

## UNIT-V

Hashing:GeneralIdea,HashFunctions, CollisionResolution-SeparateChaining, OpenAddressing- Linear probing, Quadratic Probing, Double Hashing, Rehashing, ExtendibleHashing,ImplementationofDictionaries.

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### Hashing:

- ❖ Hashing is one of the searching techniques that uses a constant time. The time complexity in hashing is  **$O(1)$** .
- ❖ Till now, we read the two techniques for searching, i.e., linear search and binary search. The worst time complexity **in linear search is  $O(n)$ , and  $O(\log n)$  in binary search**. In all these search techniques, as the number of elements increases, the time required to search an element also increases linearly.
- ❖ Using hashing data structure, a given element is searched with **constant time complexity**.
- ❖ Hashing is an effective way to reduce the number of comparisons to search an element in a data structure.

Hashing is defined as follows...

“Hashing is the process of indexing and retrieving an element (data) in a data structure to provide a faster way of finding the element using a hash key”

- ❖ Here, the hash key is a value which provides the index value where the actual data is likely to be stored in the data structure. In this data structure, we use a concept called **Hash table**.
- ❖ **Hash table:** Hash table to store data. All the data values are inserted into the hash table based on the hash key value.
- ❖ The hash key value is used to map the data with an index or slot in the hash table. And the hash key is generated for every data using a **hash function**.
- ❖ That means every entry in the hash table is based on the hash key value generated using the hash function.
- ❖ Hash tables are used to perform insertion, deletion and search operations very quickly in a data structure. Using hash table concept, insertion, deletion, and search operations are accomplished in constant time complexity. Generally, every hash table makes use of a function called **hash function** to map the data into the hash table.

Hash Table is defined as follows...

“Hash table is just an array which maps a key (data) into the data structure with the help of hash function such that insertion, deletion and search operations are performed with constant time complexity (i.e.  $O(1)$ ).”

❖ A hash function is defined as follows...

Hash function is a function which takes a piece of data (i.e. key) as input and produces an integer (i.e. hash value) as output which maps the data to a particular index in the hash table.

Basic concept of hashing and hash table is shown in the following figure...

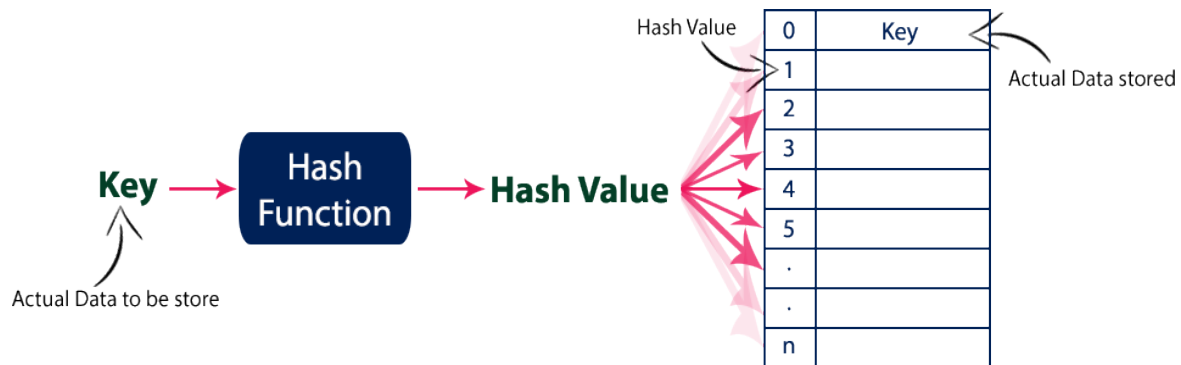
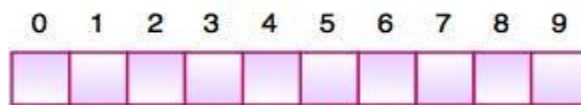
❖ **Example:**

Fig. Hash Table

- The above figure shows the hash table with the size of  $n = 10$ . Each position of the hash table is called as **Slot or Index**. In the above hash table, there are  $n$  slots in the table, names = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Slot 0, slot 1, slot 2 and so on. Hash table contains no items, so every slot is empty.
- As we know the mapping between an item and the slot where item belongs in the hash table is called the hash function. The hash function takes any item in the collection and returns an integer in the range of slot names between 0 to  $n-1$ .

❖ **Example:01**

- ❖ Suppose we have integer items {26, 70, 18, 31, 54, and 93}. One common method of determining a hash key is the division method of hashing and the formula is :
- ❖ Division method or remainder method takes an item and divides it by the table size and returns the remainder as its hash value.

Data Item	Value % No. of Slots	Hash Value
26	$26 \% 10 = 6$	6
70	$70 \% 10 = 0$	0
18	$18 \% 10 = 8$	8
31	$31 \% 10 = 1$	1

54	$54 \% 10 = 4$	4
93	$93 \% 10 = 3$	3
41	$41 \% 10 = 1$	1 (collision occurs)
74	$74 \% 10 = 4$	4 (collision occurs)

0	1	2	3	4	5	6	7	8	9
70	31		93	54		26		18	

Fig. Hash Table

**Example:02**

Suppose we have integer items {36, 18, 72, 43, and 6}.

Assume a table with 8 slots:

Hash key = key % table size

$$4 = 36 \% 8$$

$$2 = 18 \% 8$$

$$0 = 72 \% 8$$

$$3 = 43 \% 8$$

$$6 = 6 \% 8$$

[0]	72
[1]	
[2]	18
[3]	43
[4]	36
[5]	
[6]	6
[7]	

## CollisionResolutionTechniques

CollisioninHashing-

1. Inthis,thehashfunctionisusedtocomputetheindexofthearray.
2. Thehashvalue isusedtostorethe keyinthehashtable,asanindex.
3. Thehashfunctioncanreturnthesame hashvaluefortwoormorekeys.
4. When two or more keys are given the same hash value, **it is called a collision**. To handlethis collision, we use collision resolution techniques.

**Whenthehashvalueofakeymapstoanalreadyoccupiedbucket/slot/indexofthe hash table,it is called as a Collision.**

### Collision Resolution Techniques

**Separate Chaining**  
(Open Hashing)

**Open Addressing**  
(Closed Hashing)

→ **Linear Probing**

→ **Quadratic Probing**

→ **Double Hashing**

**Separate chaining(openhashing):**

Tohandlethe collision,

- Thistechniquecreatesalinkedlisttotheslotfor whichcollision occurs.
- Thenewkeyistheninsertedinthelinkedlist.
- Theselinkedliststotheslotsappearlikechains.
- Thatiswhy;thistechniqueiscalledas **separatechaining**.

**Timecomplexity:**

- Itsworst-casecomplexityforsearchingis  $O(n)$ .
- Itsworst-casecomplexityfordeletionis  $O(n)$ .

**Advantagesofseparatechaining**

- Itiseasyto implement.
- Thehashtableneverfillsfull,sowecanaddmoreelementstothechain.
- Itisless sensitiveto thefunctionofthe hashing.

**Disadvantagesofseparatechaining"**

- Inthis,cacheperformanceofchainingisnotgood.
- Thememory wastageistoomuchinthismethod.
- Itrequiresmorespaceforelementlinks.

