

Market-Making and Delta Hedging

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Introduction

- ▶ Without hedging, an active market-maker will have an arbitrary position generated by fulfilling customer orders
- ▶ An arbitrary portfolio has uncontrolled risk
 - ▶ An adverse price move has the potential to bankrupt the market-maker
- ▶ Market-makers thus attempt to hedge the risk of their positions

Delta Hedging for a Two-Day Period

- ▶ Consider a customer who wishes to buy a 91-day call option, the market-maker fills this order by selling a call option
- ▶ Take the following as given:
 - ▶ $S = \$40$
 - ▶ $K = \$40$
 - ▶ $\sigma = 0.30$
 - ▶ $r = 0.08$
 - ▶ $\delta = 0$
 - ▶ $T - t = 91/365$

Delta-Hedging Example Continued

Day 0. Consider the 40—strike call option described above, written on 100 shares of stock. The market-maker sells the option and receives \$278.04. Since $\Delta = 0.5824$, the market-maker also buys 58.24 shares.

The net investment is

$$(58.24 \times \$40) - \$278.04 = \$2051.56$$

At an 8% interest rate, the market-maker has an overnight finance charge of $\$2051.56 \times (e^{0.08/365} - 1) = \0.45 .

Delta-Hedging on Day 1 - Marking-to-Market

Day 1: Marking-to-Market. Suppose the new stock price is \$40.50. The new call option price with 1 day less to expiration and at the new stock price is \$3.0621 (via Black-Scholes-Merton). Overnight mark-to-market profit is a gain of \$0.50, computed as follows:

| | |
|-----------------------------|--|
| Gain on 58.24 shares | $58.24 \times (\$40.50 - \$40) = \$29.12$ |
| Gain on written call option | $\$278.04 - \$306.21 = -\$28.17$ |
| Interest | $-(e^{0.08/365} - 1) \times \$2051.56 = -\$0.45$ |
| Overnight Profit | \$0.50 |

Delta-Hedging on Day 1 - Rebalancing the Hedge

Day 1: Rebalancing the Portfolio. The new delta is 0.6142. Since delta has increased, we must buy $61.42 - 58.24 = 3.18$ additional shares. This will require an investment of $\$40.50 \times 3.18 = \128.79 . Since the readjustment in the number of shares entails buying at the current market price, it does not affect the mark-to-market profits for that day.

Delta-Hedging on Day 2

Day 2: Marking-to-Market. The stock price now falls to \$39.25. The market-maker makes money on the written option and loses money on the 61.42 shares. Interest increases over the previous day because additional investment was required for the extra shares. The net result from marking-to-market is a loss of $-\$3.87$:

| | |
|-----------------------------|--|
| Gain on 61.42 shares | $61.42 \times (\$39.25 - \$40.50) = -\$76.78$ |
| Gain on written call option | $\$306.21 - \$232.82 = \$73.39$ |
| Interest | $-(e^{0.08/365} - 1) \times \$2181.30 = -\$0.48$ |
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| Overnight Profit | $-\\$3.87$ |