FIN 5350

Hwk 2

Question 3 – Solution

Let $S = \$100$

$K = \$95$

$\sigma = 0.30$ or 30% per annum

$r = 0.08$ or 8%

$T = 1$ year

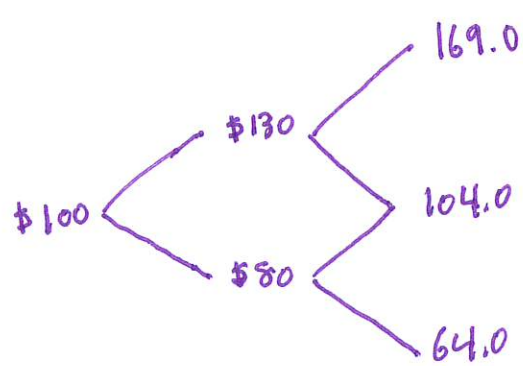
$\delta = \phi$

Also, we're given than $u = 1.3$

$d = 0.8$

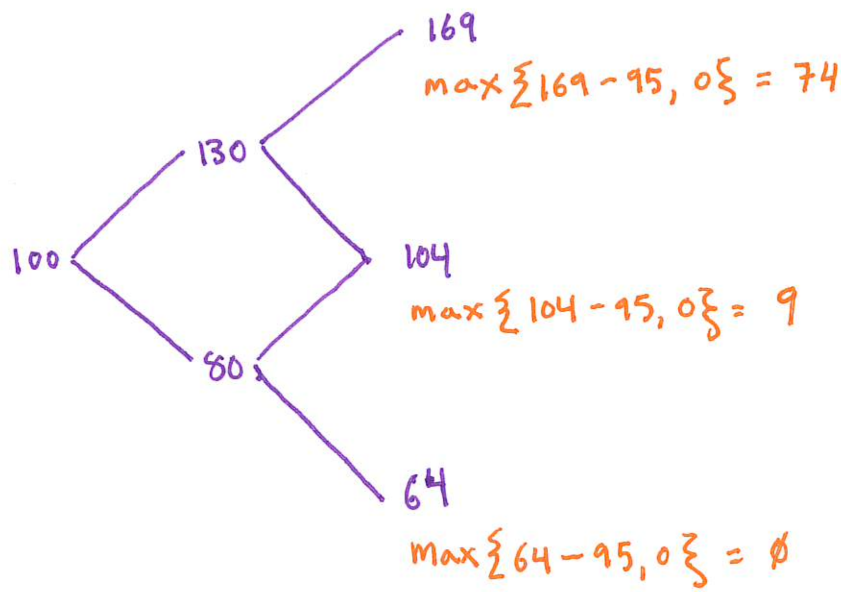
Let $n = \cancel{?}\,2$

$\boxed{\text{Step 1}}$ Construct the Stock Price Tree (forward)



Step 2 Using the boundary condition, apply the payoff function at expiry $(T=1)$ and recursively build the Option Price Tree (backwards)

N.B: Call price values in orange



Now we will use recursion with the one-period model to solve for the option values.

Recall:

$$C = \Delta S + B \quad \text{and that} \quad h = \frac{T}{n} = \frac{1}{2} = 0.5$$

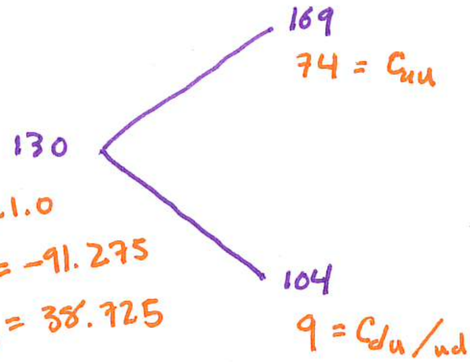
where

$$\Delta = e^{-\delta h}\left(\frac{C_u - C_d}{S(u-d)}\right)$$

$$B = e^{-rh}\left(\frac{uC_d - dC_u}{u-d}\right)$$

* See page. 296 in McDonald Chp. 9

Consider the upper nodes on the tree



$\Delta = 1.0$

$B = -91.275$

$C_u = 38.725$

$$\Delta = \underbrace{e^{-(0.07)(0.5)}}_{1}\left(\frac{74-9}{130\,(1.3-.8)}\right)$$

