Push-down Automata

Q:

q_0:

Σ:

δ:

F:

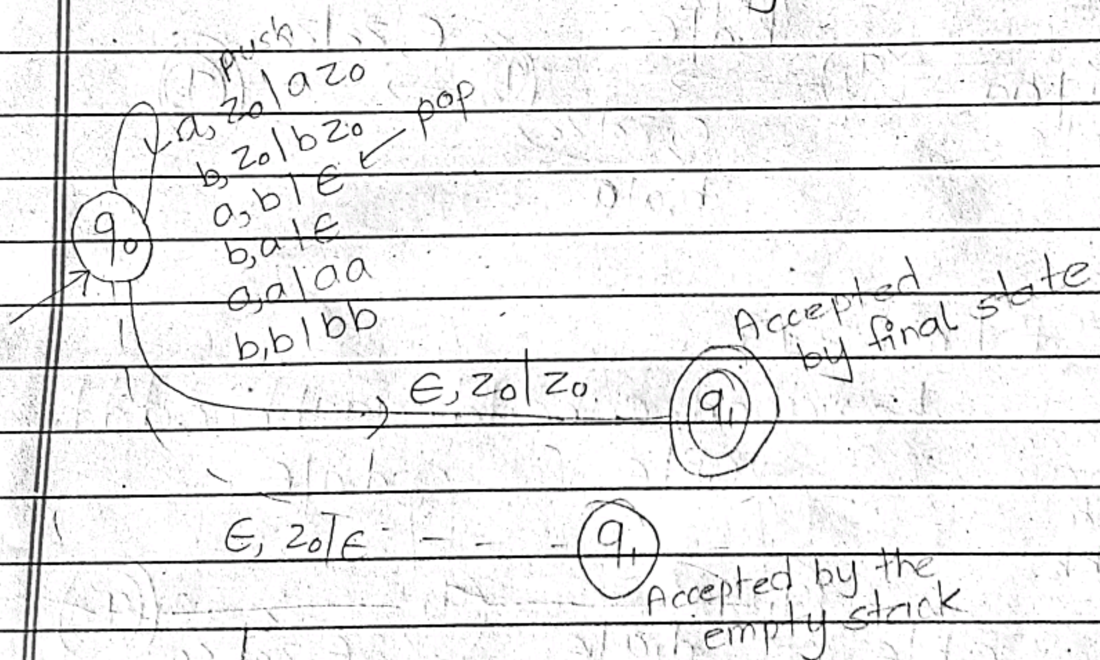
Γ: set of ~~state~~ stack alphabets

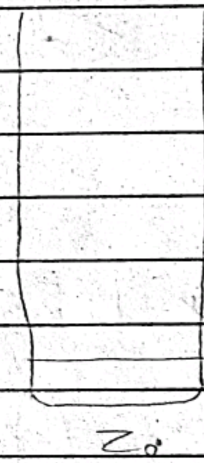
Z_0: start stack symbol

$\delta: Q \times \Sigma \times \Gamma^* \longrightarrow Q \times \Gamma^*$

or, $\delta: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma^* \longrightarrow Q \times \Gamma^*$

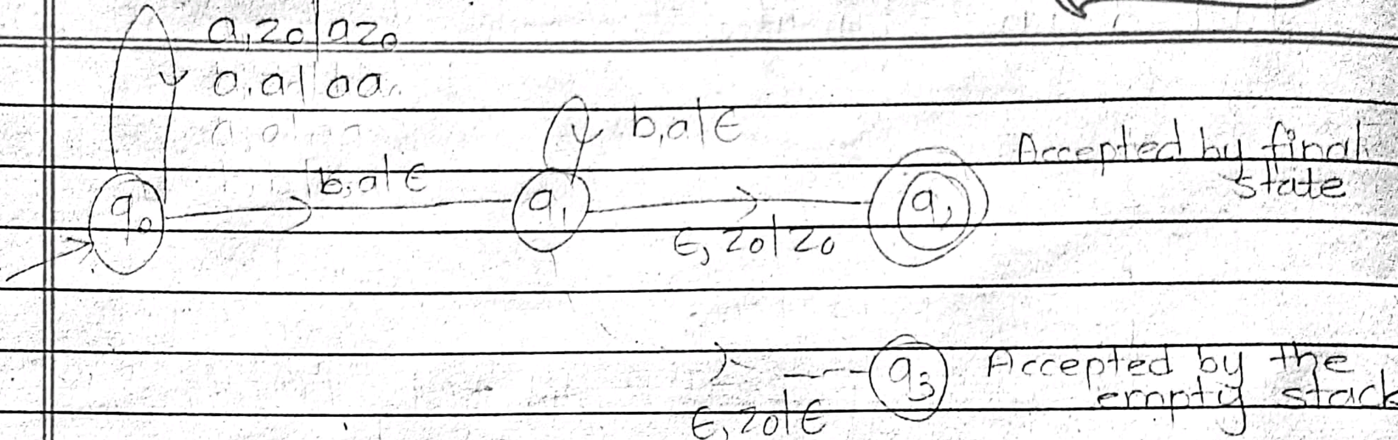
(*) Construct a PDA set of all string containing equal no. of (a&b)