**LoadFactor( $\lambda$ )**

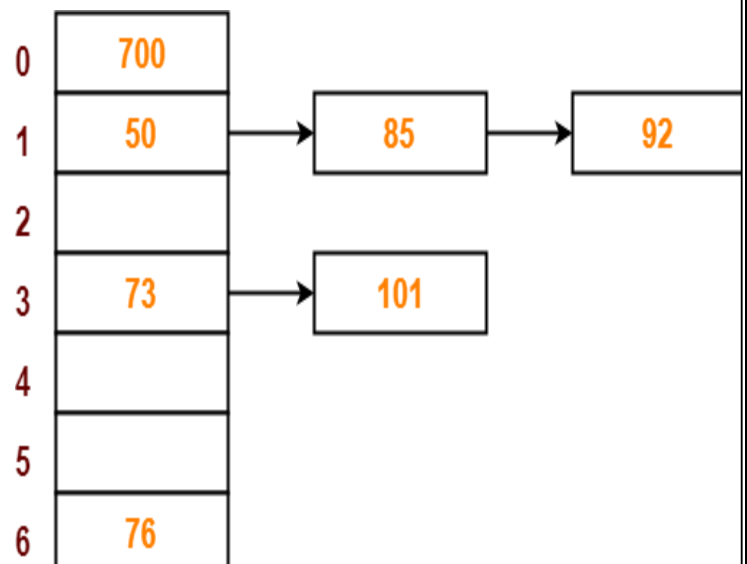
Loadfactor( $\lambda$ )isdefinedas-

$$\text{Loadfactor}(\lambda) = \frac{\text{NumberofElementspresentinthehashtable}(n)}{\text{TotalsizeoftheHashTable}(N)}$$

**Example:01**

Usingthehashfunction'keymod7',insertthefollowingsequenceofkeysinthehashtable- 50, 700, 76, 85, 92, 73 and 101. Use separate chaining technique for collision resolution.

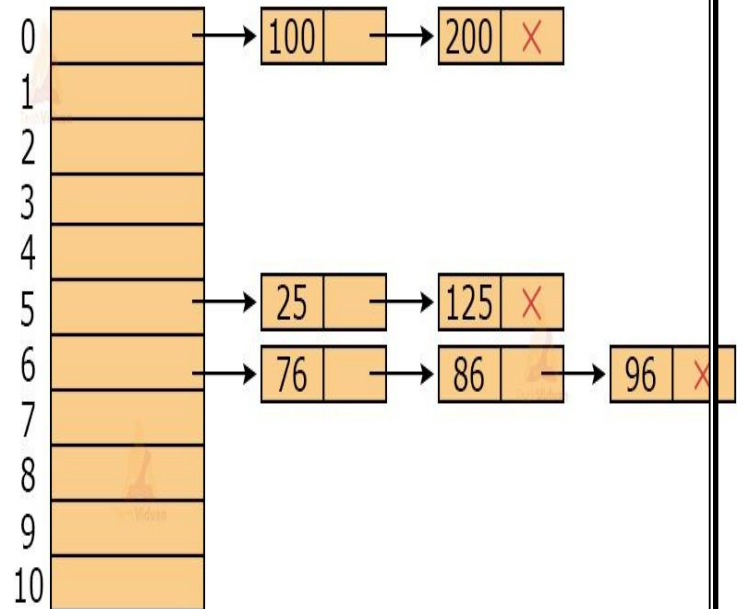
DataItem	Value%No.of Slots	HashValue
50	$50 \% 7 = 1$	1
700	$700 \% 7 = 0$	0
76	$76 \% 7 = 6$	6
85	$85 \% 7 = 1$	1 (collisionoccurs)
92	$92 \% 7 = 1$	1 (collisionoccurs)
73	$73 \% 7 = 3$	3
101	$101 \% 7 = 3$	3 (collisionoccurs)



**Example:02**

Let the keys be 100, 200, 25, 125, 76, 86, 96 and let  $m = 10$ . Given,  $h(k) = k \bmod 10$ . Use separate chaining technique for collision resolution.

DataItem	Value%No.of Slots	HashValue
100	$100 \% 10 = 0$	0
200	$200 \% 10 = 0$	0 (collision occurs)
25	$25 \% 10 = 5$	5
125	$125 \% 10 = 5$	5 (collision occurs)
76	$76 \% 10 = 6$	6
86	$86 \% 10 = 6$	6 (collision occurs)
96	$96 \% 10 = 6$	6 (collision occurs)



### Openaddressing(closed hashing)

Open addressing is collision-resolution method that is used to control the collision in the hashing table. There is no key stored outside of the hash table. Therefore, the size of the hash table is always greater than or equal to the number of keys. It is also called **closed hashing**.

#### 1.Linearprobing:

- This Hashing technique finds the hash key value through hash function and maps the key on particular position in hash table.

**key=key%size;**

- In case if key has same hash address (collision) then it will find **next empty position** in the hash table.

**key= (key+i)%size; here i is 0 to size-1**

- We take the hash table as circular array. So if table size is N then after N-1 position it will search from 0<sup>th</sup> position in the array.

**Example:01:**

Suppose we have a list of size 20 ( $m=20$ ). We want to put some elements in linear probing fashion. The elements / keys are {96, 48, 63, 29, 87, 77, 48, 65, 69, 94, 61}

DataItem	Value%No.ofSlots	HashValue	probes
96	$96\%20=16$	16	1
48	$48\%20=8$	8	1
63	$63\%20=3$	3	1
29	$29\%20=9$	9	1
87	$87\%20=7$	7	1
77	$77\%20=17$	17	1
48	$48\%20=8$ $(48+1)\%20=9$ $(48+2)\%20=10$	Collisionoccurs 10	3
65	$65\%20=5$	5	1
69	$69\%20=9$ $(69+1)\%20=10$ $(69+2)\%20=11$	Collisionoccurs 11	3
94	$94\%20=14$	14	1
61	$61\%20=1$	1	1

**HashTable**

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	61		63		65		87	48	29	48	69			94		96	77		

➤ TheorderofElements/keysinHashTableare:

--,61,--,63,--,65,--,87,48,29,48,69,--,--,94,--,96,77,--,--

➤ TotalNumberofProbesare **15**

**Example:02:**

Supposewehavealistofsize11(size=11).Wewanttoputsomeelementsinlinearprobing fashion.Theelements/keyare{29,18,43,10,36, 25,46}

DataItem	Value%No.ofSlots	HashValue	probes
29	$29\%11=7$	7	1
18	$18\%11=7$ $(18+1)\%11=8$	Collisionoccurs 8	2
43	$43\%11=10$	10	1
10	$10\%11=10$ $(10+1)\%11=0$	Collisionoccurs 0	2
36	$36\%11=3$	3	1
25	$25\%11=3$ $(25+1)\%11=4$	Collisionoccurs 4	2
46	$46\%11=2$	2	1

HashTable	
0	10
1	
2	46
3	36
4	25
5	
6	
7	29
8	18
9	
10	43

➤ TheorderofElementsinHashTableare:10,--,46,36,25,--,--,29,18,--,43

➤ TotalNumberofProbesare **10**

**Example:03 (Assignment)**

Suppose we have a list of size 10 (size=10). We want to put some elements / keys in linear probing fashion. The elements / Keys are {18, 41, 22, 32, 44, 59 }.

**Advantage-**

- It is easy to compute.

**Disadvantage-**

- The main problem with linear probing is **clustering**.
- Many consecutive elements form groups.
- Then, it takes time to search an element or to find an empty bucket.

**2. Quadratic Probing:**

- This Hashing technique finds the hash key value through hash function and maps the key on a particular position in hash table.

$$\text{key} = \text{key} \% \text{size};$$

- In case if key has same hash address (collision) then it will find **next empty position** in the hash table.

$$\text{key} = (\text{key} + i^2) \% \text{size}; \text{ here } i \text{ is } 0 \text{ to } \text{size}-1$$

- We take the hash table as circular array. So if table size is N then after N-1 position it will search from 0<sup>th</sup> position in the array.

**Example:01**

Suppose we have a list of size 20 (size = 20). We want to put some elements / keys in Quadratic Probing fashion. The elements / Keys are { 96, 48, 63, 29, 87, 77, 48, 65, 69, 94, 61 }.

