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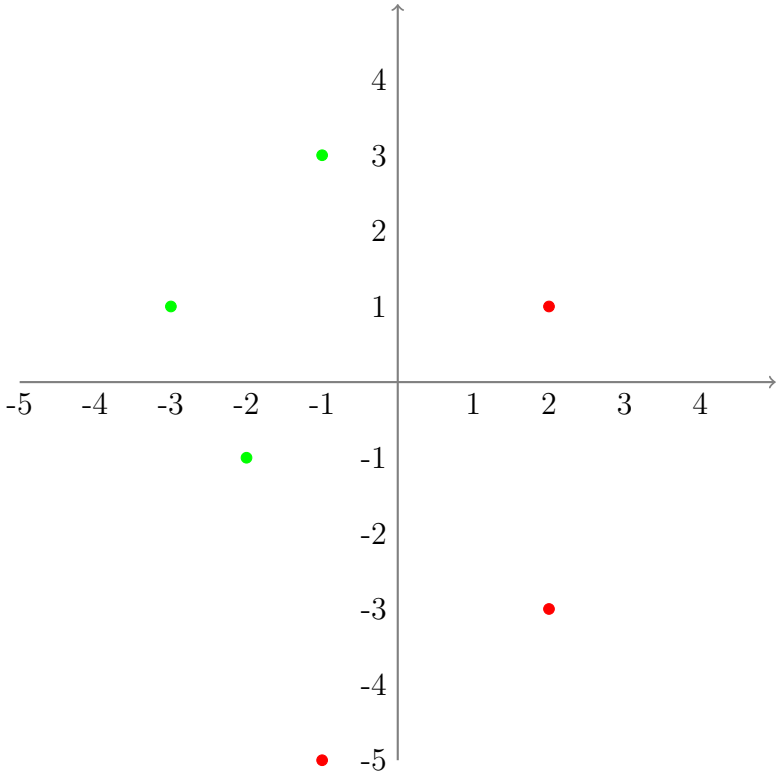
Exercise 1

(a)

(i)



(ii)

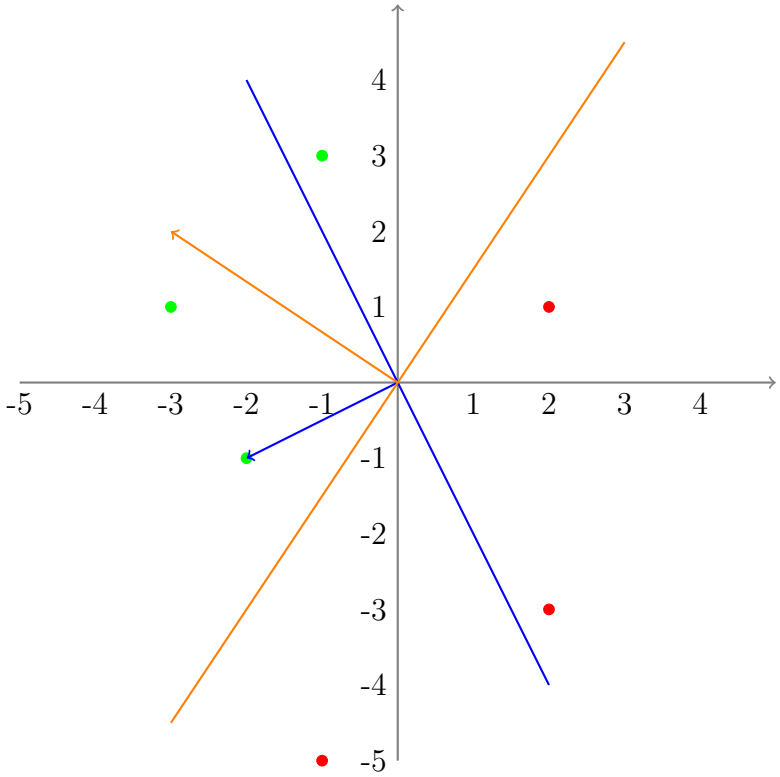
$$\text{sgn}(\langle 0, \begin{bmatrix}2\\1\end{bmatrix}\rangle) = \text{sgn}(0) = 1 \neq y_1$$

$$\Rightarrow\ w \leftarrow w + y_1 x_1 = 0 + (-1) \cdot \begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}-2\\-1\end{bmatrix}$$

$$\text{sgn}(\langle \begin{bmatrix}-2\\-1\end{bmatrix}, \begin{bmatrix}-1\\3\end{bmatrix}\rangle) = \text{sgn}(-1) = -1 \neq y_2$$

$$\Rightarrow\ w \leftarrow w + y_2 x_2 = \begin{bmatrix}-2\\-1\end{bmatrix} + 1 \cdot \begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}-3\\-2\end{bmatrix}$$

