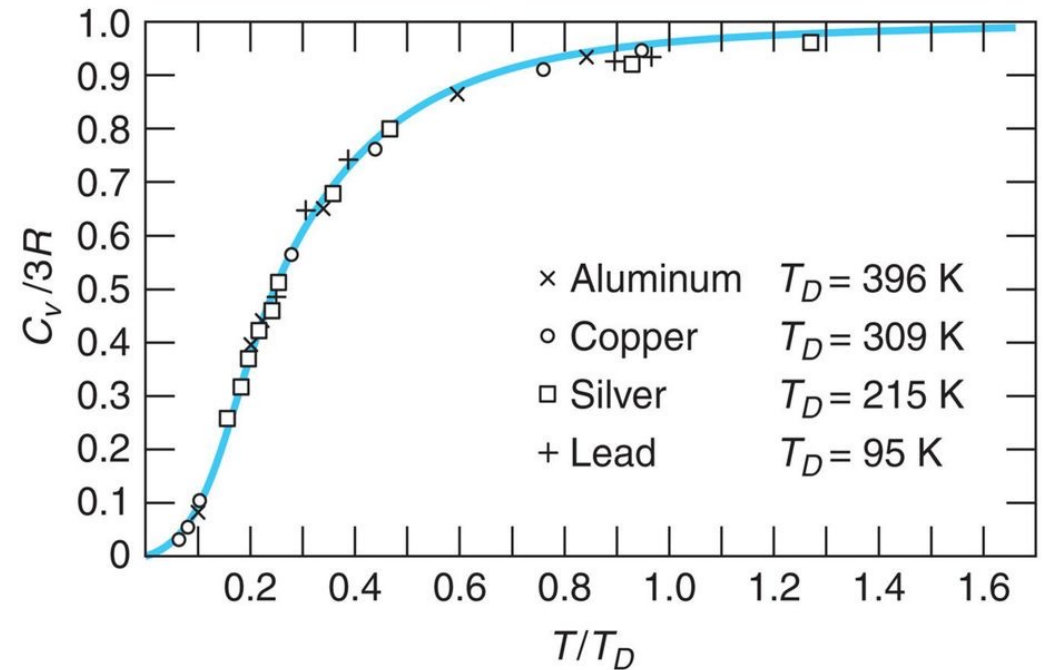
$$x \equiv \frac{h\nu}{k_B T} \equiv \frac{T_E}{T}$$

Einstein Model: C_v for Diamond

Einstein, Annalen der Physik 22 (4), 180 (1907)

