Mathematical Models - Exercises

Data Modelling and Analysis

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Population Modelling: Fish Harvesting

You are the manager of OdysSea Inc., a fish farm that specialises in harvesting Albacore fish.

Albacore can be divided into three categories, according to their age in years:

- Young: fish in the range of (0,1) years
- Adult: fish in the range of (1,2) years
- Mature: fish in the range >2 years

Your goal is to chart the fish stocks as time elapses, making sure that you suitably harvest a proportion of each category without over- or under-harvesting, as that could lead to the destruction of stocks. As a manager, you want a steady state of Albacore so that the business is under control.

Suppose that 80% of young Albacore survive into adulthood, and that 75% of adult Albacore mature. Also suppose that 40% of mature Albacore survive without dying out from one year to the next. As you farm only has Albacore and no other species in its ecosystem, depletion is taken only as natural causes (age, disease, etc).

Using this information:

1. Create a mathematical model to monitor Albacore stocks in time.
2. Solve your mathematical model if your starting population is 100 young fish, 50 adult fish, and 20 mature fish. Consider the birth rates for adult Albacore is 2 and for mature Albacore is 1. You want to study each group in the next 10 years.

Discrete Event Models: Flu Spread in Jubilee Campus[1]

A flu is spreading across Jubilee Campus. The University's Health Centre is interested in knowing and experimenting with a model for this flu, to see if it could become an epidemic[2]. They have hired you to create and use this model.

The Health Centre is interested in modelling three different populations:

- Susceptible subjects: those who haven't been infected.
- Infected subjects: those who are currently experiencing the flu and can transmit it
- Removed subjects: those who were infected by the flu, but are no longer sick with it and cannot transmit it any more.

You are asked to incorporate the following assumptions into your model:

1. No one enters or leaves the community
2. There is no contact with the outside.
3. Each person is either Susceptible (S), Infected (I), or Removed (R).
4. Initially, each person is either S or I.
5. Once someone gets the flu this year, they cannot get the flu again.
6. The average length of the flu is 5/3 weeks (1 and 2/3 weeks).
7. You need to monitor weekly changes.
8. The disease spreads in 0.14% of cases with contact

Answer the following questions:

1. According to the information given below, create a mathematical model ({S (Sr), Q, M}) to study the weekly spread of the disease.
2. If the total population of the campus is 1000 people and the number of infected subjects on Week 0 was 5, obtain the changes in each population for the first 25 weeks. Plot all of them together to visualise how they develop during that time. (Hint: You may create an R script to do this. It will be faster than calculating 25x3 population values!)

[1] I am aware of the irony of teaching remotely about how to model flu-spread in the middle of a flu-adjacent pandemic, but alas, mathematical modelling deals with these problems by nature! ¯\(•_•)/¯

[2] *sighs*