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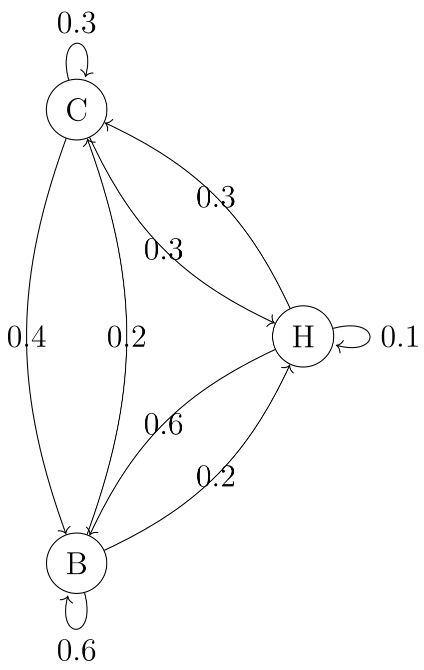
Exercise 1

(a)



Transition matrix with row order (top to bottom) and column order (left to right) $H - B - C$:

$$Q \coloneqq \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

(b)

$$\begin{aligned} \sum_{a \in H,C,B} q_{Ha} \cdot q_{aB} &= q_{HH} \cdot q_{HB} + q_{HB} \cdot q_{BB} + q_{HC} \cdot q_{CB} \\ &= 0.1 \cdot 0.6 + 0.6 \cdot 0.6 + 0.3 \cdot 0.4 \\ &= 0.06 + 0.36 + 0.12 \\ &= 0.54 \end{aligned}$$

(c)

We have the following equations (also since π is a left Eigenvector of Q with Eigenvalue 1):

$$\pi_1 + \pi_2 + \pi_3 = 1 \tag{1}$$

$$\pi \cdot Q = \pi \tag{2}$$

Thus, we have the following equation system:

$$\begin{aligned} \pi_1 + \pi_2 + \pi_3 &= 1 \\ 0.1 \cdot \pi_1 + 0.2 \cdot \pi_2 + 0.3 \cdot \pi_3 &= \pi_1 \\ 0.6 \cdot \pi_1 + 0.6 \cdot \pi_2 + 0.4 \cdot \pi_3 &= \pi_2 \\ 0.3 \cdot \pi_1 + 0.2 \cdot \pi_2 + 0.3 \cdot \pi_3 &= \pi_3 \end{aligned}$$

Solving this, we get:

$$\pi \simeq (0.204082, 0.55102, 0.244898)$$

Exercise 2

(a)

$$\begin{aligned}
&V(\mathcal{H}) \coloneqq \mathcal{VC}(G) \\
&E(\mathcal{H}) \coloneqq \{\{i, j\} \mid i, j \in V(\mathcal{H}) \wedge \exists_{v \in V(G)}(v \notin i \wedge j = i \cup \{v\})\}
\end{aligned}$$

$\mathcal{H}$ is connected, since, if U is a vertex cover, adding any additional vertex v to U ($U' \coloneqq U \cup \{v\}$), results by definition also in a vertex cover.

The maximum degree Δ of $\mathcal{H}$ is $|V(G)|$, since this is also a valid vertex cover for G, thus $V(G)$ is also in $V(\mathcal{H})$.

(b)

$$q_{U,W} \coloneqq \begin{cases} \frac{1}{|V(G)|} & \text{if } \{U, W\} \in E(\mathcal{H}) \\ 1 - \frac{|N(U)|}{|V(G)|} & \text{if } U = W \\ 0 & \text{otherwise} \end{cases}$$

Exercise 3

(a)

$$q_{ij}^{(n,r)} \coloneqq \begin{cases} \frac{1-r}{2} + \frac{r}{n} & \text{if } i \in \{1, \ldots, n-1\} \wedge (j = i+1 \vee j = n) \\ \frac{1}{n} & \text{if } i = n \\ \frac{r}{n} & \text{otherwise} \end{cases}$$

(b)

The Web graph is isometric "under rotation" around node n. Thus, the weight vectors for each $i \in \{1, \ldots, n-1\}$ must be equal. Let w_a denote the weight of each of these pages and w_n denote the weight of page n.

Similarly to Exercise 01.c), we get the following equation system:

$$\begin{aligned}
(n-1) \cdot w_a + w_n &= 1 \\
(w_a + w_n) \cdot (\frac{1-r}{2} + \frac{r}{n}) + (n-2) \cdot w_a \cdot (\frac{r}{n}) &= w_a \\
(n-1) \cdot w_a &= w_n
\end{aligned}$$

Thus:

$$\begin{aligned} w_n &= \frac{1}{2} \\ w_a &= \frac{1}{2(n-1)} \\ w_a &= \frac{r-1}{2(r+1)} \\ n &= \frac{2r}{r-1} \end{aligned}$$

Exercise 4

(a)

`MAP` function:

- On input (R, t), emit $(t, 1)$.
- On input (S, t), emit $(t, 1)$.

`REDUCE` function:

- On input $(t, values)$, emit (Q, t) $\sum_{v \in values} v$ times.

(b)

`MAP` function:

- On input (R, t), emit $(t, 1)$.

`REDUCE` function:

- On input $(t, values)$, emit (Q, t) $\sum_{v \in values} v$ times, if t satisfies C.

Exercise 5

(a)

`MAP` function:

- On input $(o, (c, p, q, d))$, emit (p, q).

`REDUCE` function:

- On input $(p, values)$, emit $(p, \sum_{q \in values} q)$.

(b)

`MAP` function:

- On input $(o, (c, p, q, d))$, emit $(c, (p, q))$.

`REDUCE` function:

- On input $(c, values)$, emit $(c, (p, q))$ for all $(p, q) \in values$, only if $\forall_{(p,q) \in values} q < 20\$$.

(c)

The algorithm is terrible, since, after Mapping all the data, we only have one $(key, values)$-pair left for the Reduction. Thus, we cannot distribute the computation of the `REDUCE` function.