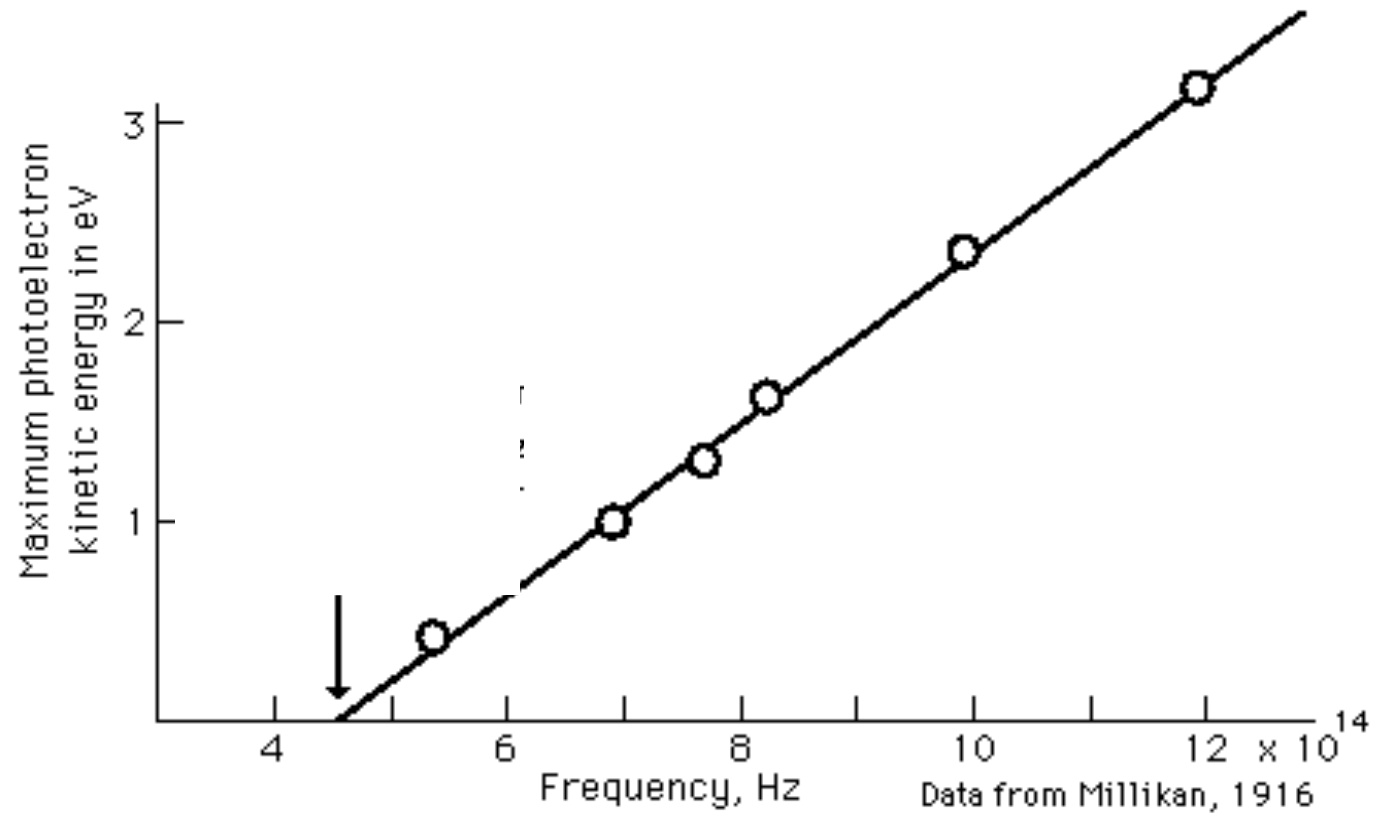
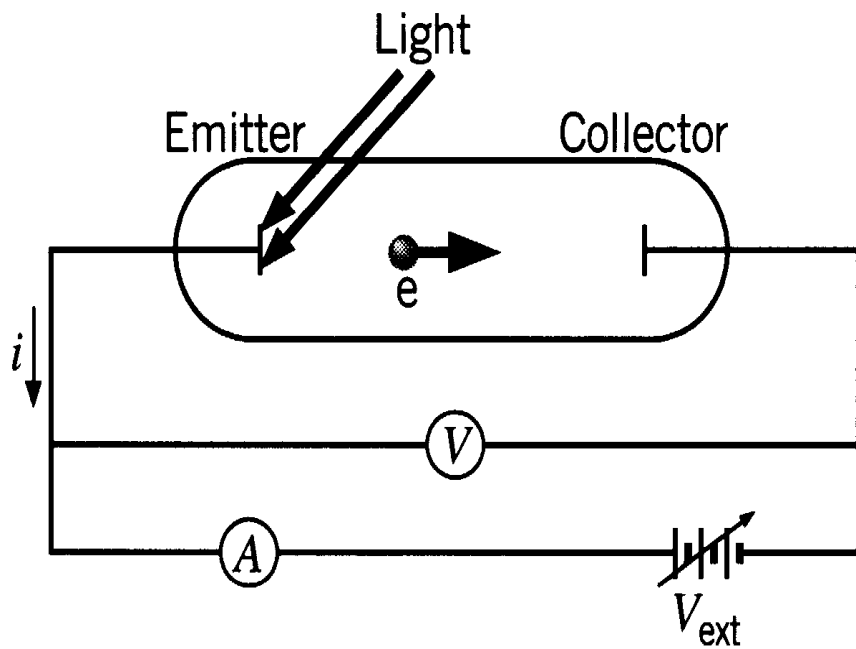


Debye Model of Heat Capacity of Solids



Photoelectric Effect :

Light consists of packet of energy : $h\nu$



Potassium : No emission below 2eV

What did Photoelectric Effect prove ?

