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AIM:	To study Supervised Learning algorithm
Program 1	
PROBLEM STATEMENT :	To implement the Supervised Learning algorithm. [BPN Algorithm]
THEORY:	<p>Supervised learning is a machine learning approach where a model learns from labeled data, meaning both input and desired output are provided. The Backpropagation Neural Network (BPN) algorithm is one of the most widely used supervised learning techniques for training multilayer feedforward neural networks. It works by minimizing the error between the actual output and the desired output using gradient descent. The algorithm consists of two phases: the forward pass and the backward pass. In the forward pass, input data is propagated through the network using weighted connections and activation functions to generate outputs. In the backward pass, the error is calculated at the output layer and then propagated backward to adjust the weights and biases.</p> <p>The BPN algorithm employs activation functions such as the sigmoid or hyperbolic tangent, along with their derivatives, to introduce non-linearity and aid in efficient error correction. Weights and biases are initialized with small random values, and a learning rate controls the magnitude of updates. Using error correction factors, weights between both hidden and output layers, as well as between input and hidden layers, are updated iteratively. Through repeated epochs, the network converges towards optimal weights, thereby minimizing error and achieving accurate predictions.</p>



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PROGRAM:	<pre>import math import matplotlib.pyplot as plt import numpy as np def sigmoid(x): return 1.0 / (1.0 + math.exp(-x)) def deriv_from_output(y): return y * (1.0 - y) v11 = 0.6 v21 = -0.1 v01 = 0.3 v12 = -0.3 v22 = 0.4 v02 = 0.5 w1 = 0.4 w2 = 0.1 w0 = -0.2 x1, x2 = 0.0, 1.0 target = 1.0 alpha = 0.25 epochs = 10 fmt = lambda v: f'{v:.6f}' epoch_list, error_list, grad_list = [], [], [] for epoch in range(1, epochs + 1): zin1 = v01 + x1 * v11 + x2 * v21 z1 = sigmoid(zin1) zin2 = v02 + x1 * v12 + x2 * v22 z2 = sigmoid(zin2)</pre>
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```
    yin = w0 + z1 * w1 + z2 * w2
    y = sigmoid(yin)

    error = target - y
    fprime_yin = deriv_from_output(y)
    delta_k = error * fprime_yin

    dw1 = alpha * delta_k * z1
    dw2 = alpha * delta_k * z2
    dw0 = alpha * delta_k * 1.0

    delta_in1 = delta_k * w1
    fprime_zin1 = deriv_from_output(z1)
    delta1 = delta_in1 * fprime_zin1

    delta_in2 = delta_k * w2
    fprime_zin2 = deriv_from_output(z2)
    delta2 = delta_in2 * fprime_zin2

    dv11 = alpha * delta1 * x1
    dv21 = alpha * delta1 * x2
    dv01 = alpha * delta1 * 1.0

    dv12 = alpha * delta2 * x1
    dv22 = alpha * delta2 * x2
    dv02 = alpha * delta2 * 1.0

    print("="*72)
    print(f"Epoch {epoch}")
    print("- Forward pass -")
    print(f" Inputs (x1, x2) = ({fmt(x1)}, {fmt(x2)}), Target =
{fmt(target)}")
    print(f" Net input z_in1 = v01 + x1*v11 + x2*v21 = {fmt(v01)} +
{fmt(x1)}*{fmt(v11)} + {fmt(x2)}*{fmt(v21)} = {fmt(zin1)}")
    print(f" z1 = f(z_in1) = {fmt(z1)}")
    print(f" Net input z_in2 = v02 + x1*v12 + x2*v22 = {fmt(v02)} +
{fmt(x1)}*{fmt(v12)} + {fmt(x2)}*{fmt(v22)} = {fmt(zin2)}")
    print(f" z2 = f(z_in2) = {fmt(z2)}")
```



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```
print(f" Net input y_in = w0 + z1*w1 + z2*w2 = {fmt(w0)} +  
{fmt(z1)}*{fmt(w1)} + {fmt(z2)}*{fmt(w2)} = {fmt(yin)}")  
print(f" Output y = f(y_in) = {fmt(y)}")  
print()  
print("- Error and deltas -")  
print(f" Error (t - y) = {fmt(error)}")  
print(f" f'(y_in) = y*(1-y) = {fmt(fprime_yin)}")  
print(f" delta_k (output) = (t - y) * f'(y_in) = {fmt(delta_k)}")  
print()  
print("- Changes in weights (hidden -> output) -")  
print(f" Δw1 = {fmt(dw1)}, Δw2 = {fmt(dw2)}, Δw0 = {fmt(dw0)}")  
print()  
print("- Backprop to hidden layer -")  
print(f" δ1 = {fmt(delta1)}, δ2 = {fmt(delta2)}")  
print()  
print("- Changes in weights (input -> hidden) -")  
print(f" Δv11 = {fmt(dv11)}, Δv21 = {fmt(dv21)}, Δv01 =  
{fmt(dv01)}")  
print(f" Δv12 = {fmt(dv12)}, Δv22 = {fmt(dv22)}, Δv02 =  
{fmt(dv02)}")  
print()  
  
grad_mag = abs(dw1) + abs(dw2) + abs(dw0) + abs(dv11) + abs(dv21) +  
abs(dv01) + abs(dv12) + abs(dv22) + abs(dv02)  
epoch_list.append(epoch)  
error_list.append(abs(error))  
grad_list.append(grad_mag)  
  
w1 += dw1  
w2 += dw2  
w0 += dw0  
  
v11 += dv11  
v21 += dv21  
v01 += dv01  
  
v12 += dv12  
v22 += dv22
```



