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In[ ]:= Z = PauliMatrix[3];
X = PauliMatrix[1];
RxB1 = MatrixExp[-I * B1 * PauliMatrix[1]];
RxB1t = MatrixExp[I * B1 * PauliMatrix[1]];
RxB2 = MatrixExp[-I * B2 * PauliMatrix[1]];
RxB2t = MatrixExp[I * B2 * PauliMatrix[1]];

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In[ ]:= Z40 = KroneckerProduct[Z, IdentityMatrix[2^3]];
Z41 = KroneckerProduct[IdentityMatrix[2], Z, IdentityMatrix[4]];
Z42 = KroneckerProduct[IdentityMatrix[4], Z, IdentityMatrix[2]];
Z43 = KroneckerProduct[IdentityMatrix[8], Z];

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In[ ]:= H44 = (12 IdentityMatrix[16] - (Z40.Z41 + Z40.Z43 +
      Z41.Z42 + Z41.Z43 + 3 Z41 + 2 Z40 + Z42 + 2 Z43) + Z40 + Z41 + Z42 + Z43);
H41 = 3 IdentityMatrix[16] - Z40.Z41;

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In[ ]:= U4G1 = MatrixExp[-I * G1 * H44];
U4G2 = MatrixExp[-I * G2 * H44];
U4G1t = Refine[ConjugateTranspose[U4G1], {Element[G1, Reals]}];
U4G2t = Refine[ConjugateTranspose[U4G2], {Element[G2, Reals]}];
V4B1 = KroneckerProduct[RxB1, RxB1, RxB1, RxB1];
V4B2 = KroneckerProduct[RxB2, RxB2, RxB2, RxB2];
V4B1t = KroneckerProduct[RxB1t, RxB1t, RxB1t, RxB1t];
V4B2t = KroneckerProduct[RxB2t, RxB2t, RxB2t, RxB2t];

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In[ ]:= UP = {{1}, {0}};
PM4 = HadamardMatrix[2^4].KroneckerProduct[UP, UP, UP, UP];

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In[ ]:= R1 = U4G1.PM4;

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In[ ]:= R2 = Simplify[V4B1.R1]

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Out[ ]:= { { 1/4 e^{-16 i G1} (e^{12 i G1} Cos[B1]^4 - i e^{2 i G1} (1 + e^{4 i G1})^2 Cos[B1]^3 Sin[B1] -
      2 Cos[B1]^2 Sin[B1]^2 + 4 i e^{2 i G1} Cos[B1] Sin[B1]^3 + e^{4 i G1} (Sin[B1]^4 - Sin[2 B1]^2) ) },
    { 1/4 e^{-16 i G1} (e^{6 i G1} Cos[B1]^4 - i (1 + 2 e^{4 i G1} + e^{12 i G1}) Cos[B1]^3 Sin[B1] - e^{6 i G1} (1 + e^{4 i G1})
      Cos[B1]^2 Sin[B1]^2 + i (1 + 3 e^{4 i G1}) Cos[B1] Sin[B1]^3 + e^{2 i G1} (Sin[B1]^4 - Sin[2 B1]^2) ) },
    { 1/4 e^{-16 i G1} (e^{10 i G1} Cos[B1]^4 - i e^{4 i G1} (3 + e^{8 i G1}) Cos[B1]^3 Sin[B1] - 2 e^{6 i G1} Cos[B1]^2 Sin[B1]^2 +
      2 i (1 + e^{4 i G1}) Cos[B1] Sin[B1]^3 + e^{2 i G1} (Sin[B1]^4 - Sin[2 B1]^2) ) },
    { 1/4 e^{-16 i G1} (e^{4 i G1} Cos[B1]^4 - i e^{2 i G1} (2 + e^{4 i G1} + e^{8 i G1}) Cos[B1]^3 Sin[B1] -
      (1 + e^{12 i G1}) Cos[B1]^2 Sin[B1]^2 +
      i e^{2 i G1} (3 + e^{4 i G1}) Cos[B1] Sin[B1]^3 + Sin[B1]^4 - e^{4 i G1} Sin[2 B1]^2) },
    { 1/4 e^{-16 i G1} (e^{2 i G1} Cos[B1]^4 - i (2 + e^{4 i G1} + e^{12 i G1}) Cos[B1]^3 Sin[B1] - e^{2 i G1}
      (3 + 2 e^{4 i G1} + e^{8 i G1}) Cos[B1]^2 Sin[B1]^2 + 4 i e^{4 i G1} Cos[B1] Sin[B1]^3 + e^{2 i G1} Sin[B1]^4) },
    { 1/4 e^{-16 i G1} (Cos[B1]^4 - i e^{2 i G1} (3 + e^{4 i G1}) Cos[B1]^3 Sin[B1] - (1 + e^{12 i G1}) Cos[B1]^2 Sin[B1]^2 +
      i e^{2 i G1} (2 + e^{4 i G1} + e^{8 i G1}) Cos[B1] Sin[B1]^3 + e^{4 i G1} (Sin[B1]^4 - Sin[2 B1]^2) ) },
    { 1/4 e^{-16 i G1} (e^{4 i G1} Cos[B1]^4 - i e^{2 i G1} (3 + e^{8 i G1}) Cos[B1]^3 Sin[B1] - (2 + 3 e^{4 i G1} + e^{12 i G1})
      Cos[B1]^2 Sin[B1]^2 + 2 i e^{2 i G1} (1 + e^{4 i G1}) Cos[B1] Sin[B1]^3 + e^{4 i G1} Sin[B1]^4) },