(iii)



Blue: weight vector (with hyperplane) after first update
Orange: weight vector (with hyperplane) after second update

(iv)

$$\min_{(x,y)\in S} \frac{|\langle w,x\rangle|}{||w||} = \min_{(x,y)\in S} \frac{|\langle w,x\rangle|}{\sqrt{(-3)^2+2^2}} = \min\{\frac{|-4|}{\sqrt{13}}, \frac{|9|}{\sqrt{13}}, \frac{|11|}{\sqrt{13}}, \frac{|4|}{\sqrt{13}}, \frac{|-7|}{\sqrt{13}}, \frac{|-12|}{\sqrt{13}}\} = \frac{4}{\sqrt{13}} = 1.109$$

(b)

Exercise 2

(a)

maj is realizable using weight vector $\hat{w} = \underbrace{(1,\ldots,1)}_{n-times}$

	Cluster(z^1)	Cluster(z^2)	Cluster(z^3)
1. Iteration	x_1	x_2	$x_3, x_4, x_5, x_6, x_7, x_8$
Updated cluster centres	$z^1 = (-3, 5)$	$z^2 = (-2, 4)$	$z^3 = (0.5, 1)$
2. Iteration	x_1	x_2, x_4	x_3, x_5, x_6, x_7, x_8
Updated cluster centres	$z^1 = (-3, 5)$	$z^2 = (-3, 2)$	$z^3 = (1.4, 1.2)$
3. Iteration	x_1, x_2	x_2, x_3	x_5, x_6, x_7, x_8
Updated cluster centres	$z^1 = (-2.5, 4.5)$	$z^2 = (-2.5, 1)$	$z^3 = (2, 1)$

(b)

(c)

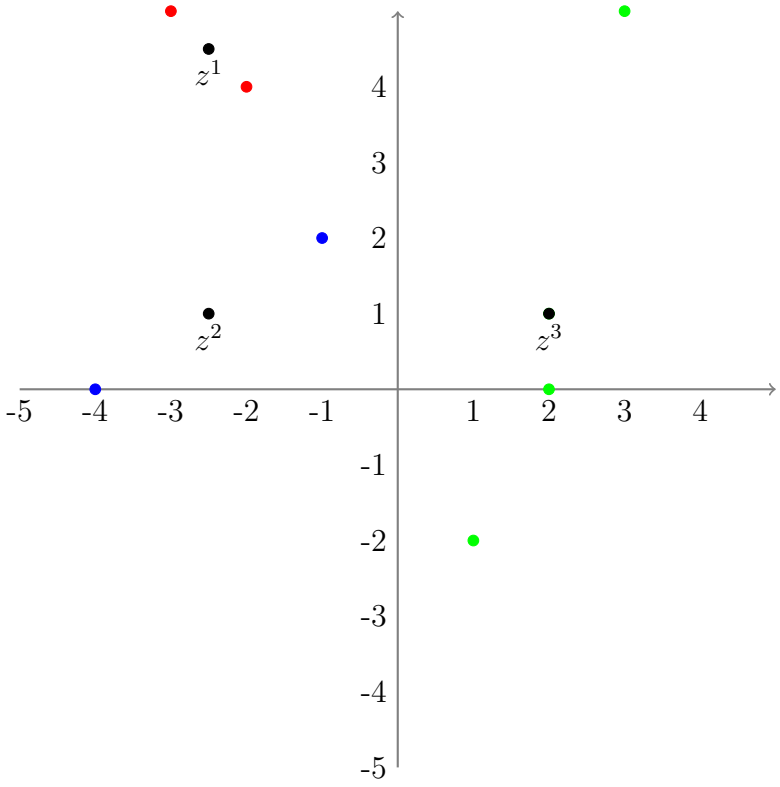
(d)

Exercise 3

Exercise 4

(a)

(b)



(c)

(d)

$z^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $z^2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $z^3 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$,

(e)

Yes, for example $z^1 = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, $z^2 = \begin{bmatrix} -1.5 \\ 3 \end{bmatrix}$, $z^3 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$,

Exercise 5