Part VI

Looking into the future

Previous lecture

- Defined the unitary property of operators. If an operator $\hat{O}$ is unitary then $\hat{O}^{\dagger}\hat{O} = I$, where $\hat{O}^{\dagger}$ is the adjoint of $\hat{O}$.
- Expressed unitary nature of operators in terms of the corresponding matrices.
- For a matrix A, its adjoint is $A^{\dagger} = A^{T*}$ and if A is unitary then $A^{\dagger}A = I$.
- It was made clear that a quantum gate should be represented by a unitary operator.

Previous lecture

- Discussed parallelism offered by a quantum computer.
- Parallelism in a quantum computer arises because of the possibility of processing a large number of qubits together by expressing them as a linear superposition.
- Reduction in processing time by a quantum computer was discussed using an example from arithmetic.
- Finally, Deutsch algorithm demonstrated how a quantum algorithm speeds up processing and learning a property of a given set of functions.

Lecture 36

Introduction to Quantum Computing-II (No cloning theorem and quantum teleportation)

What would happen if we could clone (make a perfect copy) a quantum state?

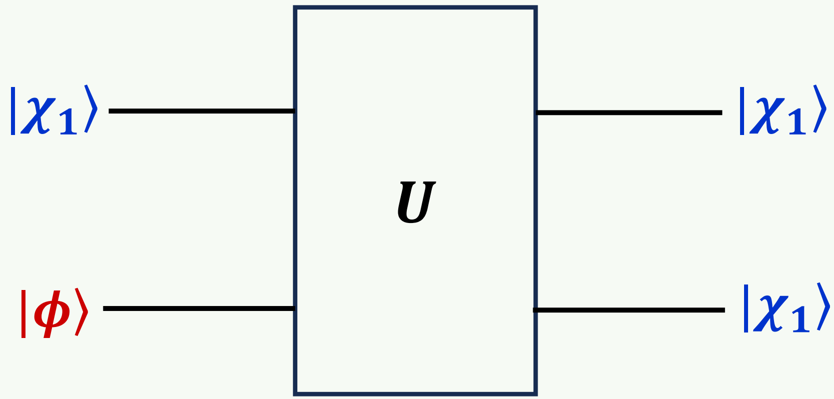
- Imagine if we could clone a quantum state

$$|\psi\rangle = \sum_i c_i |\phi_i\rangle \quad ,$$

where $|\phi_i\rangle$ are the eigenstates of an observable O with operator $\hat{O}$.

- We would then make multiple copies of the state and measure O on all copies.
- Since $|\psi\rangle$ collapses to different states $|\phi_i\rangle$ on each measurement, coefficients $|c_i|^2$ will all be measured by Born's rule.
- This way we would be able to get the wavefunction.
- Can this be done?

How is cloning to be done?