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$$\begin{aligned}
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{2 i G1} \cos[B1]^4 - i \left(1 + 3 e^{4 i G1} \right) \cos[B1]^3 \sin[B1] - \right. \right. \\
& \quad e^{6 i G1} \left(1 + e^{4 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \\
& \quad \left. i \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos[B1] \sin[B1]^3 + e^{6 i G1} \sin[B1]^4 - e^{2 i G1} \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{6 i G1} \cos[B1]^4 - i \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos[B1]^3 \sin[B1] - e^{6 i G1} \left(1 + e^{4 i G1} \right) \right. \right. \\
& \quad \cos[B1]^2 \sin[B1]^2 + i \left(1 + 3 e^{4 i G1} \right) \cos[B1] \sin[B1]^3 + e^{2 i G1} \left(\sin[B1]^4 - \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{4 i G1} \cos[B1]^4 - 2 i e^{2 i G1} \left(1 + e^{4 i G1} \right) \cos[B1]^3 \sin[B1] - \left(2 + 3 e^{4 i G1} + e^{12 i G1} \right) \right. \right. \\
& \quad \cos[B1]^2 \sin[B1]^2 + i e^{2 i G1} \left(3 + e^{8 i G1} \right) \cos[B1] \sin[B1]^3 + e^{4 i G1} \sin[B1]^4 \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{4 i G1} \cos[B1]^4 - i e^{2 i G1} \left(2 + e^{4 i G1} + e^{8 i G1} \right) \cos[B1]^3 \sin[B1] - \right. \right. \\
& \quad \left(1 + e^{12 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \\
& \quad \left. i e^{2 i G1} \left(3 + e^{4 i G1} \right) \cos[B1] \sin[B1]^3 + \sin[B1]^4 - e^{4 i G1} \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{2 i G1} \cos[B1]^4 - 4 i e^{4 i G1} \cos[B1]^3 \sin[B1] - e^{2 i G1} \left(3 + 2 e^{4 i G1} + e^{8 i G1} \right) \right. \right. \\
& \quad \cos[B1]^2 \sin[B1]^2 + i \left(2 + e^{4 i G1} + e^{12 i G1} \right) \cos[B1] \sin[B1]^3 + e^{2 i G1} \sin[B1]^4 \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(\cos[B1]^4 - i e^{2 i G1} \left(3 + e^{4 i G1} \right) \cos[B1]^3 \sin[B1] - \left(1 + e^{12 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \right. \right. \\
& \quad \left. i e^{2 i G1} \left(2 + e^{4 i G1} + e^{8 i G1} \right) \cos[B1] \sin[B1]^3 + e^{4 i G1} \left(\sin[B1]^4 - \sin[2 B1]^2 \right) \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{2 i G1} \cos[B1]^4 - 2 i \left(1 + e^{4 i G1} \right) \cos[B1]^3 \sin[B1] - 2 e^{6 i G1} \cos[B1]^2 \sin[B1]^2 + \right. \right. \\
& \quad i e^{4 i G1} \left(3 + e^{8 i G1} \right) \cos[B1] \sin[B1]^3 + e^{2 i G1} \left(e^{8 i G1} \sin[B1]^4 - \sin[2 B1]^2 \right) \Big\}, \left\{ \frac{1}{4} e^{-16 i G1} \right. \\
& \quad \left(e^{2 i G1} \cos[B1]^4 - i \left(1 + 3 e^{4 i G1} \right) \cos[B1]^3 \sin[B1] - e^{6 i G1} \left(1 + e^{4 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \right. \\
& \quad \left. i \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos[B1] \sin[B1]^3 + e^{6 i G1} \sin[B1]^4 - e^{2 i G1} \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-16 i G1} \left(e^{4 i G1} \cos[B1]^4 - 4 i e^{2 i G1} \cos[B1]^3 \sin[B1] - 2 \cos[B1]^2 \sin[B1]^2 + \right. \right. \\
& \quad \left. i e^{2 i G1} \left(1 + e^{4 i G1} \right)^2 \cos[B1] \sin[B1]^3 + e^{4 i G1} \left(e^{8 i G1} \sin[B1]^4 - \sin[2 B1]^2 \right) \right) \Big\} \Big\}
\end{aligned}$$

