6.1. Calculate probabilities using the uniform distribution rule, combinatorics, and permutation equations.

Ex: A population contains 6 women and 7 men.

✓without replacement
✓order of selection doesn't matter

1. What is the probability that we select a group of size 5 that contains 2 women and 3 men?

(out of 6) (out of 7)

$$\frac{\text{\# of ways to select a group with 2 women + 3 men}}{\text{total \# of ways to select a group of 5 (out of 13)}}$$

6W → 2

\# of combinations of 2 women

3 ← 7M

\# of combinations of 3 men

$$= \frac{\binom{6}{2}\binom{7}{3}}{\binom{13}{5}}$$

\# of combinations of 5 people

13 people → 5

$$= \frac{\dfrac{6\times5\times\cancel{4\times3\times2\times1}}{(2\times1)\times\cancel{(4\times3\times2\times1)}} \cdot \dfrac{7\times6\times5\times\cancel{4\times3\times2\times1}}{(3\times2\times1)\times\cancel{(4\times3\times2\times1)}}}{\dfrac{13\times12\times11\times10\times9\times\cancel{8\times7\times6\times5\times4\times3\times2\times1}}{(5\times4\times3\times2\times1)\cancel{(8\times7\times6\times5\times4\times3\times2\times1)}}}$$

$$= \frac{15 \times 35}{1287}$$

$$= \boxed{.411}$$

6.2. For a discrete random variable X, *that is NOT a specific family of random variables.*

Ex: Let X be the number of times that a randomly selected driver (from a large population) had to take their drivers test before passing. The probabilities for each value of X is given in the table below.

X	P(X=x)
1	0.8
2	0.15
3	0.05

1. What is the mean of X?

$$\boxed{E[X]} = 1(.8) + 2(.15) + 3(.05) = \boxed{1.25}$$

2. What is the variance of X?

$$\boxed{Var[X]} = (1-\mu)^2 P(X=1) + (2-\mu)^2 P(X=2) + (3-\mu)^2 P(X=3)$$

$$(1-1.25)^2(.8) + (2-1.25)^2(.15) + (3-1.25)^2(.05)$$

$$= \boxed{0.2875}$$

3. What is the standard deviation of X?

$$\boxed{SD[X]} = \sqrt{Var[X]}$$

$$= \boxed{0.5362}$$

6.3. For a Bernoulli random variable X.

Ex: Let Y represent a random variable that =1 if a randomly selected UIUC applicant got was accepted into UIUC and =0 if they were not accepted into UIUC. We know that P(Y=1)=0.62.

1. What kind of random variable is Y?

$Y \sim \text{Bernoulli}(p=.62)$

2. What is P(Y=0)?

$\boxed{P(Y=0)} = 1 - P(Y=1) = 1 - .62 = \boxed{0.38}$

3. What is the mean of Y?

$\boxed{E[Y]} = p = \boxed{.62}$

proof in lab

4. What is the variance of Y?

$\boxed{Var[Y]} = p(1-p) = .62(1-.62) = \boxed{.2356}$

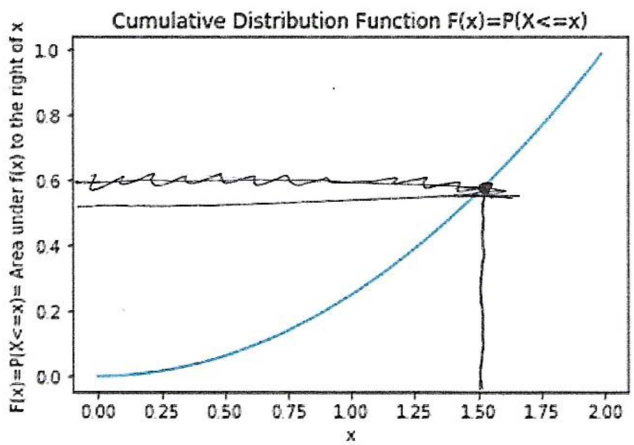
5. What is the standard deviation of Y?

$\boxed{SD[Y]} = \sqrt{Var[Y]}$

$= \boxed{0.4853}$

6.4 For a continuous random variable X, *that is not of a specific family of random variables.*

Ex: The probability density function and the cumulative distribution functions for a certain continuous random variable below.



1. Use the cdf to calculate P(X≤1.5).

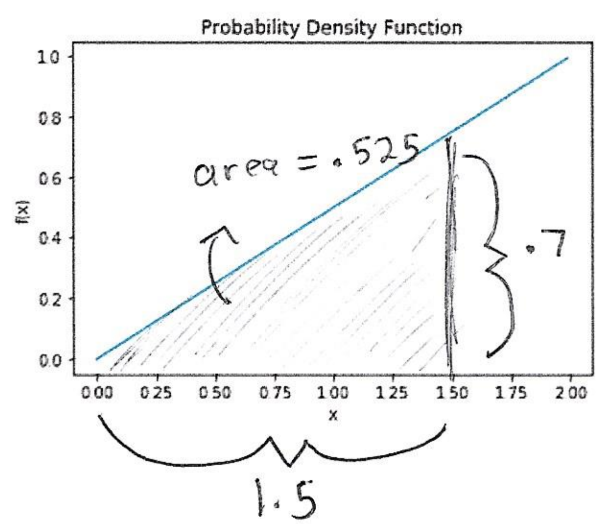
$\approx .525$

2. Use the cdf to calculate P(X>1.5).

$= 1 - P(X \leq 1.5)$

$= 1 - .525$

$= \boxed{.475}$



3. Use the pdf to calculate P(X≤1.5).

$= \boxed{.525}$

4. Use the pdf to calculate P(X>1.5).

$= 1 - P(X \leq 1.5)$

$= \boxed{0.475}$

6.7 Calculate probabilities using the rules of combining probabilities.

Ex: Let A be the event of randomly selecting a student from a calculus class that got an A on the final exam. Let B the event of randomly selecting a student from a calculus class that got a B on the final exam. Suppose the P(A)=0.2 and P(B)=0.6.

1. Are A and B mutually exclusive?

Yes. Someone cannot get an A and a B at the same time.

2. Are A and B independent or dependent?

They are dependent. Having information that you got a B, tells us information about the likelihood of getting an A. (ie $P(A \mid B)=0$)

3. What is the probability of randomly selecting a single student that got both an A and a B on the exam?

$P(A \text{ and } B) = 0$

4. What is the probability of selecting a student that got an A or a B on the exam?

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .2 + .6 - 0$
$= \boxed{.8}$

5. Finally, let the event 'passed' be the event that the randomly selected calculus student passed the final exam. Suppose that P(passed)=0.95. What is the probability that the student got an A on the final exam, given that we know that the student passed the final exam?

if you got an A we can automatically assume you pass

$$\boxed{P(A \mid \text{passed})} = \frac{P(A \text{ and passed})}{P(\text{passed})} = \frac{P(A)}{P(\text{passed})} = \frac{.2}{.95} = \boxed{0.21}$$