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```
v02 += dv02
```

```
print("- Updated weights (after epoch) -")  
print(f" w1 = {fmt(w1)}, w2 = {fmt(w2)}, w0 = {fmt(w0)}")  
print(f" v11 = {fmt(v11)}, v21 = {fmt(v21)}, v01 = {fmt(v01)}")  
print(f" v12 = {fmt(v12)}, v22 = {fmt(v22)}, v02 = {fmt(v02)}")  
print("="*72 + "\n\n")
```

```
print("\nFinal Error and Gradient Table:")  
print("Epoch | Error      | Gradient Magnitude")  
print("-----")  
for e, err, g in zip(epoch_list, error_list, grad_list):  
    print(f"{e:5d} | {err:.6f} | {g:.6f}")
```

```
plt.figure(figsize=(12,5))
```

```
plt.subplot(1,2,1)  
plt.plot(epoch_list, error_list, marker='o')  
plt.title("Error vs Epoch")  
plt.xlabel("Epoch")  
plt.ylabel("Error (|t-y|)")
```

```
plt.subplot(1,2,2)  
f = lambda w: 0.5 * w**2  
w = np.linspace(-3, 3, 400)  
E = f(w)
```

```
eta = 0.3  
w_curr = -2.5  
steps = 8  
w_hist = [w_curr]  
E_hist = [f(w_curr)]  
for _ in range(steps):  
    grad = w_curr  
    w_curr = w_curr - eta * grad  
    w_hist.append(w_curr)  
    E_hist.append(f(w_curr))
```



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```
plt.plot(w, E, label="Error Surface E(w)", color="blue")
plt.plot(w_hist, E_hist, 'ro--', label="Descent Path")
plt.title("Conceptual Gradient Descent (Error vs Weight)")
plt.xlabel("Weight (w)")
plt.ylabel("Error E(w)")
plt.legend()
plt.grid(True)

plt.tight_layout()
plt.show()
```

RESULT:

```
=====
Epoch 1
- Forward pass -
  Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
  Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.300000 + 0.000000*0.600000 + 1.000000*-0.100000 = 0.200000
  z1 = f(z_in1) = 0.549834
  Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.500000 + 0.000000*-0.300000 + 1.000000*0.400000 = 0.900000
  z2 = f(z_in2) = 0.710950
  Net input y_in = w0 + z1*w1 + z2*w2 = -0.200000 + 0.549834*0.400000 + 0.710950*0.100000 = 0.091029
  Output y = f(y_in) = 0.522741

- Error and deltas -
  Error (t - y) = 0.477259
  f'(y_in) = y*(1-y) = 0.249483
  delta_k (output) = (t - y) * f'(y_in) = 0.119068

- Changes in weights (hidden -> output) -
  Δw1 = 0.016367, Δw2 = 0.021163, Δw0 = 0.029767

- Backprop to hidden layer -
  δ1 = 0.011789, δ2 = 0.002447

- Changes in weights (input -> hidden) -
  Δv11 = 0.000000, Δv21 = 0.002947, Δv01 = 0.002947
  Δv12 = 0.000000, Δv22 = 0.000612, Δv02 = 0.000612

- Updated weights (after epoch) -
  w1 = 0.416367, w2 = 0.121163, w0 = -0.170233
  v11 = 0.600000, v21 = -0.097053, v01 = 0.302947
  v12 = -0.300000, v22 = 0.400612, v02 = 0.500612
=====
```



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Epoch 2
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.302947 + 0.000000*0.600000 + 1.000000*-0.097053 = 0.205894
z1 = f(z_in1) = 0.551292
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.500612 + 0.000000*-0.300000 + 1.000000*0.400612 = 0.901223
z2 = f(z_in2) = 0.711201
Net input y_in = w0 + z1*w1 + z2*w2 = -0.170233 + 0.551292*0.416367 + 0.711201*0.121163 = 0.145478
Output y = f(y_in) = 0.536305

