formal grammar

Context Free grammar (CFG)

- describes the structure of the programming language.
- Used in AI, syntax analysis (i.e. 2nd phase of complier)

CFG defined by 4 tuples

- V: Variable (A, B, ... by capital letter)
- T: Terminal (a, 0, 1 ... by small letter)
- P: Production (rules)
- S: Start variable

$L = \{a^n b^n \mid n \geq 1\}$

$L = \{ab, aabb, aaabbb, \ldots\}$

P = {

S → ab

S → aSb

}

Derivation

S → aSb

S → aaSbb

S → aaabbb

V = {S}

T = {a, b}

S = S

$L = a^*$

$L = \{\epsilon, a, aa, aaa, \ldots\}$

P = {

S → ϵ

S → aS

}

V = {S}

T = a

S = S

start variable

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* $r = (a+b)^*$

$L = (\epsilon, a, aa, aaa, b, bb, bbb ---)$

for $w = abbaab$

P = {
$S \to \epsilon$
$S \to aS$
$S \to bS$
}

$V = \{S\}$

$T = \{a, b\}$

$S = S$

Derivation:	Rule:
$S \to aS$	$S \to aS$
$S \to abS$	$S \to bS$
$S \to abbS$	$S \to bS$
$S \to abbaS$	$S \to aS$
$S \to abbaaS$	$S \to aS$
$S \to abbaabS$	$S \to bS$
$S \to abbaab$	$S \to \epsilon$

* $L = \{\text{palindrome} \in (a, b)^*\}$

$L = \{\epsilon, a, b, aa, bb, --\}$

P = {
or
$S \to \epsilon \mid a \mid b$
$S \to aSa \mid bSa$
}

$w = baab$

Derivation	Rule:
$S \to bSb$	$S \to bSb$
$S \to baSab$	$S \to aSa$
$S \to baab$	$S \to \epsilon$

* $L = (a+b)^* (a+b)^*$

$L = \{\epsilon, a, b, aa, bb, ----\}$

P = {
$S \to \epsilon \mid a \mid b$
$S \to aS \mid bS \mid aSaS \mid aSbS \mid bSaS \mid bSbS$
}

$((a+b)(a+b))$
$=(aa+ab+ba+bb)$

(2) $((a+b)(a+b))^*$
(3) $a(a+b)^*b$
(4) $L=\{a^n b^m \mid n,m \geq 1\}$

Homework

(1) $L=(a+b)^*(a+b)^*$
$L=\{\epsilon, a, b, aa, bb, ab \ldots\}$
$P=\{$
$S \to AA$
$A \to \epsilon \mid aS \mid bS$
$\}$

(2) $((a+b)(a+b))^*$
$L=\{\epsilon, aa, bb, ab \ldots$
$P=\{$
$S \to \epsilon \mid aaS \mid abS \mid bbS$
$\}$

(3) $a(a+b)^*b$
$L=\{ab, aab, abb, aaab \ldots\}$
$P=\{$
$S \Rightarrow \cancel{\epsilon}\, aAb$
$A \to \epsilon \mid aS \mid bS$
$\}$

(4) $L=\{a^n b^m \mid n,m \geq 1\}$
$L=\{ab, aabb, abb, aab$
$P=\{$
$S \to AB$
$A \to aS$
$B \to bS$
$\}$

Parse tree (Syntax tree)

Derivation: — Bottom-up
Derivation: — Top-down — LMD (left-most derivation)
Top-down — RMD (right-most derivation)

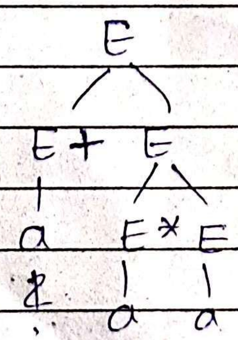
Derivation: generating string using production rule

$E \to E+E \mid E*E \mid a$

$w = a+a*a$

LMD	Rule:	RMD	Rule:
$E \to E+E$	$E \to E+E$	$E \to E+E$	$E \to E+E$
$E \to a+E$	$E \to a$	$E \to E+E*E$	$E \to E*E$
$E \to a+E*E$	$E \to E*E$	$E \to E+E*a$	$E \to a$
$E \to a+a*E$	$E \to a$	$E \to E+a*a$	$E \to a$
$E \to a+a*a$	$E \to a$	$E \to a+a*a$	$E \to a$

Parse tree:



