

Existence Proofs

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Outline

When One Example is Enough

Splitting an Octagon

Making Fun in Real Life

Know Your Rights

Nobody Can Win All The Times

Know What Are You Looking For



Proofs For Existential Statements

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- what does the proof look like?

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- it depends
- claim: *object with given properties exists*
- proof: an example
- one example is enough

Cutting Figures

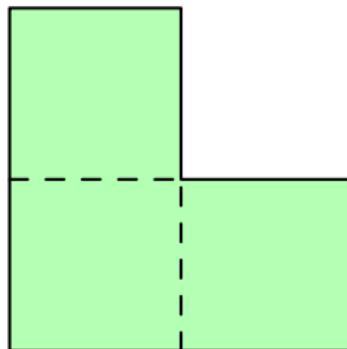
Cutting Figures

congruent pieces: of the same shape and size

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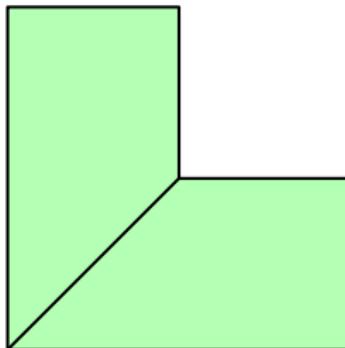
Prove that this figure can be cut into 2 congruent pieces



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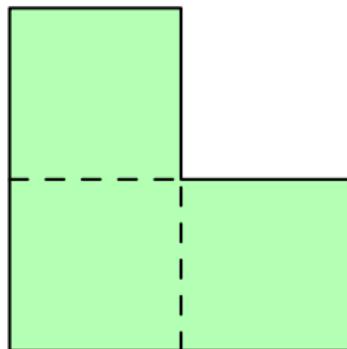
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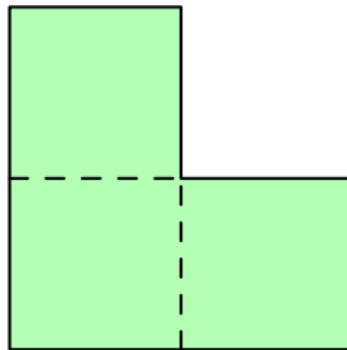
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Cutting Figures

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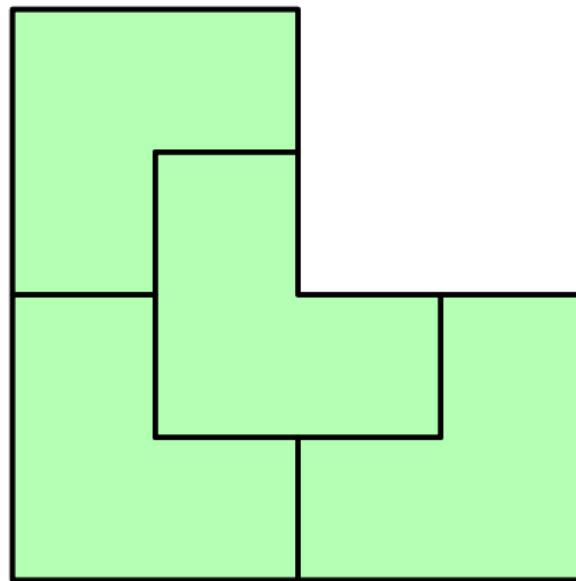
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what about 4 pieces?

Spoiler

Spoiler



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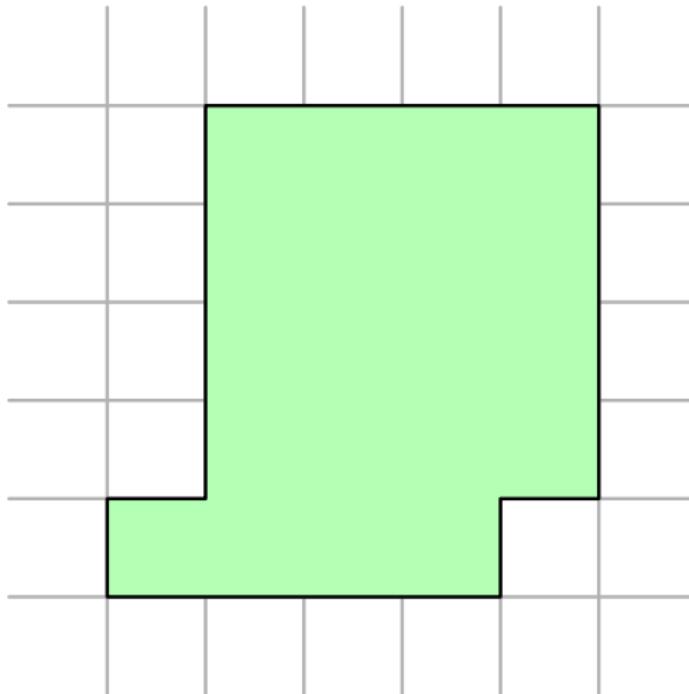
Making Fun in Real Life