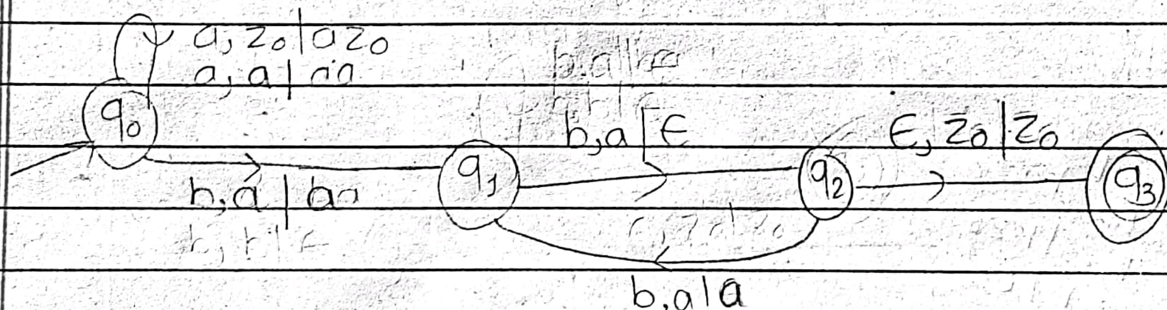
$L = \{a^n b^n \mid n \geq 1\}$

$L = \{ab, aabb, aaabbb, aaaabbbb, ---\}$



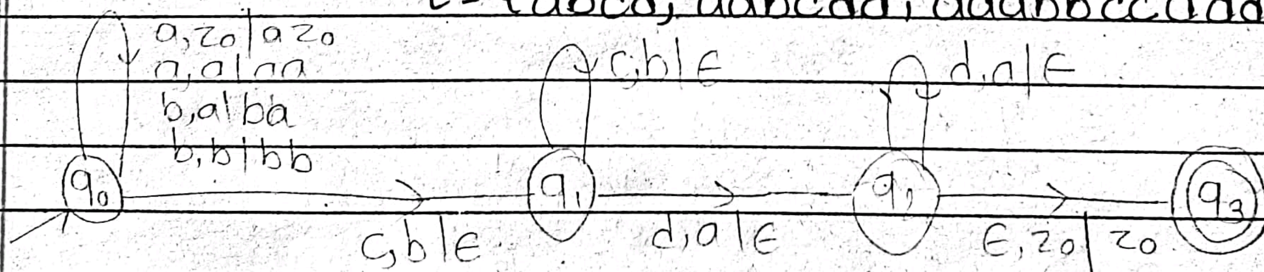
⊛ $L = \{a^n b^{2n} \mid n \geq 1\}$

$L = \{abb, aabbbb, aaabbbbbb, \ldots\}$



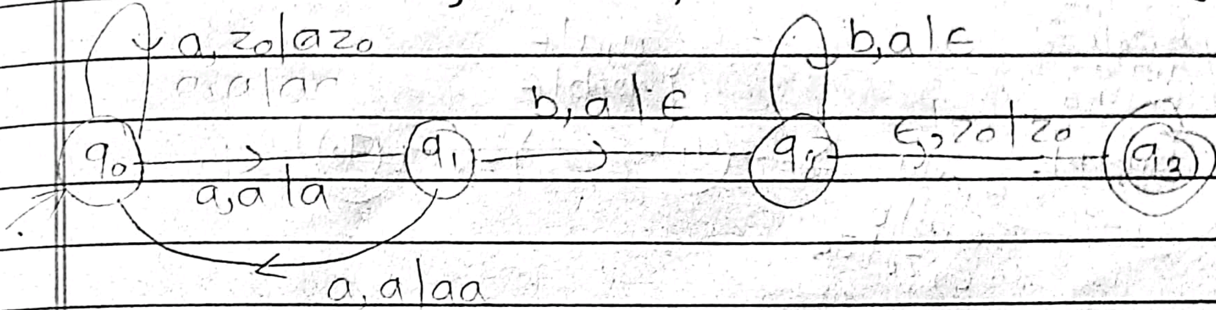
⊛ $L = \{a^n b^m c^m d^n \mid m, n \geq 1\}$