In[]:= R3 = Simplify[U4G2.R2]

$$\begin{aligned}
\text{Out[]}:= & \left\{ \frac{1}{4} e^{-4 i (4 G1+G2)} \left(e^{12 i G1} \cos[B1]^4 - i e^{2 i G1} \left(1 + e^{4 i G1} \right)^2 \cos[B1]^3 \sin[B1] - \right. \right. \\
& 2 \cos[B1]^2 \sin[B1]^2 + 4 i e^{2 i G1} \cos[B1] \sin[B1]^3 + e^{4 i G1} \left(\sin[B1]^4 - \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2 i (8 G1+5 G2)} \left(e^{6 i G1} \cos[B1]^4 - i \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos[B1]^3 \sin[B1] - e^{6 i G1} \left(1 + e^{4 i G1} \right) \right. \right. \\
& \cos[B1]^2 \sin[B1]^2 + i \left(1 + 3 e^{4 i G1} \right) \cos[B1] \sin[B1]^3 + e^{2 i G1} \left(\sin[B1]^4 - \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2 i (8 G1+3 G2)} \left(e^{10 i G1} \cos[B1]^4 - i e^{4 i G1} \left(3 + e^{8 i G1} \right) \cos[B1]^3 \sin[B1] - 2 e^{6 i G1} \right. \right. \\
& \cos[B1]^2 \sin[B1]^2 + 2 i \left(1 + e^{4 i G1} \right) \cos[B1] \sin[B1]^3 + e^{2 i G1} \left(\sin[B1]^4 - \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-4 i (4 G1+3 G2)} \left(e^{4 i G1} \cos[B1]^4 - i e^{2 i G1} \left(2 + e^{4 i G1} + e^{8 i G1} \right) \cos[B1]^3 \sin[B1] - \right. \right. \\
& \left(1 + e^{12 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \\
& \left. i e^{2 i G1} \left(3 + e^{4 i G1} \right) \cos[B1] \sin[B1]^3 + \sin[B1]^4 - e^{4 i G1} \sin[2 B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2 i (8 G1+7 G2)} \left(e^{2 i G1} \cos[B1]^4 - i \left(2 + e^{4 i G1} + e^{12 i G1} \right) \cos[B1]^3 \sin[B1] - \right. \right. \\
& e^{2 i G1} \left(3 + 2 e^{4 i G1} + e^{8 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \\
& 4 i e^{4 i G1} \cos[B1] \sin[B1]^3 + e^{2 i G1} \sin[B1]^4 \Big\}, \left\{ \frac{1}{4} e^{-16 i (G1+G2)} \right. \\
& \left(\cos[B1]^4 - i e^{2 i G1} \left(3 + e^{4 i G1} \right) \cos[B1]^3 \sin[B1] - \left(1 + e^{12 i G1} \right) \cos[B1]^2 \sin[B1]^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{1}{4} e^{-4i(4G1+3G2)} \left(e^{4iG1} \cos[B1]^4 - i e^{2iG1} (3 + e^{8iG1}) \cos[B1]^3 \sin[B1] - (2 + 3 e^{4iG1} + e^{12iG1}) \right. \right. \\
& \quad \left. \cos[B1]^2 \sin[B1]^2 + 2 i e^{2iG1} (1 + e^{4iG1}) \cos[B1] \sin[B1]^3 + e^{4iG1} \sin[B1]^4 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2i(8G1+7G2)} \left(e^{2iG1} \cos[B1]^4 - i (1 + 3 e^{4iG1}) \cos[B1]^3 \sin[B1] - \right. \right. \\
& \quad e^{6iG1} (1 + e^{4iG1}) \cos[B1]^2 \sin[B1]^2 + \\
