COL 351 : Analysis & Design of Algorithms

Lecture 8

Graph Algorithms I

Aug 07, 2024 | Rohit Vaish

REMINDERS

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WHAT DOES POLYNOMIAL-TIME MEAN?

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$$\text{input} \longrightarrow \boxed{\text{ALG}} \longrightarrow \text{output}$$

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ALG is a polynomial-time algorithm if the worst-case running time of ALG is upper bounded by a polynomial function of the size of the input.

WHAT DOES POLYNOMIAL-TIME MEAN?

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Given a number n,	$\cancel{n}$	$O(n)$ ✗
return its square.	$\log n$	$O(n^2)$

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a_1	a_2	$\cdots$	a_n

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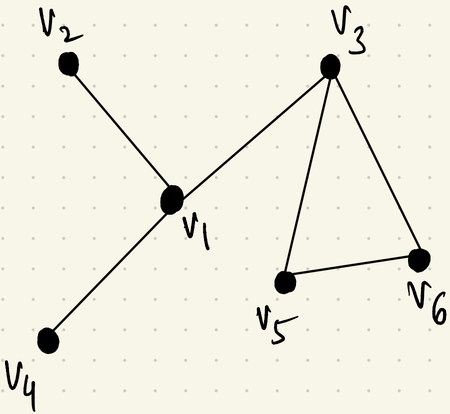
Graph Algorithms

GRAPH



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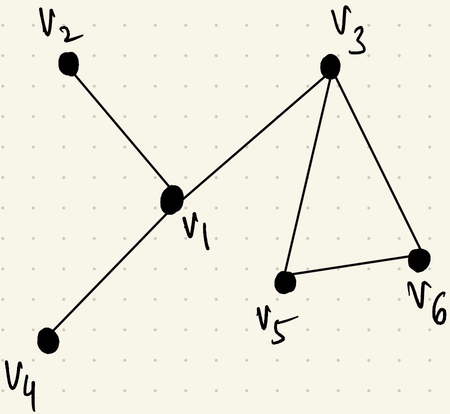

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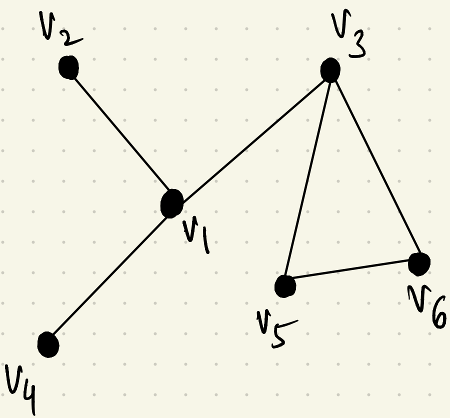

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V : non-empty set of items called vertices / nodes

E: a (possibly empty) set of 2-item subsets of V called edges

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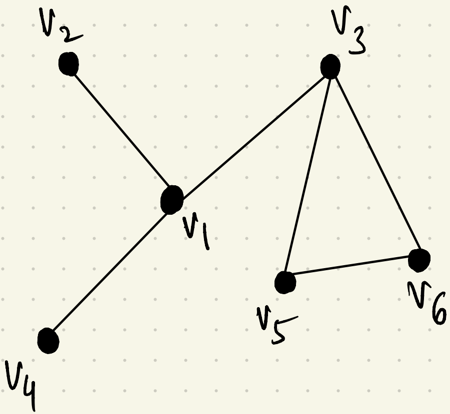

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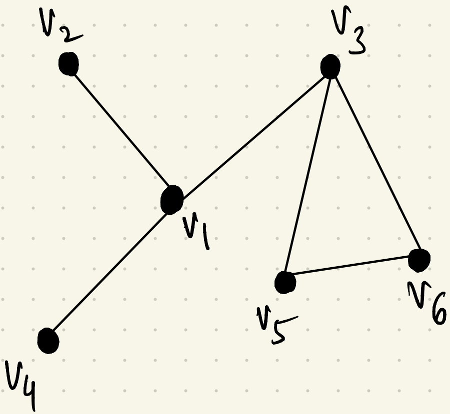

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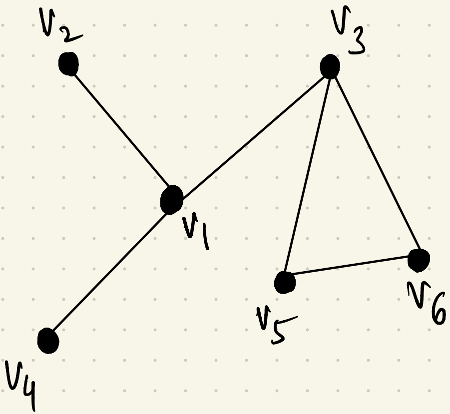

Edges can be undirected (unordered pair)

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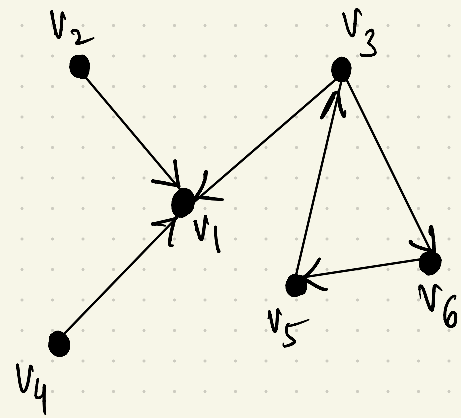

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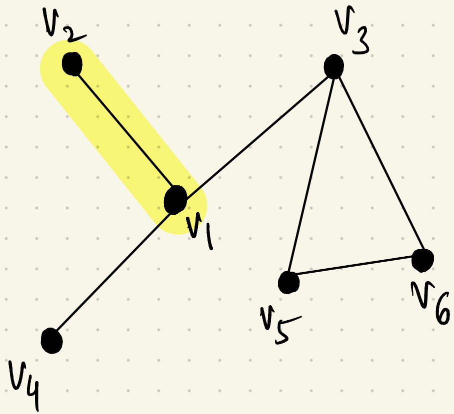
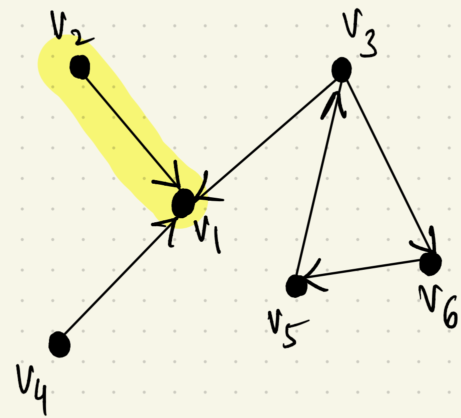


edge $\{v_1, v_2\}$ or $\{v_2, v_1\}$



edge (v_2, v_1)

APPLICATIONS

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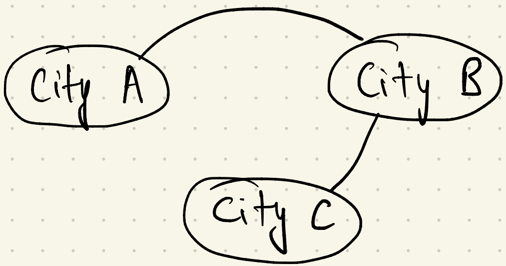
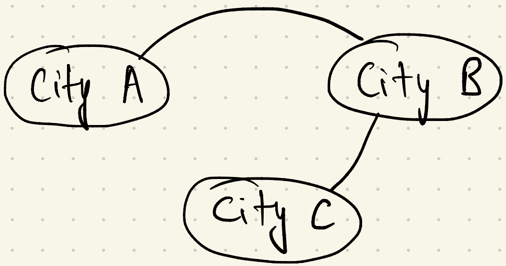