Root node is start variable / non-terminal

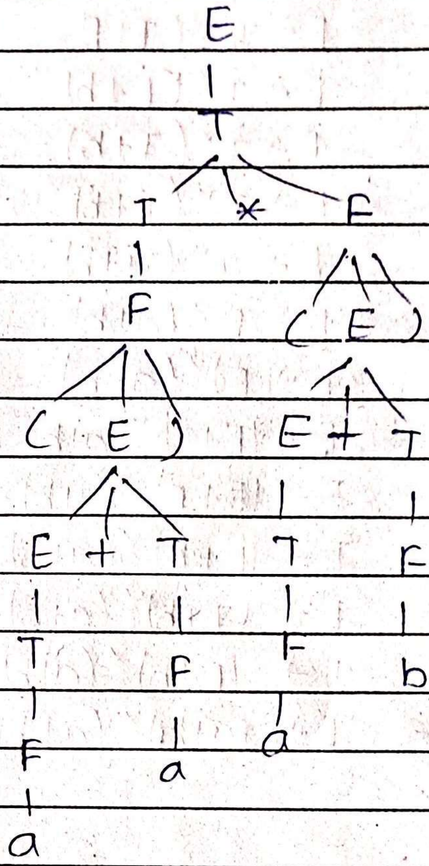
(*) $E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid a \mid b$

$w = (a+a) * (a+b)$

LMD:	Rule:
$E \rightarrow T$	$E \rightarrow T$
$E \rightarrow T*F$	$T \rightarrow T*F$
$E \rightarrow F*F$	$T \rightarrow F$
$E \rightarrow (E)*F$	$F \rightarrow (E)$
$E \rightarrow (E+T)*F$	$E \rightarrow E+T$
$E \rightarrow (T+T)*F$	$E \rightarrow T$
$E \rightarrow (F+T)*F$	$T \rightarrow F$
$E \rightarrow (a+T)*F$	$F \rightarrow A$
$E \rightarrow (a+F)*F$	$T \rightarrow F$
$E \rightarrow (a+a)*F$	$F \rightarrow A$
$E \rightarrow (a+a)*(E)$	$F \rightarrow (E)$
$E \rightarrow (a+a)*(E+T)$	$E \rightarrow E+T$
$E \rightarrow (a+a)*(T+T)$	$E \rightarrow T$
$E \rightarrow (a+a)*(F+T)$	$T \rightarrow F$
$E \rightarrow (a+a)*(a+T)$	$F \rightarrow a$
$E \rightarrow (a+a)*(a+F)$	$T \rightarrow F$
$E \rightarrow (a+a)*(a+b)$	$F \rightarrow b$



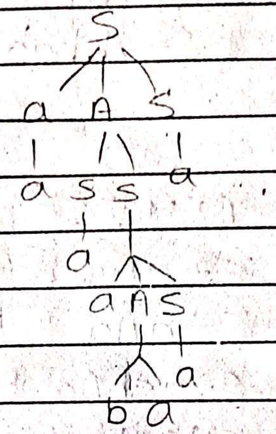
RMD:	Rule:
$E \to T$	$E \to T$
$E \to T * F$	$T \to T * F$
$E \to T * (E)$	$F \to (E)$
$E \to T * (E+T)$	$E \to E+T$
$E \to T * (E+F)$	$T \to F$
$E \to T * (E+b)$	$F \to b$
$E \to T * (T+b)$	$E \to T$
$E \to T * (F+b)$	$T \to F$
$E \to T * (a+b)$	$F \to a$
$E \to F * (a+b)$	$T \to F$
$E \to (E) * (a+b)$	$F \to (E)$
$E \to (E+T) * (a+b)$	$E \to E+T$
$E \to (E+F) * (a+b)$	$T \to F$
$E \to (E+b) * (a+b)$	$F \to b$
$E \to (T+b) * (a+b)$	$E \to T$
$E \to (F+b) * (a+b)$	$T \to F$
$E \to (a+b) * (a+b)$	$F \to a$

(*) $S \to aAS \mid a$

$A \to SbA \mid SS \mid ba$

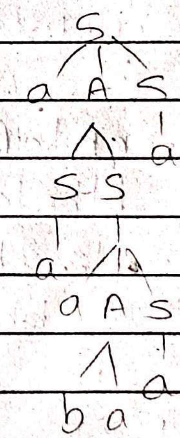
1) $w = aaabaaa$
2) $w = aabbaa$

LMD:	Rule:
$S \to aAS$	$S \to aAS$
$S \to aSSS$	$A \to SS$
$S \to aaSS$	$S \to a$
$S \to aaaASS$	$S \to aAS$
$S \to aaabaSS$	$A \to ba$
$S \to aaabaaS$	$S \to a$
$S \to aaabaaa$	$S \to a$



$\Rightarrow aaabaaa$ ✓

RMD:	Rule:
$S \to aAS$	$S \to aAS$
$S \to aAa$	$S \to a$
$S \to aSSa$	$A \to SS$
$S \to aSaASa$	$S \to aAS$
$S \to aSaAaa$	$S \to a$
$S \to aSabaaa$	$A \to ba$
$S \to aaabaaa$	$S \to a$

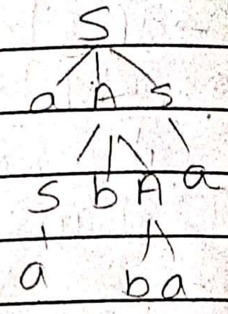


$\Rightarrow aaabaaa$ ✓

$w = aabbaa$

LMD:

