Indian Institute of Technology Kanpur
Department of Physics
PHY114: Quantum Physics

HA #1/ ynm Due for discussion on **Jan 11, 2024** 05 Jan 2024

1. Universal Constants and Scales:
   a) Estimate the largest distance of the knowable universe? Is quantum mechanics relevant in large scale phenomena that we encounter in astrophysics? Give examples if you think QM is important for such studies.
   b) What is the smallest distance and shortest time that physics can meaningfully talk about (from the universal constants that we know today)? Which one constant would you share for possible contact or communication with an extra-terrestrial civilization?
   c) *On one side of a logarithmic scale* of length, locate the typical size of the following: atomic nucleus, atomic radius, benzene molecule, lattice constant of copper crystal, transistor in an advanced CPU of a cellphone, dia of a strand of DNA, a human neuron, diameter of a human hair, height of a six storied building, cruising height of a passenger plane.
   On the other side of the above scale, identify typical wavelength of γ- rays, X-rays, ultraviolet light, green light, infrared emitted by human body, 5G cellphone carrier waves, FM radio wave, AM radio wave.

2. Use of Dirac delta function $\delta(x)$: Evaluate the following integrals.

$$\int_{-\infty}^{\infty} x^2 \delta(-3x+6)\, dx$$

$$\int_{-\infty}^{\infty} e^{-x}\, \delta(x^2-4)\, dx$$

3. Black Body Radiation:

   a) Using Planck's Law of Black body radiation distribution, obtain the Stefan constant in terms of relevant constants and determine its value. (Given $\int_0^\infty \frac{x^3}{e^x-1} dx = \frac{\pi^4}{15}$)
   b) Similarly use Planck's distribution to obtain the relation $\lambda_{max} T = C$, where C is a constant and the law is popularly known as Wien's displacement law. Obtain the expression for C and determine its value.
   c) The diameter of the sun is 1.4×10^9 m and the surface temperature 5777K. Estimate the total radiant energy being given out by the sun per second per unit area. The development of solar panels on earth crucially depends on the radiation per unit area received on earth at mean distance of $1.5 \times 10^{11} m$ (varies by 1.7% since it is an elliptical orbit) and subtends an angle of 32^0. Estimate the 'Solar Constant' (in W/m^2) which is the energy received from the sun per unit time per unit area of the surface perpendicular the direction of propagation at the mean -earth-sun distance.

   d) Suppose Planck's constant was 10% larger than its current value, what would have been the change in radiation loss percentage and the new λ_{max} of the sun as a blackbody.

4. In the successful explanation of temperature dependence of specific heat capacity of solids, does the use of Planck's distribution signify involvement of 'particles? In what sense can the oscillations in solids be considered as particles? Do they participate in collisions, for example, as in Compton scattering?

5. Probability density and variance
   a) A classical bead is free to move and occupy any position on a semi-circular horizontal track of radius R with equal probability from $0 \leq \theta < \pi$, and zero elsewhere (Fig.HA#1.5). Find the probability density $\rho(\theta)$ and sketch it. Also find the variance of the probability.
   b) Find and sketch the probability density of its $x -$ position and the variance.

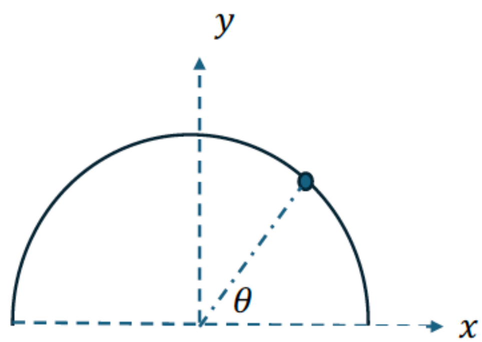

Figure HA#1.5

6. Find the complex Fourier coefficients in the expansion of the following function defined over length L.

$$f(x) = \sqrt{\frac{2}{L}} \sin \frac{4\pi x}{L}$$

7. A photon scatters from an electron at rest. The wavelength shift is observed to be triple the wavelength of the incident photon which scatters at 60^0. Calculate a) the wavelength of the incident photon, b) energy of the recoiling electron, and c) the angle at which the electron scatters.

8. Moore's Law and its limits:
   Look at the data of number of transistors per unit area in microprocessors.
   https://ourworldindata.org/grapher/transistors-per-microprocessor
   Gordon Moore (who founded Intel with Robert Noyce) in 1960s predicted that the number of transistors would double in the same area every two years (and also cost per transistor will become half in the same time frame). Semiconductor technology is showing a rough validation of this Law. In the current standard technology, the critical length called the node is 22 nm. Suppose Moore's Law continues to hold even with introduction of transistors based on spintronics, estimate by which year would you reach the fundamental limit of Compton wavelength of an electron would be reached.

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