

CSE 15

Introduction to Data Structures

Programming Assignment 5

In this project you will recreate the Dictionary ADT from pa3, but now based on a hash table instead of a linked list. You may start by reading section 18.4 on hash tables in the textbook DAPS (Carrano & Prichard 6th edition, p. 544-566.)

So far, we have seen two data structures that can form the basis of a Dictionary ADT, namely linked lists and binary search trees. In the linked list implementation, the worst case run time of the essential Dictionary operations (insert, delete, and lookup) were all in $\Theta(n)$, where n is the number of pairs in the dictionary. In the binary search tree implementation, the Dictionary operations all run in time $\Theta(\log(n))$, provided that the underlying binary search tree is “balanced”. It is possible to do better still using a hash table as the underlying data structure. In this implementation the Dictionary operations will run in constant time $\Theta(1)$. The catch is that this is the *average case* run time. The worst case run time of the Dictionary operations in a hash table implementation can be as bad as $\Theta(n)$, like a linked list.

Hash Tables

A hash table is simply an array that is used to store data associated with a set of keys. In our Dictionary ADT we wish to store a set of (key, value) pairs where `key` and `value` are C strings (i.e. null `'\0'` terminated `char` arrays). To simplify our discussion, we will suppose for the moment that the keys are integers rather than strings. It is not difficult to turn a string into an integer. Appropriate functions for doing this will be provided on the class webpage.

If the keys all happen to be in the range 0 to $N - 1$, and N is not too large, one can simply allocate an array of length N , and store the value v in array index k . This arrangement is called a *direct-address table*. Accessing a pair (k, v) then has constant cost. The problem with direct addressing is obvious: if N is too large, allocating an array of length N may be impractical, or even impossible. Think of an application in which `key` is a credit card account number, and `value` is an account balance. Such applications often have account numbers consisting of 16 or more decimal digits, which makes the universe U of *all possible keys* very large. Furthermore, the set S of *keys in use* may be so small relative to U , that most of the space allocated for the array is wasted. In the case of credit card numbers with 16 decimal digits, our universe of possible keys has on the order of one quadrillion numbers. That's a very long array, and even if everyone on the planet has an account, over 99.999% of the space is unused. For this reason, direct-address tables are rarely used to build a Dictionary ADT.

A hash table T requires much less storage than a direct address table. Specifically, the storage requirements for a hash table can be reduced to $\Theta(|S|)$, while maintaining the benefit that the dictionary operations run in (average case) constant time $\Theta(1)$. To do this we use a *hash function* $h()$ to compute the index (or *slot*) $h(k)$ where a given pair (k, v) will be stored. A suitable hash function must map the universe U of possible keys to the set $\{0, 1, \dots, m - 1\}$ of array indices:

$$h: U \rightarrow \{0, 1, \dots, m - 1\}$$

We say that the pair (k, v) *hashes to* the slot $h(k)$ in the hash table $T[0 \dots (m - 1)]$, and that $h(k)$ is the *hash value* of key k . The point of the hash function is to reduce the range of array indices that must be handled. Instead of $|U|$ indices, we need handle only m indices. Storage requirements are thereby reduced.

There is of course one problem: two keys may hash to the same slot. We call this situation a *collision*. Fortunately, there are effective techniques for resolving the conflict created by collisions. The ideal solution would be to avoid collisions altogether. We might try to achieve this goal through our choice of hash function $h()$. One possibility is to make $h(k)$ appear to be random, thus avoiding collisions by making them improbable. The very term “hash”, which evokes images of random mixing and chopping, captures the spirit of this approach. Of course, a hash function $h(k)$ must be deterministic in the sense that a given input k always produces the same output $h(k)$. Since $|U| > m$, and in general $|U|$ is *much larger* than m , there must be at least two keys that have the same hash value, and therefore avoiding collisions altogether is impossible. Thus, while a well-designed random looking hash function can minimize the number of collisions, we still need a method for resolving the collisions that do occur.

Several methods for resolving collisions will be discussed in class. The method that we will use in this project is called *chaining*, and is perhaps the simplest collision resolution technique. In chaining, we put all the pairs that hash to the same slot into a linked list. Thus, the hash table $T[0 \dots (m - 1)]$ is an array of linked lists. More precisely, $T[j]$ is a pointer to the head of a linked list storing all pairs that hash to slot j . If there are no such elements, $T[j]$ will be `NULL`. The Dictionary ADT operations on a hash table T are easy to implement when collisions are resolved by chaining. To insert a pair (k, v) into the Dictionary, we create a new Node storing this pair, then insert that Node at the head of the linked list $T[h(k)]$. To lookup a given key k , we do a linear search of the list $T[h(k)]$, and return the corresponding value if found, or `NULL` if not found. To delete the pair with key k , simply splice the corresponding Node out of the list headed by $T[h(k)]$. The remaining Dictionary operations are equally simple and are left to the student to design.