$L = \{abcd, aabcdd, aaabbccddd, \ldots\}$



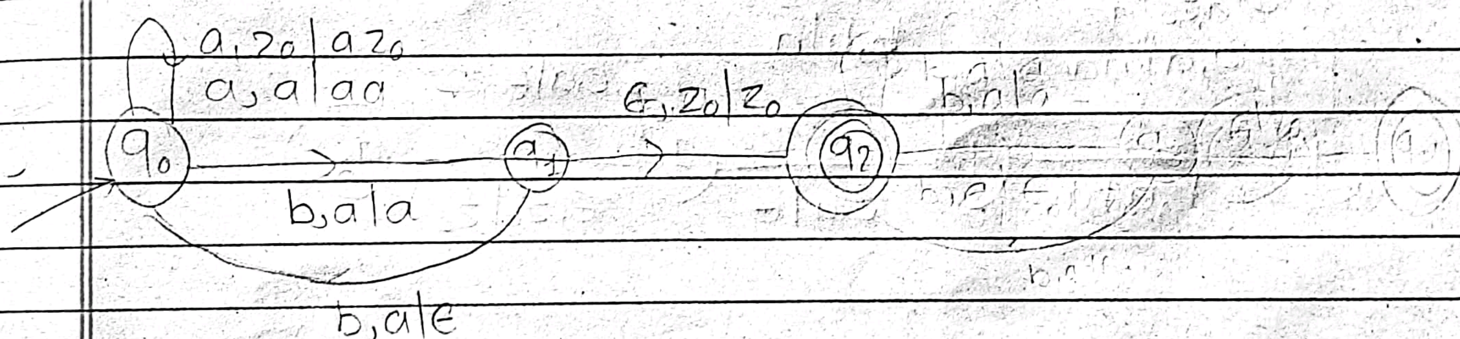
⊛ $L = \{a^{2n} b^n \mid n \geq 1\}$

$L = \{aab, aaaabb, aaaaaabbb, ---\}$



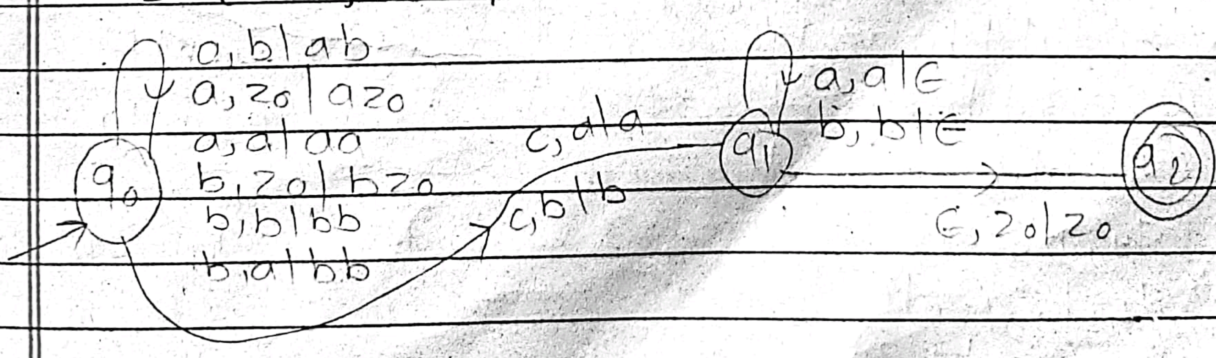
⊛ $L = \{a^n b^{2n+1} \mid n \geq 1\}$

$L = \{abbb, aabbbbb, aaabbbbbbb$



⊛ $L = \{wcw^R \mid w \in (a,b)^+\}$

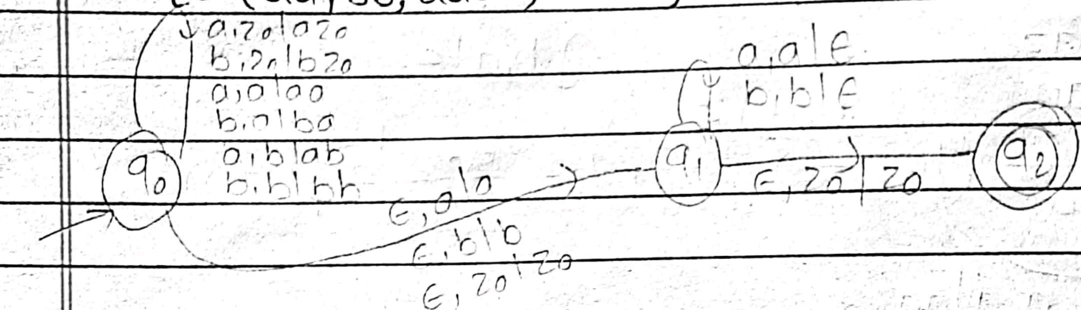
$L = \{aca, bcb, aacaa, bbcbb, bbacabb, ---\}$



Instantaneous description of PDA

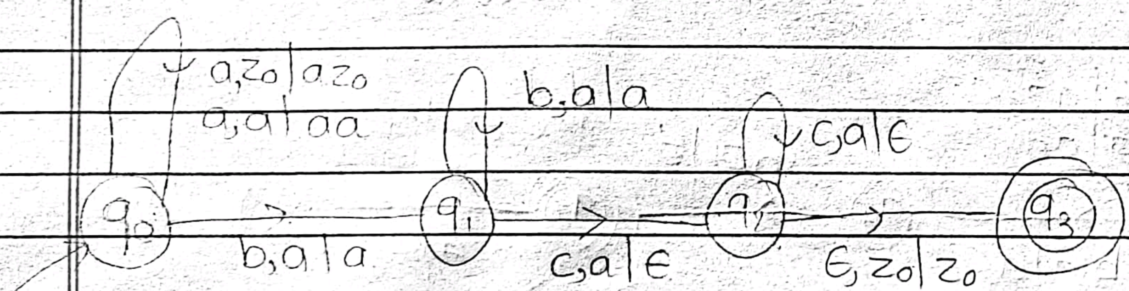
⊛ $L = \{ww^R \mid w \in (a,b)^*\}$

$L = \{aa, bb, aaaa, baab, bbbb, ...$



⊛ $L = \{a^n b^m c^n \mid m, n \geq 1\}$

$L = \{abc, aabbcc, abbc, aabbbcc ....\}$



☞ Instantaneous description of PDA

(Q, Σ, Γ)

$\delta(q_0, aaabbb, z_0)$

$\vdash (q_0, aabbb, az_0)$

$\vdash (q_0, abbb, aaz_0)$

$\vdash (q_0, bbb, aaaz_0)$

$\vdash (q_1, bb, aaz_0)$

$\vdash (q_1, b, az_0)$

$\vdash (q_1\ \epsilon, z_0)$

$\vdash (q_2\ z_0)$ or (q_2, ϵ)

accepted by final state

$\delta(q_0, w, r) \vdash^* (q_f, \Gamma)$

$\delta(q_0, w, r) \vdash^* (P, \epsilon)$

accepted by empty state

(ϵ is string)

