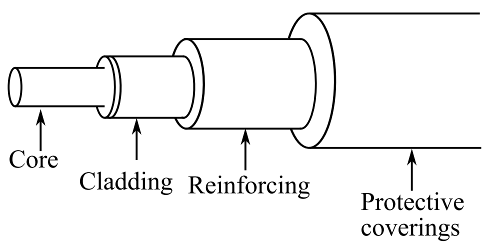
Optical Fibers

Optical fibers are thin strands or wires made of glass. As they are used to transmit light from one point to another, they are also called as "light guides" or "wave guides". Light is transmitted through them using the principle of total internal reflection of light. The loss of signal strength is considerably less in optical fiber, compared with conventional coaxial cables. Hence permits transmission over long distances. Use of light waves in place of radio and microwaves has improved the speed of communication.

Construction of an optical fiber

An optical fiber has a central cylindrical rod made of glass having a refractive index n_1, called as core. The core is surrounded by a cylindrical jacket made of glass or transparent plastic having a refractive index n_2, called as the cladding. But n_1 is always greater than n_2, $n_1 > n_2$. One more outer cylindrical jacket may also be found made of PVC for protecting the optical fiber from moisture and mechanical wear and tear.



Angle of acceptance (θ_a):

The angle of incidence on core for which total internal reflection takes places inside the core. $\theta_a = \sin^{-1}(NA)$

Numerical Aperture(NA):

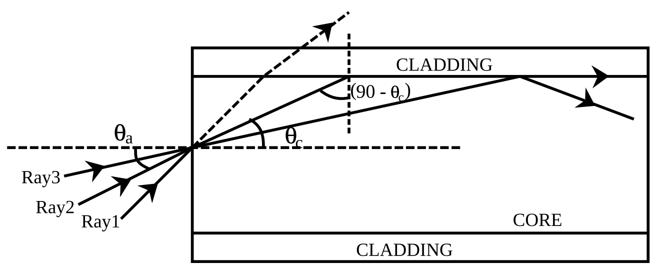
It is defined as "the sine of the acceptance angle". That is $NA = \sin\theta_a$ It gives the light gathering capability of optical fibers.

Critical angle:

The angle of incidence for which angle of refraction is 90°

Propagation of light through the fiber

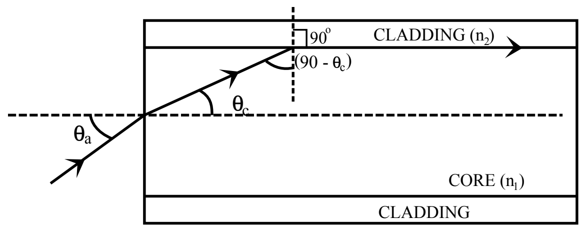
If the angle of incidence θ is greater than angle of acceptance θ_a. The light ray (ray 1) incident from air into the core of the optical fiber is refracted into the core and reaches core-cladding boundary. There it undergoes another refraction into the cladding. After travelling in the cladding suffers another refraction into the air and escapes from the fiber.



If the angle of incidence θ is equal to angle of acceptance θ_a. The light ray (ray 2) incident at an angle θ_a, is refracted into the core with an angle of refraction θ_c. It reaches the core-cladding boundary making an angle of incidence $(90-\theta_c)$. This angle is the critical angle for the core - cladding interface. Hence this ray travels parallel to the surface of the core without entering the cladding.

If the angle of incidence θ is less than angle of acceptance θ_a. The light ray is incident (ray 3 in the figure) into the core, it is refracted at an angle less than θ_c. Hence it reaches core-cladding interface with an angle of incidence greater than critical angle and undergoes total internal reflection. It reaches the opposite surface of the core and undergoes another total internal reflection. This goes on and the ray reaches the other end of the fiber without going out of the core. Thus if a ray of light is injected into the core of the fiber within an angle θ_a, it will be carried along the optical fiber without being lost.