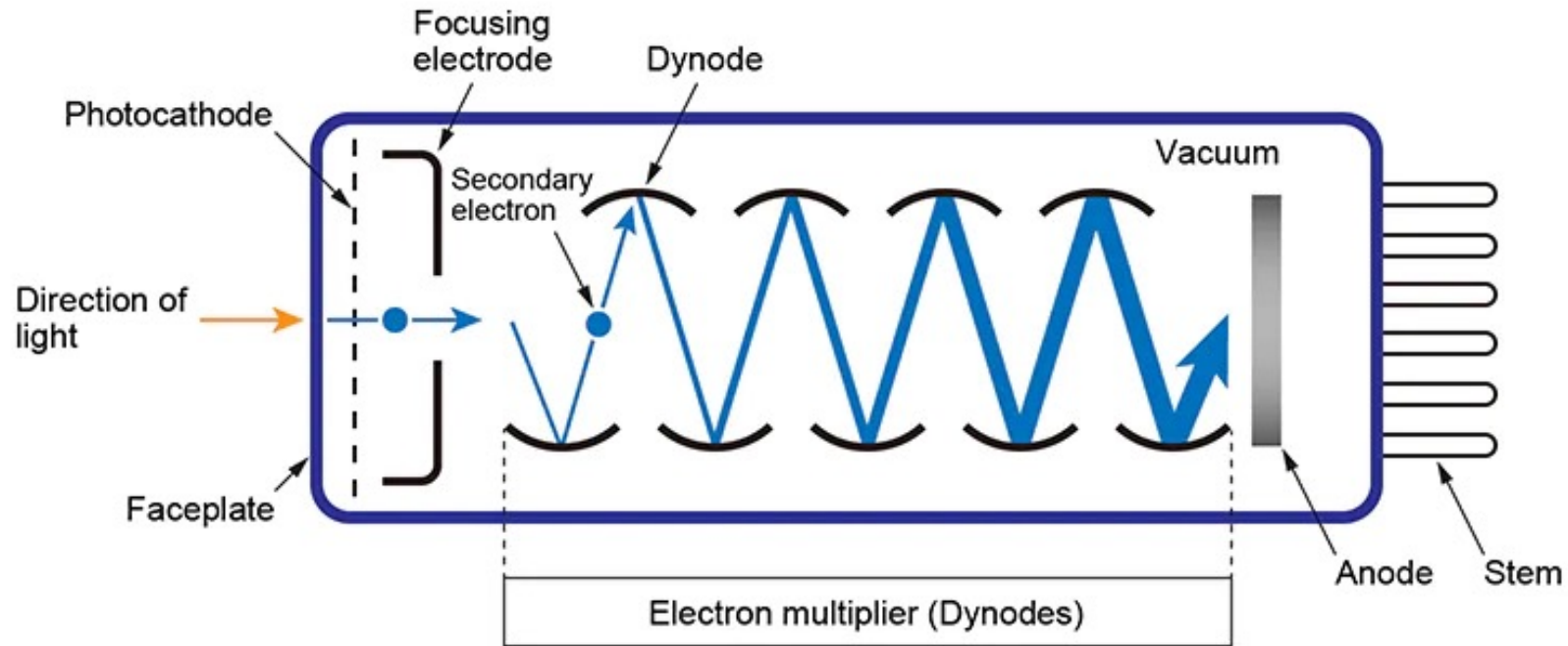
Light comes as 'packets of energy' : $h\nu$