$$= (1)\left(\frac{65}{65}\right) = 1.0$$

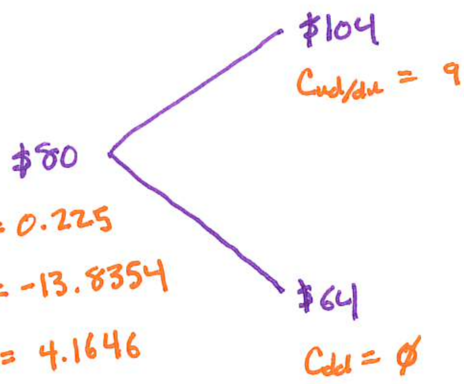
$$B = \underbrace{e^{(-.08)(.5)}}_{0.9608}\left(\frac{\overbrace{(1.3)(9)}^{11.7} - \overbrace{(.8)(74)}^{59.2}}{0.5}\right)$$

$$= -91.275$$

$$C_u = 130\,(1.0) - 91.275$$

$$= \$38.725$$

Now consider the lower nodes



$\Delta = 0.225$

$B = -13.8354$

$C_d = 4.1646$

$$\Delta = \underbrace{e^{(-.0)(0.5)}}_{1.0}\left(\frac{9-0}{80\,(1.3-.8)}\right)$$

$$= \frac{9}{40} = 0.225$$

$$B = \underbrace{e^{(-.08)(0.5)}}_{0.9608}\left(\frac{(1.3)(\emptyset)-(.8)(9)}{1.3-.8}\right)$$

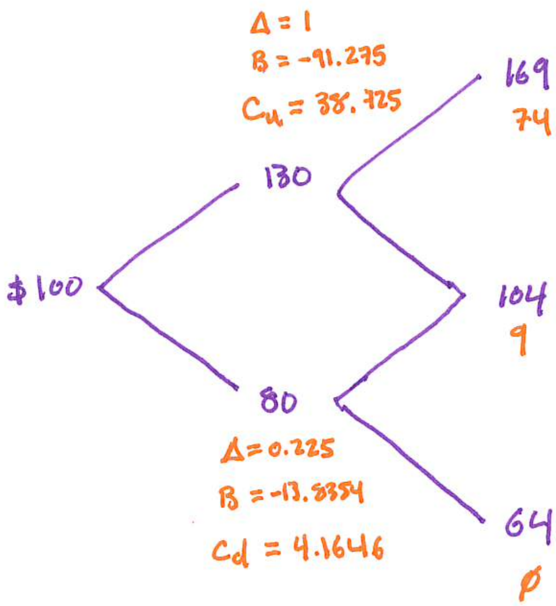
$$= .9608\,(-14.40)$$

$$= -13.8354$$

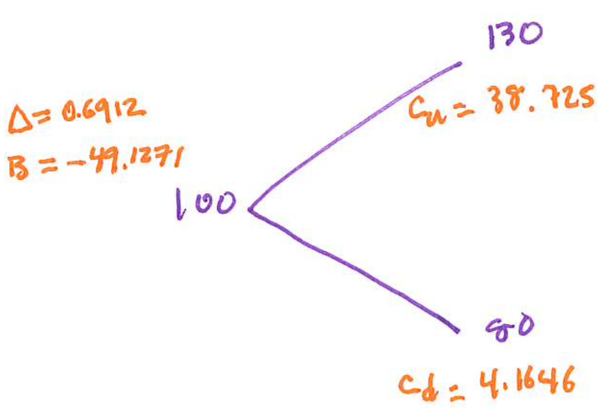
$$C_d = 80\,(.225) - 13.8354$$

$$= 4.1646$$

Now we have the following



We can now complete the recursion with one last step



$C_0 =$

$$\Delta = \underbrace{e^{(-0)(0.5)}}_{1.0} \left(\frac{38.725 - 4.1646}{100\ (1.3 - .80)} \right) = 0.691208$$