Equivalence of ~~CG~~ CFG and PDA

CFG $\rightleftarrows$ PDA

$G = (V, T, P, S)$

$\cong$ PDA $= (\{q_0\}, T, V \cup T, S, q_0, \delta, \phi)$

δ can be defined as

① $A \to \alpha$

$\delta(q_0, \epsilon, A) \ni (q_0, \alpha)$

② $a \in T$

$\delta(q_0, a, a) = (q_0, \epsilon)$

Example: $E \to E+T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid a$

PDA $= (\{q_0\}, q_0, \{+, *, (,), a\}, \{E, T, F, +, *, (,), a\}, E, \phi)$

set of state; initial state; Input alphabet; stack alphabet; initial stack symbol; Final state

δ can be defined as:

$\delta(q_0, \epsilon, E) \ni \{(q_0, E+T), (q_0, T)\}$

$\delta(q_0, \epsilon, T) \ni \{(q_0, T*F), (q_0, F)\}$

$\delta(q_0, \epsilon, F) \ni \{q_0, (E)), (q_0, a)\}$

$\delta(q_0, +, +) \ni (q_0, \epsilon)$

$\delta(q_0, *, *) \Rightarrow (q_0, \epsilon)$

$\delta(q_0, (, () \Rightarrow (q_0, \epsilon)$

$\delta(q_0,),)) \Rightarrow (q_0, \epsilon)$

$\delta(q_0, a, a) \Rightarrow (q_0, \epsilon)$

Instantaneous description of given string:

$w = a+a*a$

$\delta(q_0, a+a*a, E)$

$\vdash (q_0, a+a*a, E+T)$

$\vdash (q_0, a+a*a, T+T)$

$\vdash (q_0, a+a*a, F+T)$

$\vdash (q_0, a+a*a, a+T)$

$\vdash (q_0, +a*a, +T)$

$\vdash (q_0, a*a, T)$

$\vdash (q_0, a*a, T*F)$

$\vdash (q_0, a*a, F*F)$

$\vdash (q_0, a*a, a*F)$

$\vdash (q_0, *a, *F)$

$\vdash (q_0, a, F)$

$\vdash (q_0, a, a)$

$\vdash (q_0, \epsilon, \epsilon)$

Hence, accepted.

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(*) $S \to aSa \mid bSb \mid \epsilon$

$PDA = (\{q_0\}, q_0, \{a, b\}, \{S, a, b\}, S, \phi)$

$\delta(q_0, \epsilon, S) \ni (q_0, aSa)$
$\delta(q_0, \epsilon, S) \ni (q_0, bSb)$
$\delta(q_0, \epsilon, S) \ni (q_0, \epsilon)$
$\delta(q_0, a, a) \ni (q_0, \epsilon)$
$\delta(q_0, b, b) \ni (q_0, \epsilon)$

$\omega = abba$

$\vdash \delta(q_0, abba, S)$
$\vdash \delta(q_0, abba, aSa)$
$\vdash \delta(q_0, bba, Sa)$
$\vdash \delta(q_0, bba, bSba)$
$\vdash \delta(q_0, ba, Sba)$
$\vdash \delta(q_0, ba, \epsilon ba)$
$\vdash \delta(q_0, ba, ba)$
$\vdash \delta(q_0, a, a)$
$\vdash \delta(q_0, \epsilon, \epsilon)$

(*) PDA $\ni$ CFG

The set of variable in the grammar:

1) S is a start variable

2) All the symbols of the form $[P, X, q]$, $P, q \in Q$ and $X \in \Gamma$

i.e. $V = \{S\} \cup \{[P, X, q]\}$

Terminal $T = \Sigma$

The production of G as follows:

For all states $q \in Q$, $S \rightarrow [q_0, z_0, q]$ is a production of G

For any states $q, r \in Q$, $X \in \Gamma$ and $a \in \Sigma \cup \{\epsilon\}$

if $\delta(q, a, X) = (p, \epsilon)$ then $[q, X, p] \rightarrow a$

For any states $q, r \in Q$, $X \in \Gamma$ and $a \in \Sigma \cup \{\epsilon\}$

if $\delta(q, a, X) \Rightarrow (r, Y_1 Y_2 \cdots Y_k)$ where

$Y_1 Y_2 \cdots Y_k \in \Gamma$ and $k \geq 0$

$[q, a, q_k] \Rightarrow a[r, Y_1, q_1]\ [q_1 \cdots Y_2] \cdots [q_{k-1}\ Y_k\ q_k]$