Context: Light interacting with matter

Annus Mirabilis papers of Einstein

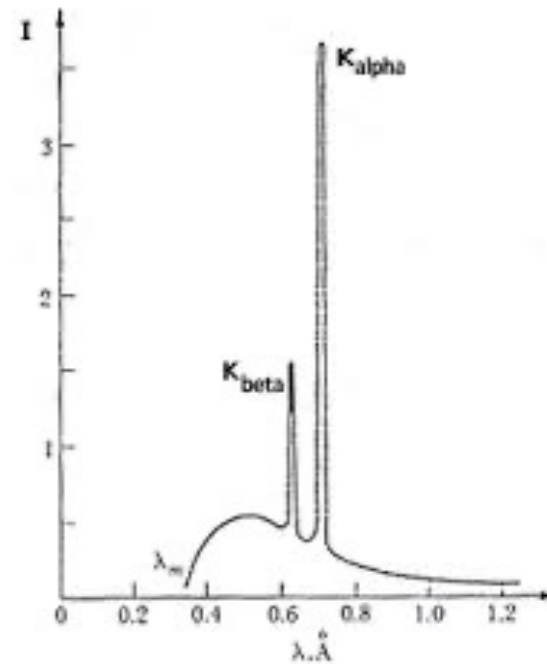
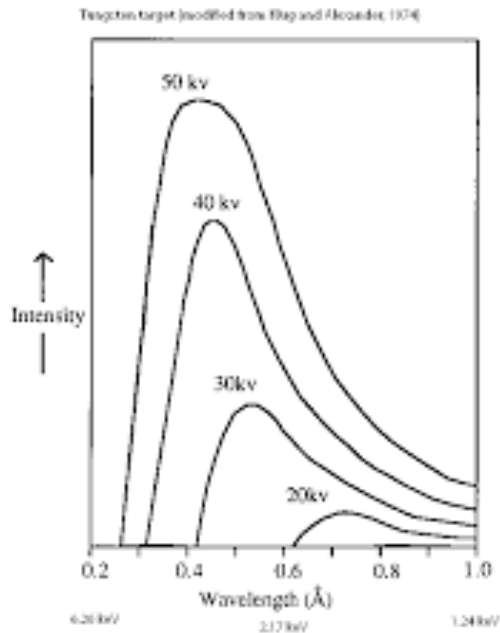
$\left\{ \begin{array}{l} \textit{Brownian Motion} \\ \textit{Specail Relativity} \\ E = mc^2 \end{array} \right.$

Photomultiplier Tube (PMT)

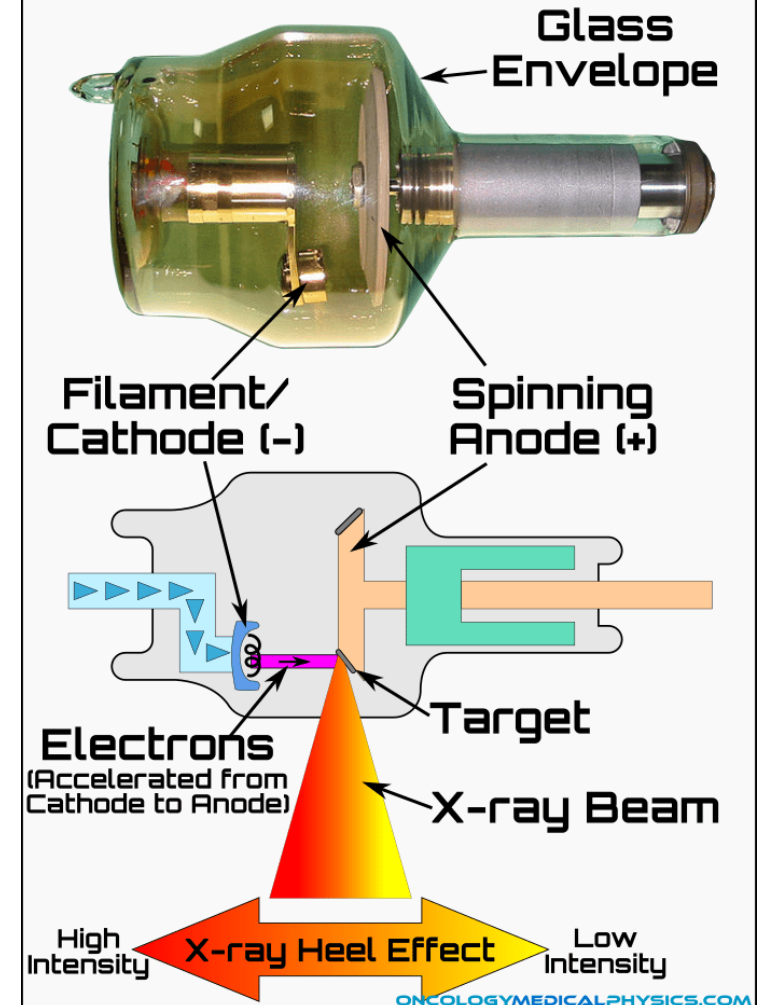


Inverse Photo-electric Effect ?

