

JSS ACADEMY OF TECHNICAL EDUCATION, NOIDA
DEPARTMENT OF MATHEMATICS
SUBJECT: ENGINEERING MATHEMATICS-II (BAS-203)
QUESTION BANK FOR CIA-II

UNIT-2 (LAPLACE TRANSFORM)	
Short Answer Type Questions	
1.	Find the Inverse Laplace Transform of $\frac{p}{p^2 + 9}$
2.	Find the Inverse Laplace Transform of $\frac{e^{-\pi p}}{(p+3)}$
3.	Find the Inverse Laplace Transform of $\frac{p+2}{(p+2)^2 - 25}$
4.	Find the Inverse Laplace Transform of $\frac{1}{p(p^2 + 1)}$
5.	Find the Inverse Laplace Transform of $\frac{1}{(p+2)^5}$
Long Answer Type Questions	
1.	Find the Inverse Laplace transform of (i) $\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}$ (ii) $\frac{p+8}{p^2+4p+5}$
2.	Find the Inverse Laplace transform of (i) $\left(\frac{e^{-p}}{\sqrt{p+1}} \right)$ (ii) $\left(\frac{e^{-2\pi p}}{p(p^2+1)} \right)$
3.	Find the Inverse Laplace transform of (i) $\frac{p-1}{p^2(p-7)}$ (ii) $\frac{4}{(p^2+2p+5)}$
4.	Find the Inverse Laplace transform of (i) $\log\left(1 + \frac{1}{p^2}\right)$ (ii) $\cot^{-1}\left(\frac{p+3}{2}\right)$
5.	Find the Inverse Laplace transform of (i) $\frac{1}{p^3(p^2+a^2)}$ (ii) $\frac{p^2-a^2}{(p^2+a^2)^2}$

6.	Apply Heaviside's expansion formula to find (i) $L^{-1} \left\{ \frac{p+5}{(p+1)(p^2+1)} \right\}$ (ii) $L^{-1} \left\{ \frac{2p^2+5p-4}{p^3+p^2-2p} \right\}$
7.	State and prove Convolution Theorem and hence evaluate $L^{-1} \left\{ \frac{p}{(p^2+1)(p^2+4)} \right\}$
8.	By using convolution theorem evaluate $L^{-1} \left\{ \frac{p^2}{(p^2+a^2)(p^2+b^2)} \right\}$.
9.	Using Laplace transform, solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos 2t$, Given that $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$
10.	Using Laplace transform, solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t$, Given that $x(0) = 0, x'(0) = 1$
11.	Using Laplace transform, solve the differential equation $\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = e^t t^2$, Given that $y(0) = 1, \left(\frac{dy}{dt}\right)_{t=0} = 0, \left(\frac{d^2y}{dt^2}\right)_{t=0} = -2$
12.	Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} + y = t \cos 2t$, Given that $y(0) = 0, \left(\frac{dy}{dt}\right)_{t=0} = 0$
13.	Solve the following simultaneous differential equation by using Laplace Transform : $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t; x(0) = 1, y(0) = 0$
14.	Solve the following simultaneous differential equation by using Laplace Transform : $\frac{dx}{dt} + \frac{dy}{dt} + x + y = 1, \frac{dy}{dt} = 2x + y; x(0) = 0, y(0) = 1$
UNIT-3 (SEQUENCE AND SERIES)	
Short Answer Type Questions	
1.	Discuss the convergence or divergence of the sequence $a_n = \frac{n}{n^2+1}$
2.	Discuss the convergence or divergence of the sequence $a_n = \frac{n+1}{n}$
3.	Discuss the convergence or divergence of the sequence $a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n}$.

4.	Test the convergence of the series $\sum (-1)^{n+1} \frac{1}{\sqrt{n}}$.
5.	Test the convergence of the series $\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \frac{5}{64} + \dots$
6.	Test the convergence of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$
7.	Find the Fourier coefficient a_0 for the function $f(x) = x \sin x$, $0 < x < 2\pi$
8.	Find the Fourier coefficient b_1 for the function $f(x) = x \sin x$, $0 < x < 2\pi$
Long Answer Type Question	
1.	Test for convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{2n-1}}{(2n+1)(2n+4)}$
2.	Test the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2+1} + \dots$
3.	Test the series $\sum_{n=1}^{\infty} \frac{\sqrt{2n-1}}{(2n+1)(2n+4)}$
4.	Test for convergence or divergence of the series $\frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots (x > 0)$
5.	Test for convergence or divergence of the series $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5.9.13} + \dots$
6.	Test the series $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \dots$
7.	Test the convergence of the series $1 + \frac{1+2}{1+2^2} + \frac{1+3}{1+3^2} + \dots$
8.	Test the convergence of the series $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots$
9.	Test the series $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots (x > 0)$
10.	Test for convergence or divergence of the series $1 + \frac{1}{2} \cdot \frac{x^2}{4} + \frac{1.3.5}{2.4.6} \cdot \frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10} \cdot \frac{x^6}{12} + \dots$
11.	Obtain the Fourier series for the function $f(x) = x^2$, $-\pi \leq x \leq \pi$. Also show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

12.	Find the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$
13.	Obtain the Fourier series expansion of the function $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$
14.	Expand $f(x) = \sin x$ in a half-range cosine series in the interval $(0, \pi)$. Also prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
15.	Find the half-range Fourier cosine series for the function $f(x) = \begin{cases} kx, & 0 \leq x \leq \frac{l}{2} \\ k(l-x), & \frac{l}{2} \leq x \leq l \end{cases}$ Also prove that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
16.	Find the half-range sine series for the function $f(x) = \begin{cases} x, & 0 < x < 2 \\ 4-x, & 2 < x < 4 \end{cases}$

UNIT-4 (Complex variable-Differentiation)

Short Answer Type Questions

1.	Show that $f(z) = z ^2$ is differentiable only at the origin.
2.	Evaluate the line integral $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x$
3.	Find p such that the function $f(z)$ expressed in polar coordinate as $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic.
4.	Write the Cauchy's Riemann equation in polar coordinate.

Long Answer Type Question

1.	Find the value of c_1 and c_2 such that the function $f(z) = x^2 + c_1 y^2 - 2xy + i(c_2 x^2 - y^2 + 2xy)$ is analytic. Also find $f'(z)$
2.	Examine the nature of function $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ in the region including origin.
3.	Show that the function $f(z) = \sqrt{ xy }$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied there.

4.	Verify if $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic at the origin or not.
5.	Show that the function defined by $f(z) = \begin{cases} \frac{x^3 y^5(x+iy)}{x^6+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic at the origin even though it satisfies Cauchy's Riemann equations at the origin.