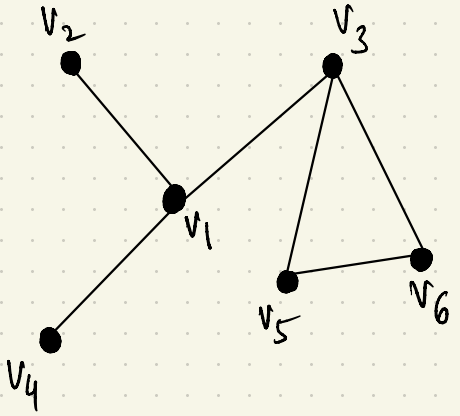
COL 351 : Analysis & Design of Algorithms

Lecture 9

Graph Algorithms II : Representation and search

Aug 09, 2024 | Rohit Vaish

GRAPHS



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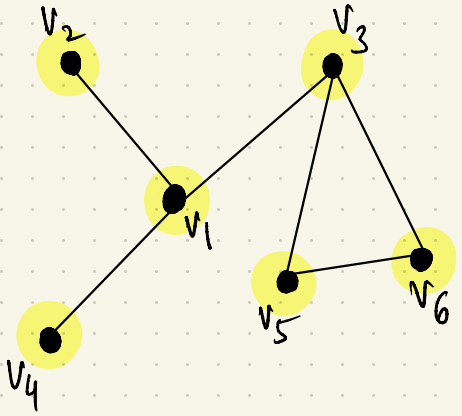
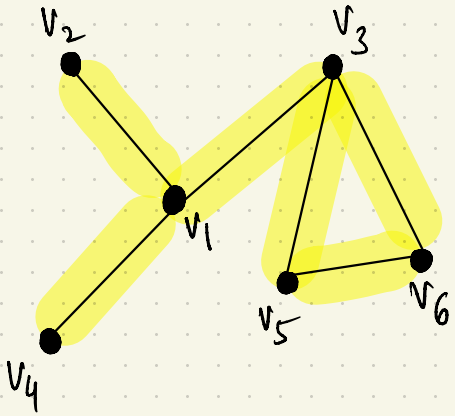


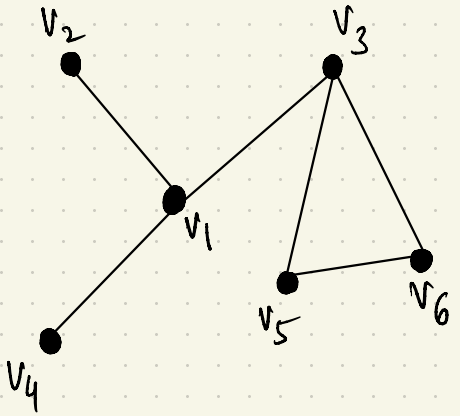
Vertices (or nodes)

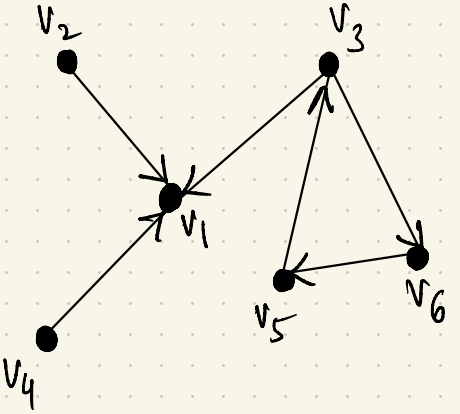
GRAPHS



Edges

GRAPHS





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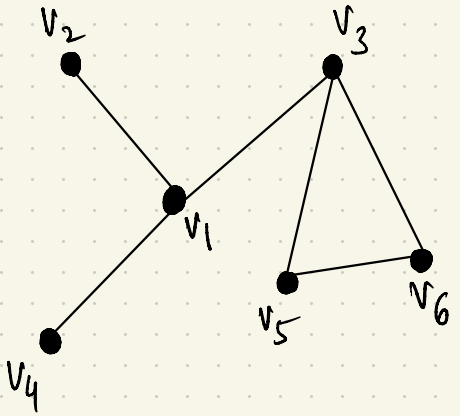
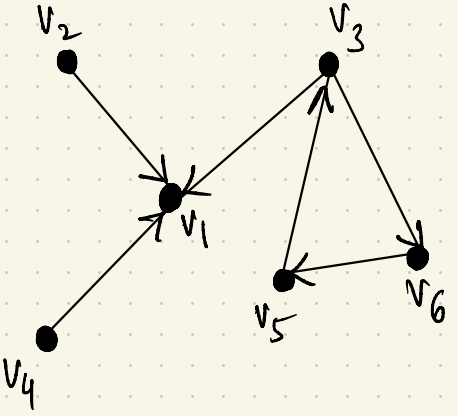
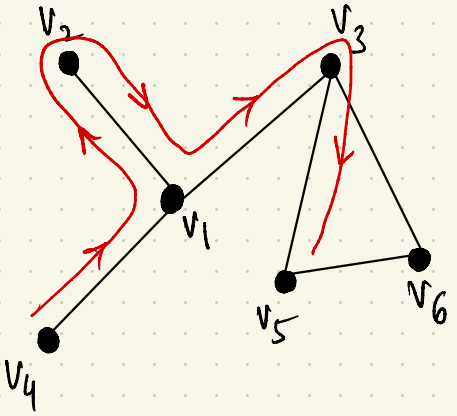


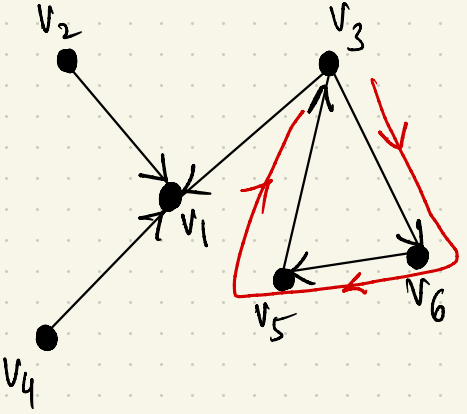


$(v_2, v_1) \in E$

$(v_1, v_2) \notin E$

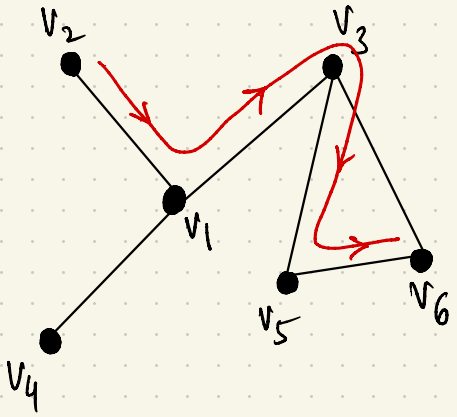
GRAPHS

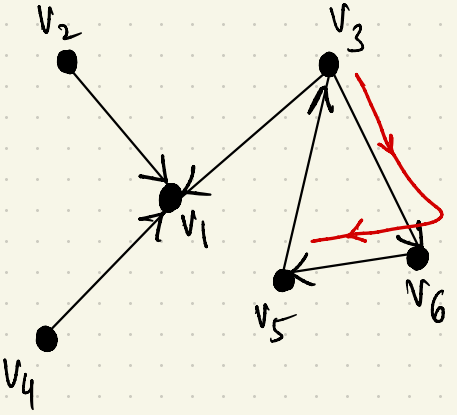




Walks

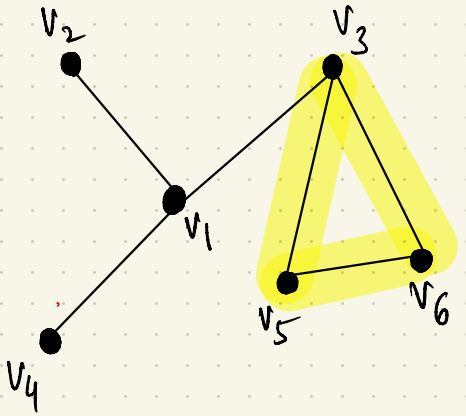
GRAPHS

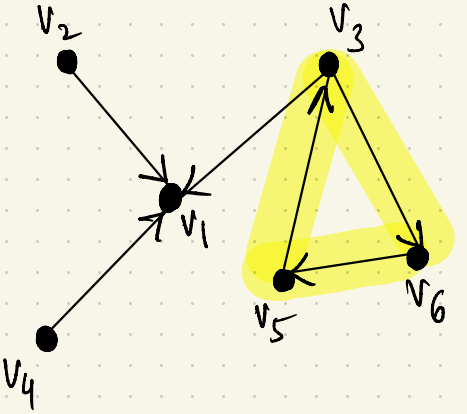




Paths

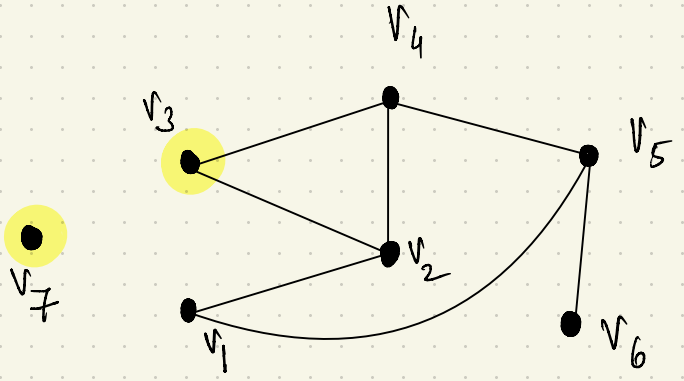
GRAPHS



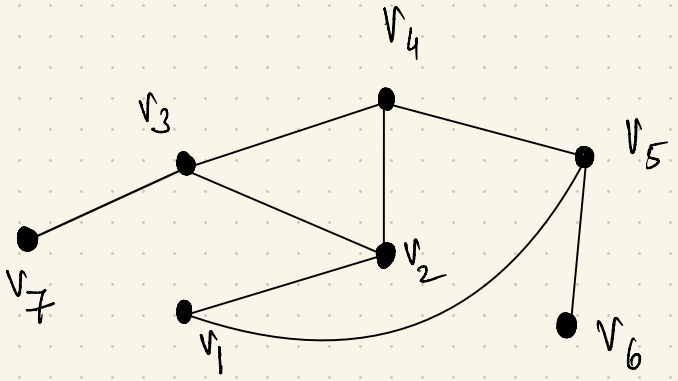


Closed walks and cycles

CONNECTIVITY



Not connected



Connected

SPARSE vs DENSE GRAPHS

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n : number of vertices

m : number of edges

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Many applications : m is $\Omega(n)$ and $O(n^2)$.
(but not always)

SPARSE vs DENSE GRAPHS

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roughly,

sparse : $m = O(n \log n)$ close to linear

dense : $m = \Omega\left(\frac{n^2}{\log n}\right)$ close to quadratic

SPARSE vs DENSE GRAPHS

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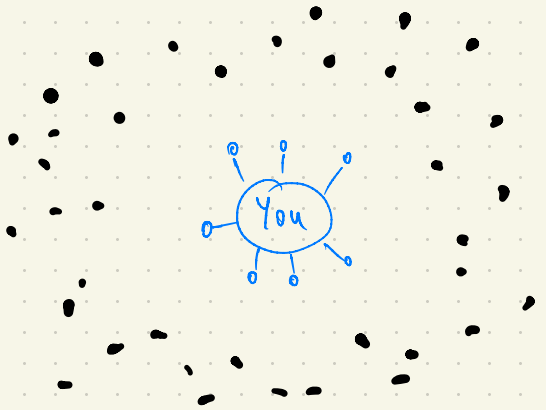
Many applications : m is $\Omega(n)$ and $O(n^2)$.
(but not always)

Facebook graph is sparse ...



