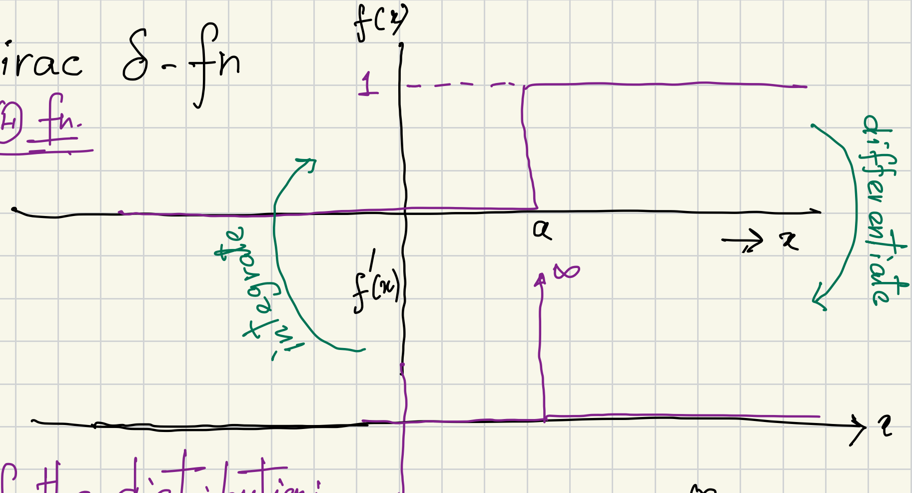
Lecture-2 05/01/24

- To introduce preliminary ideas regarding Dirac δ-function and Fourier series for later use. (i.e. Tutorial part of Lecture-1).

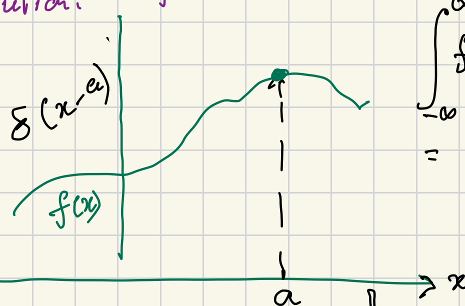
Dirac δ-fn

Derivative of Θ fn.



Integral of the distribution:

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-a) = ?$$



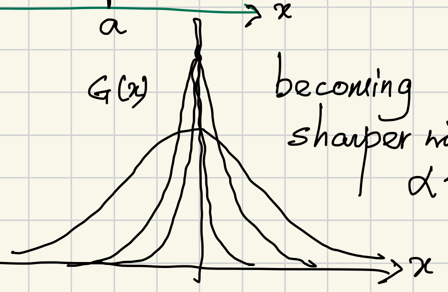
$$\int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx = f(a)$$

Gaussian Sequence:

$$G(x) = \frac{\alpha}{\sqrt{\pi}}\, e^{-\alpha x^2}$$

$$G(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-x^2/2\sigma^2}$$

$\lim_{\sigma \to 0}$



becoming sharper with $\alpha \uparrow$

sinc f^n representation

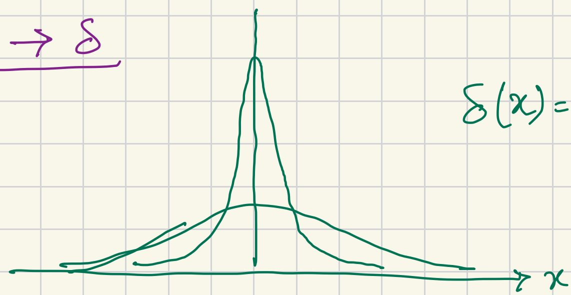
$$\lim_{g \to \infty} \frac{1}{\pi} \frac{\sin gx}{x}$$

or,
$$\frac{\sin gx}{x} = \frac{1}{2} \int_{-g}^{+g} e^{ikx} \, dk$$

$$\boxed{\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \, dk}$$

Most important Representation for us in this course.