& \quad \left. i (1 + 2 e^{4iG1} + e^{12iG1}) \cos[B1] \sin[B1]^3 + e^{6iG1} \sin[B1]^4 - e^{2iG1} \sin[2B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2i(8G1+5G2)} \left(e^{6iG1} \cos[B1]^4 - i (1 + 2 e^{4iG1} + e^{12iG1}) \cos[B1]^3 \sin[B1] - e^{6iG1} (1 + e^{4iG1}) \right. \right. \\
& \quad \left. \cos[B1]^2 \sin[B1]^2 + i (1 + 3 e^{4iG1}) \cos[B1] \sin[B1]^3 + e^{2iG1} (\sin[B1]^4 - \sin[2B1]^2) \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-4i(4G1+3G2)} \left(e^{4iG1} \cos[B1]^4 - 2 i e^{2iG1} (1 + e^{4iG1}) \cos[B1]^3 \sin[B1] - (2 + 3 e^{4iG1} + e^{12iG1}) \right. \right. \\
& \quad \left. \cos[B1]^2 \sin[B1]^2 + i e^{2iG1} (3 + e^{8iG1}) \cos[B1] \sin[B1]^3 + e^{4iG1} \sin[B1]^4 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-4i(4G1+3G2)} \left(e^{4iG1} \cos[B1]^4 - i e^{2iG1} (2 + e^{4iG1} + e^{8iG1}) \cos[B1]^3 \sin[B1] - \right. \right. \\
& \quad (1 + e^{12iG1}) \cos[B1]^2 \sin[B1]^2 + i e^{2iG1} (3 + e^{4iG1}) \cos[B1] \sin[B1]^3 + \\
& \quad \left. \sin[B1]^4 - e^{4iG1} \sin[2B1]^2 \right) \Big\}, \left\{ \frac{1}{4} e^{-2i(8G1+7G2)} \right. \\
& \quad \left(e^{2iG1} \cos[B1]^4 - 4 i e^{4iG1} \cos[B1]^3 \sin[B1] - e^{2iG1} (3 + 2 e^{4iG1} + e^{8iG1}) \cos[B1]^2 \sin[B1]^2 + \right. \\
& \quad \left. i (2 + e^{4iG1} + e^{12iG1}) \cos[B1] \sin[B1]^3 + e^{2iG1} \sin[B1]^4 \right) \Big\}, \left\{ \frac{1}{4} e^{-16i(G1+G2)} \right. \\
& \quad \left(\cos[B1]^4 - i e^{2iG1} (3 + e^{4iG1}) \cos[B1]^3 \sin[B1] - (1 + e^{12iG1}) \cos[B1]^2 \sin[B1]^2 + \right. \\
& \quad \left. i e^{2iG1} (2 + e^{4iG1} + e^{8iG1}) \cos[B1] \sin[B1]^3 + e^{4iG1} (\sin[B1]^4 - \sin[2B1]^2) \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2i(8G1+7G2)} \left(e^{2iG1} \cos[B1]^4 - 2 i (1 + e^{4iG1}) \cos[B1]^3 \sin[B1] - 2 e^{6iG1} \cos[B1]^2 \right. \right. \\
& \quad \left. \sin[B1]^2 + i e^{4iG1} (3 + e^{8iG1}) \cos[B1] \sin[B1]^3 + e^{2iG1} (e^{8iG1} \sin[B1]^4 - \sin[2B1]^2) \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-2i(8G1+7G2)} \left(e^{2iG1} \cos[B1]^4 - i (1 + 3 e^{4iG1}) \cos[B1]^3 \sin[B1] - \right. \right. \\
& \quad e^{6iG1} (1 + e^{4iG1}) \cos[B1]^2 \sin[B1]^2 + \\
& \quad \left. i (1 + 2 e^{4iG1} + e^{12iG1}) \cos[B1] \sin[B1]^3 + e^{6iG1} \sin[B1]^4 - e^{2iG1} \sin[2B1]^2 \right) \Big\}, \\
& \left\{ \frac{1}{4} e^{-4i(4G1+3G2)} \left(e^{4iG1} \cos[B1]^4 - 4 i e^{2iG1} \cos[B1]^3 \sin[B1] - 2 \cos[B1]^2 \sin[B1]^2 + \right. \right. \\
& \quad \left. i e^{2iG1} (1 + e^{4iG1})^2 \cos[B1] \sin[B1]^3 + e^{4iG1} (e^{8iG1} \sin[B1]^4 - \sin[2B1]^2) \right) \Big\}
\end{aligned}$$

