CSCI 305 Analysis of Algorithms I **Winter 2015 - Fizzano**

Homework 1
Due January 21

Turn in your assignment to Canvas by the start of class time on the due date. Late assignments are not accepted.

Before you begin to discuss anything related to this homework please refer to the syllabus to refresh your memory as to what I consider to be appropriate collaboration.

1. You walk into a casino and see a new game called Dingbat Dice. The way the game works is you are given three dice – one six-sided die, one ten-sided die and one twenty-sided die. It costs \$1 to play the game and if you roll the three dice and get a total of exactly 21 you get paid \$21 (which includes the dollar you wagered, thus you won \$20). If you do not get 21 exactly then you lose the \$1 you wagered. Answer the following questions and clearly and concisely justify your answers.

   (a) How many possible outcomes are there for rolling the three dice?

   (b) How many ways can the dice total 21 given that you rolled a 1 on the six-sided die?

   (c) How many ways can the dice total 21 given that you rolled a 6 on the six-sided die?

   (d) What is the probability of getting 21 when rolling the three dice?

   (e) What is the expected value of the winnings for a person playing this game once?

   (f) What other totals of the three dice would result in the same expected winnings as rolling 21? Explain your answer.

2. You are given a deck of ten cards numbered 1 to 10. You shuffle the deck and then turn over the top three cards one at a time. What is the probability that the three cards that you turned over appeared in increasing order?

   Now generalize this result. Suppose the deck had n cards (numbered 1 to n) and you turned over the top k cards (for $k \leq n$). What is the probability that the top k cards appeared in increasing order?

   Clearly justify your answer.

3. Fibonacci numbers F_n are defined recursively as follows: $F_0 = 0, F_1 = 1$, and for $n \geq 2, F_n = F_{n-1} + F_{n-2}$. Prove by induction that $F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$.