Numerical aperture in terms of n_1 and n_2



For the refraction at air-core boundary, refractive indices of air and core are n_0 and n_1. The angle of incidence is θ_a and angle of refraction is θ_c. From snell's law,

$$n_0 \sin\theta_a = n_1 \sin\theta_c$$

For the refraction at core-cladding interface, the refractive indices are n_1 and n_2. The angle of incidence and angle of refraction is $(90 - \theta_c)$ and $90°$ respectively. Using snell's law again

$$n_1 \sin(90 - \theta_c) = n_2 \sin 90°$$

$$n_1 \cos\theta_c = n_2$$

$$\cos\theta_c = \frac{n_2}{n_1}$$

$$\cos^2\theta_c = \frac{{n_2}^2}{{n_1}^2}$$

$$1 - \sin^2\theta_c = \frac{{n_2}^2}{{n_1}^2}$$

$$\sin^2\theta_c = 1 - \frac{{n_2}^2}{{n_1}^2} = \frac{{n_1}^2 - {n_2}^2}{{n_1}^2}$$

Substituting for $\sin\theta_c$ in (1) from equations(2),

$$NA = n_1 \times \sqrt{\frac{n_1^2 - n_2^2}{n_1}}$$

$$NA = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

The NA represents the light gathering capability of the optical fiber.

Fractional index change

It is the ratio of the difference in the refractive indices of core and cladding to the refractive index of the core of an optical fiber.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Relation between Δ and NA

We've

$$(n_1 - n_2) = n_1\Delta \tag{1}$$

and

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 + n_2)(n_1 - n_2)} = \sqrt{(n_1 + n_2)n_1\Delta} \tag{2}$$

Since for all optical fibers

$$n_1 \approx n_2,\ (n_1 + n_2) = 2n_1$$

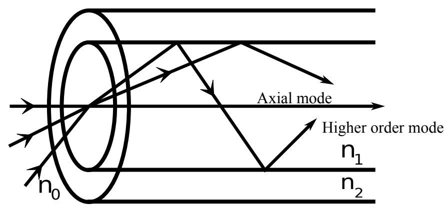
$$NA = \sqrt{2n_1^2\Delta} = n_1\sqrt{2\Delta}$$

Though an increase in the value of Δ increases NA and thus enhances the light gathering capacity of the fiber, we cannot increase Δ to a very large value, since it leads to what is called intermodal dispersion which causes signal distortion.

Modes of propagation

A mode is an allowable field configuration, for a given waveguide geometry, that satisfies Maxwell's equations (or the derived wave equations) and all of the boundary conditions of the problem. The figure below represents three modes of propagation.

A mode is a defined path in which light travels.(A mode in a fiber is basically a particular light ray travelling in a particular angle) When light is transmitted through optical fiber, it can travel along different ray paths, known as modes of propagation. The light signal can propagate through the core of the optical fiber on a single path (single-mode fiber) or on many paths (multimode fiber). The mode in which light travels depends on geometry, the index profile of the fiber and the wavelength of the light. A ray travelling along the axis is known as axial mode and the ray incident on the core cladding interface at the critical angle gives rise to higher order mode. The distance travelled by axial mode is smaller than the higher order modes due to which the different modes reach the other end of the fiber at different times. This is known as intermodal dispersion, which causes signal distortion (Inter-modal dispersion is a phenomenon, where the modes will traverse the same length of guide in different times for different ray angles). It can be reduced by reducing the number of modes travelling through the fiber by reducing the



The number of modes in a fiber is determined by a quantity known as "normalized frequency parameter" or V number. It is given by

$$V = \frac{2\pi a}{\lambda}(NA)$$

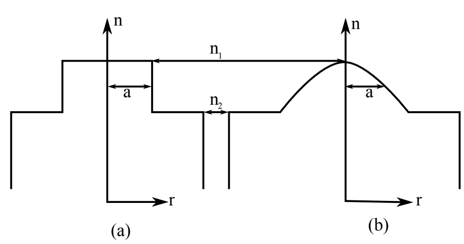
where a is the radius of the core, λ is the wavelength of light and NA is the numerical aperture

$$Number of modes\ (N) \approx \frac{V^2}{2}$$

Larger the V number more number of modes can propagate in a fiber.