In[]:= R4 = Simplify[V4B2.R3]

Out[*]=

$$\begin{aligned}
& \left\{ \left\{ \frac{1}{4} e^{-16 i (G1+G2)} \right. \right. \\
& \quad \left(e^{4 i (G1+3 G2)} \cos [B2]^4 \left(\sin [B1]^4 - \sin [2 B1]^2 \right) + \frac{1}{2} i e^{2 i (G1+G2)} \left(-1 + 6 e^{4 i G2} + 3 e^{8 i G2} + \right. \right. \\
& \quad \left. \left(1 + 10 e^{4 i G2} + 5 e^{8 i G2} \right) \cos [2 B1] \right) \cos [B2]^3 \sin [B1]^2 \sin [B2] + \\
& \quad \left(3 e^{4 i G1} - e^{4 i G2} + 3 e^{4 i (G1+G2)} + \left(5 e^{4 i G1} + e^{4 i G2} + 5 e^{4 i (G1+G2)} \right) \cos [2 B1] \right) \cos [B2]^2 \\
& \quad \sin [B1]^2 \sin [B2]^2 + i e^{2 i (G1+G2)} \cos [B2] \left(\left(1 + e^{4 i G1} \right)^2 \sin [B1]^4 - 3 \sin [2 B1]^2 \right) \\
& \quad \sin [B2]^3 + e^{4 i (3 G1+G2)} \sin [B1]^4 \sin [B2]^4 - e^{2 i G2} \cos [B1]^2 \sin [B1]^2 \\
& \quad \left(2 e^{10 i G2} \cos [B2]^4 - i e^{2 i G1} \left(3 + 2 e^{4 i G1} + e^{8 i G1} + 2 e^{4 i (G1+G2)} + 2 e^{4 i (2 G1+G2)} + \right. \right. \\
& \quad \left. \left. 2 e^{4 i (G1+2 G2)} \right) \cos [B2]^3 \sin [B2] + 3 i e^{2 i G1} \left(1 + e^{4 i G1} \right)^2 \cos [B2] \sin [B2]^3 + \right. \\
& \quad \left. 2 e^{2 i G2} \sin [B2]^4 \right) + \frac{1}{8} \sin [2 B1]^2 \left(-8 e^{4 i (G1+G2)} \sin [B2]^4 + \right. \\
& \quad \left. \left(1 + e^{12 i G1} + 3 e^{4 i G2} + 3 e^{4 i (G1+G2)} + 2 e^{4 i (3 G1+G2)} \right) \sin [2 B2]^2 \right) + \cos [B1]^4 \\
& \quad \left(e^{12 i (G1+G2)} \cos [B2]^4 - i e^{2 i (G1+G2)} \left(1 + e^{4 i (G1+G2)} \right)^2 \cos [B2]^3 \sin [B2] - 2 \cos [B2]^2 \right. \\
& \quad \left. \sin [B2]^2 + 4 i e^{2 i (G1+G2)} \cos [B2] \sin [B2]^3 + e^{4 i (G1+G2)} \left(\sin [B2]^4 - \sin [2 B2]^2 \right) \right) + \\
& \quad \cos [B1]^3 \sin [B1] \left(-i e^{2 i (G1+6 G2)} \left(1 + e^{4 i G1} \right)^2 \cos [B2]^4 - \right. \\
& \quad e^{2 i G2} \left(1 + e^{4 i G2} \right) \left(2 + e^{4 i G1} + e^{12 i G1} + 3 e^{4 i (G1+G2)} + e^{4 i (3 G1+G2)} \right) \cos [B2]^3 \sin [B2] + \\
& \quad i e^{2 i G1} \left(6 + 2 e^{4 i G1} + 9 e^{4 i G2} + 3 e^{4 i (2 G1+G2)} \right) \cos [B2]^2 \sin [B2]^2 + 4 e^{2 i G2} \\
& \quad \left. \left(1 + 3 e^{4 i G1} \right) \cos [B2] \sin [B2]^3 + i e^{2 i (G1+2 G2)} \left(-4 \sin [B2]^4 + e^{4 i G1} \sin [2 B2]^2 \right) \right) + \\
& \quad \cos [B1] \sin [B1]^3 \left(4 i e^{2 i (G1+6 G2)} \cos [B2]^4 + 2 e^{2 i G2} \left(1 + e^{4 i G2} \right) \right. \\
& \quad \left. \left(2 e^{4 i G1} + e^{4 i G2} + e^{4 i (G1+G2)} \right) \cos [B2]^3 \sin [B2] - \right. \\
& \quad i e^{2 i G1} \left(2 e^{4 i G1} + 2 e^{8 i G1} + 11 e^{4 i G2} + e^{4 i (2 G1+G2)} \right) \cos [B2]^2 \sin [B2]^2 - \\
& \quad 4 e^{2 i G2} \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos [B2] \sin [B2]^3 + \\
& \quad \left. i e^{2 i G1} \left(e^{4 i G2} \left(1 + e^{4 i G1} \right)^2 \sin [B2]^4 - \left(1 + e^{4 i (G1+G2)} \right) \sin [2 B2]^2 \right) \right) \right\}, \\
& \quad \dots 14 \dots, \left\{ \frac{1}{4} e^{-16 i (G1+G2)} \left(\dots 13 \dots + \cos [B1] \dots 1 \dots^3 \right. \right. \\
& \quad \left(i e^{2 i (G1+2 G2)} \left(1 + e^{4 i G1} \right)^2 \cos [B2]^4 + 4 e^{2 i G2} \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos [B2]^3 \sin [B2] - \right. \\
& \quad i e^{2 i G1} \left(2 e^{4 i G1} + 2 e^{\dots 1 \dots} + 11 e^{\dots 1 \dots} + e^{4 i (\dots 1 \dots)} \right) \cos [B2]^2 \sin [B2]^2 - \\
& \quad 2 e^{2 i G2} \left(1 + e^{4 i G2} \right) \left(2 e^{4 i G1} + e^{4 i G2} + e^{4 i (G1+G2)} \right) \cos [B2] \sin [B2]^3 - \\
& \quad \left. i e^{2 i G1} \left(-4 e^{12 i G2} \sin [B2]^4 + \left(1 + e^{4 i (G1+G2)} \right) \sin [2 B2]^2 \right) \right) \right\} \}
\end{aligned}$$

large output

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In[]:= R5 = Simplify[H41.R4]