DataItem	Value%No.ofSlots	HashValue	probes
96	96% 20=16	16	1
48	48% 20=8	8	1
63	63% 20=3	3	1
29	29% 20=9	9	1
87	87% 20=7	7	1
77	77% 20=17	17	1
48	48% 20=8 (48+1)%20=9 (48+4)%20=12	<b>Collision occurs 12</b>	<b>3</b>
65	65% 20=5	5	1
69	69% 20=9 (69+1)%20=10	<b>Collision occurs 10</b>	<b>2</b>



94	$94\% 20=14$	14	1
61	$61\% 20=1$	1	1

**HashTable**

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	61		63		65		87	48	29	69		48		94		96	77		

- The order of Elements/keys in HashTable are: --,61,--,63,--,65,--87,48,29,69,--,48,--94,--96,77,--
- Total Number of Probes are **14**

**Example:02**

Suppose we have a list of size 11 (size = 11). We want to put some elements/keys in Quadratic Probing fashion. The elements /Keys are {29, 18, 43, 10, 46, and 54}

Data Item	Value % No. of Slots	Hash Value	probes
29	$29\% 11=7$	7	1
18	$18\% 11=7$ $(18+1)\% 11=8$	<b>Collision occurs</b> 8	2
43	$43\% 11=10$	10	1
10	$10\% 11=10$ $(10+1)\% 11=0$	<b>Collision occurs</b> 0	2
46	$46\% 11=2$	2	1
54	$54\% 11=10$ $(54+1)\% 11=0$ $(54+4)\% 11=3$	<b>Collision occurs</b> 3	3

HashTable	
0	10
1	
2	46
3	54
4	
5	
6	
7	29
8	18
9	
10	43

- The order of Elements in HashTable are: 10,--,46,54,--,--,--,29,18,--,43
- Total Number of Probes are **10**

**Example:03 (Assignment):**

Suppose we have a list of size 10 (size=10). We want to put some elements/keys in Quadratic Probing fashion. The elements /Keys are {18,41,22,32,44,59}

### 3.DoubleHashing:

- ❖ Doublehashing use the idea of applying a second hash function to key when collision occurs.
- ❖ First hash function typically  $h_1(\text{key}) = \text{key} \% \text{size}$
- ❖ Doublehashing can be done using (When collision Occurs):

$$(h_1(\text{key}) + i * h_2(\text{key})) \% \text{Table size.}$$

Here  $h_2(\text{key}) = \text{PRIME} - (\text{key} \% \text{PRIME})$  where PRIME is a prime smaller than Table Size.

- ❖ We repeat by increasing  $i$  when collision occurs (i values from 0 to size-1)

**NOTE:** When collision occurs then find the Second Hash function

#### Example:01

Suppose we have a list of size 10 (size=10). We want to put some elements/keys in Double hashing fashion. The elements /Keys are { 3, 22, 27, 40, 42, 11, 19 }.

Data Item	$h_1 = h_1(\text{key}) \% 10$	$h_2 = 7 - (\text{key} \% 7)$	Final Hash value $= ((h_1 + i * h_2)) \% 10$	probes
3	$3 \% 10 = 3$	-		1
22	$22 \% 10 = 2$			1
27	$27 \% 10 = 7$			1
40	$40 \% 10 = 0$			1
42	$42 \% 10 = 2$ Collision Occurred	$7 - (42 \% 7) = 7$	$(2 + 1 * 7) \% 10 = 9$	2
11	$11 \% 10 = 1$			1
19	$19 \% 10 = 9$ Collision Occurred	$7 - (19 \% 7) = 2$	$(9 + 1 * 2) \% 10 = 1$ Collision Occurred $(9 + 2 * 2) \% 10 = 3$ Collision Occurred $(9 + 3 * 2) \% 10 = 5$	4

HashTable	
0	40
1	44
2	22
3	3
4	
5	19
6	
7	27
8	
9	42

- The order of Elements in Hash Table are: 40, 44, 22, 3, --, 19, --, 27, --, 42, --.
- Total Number of Probes are 10

#### Example:02

Suppose we have a list of size 20 (m=20). We want to put some elements in Double hashing fashion. The elements are {96, 48, 63, 29, 87, 77, 48, 65, 69, 94, 61}.

**SeparateChainingVsOpenAddressing-**

<u><b>SeparateChaining</b></u>	<u><b>Open Addressing</b></u>
Keys are stored inside the hash table as well as outside the hash table.	All the keys are stored only inside the hash table. No key is present outside the hash table.
The number of keys to be stored in the hash table can even exceed the size of the hash table.	The number of keys to be stored in the hash table can never exceed the size of the hash table.
Deletion is easier.	Deletion is difficult.
Extra space is required for the pointers to store the keys outside the hash table.	No extra space is required.
Cache performance is poor. This is because of linked lists which store the keys outside the hash table.	Cache performance is better. This is because here no linked lists are used.
Some buckets of the hash table are never used which leads to wastage of space.	Buckets may be used even if no key maps to those particular buckets.

**Comparison of Open Addressing Techniques-**

	Linear Probing	Quadratic Probing	Double Hashing
Primary Clustering	Yes	No	No
Secondary Clustering	Yes	Yes	No
Number of Probe Sequence (m = size of table)	m	m	m <sup>2</sup>
Cache performance	Best	Lies between the two	Poor

**Conclusions-**

- Linear Probing has the best cache performance but suffers from clustering.
- Quadratic probing lies between the two in terms of cache performance and clustering.
- Double hashing has poor cache performance but no clustering.

## Rehashing

**Rehashing** is a technique in which the table is resized, i.e., the size of the table is **doubled** by creating a new table. It is preferable if the total size of the table is a next prime number. There are situations in which the rehashing is required.

- When the table is completely full
  - With Linear probing when the table is filled with 70%
  - When insertions fail due to overflow.
- ❖ Rehashing means **hashing again**. Basically, when the Load Factor increases or it reaches to 1 or Greater than 1 then complexity increases.

$$\text{Load factor}(\lambda) = \frac{\text{Number of Elements present in the hashtable}(n)}{\text{Total size of the Hash Table}(N)}$$

i.e.  $\lambda < 1$ , When  $\lambda = 1$  or  $\lambda > 1$  we need to increase the Hash Table Size. This is known as **Rehashing**.

- ❖ In such situations, we have to transfer entries from the old table to the new table by re-computing their positions using hash functions.

### Example:1

Consider we have to insert the elements 37, 90, 55, 22, 17, 49, and 87. The table size is 10 and will use hash function:

Data Item	Value % No. of Slots	Hash Value	probes
37	$37 \% 10 = 7$	7	1
90	$90 \% 10 = 0$	0	1
55	$55 \% 10 = 5$	5	1
22	$22 \% 10 = 2$	2	1
17	$17 \% 10 = 7$ $(17+1) \% 10 = 8$	<b>Collision occurs</b> 8	2
49	$49 \% 10 = 9$	9	1
87	$87 \% 10 = 7$ $(87+1) \% 10 = 8$ $(87+2) \% 10 = 9$ $(87+3) \% 10 = 0$ $(87+4) \% 10 = 1$	<b>Collision occurs</b> 1	5

HashTable	
0	90
1	87
2	22
3	
4	
5	55
6	
7	37
8	17
9	49

- ❖ Now this table is almost full and if we try to insert more elements collisions will occur and eventually further insertions will fail.
- ❖ Hence we will rehash by doubling the table size. The old table size is 10 then we should double this size for the new table that becomes 20. But 20 is not a prime number, we will prefer to make the table size as 23.

And new hash function will be

Data Item	Value % No. of Slots	Hash Value	probes
37	$37 \% 23 = 14$	14	1
90	$90 \% 23 = 21$	21	1
55	$55 \% 23 = 9$	9	1
22	$22 \% 23 = 22$	22	1
17	$17 \% 23 = 17$	17	1
49	$49 \% 23 = 3$	3	1
87	$87 \% 23 = 18$	18	1

HashTable	
0	
1	
2	
3	49
4	
5	
6	
7	
8	
9	55
10	
11	
12	
13	
14	37
15	
16	
17	17
18	87
19	
20	
21	90
22	22



### Example:2

Consider we have to insert the elements 7, 18, 43, 10, 36, 25. The table size is 11.