Road networks

APPLICATIONS



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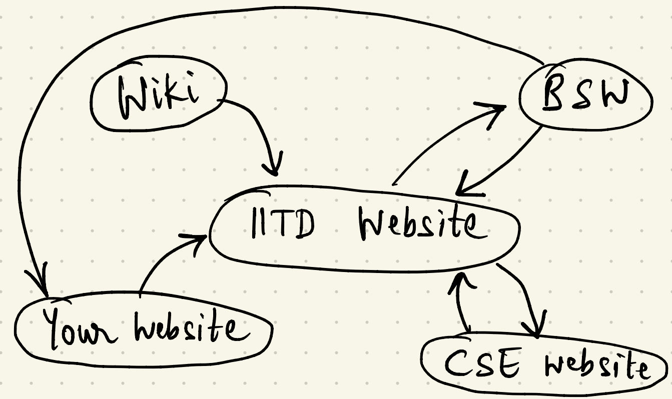
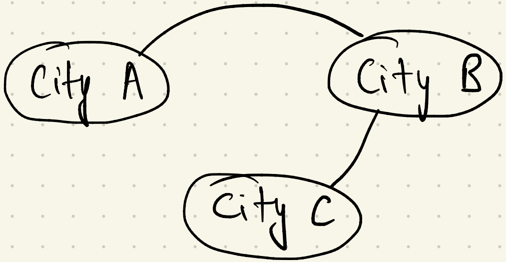


The Web

APPLICATIONS



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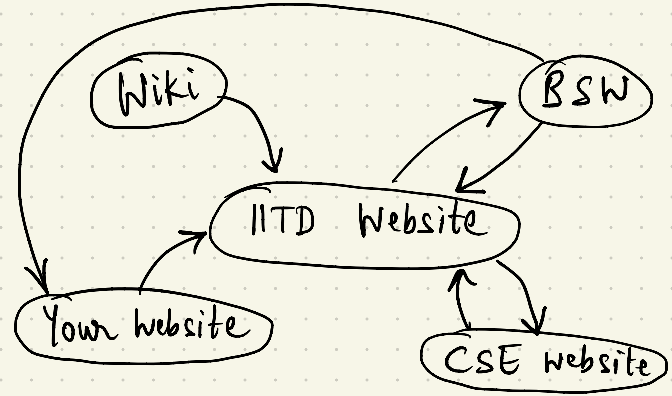

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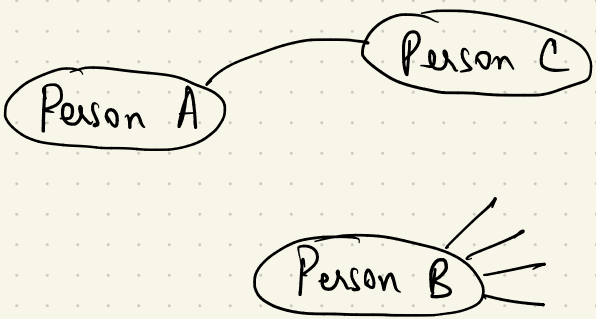
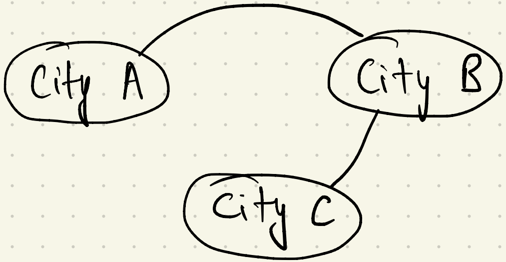


Social networks

APPLICATIONS



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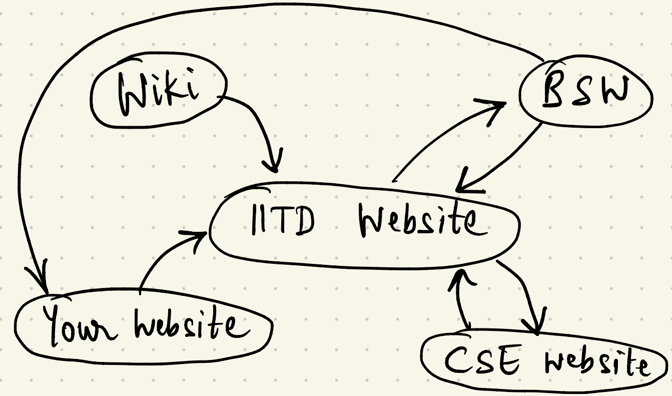

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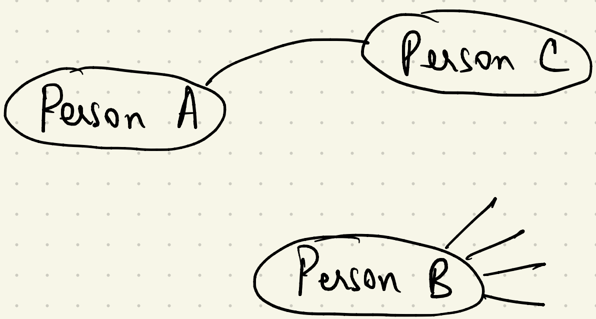

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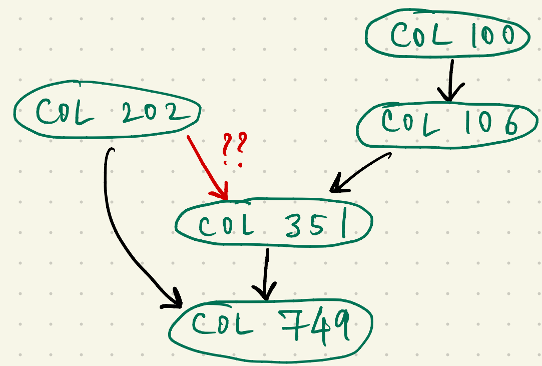
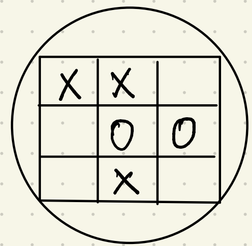


Precedence constraints

APPLICATIONS



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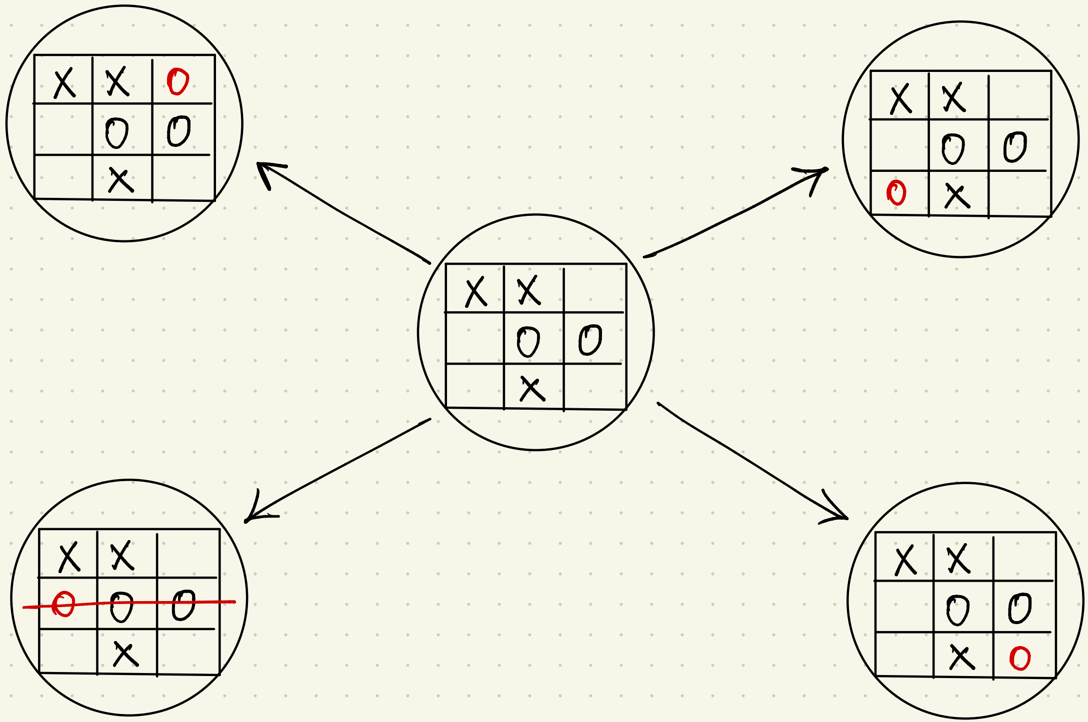

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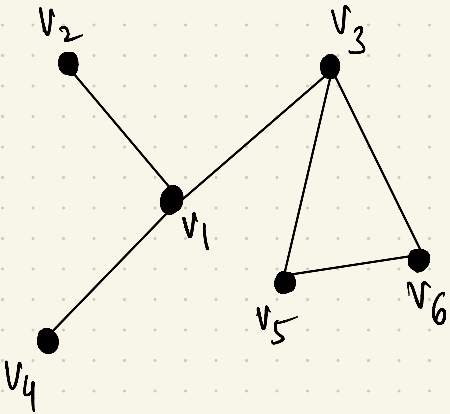

Two vertices v_i and v_j are adjacent

if $\{v_i, v_j\} \in E$.

