第一次討論區(Confidence Interval & Maximum likelihood Estimate) **Due 10/14 星期六 23:59**

依照學號最後一碼 0～9 看自己可以發文的題目。**譬如，學號為 411412345 的同學，最後一碼是 5，他可以針對 第 5 題 或者 第 14 題 發文. 這位同學可以針對其中一或二題發文，也可以不發文而回應別人的發文、回應或回覆均可（發文有限制題目，回應/回覆則不限制題目）。**

最後一碼	0	1, 3	2	4	5	6	7	8	9
題目號碼	**1, 10**	**2, 11**	**3, 12**	**4, 13**	**5, 14**	**6, 15**	**7, 16**	**8, 17**	**9, 18**

注意事項：

1. 發文時請將**題號以及題目內容**寫出來，方便其他同學閱讀/討論。
2. 發文重點是將想法、瞭解、或者**困難疑問**寫出來以供大家討論，並不是繳交作業。因此書寫時務必將**原因寫出來**。這就是所謂的 justify, justify your answer, show your work, explain, etc.
3. ChatGPT: ChatGPT 的強大功能之一就是更人性化的搜尋引擎. 你可以使用,但是絕對不允許全文複製貼到討論區. 正確方法是瞭解內容後以**你的方式重新**表達與敘述. 如果老師覺得內容看起來像ChatGPT代筆的,那麼老師會和你舉行個人線上討論來確定你是否了解所發表的內容. 發生第一次則該次討論成績零分,第二次整學期的討論區成績零分.
4. 對於某一題目，別人已經發文而且很完整了，還可以做什麼？
   a. 如果自己的想法也如出一轍。 建議看看別人的回應，有沒有你可以發表意見的。
   b. 如果自己的想法不同，找不出紕漏，或者可能是錯的？ 建議同學將你的作法也發表詢問大家的看法。
5. 對於某一題目不會寫，你可以做什麼？
   a. 去找相對應的課程內容（課本、PPT、上課錄影、網路資源等）以得到靈感。
   b. 如果沒有靈感，可以發文你對於這題是如何查找資料，卻仍然不確定或者沒有幫忙，看其他同學是否可以提供線索。
   c. 你可以嘗試著將題目重新以中文解釋一次，並分析題目中有什麼資訊，而要求什麼。
   d. 你可以看別人的發文，提出不懂的問題。
6. 千萬不要自己做完發文/回應後就沒事了，一定要看其他題目的發文/回應，彼此切磋學習。

老師希望藉由這個討論平台，讓大家對於題目知道如何下手，而不是一個人單打獨鬥束手無策。這部份的分數是所有項目中最容易取得，希望同學要把握機會。根據過去經驗，同學往往拖到期限才開始發文或者回應，突然之間就暴增了許多訊息，老師無法及時消化並回饋。所以請同學幫幫忙，盡量提早開始。最後，老師花了很多時間準備這份討論區題目，但有時難免有失漏，請同學隨時提醒老師。

分數（滿分 100）：

每次發文/回覆/回應獎勵分數: 60 上限: 92

每次**文章**被點讚獎勵分數: 1 上限: 8

1. Suppose 250 randomly selected people are surveyed to determine if they wear a Fitbit. Of the 250 surveyed, 98 reported wearing a Fitbit. Using a 95% confidence level, compute a confidence interval estimate for the true proportion of people who wear Fitbits.

2. A student polls his school to see if students in the school district are for (贊成) or against (反對) the new legislation regarding school uniforms. She surveys 600 students and finds that 480 are against the new legislation. Compute a 90% confidence interval for the true percent of students who are against the new legislation.

3. We want to estimate the population mean height in a school. Assume the population standard deviation is 8 cm. We randomly select a sample of 60 students and measure their heights in centimeters. And the average height is 165 cm.
   (a) Find a 95% confidence interval estimate for the population mean height.
   (b) Leave everything the same except the sample size is increased from 60 to 100, what happens to the confidence interval?
   (c) Leave everything the same except the sample size is decreased from 60 to 36, what happens to the confidence interval?

4. A financial officer for a company wants to estimate the percent of accounts receivable that are more than 30 days overdue. He surveys 500 accounts and finds that 300 are more than 30 days overdue. Compute a 90% confidence interval for the true percent of accounts receivable that are more than 30 days overdue, and interpret the confidence interval.
5. Suppose we want to conduct a study on the success rate of students passing an exam. We randomly select a sample of 75 students and record whether they passed (success). In this sample, 50 students passed the exam.
   (a) Find a 90% confidence interval estimate for the population passing rate.
   (b) Find a 95% confidence interval estimate for the population passing rate.

