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In[43]:= Z = PauliMatrix[3]
Out[43]= {{1, 0}, {0, -1}}

In[44]:= X = PauliMatrix[1]
Out[44]= {{0, 1}, {1, 0}}

In[45]:= Z0 = KroneckerProduct[Z, IdentityMatrix[2^8]];
In[46]:= Z1 = KroneckerProduct[IdentityMatrix[2], Z, IdentityMatrix[2^7]];
In[47]:= Z2 = KroneckerProduct[IdentityMatrix[2^2], Z, IdentityMatrix[2^6]];
In[48]:= Z3 = KroneckerProduct[IdentityMatrix[2^3], Z, IdentityMatrix[2^5]];
In[49]:= Z4 = KroneckerProduct[IdentityMatrix[2^4], Z, IdentityMatrix[2^4]];
In[50]:= Z5 = KroneckerProduct[IdentityMatrix[2^5], Z, IdentityMatrix[2^3]];
In[51]:= Z6 = KroneckerProduct[IdentityMatrix[2^6], Z, IdentityMatrix[2^2]];
In[52]:= Z7 = KroneckerProduct[IdentityMatrix[2^7], Z, IdentityMatrix[2^1]];
In[53]:= Z8 = KroneckerProduct[IdentityMatrix[2^8], Z];
In[54]:= H = (12 * 3 IdentityMatrix[2^9] -
  (Z0.Z1 + Z0.Z3 + Z1.Z2 + Z1.Z3 + Z2.Z4 + Z3.Z4 + Z4.Z5 + Z4.Z8 + Z5.Z6 + Z7.Z8 +
    Z6.Z7 + Z6.Z8) - (2 Z0 + 3 Z1 + 2 Z2 + 3 Z3 + 4 Z4 + 2 Z5 + 3 Z6 + 2 Z7 + 3 Z8)) / 4;
In[55]:= UP = {{1}, {0}};
In[56]:= DW = {{0}, {1}};
In[57]:= Test = KroneckerProduct[DW, DW, DW, DW, DW, DW, DW, DW, DW];
In[58]:= TestUp = KroneckerProduct[UP, UP, UP, UP, UP, UP, UP, UP, UP];
In[59]:= TestDw = KroneckerProduct[DW, DW, DW, DW, DW, DW, DW, DW, DW];
In[60]:= CostOfTest = Transpose[TestUp].H.Test;
In[61]:= X0 = KroneckerProduct[X, IdentityMatrix[2^8]];
In[62]:= X1 = KroneckerProduct[IdentityMatrix[2], X, IdentityMatrix[2^7]];
In[63]:= X2 = KroneckerProduct[IdentityMatrix[2^2], X, IdentityMatrix[2^6]];
In[64]:= X3 = KroneckerProduct[IdentityMatrix[2^3], X, IdentityMatrix[2^5]];
In[65]:= X4 = KroneckerProduct[IdentityMatrix[2^4], X, IdentityMatrix[2^4]];
In[66]:= X5 = KroneckerProduct[IdentityMatrix[2^5], X, IdentityMatrix[2^3]];
In[67]:= X6 = KroneckerProduct[IdentityMatrix[2^6], X, IdentityMatrix[2^2]];
In[68]:= X7 = KroneckerProduct[IdentityMatrix[2^7], X, IdentityMatrix[2^1]];
In[69]:= X8 = KroneckerProduct[IdentityMatrix[2^8], X];
In[70]:=
In[71]:= ExpCost = MatrixExp[-I * A * H];
In[72]:=

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In[73]:= Rx = MatrixExp[-I * B * PauliMatrix[1]];
In[74]:= ExpRot = KroneckerProduct[Rx, Rx, Rx, Rx, Rx, Rx, Rx, Rx, Rx];
In[75]:= ExpRotT = Refine[ConjugateTranspose[ExpRot], {Element[B, Reals]}];
In[76]:= ExpCostT = Refine[ConjugateTranspose[ExpCost], {Element[A, Reals]}];
In[77]:= PM = HadamardMatrix[2^9].KroneckerProduct[UP, UP, UP, UP, UP, UP, UP, UP, UP];
In[78]:= Result = Transpose[PM].ExpCostT.ExpRotT.H.ExpRot.ExpCost.PM

