REVIEW OF IMPORTANT DISTRIBUTIONS

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BERNOULLI DISTRIBUTION

- A random variable distributed according to Bernoulli distribution can take on two possible values:

$$X \in \{0, 1\}$$

- Distribution can be specified by a single parameter μ to be $p(X = 1) = \mu$
- We can also write the distribution as

$$p(X) = \mu^x (1 - \mu)^{1-x}$$

BINOMIAL DISTRIBUTION BIN(N,P)

- A Binomial distribution is sum of independent and identically distributed Bernoulli random variables
- The experiment consists of a sequence of n identical trials
- All possible outcomes can be classified into two categories, usually called success and failure
- The probability of an success, p, is constant from trial to trial
- The outcome of any trial is independent of the outcome of any other trial

BINOMIAL DISTRIBUTION BIN(N,P)

- The probability distribution of the random variable X is called a **binomial distribution,** and is given by the formula

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

where

- $n =$ the number of trials
- $x = 0, 1, 2, \dots n$
- $p =$ the probability of success in a single trial
- $q =$ the probability of failure in a single trial

$$E(x) = np, Var(x) = npq$$

EXAMPLE

- Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

NORMAL DISTRIBUTION

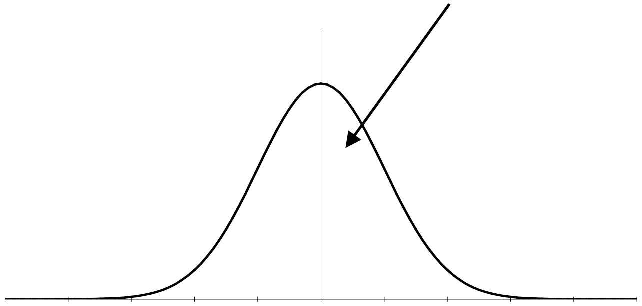
Many continuous variables are approximately normally distributed

- Error measurements
- Physical and mental properties of people
- Properties of manufactured products
- Daily revenues of investments

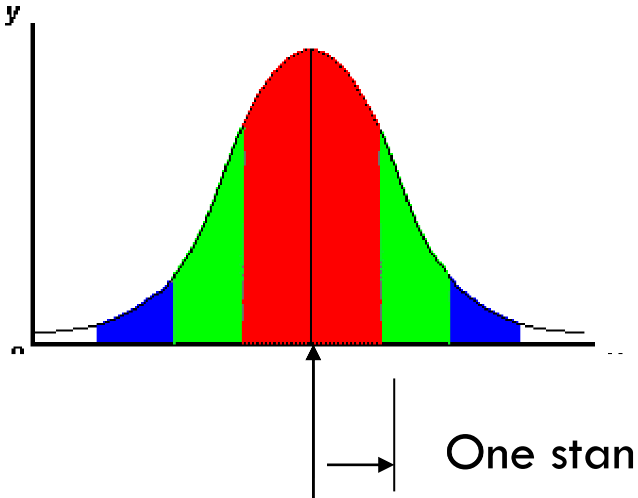
NORMAL DISTRIBUTION

Normal distribution is defined by density function

area under density function equals 1, area represents probability



expected value



Mean (μ)