$L = \{a^n b^n \mid n \geq 1\}$

1) $\delta(q_0, a, z_0) \Rightarrow (q_0, az_0)$

2) $\delta(q_0, a, a) \Rightarrow (q_0, aa)$

3) $\delta(q_0, b, a) \Rightarrow (q_1, \epsilon)$

4) $\delta(q_1, b, a) \Rightarrow (q_1, \epsilon)$

5) $\delta(q_1, \epsilon, z_0) \Rightarrow (q_1, \epsilon)$

$S \rightarrow [q_0, z_0, q_0] \mid [q_0, z_0, q_1]$

$[q_0, z_0, q_0] \rightarrow a\ [q_0, a, q_0]\ [q_0, z_0, q_0]$

$[q_0, z_0, q_0] \rightarrow a\ [q_0, a, q_1]\ [q_1, z_0, q_0]$

$[q_0, z_0, q_1] \rightarrow a\ [q_0, a, q_0]\ [q_0, z_0, q_1]$

$[q_0, z_0, q_1] \rightarrow a\ [q_0, a, q_1]\ [q_1, z_0, q_1]$

$[q_0, a, q_0] \rightarrow a\ [q_0, a, q_0]\ [q_0, a, q_0]$

$[q_0, a, q_0] \rightarrow a\ [q_0, a, q_1]\ [q_1, a, q_0]$

$[q_0, a, q_1] \rightarrow a\ [q_0, a, q_0]\ [q_0, a, q_1]$

$[q_0, a, q_1] \rightarrow a\ [q_0, a, q_1]\ [q_1, a, q_1]$

$[q_0, a, q_1] \rightarrow b$

$[q_1, a, q_1] \rightarrow b$

$[q_1, z_0, q_1] \rightarrow \epsilon$

$L = \{a^n b^n \mid n \geq 1\}$

⑤ $\delta(q_1, \epsilon, z_0) \Rightarrow (q_2, z_0)$

tz^s :