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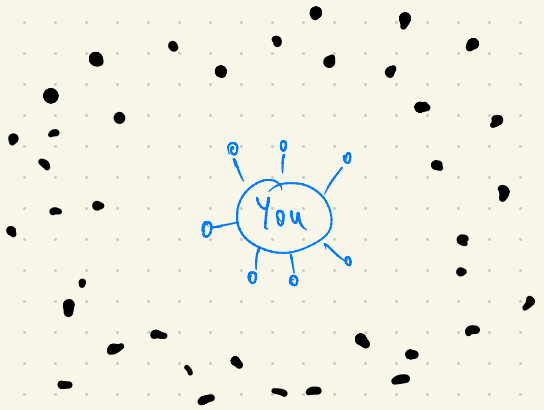
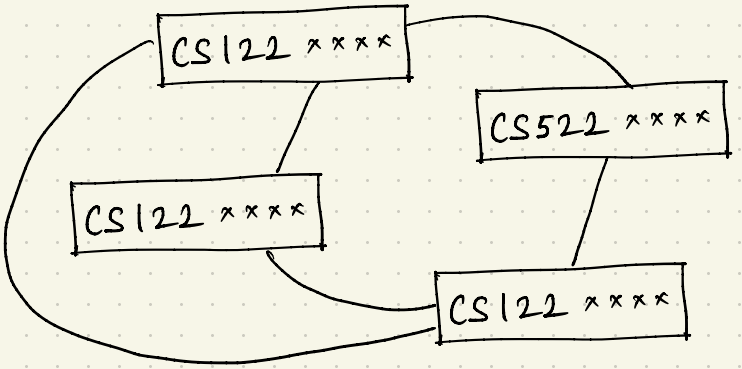


... but also dense



REPRESENTING GRAPHS

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Adjacency matrix

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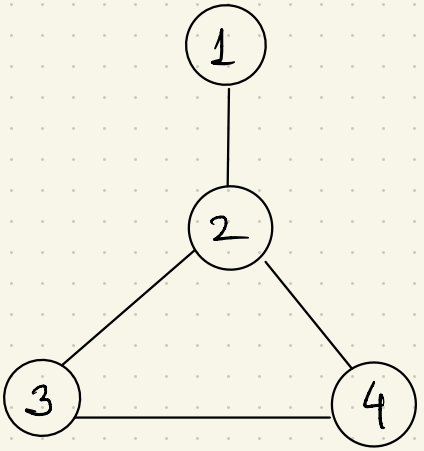
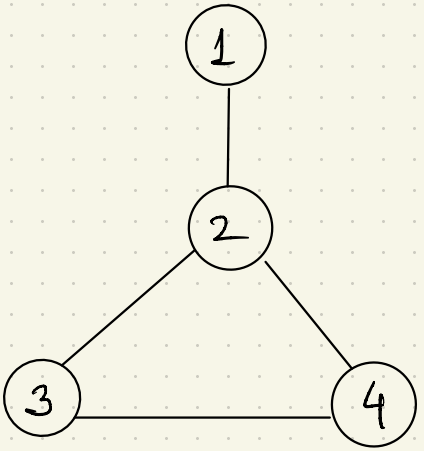


REPRESENTING GRAPHS

Adjacency matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$



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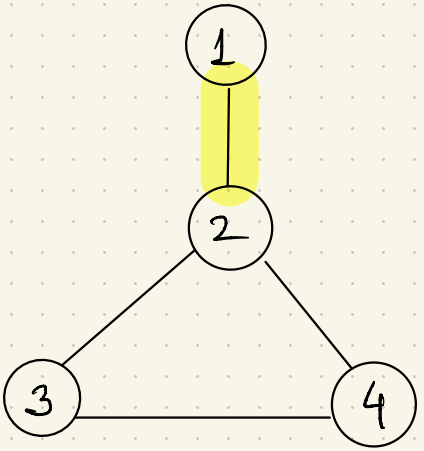

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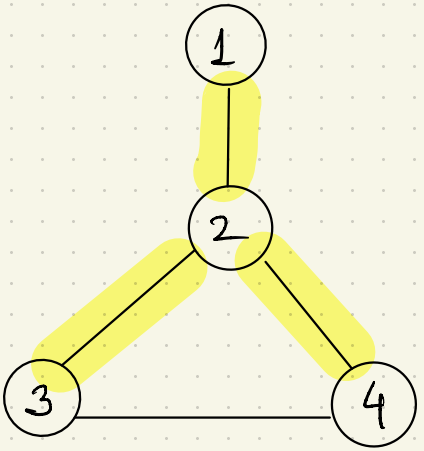

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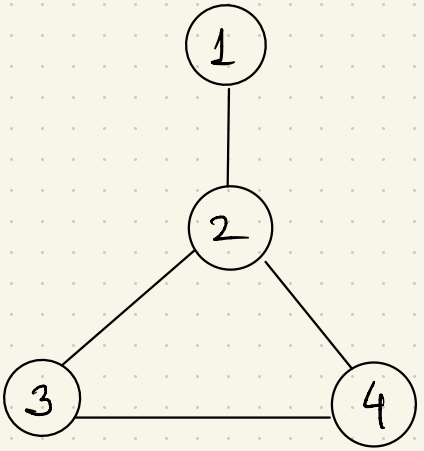

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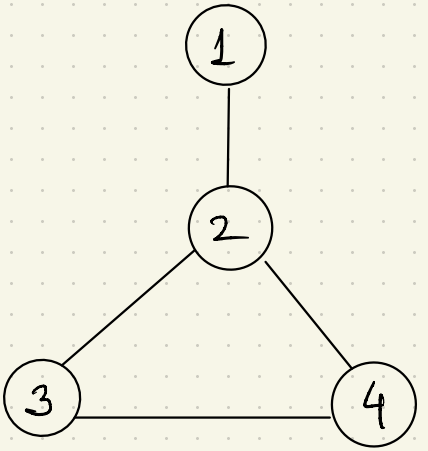
symmetric



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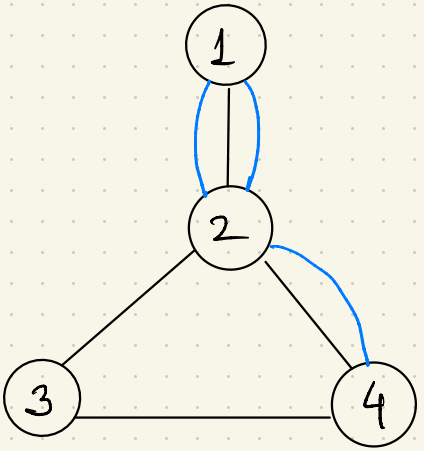

parallel edges

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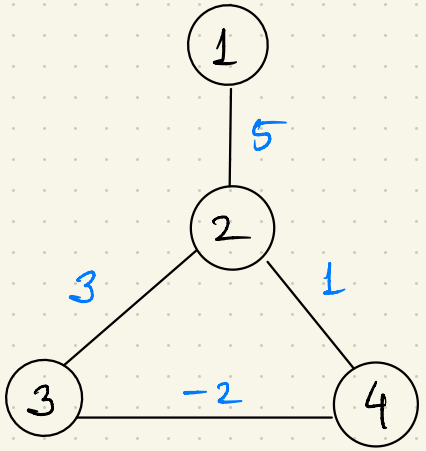

edge weights

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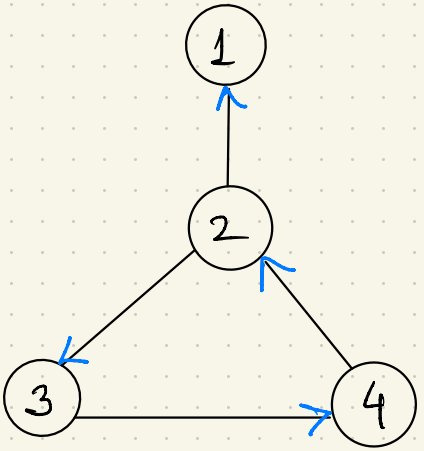
skew symmetric



Directed edges

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$\Theta(n^2)$ space

OK for dense graphs

Not OK for sparse graphs

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Adjacency list

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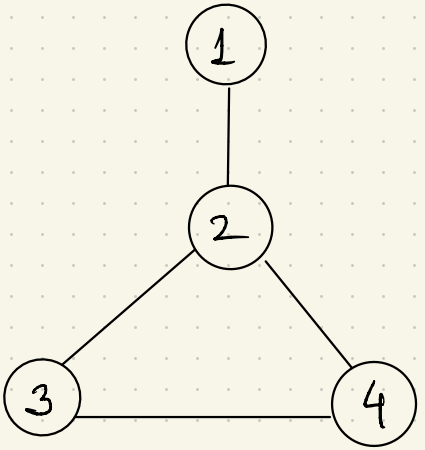
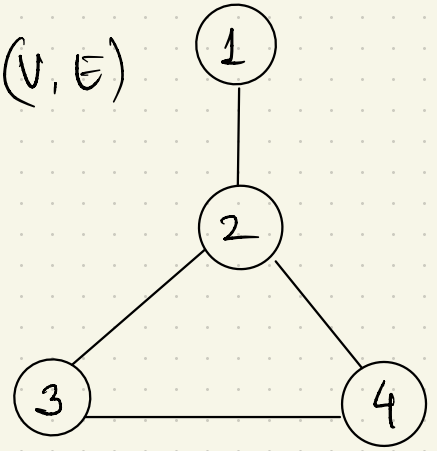


REPRESENTING GRAPHS

Adjacency list

array of $|V|$ linked lists

$G = (V, E)$



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for each vertex $u \in V$,

Adj[u] stores neighbors of u

$\{ v \in V : \{u, v\} \in E \}$.

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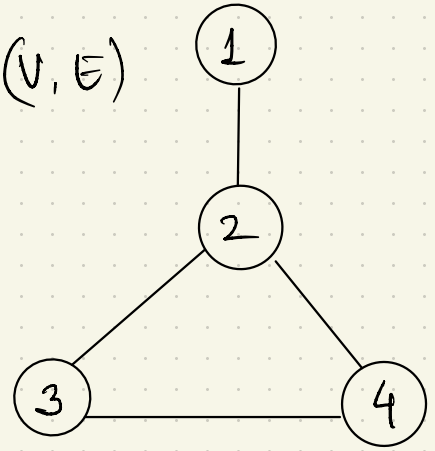

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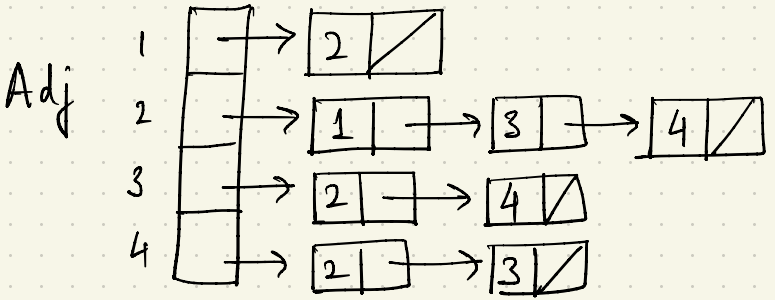

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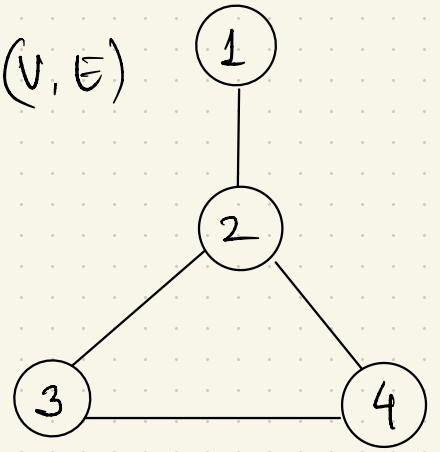

