

# Component Bound Branching in a Branch-and-Price Framework

Master Thesis in Computer Science

Til Mohr

til.mohr@rwth-aachen.de

Student ID: 405959

April 17, 2024

1<sup>st</sup> Examiner

TBD

Chair of Computer Science

2<sup>nd</sup> Examiner

Prof. Dr. Marco Lübbecke

Chair of Operations Research

## Eidesstattliche Versicherung

\_\_\_\_\_  
Name, Vorname

\_\_\_\_\_  
Matrikelnummer (freiwillige Angabe)

Ich versichere hiermit an Eides Statt, dass ich die vorliegende Arbeit/Bachelorarbeit/  
Masterarbeit\* mit dem Titel

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

selbstständig und ohne unzulässige fremde Hilfe erbracht habe. Ich habe keine anderen als die angegebenen Quellen und Hilfsmittel benutzt. Für den Fall, dass die Arbeit zusätzlich auf einem Datenträger eingereicht wird, erkläre ich, dass die schriftliche und die elektronische Form vollständig übereinstimmen. Die Arbeit hat in gleicher oder ähnlicher Form noch keiner Prüfungsbehörde vorgelegen.

\_\_\_\_\_  
Ort, Datum

\_\_\_\_\_  
Unterschrift

\*Nichtzutreffendes bitte streichen

### Belehrung:

#### § 156 StGB: Falsche Versicherung an Eides Statt

Wer vor einer zur Abnahme einer Versicherung an Eides Statt zuständigen Behörde eine solche Versicherung falsch abgibt oder unter Berufung auf eine solche Versicherung falsch aussagt, wird mit Freiheitsstrafe bis zu drei Jahren oder mit Geldstrafe bestraft.

#### § 161 StGB: Fahrlässiger Falscheid; fahrlässige falsche Versicherung an Eides Statt

(1) Wenn eine der in den §§ 154 bis 156 bezeichneten Handlungen aus Fahrlässigkeit begangen worden ist, so tritt Freiheitsstrafe bis zu einem Jahr oder Geldstrafe ein.

(2) Straflosigkeit tritt ein, wenn der Täter die falsche Angabe rechtzeitig berichtigt. Die Vorschriften des § 158 Abs. 2 und 3 gelten entsprechend.

Die vorstehende Belehrung habe ich zur Kenntnis genommen:

\_\_\_\_\_  
Ort, Datum

\_\_\_\_\_  
Unterschrift

## Abstract

This master thesis focuses on integrating the component bound branching rule, as proposed by Lübbecke et al. [citation needed], into the branch-price-and-cut solver GCG. This rule, similarly to Vanderbeck’s generic branching scheme [1], exclusively branches in the Dantzig-Wolfe reformulated problem, where no branching decision corresponds to a branching decision in the original formulation. Current limitations within GCG’s framework hinder the direct implementation of such branching rules, necessitating modifications, particularly within the pricing loop, as observed in the implementation of Vanderbeck’s method. Moreover, these implementations do not capitalize on enhancements like dual value stabilization.

A major contribution of this thesis is the enhancement of the GCG architecture to allow seamless integration of such new branching rules that solely operate on the reformulated problem. This advancement would also enable these rules to leverage current and future optimizations within the branch-price-and-cut framework, including dual value stabilization, without necessitating alterations to the branching rule itself.

To achieve this, the thesis proposes a novel interface designed to handle constraints in the master problem that lack direct counterparts in the original formulation. These constraints necessitate specific modifications to the individual pricing problems to validate them within the master problem. The newly introduced ‘generic mastercut’ interface offers a systematic approach for creating and managing these constraints. It is intricately integrated into the GCG solver, encompassing the pricing loop, column generation, and dual value stabilization. This interface is further complemented by enhancements to the existing branching rule interface in GCG, facilitating the seamless integration and effective utilization of these generic mastercuts.

Finally, I will be implementing the component bound branching rule using this new generic mastercut interface, followed by a comprehensive evaluation of its efficacy on a set of benchmark instances. The performance will be compared to the existing Vanderbeck branching rule, providing a practical analysis of both approaches.



# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
<b>2</b>	<b>Column Generation and Branch-and-Price</b>	<b>7</b>
2.1	Column Generation . . . . .	7
2.2	Dantzig-Wolfe Reformulation . . . . .	7
2.3	Dantzig-Wolfe Reformulation for Mixed Integer Programs . . . . .	7
2.4	Branch-and-Price . . . . .	7
2.5	Branch-Price-and-Cut . . . . .	7
2.6	Dual Value Stabilization . . . . .	7
<b>3</b>	<b>SCIP Optimization Suite</b>	<b>9</b>
3.1	SCIP . . . . .	9
3.2	GCG . . . . .	9
<b>4</b>	<b>Component Bound Branching</b>	<b>11</b>
4.1	Overview of the branching scheme . . . . .	11
4.2	Separation Procedure . . . . .	11
4.3	Parameterizations . . . . .	11
<b>5</b>	<b>Master Constraints without corresponding Original Problem Constraints</b>	<b>13</b>
5.1	Definition of the Generic Mastercuts . . . . .	13
5.2	Application of the Generic Mastercuts . . . . .	13
5.3	Dual Value Stabilization for Generic Mastercuts . . . . .	13
5.4	Mastervariable Synchronization across the entire B&B-Tree . . . . .	13
5.4.1	Problem Statement . . . . .	13
5.4.2	Current Approach used by the Implementation of Vanderbeck's Generic Branching . . . . .	13
5.4.3	History Tracking Approach . . . . .	13
5.4.4	History Tracking using Unrolled Linked Lists Approach . . . . .	13

<b>6</b>	<b>Implementation</b>	<b>15</b>
6.1	Generic Mastercuts . . . . .	15
6.2	Mastervariable Synchronization . . . . .	15
6.3	Component Bound Branching . . . . .	15
<b>7</b>	<b>Evaluation</b>	<b>17</b>
<b>8</b>	<b>Conclusion</b>	<b>19</b>

# Chapter 1

## Introduction





# Chapter 2

## Column Generation and Branch-and-Price

### 2.1 Column Generation

### 2.2 Dantzig-Wolfe Reformulation

### 2.3 Dantzig-Wolfe Reformulation for Mixed Integer Programs

### 2.4 Branch-and-Price

### 2.5 Branch-Price-and-Cut

### 2.6 Dual Value Stabilization



# Chapter 3

## SCIP Optimization Suite

### 3.1 SCIP

### 3.2 GCG



# Chapter 4

## Component Bound Branching

### 4.1 Overview of the branching scheme

### 4.2 Separation Procedure

### 4.3 Parameterizations

*Which block to separate in? Which components to branch on?*



# Chapter 5

## Master Constraints without corresponding Original Problem Constraints

- 5.1 Definition of the Generic Mastercuts
- 5.2 Application of the Generic Mastercuts
- 5.3 Dual Value Stabilization for Generic Mastercuts
- 5.4 Mastervariable Synchronization across the entire B&B-Tree
  - 5.4.1 Problem Statement
  - 5.4.2 Current Approach used by the Implementation of Vanderbeck's Generic Branching
  - 5.4.3 History Tracking Approach
  - 5.4.4 History Tracking using Unrolled Linked Lists Approach





# Chapter 6

## Implementation

### 6.1 Generic Mastercuts

### 6.2 Mastervariable Synchronization

### 6.3 Component Bound Branching



# Chapter 7

## Evaluation



# Chapter 8

## Conclusion



# Bibliography

- [1] François Vanderbeck. “Branching in branch-and-price: a generic scheme”. In: *Mathematical Programming* 130 (2011), pp. 249–294.