Refractive Index Profile

A refractive index profile is the distribution of refractive indices of the materials within an optical fiber. Some optical fiber has a step index profile, in which the core has uniformly distributed index and the cladding has a lower uniformly distributed index. Other optical fiber has graded index profile, in which refractive index varies gradually as a function of radial distance form the fiber center. Graded index profile include power-law index profiles and parabolic index profiles.



Refractive index profile for (a) step index and (b) graded index fibers. In both cases, the cladding refractive index is n_2 while the maximum core refractive index is n_1

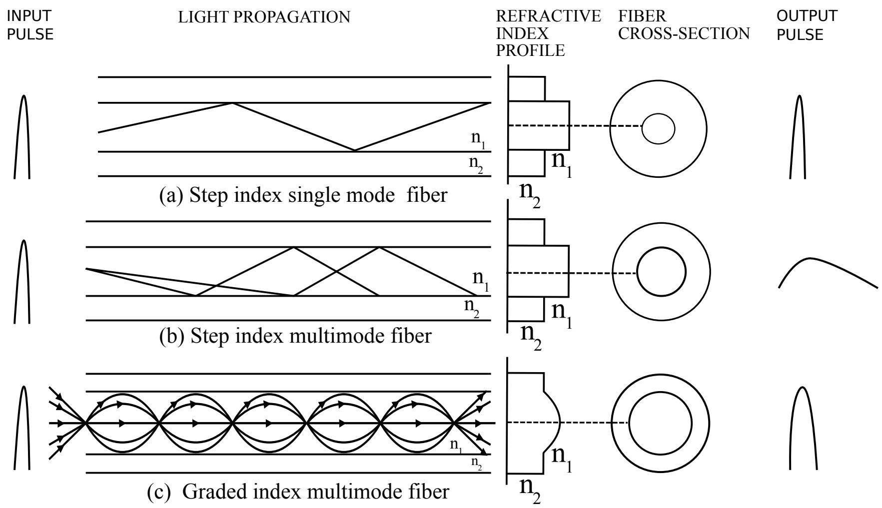
Types of optical fibers

Optical fiber are classified into two types depending upon their refractive index profile namely step index fiber and the graded index fiber. Depending upon the number of modes that are transmitted, the step index fiber is further subdivided into two types - step index single mode fiber and step index multimode fiber.

1. **Step index single mode fiber** :It has a core of constant refractive index n_1 and cladding of constant refractive index n_2. The diameter of the core is about $8 - 10$ μm and that of cladding is 100 μm. Mode of transmission is single mode. The signal overlapping and other kinds of signal losses are minimum. They are used in long distance high speed transmission lines. Bandwidth is very large > 3 GHz $-$ km. They need laser as the source of light. Splicing is very difficult due to small core. Optical fibers may be connected to each other by connectors or by splicing, that is joining two fibers together to forma continuous optical waveguide. The generally accepted splicing method is arc fusion splicing, which melts the fiber ends together with the electric arc. For quicker fastening jobs a "mechanical splice" is used. They are less expensive. Example : Submarine cable system.

2. **Step index multimode fiber** : The core and cladding has a uniform refractive index n_1 and n_2, where $n_1 > n_2$. The core diameter is about $20 - 50$ μm and that of cladding is 125 μm. Mode of transmission is multiple. This can also carry a large number of modes but with more overlapping of signals, absorption and other losses. They are mainly used for short distance communication for which signal distortion is not significant. Bandwidth is < 200 MHz $-$ km. Its typical application is in data links which has lower bandwidth. They are least expensive. It can accept either a laser or an LED as source of light. Splicing is difficult but doable.

3. **Graded index multimode fiber** : The refractive index of the core decreases gradually towards cladding from the center. The cladding has a uniform refractive index. The core diameter is about 50 μm and that of cladding is 125 μm. This can carry a large number of modes without much overlapping of signals and losses. Bandwidth is 200 MHz to 3GHz $-$ km. It is useful for premises networks, like security system etc. Splicing is difficult but doable. Laser and LED is used as the source.



