IISER Kolkata

PROJECT REPORT

(Research Methodology)

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Bell test using Quantum Computers

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1 John Bells' Experiment

Bell tests are experiments that test whether the theory of Quantum mechanics satisfies the concept of local realism, which requires the presence of hidden variables.

1.1 EPR Paradox

EPR paradox presented by Einstein, Podolsky and Rosen that Quantum Mechanics is incomplete description of reality and there must be some hidden variables, that makes the observables like momentum, position deterministic in nature.

The thought experiment involves a pair of particles prepared in an entangled states. Einstein, Podolsky, and Rosen pointed out that, in this state, if the position of the first particle were measured, the result of measuring the position of the second particle could be predicted. If, instead, the momentum of the first particle were measured, then the result of measuring the momentum of the second particle could be predicted. They argued that no action taken on the first particle could instantaneously affect the other, since this would involve information being transmitted faster than light, which is forbidden by the theory of relativity.

1.1.1 EPR criterion of reality

"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity". From this, they inferred that the second particle must have a definite value of position and of momentum prior to either being measured. This contradicted the view associated with Bohr and Heisenberg, according to which a quantum particle does not have a definite value of a property like momentum until the measurement takes place.

1.2 Bells Test: Theory

A Bell test experiment or Bell's inequality experiment, also simply a Bell test, is to test the theory of QM in relation to Einstein's concept of local realism. The experiments test whether or not the real world satisfies local realism, which requires the presence of some local variables (called "hidden")to explain the behavior of quantum particles. If nature actually operates in accord with any theory of local hidden variables, then the results of a Bell test will be constrained in a particular, quantifiable way.

If a Bell test is performed in a laboratory and the results are not thus constrained, then they are inconsistent with the hypothesis that local hidden variables exist. Such results would support the position that there is no way to explain the phenomena of quantum mechanics in terms of a more fundamental description of nature that is more in line with classical physics..

1.3 Apparatus

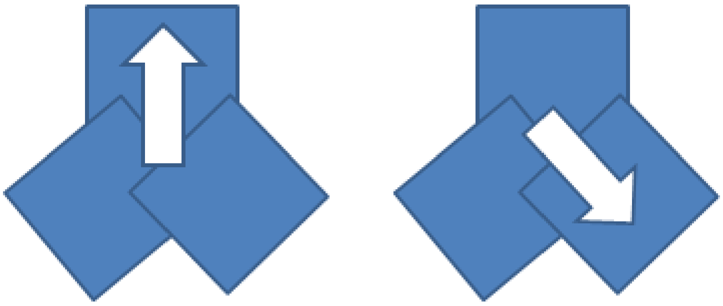

Figure 1: detectors with 3 directions of measurement

- Two spin detectors that can measure the spin of a particle in three directions.
- The direction of measurement is chosen randomly.
- We record whether the result of measurement of the spin of the particles is same, or different.

1.4 Probabilities if hidden variables exist

In this section we discuss the probability of getting opposite outcomes for the two detector's (one 'up' and the other 'down' state). The result of the measurement along each direction is pre-decided. Since the particles are entangled, the second particle will have opposite spin to the first particle corresponding to any given direction. There can only be 8 different combinations of 4 types:

- 3 $\times$ (1 up and 2 down)
- 3 $\times$ (1 down and 2 up)

- 1 × (all up)
- 1 × (all down)

1.4.1 1 up and 2 down

Consider a combination of this type where Particle 1 chooses ($\uparrow\downarrow\downarrow$), where the first,second and third symbols correspond to chosen spin along axis 1(z-axis), axis 2 (at +2pi/3 to z direction), and axis 3(at -2pi/3 to z direction).

Since the angular momentum along each axis is conserved, the second particle must choose ($\downarrow\uparrow\uparrow$).

Now there are 9 different ways in which the axis of measurement of both the detectors can be chosen :

Random choice by		
Detector 1	Detector 2	outcome
1	1	different
1	2	same
1	3	same
2	1	same
2	2	different
2	3	different
3	1	same
3	2	different
3	3	different

We observe that 5 out of 9 outcomes give different spin measurements. Therefore the probability is $\frac{5}{9}$.

1.4.2 all ups

Only combination for first particle - ($\uparrow\uparrow\uparrow$).
Hence combination for second particle-($\downarrow\downarrow\downarrow$)
Since both particles choose different spin for all directions, the probability of observing different combinations for any random choice of direction by the detectors is 1.

1.4.3 all other cases

- Since all directions randomly chosen by detectors are equivalent under ordered rotation, we will get the same result for all combinations of the same type (1 of them is up, other 2 down .etc)
- up down directions are also equivalent for the calculation of probabilities,for eg. for first particle 1 up,2 downs should give the same probability of different outcomes as 1 down, 2 ups and similarly all downs will give the same answers as all ups.