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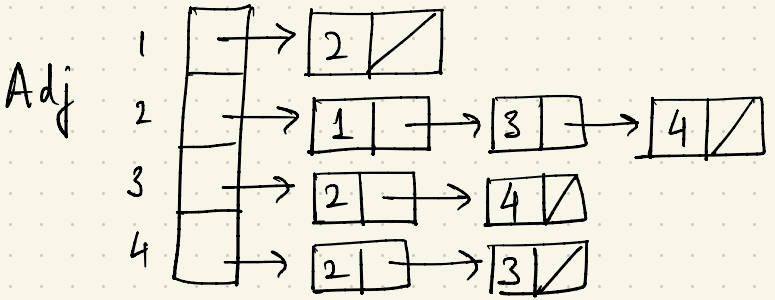

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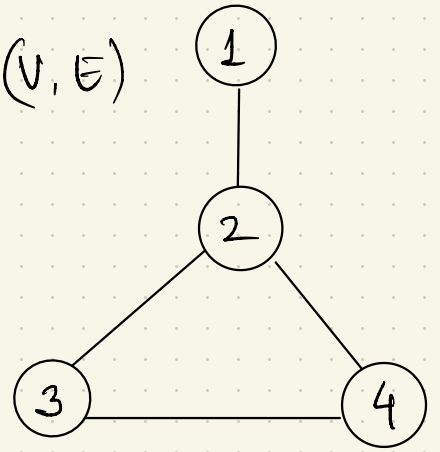

Space : ?

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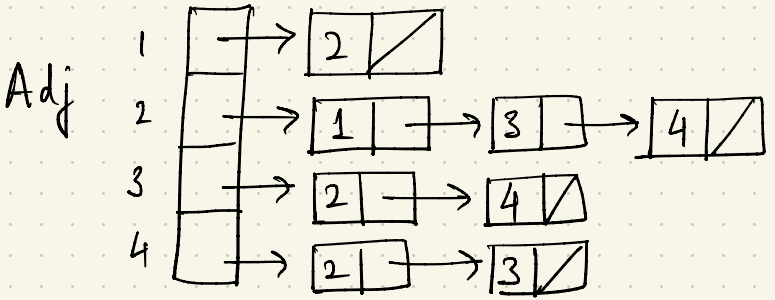

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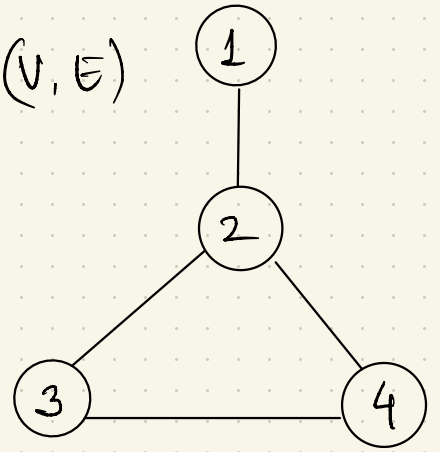

Space : $\Theta(|V| + |E|)$

or $\Theta(m+n)$

REPRESENTING GRAPHS

Which is better : adjacency matrix or adjacency list ?

REPRESENTING GRAPHS

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Depends on graph density and operations needed.

REPRESENTING GRAPHS

This course

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REPRESENTING GRAPHS

This course

Which is better : adjacency matrix or adjacency list ?

Depends on graph density and operations needed.

Adjacency lists
- great for "follow your nose" operations
- better for massive sparse graphs (e.g., the web graph)

GRAPH SEARCH

GRAPH SEARCH

input: an undirected or directed graph $G=(V, E)$ and a starting vertex s

goal: identify the vertices of V reachable from s

WHY CARE ABOUT GRAPH SEARCH

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Checking connectivity

- Web crawling, "friend finder" on social networks, garbage collection, ...

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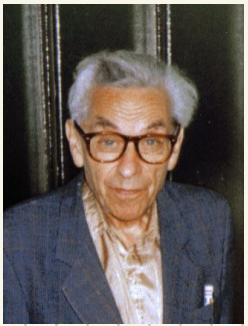

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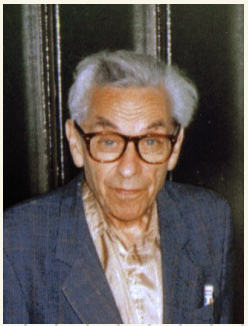

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WHY CARE ABOUT GRAPH SEARCH

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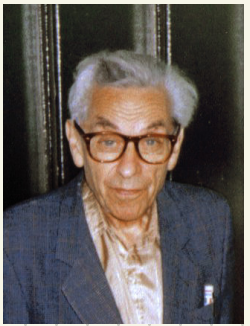

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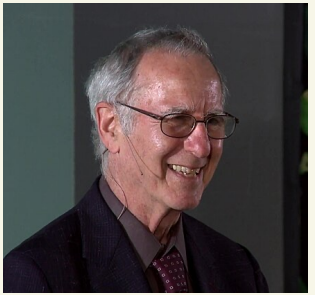

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WHY CARE ABOUT GRAPH SEARCH

Checking connectivity

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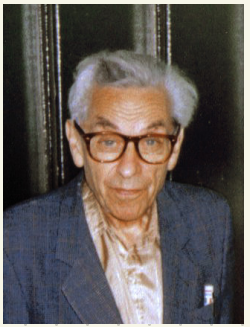

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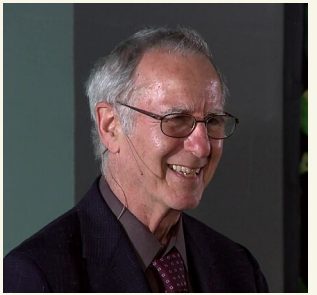

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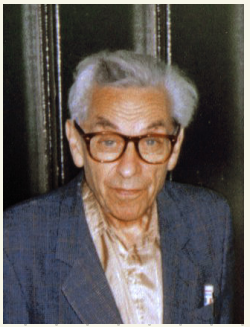

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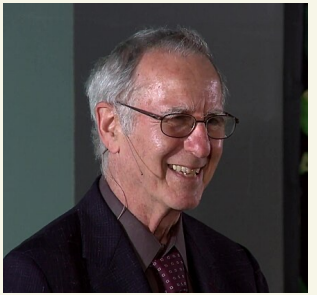

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Shortest paths

- fewest moves to solve Rubik's cube

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Revealing structure

- finding "communities" in networks

GENERIC GRAPH SEARCH

input: an undirected or directed graph $G=(V,E)$, starting vertex s

goal: identify the vertices of V reachable from s

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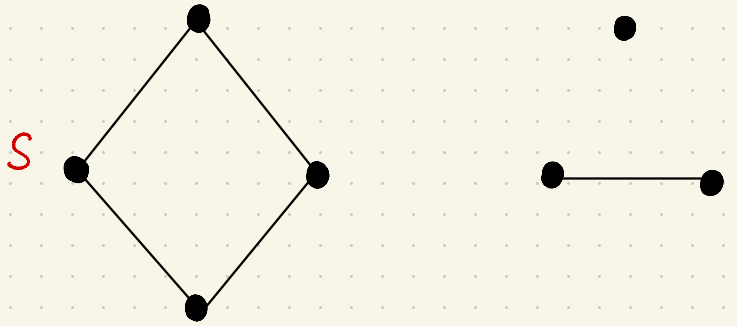

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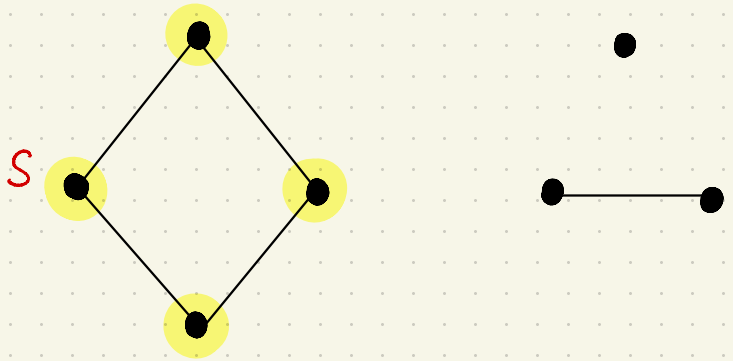

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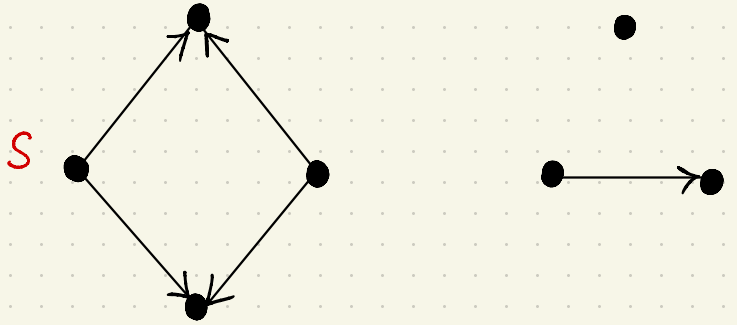

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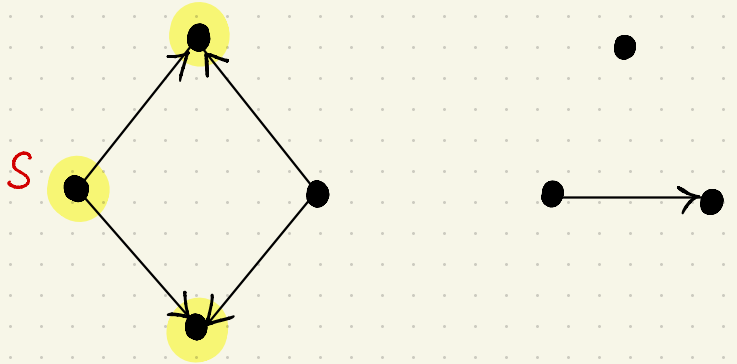

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$O(|V|+|E|)$ time

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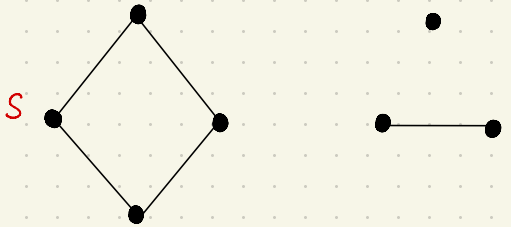

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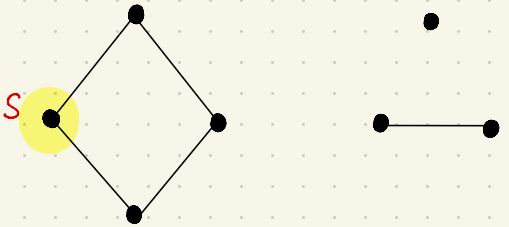

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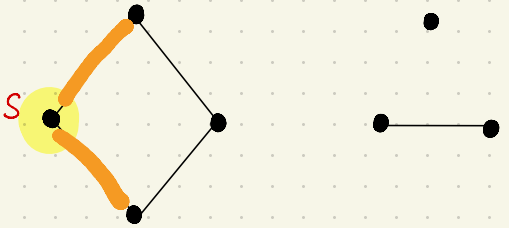