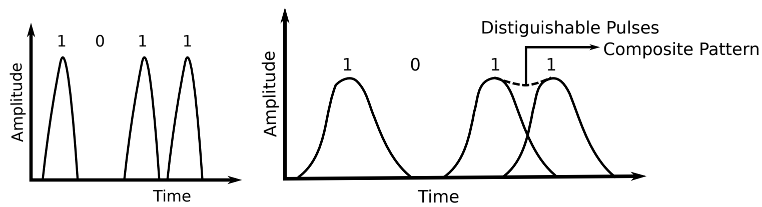
Attenuation

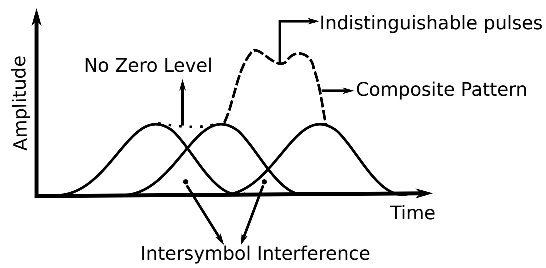
Attenuation is the loss of power suffered by the optical signal as it propagates through the fiber. It is measured in decibels(dB/km). The power of light as it propagates through fiber decays exponentially with the length.If P_i is the input power and P_o is the output power after passing through the fiber of length L, the attenuation co efficient α,

$$\alpha = -\frac{10}{L} log_{10} \left(\frac{P_o}{P_i} \right) dB/\text{km}$$

Pulse Broadening

Dispersion within the fiber causes broadening of the transmitted light pulses as they travel along the channel. It may be observed that each pulse broadens and overlaps with its neighbors, eventually becoming indistinguishable at the receiver end. This effect is known as Intersymbol Interfernece(ISI).





Basically there are three types of dispersion

1. Intermodal dispersion
2. Intramodal or Chromatic dispersion
3. Polarization mode dispersion

Intermodal dispersion

It is also referred as modal or mode dispersion. Pulse broadening due to intermodal dispersion results from propagation delay differences between modes within a multimode fiber. The pulse width at the output is dependent upon the transmission times of the slowest and fastest modes.

The multimode step index fibers exhibit a large amount of intermodal dispersion which gives the greatest pulse broadening. Their is no intermodal dispersion, under purely single mode operation. Intermodal dispersion in multimode fibers may be reduced by adoption of an optimum refractive index profile which is provided by the near parabolic profile of most graded index fibers.

Intramodal or Chromatic dispersion

It may occur in all types of optical fiber and results from the finite spectral linewidth of the optical source.The propagation delay between the different spectral components of the transmitted signal causes broadening of each transmitted mode and hence intramodal dispersion.

The delay differences may be caused by the dispersive properties of the waveguides material - Material dispersion

Guidance effects within the fiber structure - Waveguide dispersion.

Material dispersion

Due to the variation of refractive index with wavelength or frequency of light, this dispersion arises. Light waves of different wavelength travel at different speeds in a medium. Consequently, narrow pulses of light tend to broaden as they travel down the optical fiber. Pulses at different wavelength travel with different velocities in material dispersion.

Waveguide dispersion

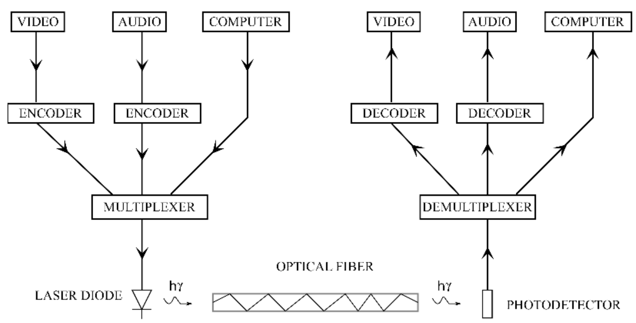
Considering the ray theory approach, it is equivalent to the angle between the ray and the fiber axis varying with wavelength which subsequently leads to variation in the transmission times for the rays. In waveguide dispersion, pulses at different wavelength propagating through in the same mode, travel at slightly different angles. It is present in both single mode and multimode fibers.