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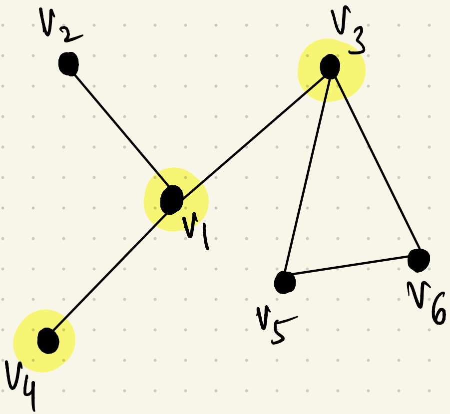

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v_3 and v_4 are NOT adjacent

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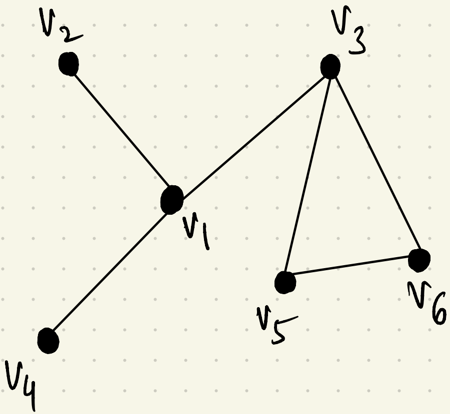

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An edge $e = \{v_i, v_j\}$ is incident

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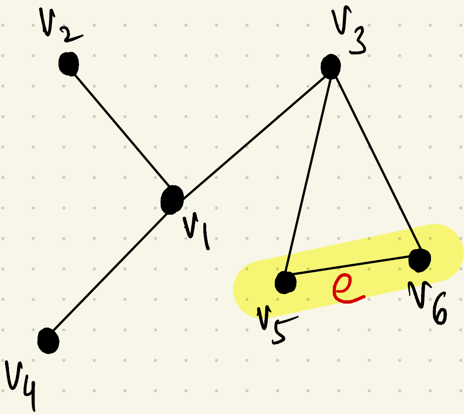

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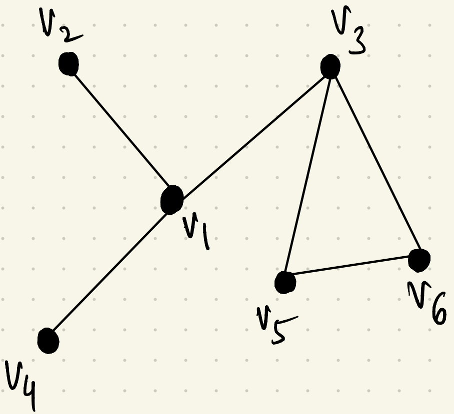
An edge $e = \{v_i, v_j\}$ is incident to the vertices v_i and v_j.

e.g., e is incident to v_5 and v_6 but not v_3.

GRAPH



The number of edges incident to a vertex is called the degree.

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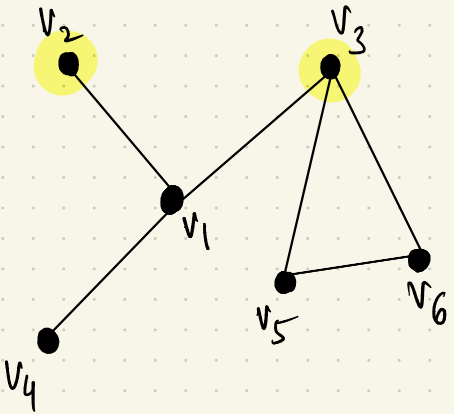

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e.g., $\deg(v_3) = 3$ $\deg(v_2) = 1$

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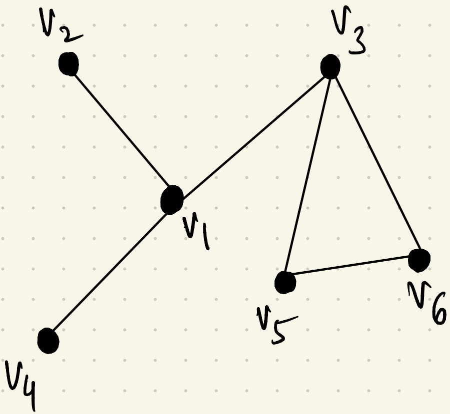

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A simple graph has no loops or multiedges.

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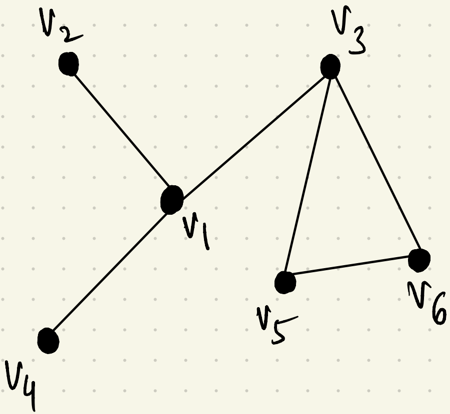

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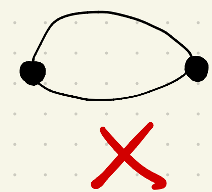

WALKS AND PATHS

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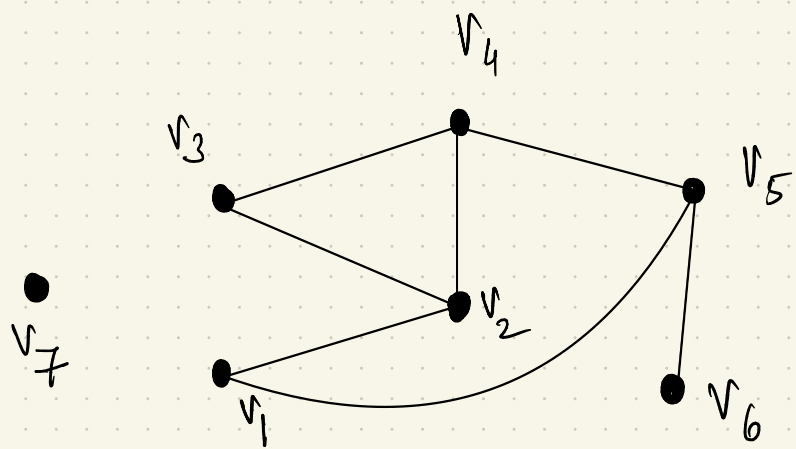
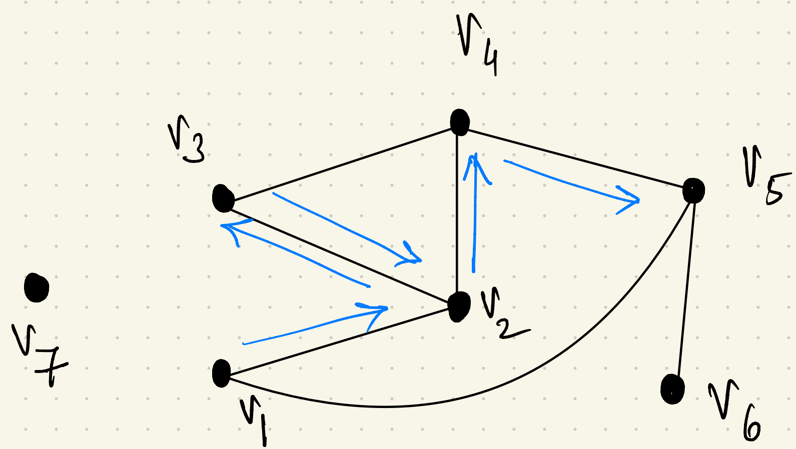


WALKS AND PATHS

A **walk** is a sequence of vertices connected by edges.

