cs113 Lab5: By Amuldeep Dhillon

Text written to file build.sh

```
doctex lab.doc
pptexenv latex lab.tex
dvipdf lab.dvi
```

Bourne Shell

```
chmod 777 build.sh
./build.sh
```

Ch 9 Problem 1

SML

```
fun f1(x) = (x*x) + x;
fun sum(k,n,f) = if k > n then 0
    else (2*f(k)) + sum(k+1,n,f);
fun id(x) = x;
"Test Case P(5000)";
val a = sum(1,5000,id); val b = f1(5000);
val result = a = b;
```

```
> fun f1(x) = (x*x) + x;
fun sum(k,n,f) = if k > n then 0 else (2*f(k)) + sum(k+1,n,f);
fun id(x) = x;
"Test Case P(5000)";
val a = sum(1,5000,id); val b = f1(5000); val result = a = b;
val f1 = fn: int -> int
> val sum = fn: int * int * (int -> int) -> int
> val id = fn: 'a -> 'a
> val it = "Test Case P(5000)": string
> val a = 25005000: int
val b = 25005000: int
val result = true: bool
```

9.1 Let $P(n) = 2 + 4 + 6 + \cdots + 2n$. We will show that $P(n) = n^2 + n$ for all $n \geq 1$ by the method of mathematical induction.

(i) (Basis of induction) $P(1) = 2 = 1^2 + 1$. So $P(n)$ holds for $n = 1$.

(ii) (Induction hypothesis) Assume $P(k)$ is true for $1 \leq k \leq n$.

(iii) (Induction step) We must show that $P(n+1) = (n+1)^2 + n + 1$. Indeed,

$$\begin{aligned} P(n+1) &= 2 + 4 + \cdots + 2n + 2(n+1) \\ &= P(n) + 2(n+1) \\ &= n^2 + n + 2n + 2 \\ &= n^2 + 2n + 1 + (n+1) \\ &= (n+1)^2 + (n+1) \end{aligned}$$

- Evaluate P(n) at P(n+1)
- 2+4+...+2n = P(n)
- Induce induction hypothesis
- Associative property of addition & Subtraction Property of Equality
- $n^2 + 2n + 1 = (n+1)^2$

Ch 9 Problem 6

SML

```
fun f1(x) = ((x*x) + x) div 2 -3;
fun sum(k,n,f) = if k > n then ~3
    else f(k) + sum(k+1,n,f);
fun id(x) = x;
val a = f1(1000); val b = sum(1,1000,id);
val result = a = b;
```

```
> fun f1(x) = ((x*x) + x) div 2 -3;
fun sum(k,n,f) = if k > n then ~3
    else f(k) + sum(k+1,n,f);
fun id(x) = x;
val a = f1(1000); val b = sum(1,1000,id);
val result = a = b;
val f1 = fn: int -> int
> # val sum = fn: int * int * (int -> int) -> int
> val id = fn: 'a -> 'a
> val a = 500497: int
val b = 500497: int
> val result = true: bool
```

$\sum_{k=1}^{n} k \equiv \frac{n(n+1)}{2}$

$(\sum_{k=1}^{n} k) - 3 \equiv \frac{n(n+1)}{2} - 3$

so, 3+4+5+6+...+1000 equals to the summation of integers from 1 - 1000 minus 3 so we can use the equation $\frac{n(n+1)}{2} - 3$ so the answer to the summation is 500497

- Given
- Subtraction Property of Equality
- Subtraction Property of Equality

Ch 9 Problem 17

SML

```
fun f1(x) = (x*x);
fun sum(k,n,f) = if k > n then 0
    else (2*f(k)-1) + sum(k+1,n,f);
fun id(x) = x;
"Test Case P(1500)";
val a = sum(1,1500,id); val b = f1(1500);
val result = a = b;
```

```
> fun f1(x) = (x*x);
fun sum(k,n,f) = if k > n then 0
    else (2*f(k)-1) + sum(k+1,n,f);
fun id(x) = x;
"Test Case P(1500)";
val a = sum(1,1500,id); val b = f1(1500);
val result = a = b;
val f1 = fn: int -> int
> # val sum = fn: int * int * (int -> int) -> int
> val id = fn: 'a -> 'a
> val it = "Test Case P(1500)": string
> val a = 2250000: int
val b = 2250000: int
> val result = true: bool
```

9.17 Let $P(n)\,:\,1+3+\cdots+2(n-1)+1$ for $n\geq 1$ represent the sum of the first n odd positive integers. We use mathematical induction.
(i) (Basis of induction) $P(1)$ is true since $1=1^2$.
(ii) (Induction hypothesis) Assume $P(k)$ is true for $1\leq k\leq n$. (iii) (Induction step) We want to show that $P(n+1)$ is also true. That is, the sum of the first $n+1$ odd positive integers is $(n+1)^2$. Indeed, the sum of the first $n+1$ odd positive integers is

$$\begin{aligned}1+3+5+\cdots+2(n-1)+1+2n+1 =&[1+3+5+\cdots+2(n-1)+1]+2n+1\\ =&n^2+2n+1=(n+1)^2.\end{aligned}$$

- evalute the funtion at n+1
- associative property of addition
- using the induction hypothesis
- factoring