DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
SSC106: MATHEMATICS FOR SOCIAL SCIENCES II
RAIN SEMESTER EXAMINATION (2008/2009 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A**
- Attempt all questions in **Section B**
- Do not cheat

Time allowed: 2 hours

SECTION A

1. Distinguish between a matrix and a determinant.

2. ………………… is a function which expresses the ratio of two distances as a function of an angle.

3. Differentiate between the functions: $y = b^x$ $(b > 1)$ and $y = x^b$ $(b \neq 0)$

4. What is a monotonic function? Give appropriate examples.

5. A function which satisfies Laplace equation is in general called ………………

6. Find $\frac{dy}{dx}$ when $y = \log(e^x + e^{-x})$

7. State Young's Theorem.

8. If $x = r\sin\theta$and $y = r\cos\theta$,

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 =. \dots \dots \dots \dots ..$$

9. Seyi asserts that the following functions are unrelated: $Z_1 = 3x - y$ and $Z_2 = 9x^2 - 6xy + y^2$. Is Seyi's assertion correct?

10. Using Euler's theorem, the degree of homogeneity of the function: $f(x, y) = 2x^2 + xy - y^2$ is ……………

11. Outline any two techniques used in solving constrained optimization problem.

12. If $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 8 & 8 & 8 & 0 \\ 16 & 16 & 16 & 16 \end{bmatrix}$, find the spur$A$.

SECTION B

1. *(a)* If a firm's average production function is $AP = \frac{200}{L} + 5L - \frac{L^2}{4}$, determine the firm's production function.

   (b) A company determines its marginal cost $C'(x) = x^2 - 3x$. Find the cost function, $C(x)$ if its fixed cost (cost of producing zero units) is #1000.

   (c) If $q_d = 18 - 2p$ and $q_x = -2 + 2p$ are the respective demand and supply functions for a product. Compute the equilibrium prices and the point elasticity of demand and supply at the price.

2. *(a)* Why is some knowledge of mathematics useful in social sciences?

(b) Integrate the following expressions:

(i) $\frac{x^n}{1+n}$

(ii) $x^4 + x^3 + x^2 + x + 4$

(iii) $\frac{\cos x}{1+\sin x}$

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2008/2009 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and all questions from Section B. The instruction of answering all is perhaps nice; the questions are too simple and too nice.

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

Question 1

Distinguish between a matrix and a determinant.

Such a weird question, a determinant doesn't even have any fixed definition as it were; however, we'll tackle the question anyhow.

A matrix is an array of numbers in rows and columns while a determinant is a number calculated from a matrix.

Full stop! I don't think there is anything else to say!

Question 2

………………… is a function which expresses the ration of two distances as a function of an angle.

Fill in the gap, the answer is a definition of a type of function which is the **trigonometric functions.**

Question 3

Differentiate between the functions:
$y = b^x\ (b > 1)$ and $y = x^b\ (b \neq 0)$

Of course, we even emphasized this when in the study of functions.

The first function: $y = b^x\ (b > 1)$ is an exponential function; since x, the independent variable is an exponent, while the second one; $y = x^b\ (b \neq 0)$ is a power function since x, the independent variable is being raised to a constant power.

The stuffs in bracket are formalities that describe the values for which those functions are valid;

Question 4

What is a monotonic function? Give appropriate examples.

Monotonic functions are functions that are increasing or decreasing for all values of the function. And hence, not only within an interval, monotonic functions are either **strictly increasing** or **strictly decreasing** for all values of the independent variable.

Examples of monotonic functions are linear functions and exponential functions.

Question 5

A function which satisfies Laplace equation is in general called ………………

As stated in the notes, a function which satisfies the Laplace equation is a harmonic function.

