Scotty Felch
21 January 2015
CSCI 305
HW #1

1. a. How many possible outcomes are there for rolling the three (6-sided, 10-sided, 20-sided) dice?
   6 * 10 * 20 = 1200 possible outcomes
   b. How many ways can the dice total 21 given a 1 rolled on the 6-sided die?

6 side	10 side	20 side
1	10	10
	9	11
	8	12
	7	13
	6	14
	5	15
	4	16
	3	17
	2	18
	1	19

There are 10 possible ways.

c. How many different ways to win with a 6 on the 6-sided die?

6 side	10 side	20 side
6	10	5
	9	6
	8	7
	7	8
	6	9
	5	10
	4	11
	3	12
	2	13
	1	14

d. What is the probability of getting 21 when rolling dice?

Probability = # ways to add to 21 / total # ways to roll = (6 * 10) / (6 * 10 * 20) = 1/20

e. What is the expected value of the winnings for a person playing the game once?

E(X) = x*P(X)

X	Win	Lose
X	+$20	-$1
E(x)=x * P(X)	20 * 0.05 = 1	-1 * 0.95 = -0.95

E(X) = (0.95)($-1) + (0.05)($20) = $0.05

f. What other totals of the three dice would result in the same expected winnings as rolling 21?
Minimum to maximum values you can achieve by setting the six-die and ten-die to 1 and 1, or 6 and 10.

(6 + 10 + 1) to (1 + 1 + 20) => 17 to 22 min and max

2. a. What is the probability that the top three cards that you turned over appeared in increasing order?

1st card	2nd card	3rd card
1	9/9	8/8
	8/9	7/8
	7/9	6/8
	...	...
	2/9	1/8
2	8/9	7/8
	7/9	6/8
	...	...
	3/9	2/8
	2/9	1/8
3	7/9	6/8
	6/9	5/8
	...	...
	2/9	1/8
4	6/9	5/8
	5/9	4/8
	...	...

	2/9	1/8
	...	...

Using this pattern and a summation formula I can tell how many possible ways there are to win.

For the 1st card being 1, I get (8(8+1))/2 = 36 possibilities.
For the 1st card being 2, I get (7(7+1))/2 = 28 possibilities.
For the 1st card being 2, I get (6(6+1))/2 = 21 possibilities.
For the 1st card being 2, I get (5(5+1))/2 = 15 possibilities.
For the 1st card being 2, I get (4(4+1))/2 = 10 possibilities.
For the 1st card being 2, I get (3(3+1))/2 = 6 possibilities.
For the 1st card being 2, I get (2(2+1))/2 = 3 possibilities.
For the 1st card being 2, I get (1(1+1))/2 = 1 possibilities.

Or in other words,

$$\sum_{i=1}^{8} \frac{1}{2}\, i\,(i+1) = 120$$

Summing that up you get a total of 120 possible ways to win.
Dividing by the total number of ways to draw, you get:

(120 ways to win) / (720 ways to draw) = (1/6) chance to win.

In a general format, you get

$$\sum_{i=1}^{n-k} \frac{1}{2}\, i\,(i+1) = 120$$

where n = total cards and k = amount to draw.

3. Prove by induction that $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$.

<u>Base case:</u>

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

Assume for all $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$.
Prove $F_{n+(k+1)} = F_{k+1}F_{n+1} + F_kF_n$

<u>Inductive case:</u>

$F_{n+(k+1)} = F_{n+k} + F_{n+(k-1)}$	by Fibonacci definition
$= (F_kF_{n+1} + F_{k-1}F_n) + F_{n+(k-1)}$	by initial assumption
$= F_kF_{n+1} + F_{k-1}F_n + (F_{k-1}F_{n+1} + F_{k-2}F_n)$	by assumption
$= F_kF_{n+1} + F_{k-1}F_{n+1} + F_{k-1}F_n + F_{k-2}F_n$	commutative property
$= F_{n+1}(F_k + F_{k-1}) + F_n(F_{k-1} + F_{k-2})$	factoring
$F_{n+(k+1)} = F_{k+1}F_{n+1} + F_kF_n$	QED!