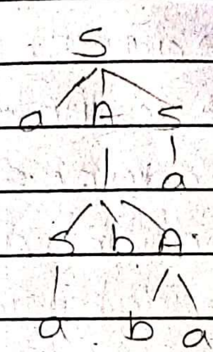
LMD	Rule
$S \to aAS$	$S \to aAS$
$S \to asbAS$	$A \to sbA$
$S \to aabAS$	$S \to a$
$S \to aabbaS$	$A \to ba$
$S \to aabbaa$	$S \to a$



$\Rightarrow aabbaa$

RMD:

RMD	Rule
$S \to aAS$	$S \to aAS$
$S \to aAa$	$S \to a$
$S \to asbAa$	$A \to sbA$
$S \to asbbaa$	$A \to ba$
$S \to aabbaa$	$S \to a$



$\Rightarrow aabbaa$

$S \to bB \mid aA$

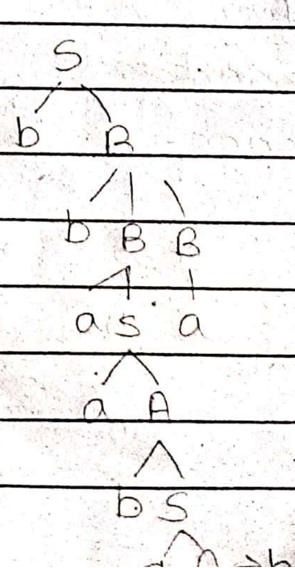
$A \to b \mid bS \mid aAA$

$B \to a \mid aS \mid bBB$

$w = bbaababa$

LMD:

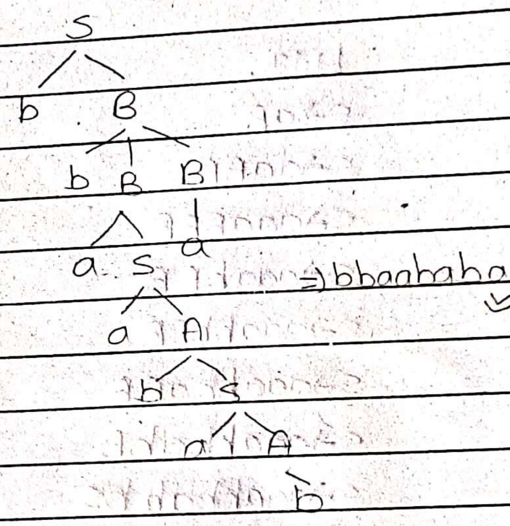
LMD	Rule
$S \to bB$	$S \to bB$
$S \to bbBB$	$B \to bBB$
$S \to bbaSB$	$B \to aS$
$S \to bbaaAB$	$S \to aA$
$S \to bbaabSB$	$A \to bS$



$\Rightarrow bbaababa$

S→bbaabaAB	S→aA
S→bbaababB	A→b
S→bbaababa	B→a

RMD:	Rule:
S→bB	S→bB
S→bbBB	B→bBB
S→bbBa	B→a
S→bbaSa	B→aS
S→bbaaAa	S→aA
S→bbaabSa	A→bS
S→bbaabaAa	S→aA
S→bbaababa	A→b



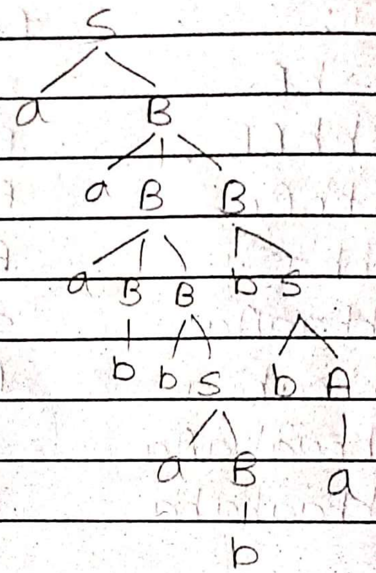
Q) $S \to aB \mid bA$

$A \to aS \mid bAA \mid a$

$B \to bS \mid aBB \mid b$

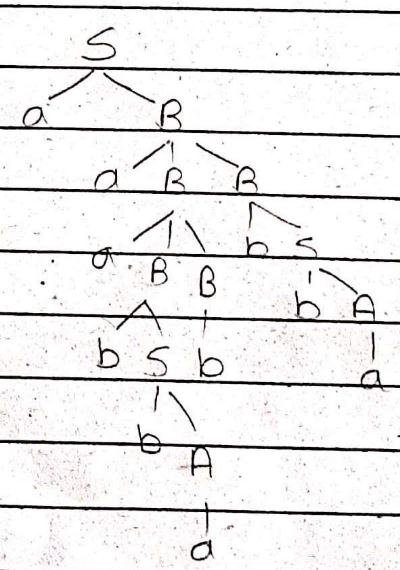
$w = aaabbabbba$

LMD:	Rule
$S \to aB$	$S \to aB$
$S \to aaBB$	$B \to aBB$
$S \to aaaBBB$	$B \to aBB$
$S \to aaabBB$	$B \to b$
$S \to aaabbSB$	$B \to bS$
$S \to aaabbaBB$	$S \to aB$
$S \to aaabbabB$	$B \to b$
$S \to aaabbabbS$	$B \to bS$
$S \to aaabbabbbA$	$S \to bA$
$S \to aaabbabbba$	$A \to a$