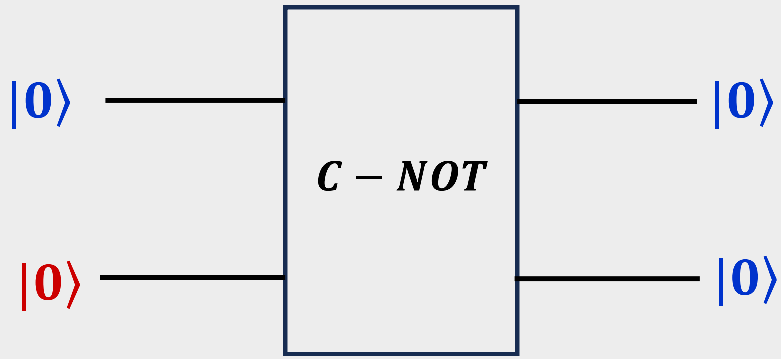
- In the cloning gate shown above

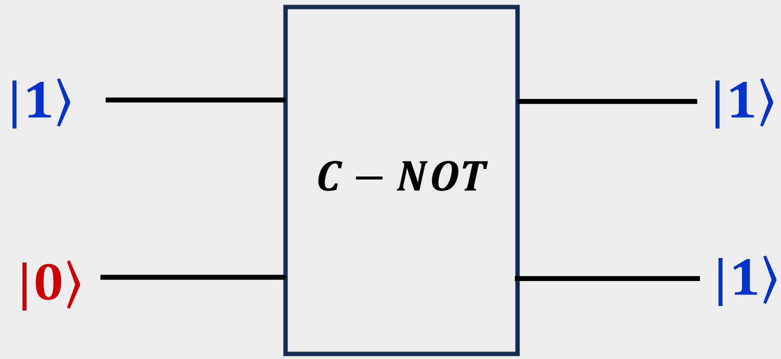
$$U|\chi_1\rangle|\phi\rangle = |\chi_1\rangle|\chi_1\rangle$$

- This means state $|\chi_1\rangle$ has been reproduced.

Examples of cloning using C-NOT gate

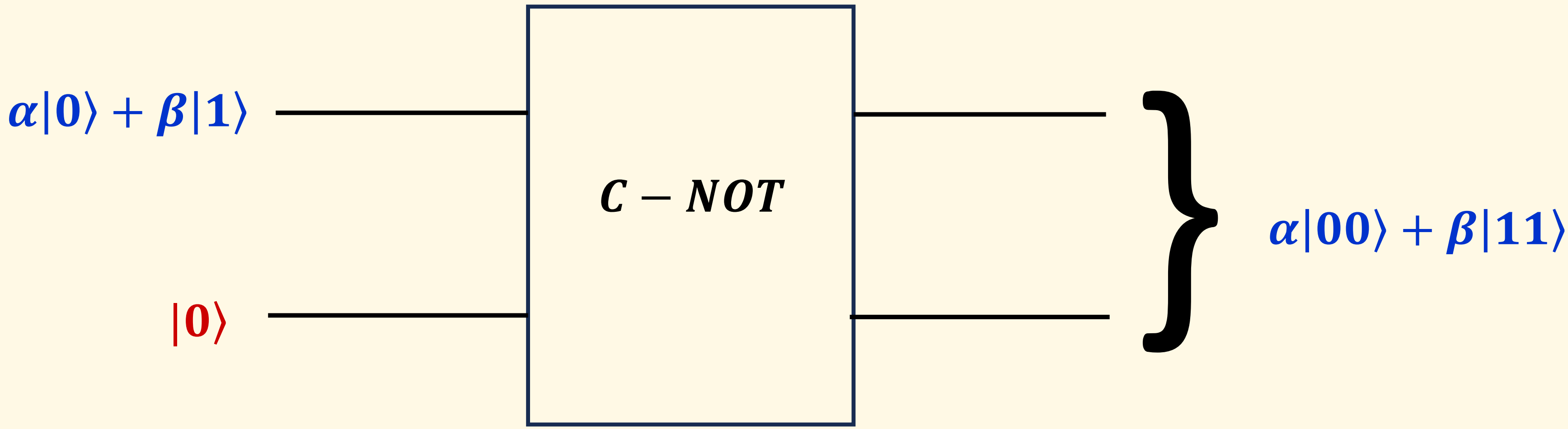
$$U(|x\rangle|y\rangle) = (|x\rangle|x \oplus y\rangle)$$





What happens when a mixed state is an input to C-NOT gate?

$$U(\alpha|0\rangle + \beta|1\rangle)|0\rangle = U(\alpha|00\rangle + \beta|10\rangle) = \alpha|00\rangle + \beta|11\rangle$$



- Output of C-NOT gate is an entangled state; it cannot be written as a product of one state for the upper port and one for the lower port .
- Cloning of state $\alpha|0\rangle + \beta|1\rangle$ has not been done.

No-cloning theorem

Statement: It is impossible to construct a gate that clones (reproduces) general qubits; Furthermore, we can clone two orthogonal states but not a general superposition of these.