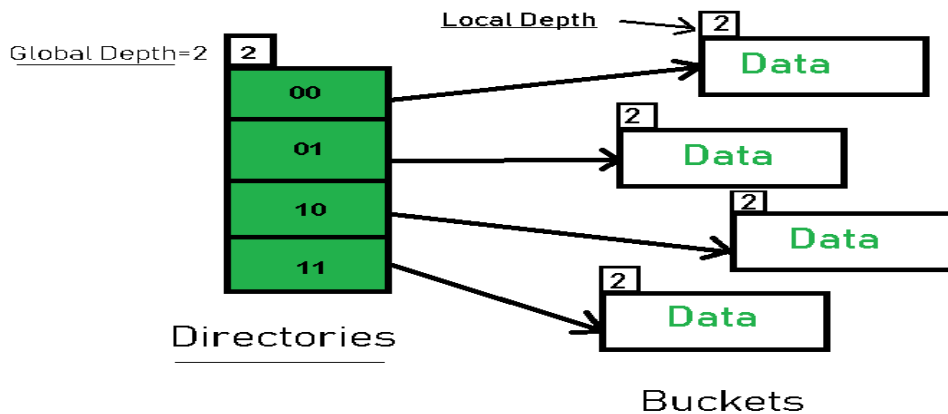
## Extendible hashing (Dynamic Hashing)

- In open addressing or separate chaining, the major problem is collision could cause several buckets to be examined to find an alternative empty cell.
- When the table gets too filled, rehashing must be performed which is expensive.

### Extendible hashing:

- ❖ This is a very simple strategy which provides quick access time for inserts and search operations on large database.
- ❖ It is a dynamic hashing method. The **directories** and **buckets** are used to hash data.
- ❖ The cost of expanding and updating is very low.
- ❖ **Directories:** The directories store addresses of the buckets in pointers. An id is assigned to each directory which may change each time when Directory Expansion takes place.
- ❖ **Buckets:** The buckets are used to hash the actual data.

### Basic Structure of Extendible Hashing:



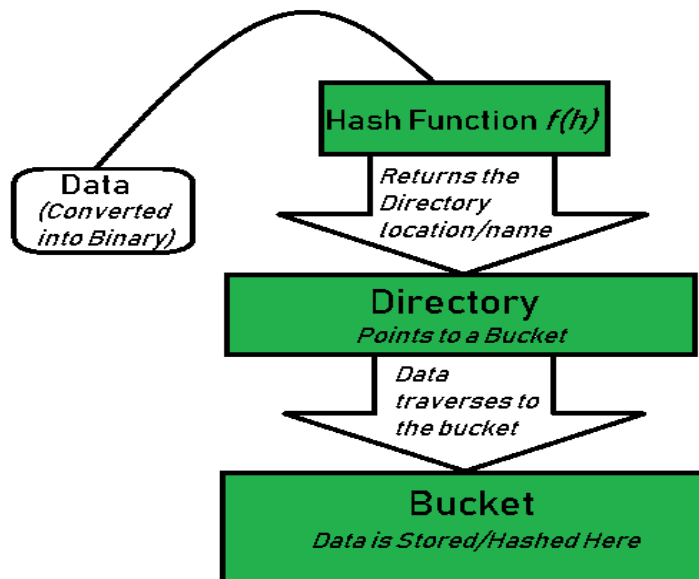
### Extendible Hashing

### Frequently used terms in Extendible Hashing:

- **Directories:** These containers store pointers to buckets. Each directory is given a unique id which may change each time when expansion takes place. The hash function returns this directory id which is used to navigate to the appropriate bucket. Number of Directories =  $2^{\text{Global Depth}}$ .
- **Buckets:** They store the hashed keys. Directories point to buckets. A bucket may contain more than one pointer to it if its local depth is less than the global depth.
- **Global Depth:** It is associated with the Directories. They denote the number of bits which are used by the hash function to categorize the keys.  $\text{Global Depth} = \text{Number of bits in directory id}$ .

- **Local Depth:** It is the same as that of Global Depth except for the fact that Local Depth is associated with the buckets and not the directories. Local depth in accordance with the global depth is used to decide the action that to be performed in case an overflow occurs. Local Depth is always less than or equal to the Global Depth.
- **Bucket Splitting:** When the number of elements in a bucket exceeds a particular size, then the bucket is split into two parts.
- **Directory Expansion:** Directory Expansion Takes place when a bucket overflows. Directory Expansion is performed when the local depth of the overflowing bucket is equal to the global depth.

#### Basic Working of Extendible Hashing:



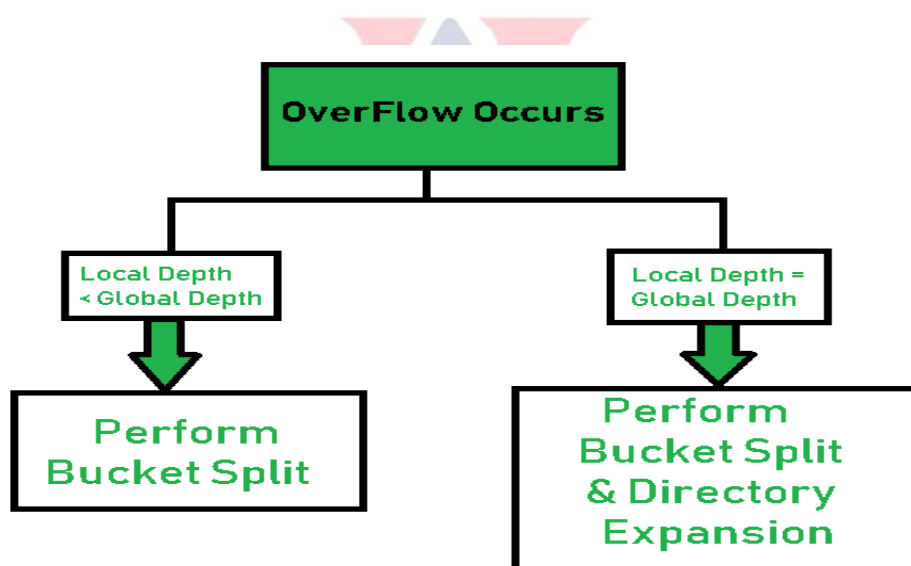
- **Step 1 – Analyze Data Elements:** Data elements may exist in various forms e.g. Integer, String, Float, etc.. Currently, let us consider data elements of type integer. eg: 49.
- **Step 2 – Convert into binary format:** Convert the data element in Binary form. For string elements, consider the ASCII equivalent integer of the starting character and then convert the integer into binary form. Since we have 49 as our data element, its binary form is 110001.
- **Step 3 – Check Global Depth of the directory.** Suppose the global depth of the Hash-directory is 3.
- **Step 4 – Identify the Directory:** Consider the 'Global-Depth' number of LSBs in the binary number and match it to the directory id.  
Eg. The binary obtained is: 110001 and the global-depth is 3. So, the hash function will return 3 LSBs of 110001 viz. 001.
- **Step 5 – Navigation:** Now, navigate to the bucket pointed by the directory with directory-id 001.

➤ **Step 6 - Insertion and Overflow Check:** Insert the element and check if the bucket overflows. If an overflow is encountered, go to **step 7** followed by **Step 8**, otherwise, go to **step 9**.

➤ **Step 7 - Tackling Over Flow Condition during Data Insertion:** Many times, while inserting data in the buckets, it might happen that the Bucket overflows. In such cases, we need to follow an appropriate procedure to avoid mishandling of data.

First, Check if the local depth is less than or equal to the global depth. Then choose one of the cases below.

- **Case1:** If the local depth of the overflowing Bucket is equal to the global depth, then Directory Expansion, as well as Bucket Split, needs to be performed. Then increment the global depth and the local depth value by 1. And, assign appropriate pointers. Directory expansions will double the number of directories present in the hash structure.
- **Case2:** In case the local depth is less than the global depth, then only Bucket Split takes place. Then increment only the local depth value by 1. And, assign appropriate pointers.



- **Step 8 - Rehashing of Split Bucket Elements:** The Elements present in the overflowing bucket that is split are rehashed w.r.t the new global depth of the directory.
- **Step 9 - The element is successfully hashed.**



**Example-1:** Now, let us consider a prominent example of hashing the following elements: **16,4,6,22,24,10,31,7,9,20,26**.

**BucketSize:**3(Assume)

**HashFunction:** Suppose the global depth is X. Then the HashFunction returns X LSBs.

**Solution:**

First, calculate the binary forms of each of the given numbers.

