Type-Level Property Based Testing

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- These can be modelled using dependent types
  - Dependent types are difficult to get right
- How do we increase confidence in our dependent types?

Disclaimer: “increase confidence”

This is not a proof technique

But hopefully, it helps us catch errors faster and provides guarantees that our model behaves as intended

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- These are all stateful
- And we would very much like them to be correct

Spectrum of Verification

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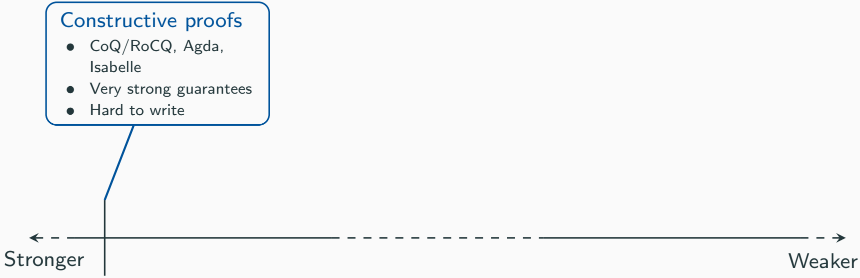

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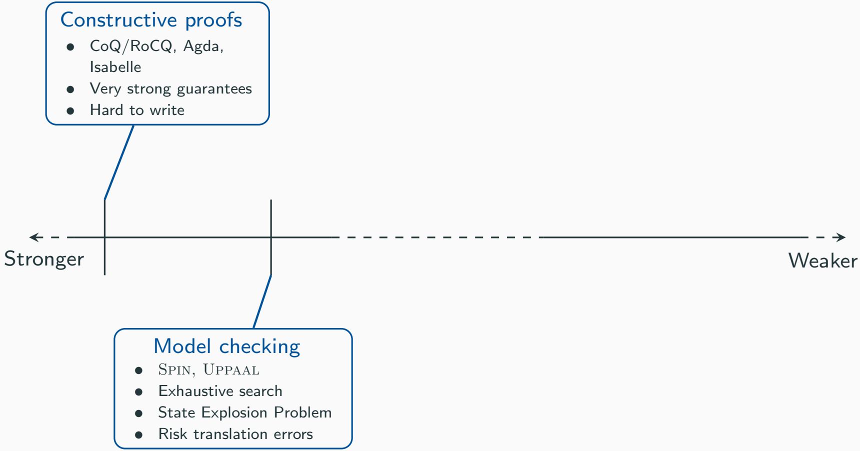

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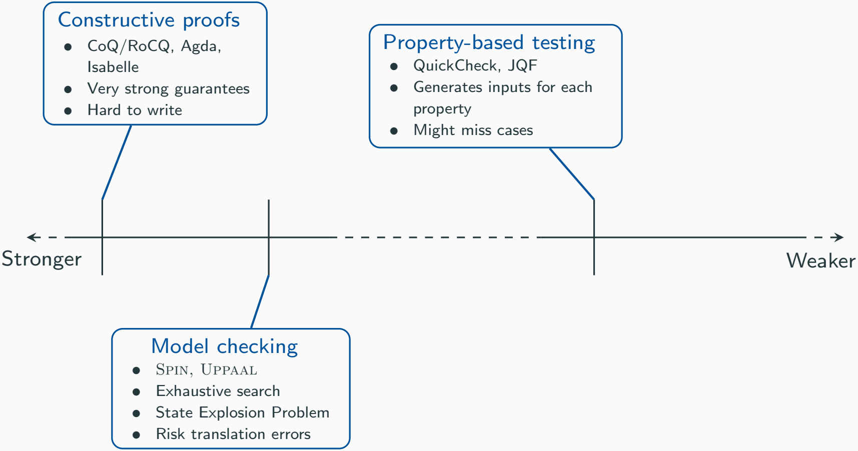