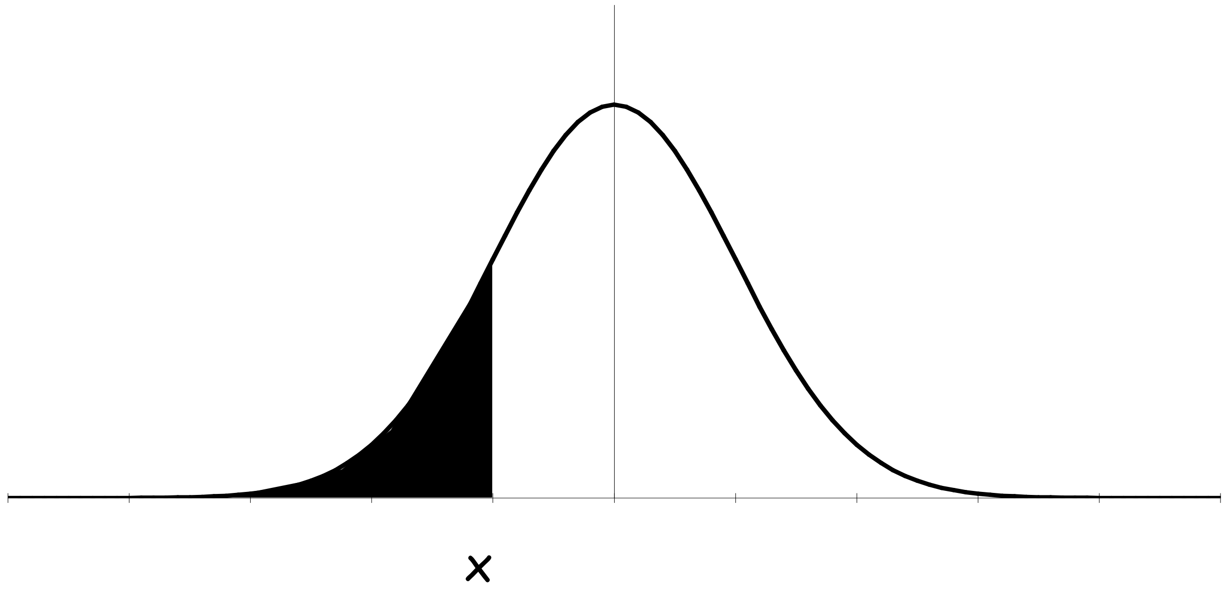
One standard deviation from the mean (σ)

CUMULATIVE PROBABILITY FUNCTION

- Cumulative function for x

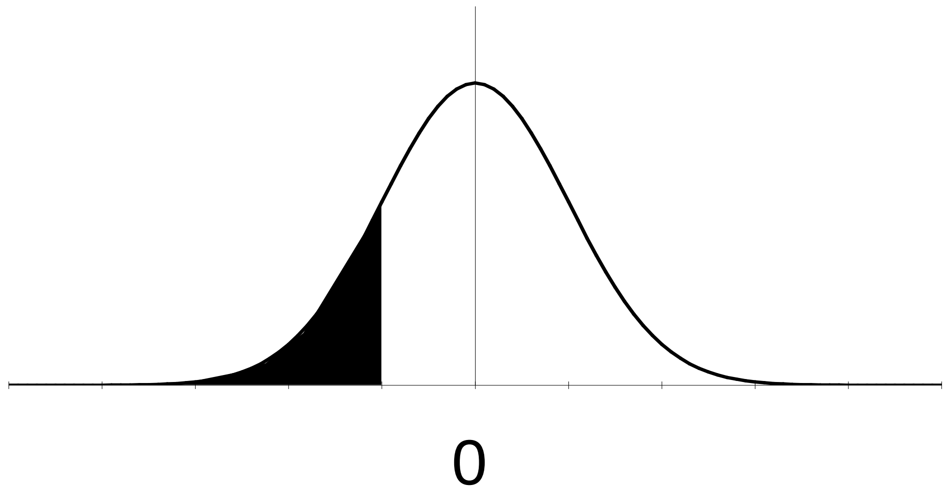
  = area to the left of x

  = probability to get at most x:



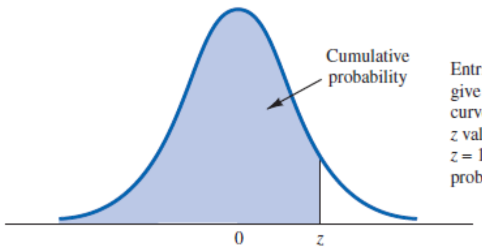
STANDARDIZED DISTRIBUTION N(0,1)

- Cumulative function values have been tabulated (in most statistics textbooks) for normal distribution with expected value 0 and standard deviation 1
- This distribution is called standardized distribution and is denoted by $N(0,1)$.