$\Rightarrow aaabbabbba$

RMD:	Rule
$S \to aB$	$S \to aB$
$S \to aaBB$	$B \to aBB$
$S \to aaBbS$	$B \to bS$
$S \to aaBbbA$	$S \to bA$
$S \to aaBbba$	$A \to a$
$S \to aaaBBbba$	$B \to aBB$
$S \to aaaBbbba$	$B \to b$
$S \to aaabSbbba$	$B \to bS$
$S \to aaabbAbbba$	$S \to bA$
$S \to aaabbabbba$	$A \to a$



$\Rightarrow aaabbabbba$

Ambiguity in CFG

(L) better do by parse tree.

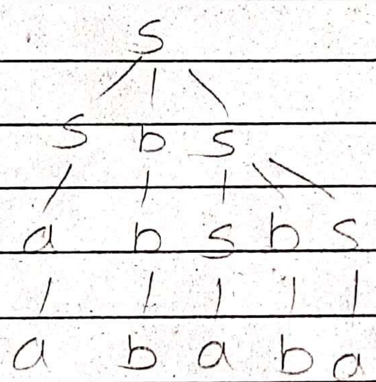
* $S \to aS \mid Sa \mid a$

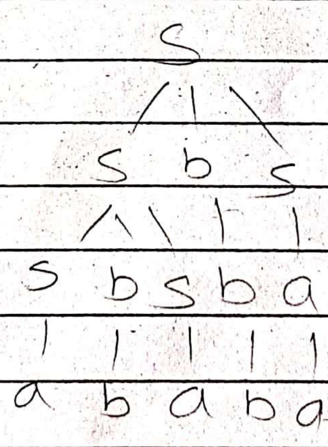
aaaaa

LMD:	LMD:
$S \to aS$	$S \to aS$
$S \to aSa$	$S \to aaS$
$S \to aaSa$	$S \to aaaS$
$S \to aaSaa$	$S \to aaaaS$
$S \to aaaaa$	$S \to aaaaa$

Hence, Ambiguity occurs.

* $S \to SbS \mid a$





* $S \to aB \mid ab$
  $A \to aAB \mid a$
  $B \to ABb \mid b$

$S \to AB \mid aaB$
$A \to a \mid Aa$
$B \to b$

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Simplication of CFG

$A \xrightarrow{*} \epsilon$ (nullable variable)

Q) $S \to AB$

$A \to aA | \epsilon$

$B \to bbB | \epsilon$

Here, $A \to \epsilon$ (A is a nullable variable)

$B \to \epsilon$ (A is a nullable variable)

$S \to AB$ (A and B are nullable so, S is also nullable)

Now, removing nullable variable, we get:

$S \to AB | A | B$

$A \to aA | a$

$B \to bbB | bb$

Q) $S \to ABC$ $\Rightarrow$ $S \to ABC | BC | CA | AB | A | B | C$

$A \to BB | \epsilon$ $\qquad$ $A \to BB | B$

$B \to cc | a$ $\qquad$ $B \to cc | c | a$

$C \to AA | b$ $\qquad$ $C \to AA | A | b$

all nullable

Unit production

(*) $E \to E+T | T$

$T \to T*F | F$

$F \to (E) | a$

Here, $E \to T$ $\quad$ (E, T) unit pair

$T \to F$ $\quad$ (T, F) unit pair

$E \to T \to F$ $\quad$ (E, F) unit pair

Now, removing unit pair, we get:

$E \to E+T | T*F | (E) | a$

$T \to T*F | (E) | a$

$F \to (E) | a$

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Remove the unit production:

$S \to AB$
$A \to aA \mid a$
$B \to \cancel{C}^{e} \mid b$ $\Rightarrow$ (B,C) (C,D), (D,E), (B,D), (B,E)
$C \to \cancel{D}^{e}$ (C,E)
$D \to \cancel{E}^{e}$
$E \to e$

$S \to AB$
$A \to aA \mid a$
$B \to e \mid b$
$\boxed{C \to e}$
$\boxed{D \to e}$ since, starting variable doesn't use C, D, E, we can remove it.
$\boxed{E \to e}$

Removing useless variable

$S \to aB \mid bX$
$A \to BAd \mid bSX \mid a$
$B \to aSB \mid bBX$
$X \to SB\cancel{D} \mid aBx \mid ad$

* generating variable
$A \xrightarrow{*} w$ $\Rightarrow$ gives terminal value
* Reachable variable
$S \xrightarrow{*} \alpha A \beta,\ \alpha, \beta \in (V \cup T)^*$
starting variable

Here, A and X are generating variable
since, $A \to a$
$X \to ad$

S is also generating since, $X \to ad$
$S \to bad$

But B is not generating variable.
After removing non-generating variable,
CFG becomes:

$S \to bX$

$A \rightarrow bSX \mid a$

$B \rightarrow X \rightarrow ad$

Here, A is a unreachable variable since A cannot be reached from starting variable.