$S \rightarrow [q_0, z_0\, q_0] \quad [q_0, z_0, q_1] \quad [q_0, z_0, q_2]$

$[q_0, z_0, q_0] \rightarrow a \quad [q_0, a, q_0]\ [q_0, z_0, q_0]$

$[q_0, z_0, q_0] \rightarrow a \quad [q_0, a, q_1]\ [q_1, z_0, q_0]$

$[q_0, z_0, q_0] \rightarrow a \quad [q_0, a, q_2]\ [q_2, z_0, q_0]$

$[q_0, z_0, q_1] \rightarrow a \quad [q_0, a, q_0]\ [q_0, z_0, q_1]$

$[q_0, z_0, q_1] \rightarrow a \quad [q_0, a, q_1]\ [q_1, z_0, q_1]$

$[q_0, z_0, q_1] \rightarrow a \quad [q_0, a, q_2]\ [q_2, z_0, q_1]$

$[q_0, z_0, q_2] \rightarrow a \quad [q_0, a, q_0]\ [q_0, z_0, q_2]$

$[q_0, z_0, q_2] \rightarrow a \quad [q_0, a, q_1]\ [q_1, z_0, q_2]$

$[q_0, z_0, q_2] \rightarrow a \quad [q_0, a, q_2]\ [q_2, z_0, q_2]$

$[q_0, a, q_0] \rightarrow a \quad [q_0, a, q_0]\ [q_0, a, q_0]$

$[q_0, a, q_0] \rightarrow a \quad [q_0, a, q_1]\ [q_1, a, q_0]$

$[q_0, a, q_0] \rightarrow a\ [q_0, a, q_2]\ [q_2, a, q_0]$

$[q_0, a, q_1] \rightarrow a\ [q_0, a, q_0]\ [q_0, a, q_1]$

$[q_0, a, q_1] \rightarrow a\ [q_0, a, q_1]\ [q_1, a, q_1]$

$[q_0, a, q_1] \rightarrow a\ [q_0, a, q_2]\ [q_2, a, q_1]$

$[q_0, a, q_2] \rightarrow a\ [q_0, a, q_0]\ [q_0, a, q_2]$

$[q_0, a, q_2] \rightarrow a\ [q_0, a, q_1]\ [q_1, a, q_2]$

$[q_0, a, q_2] \rightarrow a\ [q_0, a, q_2]\ [q_2, a, q_2]$

$[q_0, a, q_1] \rightarrow b$

$[q_1, a, q_1] \rightarrow b$

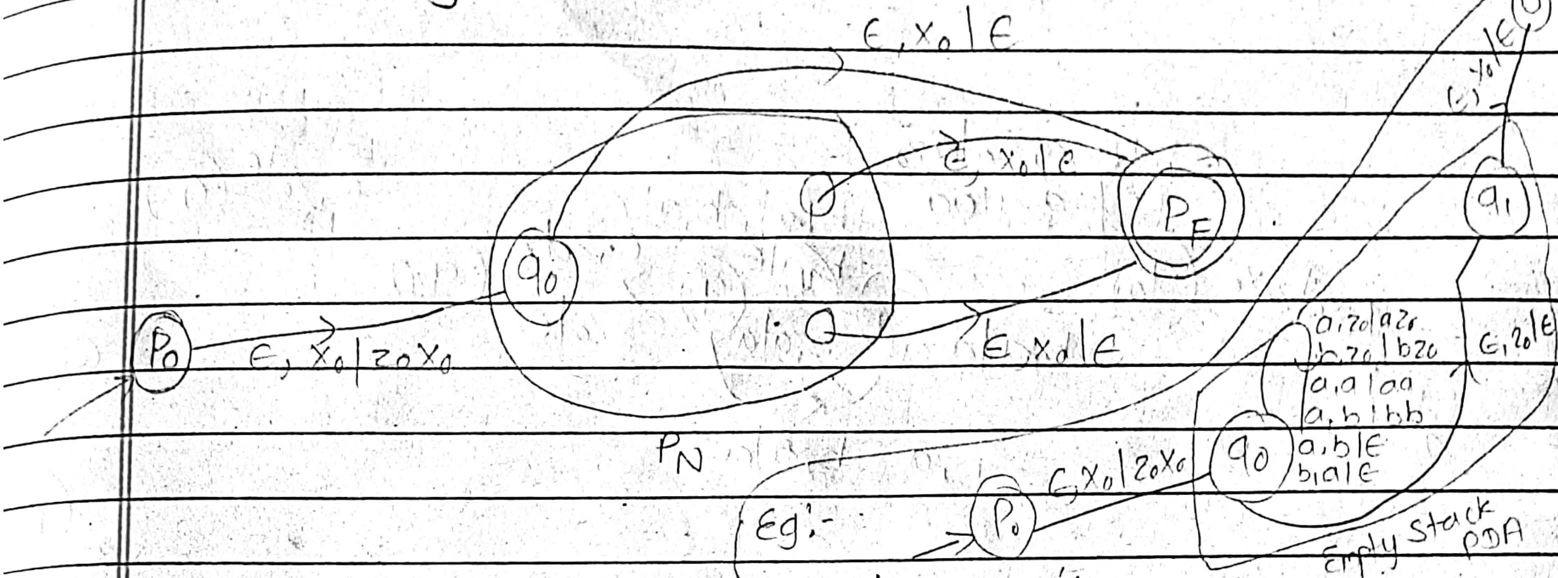
$[q_1, z_0, q_0] \rightarrow \epsilon \mid [q_2, z_0, q_0]$