CUMULATIVE STANDARD NORMAL DIST. TABLE

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)

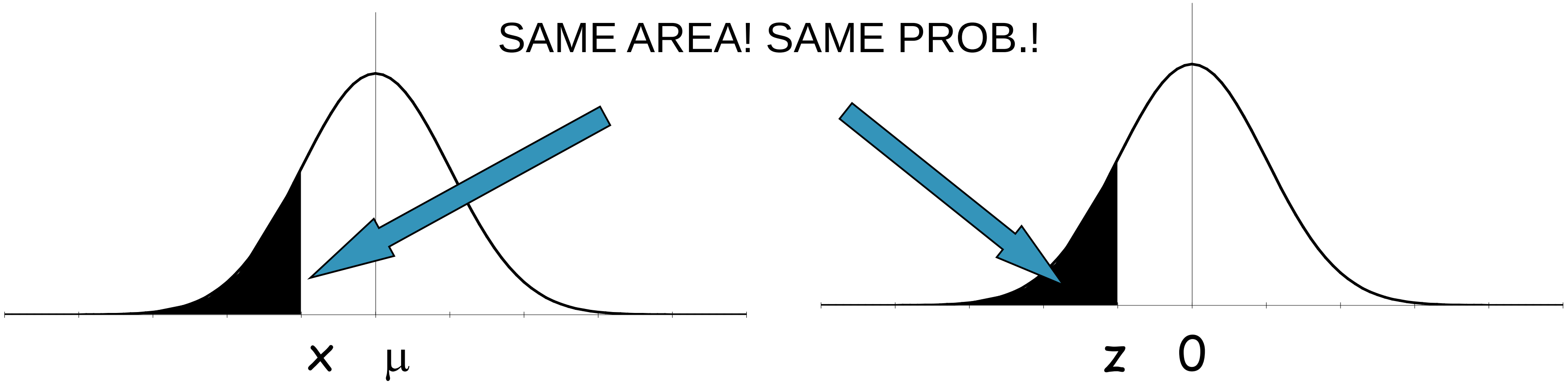


Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

STANDARDIZING

You can standardize any normal distribution $N(\mu, \sigma)$ variable to a standardized distribution $N(0,1)$ variable



$$z = \frac{x-\mu}{\sigma}$$

EXAMPLE

- On a recent English test, the scores were normally distributed with a mean of 74 and a standard deviation of 7. What proportion of the class would be expected to score between 60 and 80 points? Use normal table to answer this question.

GAUSSIAN DISTRIBUTION

- Gaussian distribution is also known as normal distribution
- The distribution is determined by two parameters: the mean μ and the variance σ^2.
- The probability density function is given by

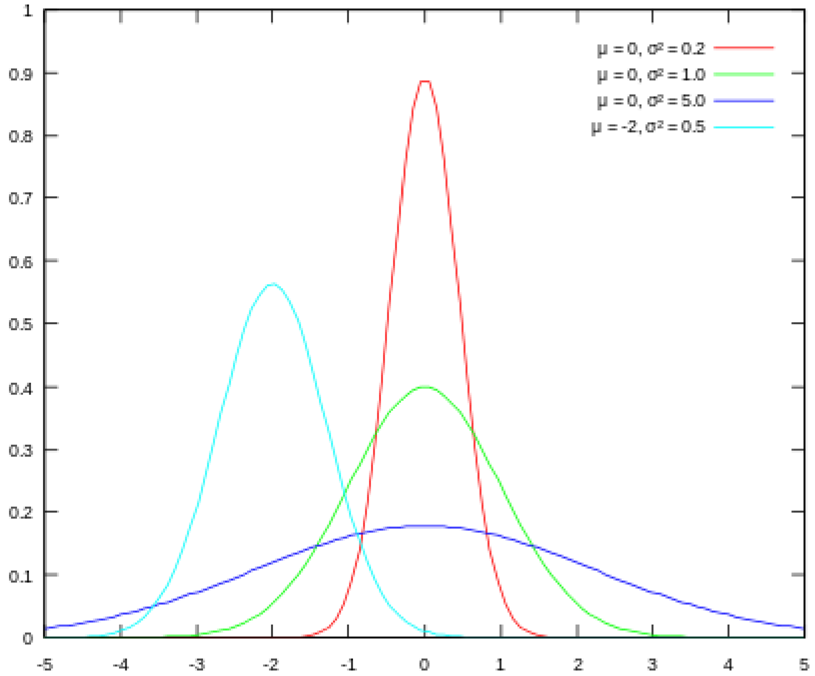
Causes pdf to decrease as distance from center increases

$$f(x) = N(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Normalized constant. Ensures that the distribution integrates to 1

