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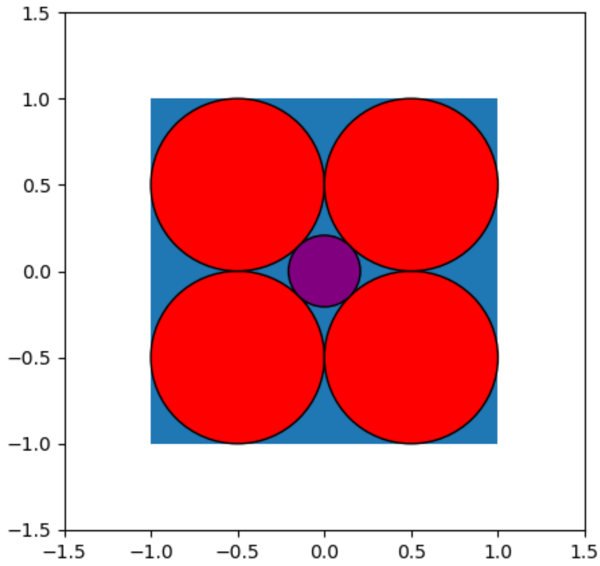
Exercise 1

(a)

Situation for l=2 and s=2:

$$Q_{2,2} = \{[x_1 x_2]^T \in \mathbb{R}^2 \mid |x_i| \leq 1 \text{ for all } i = 1,2\} = [-1,1]^2$$



(b)

First, let's calculate the radius of the inner hyperball for any $l \in \mathbb{N}, s \in \mathbb{R}_{>0}$:
The distance from the center of the inner hyperball (equal to the center of the hypercube) to the center of one of the 2^l outter hyperballs (doesn't matter which one) can be calculated the following:

$$d := \sqrt{l \cdot \left(\frac{s}{4}\right)^2}$$

Thus, the radius of the inner hyperball is equal to:

$$r := d - \frac{s}{4} = \sqrt{l \cdot \left(\frac{s}{4}\right)^2} - \frac{s}{4}$$

Now, we simply must solve the following inequality to find an $l \in \mathbb{N}$ for an arbitrary but fixed $s \in \mathbb{R}_{>0}$ such that $B(Q_{l,s}) \not\subseteq Q_{l,s}$:

$$\begin{aligned}
\frac{s}{2} &< r \\
\frac{s}{2} &< d - \frac{s}{4} \\
\frac{s}{2} &< \sqrt{l \cdot \left(\frac{s}{4}\right)^2} - \frac{s}{4} \\
\frac{3 \cdot s}{4} &< \sqrt{l} \cdot \frac{s}{4} \\
3 &< \sqrt{l} \\
l &> 9
\end{aligned}$$

Exercise 2

(a)

Let's first find all eigenvalues of A_c:

$$\begin{aligned}
&\{\lambda \in \mathbb{R} \mid \det(A_c - \lambda \cdot I_3) = 0\}\\
&\{\lambda \in \mathbb{R} \mid \det \begin{bmatrix} 2-\lambda & 0 & c \\ 0 & 1-\lambda & 0 \\ c & 0 & 1-\lambda \end{bmatrix} = 0\}\\
&\{\lambda \in \mathbb{R} \mid -\lambda^3 + 4 \cdot \lambda^2 + c^2 \cdot \lambda - 5 \cdot \lambda + 2 - c^2\}\\
&\{\lambda \in \mathbb{R} \mid (\lambda - 1) \cdot (-\lambda^2 + 3\dot{\lambda} + c^2 - 2)\}\\
\{1, \frac{3-\sqrt{4 \cdot c^2+1}}{2}, \frac{3+\sqrt{4 \cdot c^2+1}}{2}\} =: \Lambda_{A_c}&
\end{aligned}$$

So, if for all $\lambda \in \Lambda_{A_c}$ it must hold that $\lambda \geq 0$, we must set c such that the following inequality holds:

$$\begin{aligned}
\frac{3-\sqrt{4 \cdot c^2+1}}{2} &\geq 0\\
3-\sqrt{4 \cdot c^2+1} &\geq 0\\
3 &\geq \sqrt{4 \cdot c^2+1}\\
9 &\geq 4 \cdot c^2 + 1\\
8 &\geq 4 \cdot c^2\\
2 &\geq c^2\\
c &\in (-\sqrt{2}, \sqrt{2}) \subseteq \mathbb{R}
\end{aligned}$$

(b)

Exercise 3

(a)

(b)

(c)

(d)

Exercise 4

(a)

(b)

(c)

(d)

(e)

Exercise 5