Lorentzian $\to \delta$



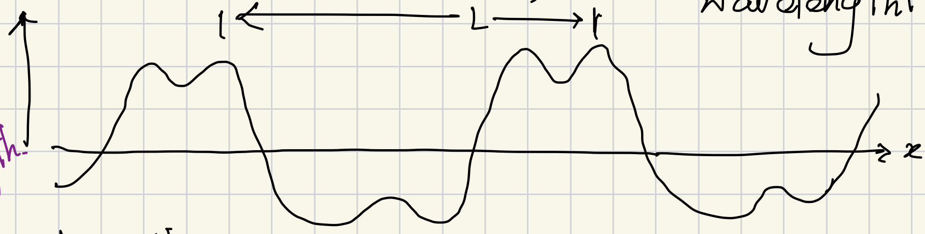
$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\pi} \left(\frac{\epsilon}{x^2 + \epsilon^2} \right)$$

$$\delta(-x) = \delta(x)$$

$$\delta(ax) = \frac{1}{|a|} \delta(x)$$

$$\delta(x-a) = \begin{cases} 0, & \text{every.} \\ \infty, & x = a \end{cases} \quad \text{s.t} \quad \int_{-\infty}^{\infty} \delta(x-a)\, dx = 1$$

Fourier Series : Only sin & cos have wavelength.



a periodic f(x) function but no wavelength.

Wavelength $\qquad \sin\frac{2\pi x}{\lambda} \qquad \cos\frac{2\pi x}{\lambda}$

Choose all those λ_s whose integral multiple fits the periodicity $\boxed{k = \frac{2\pi}{\lambda}}$

$\boxed{m\lambda = L}$ Periodicity

$m = 0, 1, 2, \ldots$

$k_m = \frac{2\pi}{L} m$

$$\boxed{f(x) = \sum_k A_k\, e^{ikx}}$$

complex

$$\underbrace{\frac{1}{L}\int_0^L e^{-ik'x} f(x)\,dx}_{} = \sum_k \underbrace{\frac{1}{L}\int_0^L A_k\, e^{i(k-k')x}\,dx}_{}$$

$$= A_{k'}$$

$k = \frac{2\pi}{L} m$

$k' = \frac{2\pi}{L} m'$

Given f(x), I can find all A_k needed.

$\boxed{f(x) = 3\sin 2x}$

Either Table

x	$f(x)$
0	
0.001	–
⋮	–
	–
	–

Or, Fourier Coefficients

λ	A_λ
π	3

$\frac{2\pi}{\lambda} = 2$

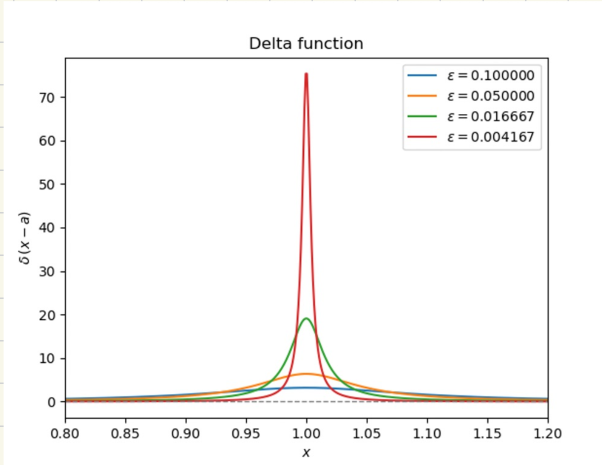
$\Rightarrow \lambda = \pi$

Post-Class Additions

ynm/2-14

Lorentzian Sequence:

$$L(x) = \frac{1}{\pi}\,\frac{\epsilon}{\epsilon^2 + x^2}$$



Consider:

$$\lim_{\epsilon \to 0} L(x) = \lim_{\epsilon \to 0} \frac{1}{\pi \epsilon}\left(\frac{1}{1 + x^2/\epsilon^2}\right)$$

Sinc function sequence.

$$S(x) = \frac{\sin\left(\frac{x}{\epsilon}\right)}{\pi x}$$

$$\delta(x) = \lim_{\epsilon \to 0} \frac{\sin(x/\epsilon)}{\pi x}$$