Question 6

Find $\frac{dy}{dx}$ when $y = \log(e^{x} + e^{-x})$

We have:

$$y = \log(e^x + e^{-x})$$

Here;

We have a function of function situation;

Hence; put:

$$u = e^x + e^{-x}$$

To find:

$$\frac{du}{dx} = \frac{d}{dx}(e^x) + \frac{du}{dx}(e^{-x})$$

$$\frac{d}{dx}(e^x) = e^x$$

Above is standard integral;

$$\frac{d}{dx}(e^{-x})$$

Substitution;

$$z = -x$$

$$\frac{dz}{dx} = -1$$

Hence, we have:

$$e^z$$

$$\frac{d}{dz}(e^z) = e^z$$

Chain rule;

$$\frac{d}{dx}(e^{-x}) = \frac{d}{dz}(e^{z}) \times \frac{dz}{dx}$$

We have:

$$\frac{d}{dx}(e^{-x}) = e^{z} \times -1$$

Return z;

$$\frac{d}{dx}(e^{-x}) = -e^{-x}$$

Hence;

We have:

$$\frac{du}{dx} = e^{x} + (-e^{-x})$$

$$\frac{du}{dx} = e^{x} - e^{-x}$$

Here;

$$y = \log u$$

Meaning;

$$y = \log_{10} u$$

Hence;

$$\frac{dy}{du} = \frac{1}{u \ln 10}$$

Chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence;

$$\frac{dy}{dx} = \frac{1}{u \ln 10} \times (e^x - e^{-x})$$

Return u and combine fractions;

$$\frac{dy}{dx} = \frac{e^x - e^{-x}}{\ln 10\,(e^x + e^{-x})}$$

Question 7

State Young's Theorem.

It states that two complementary second order mixed partials of a continuous and twice differentiable function are equal; for a multivariate function dependent on x and y;

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

In function notation;

$$f_{xy} = f_{yx}$$

Question 8

If $x = r\sin\theta$ and $y = r\cos\theta$,

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \ldots\ldots\ldots\ldots\ldots$$

The question has been sorted out in the notes already!

So we have two functions here:

$$x = r\sin\theta$$
$$y = r\cos\theta$$

$$x = r\sin\theta$$

$$\frac{dx}{d\theta} = r \times \frac{d}{d\theta}(\sin\theta)$$

$$\frac{dx}{d\theta} = r \times \cos\theta = r\cos\theta$$

$$y = r\cos\theta$$

$$\frac{dy}{d\theta} = r \times \frac{d}{d\theta}(\cos\theta)$$

$$\frac{dy}{d\theta} = r \times -\sin\theta = -r\sin\theta$$

We're told to evaluate this:

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$$

To do this, we take the squares of our derivatives just like it is in the question;

$$\left(\frac{dx}{d\theta}\right)^2 = (r\cos\theta)^2 = r^2(\cos\theta)^2 = r^2(\cos^2\theta)$$

$$\left(\frac{dy}{d\theta}\right)^2 = (-r\sin\theta)^2 = r^2(\sin\theta)^2 = r^2(\sin^2\theta)$$

Taking their sum, we have:

$$r^2(\cos^2\theta) + r^2(\sin^2\theta)$$

r^2 is common between both, factorize it:

$$r^2[\cos^2\theta + \sin^2\theta]$$

From trigonometry;

$$\sin^2\theta + \cos^2\theta = 1$$

Hence, we have that this reduces to:

$$r^2(1) = r^2$$

Hence,

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = r^2$$

Question 9

Seyi asserts that the following functions are unrelated: $Z_1 = 3x - y$ and $Z_2 = 9x^2 - 6xy + y^2$. Is Seyi's assertion correct?

The Seyi story is just formalities, it was Toyin's name that was used in the previous year with this same values hence, you should know it's just names. You're just asked to check if the functions are related (or dependent). If you find out that they're related, you conclude Seyi's assertion is incorrect and if they aren't related, you conclude it is a correct assertion.