$$B = e^{(-.08)(.5)}\left(\frac{(1.3)(4.1646) - (.8)(38.725)}{1.3 - 0.80}\right)$$

$$= (.9608)\Big($$

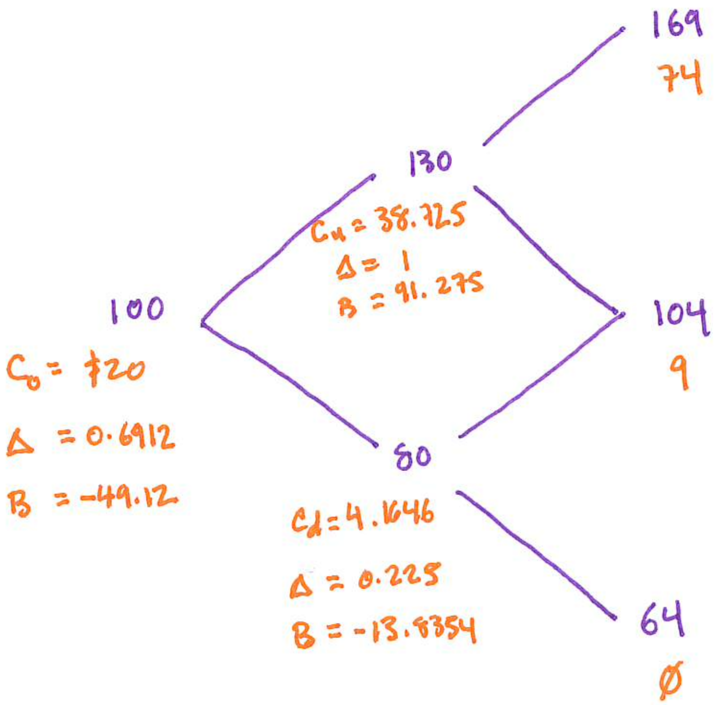
$$= -49.1271$$

$$C_0 = (100)(.6912) - 49.1271$$

$$= 69.12 - 49.1271$$

$$= \$20.00 \text{ (ish)}$$

So recapitulating



Now let's see how to use the Binomial Probability Mass Function to see if we can use a short cut to pricing European call options.

~~Take~~ Recall that the Binomial PMF is given as

$$\binom{n}{i} (p^*)^i (1-p^*)^{n-i}$$

with $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ read as "n choose i"

Take the final set of nodes on the stock tree at expiry:

	$S_{T,i}$	Prob
$i=2$	169	$\binom{2}{2}(p^*)^2(1-p^*)^{2-2} = \underline{(p^*)^2}$
$i=1$	104	$\binom{2}{1}(p^*)^1(1-p^*)^{2-1} = \underline{2(p^*)(1-p^*)}$
$i=0$	$\emptyset$	$\binom{2}{0}(p^*)^0(1-p^*)^{2-0} = \underline{(1-p^*)^2}$

$i =$ success or "up move"

Recall that:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$ (McDonald p. 299)

$$= \frac{e^{(.08)(0.5)} - .8}{1.3 - .80} = 0.4816$$

So that:

		Prob	
•	169 74	$(p^*)^2 =$	0.2320
•	104 9	$2(p^*)(1-p^*) =$	0.4993
•	64 0	$(1-p^*)^2 =$	0.2687

total = 1.0

$.2320 + .4993 + .2687 = 1.0$

Notice that we can now write the option price as

$$e^{-\delta T}\left[C_{uu}(p^*)^2 + C_{ud/du}\,2(p^*)(1-p^*) + C_{dd}(1-p^*)^2\right]$$

N.B. some numerical error

$C = \$20$ (ish)

* just as with the fully recursive approach!