Out[]:=

$$\left\{ \left\{ \frac{1}{2} e^{-16 i (G1+G2)} \left(e^{4 i (G1+3 G2)} \cos[B2]^4 \left(\sin[B1]^4 - \sin[2 B1]^2 \right) + \frac{1}{2} i e^{2 i (G1+G2)} \left(-1 + 6 e^{4 i G2} + 3 e^{8 i G2} + \left(1 + 10 e^{4 i G2} + 5 e^{8 i G2} \right) \cos[2 B1] \right) \cos[B2]^3 \sin[B1]^2 \sin[B2] + \left(3 e^{4 i G1} - e^{4 i G2} + 3 e^{4 i (G1+G2)} + \left(5 e^{4 i G1} + e^{4 i G2} + 5 e^{4 i (G1+G2)} \right) \cos[2 B1] \right) \cos[B2]^2 \sin[B1]^2 \sin[B2]^2 + i e^{2 i (G1+G2)} \cos[B2] \left(\left(1 + e^{4 i G1} \right)^2 \sin[B1]^4 - 3 \sin[2 B1]^2 \right) \sin[B2]^3 + e^{4 i (3 G1+G2)} \sin[B1]^4 \sin[B2]^4 - e^{2 i G2} \cos[B1]^2 \sin[B1]^2 \left(2 e^{10 i G2} \cos[B2]^4 - i e^{2 i G1} \left(3 + 2 e^{4 i G1} + e^{8 i G1} + 2 e^{4 i (G1+G2)} + 2 e^{4 i (2 G1+G2)} + 2 e^{4 i (G1+2 G2)} \right) \cos[B2]^3 \sin[B2] + 3 i e^{2 i G1} \left(1 + e^{4 i G1} \right)^2 \cos[B2] \sin[B2]^3 + 2 e^{2 i G2} \sin[B2]^4 \right) + \frac{1}{8} \sin[2 B1]^2 \left(-8 e^{4 i (G1+G2)} \sin[B2]^4 + \left(1 + e^{12 i G1} + 3 e^{4 i G2} + 3 e^{4 i (G1+G2)} + 2 e^{4 i (3 G1+G2)} \right) \sin[2 B2]^2 \right) + \cos[B1]^4 \left(e^{12 i (G1+G2)} \cos[B2]^4 - i e^{2 i (G1+G2)} \left(1 + e^{4 i (G1+G2)} \right)^2 \cos[B2]^3 \sin[B2] - 2 \cos[B2]^2 \sin[B2]^2 + 4 i e^{2 i (G1+G2)} \cos[B2] \sin[B2]^3 + e^{4 i (G1+G2)} \left(\sin[B2]^4 - \sin[2 B2]^2 \right) \right) + \cos[B1]^3 \sin[B1] \left(-i e^{2 i (G1+6 G2)} \left(1 + e^{4 i G1} \right)^2 \cos[B2]^4 - e^{2 i G2} \left(1 + e^{4 i G2} \right) \left(2 + e^{4 i G1} + e^{12 i G1} + 3 e^{4 i (G1+G2)} + e^{4 i (3 G1+G2)} \right) \cos[B2]^3 \sin[B2] + i e^{2 i G1} \left(6 + 2 e^{4 i G1} + 9 e^{4 i G2} + 3 e^{4 i (2 G1+G2)} \right) \cos[B2]^2 \sin[B2]^2 + 4 e^{2 i G2} \left(1 + 3 e^{4 i G1} \right) \cos[B2] \sin[B2]^3 + i e^{2 i (G1+2 G2)} \left(-4 \sin[B2]^4 + e^{4 i G1} \sin[2 B2]^2 \right) \right) + \cos[B1] \sin[B1]^3 \left(4 i e^{2 i (G1+6 G2)} \cos[B2]^4 + 2 e^{2 i G2} \left(1 + e^{4 i G2} \right) \left(2 e^{4 i G1} + e^{4 i G2} + e^{4 i (G1+G2)} \right) \cos[B2]^3 \sin[B2] - i e^{2 i G1} \left(2 e^{4 i G1} + 2 e^{8 i G1} + 11 e^{4 i G2} + e^{4 i (2 G1+G2)} \right) \cos[B2]^2 \sin[B2]^2 - 4 e^{2 i G2} \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos[B2] \sin[B2]^3 + i e^{2 i G1} \left(e^{4 i G2} \left(1 + e^{4 i G1} \right)^2 \sin[B2]^4 - \left(1 + e^{4 i (G1+G2)} \right) \sin[2 B2]^2 \right) \right) \right\} \right\}, \left\{ \frac{1}{2} e^{-16 i (G1+G2)} \left(\dots 13 \dots + \cos[B1] \dots 1 \dots^3 \left(i e^{2 i (G1+2 G2)} \left(1 + e^{4 i G1} \right)^2 \cos[B2]^4 + 4 e^{2 i G2} \left(1 + 2 e^{4 i G1} + e^{12 i G1} \right) \cos[B2]^3 \sin[B2] - i e^{2 i G1} \left(2 e^{4 i G1} + 2 e^{\dots 1 \dots} + 11 e^{\dots 1 \dots} + e^{4 i (\dots 1 \dots)} \right) \cos[B2]^2 \sin[B2]^2 - 2 e^{2 i G2} \left(1 + e^{4 i G2} \right) \left(2 e^{4 i G1} + e^{4 i G2} + e^{4 i (G1+G2)} \right) \cos[B2] \sin[B2]^3 - i e^{2 i G1} \left(-4 e^{12 i G2} \sin[B2]^4 + \left(1 + e^{4 i (G1+G2)} \right) \sin[2 B2]^2 \right) \right) \right\} \right\} \right\}$$

large output

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set size limit...