- Error and deltas -
Error (t - y) = 0.463695
f'(y_in) = y*(1-y) = 0.248682
delta_k (output) = (t - y) * f'(y_in) = 0.115312

- Changes in weights (hidden -> output) -
Δw1 = 0.015893, Δw2 = 0.020503, Δw0 = 0.028828

- Backprop to hidden layer -
δ1 = 0.011877, δ2 = 0.002870

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002969, Δv01 = 0.002969
Δv12 = 0.000000, Δv22 = 0.000717, Δv02 = 0.000717

- Updated weights (after epoch) -
w1 = 0.432260, w2 = 0.141665, w0 = -0.141405
v11 = 0.600000, v21 = -0.094084, v01 = 0.305916
v12 = -0.300000, v22 = 0.401329, v02 = 0.501329
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Epoch 3
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.305916 + 0.000000*0.600000 + 1.000000*-0.094084 = 0.211833
z1 = f(z_in1) = 0.552761
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.501329 + 0.000000*-0.300000 + 1.000000*0.401329 = 0.902658
z2 = f(z_in2) = 0.711495
Net input y_in = w0 + z1*w1 + z2*w2 = -0.141405 + 0.552761*0.432260 + 0.711495*0.141665 = 0.198326
Output y = f(y_in) = 0.549420

- Error and deltas -
Error (t - y) = 0.450580
f'(y_in) = y*(1-y) = 0.247558
delta_k (output) = (t - y) * f'(y_in) = 0.111545

- Changes in weights (hidden -> output) -
Δw1 = 0.015414, Δw2 = 0.019841, Δw0 = 0.027886

- Backprop to hidden layer -
δ1 = 0.011920, δ2 = 0.003244

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002980, Δv01 = 0.002980
Δv12 = 0.000000, Δv22 = 0.000811, Δv02 = 0.000811

- Updated weights (after epoch) -
w1 = 0.447674, w2 = 0.161506, w0 = -0.113519
v11 = 0.600000, v21 = -0.091104, v01 = 0.308896
v12 = -0.300000, v22 = 0.402140, v02 = 0.502140
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Epoch 4
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.308896 + 0.000000*0.600000 + 1.000000*-0.091104 = 0.217793
z1 = f(z_in1) = 0.554234
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.502140 + 0.000000*-0.300000 + 1.000000*0.402140 = 0.904280
z2 = f(z_in2) = 0.711828
Net input y_in = w0 + z1*w1 + z2*w2 = -0.113519 + 0.554234*0.447674 + 0.711828*0.161506 = 0.249562
Output y = f(y_in) = 0.562069

- Error and deltas -
Error (t - y) = 0.437931
f'(y_in) = y*(1-y) = 0.246147
delta_k (output) = (t - y) * f'(y_in) = 0.107796

- Changes in weights (hidden -> output) -
Δw1 = 0.014936, Δw2 = 0.019183, Δw0 = 0.026949

- Backprop to hidden layer -
δ1 = 0.011922, δ2 = 0.003571

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002981, Δv01 = 0.002981
Δv12 = 0.000000, Δv22 = 0.000893, Δv02 = 0.000893

- Updated weights (after epoch) -
w1 = 0.462610, w2 = 0.180689, w0 = -0.086570
v11 = 0.600000, v21 = -0.088123, v01 = 0.311877
v12 = -0.300000, v22 = 0.403033, v02 = 0.503033
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Epoch 5
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.311877 + 0.000000*0.600000 + 1.000000*-0.088123 = 0.223754
z1 = f(z_in1) = 0.555706
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.503033 + 0.000000*-0.300000 + 1.000000*0.403033 = 0.906066
z2 = f(z_in2) = 0.712194
Net input y_in = w0 + z1*w1 + z2*w2 = -0.086570 + 0.555706*0.462610 + 0.712194*0.180689 = 0.299191
Output y = f(y_in) = 0.574245

- Error and deltas -
Error (t - y) = 0.425755
f'(y_in) = y*(1-y) = 0.244488
delta_k (output) = (t - y) * f'(y_in) = 0.104092

- Changes in weights (hidden -> output) -
Δw1 = 0.014461, Δw2 = 0.018533, Δw0 = 0.026023

- Backprop to hidden layer -
δ1 = 0.011889, δ2 = 0.003855

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002972, Δv01 = 0.002972
Δv12 = 0.000000, Δv22 = 0.000964, Δv02 = 0.000964