16- 10000

4-00100

6-00110

22-10110

24-11000

10-01010

31-11111

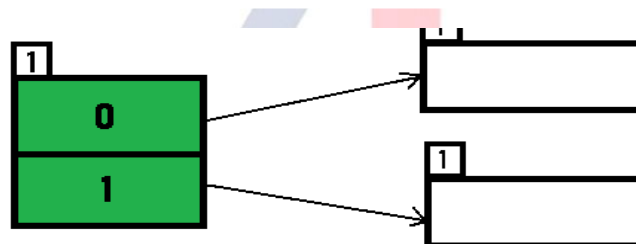
7-00111

9-01001

20-10100

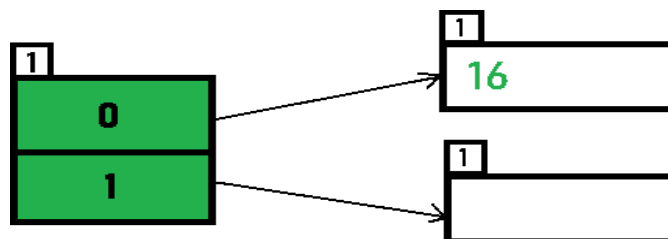
26-11010

- Initially, the global-depth and local-depth is always 1. Thus, the hashing frame looks like this:



**Inserting16:**

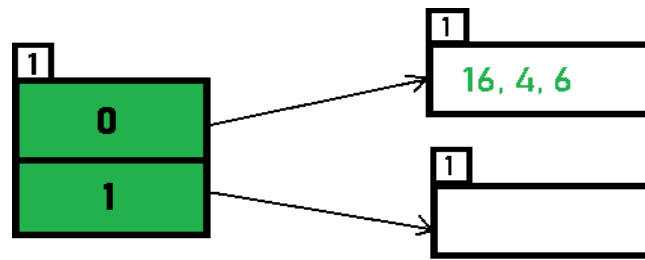
The binary format of 16 is 10000 and global-depth is 1. The hash function returns 1 LSB of 10000 which is 0. Hence, 16 is mapped to the directory with id=0.



*Hash(16)= 10000*

**Inserting4and6:**

Both 4(100) and 6(110) have 0 in their LSB. Hence, they are hashed as follows:

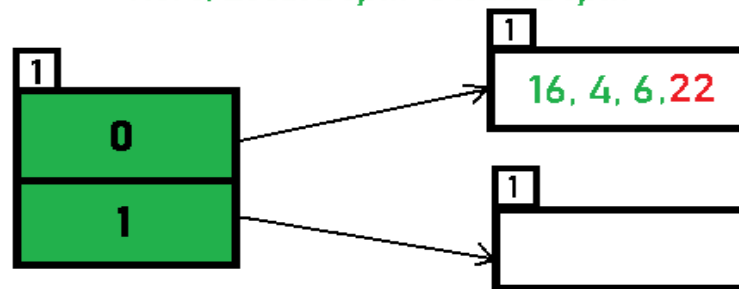


$Hash(4) = 100$   
 $Hash(6) = 110$

**Inserting 22:** The binary form of 22 is 10110. Its LSB is 0. The bucket pointed by directory 0 is already full. Hence, Over Flow occurs.

### OverFlow Condition

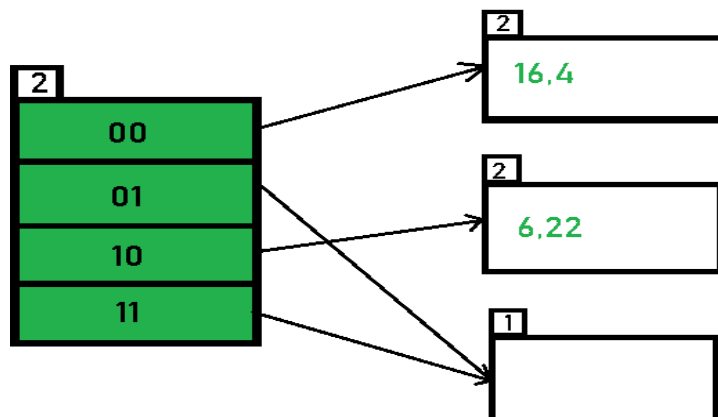
*Here, Local Depth = Global Depth*



$Hash(22) = 10110$

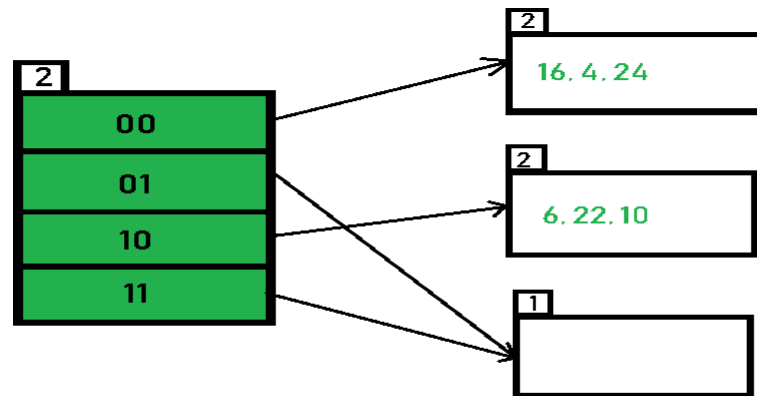
As directed by **Step 7-Case 1**, Since Local Depth = Global Depth, the bucket splits and directory expansion takes place. Also, rehashing of numbers present in the overflowing bucket takes place after the split. And, since the global depth is incremented by 1, now, the global depth is 2. Hence, 16, 4, 6, 22 are now rehashed w.r.t 2 LSBs. [16(10000), 4(100), 6(110), 22(10110)].

### *After Bucket Split and Directory Expansion*



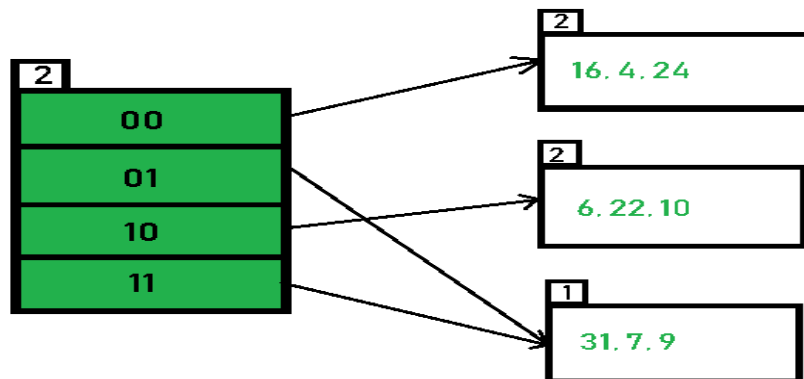
*\*Notice that the bucket which was underflow has remained untouched. But, since the number of directories has doubled, we now have 2 directories 01 and 11 pointing to the same bucket. This is because the local-depth of the bucket has remained 1. And, any bucket having a local depth less than the global depth is pointed-to by more than one directory.*

**Inserting 24 and 10:** 24(11000) and 10 (1010) can be hashed based on directories with id 00 and 10. Here, we encounter no overflow condition.



$Hash(24) = 11000$   
 $Hash(10) = 1010$

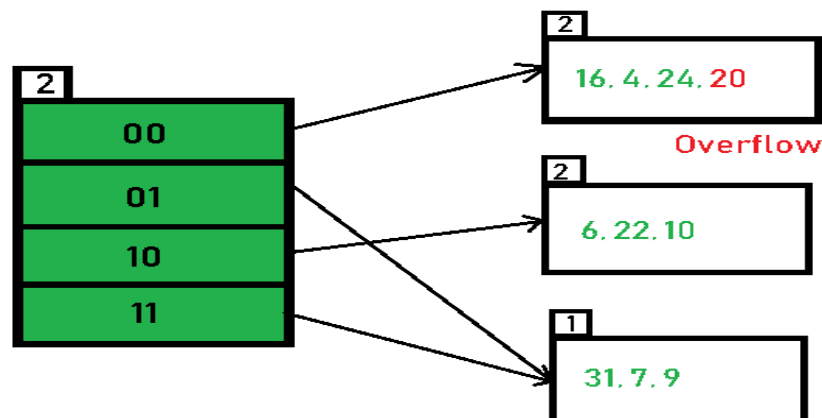
**Inserting 31,7,9:** All of these elements[31(11111), 7(111), 9(1001) ] have either 01 or 11 in their LSBs. Hence, they are mapped on the bucket pointed out by 01 and 11. We do not encounter any overflow condition here.