Proof: Take two states $|\chi_1\rangle$ and $|\chi_2\rangle$. Then

$$U|\chi_1\rangle|\phi\rangle| = |\chi_1\rangle|\chi_1\rangle \quad \text{and} \quad U|\chi_2\rangle|\phi\rangle| = |\chi_2\rangle|\chi_2\rangle$$

Calculate

$$\langle\phi|\chi_1|U^{\dagger}U|\chi_2\rangle|\phi\rangle = \langle\chi_1|\langle\chi_1|\chi_2\rangle|\ \chi_2\rangle = \langle\chi_1|\chi_2\rangle^2$$

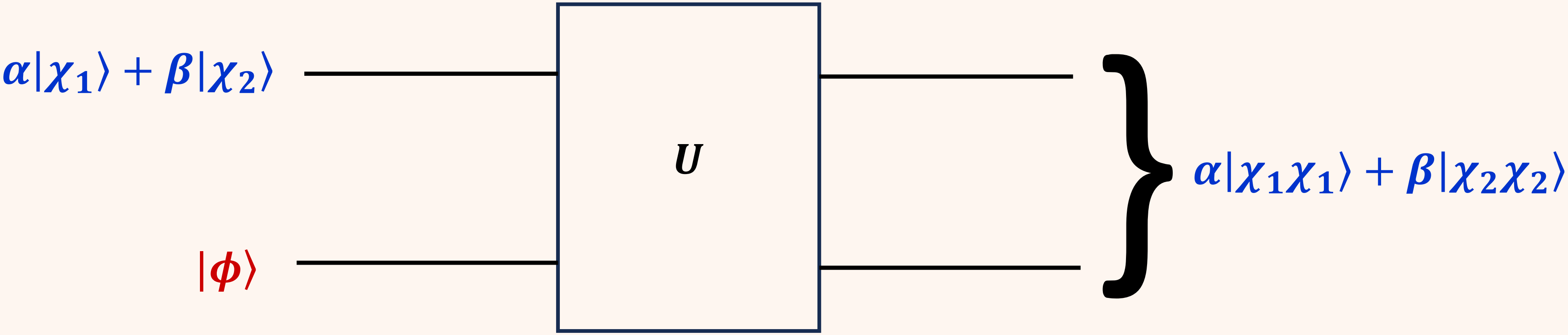
Since U is a unitary operator we have $U^{\dagger}U = I$ so that

$$\langle\phi|\chi_1|U^{\dagger}U|\chi_2\rangle|\phi\rangle = \langle\phi|\langle\chi_1|\chi_2\rangle|\ \phi\rangle = \langle\chi_1|\chi_2\rangle$$

This gives $\langle\chi_1|\chi_2\rangle = \langle\chi_1|\chi_2\rangle^2 \Rightarrow \textbf{either } |\chi_1\rangle = |\chi_2\rangle \textbf{ or } \langle\chi_1|\chi_2\rangle = 0$

What happens when a mixed state is an input?

$$U(\alpha|\chi_1\rangle + \beta|\chi_2\rangle)|\phi\rangle = U(\alpha|\chi_1\phi\rangle + \beta|\chi_2\phi\rangle) = \alpha|\chi_1\chi_1\rangle + \beta|\chi_2\chi_2\rangle$$



- Output is an entangled state; it cannot be written as a product of one state for the upper port and one for the lower port .
- Cloning of state $\alpha|\chi_1\rangle + \beta|\chi_2\rangle$ has not been done.