- Updated weights (after epoch) -
w1 = 0.477071, w2 = 0.199223, w0 = -0.060547
v11 = 0.600000, v21 = -0.085151, v01 = 0.314849
v12 = -0.300000, v22 = 0.403997, v02 = 0.503997
=====
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Epoch 6
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.314849 + 0.000000*0.600000 + 1.000000*-0.085151 = 0.229698
z1 = f(z_in1) = 0.557173
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.503997 + 0.000000*-0.300000 + 1.000000*0.403997 = 0.907993
z2 = f(z_in2) = 0.712589
Net input y_in = w0 + z1*w1 + z2*w2 = -0.060547 + 0.557173*0.477071 + 0.712589*0.199223 = 0.347228
Output y = f(y_in) = 0.585945

- Error and deltas -
Error (t - y) = 0.414055
f'(y_in) = y*(1-y) = 0.242613
delta_k (output) = (t - y) * f'(y_in) = 0.100455

- Changes in weights (hidden -> output) -
Δw1 = 0.013993, Δw2 = 0.017896, Δw0 = 0.025114

- Backprop to hidden layer -
δ1 = 0.011824, δ2 = 0.004099

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002956, Δv01 = 0.002956
Δv12 = 0.000000, Δv22 = 0.001025, Δv02 = 0.001025

- Updated weights (after epoch) -
w1 = 0.491064, w2 = 0.217119, w0 = -0.035433
v11 = 0.600000, v21 = -0.082195, v01 = 0.317805
v12 = -0.300000, v22 = 0.405021, v02 = 0.505021
=====
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=====
Epoch 7
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.317805 + 0.000000*0.600000 + 1.000000*-0.082195 = 0.235610
z1 = f(z_in1) = 0.558632
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.505021 + 0.000000*-0.300000 + 1.000000*0.405021 = 0.910043
z2 = f(z_in2) = 0.713009
Net input y_in = w0 + z1*w1 + z2*w2 = -0.035433 + 0.558632*0.491064 + 0.713009*0.217119 = 0.393698
Output y = f(y_in) = 0.597173

- Error and deltas -
Error (t - y) = 0.402827
f'(y_in) = y*(1-y) = 0.240557
delta_k (output) = (t - y) * f'(y_in) = 0.096903

- Changes in weights (hidden -> output) -
Δw1 = 0.013533, Δw2 = 0.017273, Δw0 = 0.024226

- Backprop to hidden layer -
δ1 = 0.011733, δ2 = 0.004305

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002933, Δv01 = 0.002933
Δv12 = 0.000000, Δv22 = 0.001076, Δv02 = 0.001076