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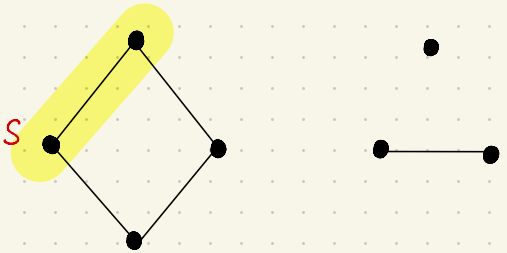

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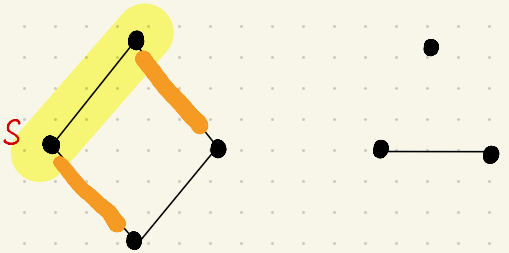

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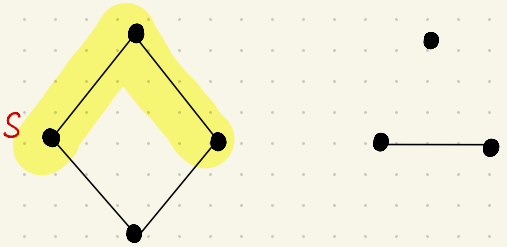

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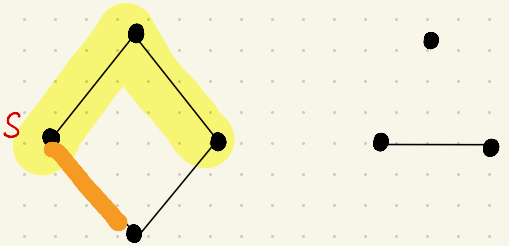

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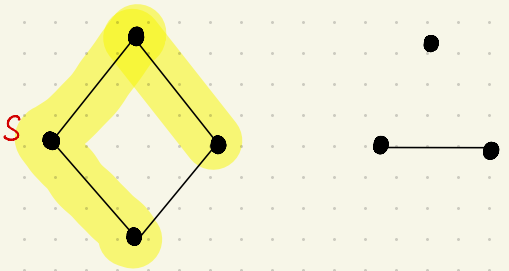

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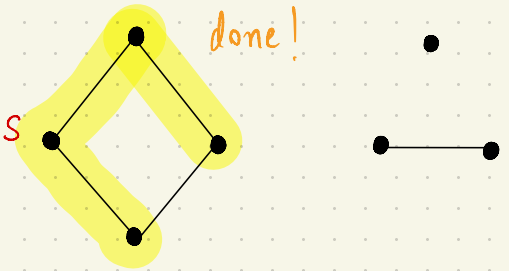

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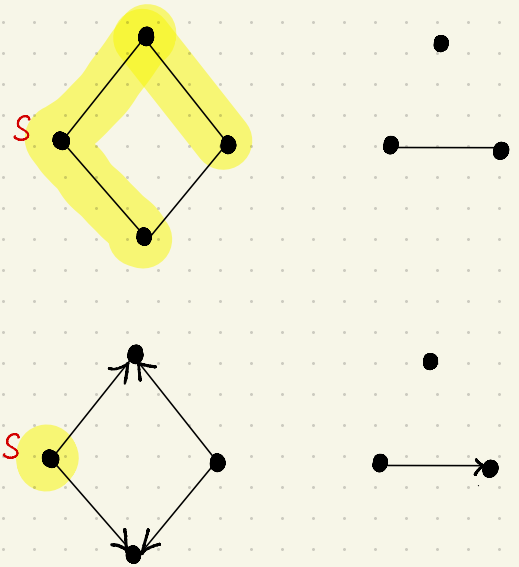

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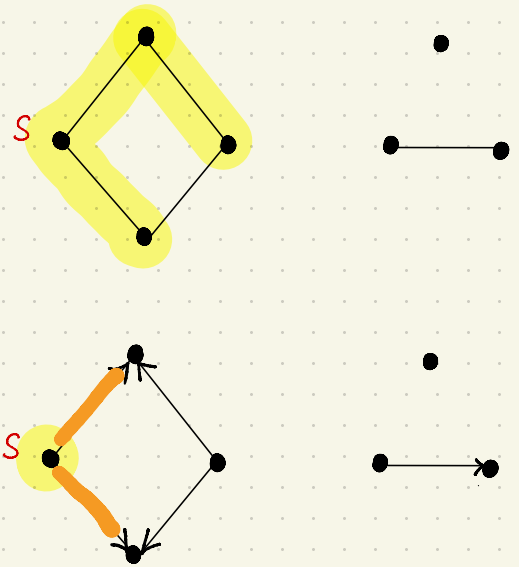

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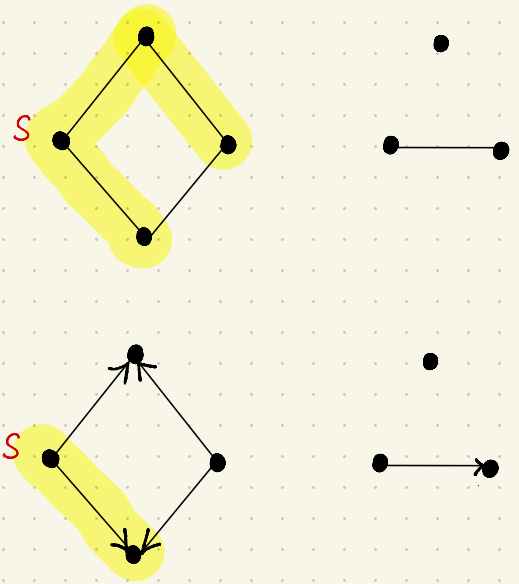

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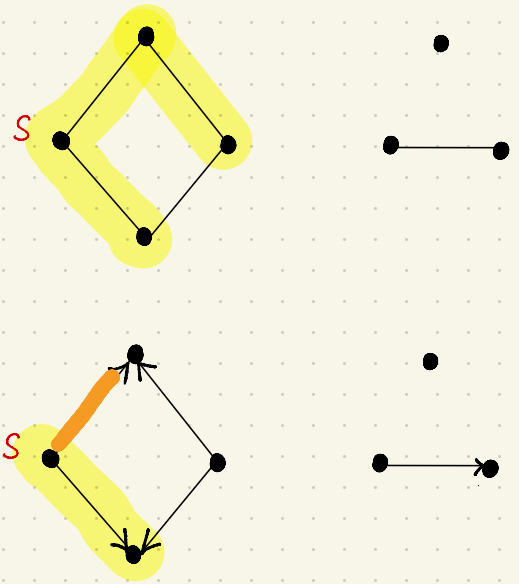

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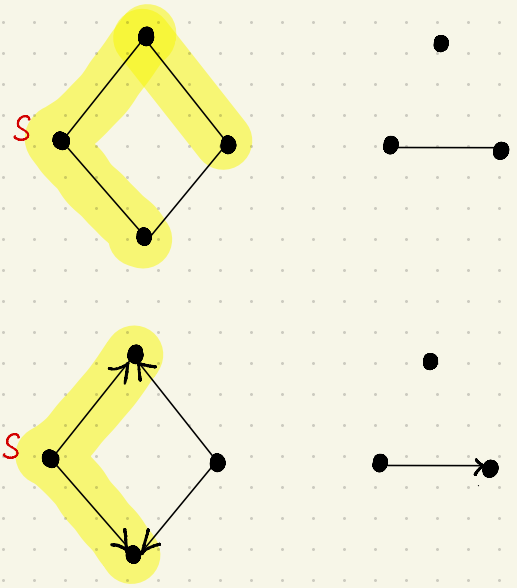

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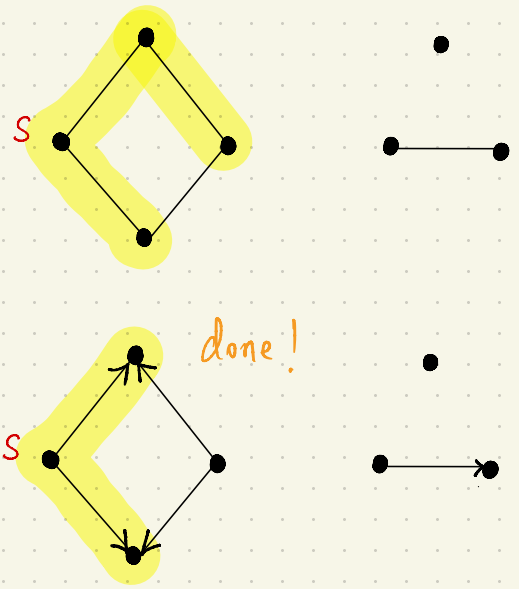

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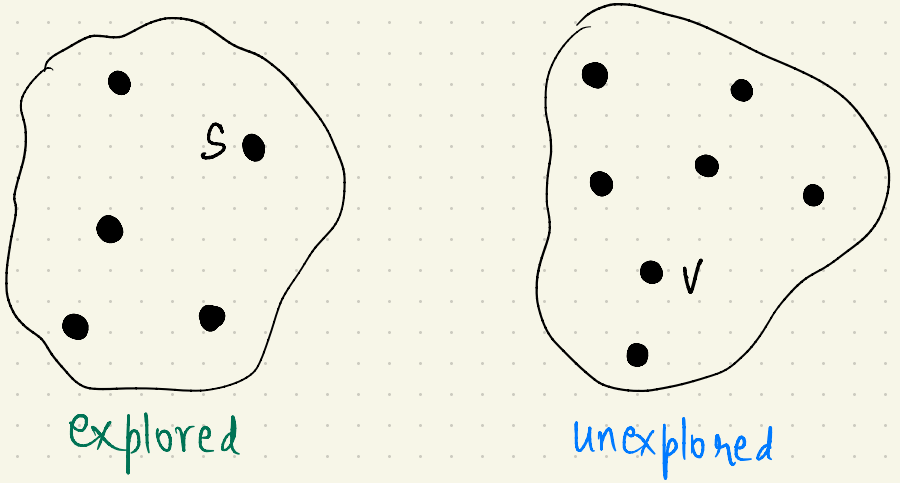

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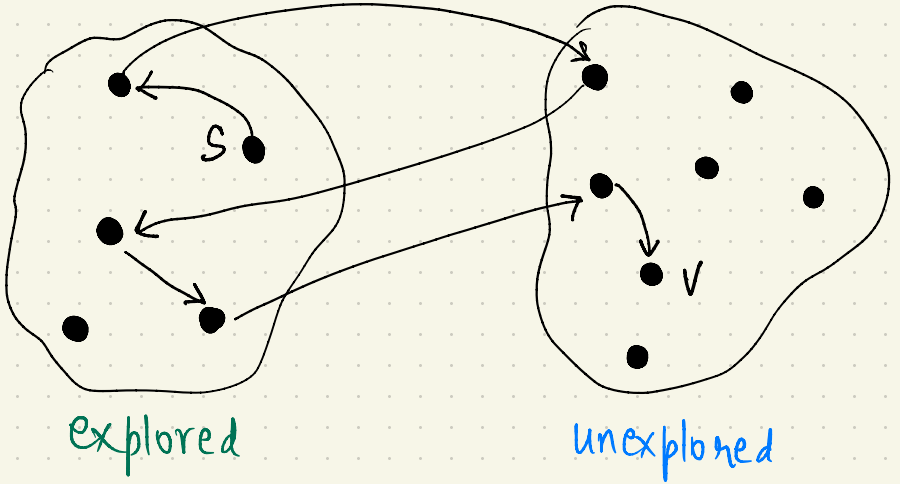