E.g., $v_1 — v_2 — v_3 — v_2 — v_4 — v_5$

start end

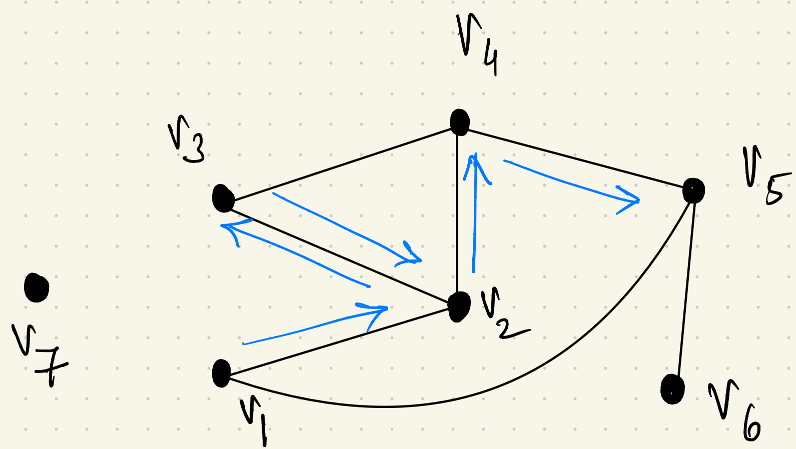


WALKS AND PATHS

A **walk** is a sequence of vertices connected by edges.

E.g., $v_1 - v_2 - v_3 - v_2 - v_4 - v_5$ } walk of length 5

start ... end



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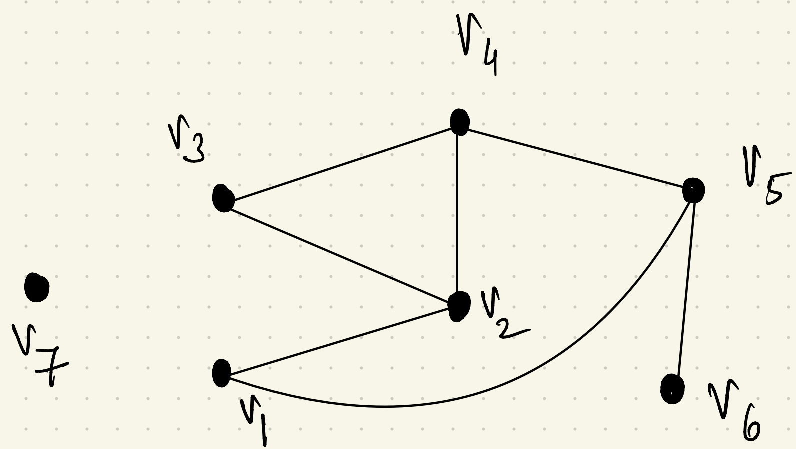
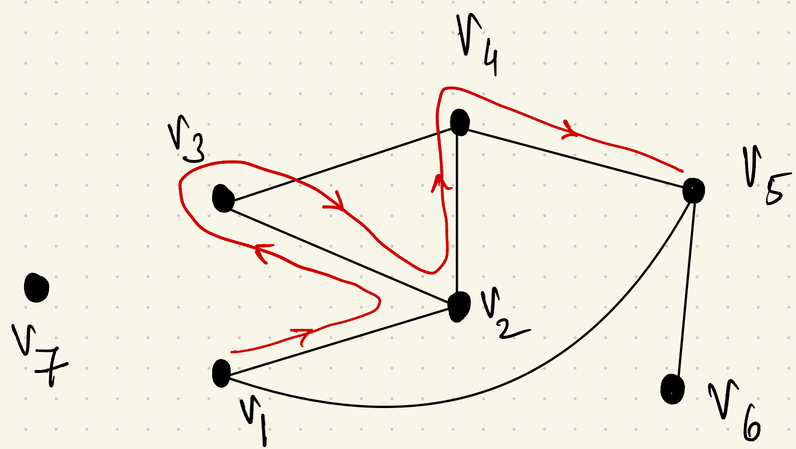


WALKS AND PATHS

A **path** is a walk where all vertices are different.

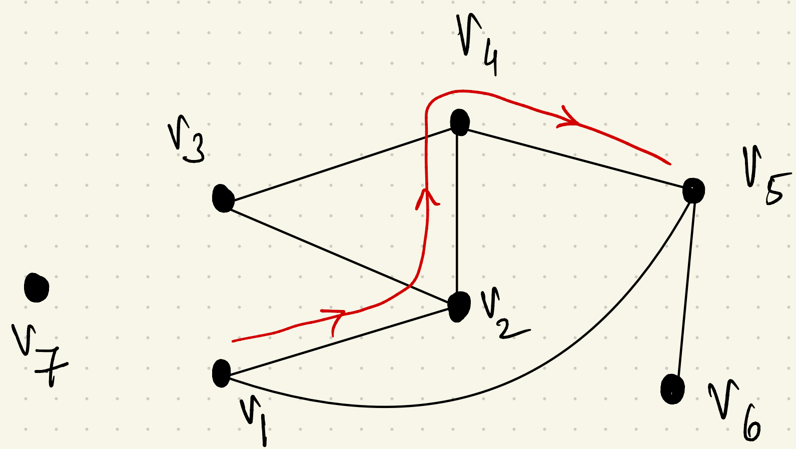
E.g., $v_1 - v_2 - v_3 - v_2 - v_4 - v_5$ ✗ Not a path



WALKS AND PATHS

A **path** is a walk where all vertices are different.

E.g., v_1 —— v_2 —— v_4 —— v_5 A valid path (length = 3)



Lemma: For any two distinct vertices u and v,

there exists a walk between u and v $\iff$ there exists a path between u and v

Proof: Exercise

CONNECTIVITY

A pair of vertices u and v are connected if there is a path between u and v.

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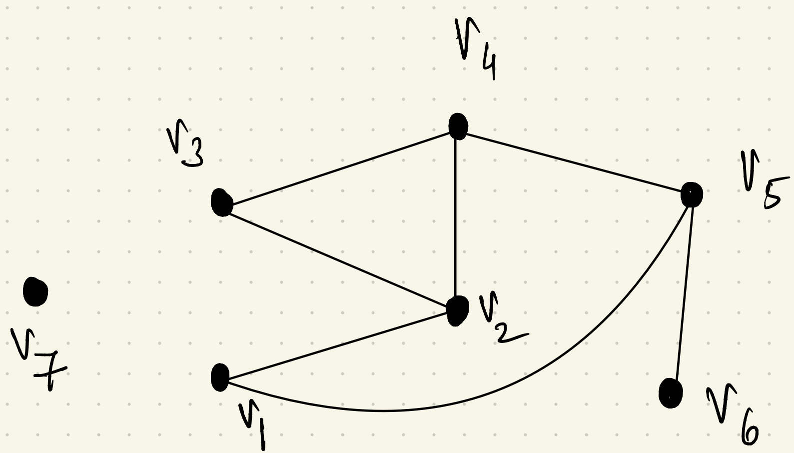
A pair of vertices u and v are **connected** if there is a path between u and v.



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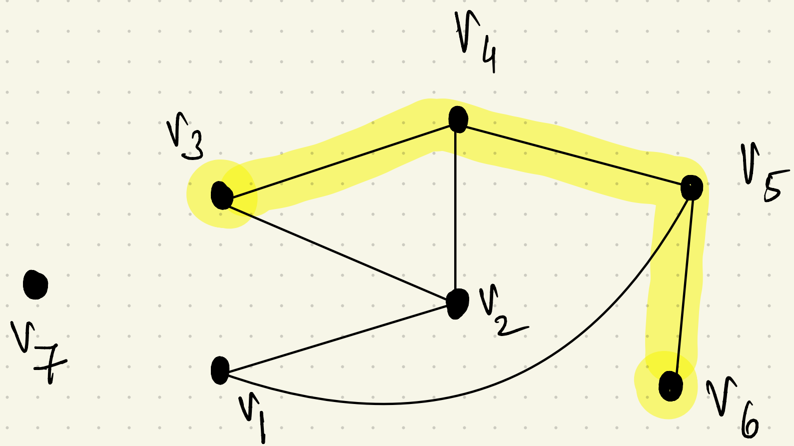