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Out[78]=

$$\begin{aligned}
& \left\{ \left\{ \frac{-12 i \sin[B]^9 \left(\frac{e^{12 i A} \cos[B]^9}{16 \sqrt{2}} + \dots 44 \dots + \frac{e^{2 i A} \cos[B] \sin[B]^8}{4 \sqrt{2}} + \frac{e^{3 i A} \cos[B] \sin[B]^8}{4 \sqrt{2}} + \frac{e^{4 i A} \cos[B] \sin[B]^8}{16 \sqrt{2}} + \frac{i \sin[B]^9}{16 \sqrt{2}} \right) + \dots 405 \dots}{16 \sqrt{2}} + \right. \right. \\
& \left. \frac{e^{-4 i A} \left(12 i \cos[B]^2 \sin[B]^7 \left(\frac{e^{12 i A} \cos[B]^9}{16 \sqrt{2}} + \dots 45 \dots + \frac{e^{3 i A} \cos[B] \sin[B]^8}{4 \sqrt{2}} + \frac{e^{4 i A} \cos[B] \sin[B]^8}{16 \sqrt{2}} + \frac{i \sin[B]^9}{16 \sqrt{2}} \right) + \dots 405 \dots \right)}{16 \sqrt{2}} + \right. \\
& \frac{e^{-4 i A} \left(\dots 1 \dots \right)}{16 \sqrt{2}} + \frac{e^{-3 i A} \left(\dots 1 \dots \right)}{8 \sqrt{2}} + \frac{e^{-3 i A} \left(12 \cos[B] \left(\dots 1 \dots \right)^8 \left(\frac{\dots 1 \dots}{16 \sqrt{2}} + \dots 48 \dots \right) + \dots 597 \dots \right)}{8 \sqrt{2}} + \\
& \frac{e^{\dots 1 \dots} \left(\dots 1 \dots \right)}{8 \sqrt{2}} + \dots 260 \dots + \frac{e^{\dots 1 \dots} \left(\dots 1 \dots \right)}{8 \sqrt{2}} + \\
& \frac{e^{\dots 1 \dots} \left(\dots 1 \dots \right)}{8 \sqrt{2}} + \frac{e^{-12 i A} \left(\dots 1 \dots \right)}{8 \sqrt{2}} + \frac{e^{-12 i A} \left(\dots 1 \dots \right)}{16 \sqrt{2}} + \\
& \left. \frac{e^{-12 i A} \left(\dots 405 \dots + 4 \dots 2 \dots \left(\frac{e^{2 i A} \cos[B]^9}{16 \sqrt{2}} + \frac{\dots 1 \dots}{16 \sqrt{2}} + \dots 61 \dots + \frac{\dots 1 \dots}{16 \sqrt{2}} + \frac{i e^{\dots 1 \dots} \sin[\dots 1 \dots] \dots 1 \dots}{16 \sqrt{2}} \right) \right)}{16 \sqrt{2}} + \right. \\
& \left. \frac{1}{8 \sqrt{2}} e^{-12 i A} \left(-12 i \cos[B]^8 \sin[B] \left(\frac{e^{12 i A} \cos[B]^9}{16 \sqrt{2}} + \dots 44 \dots + \frac{e^{2 i A} \cos[B] \sin[B]^8}{4 \sqrt{2}} + \right. \right. \right. \\
& \left. \left. \frac{e^{3 i A} \cos[B] \sin[B]^8}{4 \sqrt{2}} + \frac{e^{4 i A} \cos[B] \sin[B]^8}{16 \sqrt{2}} + \frac{i \sin[B]^9}{16 \sqrt{2}} \right) + \dots 597 \dots \right) \left. \right\} \left. \right\}
\end{aligned}$$

large output show less show more show all set size limit...

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In[82]:= F1 = Simplify[Result]

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$$\begin{aligned}
\text{Out[82]} = & \left\{ \left\{ \frac{1}{512} e^{-6 i A} \left(1 + 11 e^{i A} + 46 e^{2 i A} + 55 e^{3 i A} + 31 e^{4 i A} - 66 e^{5 i A} + 4452 e^{6 i A} - \right. \right. \right. \\
& 66 e^{7 i A} + 31 e^{8 i A} + 55 e^{9 i A} + 46 e^{10 i A} + 11 e^{11 i A} + e^{12 i A} - (-1 + e^{2 i A})^2 \\
& \left. \left(1 + 11 e^{i A} + 48 e^{2 i A} + 77 e^{3 i A} + 126 e^{4 i A} + 77 e^{5 i A} + 48 e^{6 i A} + 11 e^{7 i A} + e^{8 i A} \right) \cos[4 B] - \right. \\
& 16 i e^{2 i A} (1 + e^{i A})^3 (-1 - 7 e^{i A} - 8 e^{2 i A} + 8 e^{3 i A} + 7 e^{4 i A} + e^{5 i A}) \sin[2 B] + \\
& 8 i e^{2 i A} \sin[4 B] + 64 i e^{3 i A} \sin[4 B] + 112 i e^{4 i A} \sin[4 B] - 64 i e^{5 i A} \sin[4 B] + \\
& \left. \left. 64 i e^{7 i A} \sin[4 B] - 112 i e^{8 i A} \sin[4 B] - 64 i e^{9 i A} \sin[4 B] - 8 i e^{10 i A} \sin[4 B] \right) \right\} \left. \right\}
\end{aligned}$$