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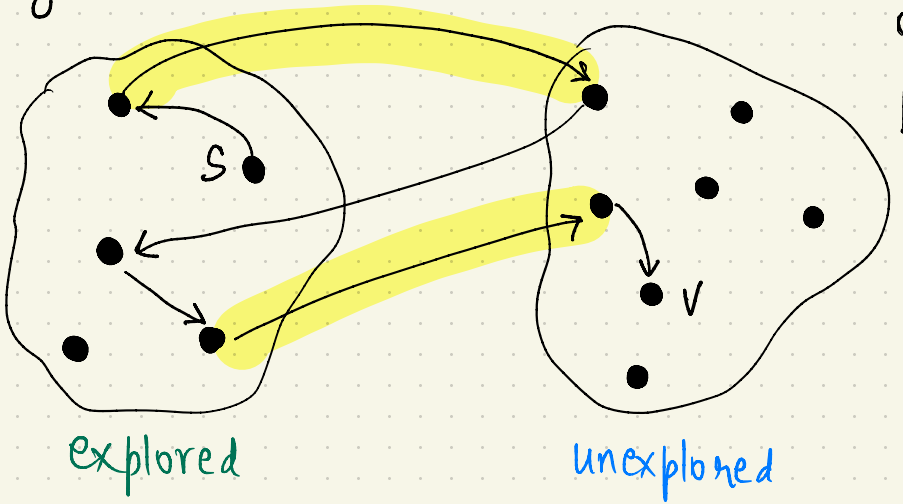

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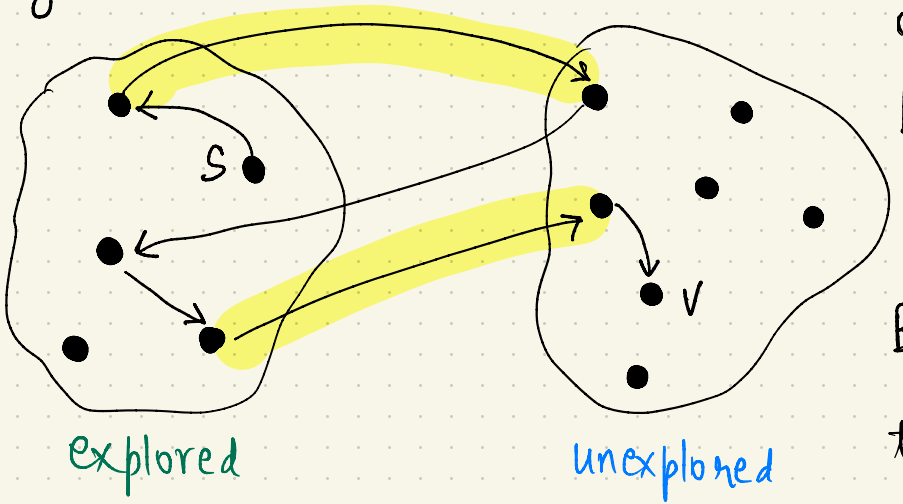

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But then, algo. wouldn't have terminated.

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while some edge $(u, v) \in E$ with u explored
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How to choose among the "crossing" edges?

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Depth-First Search: explores aggressively like a maze
- topological ordering in DAGs
- strongly connected components in directed graphs

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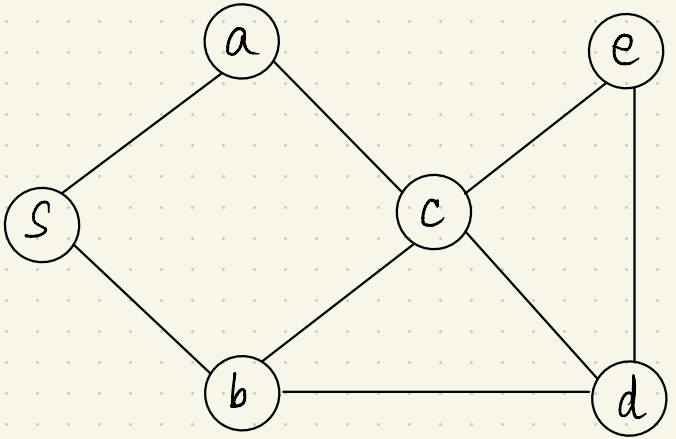
Breadth-First Search

Depth-First Search

Linear time: $O(m+n)$

BREADTH - FIRST SEARCH



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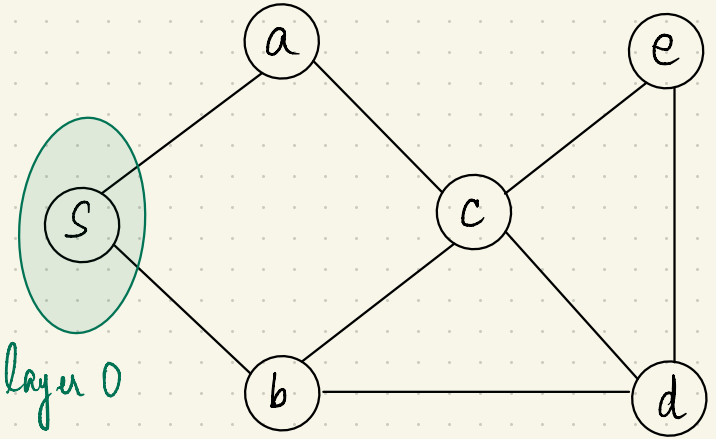

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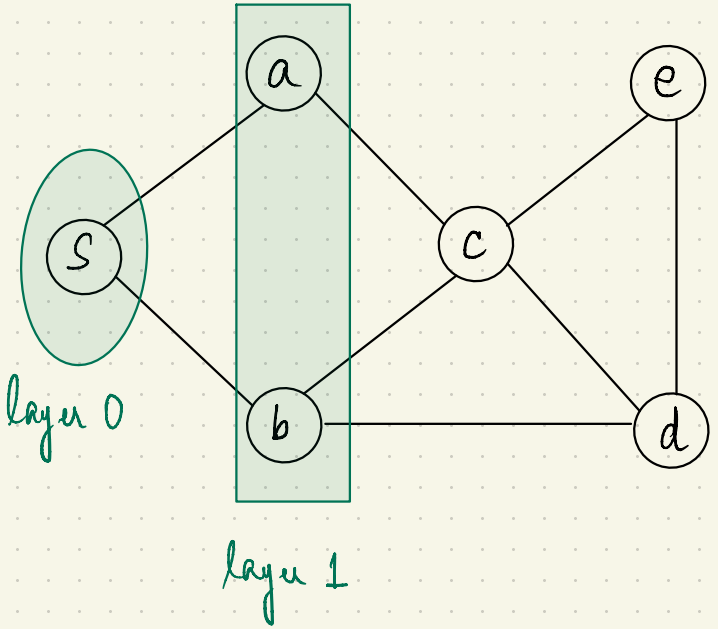

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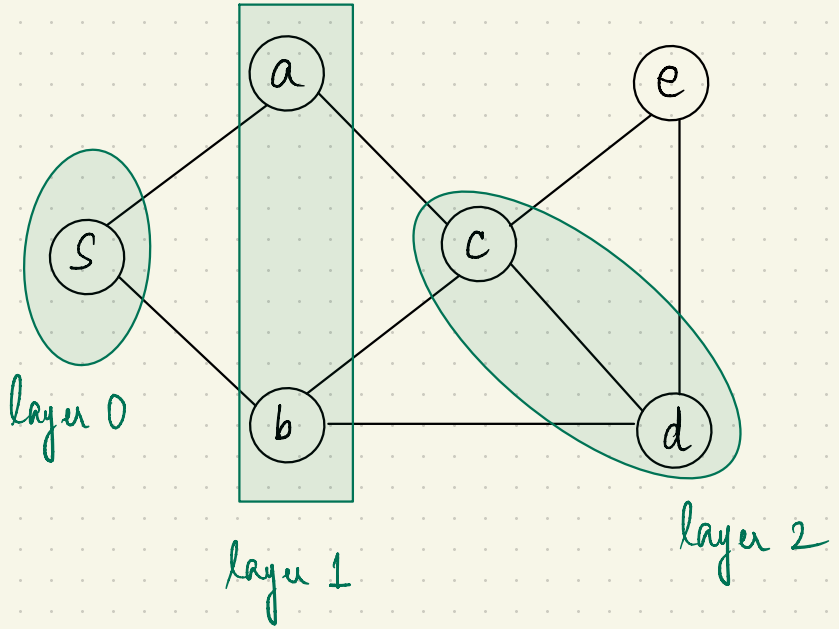

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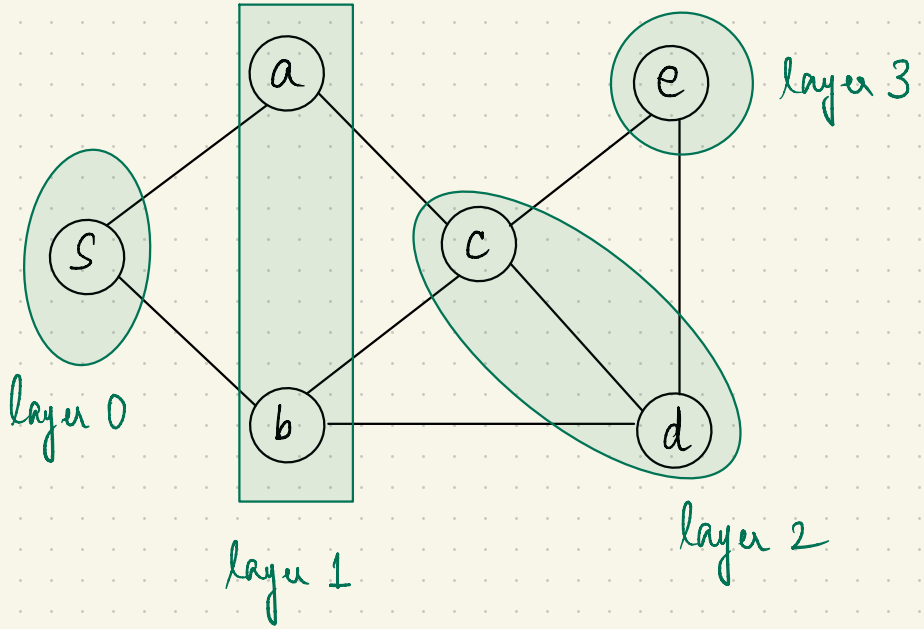

BREADTH-FIRST SEARCH

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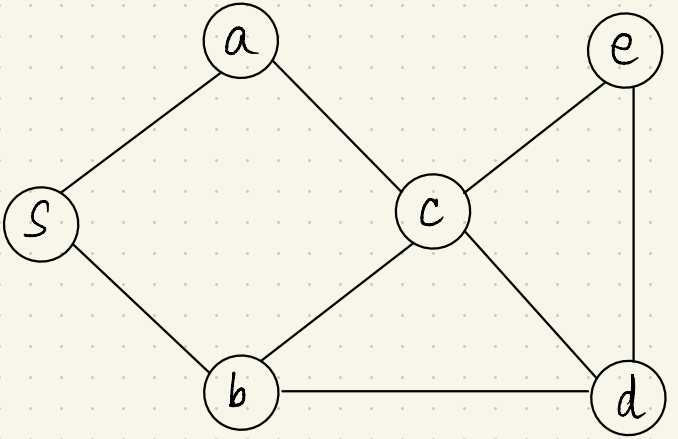