If we assume that the particles choose a combination randomly. Then the probability of observing different outcomes of the particles will be :

$$\frac{3}{8} \times \frac{5}{9} \times 2 + \frac{1}{8} \times 1 \times 2 = \frac{2}{3} \tag{1}$$

i.e. two thirds of the measurements should give different outcomes of the particles.

2 Experiment on IBM Quantum Computer "ibmq-vigo"

2.1 Steps

Steps we need to perform:-

- First we need to form the Entangled Particle.
- Choose 3 different axis to measure along.
- Repeat the experiment with every possible unique combination.
- Take all the results and find the probability of finding the particle in opposite spin.

2.2 Entangled Particle

We need 2 particle that are in entangled state that too with opposite spins. So we use H gate, CX gate and then X gate to accomplish that making a bell state.

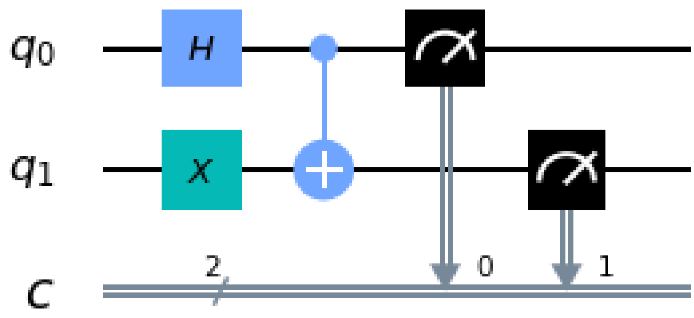

Figure 2: Circuit for the Bell State formation

Bell State : $|\Psi >= \frac{1}{2}(|10 > +|01 >)$

2.3 Axis of Measurement

According to the experiment we need 3 Axis for measurement.

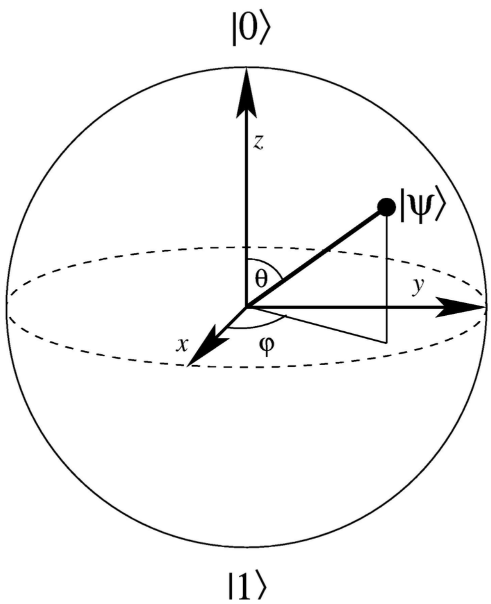

Figure 3: Representation of states on Bloch Sphere

We can choose one axis to be Z axis and other to be 2pi/3 and -2pi/3 angle with Z axis in XZ plane. We got direction one as $\theta = \phi = 0$, second as $\theta =$ 2pi/3 and $\phi =$ 0 and last direction as theta = -$2\pi/3$ and $\phi = \pi$.

2.4 Finding Similarities

We can Clearly state that when the first particle and second particle are measured in same direction they behave same irrespective of the direction they are being measured.

So the direction pair [0,0],[$2\pi/3$,$2\pi/3$] and [-$2\pi/3$,-$2\pi/3$] can be summarised by just taking measurement for one of the case. We choose it to be [0,0].

Same goes for [0,$2\pi/3$], [$2\pi/3$,-$2\pi/3$], [-$2\pi/3$,0] set as well. We choose [0,$2\pi/3$] to experiment with.

And at last [0,-$2\pi/3$], [-$2\pi/3$,$2\pi/3$], [$2\pi/3$,0] and for this set we choose [0,-$2\pi/3$] to be experimented on.

2.5 Calculating Probability

Taking,

P[0,0] as the probability of particle being detected in opposite spin when detector is in [0,0] state.

P[0,$2\pi/3$] as the probability of particle being detected in opposite spin when detector is in [0,$2\pi/3$] state.

P[0,-$2\pi/3$] as the probability of particle being detected in opposite spin when detector is in [0,-$2\pi/3$] state.