$[q_1, z_0, q_1] \rightarrow \epsilon \mid [q_2, z_0, q_1]$

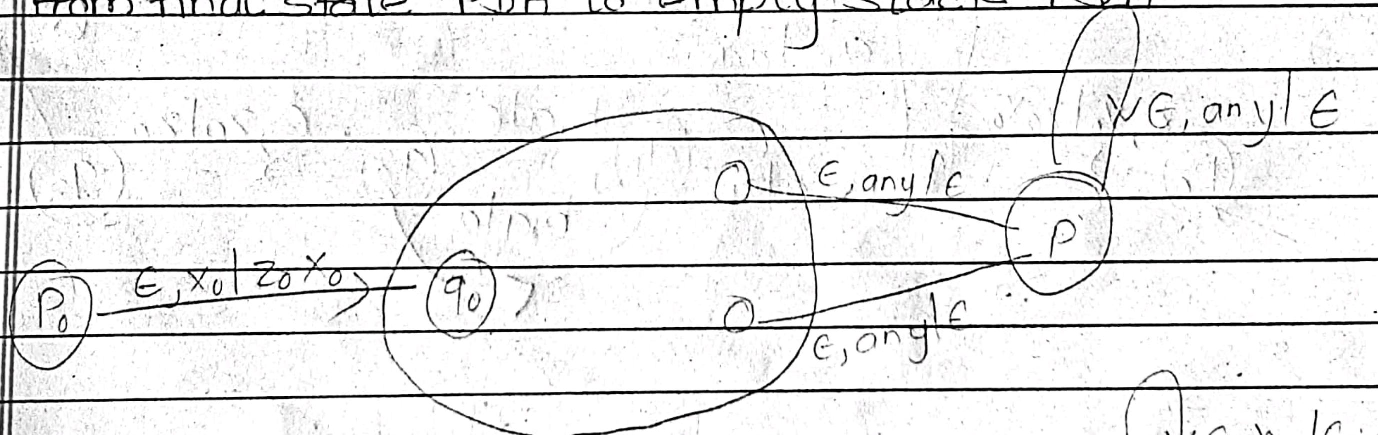
$[q_2, z_0, q_2] \rightarrow \epsilon \mid [q_2, q_2, q_2]$

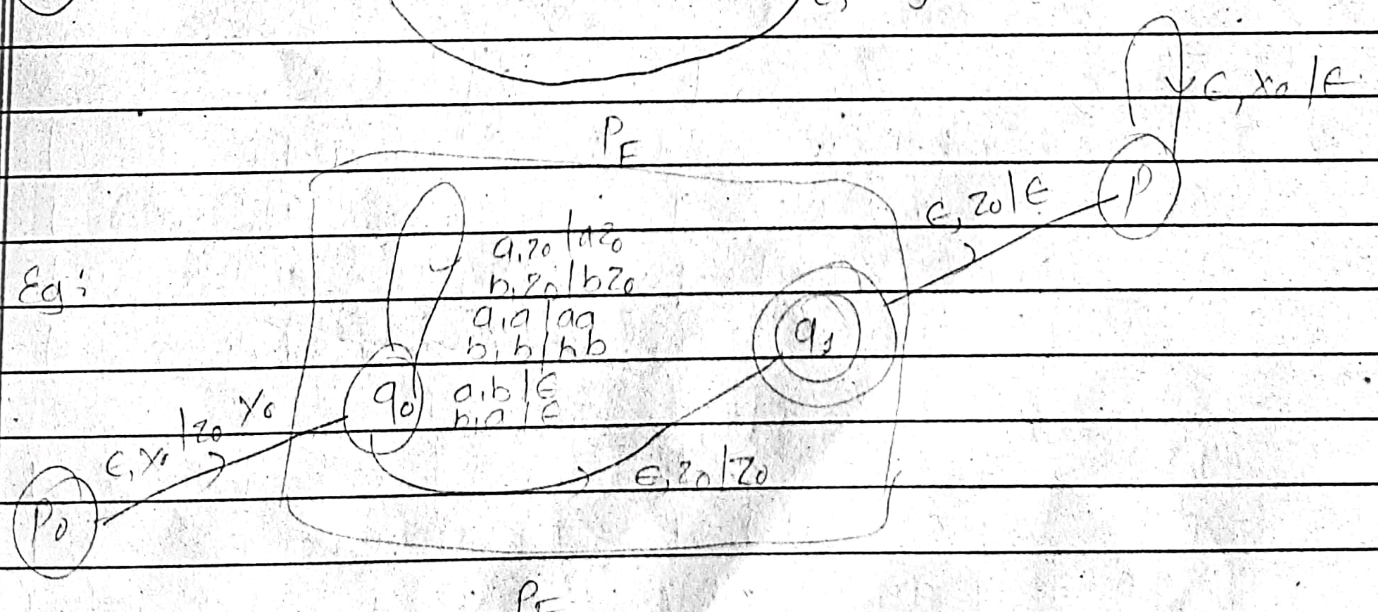
Conversion

From empty stack PDA to final state PDA



From final state PDA to empty stack PDA





a b b

L = {$a^n b^{2n}$ | $n \geq 1$}