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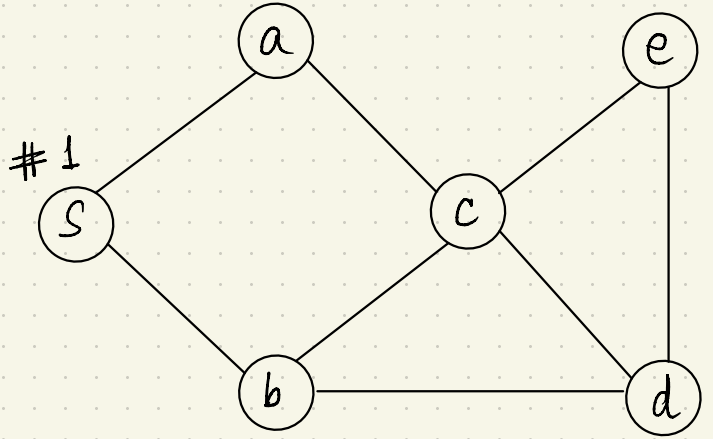

Q : S

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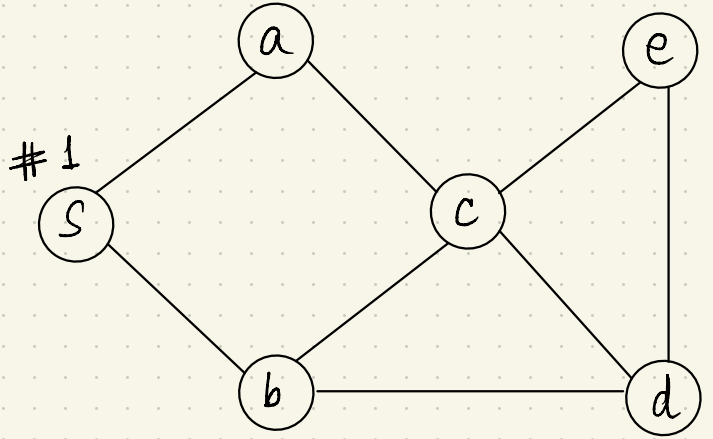

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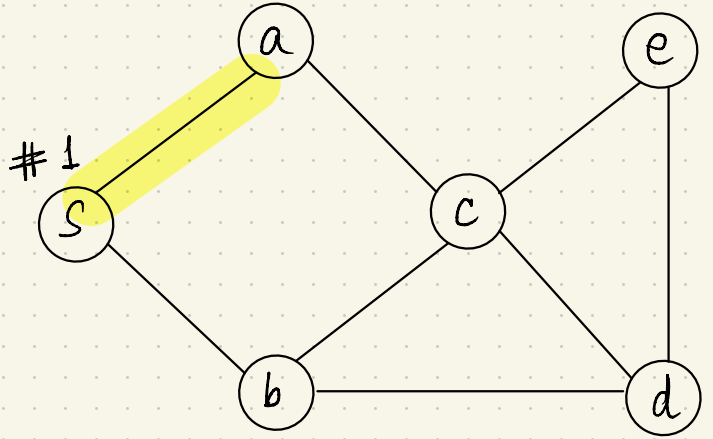

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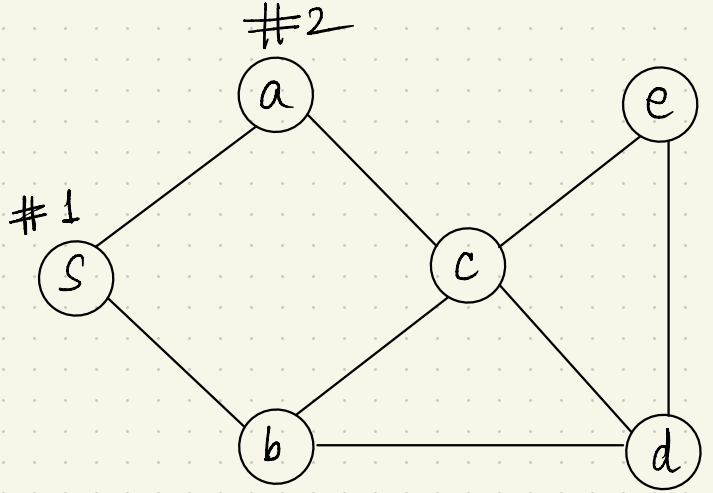

Q : ~~s~~ a

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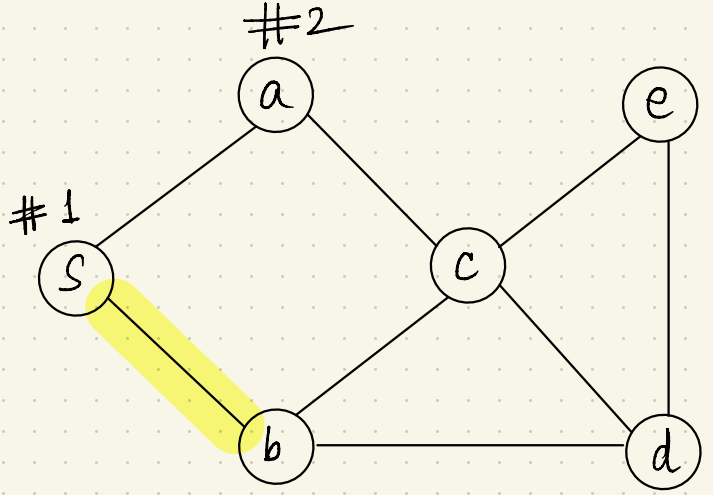

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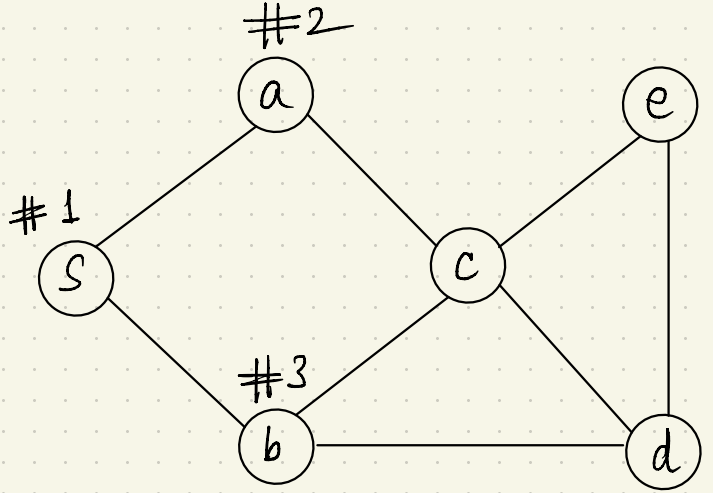

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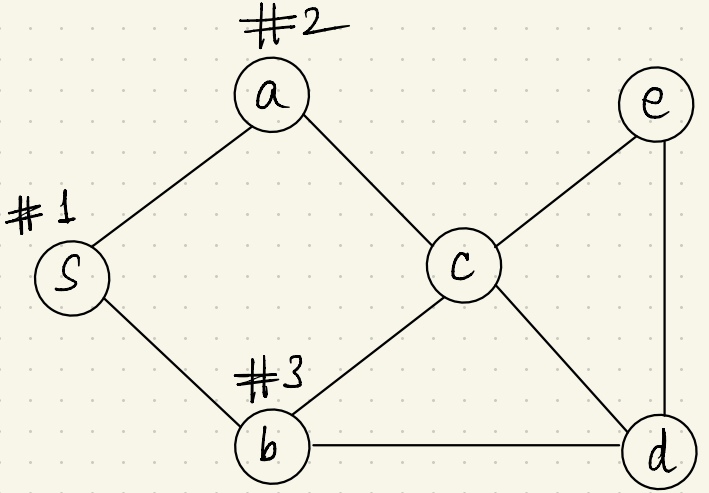

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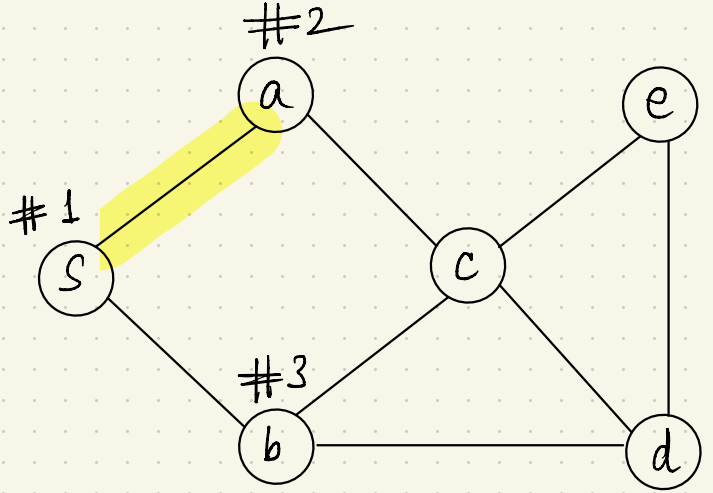

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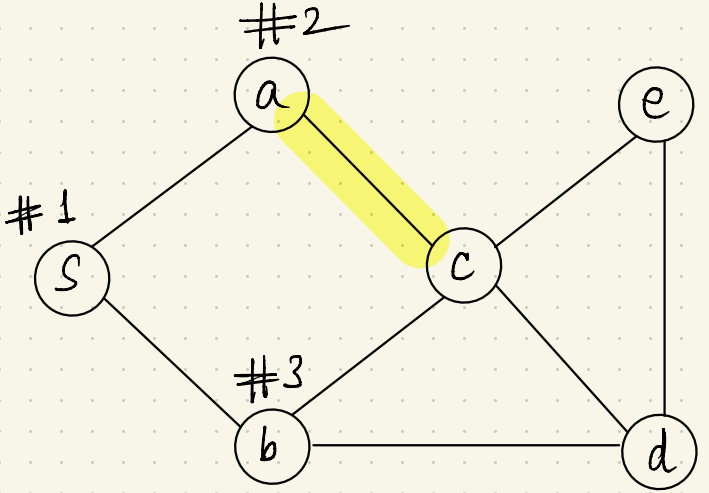

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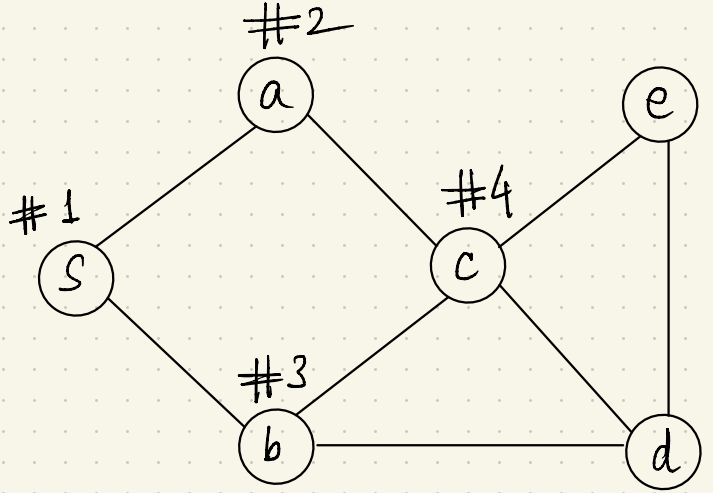