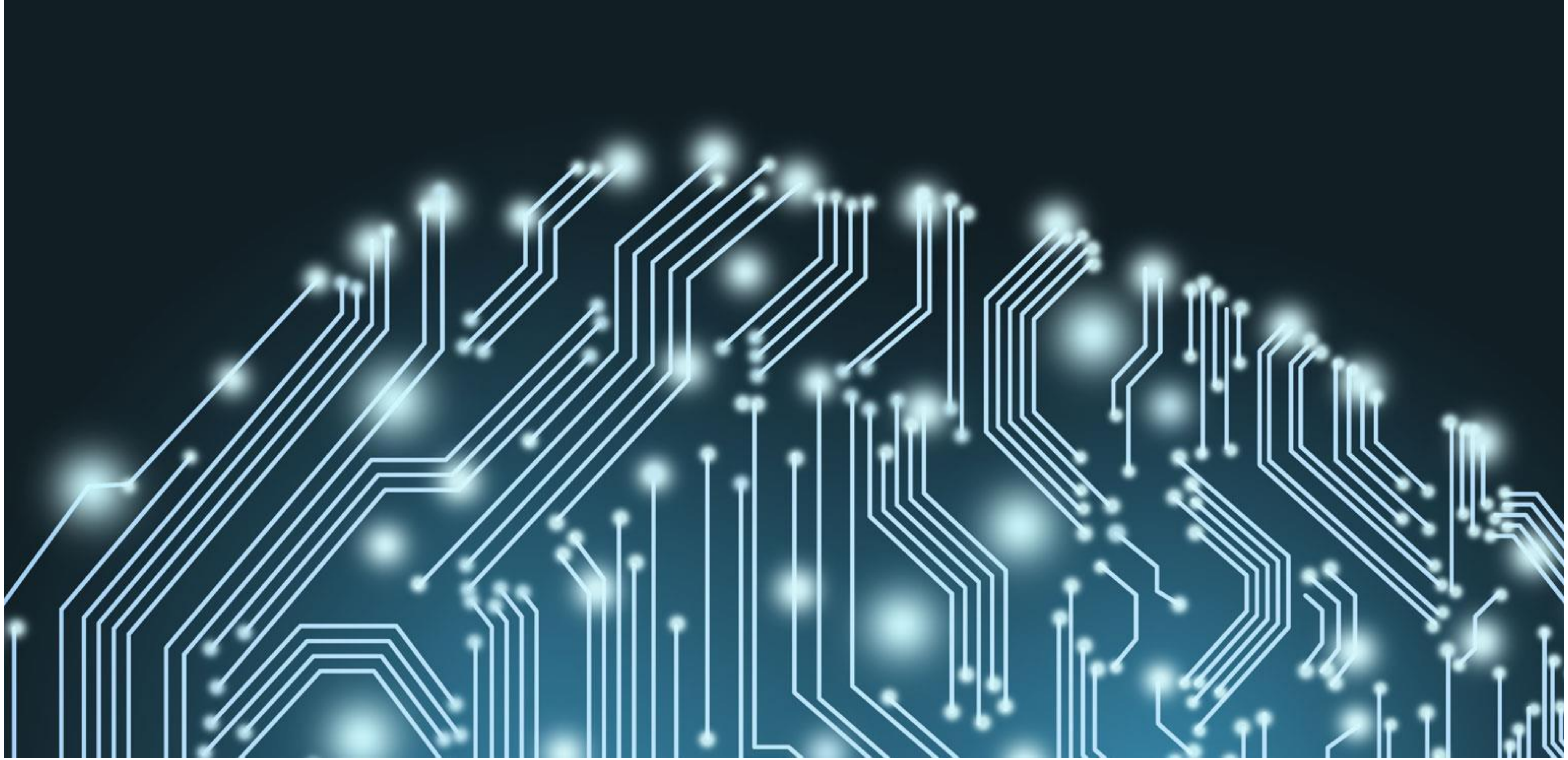
Controls width of the curve

GAUSSIAN DISTRIBUTION WITH DIFFERENT μ & Σ



IMPORTANT PROPERTIES OF GAUSSIANS

- All marginal of a Gaussian are again Gaussian
- Any conditional of a Gaussian is Gaussian
- The product of two Gaussians is again Gaussian
- Even the sum of two independent Gaussian random variables is a Gaussian



QUESTIONS?