

# Advanced Computer Graphics Assignment 1: Part 1: Curves (40pts)

September 25, 2022

## 1 Problem Description

A  $n$ -th order Bezier curve  $\mathbf{B}$  parametrized by  $n + 1$  control points  $\{\mathbf{P}_i\}_{i=0}^n$  is defined as:

$$\mathbf{B}(t) = \sum_{i=0}^n b_{i,n}(t) \mathbf{P}_i, 0 \leq t \leq 1.$$

where  $b_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i}$  is the Bernstein polynomial.

Please prove the following statements:

1) (20pts) The tangent vector for this curve is

$$\mathbf{B}'(t) = n \sum_{i=0}^{n-1} b_{i,n-1}(t) (\mathbf{P}_{i+1} - \mathbf{P}_i).$$

In particular,  $\mathbf{B}'(0) = n(\mathbf{P}_1 - \mathbf{P}_0)$  and  $\mathbf{B}'(1) = n(\mathbf{P}_n - \mathbf{P}_{n-1})$ .

2) (20pts) The curve  $\mathbf{B}$  is included in the convex hull of control points.

Tips: The convex hull  $\mathbf{C}$  of a set of points  $\{\mathbf{P}_i\}_{i=1}^n$  is defined as

$$\mathbf{C} = \left\{ \sum_{i=1}^n a_i \mathbf{P}_i \mid \sum_{i=1}^n a_i = 1 \text{ and } \forall i, a_i \geq 0 \right\}.$$