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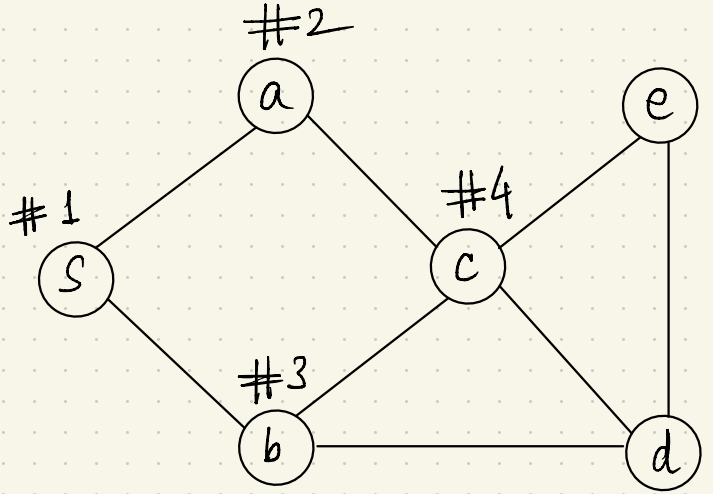

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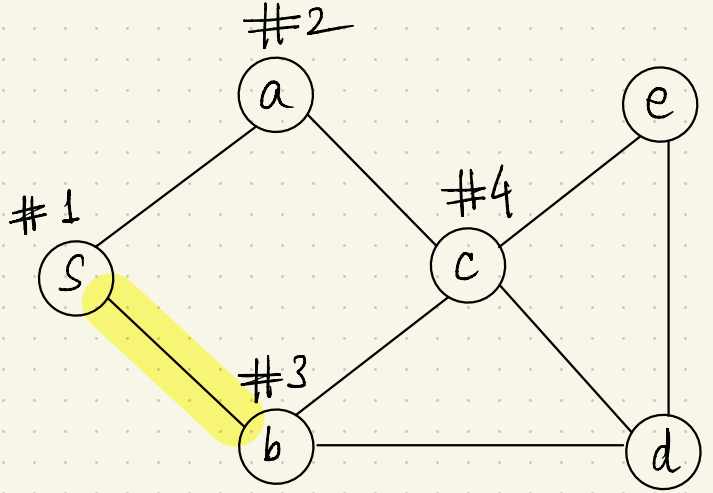

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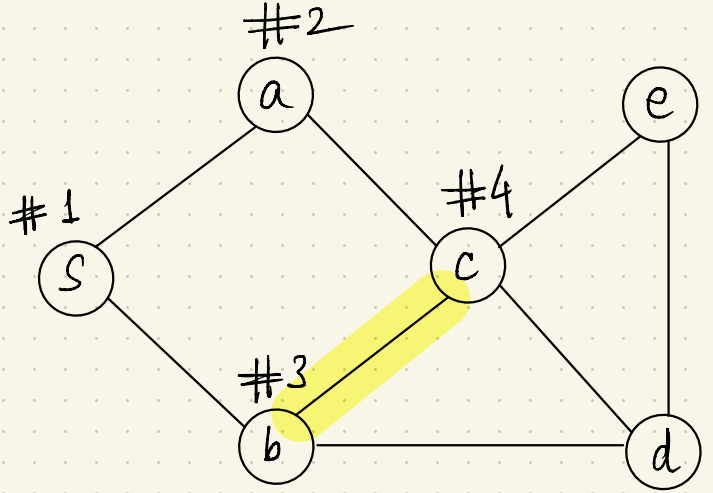

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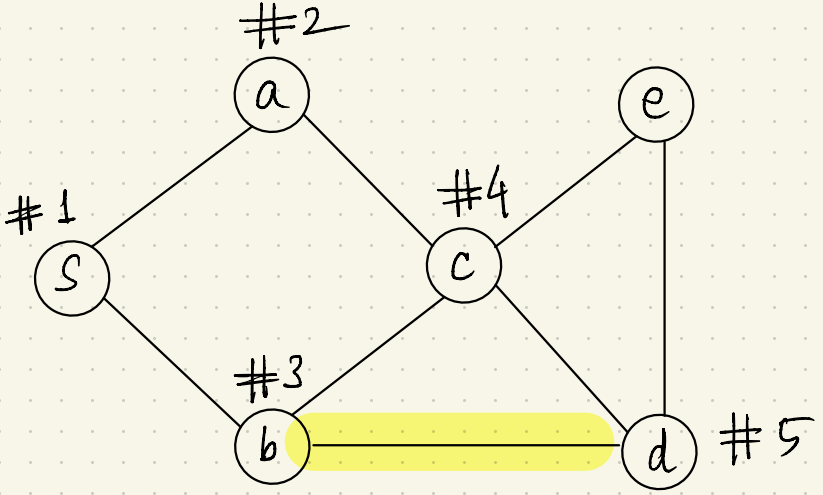

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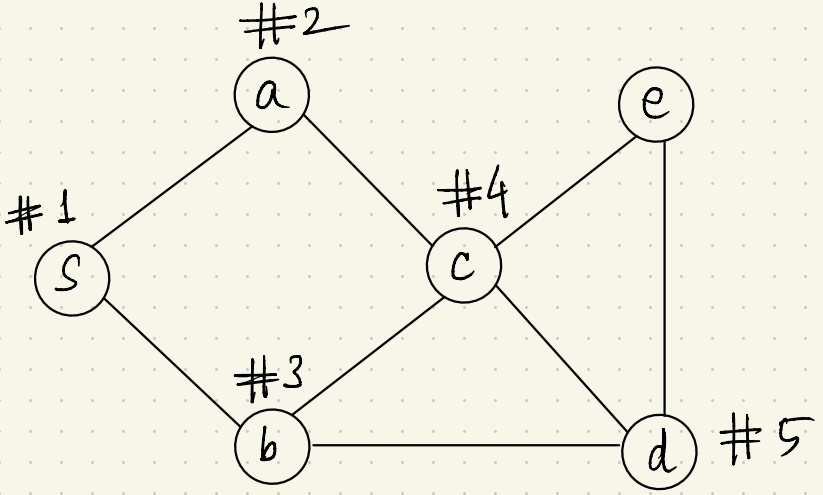

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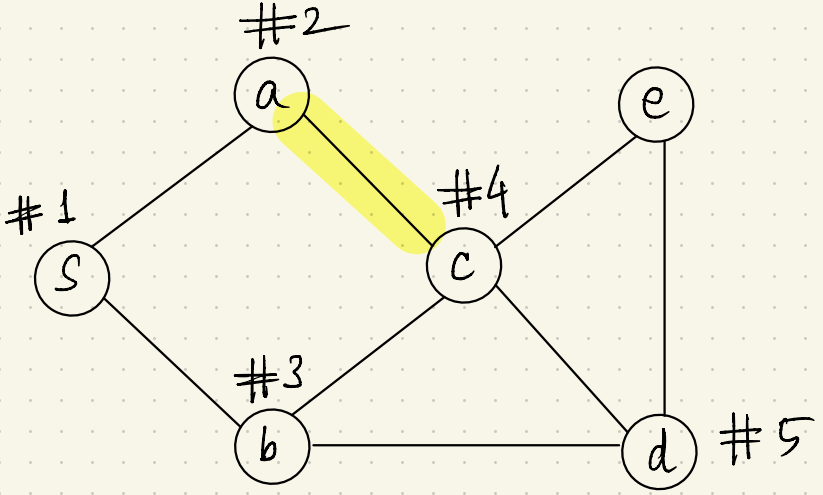

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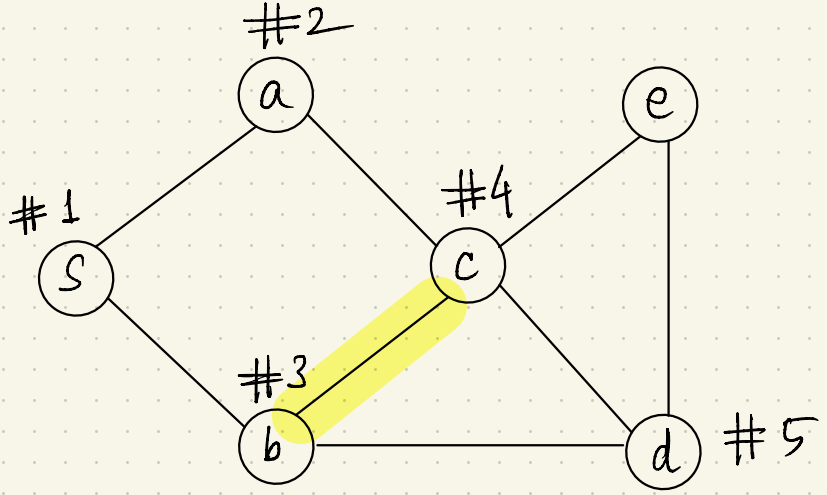

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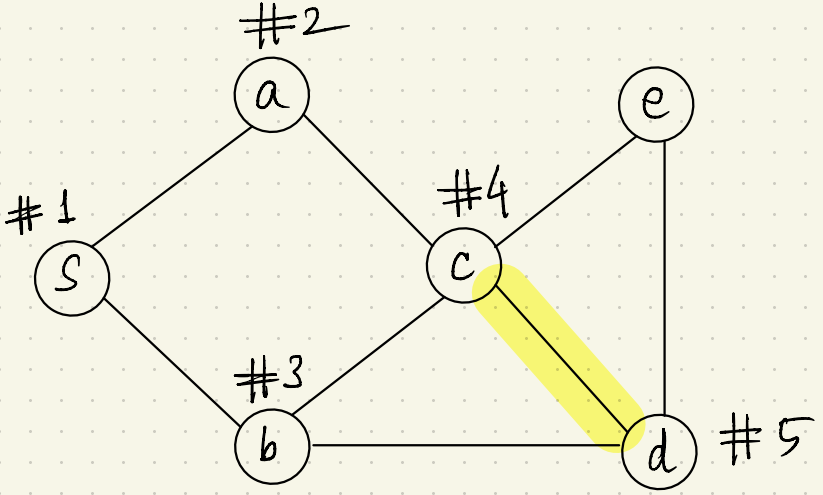

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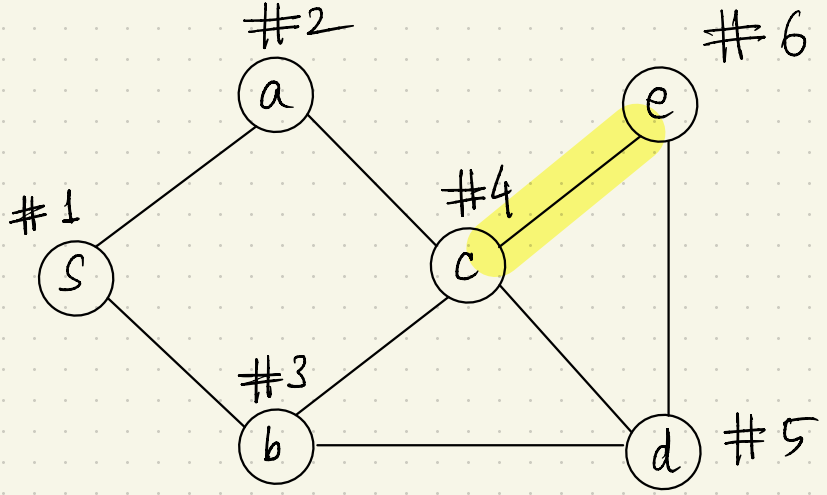