Generating X-Rays

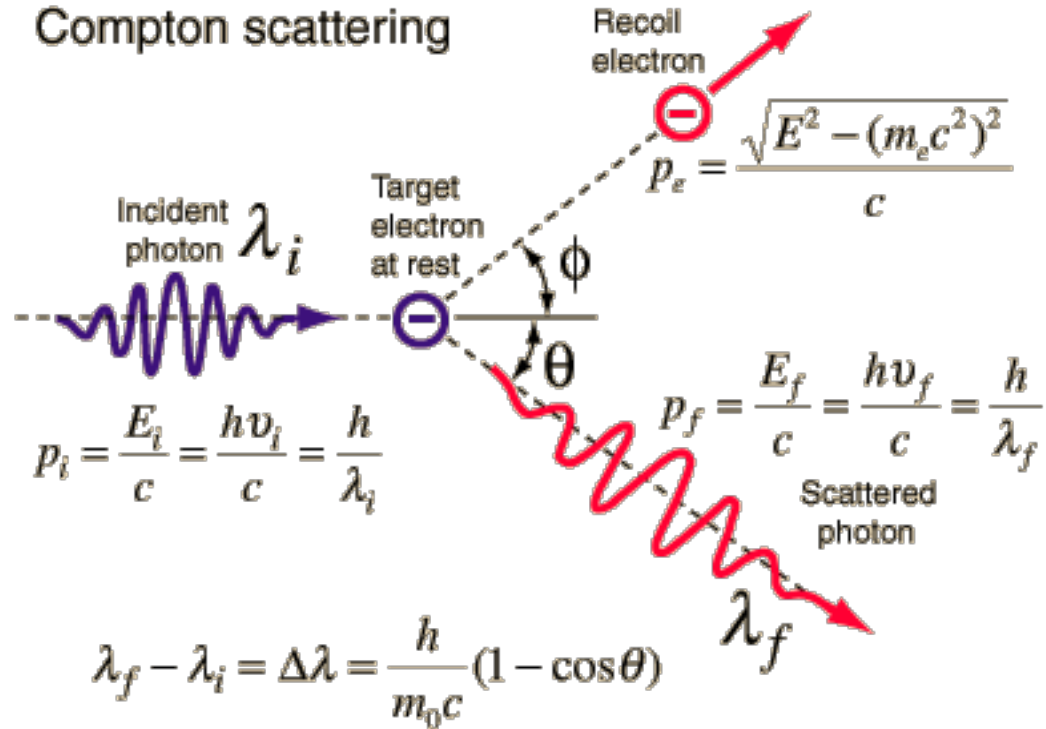


X-ray Tube Design



Photon as a particle ? Need solid proof?

Compton Effect (1923) Nobel in 1927



$$m_e c^2 + h\nu_i = h\nu_f + \sqrt{(m_e c^2)^2 + (p_e c)^2}$$

$$\vec{p}_i = \vec{p}_f + \vec{p}_e$$

$$p_e^2 = (\vec{p}_i - \vec{p}_f) \cdot (\vec{p}_i - \vec{p}_f) = p_i^2 + p_f^2 - 2p_i p_f \cos\theta$$

$$(p_e c)^2 = (h\nu_i)^2 + (h\nu_f)^2 - 2h^2 \nu_i \nu_f \cos\theta$$

$$(p_e c)^2 = (h\nu_i)^2 + (h\nu_f)^2 - 2h^2 \nu_i \nu_f + 2m_e c^2 (h\nu_i - h\nu_f)$$

