Assignment 1: Applications of Integration, Pappus Theorem

1. Compute area of the region bounded by the curves $y = x^3$ and $y = 3x - 2$.

2. Sketch the graphs of $r = f(\theta) = \cos(2\theta)$ and $r = g(\theta) = \sin(2\theta)$. Find the values of $\theta \in [0, \pi]$ such that $f(\theta) = g(\theta)$.

3. Find the area of the region that lies inside both the curves $r = 3,\ r = 6\cos 2\theta$.

4. Let C denote the circular disc of radius b centered at $(a, 0)$ where $0 < b < a$. Find the volume of the torus that is generated by revolving C around the y-axis using

   (a) the Washer Method

   (b) the Shell Method.

5. Find the volume of the solid generated by revolving the region bounded by the lines $y = 0$, $x = 4$ and the curve $y = \sqrt{x}$ about the line $x = 6$.

6. Consider the curve C defined by $x(t) = \cos^3 t,\ y(t) = \sin^3 t,\ 0 \le t \le \frac{\pi}{2}$.

   (a) Find the length of the curve.

   (b) Find the area of the surface generated by revolving C about the x-axis.

   (c) If $(\overline{x}, \overline{y})$ is the centroid of C then find $\overline{y}$.

7. Find the centroid of the semicircular arc $(x - r)^2 + y^2 = r^2,\ r > 0$ described in the first quadrant. If this arc is rotated about the line $y + mx = 0,\ m > 0$, determine the generated surface area A and show that A is maximum when $m = \pi/2$.

8. Consider an equilateral triangle of each side 2 cm with its base on the x-axis and the triangle is on the positive side of the y-axis. Compute the volume of the solid generated by revolving the triangle about the line $y = -2$.

Assignment 2 : Vectors, Curves, Surfaces, Vector Functions

1. Find a parametric equation of the line of intersection of the planes $x - 2z = 3$ and $y + 2z = 5$.

2. Find an equation of the plane that passes through the point $(6, 0, 0)$ and contains the line $x = 4 - 2t, y = 2 + 3t, z = 3 + 5t$.

3. Determine the equation of a cone with vertex $(0, -a, 0)$ and base curve $x^2 = 2y,\ z = h$.

4. The velocity of a particle moving in space is $\frac{d}{dt}c(t) = (\cos t)\vec{i} - (\sin t)\vec{j} + \vec{k}$. Find the particle's position as a function of t if $c(0) = 2\vec{i} + \vec{k}$. Also find the angle between its position vector and the velocity vector.

5. Show that $c(t) = \sin(t^2)\vec{i} + \cos(t^2)\vec{j} + 5\vec{k}$ has constant magnitude and is orthogonal to its derivative. Is the velocity vector of constant magnitude?

6. Find the point on the curve $c(t) = (5\sin t)\vec{i} + (5\cos t)\vec{j} + (12t)\vec{k}$ at a distance 26π units from $(0, 5, 0)$ along the curve in the direction of increasing arc length.

7. Reparametrize the following curves in terms of arc length.

   (a) $c(t) = \frac{t^2}{2}\vec{i} + \frac{t^3}{3}\vec{k},\quad 0 \leq t \leq 2,$

   (b) $c(t) = 2\cos t\ \vec{i} + 2\sin t\ \vec{j},\quad 0 \leq t \leq 2\pi$

8. Show that the parabola $y = ax^2,\quad a \neq 0$ has its largest curvature at its vertex and has no minimum curvature.