Polarization mode dispersion(PMD)

It is a result from birefringence and it can become a limiting factor for optical fiber communication at high rates. Two different polarization states of light travel at slightly different velocities. This is caused by imperfections and asymmetries in the glass fiber core itself and results in random spreading of optical signal. Single mode optical fiber consists of one propagation mode, which in turn is comprised of two orthogonal polarization modes. Asymmetrical differences due to both intrinsic and extrinsic factors which is induced at the time of fiber manufacture, introduce a small refractive index variations between the two states. This is known as birefringence or double refraction

Applications

Optical fiber communication : A typical communication system has the following layout: Signals from the audio and video sources are coded to binary form and fed to a multiplexer along with the computer signals. From the multiplexer the coded electrical signals are sent to a laser diode. The optical signals from the laser diode is coupled into the optical fiber. At the other end of the fiber the optical signals are picked up by a photo detector. The electrical signals from the photo detector then reach the demultiplexer. Here the signals are separated into audio, video and computer signals. After decoding the signals reach the respective output devices.



Advantages:

1. They can transmit a large number of signals. As many as 1500 signals can be transmitted in an fiber having the diameter of a human hair.
2. Since the materials used for manufacturing the fibers like glass and plastic are cheap cost per channel becomes cheaper.
3. Once established, the lines need not be dug up again and again as the fiber can support future heavier traffic.
4. Since electric and magnetic fields do not effect them, these lines can be established close to the high-tension power lines also.
5. Leakage of signals does not take place. Hence tapping of information and spying is prevented.
6. Cross talk is also avoided, as light signals do not overlap.

Disadvantage

1. In case of breakdown, the repair work requires skilled personnel and hence can be costly and time consuming.
2. As the fibers are made of glass, excessive twisting and bending of the cables should be avoided.

Problems

1. Calculate the numerical aperture, angle of acceptance and fractional index change for an optical fiber having refractive indices 1.563 and 1.498 for the core and the cladding respectively.

   Solution:

   Refractive index of the core, $n_1 = 1.563$

   Refractive index of the cladding, $n_2 = 1.498$

   NA = ?

   $\theta_a = ?$

   $\Delta = ?$

   $$NA = \sqrt{{n_1}^2 - {n_2}^2} = \sqrt{1.563^2 - 1.498^2} = 0.446$$

   which means only 44.6% of light is gathered inside the optical fiber.

   $$\theta_a = \sin^{-1}(NA) = \sin^{-1}(0.446) = 26.49^\circ$$

   $$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.563 - 1.498}{1.563} = 0.0415$$

2. An optical fiber of refractive index 1.50 is to be clad with another glass to ensure internal reflection that will contain light travelling within 5° of the fiber axis. What maximum index of refraction is allowed for the cladding.

   Solution:

   Angle between the ray and the fiber axis = 5°.

   Angle of incidence of the ray at the core - cladding interface = 85°

   For grazing incidence,

   $$n_1 \sin 85 = n_2 \sin 90^\circ$$
   $$1.50 \sin 85^\circ = n_2$$
   $$n_2 = 1.494$$

   if $n_2 = 1.494$, the refracted ray grazes along the interface . The ray suffers total internal reflection for $n_2 < 1.494$. The maximum index of refraction allowed for cladding should be < 1.494.

3. The angle of acceptance of an optical fiber is 30° when kept in air. Find the angle of acceptance when it is in a medium of refractive index 1.33.

   Solution:

   $$(\theta_a)_{air} = 30^\circ, n_o = 1.33, (\theta_a)_{medium}$$

   In air,

   $$NA = \sqrt{{n_1}^2 - {n_2}^2} = \sin(\theta_a)_{air} = \sin(30) = 0.5$$

In medium of refractive index n_o

$$n_o \sin_a = \sqrt{{n_1}^2 - {n_2}^2}$$

$$(1.33)\sin(\theta_a) = 0.5$$

$$\theta_a = 22.08^\circ$$

4. The numerical aperture of an optical fiber is 0.2 when surrounded by air. Determined the refractive index of its core given the refractive index of cladding as 1.59. Also, find the acceptance angle when it is in a medium of refractive index 1.33.