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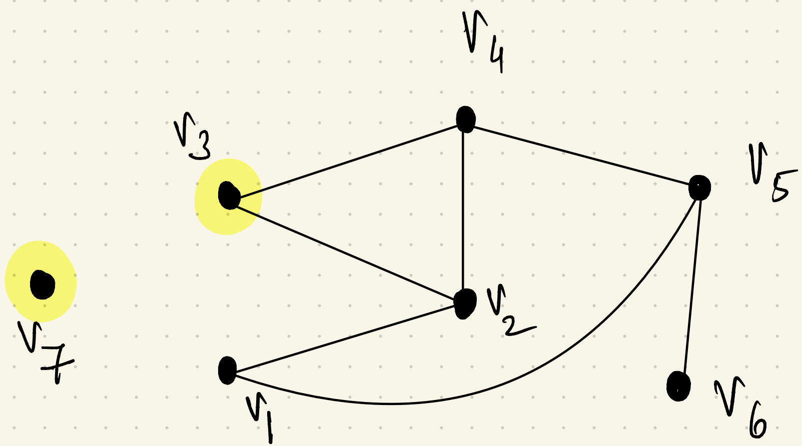
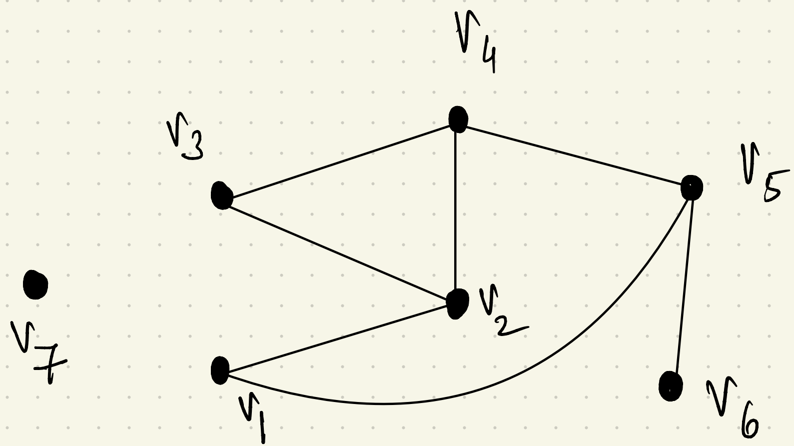


$v_3, v_6 \rightarrow$ Connected

$v_3, v_7 \rightarrow$ Not connected

CONNECTIVITY

A pair of vertices u and v are connected if there is a path between u and v.



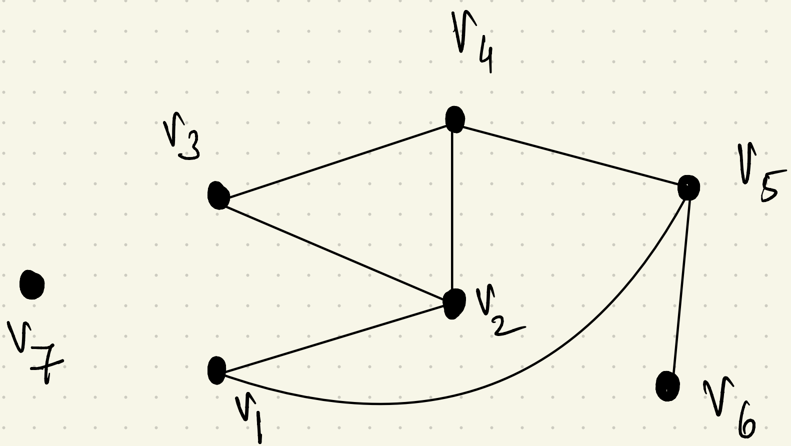
$v_3, v_6 \rightarrow$ Connected

$v_3, v_7 \rightarrow$ Not connected

A graph is connected if every pair of vertices are connected.

CONNECTIVITY

A pair of vertices u and v are connected if there is a path between u and v.



Not a connected graph

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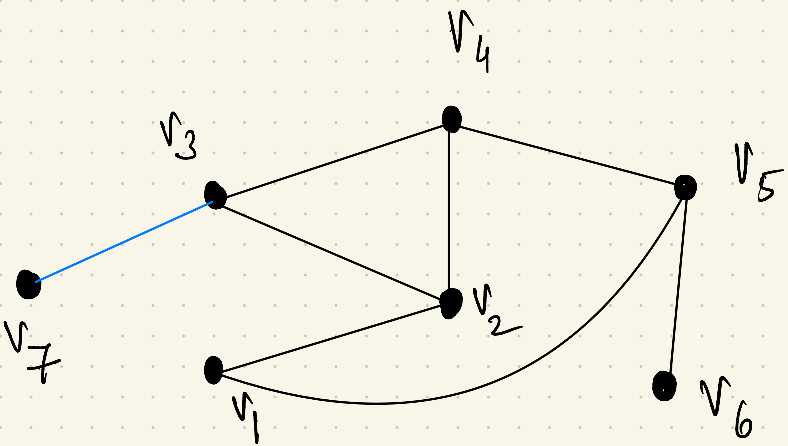


Connected

A graph is connected if every pair of vertices are connected.

CONNECTIVITY

Given an undirected and connected graph on n vertices

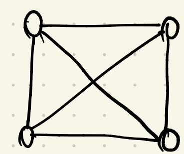
Maximum # edges =

Minimum # edges =

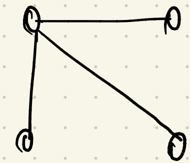
CONNECTIVITY

Given an undirected and connected graph on n vertices

Maximum # edges = ${}^{n}C_2$



Minimum # edges = $n-1$



CONNECTIVITY

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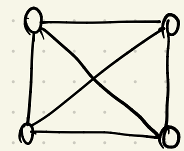
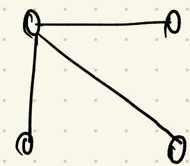


complete graph

Minimum # edges = $n-1$



tree

CYCLES AND CLOSED WALKS

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A closed walk is a walk that starts and ends at the same vertex.

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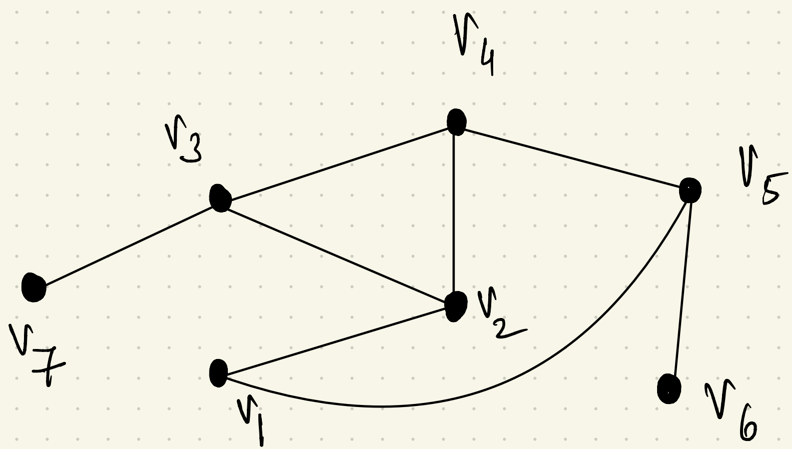
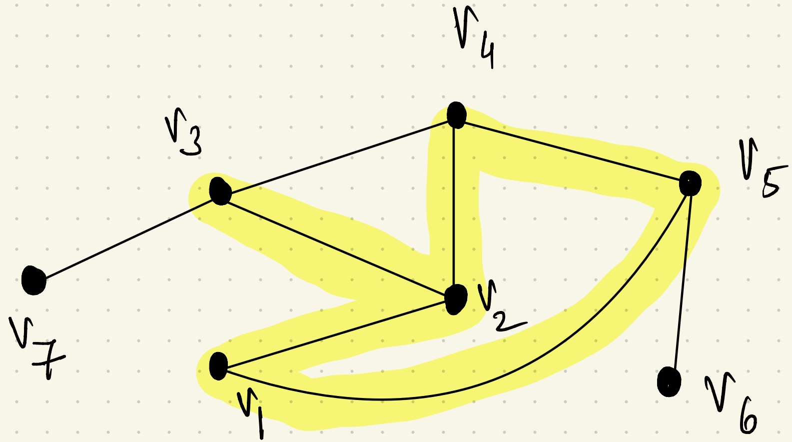


CYCLES AND CLOSED WALKS

A closed walk is a walk that starts and ends at the same vertex.

$v_1 - v_2 - v_3 - v_2 - v_4 - v_5 - v_1$



CYCLES AND CLOSED WALKS

A closed walk is a walk that starts and ends at the same vertex.