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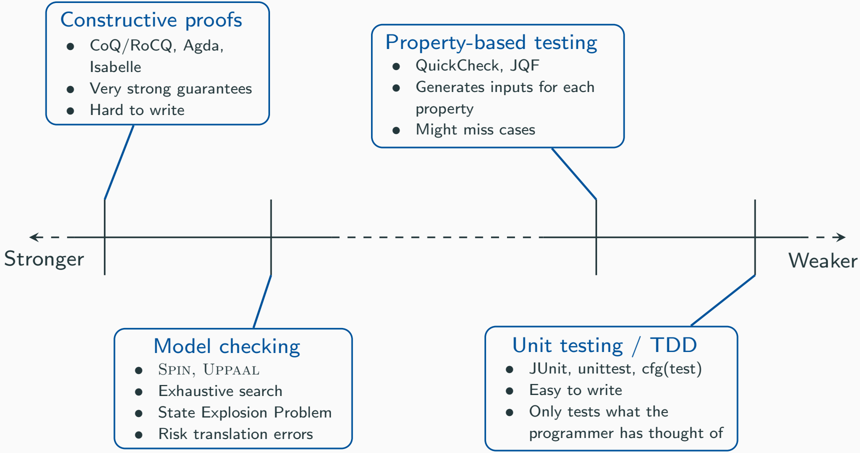

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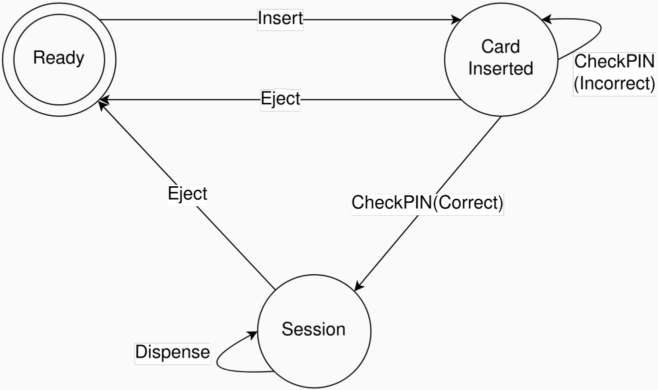
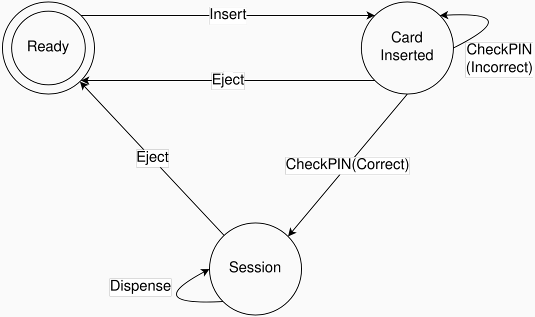
Fits somewhere in the middle

The ATM state machine



Datatype for the ATM states

```
data ATMState
  = Ready
  | CardInserted
  | Session
```



Datatype for ATM operation results

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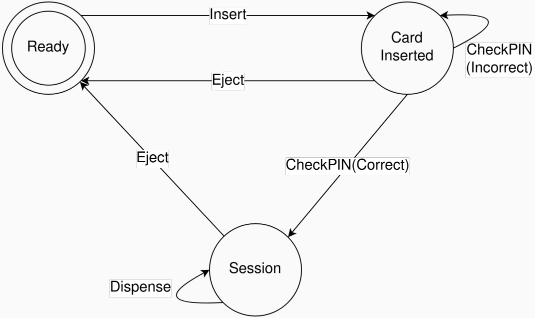
data PINok
  = Correct
  | Incorrect
```



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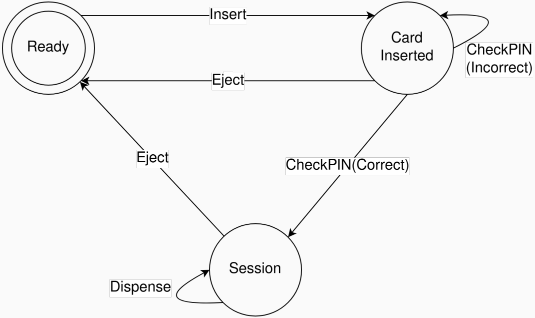
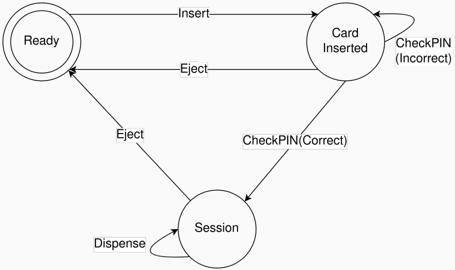


Everything which does not have a result returns Unit — `()`

State Transition Function