- Updated weights (after epoch) -
w1 = 0.504597, w2 = 0.234392, w0 = -0.011207
v11 = 0.600000, v21 = -0.079262, v01 = 0.320738
v12 = -0.300000, v22 = 0.406098, v02 = 0.506098
=====
```



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=====
Epoch 8
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.320738 + 0.000000*0.600000 + 1.000000*-0.079262 = 0.241477
z1 = f(z_in1) = 0.560078
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.506098 + 0.000000*-0.300000 + 1.000000*0.406098 = 0.912195
z2 = f(z_in2) = 0.713449
Net input y_in = w0 + z1*w1 + z2*w2 = -0.011207 + 0.560078*0.504597 + 0.713449*0.234392 = 0.438633
Output y = f(y_in) = 0.607933

- Error and deltas -
Error (t - y) = 0.392067
f'(y_in) = y*(1-y) = 0.238350
delta_k (output) = (t - y) * f'(y_in) = 0.093449

- Changes in weights (hidden -> output) -
Δw1 = 0.013085, Δw2 = 0.016668, Δw0 = 0.023362

- Backprop to hidden layer -
δ1 = 0.011618, δ2 = 0.004478

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002905, Δv01 = 0.002905
Δv12 = 0.000000, Δv22 = 0.001119, Δv02 = 0.001119

- Updated weights (after epoch) -
w1 = 0.517682, w2 = 0.251060, w0 = 0.012155
v11 = 0.600000, v21 = -0.076357, v01 = 0.323643
v12 = -0.300000, v22 = 0.407217, v02 = 0.507217
=====
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=====
Epoch 9
- Forward pass -
Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000
Net input z_in1 = v01 + x1*v11 + x2*v21 = 0.323643 + 0.000000*0.600000 + 1.000000*-0.076357 = 0.247286
z1 = f(z_in1) = 0.561508
Net input z_in2 = v02 + x1*v12 + x2*v22 = 0.507217 + 0.000000*-0.300000 + 1.000000*0.407217 = 0.914434
z2 = f(z_in2) = 0.713907
Net input y_in = w0 + z1*w1 + z2*w2 = 0.012155 + 0.561508*0.517682 + 0.713907*0.251060 = 0.482071
Output y = f(y_in) = 0.618237

- Error and deltas -
Error (t - y) = 0.381763
f'(y_in) = y*(1-y) = 0.236020
delta_k (output) = (t - y) * f'(y_in) = 0.090104

- Changes in weights (hidden -> output) -
Δw1 = 0.012649, Δw2 = 0.016081, Δw0 = 0.022526

- Backprop to hidden layer -
δ1 = 0.011485, δ2 = 0.004620

- Changes in weights (input -> hidden) -
Δv11 = 0.000000, Δv21 = 0.002871, Δv01 = 0.002871
Δv12 = 0.000000, Δv22 = 0.001155, Δv02 = 0.001155

- Updated weights (after epoch) -
w1 = 0.530330, w2 = 0.267141, w0 = 0.034681
v11 = 0.600000, v21 = -0.073486, v01 = 0.326514
v12 = -0.300000, v22 = 0.408372, v02 = 0.508372
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Epoch 10

- Forward pass -

Inputs (x1, x2) = (0.000000, 1.000000), Target = 1.000000