Hence, final CFG becomes:

$S \rightarrow bX$

$X \rightarrow ad$

Q) Simply the given grammar:

$S \rightarrow ASB \mid \epsilon$

$A \rightarrow aSA \mid a$

$B \rightarrow SbS \mid A \mid bb \mid \epsilon$

(i) Removing nullable variable, we get:

~~S → ASB | ASB~~

~~A → aSA~~

Here, $S \rightarrow \epsilon$ (S is a nullable variable)

$B \rightarrow \epsilon$ (B is a nullable variable)

Now, we get after ϵ-production:

$S \rightarrow ASB \mid AS \mid AB \mid A$

$A \rightarrow aSA \mid a$

$B \rightarrow SbS \mid Sb \mid bS \mid b \mid A \mid bb$

(ii) Unit production:

$S \rightarrow A$ (S, A) unit pair

$B \rightarrow A$ (B, A) unit pair

removing unit production:

$S \rightarrow ASB \mid AS \mid AB \mid aSA \mid a$

$A \rightarrow aSA \mid a$

$B \rightarrow SbS \mid Sb \mid bS \mid bb \mid b \mid aSA \mid a$

Since, the obtained CFS doesnot have useless variable, It is the final simplify grammar.

Q) $S \to 0A0 \mid 1B1 \mid BB$
$A \to C$
$B \to S \mid A$
$C \to S \mid \epsilon$

Step 1: Eliminating ϵ-productions.

Here, $C \to \epsilon$ — C is nullable
$A \to C \xrightarrow{*} \epsilon$ — A is nullable
$B \to A \xrightarrow{*} \epsilon$ — B is nullable
$S \to BB \xrightarrow{*} \epsilon$ — S is nullable

Now, Removing of ϵ-production:
$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B$
$A \to C$
$B \to S \mid A$
$C \to S$

Step 2: Eliminating unit productions

Here, $S \to B$ — (S,B) is unit pair.
$A \to C$ — (A,C) is unit pair.
$B \to S \mid A$ — (B,S) or (B,A) is unit pair.
$C \to S$ — (C,S) is unit pair

All together, (S,A), (A,S), (B,C), (C,B) are unit pair

Now, Removing unit production:
$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid S \mid A$
$A \to S$
$B \to S \mid A$
$C \to S$

Step 3: Eliminating useless symbols

Here, S is generating symbol since it produces 00. Hence, A, B, C are also generating symbol.
Also, C is not reachable. Hence, removing it.

Hence, the required grammar is:

$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid S \mid \cancel{A}$

$A \rightarrow S \quad 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid$

$B \rightarrow S \mid A \quad 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$C \rightarrow S \quad 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$\boxed{A \to BC|a} \Rightarrow$ chomsky Normal form

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Chomsky Normal form

Theorem: Every CFL without ϵ-production can be generated grammar in CNF.

(*) Convert in CNF:

$S \to AAC$
$A \to aAb|\epsilon$
$C \to aC|a$

Step 1: Removing ϵ-production

$A \to \epsilon$ nullable variable

$S \to AAC|AC|C$
$A \to aAb|ab$
$C \to aC|a$

Step 2: Removing unit production

$S \to C$ (S,C) is a unit pair

Then, $S \to AAC|AC|aC|a$
$A \to aAb|ab$
$C \to aC|a$

Step 3: Here, useless variable doesn't occur. Hence, the above can be converted to CNF as:

(*) Firstly, replacing terminal by new non-terminal, then grammar becomes:

$S \to AAC|AC|C_1C|a$
$A \to C_1AC_2|C_1C_2$
$C \to C_1C|a$
$C_1 \to a$
$C_2 \to b$

* Replacing sequence of non-terminal by new non-terminal and introducing new production:
Replace: $S \to AAC$ by $S \to AS_1$ with $S_1 \to AC$
Replace: $A \to C_1AC_2$ by $A \to C_1S_2$ with $S_2 \to AC_2$

Final grammar of the form
$S \to AS_1 \mid AC \mid C_1C \mid a$
$A \to C_1S_2 \mid C_1C_2$
$C \to C_1C \mid a$
$S_1 \to AC$
$S_2 \to AC_2$
$C_1 \to a$
$C_2 \to b$

* $S \to ASB \mid \epsilon$
$A \to aAs \mid a$
$B \to SbS \mid A \mid bb$
Step 1: Eliminating ϵ-production
$S \to \epsilon$
$S \to ASB \mid AB$
$A \to aAS \mid aA \mid a$
$B \to SbS \mid Sb \mid bS \mid b \mid A \mid bb$
Step 2: Removing unit production:
(B, A) is unit pair $\Rightarrow$ $B \to SbS \mid Sb \mid bS \mid b \mid aAS \mid aA \mid a \mid bb$
Step 3: Removing useless variable
No useless variable.