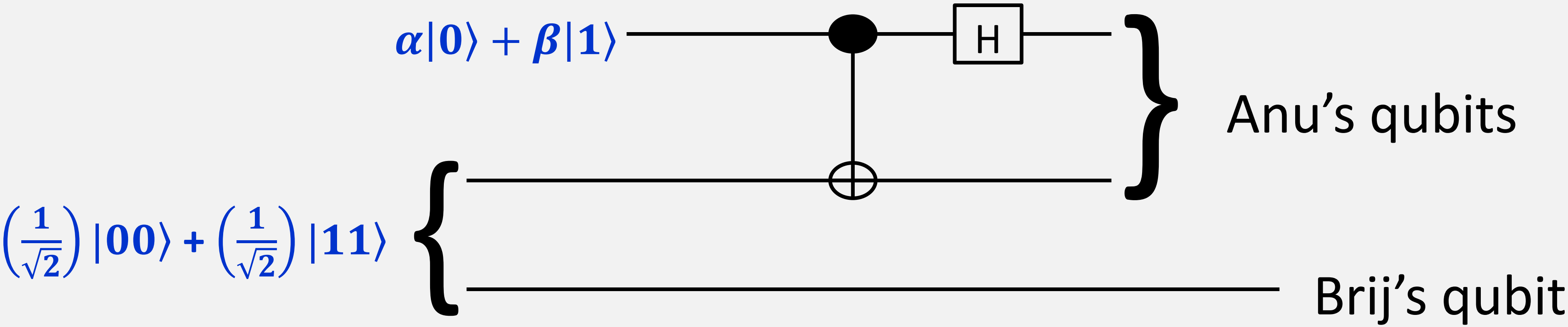
Implications of no cloning theorem

- Our present programmable computers are based on ability to copy. Quantum computers have to be designed differently since copying of quantum states is not possible.
- It also means that if a quantum state is being transferred , it cannot be copied and read by someone else along the way.

Question: Since classical world is an approximation to the quantum world, how is copying possible classically?

Quantum teleportation

We consider the three qubit circuit



Input qubit

$$(\alpha|0\rangle + \beta|1\rangle)\left[\left(\frac{1}{\sqrt{2}}\right)|00\rangle + \left(\frac{1}{\sqrt{2}}\right)|11\rangle\right] = \frac{\alpha}{\sqrt{2}}|00\rangle|0\rangle + \frac{\alpha}{\sqrt{2}}|01\rangle|1\rangle + \frac{\beta}{\sqrt{2}}|10\rangle|0\rangle + \frac{\beta}{\sqrt{2}}|11\rangle|1\rangle$$

Quantum teleportation……

Input qubit

$$\frac{\alpha}{\sqrt{2}}|00\rangle|0\rangle + \frac{\alpha}{\sqrt{2}}|01\rangle|1\rangle + \frac{\beta}{\sqrt{2}}|10\rangle|0\rangle + \frac{\beta}{\sqrt{2}}|11\rangle|1\rangle$$

Apply C-NOT to Anu's qubits to get

$$\frac{\alpha}{\sqrt{2}}|00\rangle|0\rangle + \frac{\alpha}{\sqrt{2}}|01\rangle|1\rangle + \frac{\beta}{\sqrt{2}}|11\rangle|0\rangle + \frac{\beta}{\sqrt{2}}|10\rangle|1\rangle$$

Now the Hadamard gate acts only on the first bit. Applying this and doing a bit of algebra gives the final state to be

$$\frac{1}{2}|00\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}|01\rangle(\alpha|1\rangle + \beta|0\rangle)$$
$$+\frac{1}{2}|10\rangle(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2}|11\rangle(\alpha|1\rangle - \beta|0\rangle)$$

Quantum teleportation……

- The final state

$$\frac{1}{2}|\mathbf{00}\rangle(\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle) + \frac{1}{2}|\mathbf{01}\rangle(\boldsymbol{\alpha}|\mathbf{1}\rangle + \boldsymbol{\beta}|\mathbf{0}\rangle)$$
$$+\frac{1}{2}|\mathbf{10}\rangle(\boldsymbol{\alpha}|\mathbf{0}\rangle - \boldsymbol{\beta}|\mathbf{1}\rangle) + \frac{1}{2}|\mathbf{11}\rangle(\boldsymbol{\alpha}|\mathbf{1}\rangle - \boldsymbol{\beta}|\mathbf{0}\rangle)$$

- Anu now makes measurement of the two qubits. She gets one of the four $|\mathbf{00}\rangle$, $|\mathbf{01}\rangle$, $|\mathbf{10}\rangle$ or $|\mathbf{11}\rangle$.
- Anu tells Brij about the result of her measurement.
- Depending upon the qubits of Anu, Brij now knows which of the four $(\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle)$, $(\boldsymbol{\alpha}|\mathbf{1}\rangle + \boldsymbol{\beta}|\mathbf{0}\rangle)$, $(\boldsymbol{\alpha}|\mathbf{1}\rangle - \boldsymbol{\beta}|\mathbf{0}\rangle)$ or $(\boldsymbol{\alpha}|\mathbf{0}\rangle - \boldsymbol{\beta}|\mathbf{1}\rangle)$ he has received.
- By applying appropriate single qubit gate $\boldsymbol{I}, \boldsymbol{\sigma_x}, \boldsymbol{\sigma_y}$ **or** $\boldsymbol{\sigma_z}$ (depending upon the bits received from Anu), he can get back the original state $(\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle)$.
- **The state has been teleported using entangled qubits.**

What is being done physically?