At this point the only question left is what to choose as our hash function $h()$. A good hash function should satisfy (at least approximately) the assumption of *simple uniform hashing*: each key is equally likely to hash to any of the m slots, independently of the slot to which any other key has hashed. Unfortunately, it is often not possible to check this condition in practice, since one may not know the probability distribution on the universe U from which the keys are drawn. Furthermore, the keys may not be drawn independently from U , i.e. the next key used may depend (probabilistically) on the previous key.

In this project U will be the set of non-negative integers up to the maximum value of the `unsigned int` data type in C (typically $2^{32} - 1 = 4,294,967,295$ in most implementations.) We will assume that keys are uniformly distributed over this universe. (We will try to enforce this assumption, at least approximately, by the way we turn strings into `unsigned ints`.) Under these conditions the function

$$h(k) = k \bmod m$$

satisfies the simple uniform hashing property. Here m is the length of the hash table T , and **mod** denotes the remainder operation, i.e. $h(k)$ is simply the remainder of k upon division by m . Note that this quantity necessarily lies in the range from 0 to $m - 1$, as required. Other possible hash functions with their pros and cons, will be discussed in class. It is suggested that your `Dictionary.c` file contain a private constant integer

```
const int tableSize=101; // or some prime other than 101
```

establishing the length of the hash table array. The choice here of length 101 is somewhat arbitrary and should be changed during testing of your project. In particular, you should see how your Dictionary ADT performs with small table lengths, i.e. no more than half the number of pairs in the Dictionary, so that collisions are guaranteed. You will include the following functions unaltered in your `Dictionary.c` file.

```

// rotate_left()
// rotate the bits in an unsigned int
unsigned int rotate_left(unsigned int value, int shift) {
    int sizeInBits = 8*sizeof(unsigned int);
    shift = shift & (sizeInBits - 1);
    if ( shift == 0 )
        return value;
    return (value << shift) | (value >> (sizeInBits - shift));
}

// pre_hash()
// turn a string into an unsigned int
unsigned int pre_hash(char* input) {
    unsigned int result = 0xBAE86554;
    while (*input) {
        result ^= *input++;
        result = rotate_left(result, 5);
    }
    return result;
}

// hash()
// turns a string into an int in the range 0 to tableSize-1
int hash(char* key){
    return pre_hash(key)%tableSize;
}

```

Functions `rotate_left()` and `pre_hash()` turn a string into an unsigned int, and function `hash()` converts that number into an int in the range 0 to `tableSize - 1`. We will discuss the operational details of these functions in class, but try to figure out how they work on your own by consulting the usual sources (<https://en.cppreference.com/w/c>, Google, Stack-Overflow).

These functions are used to compute the array index `hash(k)` in which a pair with key `k` is stored. The header file for this project is identical to that found in `pa3` and to that of the BST based Dictionary discussed in lecture. There is however one ADT operation whose requirements are relaxed from those in `pa3`, namely the `DictionaryToString()` function. Recall that in `pa3`, a text representation of a Dictionary was required to present `(key, value)` pairs in the order that they were originally inserted into the Dictionary. This was easy and natural for the linked list representation since the list itself maintained the insertion order. Your `DictionaryToString()` function in this project should return a text representation in which pairs occur in the order that they appear in the table, i.e. print out list `T[0]` in order (i.e. head to tail), then list `T[1]` in order, ..., then end by printing list `T[m - 1]`.

What to turn in

Write the implementation file `Dictionary.c` as described above based on a hash table data structure. Also write your own set of test procedures in a file called `DictionaryTest.c`, just as for `pa3`. The webpage will include files `Dictionary.h`, as well as `DictionaryClient1.c`, `DictionaryClient2.c` and `DictionaryClient3.c`. Submit `Dictionary.h` unaltered. Note that although the same Dictionary Clients from `pa3` are included as examples, their output will be different in this project because of the altered specification of function `DictionaryToString()`. Correct output for these clients will also be included on the webpage. Note if you change `tableSize` to something other than 101 you will get different output from `DictionaryToString()`, while lookups and deletes will give the same results.

A Makefile will also be provided which you may alter as needed. (Actually it is the same Makefile as for pa3, except it makes your DictionaryTest rather than DictionaryClient2.) To receive full credit, your project must compile without errors or warnings, produce correct output, and cause no memory leaks when run under `valgrind`. Submit the following files to the assignment name pa4.

README	Created by you
Dictionary.c	Created by you
Dictionary.h	Provided, do not change
DictionaryTest.c	Created by you
Makefile	Provided, alter as needed, must make DictionaryTest

As always start early and ask plenty of questions.