6. Suppose we are interested in the mean scores on an exam. A random sample of 36 scores is taken and gives a sample mean (sample mean score) of 68. If we know that the population standard deviation is 3 points.
   (a) Find a 90% confidence interval estimate for the population mean exam score.
   (b) Leave everything the same except the sample size is increased from 36 to 64, what happens to the confidence interval?
   (c) Leave everything the same except the sample size is decreased from 36 to 25, what happens to the confidence interval?

7. Assume an observation of (x1, x2, …, x6) = (4,4,5,4,7,6) is drawn independently from a **Poisson distribution** with a parameter λ such that Poisson PMF is $f(x) = (\lambda^x/x!)\, e^{-\lambda}$ $\lambda > 0$, and x = 0, 1, 2, …
   (a) Write the likelihood function and the log likelihood of λ, i.e, write $L(\lambda)$, and $\log(L(\lambda))$.
   (b) find the MLE of λ, λ^.

8. The number of threes made during an NBA game is **Poisson** distributed. Last Saturday, the number of

threes made were 14, 26, 25, and 13.

(a) Write the likelihood function and the log likelihood of λ, i.e, write $L(\lambda)$, and $\log(L(\lambda))$.

(b) find the MLE of λ, λ^.

9. Flip a coin 8 times and get the sequence "H H T H T T H T". Let p denote the probability of getting heads.

(a) Write the likelihood function and log likelihood function of p.

(b) Find the MLE of p, p^.

(c) Find a 95% confidence interval.

10. To determine the percentage of mint-flavored candies in a large bag of candies, we continuously select the candies without replacement until the first mint-flavored one is found. Repeat the process four times and the first mint-flavored candy occurs in the 28^{th} , 18^{th} , 30^{th} , 46^{th} attempts, respectively. Estimate the percentage of mine-flavored candies p by MLE. (Note that the bag contains a large number of candies, so you can assume the percentage is a fixed number.)

(a) Write the likelihood function and log likelihood function of p.

(b) Find the MLE of p, p^.

11. A bag contains 3 balls. Each ball is either red (R) or blue (B). To estimate the number of blue balls, call it θ, $\theta \in \{0,1,2,3\}$, I choose 4 balls at random from the bag with replacement and a sequence of "B R B B" is yielded. Estimate the number of blue balls, θ, by MLE.

12. If the random samples are from a Binomial distribution with n = 3 and the unknown rate of success θ. If we have observed (x1, x2, x3, x4) = (1, 3, 2, 2), find the likelihood function and the MLE of θ, θ^.

13. If the random samples are from an Exponential distribution with unknown parameter θ. If we have observed (x1, x2, x3, x4) = (1.23, 3.32, 1.98, 2.12), find the likelihood function and the MLE of θ, θ^.
Hint. The pdf of exponential RV $f(x) = \theta e^{-\theta x}$ for $x \geq 0$, and $f(x) = 0$ otherwise.

14. Suppose our samples are X = (0, 0, 1, 1, 0) from Bernoulli (θ), $\theta \in (0, 1)$.

(a) Write the log maximum likelihood function of θ, $\log(L(\theta))$;

(b) find the MLE of θ, θ^.

(c) What would be the MLE of θ, θ^, if $\theta \in \{0.2, 0.5, 0.7\}$.

15. Define a PDF of X as $f(x) = (1-\theta) + 2\theta x$ for $0 < x < 1$, and $f(x) = 0$ otherwise, for $-1 < \theta < 1$.
If 5 independent observations X = (0.4. 0.4, 0.6, 0.4, 0.6) are from this distribution,

(a) Write the likelihood function and the log maximum likelihood function of θ.

(b) find the MLE of θ, θ^.

16. Suppose X is a RV with the PMF given as below with the parameter θ, $0 \le \theta \le 1$.

X	0	1	2	3
f(X)	$2\theta / 3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

Eight independent samples were taken from such a distribution: (3, 0, 2, 1, 3, 2, 3, 2)

(a) Write the likelihood function and the log maximum likelihood function of θ.

(b) find the MLE of θ, θ^.

17. We want to estimate the mean age of TKU students. From previous information, the standard deviation of the ages of the students is 15 years. We want to be 95% confident that the sample mean age is within two years of the population mean age. How many randomly selected students must be surveyed to achieve the desired level of accuracy?

18. Suppose a mobile phone company wants to determine the current percentage of customers aged 50+ who use text messaging on their cell phones. How many customers aged 50+ should the company survey in order to be 90% confident with a margin of error of 3%? (Hint. When there is no estimate of the population proportion, we use p^ = 0.5.)