A cycle is a closed walk in which all vertices (except start and end) are different.

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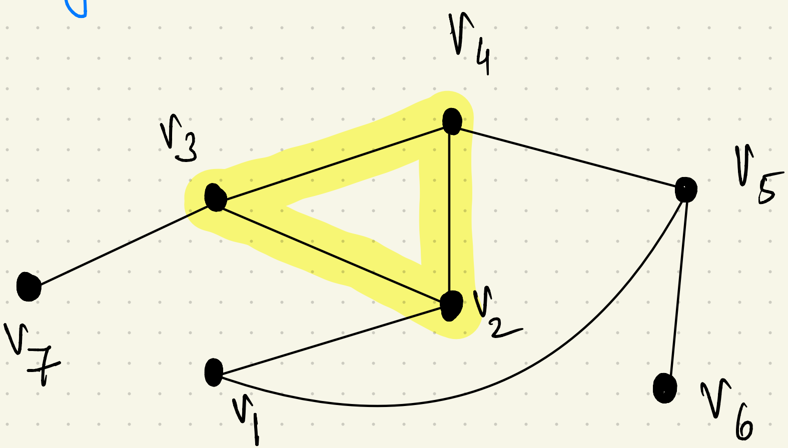

$v_2 - v_3 - v_4 - v_2$ is a cycle of length three

CYCLES AND CLOSED WALKS

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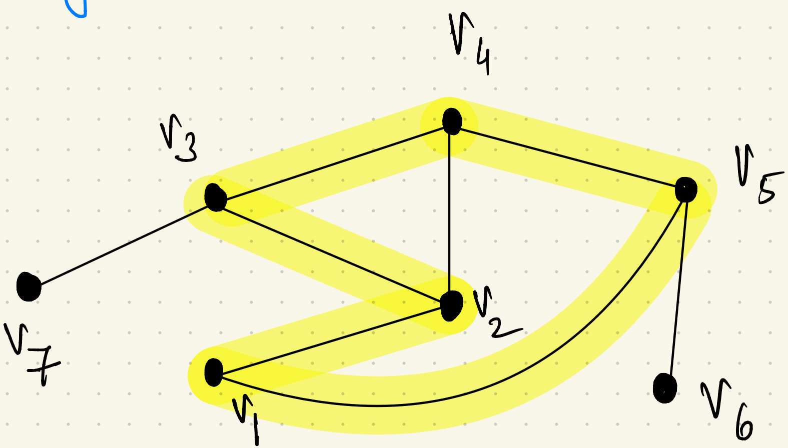
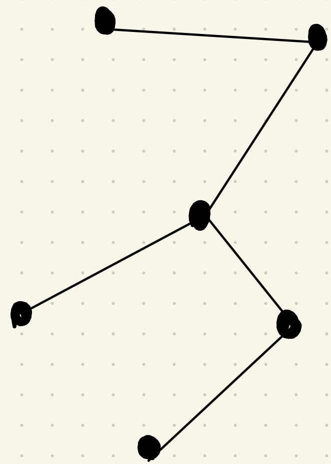


$v_1 - v_2 - v_3 - v_4 - v_5 - v_1$ is a cycle of length five

TREES

A tree is a connected and acyclic graph.



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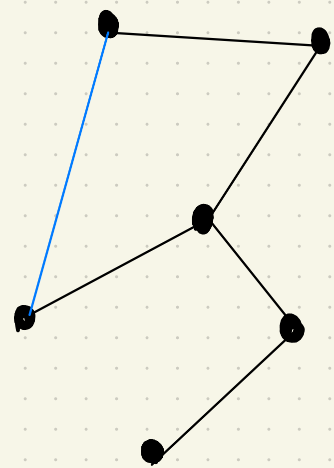

Not a tree

TREES

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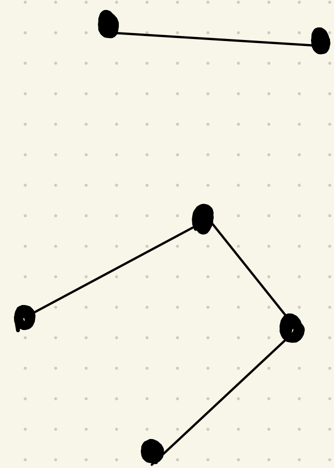

Not a tree

(It's a forest!)

SPARSE vs DENSE GRAPHS

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n : number of vertices

m : number of edges

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Many applications : m is $\Omega(n)$ and $O(n^2)$.
(but not always)

SPARSE vs DENSE GRAPHS

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Many applications : m is $\Omega(n)$ and $O(n^2)$.
(but not always)

roughly,

sparse : $m = O(n \log n)$ $\qquad$ close to linear

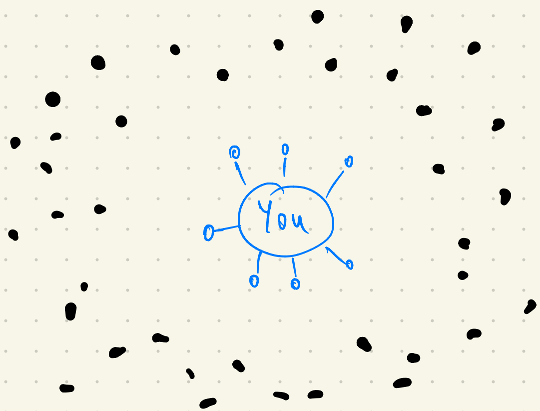
dense : $m = \Omega\left(\frac{n^2}{\log n}\right)$ $\qquad$ close to quadratic

SPARSE vs DENSE GRAPHS

n : number of vertices $\qquad$ m : number of edges

Many applications : m is $\Omega(n)$ and $O(n^2)$.
(but not always)

Facebook graph is sparse ...

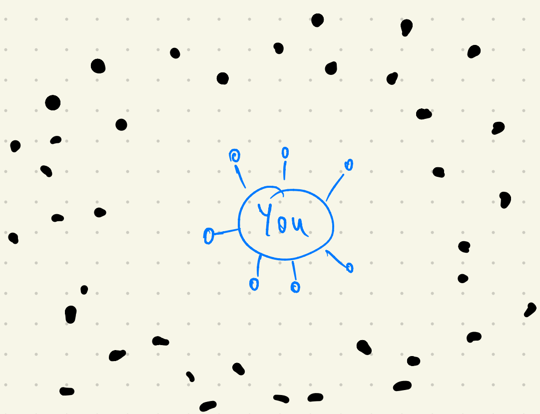


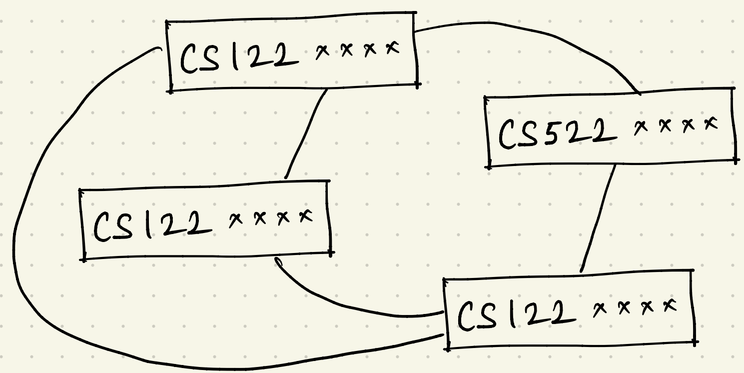
SPARSE vs DENSE GRAPHS

n : number of vertices $\qquad$ m : number of edges

Many applications : m is $\Omega(n)$ and $O(n^2)$.
(but not always)

Facebook graph is sparse ... $\qquad$ ... but also dense





REPRESENTING GRAPHS

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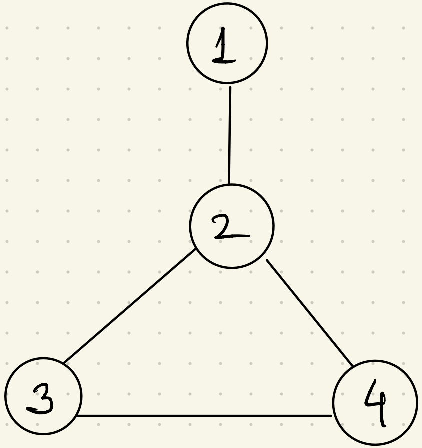
Adjacency matrix