- Two electrons/photons are prepared in entangled state $\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right)|\mathbf{00}\rangle + \left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right)|\mathbf{11}\rangle$ and one of them is sent to Brij. Particles do not have to be close to each other to remain entangled.
- The particle sent to Brij is not in a definite state because it is entangled with the particle left with Anu. The particles remain entangled no matter how far apart Anu and Brij are.
- Now Anu prepares another particle in a state $\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle$ that is to be teleported.

What is being done ....

- The two particles with Anu are now put through a C-NOT and Hadamard gate combination and the output state of three particles is superposition of states with all the particles entangled. One of these particles is with Brij.
- As soon as Anu makes a measurement, the particle with Brij gets into one of the states $(\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle)$, $(\boldsymbol{\alpha}|\mathbf{1}\rangle + \boldsymbol{\beta}|\mathbf{0}\rangle)$, $(\boldsymbol{\alpha}|\mathbf{1}\rangle - \boldsymbol{\beta}|\mathbf{0}\rangle)$ or $(\boldsymbol{\alpha}|\mathbf{0}\rangle - \boldsymbol{\beta}|\mathbf{1}\rangle)$. That is, information about $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ has reached Brij.
- After hearing from Anu about the measurement she made, Brij puts the state through appropriate gate to get the original state.

Matrix representation

- Write the eight column matrices for the basis $|000\rangle\ |001\rangle \ldots \ldots |110\rangle|111\rangle$
- Find the operation of the teleporter on each
- Write the 8×8 matrix for it
- Apply it on the state taken and check your answer

Two points about teleportation

(**From "Quantum Computation for Everyone" by C. Bernhardt**)

- As soon as Anu makes her measurement, Brij's qubit instantaneously jumps to one of the four state.

Does it mean that information has been transferred at infinite speed?

The answer is no because the information is not complete until Brij hears from Anu about the result of her measurement, and that flow of information is not instantaneous.

- By no-cloning theorem, there can be only one copy anytime during the process. Note that there is only one qubit in state $\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle$ at any time. Initially Anu has it and finally Brij has it.

Experimental realization of quantum teleportation

Published: 03 June 2007

Entanglement-based quantum communication over 144 km

R. Ursin, F. Tiefenbacher, T. Schmitt-Manderbach, H. Weier, T. Scheidl, M. Lindenthal, B. Blauensteiner, T. Jennewein, J. Perdigues, P. Trojek, B. Ömer, M. Fürst, M. Meyenburg, J. Rarity, Z. Sodnik, C. Barbieri, H. Weinfurter & A. Zeilinger

Nature Physics **3**, 481–486 (2007) | Cite this article

8720 Accesses | **772** Citations | **69** Altmetric | Metrics

Published: 05 September 2012

Quantum teleportation over 143 kilometres using active feed-forward

Xiao-Song Ma, Thomas Herbst, Thomas Scheidl, Daqing Wang, Sebastian Kropatschek, William Naylor, Bernhard Wittmann, Alexandra Mech, Johannes Kofler, Elena Anisimova, Vadim Makarov, Thomas Jennewein, Rupert Ursin & Anton Zeilinger

Nature **489**, 269–273 (2012) | Cite this article

16k Accesses | **434** Citations | **507** Altmetric | Metrics

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2022 to **Alain Aspect** **John F. Clauser** **Anton Zeilinger**

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Using refined tools and long series of experiments, **Anton Zeilinger** started to use entangled quantum states. Among other things, his research group has demonstrated a phenomenon called quantum teleportation, which makes it possible to move a quantum state from one particle to one at a distance.