

a) Sobel in x-direction:

$$\overset{\text{smoothing}}{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}} \otimes \overset{\text{differentiation}}{\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

↓ filter

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Prewitt in x-direction:

$$\overset{\text{smoothing}}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \otimes \overset{\text{differentiation}}{\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

↳ same computation

b) I_m $\xrightarrow{\text{x-der}}$ $\begin{bmatrix} 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 & -2 & -1 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & -1 & 0 \end{bmatrix}$

padding \rightarrow $\begin{bmatrix} 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 & -2 & -1 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & -1 & 0 \end{bmatrix}$

$-1+1$

c) I_m $\xrightarrow{\text{x-der}}$ $\begin{bmatrix} 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 & -2 & -1 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & -1 & 0 \end{bmatrix}$

padding \rightarrow $\begin{bmatrix} 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 2 & 2 & -1 & -3 & -1 \\ 3 & 2 & 0 & 1 & -3 & -3 \\ 3 & 3 & -1 & 0 & -2 & -3 \\ 1 & 3 & 1 & -2 & -2 & -1 \\ 0 & -1 & -3 & -3 & -1 & 0 \end{bmatrix}$

$-1+2$

d) Prewitt only - Sobel analogous

$I_{m-x} = \begin{bmatrix} 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 & -2 & -1 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & -1 & 0 \end{bmatrix}$

$I_{m-y} = \begin{bmatrix} 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 & -2 & -1 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 2 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & -1 & 0 \end{bmatrix}$

$I_{m-x^2} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 4 & 1 \\ 4 & 4 & 0 & 0 & 4 & 4 \\ 4 & 4 & 0 & 0 & 4 & 4 \\ 1 & 4 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

$I_{m-y^2} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 4 & 1 \\ 4 & 4 & 0 & 0 & 4 & 4 \\ 4 & 4 & 0 & 0 & 4 & 4 \\ 1 & 4 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

$\downarrow +$

$\begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 & 8 & 2 \\ 8 & 8 & 0 & 0 & 8 & 8 \\ 8 & 8 & 0 & 0 & 8 & 8 \\ 2 & 8 & 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 & 2 & 0 \end{bmatrix}$

$\downarrow \cdot$

$\begin{bmatrix} 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & 0 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & 0 & 2\sqrt{2} & \sqrt{2} \\ 2\sqrt{2} & 2\sqrt{2} & 0 & 0 & 2\sqrt{2} & 2\sqrt{2} \\ 2\sqrt{2} & 2\sqrt{2} & 0 & 0 & 2\sqrt{2} & 2\sqrt{2} \\ \sqrt{2} & 2\sqrt{2} & 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & 0 \end{bmatrix}$