```
ChkPINfn : PINok -> ATMState
ChkPINfn Correct = Session
ChkPINfn Incorrect =
↪  CardInserted
```



Dependent State Transition

```
data ATM : (t : Type) -> ATMState -> (t -> ATMState)
         -> Type where
  CheckPIN :  (pin : Int)
           -> ATM PINok CardInserted ChkPINfn
     ⋮

  (>>=) :  ATM a s1 s2f
        -> ((x : a) -> ATM b (s2f x) s3f)
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```

ATM Indexed State Monad

```
data ATM : (t : Type) -> ATMState -> (t -> ATMState) ->
↪  Type where
  CheckPIN :  (pin : Int)
           -> ATM PINok CardInserted ChkPINfn
  Insert : ATM () Ready (const CardInserted)
  Dispense : (amt : Nat) -> ATM () Session (const Session)
  Eject : ATM () st (const Ready)
  Pure : (x : t) -> ATM t (stFn x) stFn
  (>>=) : ATM a s1 s2f -> ((x : a) -> ATM b (s2f x) s3f)
  ↪  -> ATM b s1 s3f
```

Why Is This Neat?

- We declare our intended start and end state in the type
  `prog : ATM () Ready (const Ready)`

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```
prog : ATM () Ready (const Ready)
```

- And the type-checker verifies that we don't use operations incorrectly

```
prog = do                      -- We start in Ready
  Insert  --------------------- Ready to CardInserted
  Correct <- CheckPIN 1234  --- CI to Session
    | Incorrect => <...>  ----- (or stay in CI)
  Dispense 42  ---------------- Stay in Session
  Eject  ---------------------- Return to Ready
```

Dependent Types Only Get Some Things Right

Rejected by the type-checker:

```
badProg : ATM ()
            Ready (const Ready)
badProg = do
  Insert
  let pin = 1234
  Correct <- CheckPIN pin
    | Incorrect => InsertCard
  Dispense 42
  -- We never Eject, so we
  -- never come back to
  -- `Ready'
```

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  Correct <- CheckPIN pin
    | Incorrect => InsertCard
  Dispense 42
  -- We never Eject, so we
  -- never come back to
  -- `Ready'
```

Accepted by the type-checker:

```
loopProg : ATM ()
             Ready (const Ready)
loopProg = do
    InsertCard
    let pin = 4321
    loopIncorrect pin
  where
    loopIncorrect : Nat -> ATM ()
                      CardInserted
                      (const Ready)
    loopIncorrect p = do
      Incorrect <- CheckPIN p
        | Correct => -- <...>
      loopIncorrect p
```

Property Based Testing

- Generate instances of types from random numbers

```
data Gen a : (Int -> a) -> Type
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- We express boolean properties, which are then tested over random inputs (typically 100)

  ```
  reverse (reverse xs) == xs
  ```
  ```

What About Dependent Types?

QuickCheck works very well for regular types, what about dependent types?

Generating Dependent Types is Tricky

Consider generating arbitrary vectors:

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Arbitrary t => Arbitrary (Vect n t) where
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How do we solve this?

Arbitrary Dependent Types

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- Specifically: dependent pairs

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  snd : p fst
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- As long as we know how to generate an `Arbitrary a`, we can generate an `Arbitrary (x : a ** p x)`

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    pure (length_ ** arbVect)
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Arbitrary (n : Nat ** Vect n a) where
  arbitrary = do
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    arbVect <- nArbitrary length_
    pure (_ ** arbVect)
```

Arbitrary ATMs?

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With a bit of work, yes!

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```
options CardInserted = do
  -- we need a PIN, even though we control the result
  let arbPIN = 0
  let op1 = (_ ** _ ** MkOpRes (CheckPIN arbPIN) Correct)
  let op2 = (_ ** _ ** MkOpRes (CheckPIN arbPIN) Incorrect)
  let op3 = (_ ** _ ** MkOpRes Eject ())

  frequency $ [(1, pure op1), (4, pure op2), (1, pure op3)]
```

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  ```
  So (quickCheck <property>)
  ```

- Idris2 is built on Quantitative Type Theory, which has erasure, meaning the tests can be removed from the compiled program

QuickCheck Spots the Error!