REPRESENTING GRAPHS

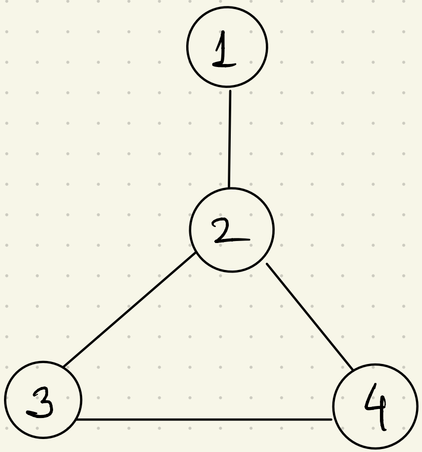
Adjacency matrix



REPRESENTING GRAPHS

Adjacency matrix

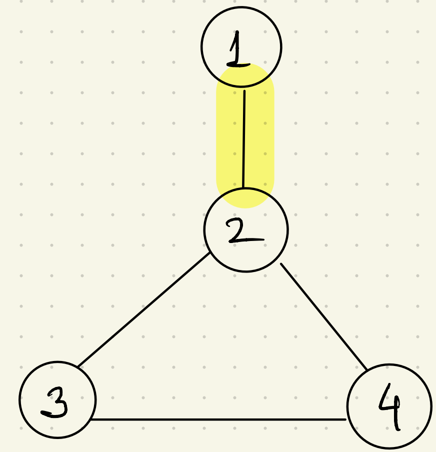
$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array}$$



REPRESENTING GRAPHS

Adjacency matrix

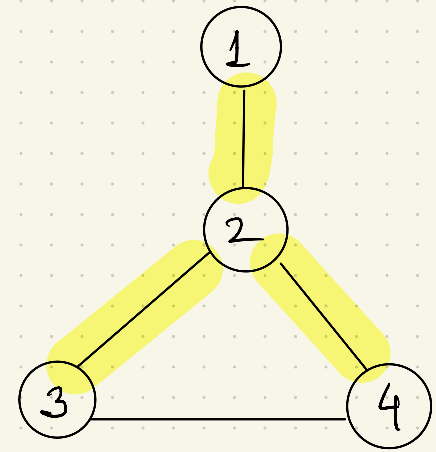
$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ & & & \\ & & & \\ & & & \end{array}\right] \end{array}$$



REPRESENTING GRAPHS

Adjacency matrix

$$A = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ & & & \\ & & & \end{bmatrix} \end{array}$$



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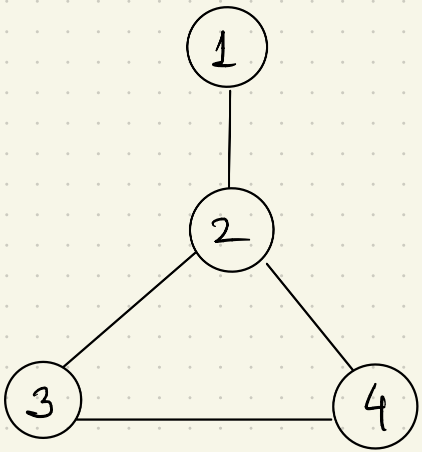

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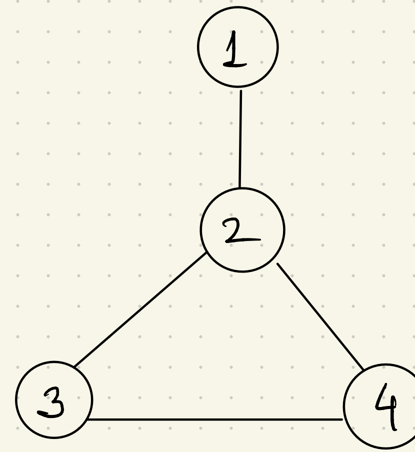

symmetric

REPRESENTING GRAPHS

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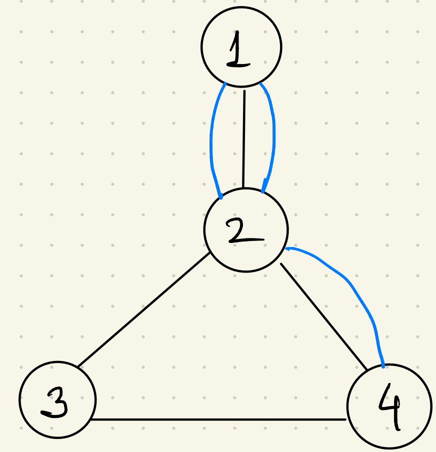
$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

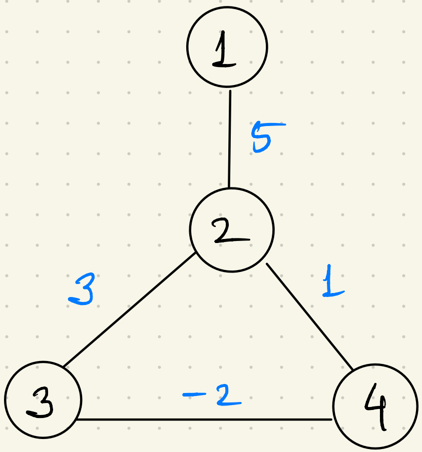


parallel edges

REPRESENTING GRAPHS

Adjacency matrix

$$A = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 5 & 0 & 0 \\ 5 & 0 & 3 & 1 \\ 0 & 3 & 0 & -2 \\ 0 & 1 & -2 & 0 \end{bmatrix} \end{array}$$



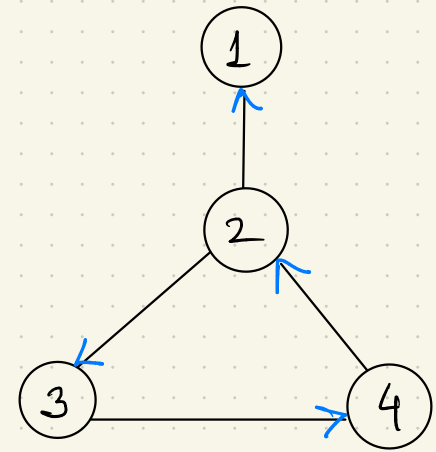
edge weights

REPRESENTING GRAPHS

Adjacency matrix

$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & -1 & 0 & 0 \\ 2 & +1 & 0 & +1 & -1 \\ 3 & 0 & -1 & 0 & +1 \\ 4 & 0 & +1 & -1 & 0 \end{array}$$

skew symmetric



Directed edges

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$\Theta(n^2)$ space

OK for dense graphs

Not OK for sparse graphs

REPRESENTING GRAPHS

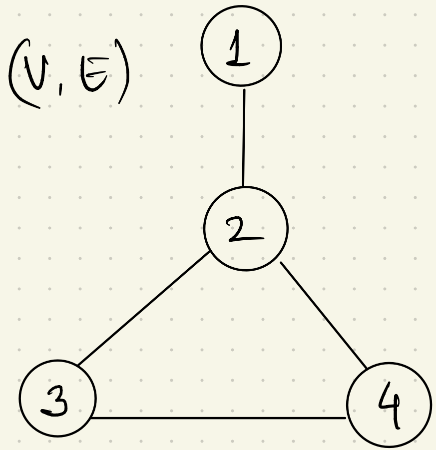
Adjacency list

REPRESENTING GRAPHS

Adjacency list

array of $|V|$ linked lists

$G = (V, E)$



REPRESENTING GRAPHS

Adjacency list

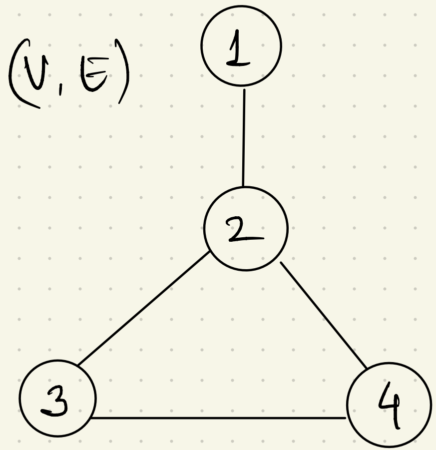
array of $|V|$ linked lists

for each vertex $u \in V$,

Adj[u] stores neighbors of u

$\{ v \in V : \{u, v\} \in E \}$.