Now, converting the grammar to CNF as:

i) Replacing terminal by new non-terminal

$S \rightarrow ASB \mid AB$

$A \rightarrow C_1AS \mid C_1A \mid a$

$B \rightarrow SC_2S \mid SC_2 \mid C_2S \mid b \mid C_1AS \mid C_1A \mid a \mid C_2C_2$

$C_1 \rightarrow a$

$C_2 \rightarrow b$

ii) Replacing sequence of non-terminal by new non-terminal and introducing new production

$A \to A\alpha$
$A \to \beta A'$
$A' \to \alpha A' | \epsilon$

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Left recursive grammar:

$A \to A\alpha_1 | A\alpha_2 | A\alpha_3 | \dots | A\alpha_n | \beta_1 | \beta_2 | \dots | \beta_m$

$\Downarrow$

$A \to \beta_1 A' | \beta_2 A' | \dots | \beta_m A' | \beta_1 | \beta_2 | \dots | \beta_m$
$A' \to \alpha_1 A' | \alpha_2 A' | \dots | \alpha_n A' | \alpha_1 | \alpha_2 | \dots | \alpha_n$

Q) $\overset{A}{E} \to \overset{A}{E}\overset{\alpha}{+T} | \overset{\beta}{T}$
$E \to TE' | T$
$E' \to +TE' | +T$

Q) $T \to T*F | F$
$T \to FT' | F$
$T' \to *FT' | *F$

Q) $S \to Aa | b$
$A \to Ac | Sd | \epsilon$

~~$S \to Aca | Sda | Aa | b$~~
~~$A \to Ac | Sd | \epsilon$~~

~~$S \to Sda | Aca | Aa | b$~~
~~$S \to AcaS' | AaS' | bS' | Aca | Aa | b$~~
~~$S' \to daS' | da$~~

$A \to Ac | Sd | \epsilon$
$A \to SdA'$
$A' \to cA'$

~~$S \to Aa | b$~~
~~$A \to Ac | Sd | \epsilon$~~
~~$\alpha_1 = c$~~
~~$\beta_1 = Sd$~~
~~$\beta_2 = \epsilon$~~
~~$A \to SdA' | A'$~~
~~$A' \to cA'$~~

⊛ $S \to Aa | b$
$A \to Ac | Aad | bd | \epsilon$
$A \to bdA' | A'' | bd | \epsilon$
$A' \to cA' + adA' | c | ad$

Q) $S \rightarrow AA \mid 0$

$A \rightarrow SS \mid 1$

$S \rightarrow AA \mid 0$

$A \rightarrow AAS \mid \underbrace{0S}_{\beta_1} \mid \underbrace{1}_{\beta_2}$

Removing the left-recursive and equivalenting

$S \rightarrow AA \mid 0$

$A \rightarrow 0SA' \mid 1A' \mid 0S \mid 1$

$A' \rightarrow ASA' \mid AS$

Q) $S \rightarrow (L) \mid a$

$L \rightarrow L\underbrace{,S}_{\alpha} \mid \underbrace{S}_{\beta}$

~~$S \rightarrow (L) \mid a$~~

~~$L \rightarrow SL' \mid S$~~

~~$L' \rightarrow ,SL' \mid ,S$~~

$S \rightarrow (L) \mid a$

$L \rightarrow L\underbrace{,(L)}_{\alpha_1} \mid L\underbrace{,a}_{\alpha_2} \mid \underbrace{(L)}_{\beta_1} \mid \underbrace{a}_{\beta_2}$

Now, removing the left-recursive

$L \rightarrow (L)L' \mid aL'$

$L' \rightarrow ,(L)L' \mid ,aL' \mid \epsilon$

Hence, final equivalent grammar is ::

$L \rightarrow (L)L' \mid aL' \mid (L) \mid a$

$L' \rightarrow ,(L)L' \mid ,aL' \mid ,(L) \mid ,a$

$S \rightarrow (L) \mid a$

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Greiback Normal form ($A \to aV^*$)

$\Rightarrow$ simplify $\Rightarrow$ CNF $\Rightarrow$ left (remove) recursive $\Rightarrow$ Equivalent
$\Downarrow$
GNF

* Convert given grammar into GNF

$S \to AA \mid 0$
$A \to SS \mid 1$ $\quad AAS \mid 0S \mid 1$

Now, removing the left recursive:

$S \to AA \mid 0$
$A \to AAS \mid 0S \mid 1$

$S \to AA \mid 0$
$A \to 0SA' \mid 1A' \mid 0S \mid 1$ $\Rightarrow$ GNF Form
$A' \to ASA' \mid AS$

Here, replacing the first non-terminal of S and A' production by A-production

$S \to 0SA'A \mid 1A'A \mid 0SA \mid 1A \mid 0$
$A \to 0SA' \mid 1A' \mid 0S \mid 1$
$A' \to 0SA'SA' \mid 1A'SA' \mid 0SSA' \mid 1SA' \mid 0SA'S \mid 1A'S \mid 0SS \mid 1S$

$S \to AB \mid BC$
$A \to aB \mid bA \mid a$
$B \to bB \mid cC \mid b$
$C \to c$

$\Rightarrow$ $S \to aBB \mid bAB \mid aB \mid bBC \mid cCC \mid bC$
$A \to aB \mid bA \mid a$
$B \to bB \mid cC \mid b$
$C \to c$

Q. $S \to AB$ $\qquad S \to BSB \mid aB$

$A \to BS \mid a \quad \Rightarrow \quad A \to BS \mid a$

$B \to SA \mid b \qquad B \to \underbrace{BSBA}_{\alpha} \mid \underbrace{aBA}_{\beta_1} \mid \underbrace{b}_{\beta}$

$S \to BSB \mid aB$

$A \to BS \mid a$

$B \to aBAB' \mid bB' \mid aBA \mid b$

$B' \to SBAB' \mid SBA$

$S \to aBAB'SB \mid bB'SB \mid aBASB \mid bSB \mid aB$

$A \to aBAB'S \mid bB'S \mid aBAS \mid bS \mid a$

$B \to aBAB' \mid bB' \mid aBA \mid b$

$B' \to aBAB'SBBAB' \mid bB'SBBAB' \mid aBASBBAB' \mid$
$bSBBAB' \mid aBBAB' \mid aBAB'SBBA \mid bB'SBBA \mid$
$aBASBBA \mid bSBBA \mid aBBA$

$\epsilon \to$ always final state

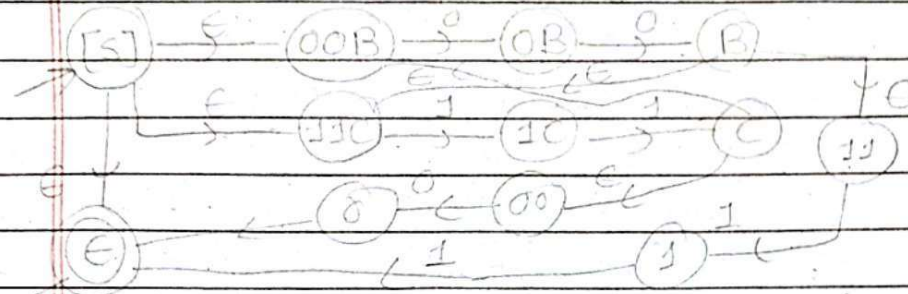
$\alpha \in (V \cup T)^*$

Regular Grammar (type 3 grammar) $\to$ Right linear $\to wB$; Left linear $\to Bw$