All we need to do is to solve for the Jacobian determinant and see its nature;

Taking the first order partials;

$$\frac{\partial Z_1}{\partial x} = 3x^{1-1} - 0 = 3$$

$$\frac{\partial Z_1}{\partial y} = 0 - 1 \times 1y^{1-1} = -1$$

$$\frac{\partial Z_2}{\partial x} = 2 \times 9x^{2-1} - 1 \times 6x^{1-1}y + 0$$

$$\frac{\partial Z_2}{\partial x} = 18x - 6y$$

$$\frac{\partial Z_2}{\partial y} = 0 - 1 \times 6xy^{1-1} + 2 \times y^{2-1}$$

$$\frac{\partial Z_2}{\partial y} = -6x + 2y$$

Form the Jacobian matrix;

$$J = \begin{pmatrix} \frac{\partial Z_1}{\partial x} & \frac{\partial Z_1}{\partial y} \\ \frac{\partial Z_2}{\partial x} & \frac{\partial Z_2}{\partial y} \end{pmatrix}$$

$$J = \begin{pmatrix} 3 & -1 \\ 18x - 6y & -6x + 2y \end{pmatrix}$$

Evaluate the Jacobian determinant;

$$|J| = 3(-6x + 2y) - (-1)(18x - 6y)$$

$$|J| = -18x + 6y + 18x - 6y$$

$$|J| = 0$$

Hence, the two functions are dependent;;

And then Seyi's assertion is incorrect!

Question 10

Using Euler's theorem, the degree of homogeneity of the function:
$f(x, y) = 2x^2 + xy - y^2$ is ……………

$$f(x, y) = 2x^2 + xy - y^2$$

Euler's theorem;

The degree of homogeneity, n, is:

$$xf_x + yf_y = nf(x, y)$$

Hence; solving for the first order partials;

$$f(x, y) = 2x^2 + xy - y^2$$

$$f_x = 2 \times 2x^{2-1} + 1 \times x^{1-1}y - 0$$

$$f_x = 4x + y$$

Also;

$$f_y = 0 + 1 \times xy^{1-1} - 2 \times y^{2-1}$$

$$f_y = x - 2y$$

Slotting into Euler's theorem;

$$x(4x + y) + y(x - 2y) = n(2x^2 + xy - y^2)$$

Expanding the left hand side;

$$4x^2 + xy + xy - 2y^2 = n(2x^2 + xy - y^2)$$

$$4x^2 + 2xy - 2y^2 = n(2x^2 + xy - y^2)$$

Factorize the LHS;

$$2(2x^2 + xy - y^2) = n(2x^2 + xy - y^2)$$

Hence; by comparison; the degree of homogeneity, n is:

$$n = 2$$

Question 11

Outline any two techniques used in solving constrained optimization problem.

A direct question from the notes; the two methods are:

- Direct substitution
- Method of the Lagrangean multipliers

Question 12

If $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 8 & 8 & 8 & 0 \\ 16 & 16 & 16 & 16 \end{bmatrix}$, find the spur$A$.

The Spur of any square matrix is the sum of the elements on its main diagonal;

Hence; for the matrix above;

$$\text{spur}\, A = 2 + 4 + 8 + 16$$

$$\text{spur}\, A = 30$$

SECTION B

Question 1

(a) If a firm's average production function is $AP = \frac{200}{L} + 5L - \frac{L^2}{4}$, determine the firm's production function.

(b) A company determines its marginal cost $C'(x) = x^2 - 3x$. Find the cost function, $C(x)$ if its fixed cost (cost of producing zero units) is #1000.

(c) If $q_d = 18 - 2p$ and $q_x = -2 + 2p$ are the respective demand and supply functions for a product. Compute the equilibrium prices and the point elasticity of demand and supply at the price.

(a)

We know that the average function for any function is gotten by dividing the function by the unit it depends on, hence, the average production function is given by;

$$AP = \frac{P(L)}{L}$$

Hence;

$$P(L) = AP \times L$$

$$P(L) = \left(\frac{200}{L} + 5L - \frac{L^2}{4}\right) \times L$$

Expanding;

$$P(L) = 200 + 5L^2 - \frac{L^3}{4}$$

Hence, we have gotten the firm's production form, $P(L)$;

(b)

A company determines its marginal cost $C'(x) = x^2 - 3x$. Find the cost function, $C(x)$ if its fixed cost (cost of producing zero units) is #1000.