$G = (V, E)$



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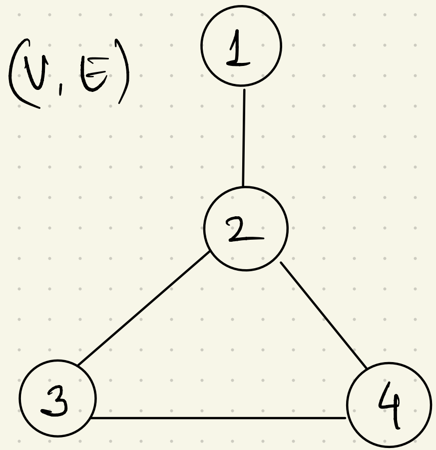
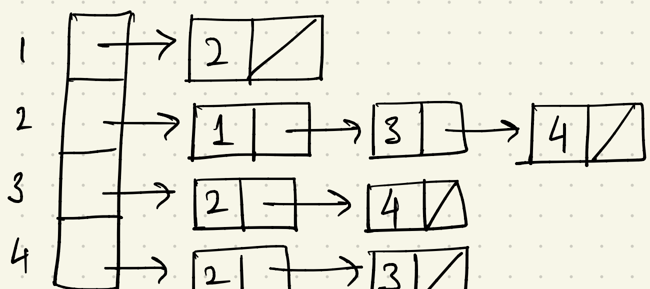


Adj



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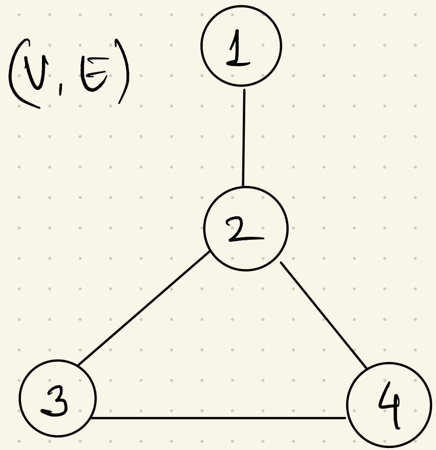

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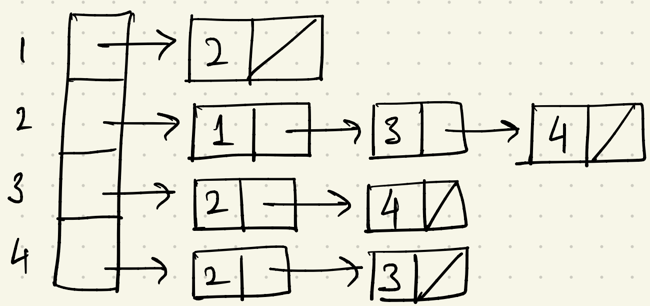

Space : ?

REPRESENTING GRAPHS

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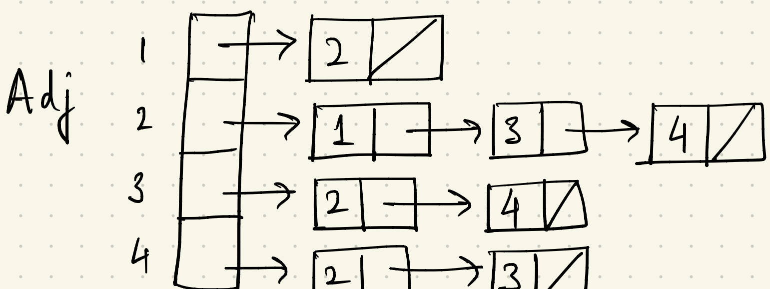
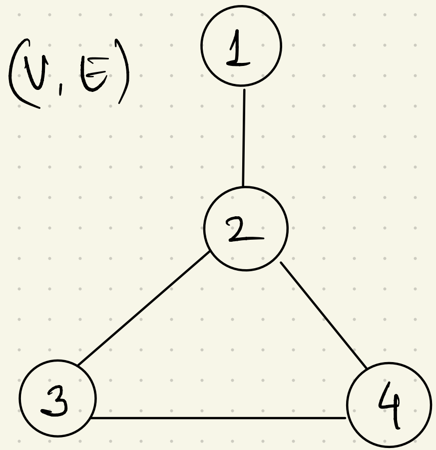


$G = (V, E)$



Space : $\Theta(|V| + |E|)$

or $\Theta(m+n)$

REPRESENTING GRAPHS

Which is better : adjacency matrix or adjacency list ?

REPRESENTING GRAPHS

Which is better : adjacency matrix or adjacency list ?

Depends on graph density and operations needed.

REPRESENTING GRAPHS

This course

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Depends on graph density and operations needed.

REPRESENTING GRAPHS

This course

Which is better : adjacency matrix or adjacency list?

Depends on graph density and operations needed.

Adjacency lists

- great for "follow your nose" operations
- better for massive sparse graphs (e.g., the web graph)

GRAPH SEARCH

GRAPH SEARCH

Checking connectivity

- Is there a way to reach Hauz Khas from Connaught Place?

GRAPH SEARCH

Checking connectivity

- Is there a way to reach Hauz Khas from Connaught Place?
- Is my Erdös number ≤ 3?

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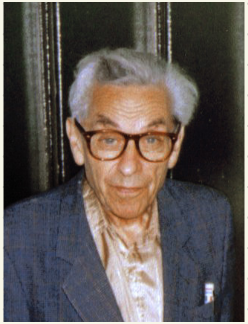

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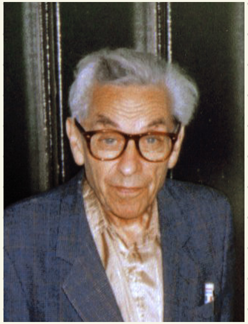

0

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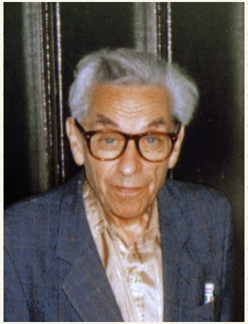

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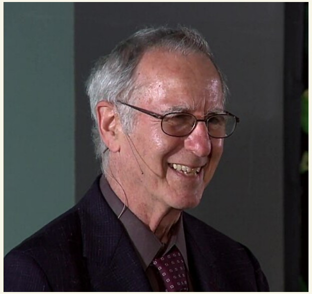

1

GRAPH SEARCH

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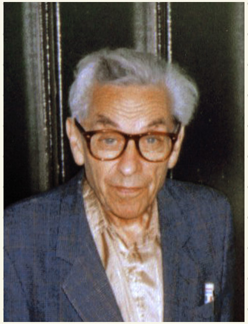

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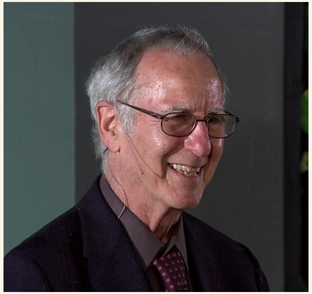

1

2

GRAPH SEARCH

Checking connectivity

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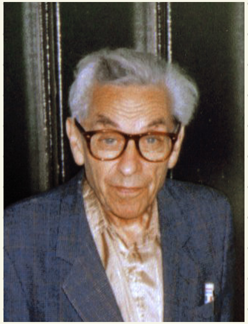
- Is my Erdös number ≤ 3 ?



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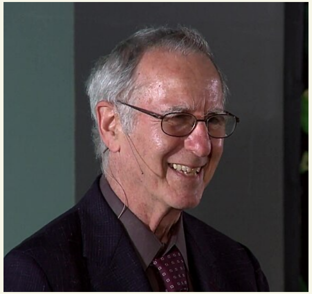

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2

3

GRAPH SEARCH

Checking connectivity

- Is there a way to reach Hauz Khas from Connaught Place?
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GRAPH SEARCH

Checking connectivity

- Is there a way to reach Hauz Khas from Connaught Place?
- Is my Erdös number ≤ 3?

Shortest paths

- cheapest combination of flight tickets

GRAPH SEARCH

Checking connectivity

- Is there a way to reach Hauz Khas from Connaught Place?
- Is my Erdös number ≤ 3?

Shortest paths

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Planning (abstract)

- e.g., Sudoku, Rubik's cube, locomotion, painting