$S \to 00B \mid 11C \mid \epsilon$

$B \to 11C \mid 11$

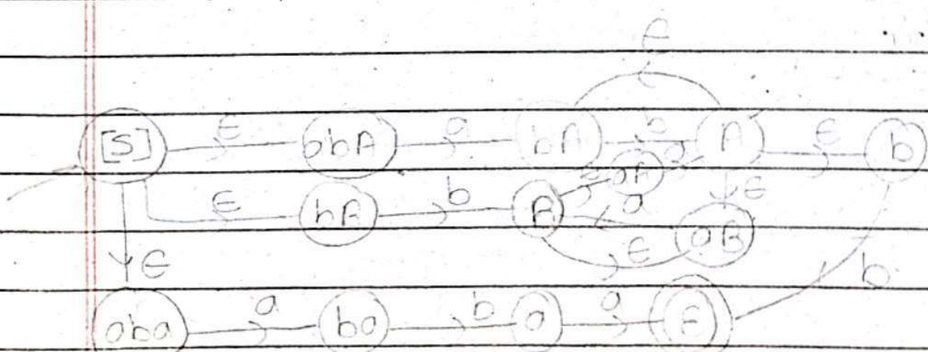
$C \to 00B \mid 00$



$S \to abA \mid bB \mid aba$

$A \to b \mid aB \mid bA$

$B \to aB \mid aA$



For production of form aB i.e. A→wB

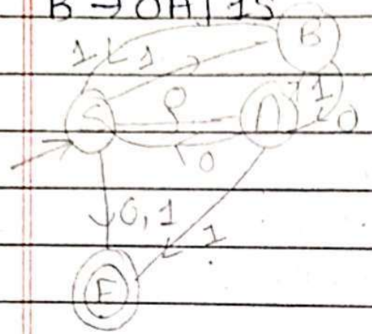
no of variable → (n) —— (n+1) state

A→aB   δ(A,a) = B

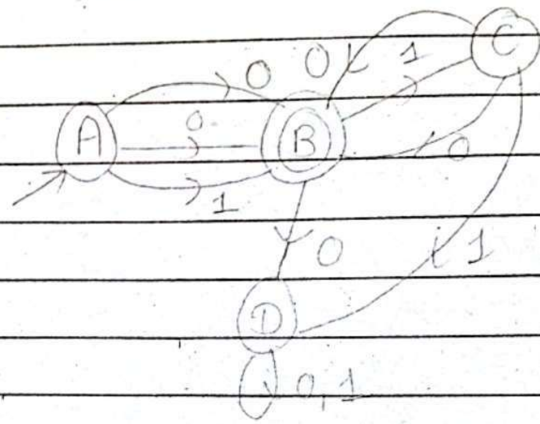
A→a   δ(A,a) → Final state

becomes final state (A)→ε

Q) S→0A|1B|0|1
A→0S|1B|1
B→0A|1S

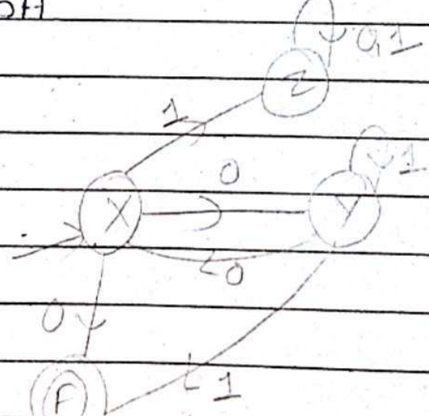


Q) A→0B|1B|0
B→0D|1C|ε
C→0B|1D|0
D→0D|1D



Q) S→abA|bB|aba
A→b|aB|bA
B→aB|aA

Q) X→0Y|0|1Z
Y→0X|1Y|1
Z→0Z|1Z



Pumping Lemma for CFL

Statement: Suppose L is a context free language. Then there exist an integer n such that for every string $z \in L$ with $|z| \geq n$, z can be written as $z = uvwxy$ for some strings u, v, w, x and y satisfying following condition:

1. $|vx| > 0$
2. $|vwx| \leq n$
3. $uv^k w x^k y \in L \ \forall k \geq 0$

Q) $L = \{a^n b^n c^n \mid n \geq 1\}$ is not a CFL

$L = \{abc, aabbcc, aaabbbccc, \ldots\}$

Suppose given language is a context free language.
Lets take a string (z) $= a^3 b^3 c^3$

Then, pumping length = 3

Here, $z = \underbrace{a}_{u}\underbrace{a}_{v}\underbrace{a}_{w}\underbrace{b}_{x}\underbrace{bbccc}_{y}$

(i) Here, $|vx| > 0$ ✓
(ii) $|vwx| \leq 3$ ✓
(iii) when $k = 0$

$z = a(a)^k a(b)^k bbccc$
$= aabbccc \notin L$ (X)

Hence, given language is not a CFL.

Q) $L = \{ww \mid w \in (a,b)^*\}$ $\Rightarrow a^n b^n a^n b^n$

$L = \{\epsilon, ~~a, b~~, ~~ab~~, aa, ~~ba~~, bbabab, aaaa, \ldots\}$

Suppose, given language is a context free language.
Lets take a string (z) = ababab~~a~~babab

Then, pumping length = ~~4~~ 3

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Here, $z = \underset{u}{a}\underset{v}{b}\underset{w}{a}\underset{x}{b}\underset{y}{ababababab}$

(i) $|vx| > 0$
(ii) $|vwx| = 3 \leq n$
(iii) when $k=0$,
uv^0wx^0y
$aaabababab \notin L$
Hence it is not CFL.

* Closure propertys of CFL

$G_1 \quad G_2$

$G_1 + G_2 = \_\_$
$G_1 \cdot G_2 = \_\_$
$G_1^* = \_\_$

let one grammar be $G_1 : \{V_1, T_1, P_1, S_1\}$
Another be $G_2 : \{V_2, T_2, P_2, S_2\}$

~~for eg.~~ $S \rightarrow S_1 / S_2$
$S_1 \rightarrow aS_1 \mid \epsilon$
$S_2 \rightarrow bS_2 \mid \epsilon$

for example $S \rightarrow S_1 \mid S_2$
$S \rightarrow S_1 S_2$
$S \rightarrow S_1^*$

$S_1 \rightarrow aS_1b \mid \epsilon$
$S_2 \rightarrow bS_2a \mid \epsilon$
$S \rightarrow S_1 \mid S_2$
$S \rightarrow S_1 S_2$
$S \rightarrow S_1^*$