Know Your Rights

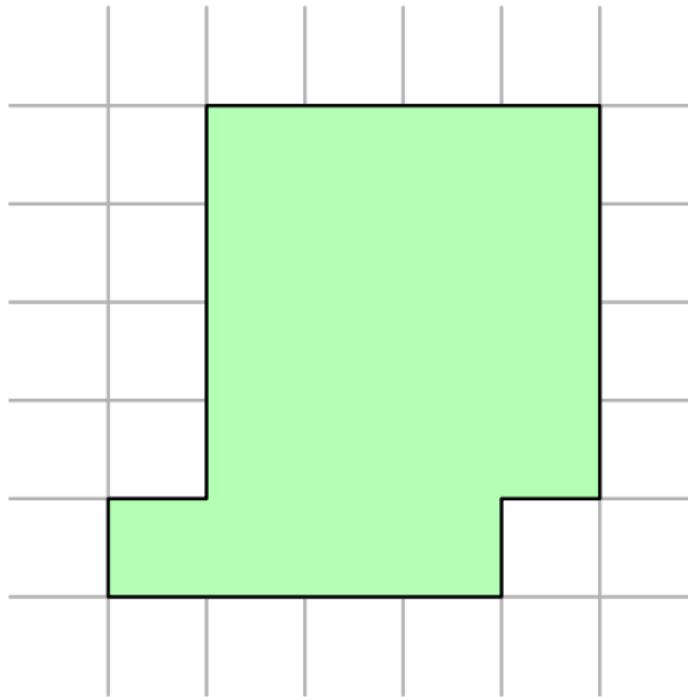
Nobody Can Win All The Times

The Octagon

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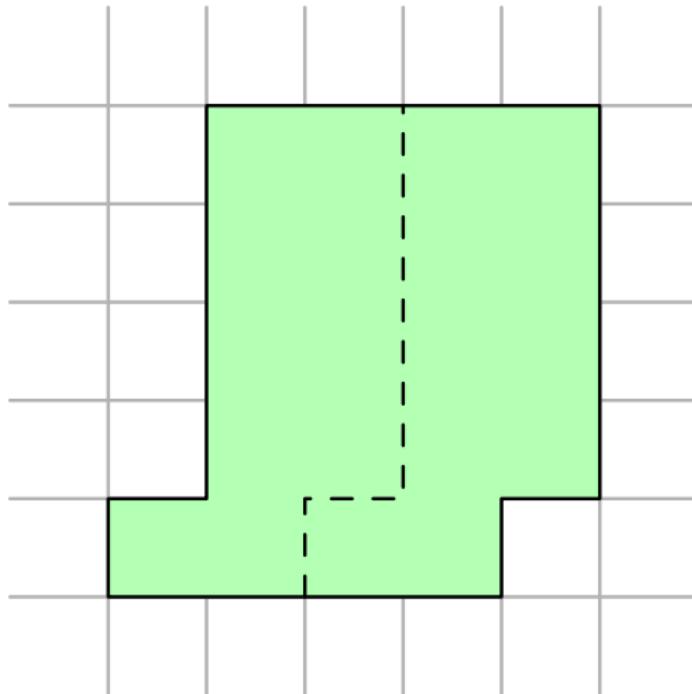
The Octagon



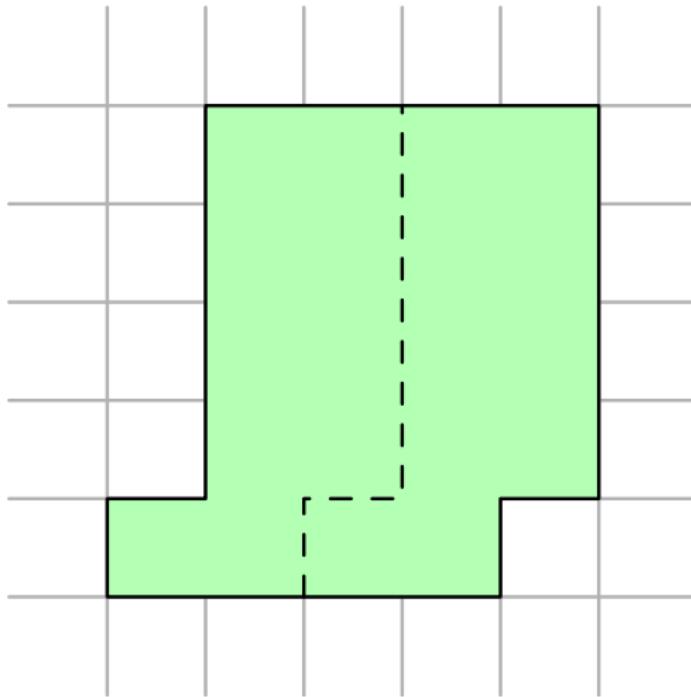
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what about three congruent pieces?

