The Heston Model

Tyler J. Brough

November 17, 2016

The Heston Model

The Heston (1993) model assumes the underlying stock price follows the diffusion process given by:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dz_{1,t}$$

where μ is the drift parameter, and $z_{1,t}$ is a standard Wiener process. The volatility $\sqrt{v_t}$ itself follows a diffusion process:

$$d\sqrt{v_t} = -\beta\sqrt{v_t}dt + \delta dz_{2,t}$$

where $z_{2,t}$ is a Wiener process, and ρ defines the correlation between $z_{1,t}$ and $z2,t$.

Using Ito's lemma the variance process can be written as an Orstein-Uhlenbeck (mean-reverting) process:

$$dv_t = \kappa[\theta - v_t]dt + \sigma\sqrt{v_t}dz_{2,t}$$

where

- θ is the long-run mean of the variance
- κ is a mean reversion parameter
- σ is the volatility of volatility

Given this set up the Heston (1993) gives the price of the call option as:

$$Call(S, v, t) = S_t P_1 - KP(t, T)P_2$$

where

- $S_{t} = $ the spot price of the asset
- $K = $ the strike price of the option
- $P(t,T) = $ a discount factor from time t to time T

If we assume a constant rate of interest r, we can write this as

$$P(t, T) = \exp -r \times (T - t)$$

P_1 and P_2 are the probabilities that the call option will expire in-the-money, conditional on the log stock price $x_t = ln[S_t]$, and on the volatility v_t, each at time t.

The risk-neutral versions are given by:

$$dx_t = [r - \frac{1}{2}v_t]dt + \sqrt{v_t}dz^*_{1,t}$$
$$dv_t = \kappa[\theta - v_t]dt + \sigma\sqrt{v_t}dz^*_{2,t}$$