   Solution:

   NA = 0.2, n_2 = 1.59, n_1 =?θ_a =? In air,

$$NA = \sqrt{{n_1}^2 - {n_2}^2};$$

$$n_1 = \sqrt{(NA)^2 + (n_2)^2} = \sqrt{(0.2)^2 + (1.59)^2} = 1.602$$

   In medium of refractive index n_o

$$n_o \sin\theta_a = \sqrt{{n_1}^2 - {n_2}^2}; \theta_a = 8.65^\circ$$

5. Calculate the V-number for a fiber of core diameter 40µm and with refractive indices of 41.55 and 41.50 respectively for core and cladding when the wavelength of the propagation wave is 1400nm. Also calculate the number of modes that the fiber can support for propagation. Assume that the fiber is in air.

   Solution:

$$n_1 = 1.55, n_2 = 1.50, d = 40 \times 10^{-6}\text{m}, \lambda = 1400 \times 10^{-9}\text{m}, V = ?$$

$$V = \frac{\pi d}{\lambda}\sqrt{{n_1}^2 - {n_2}^2} = \frac{\pi \times 40 \times 10^{-6}}{1400 \times 10^{-9}}\sqrt{1.55^2 - 1.50^2} = 35.05$$

   Therefore the number of modes of the fiber support

$$\frac{V^2}{2} = \frac{35^2}{2} = 614$$

6. Find the attenuation in an optical fiber of length 500 m, when a light signal of power 100 mW emerges out of the fiber with a power 90 mW

   Solution:

$$L = 500\text{ m} = 0.5\text{ km}, P_i = 100\text{ mW}, P_o = 90\text{ mW}, \alpha = ?$$

$$\alpha = -\frac{10}{L} log_{10}\left(\frac{P_o}{P_i}\right) = \frac{10}{0.5} log_{10}\left(\frac{90}{100}\right) = 0.915\ dB/\text{km}$$

7. A fiber with an input power of 9 μW has a loss of 1.5 dB/km. If the fiber is 3000 m long. What is the output power.

Solution:

$$L = 3 \text{ km}, \alpha = 1.5\ dB/\text{km}, P_i = 9\ \mu\text{W}, P_o = ?$$

$$\alpha = -\frac{10}{L} log_{10}\left(\frac{P_o}{P_i}\right)$$

$$P_o = \frac{P_i}{10^{\left(\frac{\alpha \times L}{10}\right)}} = \frac{9 \times 10^{-6}}{10^{\left(\frac{1.5 \times 3 \times 10^3}{10}\right)}} = 3.193\ \mu\text{W}$$

8. The attenuation of light in an optical fiber is 3.6 dB/km. What fractional initial intensity remains that after (i) 1 km and (ii) 3 km.

Solution:

$$L = 1\&3 \text{ km}, \alpha = 3.6\ dB/\text{km}, \frac{P_o}{P_i} = ?$$

$$\alpha = -\frac{10}{L} log_{10}\left(\frac{P_o}{P_i}\right)$$

$$\left(\frac{P_o}{P_i}\right)_{1 \text{ km}} = 10^{-\frac{\alpha L}{10}} = 10^{-\frac{3.6 \times 1}{10}} = 0.4365$$

$$\left(\frac{P_o}{P_i}\right)_{3 \text{ km}} = 10^{-\frac{\alpha L}{10}} = 10^{-\frac{3.6 \times 3}{10}} = 0.0831$$

9. A multimode step index fiber with a core diameter of 80 μm and a relative index difference of 1.5% is operating at a wavelength of 0.85 μm. If the core refractive index is 1.48 estimate (a) the normalized frequency of the fiber. (b) the number of guided modes.

Solution:

$$n_1 = 1.48, \lambda = 0.85\ \mu\text{m}, d = 8\ \mu\text{m}, \Delta = 1.5\% = \frac{1.5}{100} = 0.015$$

$$V \approx \frac{\pi d n_1 \sqrt{2\Delta}}{\lambda} = \frac{\pi \times 80 \times 10^{-6} \times 1.48\sqrt{2 \times 0.015}}{0.85 \times 10^{-6}} = 75.8$$

The total number of guided modes is given by

$$N \approx \frac{V^2}{2} = \frac{5745.6}{2} = 2873$$

Hence, this fiber has a V number of approximately 76, giving nearly 3000 guided modes.

