Exercise 1

We can define the *edit distance* $d_{\text{edit}}(w, w') : \Sigma^2 \to \mathbb{R}$ as follows. (Let $w = w_1 \dots w_n$ and $w' = w'_1 \dots w'_m$)

$$d_{\text{edit}}(w,w') \mapsto \begin{cases} |w| & \text{if } |w'| = 0 \\ |w'| & \text{if } |w| = 0 \\ d_{\text{edit}}(w_2 \dots w_n, w'_2 \dots w'_m) & \text{if } w_1 = w'_1 \\ 1 + \min \begin{cases} d_{\text{edit}}(w_2 \dots w_n, w') \\ d_{\text{edit}}(w, w'_2 \dots w'_m) \\ (w_2 \dots w_n, w'_2 \dots w'_m) \end{cases} & \text{otherwise} \end{cases}$$

As this definition of d_{edit} works by removing at most the first character of each word, we can proof by induction the length of $x, y, z \in \Sigma$, that d_{edit} is a metric on Σ:

- Let $|x| = |y| = |z| = 0$. Therefore, also $x = y = z$.
  Then $0 \leq d_{\text{edit}} = 0$. Thus, Nonnegativity is given.
  Since $x = y$, also $d_{\text{edit}}(x, y) = d_{\text{edit}}(y, x)$. Thus, Symmetry is given.
  Since $x = y = z$, the Triangle Inequality $d_{\text{edit}}(x, z) \leq d_{\text{edit}}(x, y) + (d_{\text{edit}})(y, z) \Leftrightarrow 0 \leq 0 + 0$ is given.

- Let $x = x_1 \dots x_n$, $y = y_1 \dots y_m$, and $z = z_1 \dots z_o$, $n, m, o \geq 1$. For $x' = x_2 \dots x_n$, $y' = y_2 \dots y_m$, and $z' = z_2 \dots z_o$ Nonnegativity, Symmetry, and the Triangle Inequality of d_{edit} is given.

- Since $n, m \geq 1$, the second rule of Nonnegativity, namely $d_{\text{edit}}(x, y) \Leftrightarrow x = y$ does not apply here.
  Since all $d_{\text{edit}}(x', y'), d_{\text{edit}}(x', y), d_{\text{edit}}(x, y')$ are non-negative, by definition of d_{edit}, $d_{\text{edit}}(x, y)$ must be non-negative as well. Therefore, the Nonnegativity of d_{edit} is proven.

- Symmetry

- Triangle Inequality