```
0 PROP_eventuallyReady : Fn (ATMTrace Ready 10) Bool
PROP_eventuallyReady = MkFn
  (\case (MkATMTrace _ trace) => elem Ready (map (.resSt) trace))
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```
-- Error: While processing right hand side of
--        EventuallyReady_OK. When unifying:
    So True
-- and:
    So (quickCheck PROP_eventuallyReady)
-- Mismatch between: True and False.
```

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```
MkQCRes (Just False) <log> """
Falsifiable, after 4 tests:
Starting @ Ready:
[ (<ATM 'Insert ~ ()'>, CardInserted)
, (<ATM 'CheckPIN 0 ~ Incorrect'>, CardInserted)
, (<ATM 'CheckPIN 0 ~ Incorrect'>, CardInserted)
, (<ATM 'CheckPIN 0 ~ Incorrect'>, CardInserted)
, (<ATM 'CheckPIN 0 ~ Incorrect'>, CardInserted)
<...>
]\n"""
```

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ChkPINfn : (retries : Nat) -> PINok -> ATMState
ChkPINfn 0     Correct   = Session
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```

- Carrying this through to the generators, our property passes: the file reloads successfully and the REPL reports

```
> quickCheck PROP_eventuallyReady
MkQCRes (Just True) <log> "OK, passed 100 tests"
```

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- With most verification tools, we have to translate between representations
  - Spec, model, and implementation are independent
- This results in the risk of translation mistakes
  - The verification tool might not support the same types as the implementation language
  - Might think we're verifying the same thing, when in actual fact the semantics have changed between representations

All in one

In our case, the specification *is* the model; *everywhere*

```
trace : (steps : Nat) -> (st : stT) -> Gen (Vect steps
↪  (TraceStep opT))
trace 0 _ = pure []
trace (S j) st = do
  (_ ** stFn ** opR@(MkOpRes op res)) <- arbitrary
  let nextSt = stFn res
  pure $ (MkTS opR nextSt) :: !(trace j nextSt)
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And this works for anything expressed in terms of states and operations with results — ISMs generalise

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- We have tested the dependent types which help guide us when writing the implementation, not the implementation itself since it is being kept in check by the types
- Testing gives us confidence that our dependent types are not misleading
- Dependent types are type-level programs, let's test them!

Thank You

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Paper

doi:10.1145/3678000.3678206

Further Work

- Running tests at the type level puts a lot of strain on the compiler, so there may be interesting optimisations to explore there
- Can we do more? ARQ with Sliding Window? Protocols with crash-stop failures?
- What kinds of properties can we test? There are parallels to Model Checking, so how does this compare to LTL or TLA^+?

Generic ISM Datatype

```
op : forall st . (t' : Type) -> st -> (t' -> st) -> Type

data Prog : {0 stT : _} -> (opT : (t' : _) -> stT -> (t'
↪  -> stT) -> Type) -> (t : Type) -> (from : stT) -> (to
↪  : t -> stT) -> Type where
  Pure : (x : t) -> Prog opT t (stFn x) stFn
  Op : {0 opT : (t' : _) -> stT -> (t' -> stT) -> Type} ->
  ↪  opT t st stFn -> Prog opT t st stFn
  (>>=) : Prog opT resT1 st1 stFn1 -> ((x : resT1) -> Prog
  ↪  opT resT2 (stFn1 x) stFn2) -> Prog opT resT2 st1
  ↪  stFn2
```

Operation-Result Pairs

```
record OpRes {0 stT : _} (opT : (t' : _) -> stT -> (t' ->
↪  stT) -> Type) (resT : Type) (currSt : stT) (0 nsFn :
↪  resT -> stT) where
  constructor MkOpRes
  op : opT resT currSt nsFn
  res : resT
  {auto opShow : Show (opT resT currSt nsFn)}
  {auto rShow : Show resT}
```

TraceStep

```
record TraceStep (opT : (t' : _) -> stT -> (t' -> stT) ->
↪  Type) where
  constructor MkTS
  {0 stepRT : _}
  {0 stepSt : stT}
  {0 stepFn : stepRT -> stT}

  opRes : OpRes opT stepRT stepSt stepFn
  resSt : stT

  {auto showStT : Show stT}
```

Trace

```
data Trace : (opT : (t' : _) -> stT -> (t' -> stT) ->
↪  Type) -> stT -> Nat -> Type where
  MkTrace : Show stT => (initSt : stT) -> {bound : Nat}
          -> (trace : Vect bound (TraceStep opT))
          -> Trace opT initSt bound
```