10. A graded index fiber has a core with a parabolic refractive index profile which has a diameter of 50µm. The fiber has a numerical aperture of 0.2. Estimate the total number of guided modes propagating in the fiber when it is operating at a wavelength of 1 µm.

    Solution:

    $$d = 50\ \mu\text{m}, NA = 0.2, \lambda = 1\ \mu\text{m}$$

    The normalized frequency parameter for the fiber is

    $$V = \frac{\pi\, d\, NA}{\lambda} = \frac{\pi \times 50 \times 10^{-6} \times 0.2}{1 \times 10^{-6}} = 31.4$$

    The mode volume may be obtained, where for a parabolic profile (graded profile)

    $$N \approx \frac{V^2}{4} = \frac{986}{4} = 246.49 \approx 247$$

    Hence, the fiber supports approximately 247 guided modes.

11. Estimate the maximum core diameter of an optical fiber with the same relative refractive index difference 1.5% and core refractive index 1.48 as the fiber given in order that it may be suitable for single mode operation. It may be assumed that the fiber is operating at the same wavelength 0.85 µm. Further, estimate the new maximum core diameter for single mode operation when the relative refractive index difference is reduced by a factor of 10.

    Solution:

    $$d =?, \Delta = 1.5\% = \frac{1.5}{100} = 0.015, n_1 = 1.48, \lambda = 0.85\ \mu\text{m}$$

    $$V = \frac{\pi\, d\, (NA)}{\lambda}$$

    $$a = \frac{V\lambda}{2\pi\, n_1 \sqrt{2\Delta}} = \frac{2.405 \times 0.85 \times 10^{-6}}{2\pi \times 1.48 \times \sqrt{2 \times 0.015}} = 1.266\ \mu\text{m}$$

    Therefore the maximum core diameter for single mode operation is approximately 2.6 µm.

    Reducing the relative refractive index difference by a factor of 10 and again $\Delta = \frac{0.015}{10} = 0.0015$

    $$d = \frac{V\lambda}{\pi n_1 \sqrt{2\Delta}} = \frac{2.405 \times 0.85 \times 10^{-6}}{\pi \times 1.48 \times \sqrt{2 \times 0.0015}} = 8.02\ \mu\text{m}$$

12. Determine the cut off wavelength for a step index fiber to exhibit single mode operation when the core refractive index and radius are 1.46 and 4.5 µm respectively, with the relative index difference being 0.25%.

Solution: With $V = 2.405$, gives

$$V = \frac{d\pi(NA)}{\lambda} = \frac{d\pi(n\sqrt{2\Delta})}{\lambda}$$

$$\lambda_c = \frac{2a\pi n\sqrt{(2\Delta)}}{V} = \frac{2 \times 4.5 \times 10^{-6} \times \pi \times 1.46\sqrt{2 \times 0.0025}}{2.405}$$
$$\lambda_c = 1.214 \text{ µm} = 1214 \text{ nm}$$

13. A typical relative refractive index difference for an optical fiber designed for long distance transmission is 1%. Estimate the NA and acceptance angle in air for the fiber when the core index is 1.46. Further, calculate the critical angle at the core-cladding interface within the fiber.

    Solution: $\Delta = 0.01$, $n_1 = 1.46$, $NA = ?$, $\theta_c = ?$

    $$NA = n_1\sqrt{2\Delta} = 1.46\sqrt{2 \times 0.01} = 0.21$$

    $$\theta_c = \sin^{-1} NA = \sin^{-1} 0.21 = 12.12$$

    For relative refractive index difference Δ gives,

    $$\Delta = \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$$

    Hence,
    $$\frac{n_2}{n_1} = 1 - \Delta = 1 - 0.01 = 0.99$$

    The critical angle at core-cladding interface is

    $$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(0.99) = 81.9^\circ$$

Note

1. The maximum radius of the single mode fiber is $a < 1.27$ µm and V is always 2.405.

2. For a parabolic index profile core fiber ($\alpha = 2$), $N \approx \frac{V^2}{4}$, which half the number supported by a step index fiber ($\alpha = \infty$) with the same V value.