$Hash(31) = 11111$   
 $Hash(7) = 111$   
 $Hash(9) = 1001$

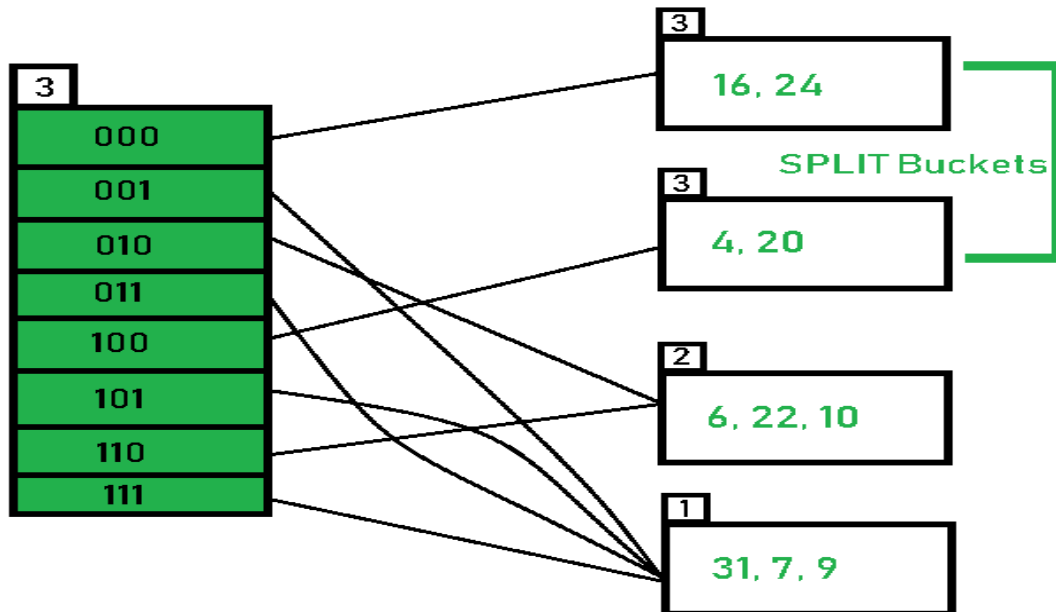
**Inserting 20:** Insertion of data element 20(10100) will again cause the overflow problem.

**Overflow, Local Depth = Global Depth**



$Hash(20) = 10100$

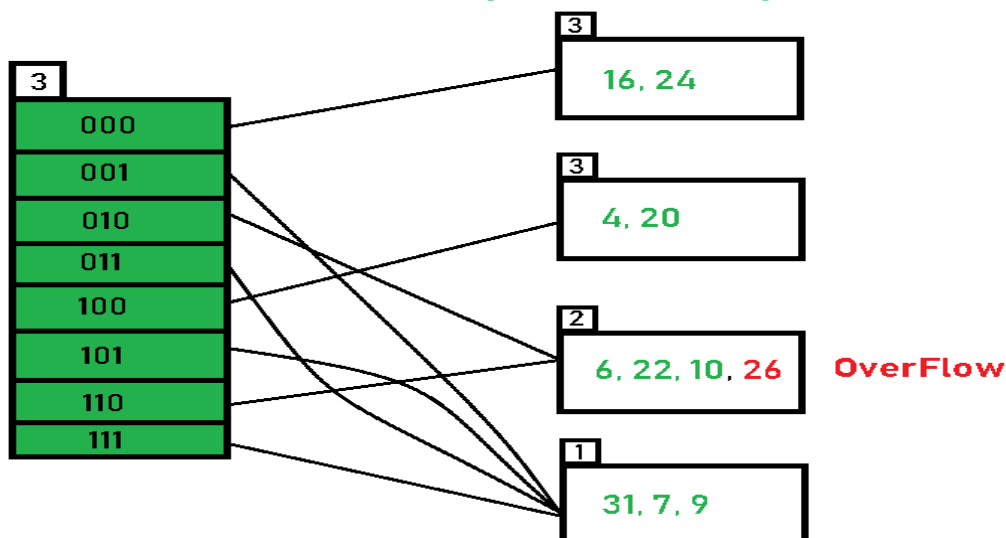
20 is inserted in bucket pointed out by 00. As directed by **Step 7-Case 1**, since the **local depth of the bucket = global-depth**, directory expansion (doubling) takes place along with bucket splitting. Elements present in overflowing bucket are rehashed with the new global depth. Now, the new Hash table looks like this:



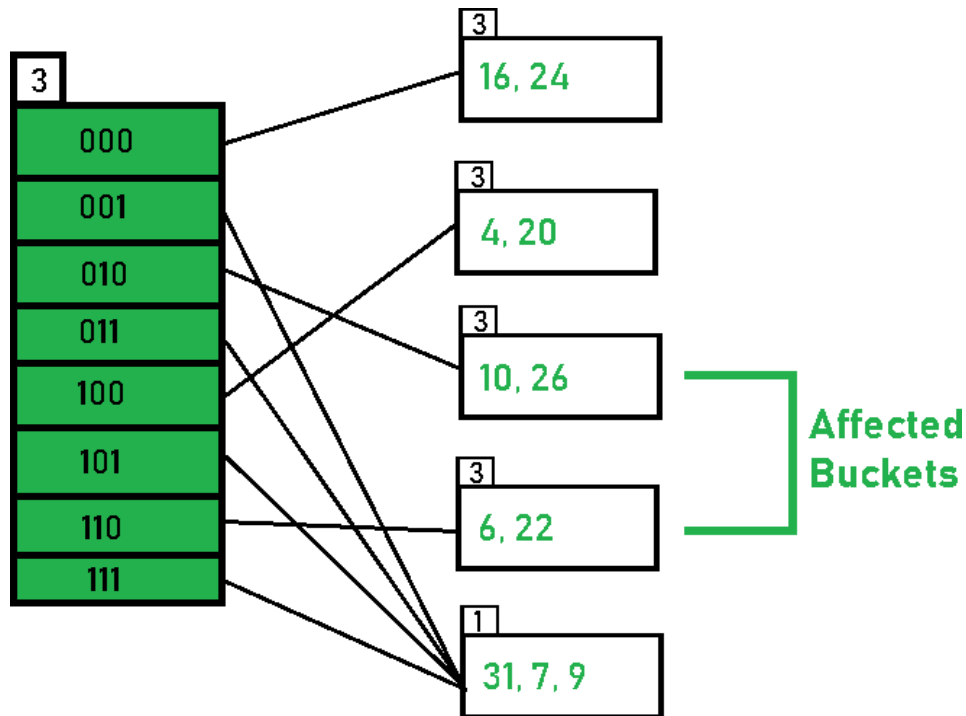
**Inserting 26:** Global depth is 3. Hence, 3 LSBs of 26 (11010) are considered. Therefore 26 best fits in the bucket pointed out by directory 010.

*Hash(26) = 11010*

*OverFlow, Local Depth < Global Depth*



The bucket overflows, and, as directed by **Step 7-Case 2**, since the **local depth of bucket < Global depth (2 < 3)**, directories are not doubled but, only the bucket is split and elements are rehashed. Finally, the output of hashing the given list of numbers is obtained.



➤ Hashing of 11 Numbers is thus completed.

### Advantages

Data retrieval is less expensive (in terms of computing).

No problem of Data-loss since the storage capacity increases dynamically.

With dynamic changes in hashing function, associated old values are rehashed w.r.t the new hash function.

### Limitations of Extendible Hashing:

The directory size may increase significantly if several records are hashed on the same directory while keeping the record distribution non-uniform.

Size of every bucket is fixed and this method is complicated to code.