$$\frac{1}{h\nu_f} - \frac{1}{h\nu_i} = \frac{1}{m_e c^2} (1 - \cos\theta)$$

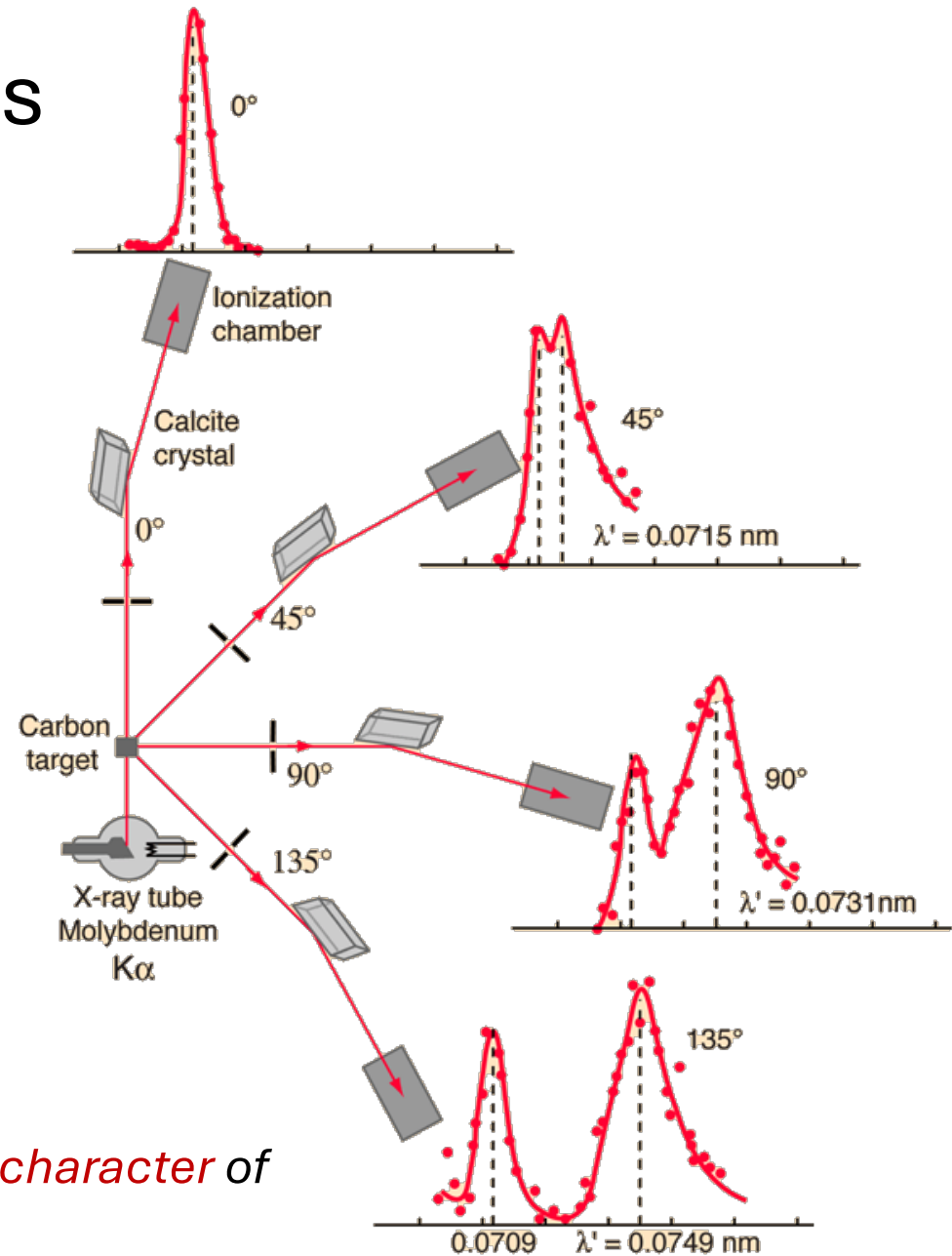
$$\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos\theta)$$

- Shift $\Delta\lambda$ is independent of λ .
- Lighter the target particle larger is the shift.
- For electron, the Compton wavelength is $2.42 \times 10^{-3} \text{ nm}$.

Compton's Experimental Results

- For convenient detection, X-rays (0.1 to 0.1) convenient.
- Compton used 0.0709 nm (Molybdenum K_{α}). The first peak is as per classical expectation due to Rayleigh scattering.
- The Compton shift increases with scattering angle with excellent agreement with prediction.

*Though wavelength is measured, it is the most convincing demonstration of **particle-like character** of **Electromagnetic radiation**.*



In Summary,

Mounting evidence of EM radiation and waves behave like packets of energy or particles :

- Blackbody Radiation energy density spectrum (Oscillators)
- Specific Heat of solids: T- dependence. (Oscillators)
- Photoelectric Effect
- X-ray production
- Compton Scattering

Above all, birth of Planck's Constant

Do particles also behave like waves?