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Introduction to Artificial Intelligence
Assignment 02

Exercise 2.1

(a) $g(n)$ denotes the path cost from the root of the search tree to node n. Since the Greedy Search always expands the node n^* where $h(n^*) = g(n^*)$ is minimal, the resulting algorithm can be described as a shortest path search (or lowest cost search) algorithm.

(b) Since $d(n)$ denotes the depth of a node n in the search tree and the Greedy Search always expands the node n^* where $h(n^*) = d(n^*)$ is minimal (so minimal depth), the resulting algorithm is a BFS.

(c) $\frac{1}{1+d(n)}$ is inversely proportional to the depth of node n in the search tree. Therefore, the Greedy Search always expands the node n^* with the largest depth. So we get an DFS algorithm.

Exercise 2.2

1)

An admissible heuristic is a function that never overestimates the cost of the minimum cost path from a node to the goal node. Let $h^*(n)$ be the optimal cost to reach the goal from n. If h is admissible, then $\forall n : h(n) \leq h^*(n)$.

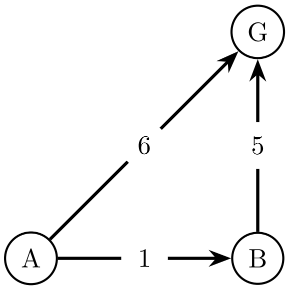
Let h be consistent.
Suppose there is not path from n to the goal state. Then $h^*(n) = \infty$, therefore $h(n) \leq h^*(n)$.
Now suppose that there is some path from n to te goal state g. Let $(n, x_1, \dots, x_m, g)$ be the shortest path from n to the goal state g with corresponding actions $(a_n, a_{x_1}, \dots, a_{x_m})$.

Induction:
Consider x_m. $h(x_m) \leq c(x_m, a_{x_m}, g) + h(g) \overset{h(g)=0}{=} c(x_m, a_{x_m}, g) = h^*(x_m)$.
Similarly x_{m-1}. $h(x_{m-1}) \leq c(x_{m-1}, a_{x_{m-1}}, x_m) + h(x_m) = c(x_{m-1}, a_{x_{m-1}}, x_m) + c(x_m, a_{x_m}, g) = h^*(x_{m-1})$.
By induction on the shortest past we get $h(n) \leq c(n, a_n, x_1) + c(x_m, a_{x_m}, g) = h^*(n)$, which is of course the shortest path cost.

Therefore when a heuristic is consistent, it is admissible.

2)

Let there be the graph with goal node G:



Node n	$h^*(n)$
A	6
B	5
G	0

An admissible heuristic, that is not consistent:

Node n	$h(n)$
A	6
B	1
G	0

Exercise 2.3

Exercise 2.4

(a) First we create a search graph

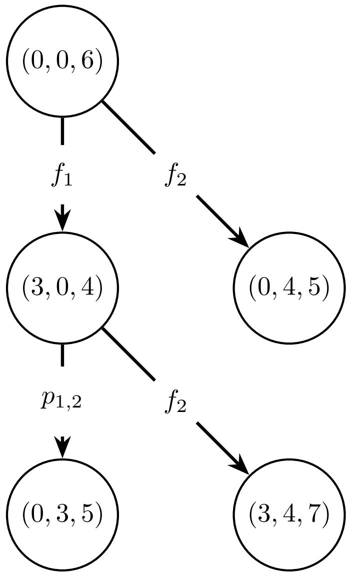
- States/Nodes V: $(j_1, j_2) \in \{0,1,2,3\} \times \{0,1,2,3,4\}$
  A state (j_i, j_2) reflects how much liter of water is in jug 1 and jug 2.
- Actions $A \coloneqq \{f_i, e_i, p_{i,j} | i, j \in \{1,2\} \wedge i \neq j\}$
  f_i denotes the action of filling jug i. e_i denotes the action of emptying jug i. $p_{i,j}$ denotes the action of pouring all the possible water from jug i into jug j.
- Edges $E \subseteq V \times V$: $(v_1, v_2, a) \in V \times V \times A$ Each edge (directed) (v_1, v_2, a) denotes a change of state v_1 into v_2 by action a.
  There should only be possible edges in our graph: (the restrictions prevent self loops)
  - $((j_1, j_2), (3, j_2), f_1)$, if $j_1 \neq 3$
  - $((j_1, j_2), (j_1, 4), f_2)$, if $j_2 \neq 4$
  - $((j_1, j_2), (0, j_2), e_1)$, if $j_1 \neq 0$
  - $((j_1, j_2), (j_1, 0), e_2)$, if $j_2 \neq 0$
  - $((j_1, j_2), (\max(0, j_1 - \min(3, 4 - j_2)), j_2 + (j_1 - \max(0, j_1 - \min(3, 4 - j_2))), p_{1,2})$, if $j_1 \neq 0 \wedge j_2 \neq 4$
  - $((j_1, j_2), (j_1 + (j_2 - \max(0, j_2 - (3 - j_1)))), \max(0, j_2 - (3 - j_1))), p_{2,1})$, if $j_2 \neq 0 \wedge j_1 \neq 3$

Now we can search from starting position $(0, 0)$ to goal position $(0, 2)$ in this graph.

(b)

j_1	j_2	$h(n)$	reason
$*$	2	0	given (final)
0	0	5	given
0	$0 < y \neq 2$	y	given
3	$y < 4 \neq 2$	3	given
3	4	5	given
$0 < x < 3$	$y < 4 \neq 2$	4	f_1
$0 < x < 3$	4	5	e_1

(c) Node Encoding: (j_1, j_2, f)



TODO NOT DONE YET