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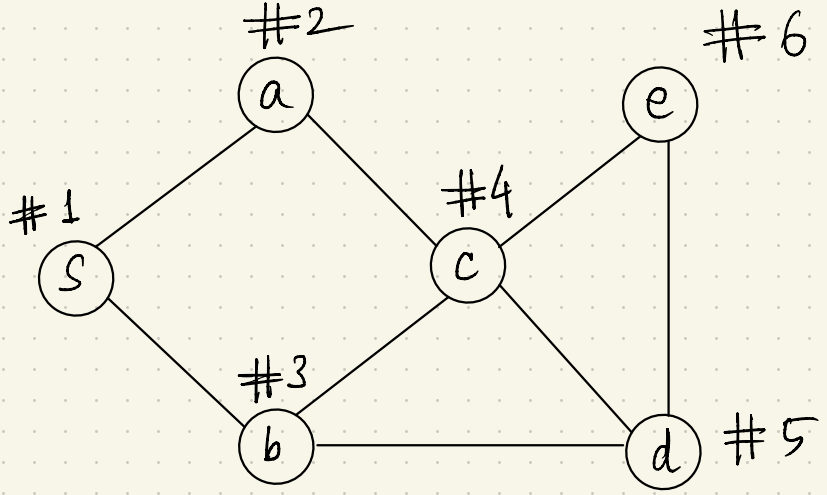

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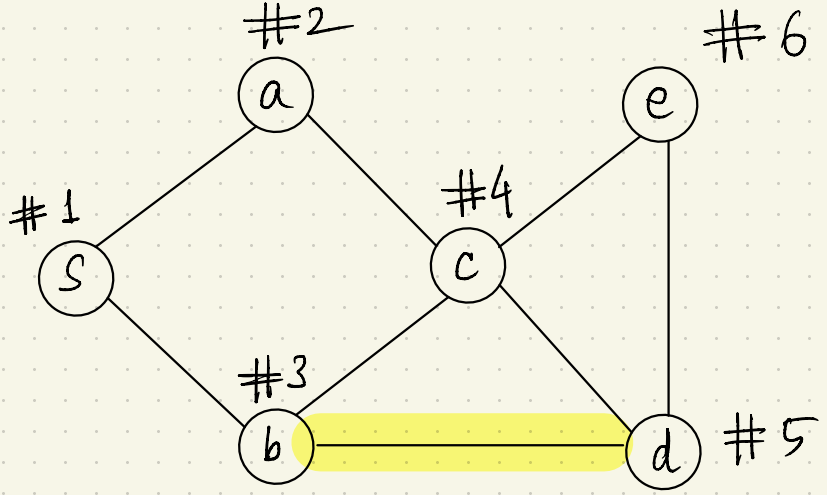

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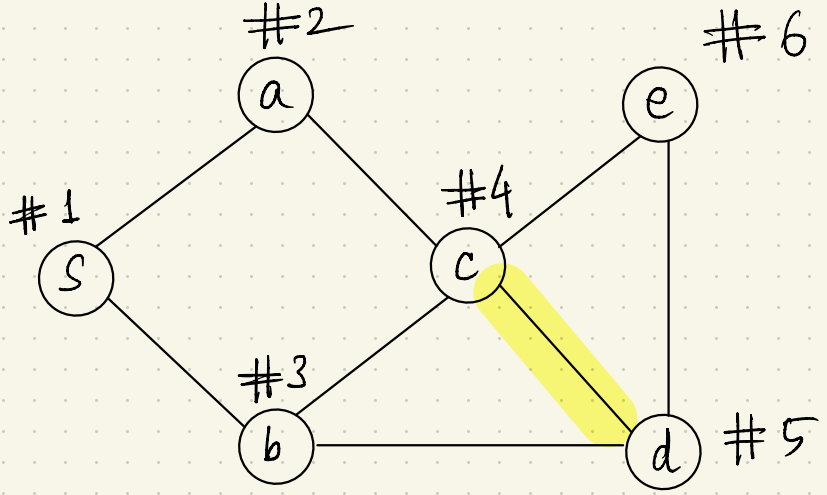

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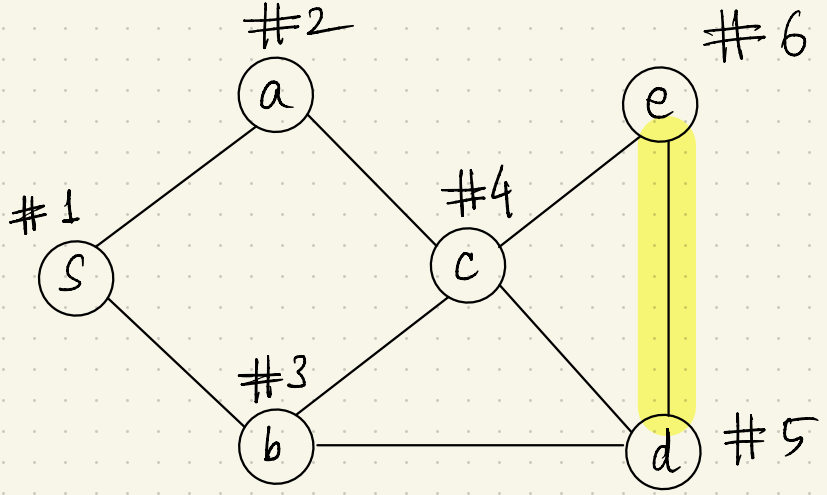

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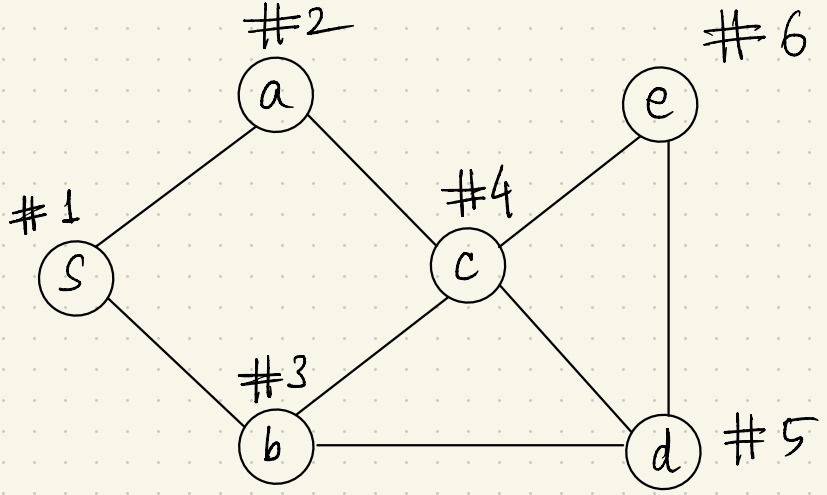

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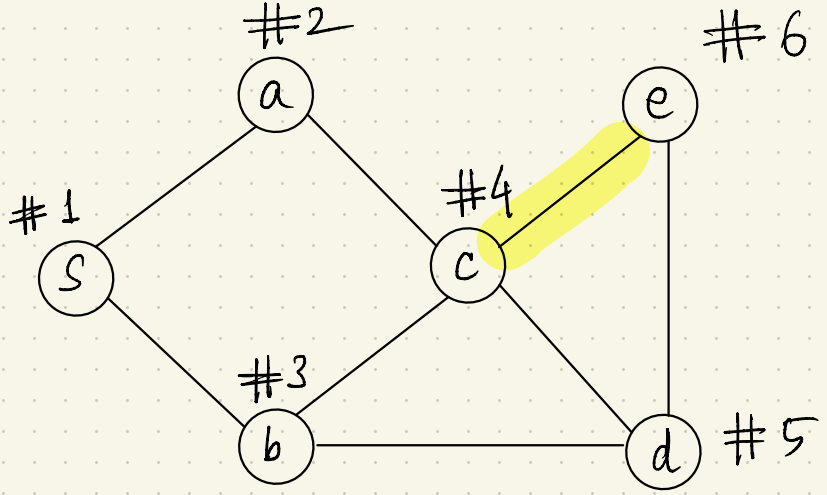

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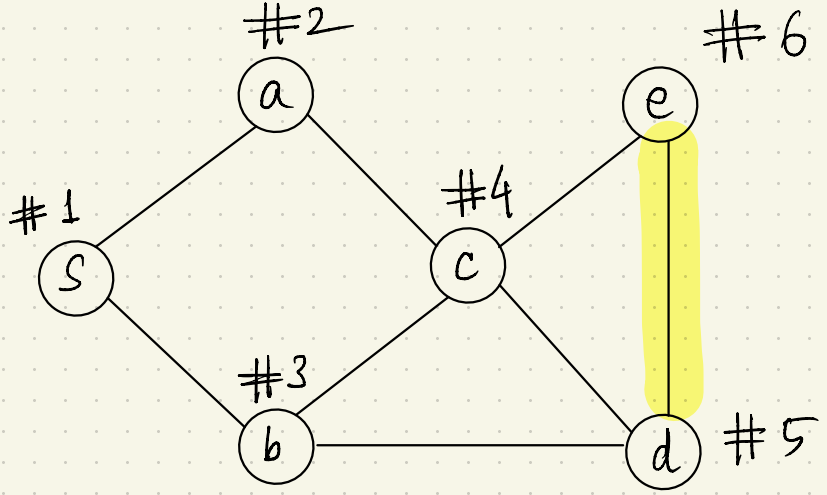

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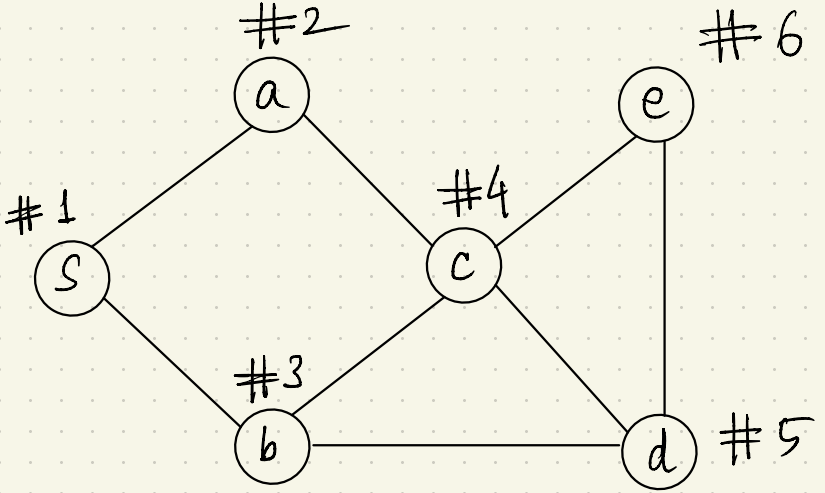

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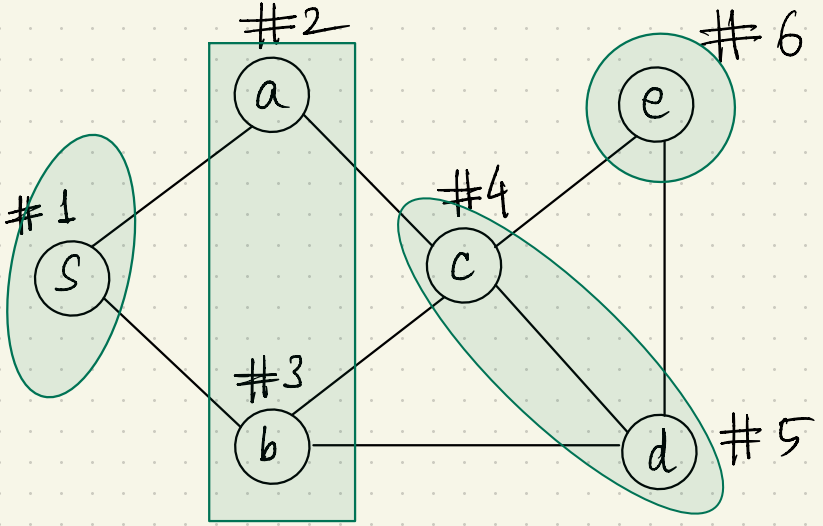

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Claim 2: Running time of while-loop of BFS is

$$O(n_s + m_s)$$

where n_s = # vertices reachable from s

m_s = # edges " " "

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