Net input $z_{in1} = v_{01} + x1*v_{11} + x2*v_{21} = 0.326514 + 0.000000*0.600000 + 1.000000*-0.073486 = 0.253028$

$z1 = f(z_{in1}) = 0.562922$

Net input $z_{in2} = v_{02} + x1*v_{12} + x2*v_{22} = 0.508372 + 0.000000*-0.300000 + 1.000000*0.408372 = 0.916744$

$z2 = f(z_{in2}) = 0.714378$

Net input $y_{in} = w_0 + z1*w1 + z2*w2 = 0.034681 + 0.562922*0.530330 + 0.714378*0.267141 = 0.524055$

Output $y = f(y_{in}) = 0.628096$

- Error and deltas -

Error (t - y) = 0.371904

$f'(y_{in}) = y*(1-y) = 0.233592$

$\text{delta}_k (\text{output}) = (t - y) * f'(y_{in}) = 0.086874$

- Changes in weights (hidden -> output) -

$\Delta w1 = 0.012226, \Delta w2 = 0.015515, \Delta w0 = 0.021718$

- Backprop to hidden layer -

$\delta1 = 0.011336, \delta2 = 0.004735$

- Changes in weights (input -> hidden) -

$\Delta v11 = 0.000000, \Delta v21 = 0.002834, \Delta v01 = 0.002834$

$\Delta v12 = 0.000000, \Delta v22 = 0.001184, \Delta v02 = 0.001184$

- Updated weights (after epoch) -

$w1 = 0.542556, w2 = 0.282656, w0 = 0.056399$

$v11 = 0.600000, v21 = -0.070652, v01 = 0.329348$

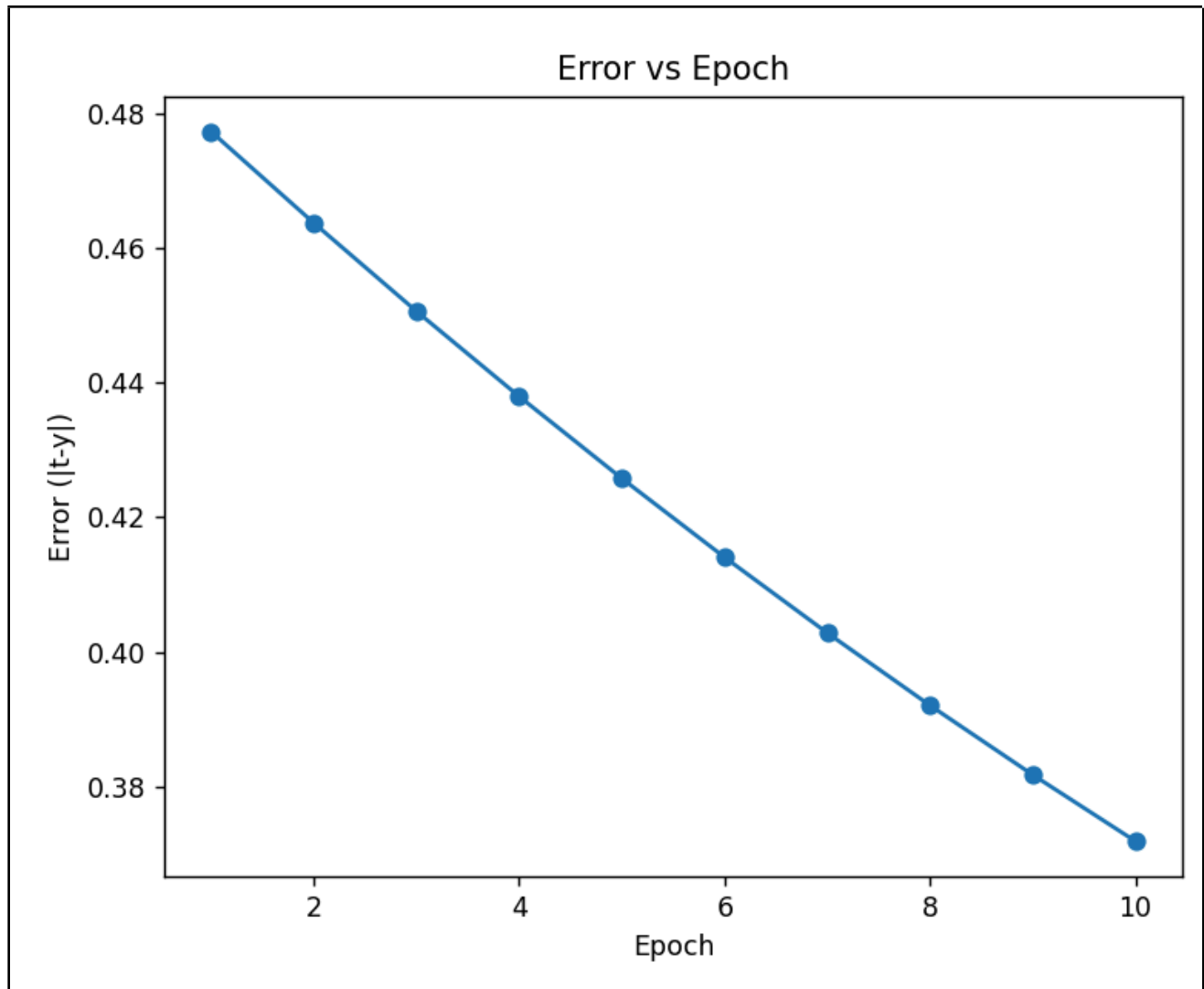
$v12 = -0.300000, v22 = 0.409556, v02 = 0.509556$

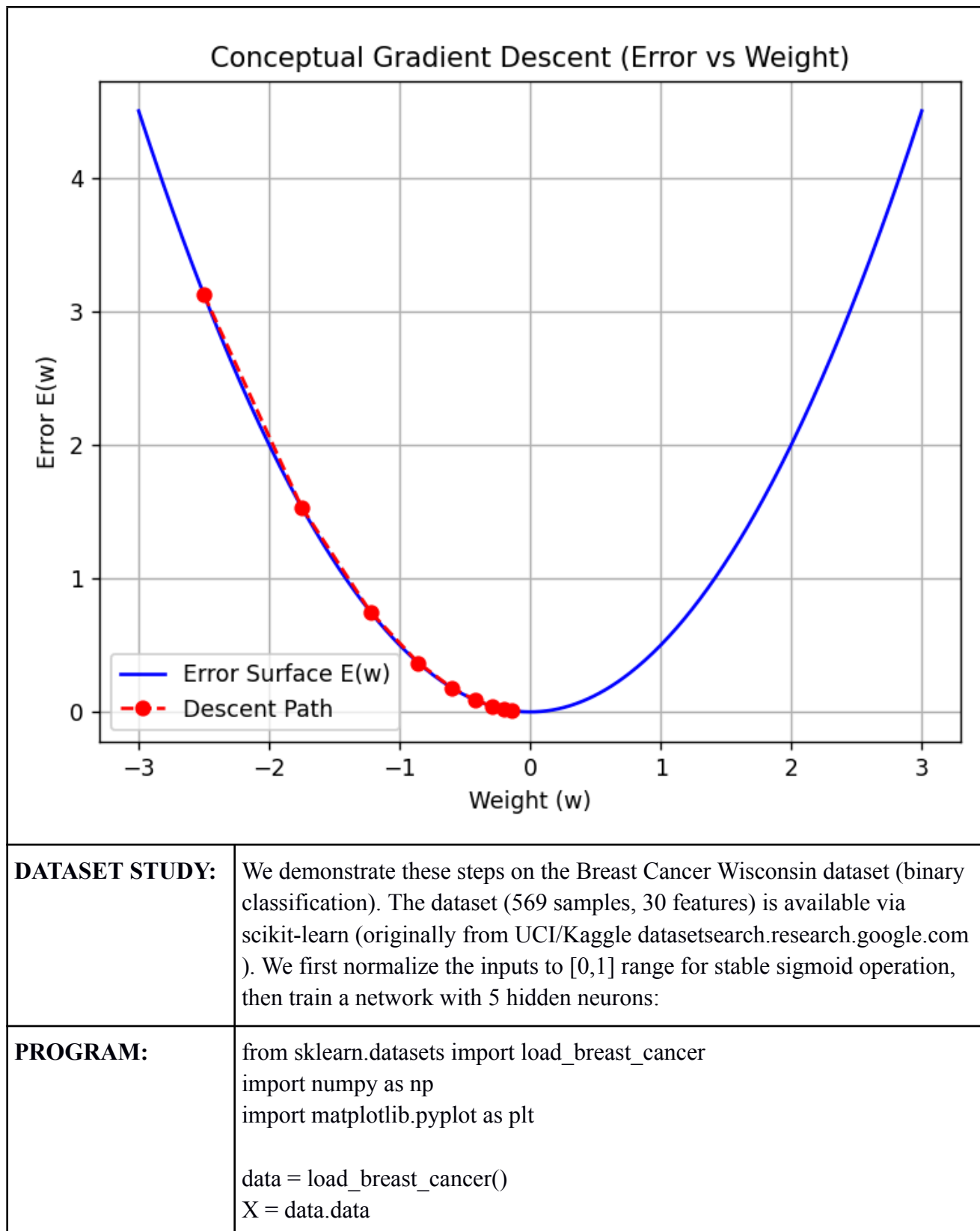
Final Error and Gradient Table:

Epoch	Error	Gradient Magnitude
1	0.477259	0.074414
2	0.463695	0.072597
3	0.450580	0.070723
4	0.437931	0.068815
5	0.425755	0.066890
6	0.414055	0.064964
7	0.402827	0.063051
8	0.392067	0.061163
9	0.381763	0.059308
10	0.371904	0.057495



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```
y = data.target
X_min = X.min(axis=0); X_max = X.max(axis=0)
X_norm = (X - X_min) / (X_max - X_min)

n_input = X_norm.shape[1]
n_hidden = 5
n_output = 1
np.random.seed(42)
W_input_hidden = np.random.normal(scale=0.1, size=(n_input,
n_hidden))
b_hidden      = np.zeros(n_hidden)
W_hidden_output = np.random.normal(scale=0.1, size=(n_hidden,
n_output))
b_output      = np.zeros(n_output)

def sigmoid(x):
    return 1.0 / (1.0 + np.exp(-x))
def sigmoid_derivative(y):
    return y * (1.0 - y)

learning_rate = 0.1
epochs = 1000
loss_history = []

for epoch in range(epochs):
    total_loss = 0.0
    for i in range(X_norm.shape[0]):
        x_i = X_norm[i]
        t_i = y[i]

        z_hidden = np.dot(x_i, W_input_hidden) + b_hidden
        a_hidden = sigmoid(z_hidden)
        z_out  = np.dot(a_hidden, W_hidden_output) + b_output
        a_out  = sigmoid(z_out)

        error = t_i - a_out[0]
        total_loss += error**2
```