Memory is wasted in pointers when the global depth and local depth difference becomes drastic.

**Example-2:** consider a prominent example of hashing the following elements:

26,14,16,12,14,10,21,17,19,20.

**BucketSize:** 2 (Assume)

**HashFunction:** Suppose the global depth is X. Then the HashFunction returns X LSBs.

**Solution:**

First, calculate the binary forms of each of the given numbers.

26-11010

14-01110

16-10000

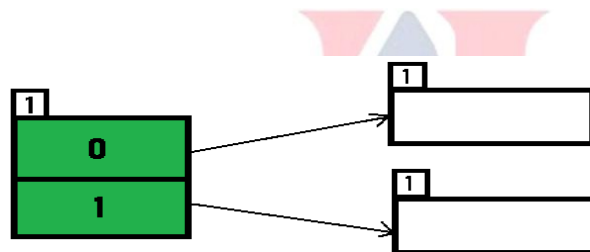
12-01100

10-01010

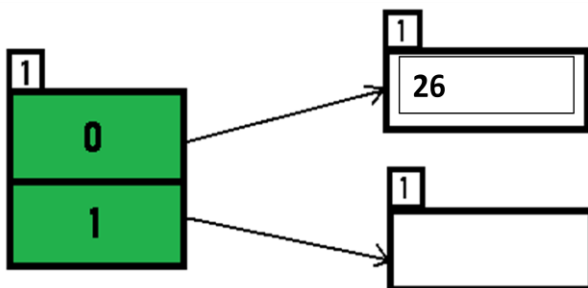
21-10101

17-10001

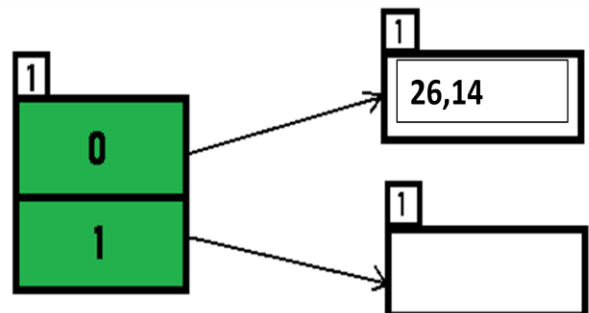
- Initially, the global-depth and local-depth is always 1. Thus, the hashing frame looks like this:



**Inserting 26:**

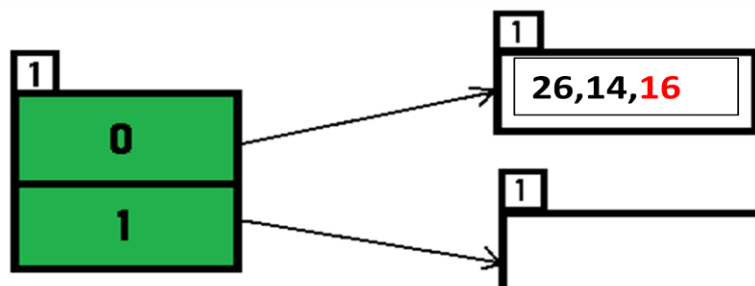


**Inserting 14:**



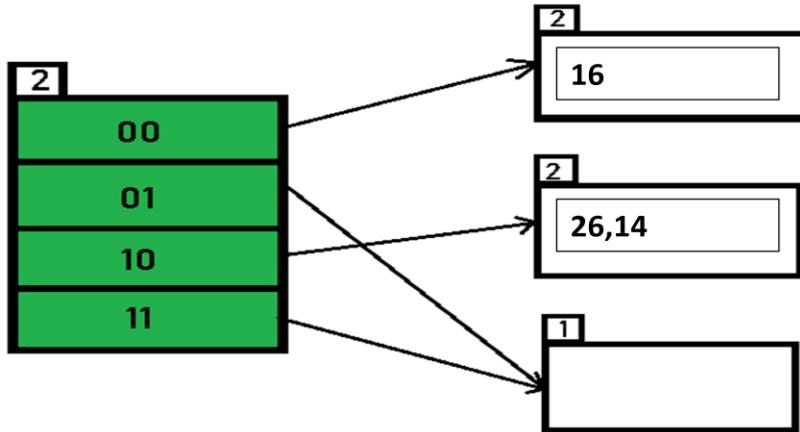
**Inserting 16:**

**Overflow Condition**  
Here, Local Depth = Global Depth



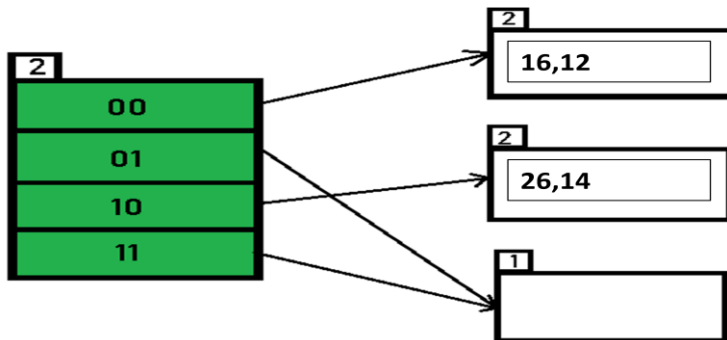
As directed by **Step 7-Case 1**, Since Local Depth = Global Depth, the bucket splits and directory expansion takes place. Also, rehashing of numbers present in the overflowing bucket takes place after the split. And, since the global depth is incremented by 1, now, the global depth is 2. Hence, 26,14,16 are now rehashed w.r.t 2 LSBs. [ 26(11010),14(01110),16(10000)] .

***After Bucket Split and Directory Expansion***



Inserting 12:

***After Bucket Split and Directory Expansion***

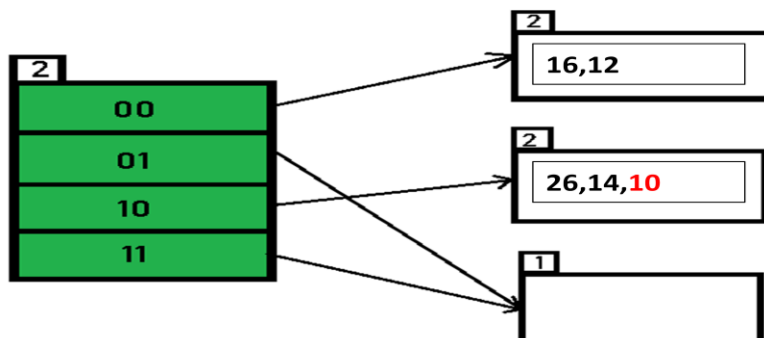


Inserting 10:

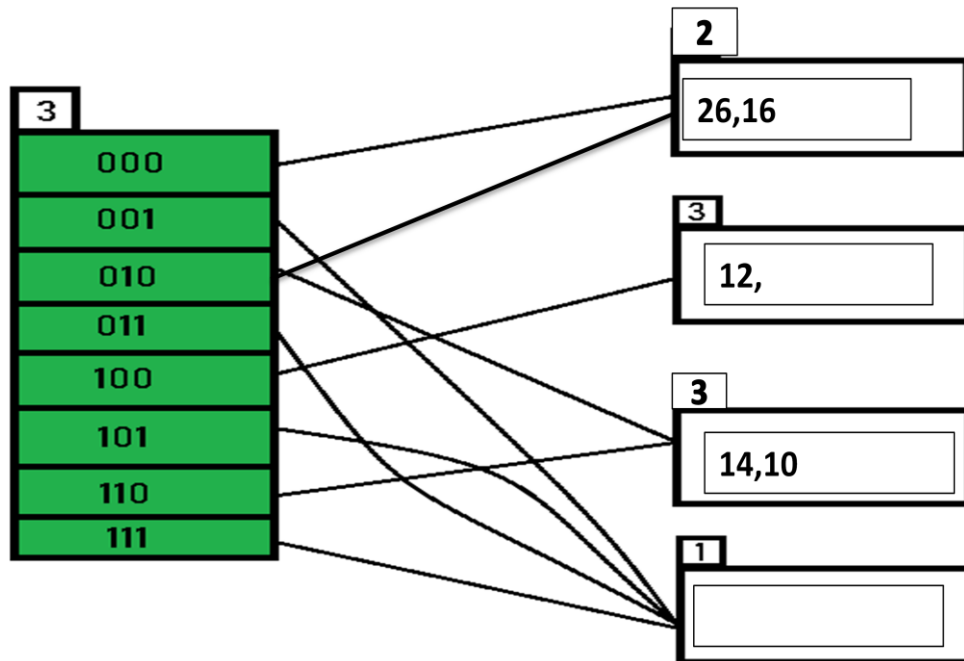
**OverFlow Condition**

*Here. Local Depth=Global Depth*

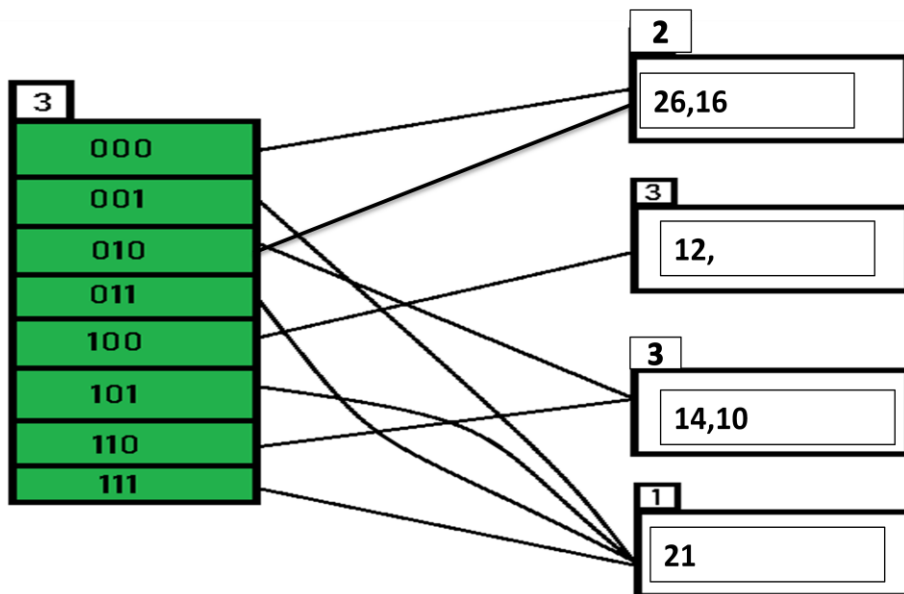
***After Bucket Split and Directory Expansion***



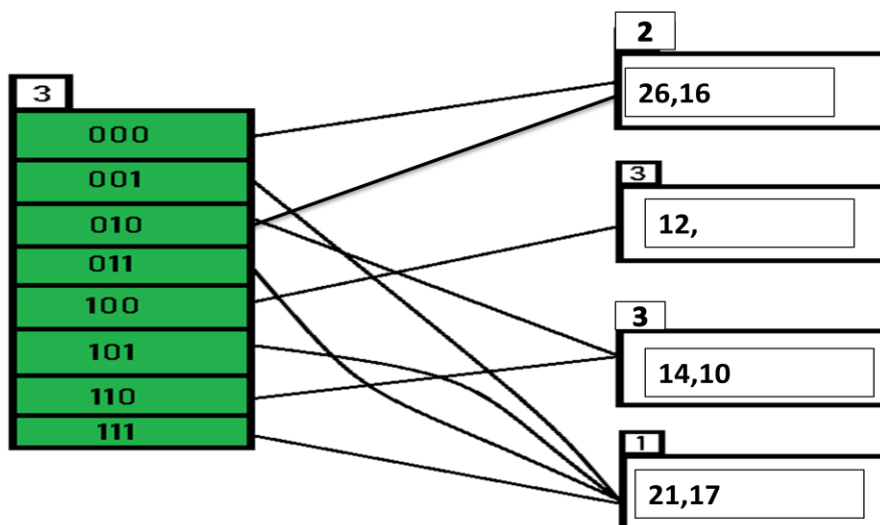
10 is inserted in bucket pointed out by 10. As directed by **Step 7-Case 1**, since the **local depth of the bucket = global-depth**, directory expansion (doubling) takes place along with bucket splitting. Elements present in overflowing bucket are rehashed with the new global depth. Now, the new Hash table looks like this:



Inserting



Inserting 17:



Insertion is completed



## Implementation of Dictionaries:

In computer science, a dictionary is an abstract data type that represents an ordered or unordered list of key-value pair elements where keys are used to search/locate the elements in the list. In a dictionary ADT, the data to be stored is divided into two parts:

- **Key**
- **Value.**

Each item stored in a dictionary is represented by a key-value pair. Key is used to access the elements in the dictionary. With the key we can access value which has more information about the element.

## Characteristics of Dictionary

- **Key-Value Pairs:** Dictionaries store data as key-value pairs where each key is unique and maps to exactly one value.
- **Direct Access:** The primary feature of dictionaries is to provide fast access to elements not by their position, as in lists or arrays, but by their keys.
- **Dynamic Size:** Like many abstract data types, dictionaries typically allow for dynamic resizing. New key-value pairs can be added, and existing ones can be removed.
- **Ordering:** Some dictionaries maintain the order of elements, such as ordered maps or sorted dictionaries. Others, like hash tables, do not maintain any particular order.
- **Key Uniqueness:** Each key in a dictionary must be unique, though different keys can map to the same value.

## Types of Dictionary

There are two major variations of dictionaries:

### Ordered dictionary.

- In an ordered dictionary, the relative order is determined by comparison on keys.
- The order should be completely dependent on the key.

### Unordered dictionary.

- In an unordered dictionary, no order relation is assumed on keys.
- Only equality operation can be performed on the keys.

**Example1:**

Key	Value
FirstName	Mahesh
LastName	Babu
Address	Hyderabad
Age	45

**Example2:**

The results of a classroom test could be represented as a dictionary with student's names as keys and their scores as the values:

```
results={"Sachin":65,"Dhoni":70,"Kohli":55,"Irfan":50,"Laxman":40}
```

**Basic Dictionary Operations**

The dictionary ADT provides operations for inserting the records, deleting the records and searching the records in the collection of databases. Dictionaries typically support so many operations such as:

➤ **Insert(x,D)** → insertion of element x (key & value) into dictionary D.

```
Insert(key, value)
```

**Example:** Insert(age, 40)

➤ **Delete(x,D)** → deletion of element x (key & value) from the dictionary D.

```
delete(key)
```

**Example:** delete(age)

➤ **Search(x,D)** → searching the prescribed value of x in the dictionary D with a key of an element x.

```
search(key) → value
```

**Example:** search(age) → 40

➤ **Member(x,D)** → It returns "true" if x belongs to D else returns "false".

➤ **size(D)** → It returns the count of total number of elements in dictionary D.

➤ **Max(D)** → It returns the maximum element in the dictionary D.

➤ **Min(D)** → It returns the minimum element in the dictionary D.

**Example:** Consider an empty unordered dictionary and the following set of operations:

<u>Operation</u>	<u>Dictionary</u>	<u>Output</u>
insertItem(5,A)	{(5,A)}	
insertItem(7,B)	{(5,A), (7,B)}	
insertItem(2,C)	{(5,A), (7,B), (2,C)}	
insertItem(8,D)	{(5,A), (7,B), (2,C), (8,D)}	
insertItem(2,E)	{(5,A), (7,B), (2,C), (8,D), (2,E)}	
findItem(7)	{(5,A), (7,B), (2,C), (8,D), (2,E)}	B
findItem(4)	{(5,A), (7,B), (2,C), (8,D), (2,E)}	NO_SUCH_KEY
findItem(2)	{(5,A), (7,B), (2,C), (8,D), (2,E)}	C
size()	{(5,A), (7,B), (2,C), (8,D), (2,E)}	5
removeItem(5)	{(7,B), (2,C), (8,D), (2,E)}	A
removeAllItems(2)	{(7,B), (8,D)}	C, E
findItem(4)	{(7,B), (8,D)}	NO_SUCH_KEY