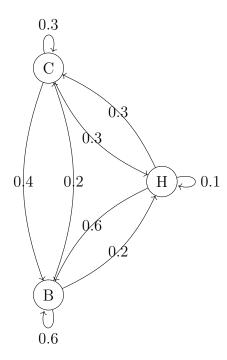
Exercise 1

(a)



Transition matrix with row order (top to bottom) and column order (left to right) H - B - C:

$$Q := \left[\begin{array}{ccc} 0.1 & 0.6 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{array} \right]$$

(b)

$$\sum_{a \in H,C,B} q_{Ha} \cdot q_{aB} = q_{HH} \cdot q_{HB} + q_{HB} \cdot q_{BB} + q_{HC} \cdot q_{CB}$$

$$= 0.1 \cdot 0.6 + 0.6 \cdot 0.6 + 0.3 \cdot 0.4$$

$$= 0.06 + 0.36 + 0.12$$

$$= 0.54$$

(c)

We have the following equations (also since π is a left Eigenvector of Q with Eigenvalue 1):

$$\pi_1 + \pi_2 + \pi_3 = 1 \tag{1}$$

$$\pi \cdot Q = \pi \tag{2}$$

Thus, we have the following equation system:

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$$0.1 \cdot \pi_1 + 0.2 \cdot \pi_2 + 0.3 \cdot \pi_3 = \pi_1$$

$$0.6 \cdot \pi_1 + 0.6 \cdot \pi_2 + 0.4 \cdot \pi_3 = \pi_2$$

$$0.3 \cdot \pi_1 + 0.2 \cdot \pi_2 + 0.3 \cdot \pi_3 = \pi_3$$

Solving this, we get:

$$\pi \simeq (0.204082, 0.55102, 0.244898)$$

Exercise 2

- (a)
- (b)

Exercise 3

SOLUTION IS WRONG

(a)

$$q_{ij}^{(n,r)} \coloneqq \begin{cases} \frac{1-r}{2} + \frac{r}{n} & \text{if } i \in \{1,\dots,n-1\} \land (j=i+1 \lor j=n) \\ \frac{1}{n} & \text{if } i=n \\ \frac{r}{n} & \text{otherwise} \end{cases}$$

(b)

The Web graph is isometric "under rotation" around node n. Thus, the weight vectors for each $i \in \{1, ..., n-1\}$ must be equal. Let w_a denote the weight of each of these pages and w_n denote the weight of page n.

Similarly to Exercise 01.c), we get the following equation system:

$$(n-1) \cdot w_a + w_n = 1$$

$$(w_a + w_n) \cdot (\frac{1-r}{2} + \frac{r}{n}) + (n-2) \cdot w_a \cdot (\frac{r}{n}) = w_a$$

$$(n-1) \cdot w_a = w_n$$

Thus:

$$w_n = \frac{1}{2}$$

$$w_a = \frac{1}{2(n-1)}$$

$$w_a = \frac{r-1}{2(r+1)}$$

$$n = \frac{2r}{r-1}$$

Exercise 4

(a)

MAP function:

• On input (R, t), emit (t, 1).

• On input (S, t), emit (t, 1).

REDUCE function:

• On input (t, values), emit $(Q, t) \sum_{v \in values} v$ times.

(b)

MAP function:

- On input (R, t), emit (t, 1).
- On input (S, t), emit (t, 1).

REDUCE function:

• On input (t, values), emit $(Q, t) \sum_{v \in values} v$ times, if t satisfies C.

Exercise 5

(a)

MAP function:

• On input (o, (c, p, q, d)), emit (p, q).

REDUCE function:

• On input (p, values), emit $(p, \sum_{q \in values} q)$.

(b)

MAP function:

• On input (o, (c, p, q, d)), emit (c, (p, q)).

REDUCE function:

• On input (c, values), emit (c, (p, q)) for all $(p, q) \in values$, only if $\forall_{(p,q) \in values} q < 20$ \$.

(c)

The algorithm is terrible, since, after Mapping all the data, we only have one (*key*, *values*)-pair left for the Reduction. Thus, we cannot distribute the computation of the REDUCE function.