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	<pre> delta_out = error * sigmoid_derivative(a_out)[0] grad_hidden_output = np.expand_dims(a_hidden,1) * delta_out W_hidden_output += learning_rate * grad_hidden_output b_output += learning_rate * delta_out delta_hidden = (W_hidden_output.flatten() * delta_out) * sigmoid_derivative(a_hidden) grad_input_hidden = np.outer(x_i, delta_hidden) W_input_hidden += learning_rate * grad_input_hidden b_hidden += learning_rate * delta_hidden mse = total_loss / X_norm.shape[0] loss_history.append(mse) if epoch % 100 == 0 or epoch == epochs-1: print(f'Epoch {epoch:4d}, MSE = {mse:.4f}') plt.plot(range(epochs), loss_history) plt.xlabel("Epochs") plt.ylabel("Mean Squared Error") plt.title("Error vs Epochs") plt.grid(True) plt.show() </pre>
PROGRAM-2:	<pre> X_xor = np.array([[0,0],[0,1],[1,0],[1,1]]) y_xor = np.array([0,1,1,0]) np.random.seed(1) W_input_hidden = np.random.uniform(-1,1,(2, 2)) b_hidden = np.random.uniform(-1,1,2) W_hidden_output = np.random.uniform(-1,1,(2, 1)) b_output = np.random.uniform(-1,1,1) def sigmoid(x): return 1/(1+np.exp(-x)) def sigmoid_derivative(y): return y*(1-y) learning_rate = 0.5 epochs = 10000 </pre>