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Tensegrities

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- drinking straws and thread



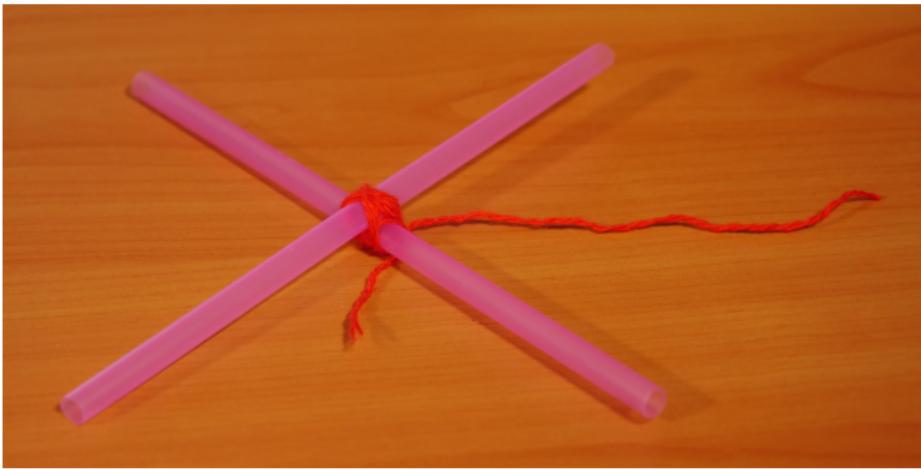
Tensegrities

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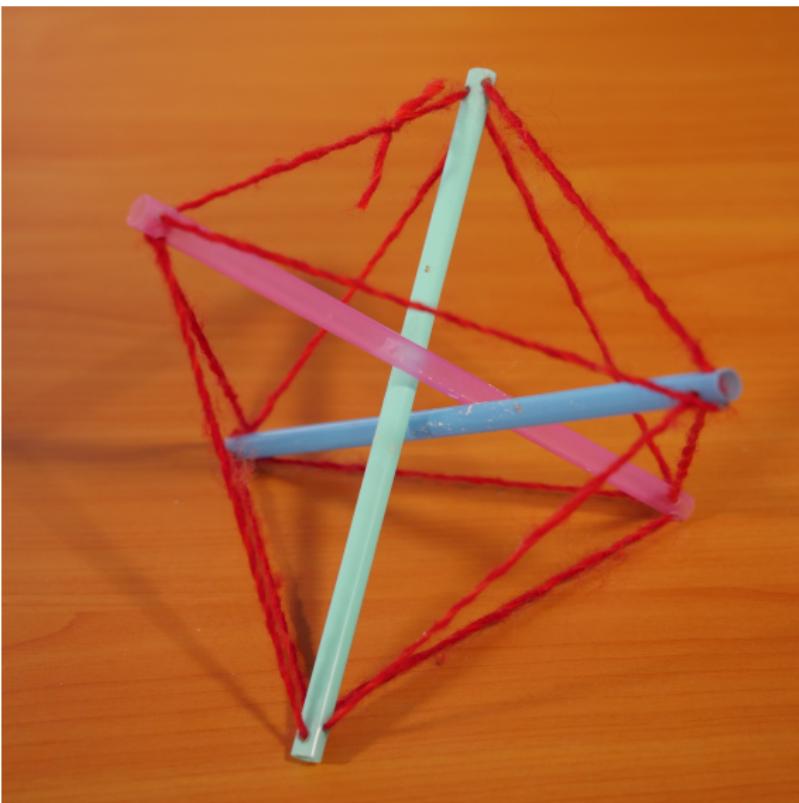


- there exists a “tensegrity”: a solid construction; straws do not touch each other; connected by threads

Not Allowed



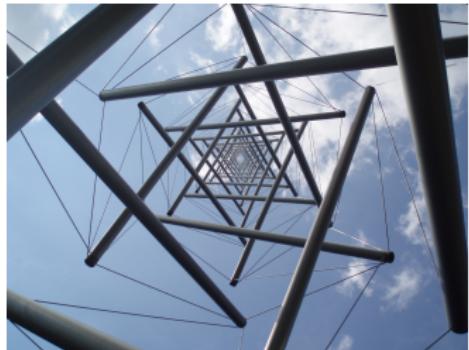
Tensegrity Finished



A Tensegrity: Animation

[Source: https://commons.wikimedia.org/wiki/File:Tensegrity_simple_3.gif]

Tensegrities in the Real Life



[Source:https://en.wikipedia.org/wiki/Needle_Tower]

made by Kenneth Snelson, a student of Buckminster Fuller (who invented the word and made many of them)

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- divisible by 7: 14, 21, 28, 35, 42, 49, 56, 63
- but what if we asked for a number that becomes 57 times smaller?
- you say: $7125 = 57 \cdot 125$
- no need to explain how you found it

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- a divisible by 7: only $a = 7$ works
- $10^k = 8 \times X$; 10 and 100 not multiples of 8
- 1000 works, $X = 125$
- also $71250 = 57 \times 1250$, etc.

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- three weights: 1, 2, 3

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- $+1 + 2 - 3 = 0$

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- what about weights 2, 4, 6, 8, 10, 12?

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- total weight 21: not a multiple of 2
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- what about weights 2, 4, 6, 8, 10, 12?
- hint: just changing the units

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- no complete list
- NP-complete \approx infeasible

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- claim: *an object with some property exists*
- proof: *an example*
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- no need to disclose the sources
- beware: claim may be false!