$$\begin{aligned} \text{In[90]:= Cost} &= \text{Simplify}\left[\frac{1}{512} (\cos[6A] - i \sin[6A]) \right. \\ &\quad \left(1 + 11 (\cos[A] + i \sin[A]) + 46 (\cos[2A] + i \sin[2A]) + 55 (\cos[3A] + i \sin[3A]) + \right. \\ &\quad \left. 31 (\cos[4A] + i \sin[4A]) - 66 (\cos[5A] + i \sin[5A]) + 4452 (\cos[6A] + i \sin[6A]) - \right. \\ &\quad \left. 66 (\cos[7A] + i \sin[7A]) + 31 (\cos[8A] + i \sin[8A]) + 55 (\cos[9A] + i \sin[9A]) + \right. \\ &\quad \left. 46 (\cos[10A] + i \sin[10A]) + 11 (\cos[11A] + i \sin[11A]) + (\cos[12A] + i \sin[12A]) - \right. \\ &\quad \left. (-1 + (\cos[2A] + i \sin[2A]))^2 (1 + 11 (\cos[A] + i \sin[A]) + 48 (\cos[2A] + i \sin[2A]) + \right. \\ &\quad \left. 77 (\cos[3A] + i \sin[3A]) + 126 (\cos[4A] + i \sin[4A]) + 77 (\cos[5A] + i \sin[5A]) + \right. \\ &\quad \left. 48 (\cos[6A] + i \sin[6A]) + 11 (\cos[7A] + i \sin[7A]) + (\cos[8A] + i \sin[8A]) \right) \\ &\quad \cos[4B] - 16i (\cos[2A] + i \sin[2A]) (1 + (\cos[A] + i \sin[A]))^3 \\ &\quad (-1 - 7 (\cos[A] + i \sin[A]) - 8 (\cos[2A] + i \sin[2A]) + 8 (\cos[3A] + i \sin[3A]) + \\ &\quad \left. 7 (\cos[4A] + i \sin[4A]) + (\cos[5A] + i \sin[5A]) \right) \sin[2B] + \\ &\quad 8i (\cos[2A] + i \sin[2A]) \sin[4B] + 64i (\cos[3A] + i \sin[3A]) \sin[4B] + \\ &\quad 112i (\cos[4A] + i \sin[4A]) \sin[4B] - 64i (\cos[5A] + i \sin[5A]) \sin[4B] + \\ &\quad 64i (\cos[7A] + i \sin[7A]) \sin[4B] - 112i (\cos[8A] + i \sin[8A]) \sin[4B] - \\ &\quad \left. 64i (\cos[9A] + i \sin[9A]) \sin[4B] - 8i (\cos[10A] + i \sin[10A]) \sin[4B] \right) \end{aligned}$$

$$\begin{aligned} \text{Out[90]=} &\frac{1}{512} (4452 - 132 \cos[A] + 62 \cos[2A] + 110 \cos[3A] + 92 \cos[4A] + 22 \cos[5A] + \\ &2 \cos[6A] + 2 \cos[A - 4B] + 9 \cos[3A - 4B] - 11 \cos[5A - 4B] - \cos[6A - 4B] + \\ &608 \cos[A - 2B] + 81 \cos[2(A - 2B)] + 160 \cos[3A - 2B] + 16 \cos[4A - 2B] + \\ &512 \cos[2(A - B)] - 38 \cos[4(A - B)] + 156 \cos[4B] - 512 \cos[2(A + B)] - \\ &54 \cos[4(A + B)] - 16 \cos[2(2A + B)] - 608 \cos[A + 2B] - 143 \cos[2(A + 2B)] - \\ &160 \cos[3A + 2B] + 130 \cos[A + 4B] - 119 \cos[3A + 4B] - 11 \cos[5A + 4B] - \cos[6A + 4B]) \end{aligned}$$

$$\text{In[91]:= SolutionCord}[A_, B_] = \text{Cost}$$

$$\begin{aligned} \text{Out[91]=} &\frac{1}{512} (4452 - 132 \cos[A] + 62 \cos[2A] + 110 \cos[3A] + 92 \cos[4A] + 22 \cos[5A] + \\ &2 \cos[6A] + 2 \cos[A - 4B] + 9 \cos[3A - 4B] - 11 \cos[5A - 4B] - \cos[6A - 4B] + \\ &608 \cos[A - 2B] + 81 \cos[2(A - 2B)] + 160 \cos[3A - 2B] + 16 \cos[4A - 2B] + \\ &512 \cos[2(A - B)] - 38 \cos[4(A - B)] + 156 \cos[4B] - 512 \cos[2(A + B)] - \\ &54 \cos[4(A + B)] - 16 \cos[2(2A + B)] - 608 \cos[A + 2B] - 143 \cos[2(A + 2B)] - \\ &160 \cos[3A + 2B] + 130 \cos[A + 4B] - 119 \cos[3A + 4B] - 11 \cos[5A + 4B] - \cos[6A + 4B]) \end{aligned}$$

$$\text{In[92]:= FindMaximum}[\{\text{SolutionCord}[A, B], 0 < A < 3.14 \&\& 0 < B < 3.14\}, \{A, B\}]$$

$$\text{Out[92]=} \{9.26982, \{A \rightarrow 2.26383, B \rightarrow 2.85107\}\}$$