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```
for epoch in range(epochs):
    total_loss = 0.0
    for i in range(X_xor.shape[0]):
        x_i = X_xor[i]
        t_i = y_xor[i]

        z_hidden = np.dot(x_i, W_input_hidden) + b_hidden
        a_hidden = sigmoid(z_hidden)
        z_out = np.dot(a_hidden, W_hidden_output) + b_output
        a_out = sigmoid(z_out)

        error = t_i - a_out[0]
        total_loss += error**2
        delta_out = error * sigmoid_derivative(a_out)[0]

        grad_hidden_output = np.expand_dims(a_hidden,1) * delta_out
        W_hidden_output += learning_rate * grad_hidden_output
        b_output += learning_rate * delta_out

        delta_hidden = (W_hidden_output.flatten() * delta_out) *
sigmoid_derivative(a_hidden)
        grad_input_hidden = np.outer(x_i, delta_hidden)
        W_input_hidden += learning_rate * grad_input_hidden
        b_hidden += learning_rate * delta_hidden

    if epoch % 2000 == 0:
        mse = total_loss / X_xor.shape[0]
        print(f'Epoch {epoch:4d}, MSE = {mse:.6f}')

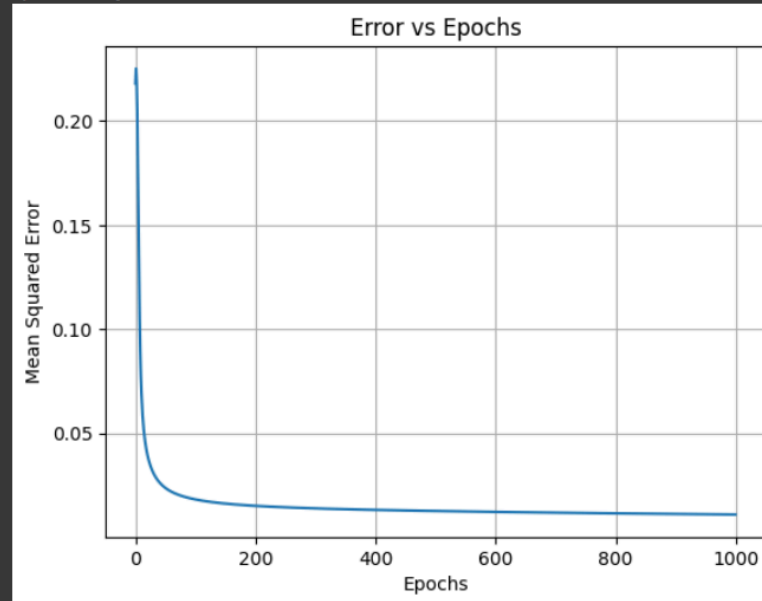
print("\nFinal network outputs for XOR:")
for x_i, t_i in zip(X_xor, y_xor):
    a_hidden = sigmoid(np.dot(x_i, W_input_hidden) + b_hidden)
    a_out = sigmoid(np.dot(a_hidden, W_hidden_output) + b_output)[0]
    print(f'Input {x_i.tolist()} -> Predicted {a_out:.3f}, Target {t_i}')
```




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OUTPUT 1:

```
Epoch 0, MSE = 0.2181
Epoch 100, MSE = 0.0181
Epoch 200, MSE = 0.0151
Epoch 300, MSE = 0.0139
Epoch 400, MSE = 0.0131
Epoch 500, MSE = 0.0126
Epoch 600, MSE = 0.0122
Epoch 700, MSE = 0.0118
Epoch 800, MSE = 0.0115
Epoch 900, MSE = 0.0112
Epoch 999, MSE = 0.0109
```



OUTPUT 2:

```
Epoch 0, MSE = 0.269411
Epoch 2000, MSE = 0.004854
Epoch 4000, MSE = 0.001108
Epoch 6000, MSE = 0.000607
Epoch 8000, MSE = 0.000414
```

Final network outputs for XOR:

```
Input [0, 0] -> Predicted 0.016, Target 0
Input [0, 1] -> Predicted 0.983, Target 1
Input [1, 0] -> Predicted 0.983, Target 1
Input [1, 1] -> Predicted 0.021, Target 0
```

CONCLUSION:

The experiment on implementing the Backpropagation Neural Network (BPN) highlights the effectiveness of supervised learning in training multilayer neural networks. By systematically propagating inputs forward and errors backward, the algorithm adjusts weights and biases to minimize



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the difference between predicted and actual outputs. This iterative process ensures that the network gradually improves its ability to recognize patterns and relationships within the dataset. The use of activation functions introduces non-linearity, enabling the model to handle complex problems beyond linear separability.

Overall, the BPN algorithm demonstrates how supervised learning can be applied to achieve accurate predictions and classification results. The step-by-step computation of net inputs, activations, error correction factors, and weight updates reinforces the importance of mathematical rigor in neural network training. The experiment confirms that with proper learning rate and sufficient iterations, the network converges towards optimal weights, showcasing the practical significance of backpropagation in modern artificial intelligence applications.