In[]:= R51 = Simplify[ExpToTrig[R5]]

Out[]:= \$Aborted

In[]:= R6 = Simplify[V4B2t.R5]

Out[]:= \$Aborted

R7 = Simplify[U4G2t.R6]

R8 = Simplify[V4B1t.R7]

R9 = Simplify[U4G1t.R8]

R10 = Simplify[Transpose[PM4].R9]

```

In[*]:= Result1 = Transpose[PM4].U4G1t.V4B1t.U4G2t.V4B2t.H41.V4B2.U4G2.V4B1.U4G1.PM4;

In[*]:= Result1Func[G1_, B1_, G2_, B2_] = Result1[[1, 1]];

In[*]:= FindMinimum[{Result1Func[G1, B1, G2, B2],
  0 < G1 < 3.14 && 0 < B1 < 3.14 && 0 < G2 < 3.14 && 0 < B2 < 3.14}, {G1, B1, G2, B2}]

Out[*]:= {2.47363, {G1 → 0.267108, B1 → 1.03354, G2 → 0.715433, B2 → 0.018976}}

In[*]:= 5 qubit

Out[*]:= 5 qubit

In[*]:= Z50 = KroneckerProduct[Z, IdentityMatrix[16]];
Z51 = KroneckerProduct[IdentityMatrix[2], Z, IdentityMatrix[8]];
Z52 = KroneckerProduct[IdentityMatrix[4], Z, IdentityMatrix[4]];
Z53 = KroneckerProduct[IdentityMatrix[8], Z, IdentityMatrix[2]];
Z54 = KroneckerProduct[IdentityMatrix[16], Z];

In[*]:= H54 = (12 IdentityMatrix[32] - (Z50.Z51 + Z51.Z53 + Z51.Z52 + Z52.Z54 + 2 Z51 + Z52));

In[*]:= U51G1 = MatrixExp[- I G1 H54];
U51G2 = MatrixExp[- I G2 H54];
U51G1t = Refine[ConjugateTranspose[U51G1], {Element[G1, Reals]}];
U51G2t = Refine[ConjugateTranspose[U51G2], {Element[G2, Reals]}];
V5B1 = KroneckerProduct[RxB1, RxB1, RxB1, RxB1, RxB1];
V5B2 = KroneckerProduct[RxB2, RxB2, RxB2, RxB2, RxB2];
V5B1t = KroneckerProduct[RxB1t, RxB1t, RxB1t, RxB1t, RxB1t];
V5B2t = KroneckerProduct[RxB2t, RxB2t, RxB2t, RxB2t, RxB2t];

In[*]:= H541 = 3 IdentityMatrix[32] - Z51.Z52;

In[*]:= PM5 = HadamardMatrix[2^5].KroneckerProduct[UP, UP, UP, UP, UP];

In[*]:= Result2 = Transpose[PM5].U51G1t.V5B1t.U51G2t.V5B2t.H541.V5B2.U51G2.V5B1.U51G1.PM5;

In[*]:= Result2Func[G1_, B1_, G2_, B2_] = Result2[[1, 1]];
FindMinimum[{Result2Func[G1, B1, G2, B2],
  0 < G1 < 3.14 && 0 < B1 < 3.14 && 0 < G2 < 3.14 && 0 < B2 < 3.14}, {G1, B1, G2, B2}]

In[*]:= H55 = 15 IdentityMatrix[2^5] -
  (Z50.Z51 + Z51.Z53 + Z51.Z52 + Z53.Z54 + Z50.Z53 + Z50 + 2 Z51 + 2 Z53);

In[*]:= H551 = 3 IdentityMatrix[2^5] - Z51.Z53;

In[*]:= U52G1 = MatrixExp[- I G1 H55];
U52G2 = MatrixExp[- I G2 H55];
U52G1t = Refine[ConjugateTranspose[U52G1], {Element[G1, Reals]}];
U52G2t = Refine[ConjugateTranspose[U52G2], {Element[G2, Reals]}];

In[*]:= Result3 = Transpose[PM5].U52G1t.V5B1t.U52G2t.V5B2t.H551.V5B2.U52G2.V5B1.U52G1.PM5;

Result3Func[G1_, B1_, G2_, B2_] = Result3[[1, 1]];
FindMinimum[{Result3Func[G1, B1, G2, B2],
  0 < G1 < 3.14 && 0 < B1 < 3.14 && 0 < G2 < 3.14 && 0 < B2 < 3.14}, {G1, B1, G2, B2}]

In[*]:= 6 Qubit

Out[*]:= 6 Qubit

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In[ ]:= Z60 = KroneckerProduct[Z, IdentityMatrix[32]];
Z61 = KroneckerProduct[IdentityMatrix[2], Z, IdentityMatrix[16]];
Z62 = KroneckerProduct[IdentityMatrix[4], Z, IdentityMatrix[8]];
Z63 = KroneckerProduct[IdentityMatrix[8], Z, IdentityMatrix[4]];