We got,

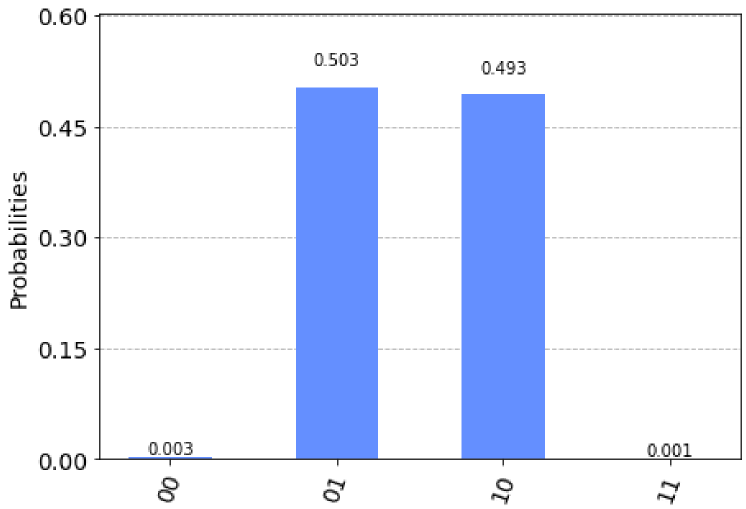

Figure 4: Representation of states on Bloch Sphere

P[0,0] = 0.993

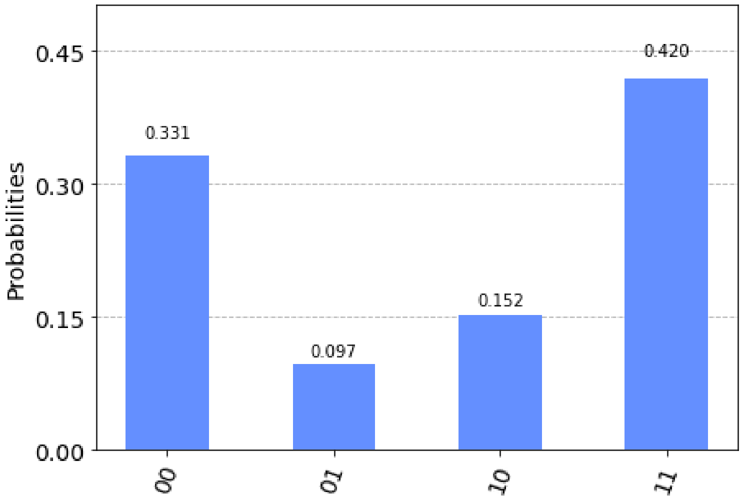

Figure 5: Representation of states on Bloch Sphere

P[0,$2\pi/3$] = 0.249

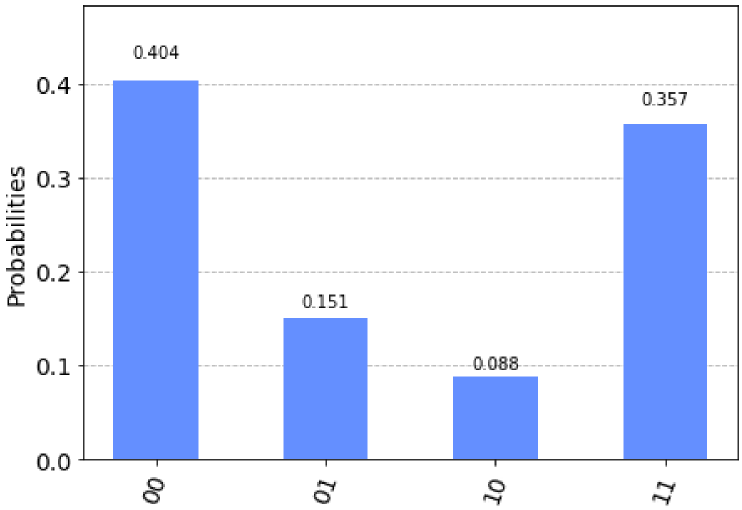

Figure 6: Representation of states on Bloch Sphere

P[0,-$2\pi/3$] = 0.238

So,

$$P = \frac{3}{9}P[0,0] + \frac{3}{9}P[0,2\pi/3] + \frac{3}{9}P[0,-2\pi/3] \tag{2}$$

$$\tag{3}$$

$$= \frac{1}{3}P[0,0] + \frac{1}{3}P[0,2\pi/3] + \frac{1}{3}P[0,-2\pi/3] \tag{4}$$

$$\tag{5}$$

$$= \frac{0.993 + 0.249 + 0.238}{3} \tag{6}$$

$$= 0.494 \tag{7}$$

So P total will be equal to 0.494 which is no where near 0.66. And it can be rounded of to 0.5.

P = 0.5

So it Falsifies the Theory of Hidden Variable and strengthen the Theory of Entanglement.

Resource

- `https://github.com/CodieKev/PH4109_FInal_Project.git`, Github link to all codes and presentation.
- `https://www.ibm.com/quantum-computing/`, Quantum Computing IBM.
- BELL'S THEOREM : THE NAIVE VIEW OF AN EXPERIMENTALIST.- Alain Aspect