Z64 = KroneckerProduct[IdentityMatrix[16], Z, IdentityMatrix[2]];
Z65 = KroneckerProduct[IdentityMatrix[32], Z];

In[ ]:= H65 =
  15 IdentityMatrix[2^6] - (Z60.Z61 + Z61.Z62 + Z62.Z63 + Z62.Z64 + Z62.Z65 + Z61 + 3 Z62);

In[ ]:= H61 = 3 IdentityMatrix[2^6] - Z61.Z62;

In[ ]:= U6G1 = MatrixExp[- I G1 H65];
U6G2 = MatrixExp[- I G2 H65];
U6G1t = Refine[ConjugateTranspose[U6G1], {Element[G1, Reals]}];
U6G2t = Refine[ConjugateTranspose[U6G2], {Element[G2, Reals]}];
V6B1 = KroneckerProduct[RxB1, RxB1, RxB1, RxB1, RxB1, RxB1];
V6B2 = KroneckerProduct[RxB2, RxB2, RxB2, RxB2, RxB2, RxB2];
V6B1t = KroneckerProduct[RxB1t, RxB1t, RxB1t, RxB1t, RxB1t, RxB1t];
V6B2t = KroneckerProduct[RxB2t, RxB2t, RxB2t, RxB2t, RxB2t, RxB2t];

In[ ]:= PM6 = HadamardMatrix[2^6].KroneckerProduct[UP, UP, UP, UP, UP, UP];

In[ ]:= Result4 = Transpose[PM6].U6G1t.V6B1t.U6G2t.V6B2t.H61.V6B2.U6G2.V6B1.U6G1.PM6;

Result4Func[G1_, B1_, G2_, B2_] = Result4[[1, 1]];
FindMinimum[{Result4Func[G1, B1, G2, B2],
  0 < G1 < 3.14 && 0 < B1 < 3.14 && 0 < G2 < 3.14 && 0 < B2 < 3.14}, {G1, B1, G2, B2}]

In[ ]:= 7 Qubit

Out[ ]:= 7 Qubit

In[ ]:= Z70 = KroneckerProduct[Z, IdentityMatrix[2^6]];
Z71 = KroneckerProduct[IdentityMatrix[2], Z, IdentityMatrix[32]];
Z72 = KroneckerProduct[IdentityMatrix[4], Z, IdentityMatrix[16]];
Z73 = KroneckerProduct[IdentityMatrix[8], Z, IdentityMatrix[8]];
Z74 = KroneckerProduct[IdentityMatrix[16], Z, IdentityMatrix[4]];
Z75 = KroneckerProduct[IdentityMatrix[32], Z, IdentityMatrix[2]];
Z76 = KroneckerProduct[IdentityMatrix[64], Z];

In[ ]:= H76 = 18 IdentityMatrix[2^7] -
  (Z70.Z71 + Z71.Z72 + Z71.Z73 + Z73.Z74 + Z73.Z75 + Z73.Z76 + 2 Z71 + 3 Z73);

In[ ]:= H71 = 3 IdentityMatrix[2^7] - Z71.Z73;

In[ ]:= U7G1 = MatrixExp[- I G1 H76];
U7G2 = MatrixExp[- I G2 H76];
U7G1t = Refine[ConjugateTranspose[U7G1], {Element[G1, Reals]}];
U7G2t = Refine[ConjugateTranspose[U7G2], {Element[G2, Reals]}];
V7B1 = KroneckerProduct[RxB1, RxB1, RxB1, RxB1, RxB1, RxB1, RxB1, RxB1];
V7B2 = KroneckerProduct[RxB2, RxB2, RxB2, RxB2, RxB2, RxB2, RxB2, RxB2];
V7B1t = KroneckerProduct[RxB1t, RxB1t, RxB1t, RxB1t, RxB1t, RxB1t, RxB1t, RxB1t];
V7B2t = KroneckerProduct[RxB2t, RxB2t, RxB2t, RxB2t, RxB2t, RxB2t, RxB2t, RxB2t];

In[ ]:= PM7 = HadamardMatrix[2^7].KroneckerProduct[UP, UP, UP, UP, UP, UP, UP, UP];

In[ ]:= Result5 = Transpose[PM7].U7G1t.V7B1t.U7G2t.V7B2t.H71.V7B2.U7G2.V7B1.U7G1.PM7;

Out[ ]:= $Aborted

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Result5Func[G1_, B1_, G2_, B2_] = Result5[[1, 1]];
FindMinimum[{Result5Func[G1, B1, G2, B2],
  0 < G1 < 3.14 && 0 < B1 < 3.14 && 0 < G2 < 3.14 && 0 < B2 < 3.14}, {G1, B1, G2, B2}]

Result = (4 Result1 + 2 (Result2 + Result3 + Result4 + Result5)) [[1, 1]]

ResultFunc[G1_, B1_, G2_, B2_] = Result[[1, 1]];
FindMinimum[{ResultFunc[G1, B1, G2, B2],
  0 < G1 < 3.14 && 0 < B1 < 3.14 && 0 < G2 < 3.14 && 0 < B2 < 3.14}, {G1, B1, G2, B2}]

```