Simple stuff;

We have the marginal cost function, integrate it to get the cost function;

$$C'(x) = x^2 - 3x$$

$$C(x) = \int C'(x)dx = \int (x^2 - 3x)dx$$

$$C(x) = \frac{x^{2+1}}{2+1} - \frac{3x^{1+1}}{1+1} + C$$

$$C(x) = \frac{x^3}{3} - \frac{3x^2}{2} + C$$

We are given the fixed cost as #1000, it means even when zero units are produced, hence, we can the constant in the cost function is 1000; hence, we have our cost function as;

The fixed cost is like when in normal integration, you're told that when $\boldsymbol{x = 0}$, $\boldsymbol{C(x) = 1000}$

$$C(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 1000$$

(c)

A similar question is in the notes section.

We have the demand and supply functions respectively:

$$q_d = 18 - 2p$$

And

$$q_x = -2 + 2p$$

Firstly, the equilibrium price is:

$$q_d = q_x$$

$18 - 2p = -2 + 2p;$

Solving for the price, p,

$20 = 4p;$

$p = 5.$

Hence, $p = 5$ is the price at which we'll be evaluating the point elasticities for both the demand and the supply functions as the question demands that we compute the point elasticities at the equilibrium price.

Hence, we'll also solve for the corresponding values of q.

For the demand function;

$q_d = 18 - 2p$; at $p = 5$

$$q_d = 18 - 2(5)$$
$$q_d = 18 - 10$$
$$q_d = 8$$

For the supply function;

$q_x = -2 + 2p$; at $p = 5$

$$q_x = -2 + 2(5)$$
$$q_x = -2 + 10$$
$$q_x = 8$$

Don't mind the slight discrepancy in the question, the supply function as in the notes is q_s but the question stated it as q_x and we have to use it that way. Hence;

Now, to find their different price elasticities; take each function on its own and find their separate derivatives with respect to p;

For the demand function;

$q_d = 18 - 2p$ $\qquad\qquad\qquad$ $q_x = -2 + 2p$

Differentiating q_d and q_x respectively with respect to p;

$$\frac{dq_d}{dp} = -2 \qquad\qquad \frac{dq_s}{dp} = 2$$

We now find the price elasticity of demand (PED) and the price elasticity of supply (PES) for the respective functions at $p = 5$;

$$PED = \left(\frac{dq_d}{dp}\right) \times \left(\frac{p}{q}\right)$$

Now p and q have corresponding values as: $p = 5$; $q = 8$.

Hence, we have PED thus:

$$PED = -2 \times \left(\frac{5}{8}\right) = -\frac{10}{8} = -1.25$$

$$PES = \left(\frac{dq_x}{dp}\right) \times \left(\frac{p}{q}\right)$$

Now p and q have corresponding values as: $p = 5$; $q = 8$.

Hence, we have PES thus:

$$PES = 2 \times \left(\frac{5}{8}\right) = \frac{10}{8} = 1.25$$

Question 2

(a) Why is some knowledge of mathematics useful in social sciences?

(b) Integrate the following expressions:

(i) $\frac{x^n}{1+n}$

(ii) $x^4 + x^3 + x^2 + x + 4$

(iii) $\frac{\cos x}{1+\sin x}$

(a)

Very common question in SSC106 exams!

The knowledge of mathematics is very useful in the field of Social Sciences because:

- The use of matrices are very useful in solving cases of multiple inputs;

- With calculus, we can find the relative optima of different economical functions to easily find desired optimum results;
- We can find desired equilibrium points for economical situations for market equilibrium such as when demand is equal to supply.
- With the concept of partial differentiation, we can find optimal points for functions of multiple situations.
- Mathematical optimization helps consumers to maximize their utilities; this helps consumers in making decisions.
- Mathematical optimization helps producers to make optimum number of products to help minimize loss.
- Mathematical optimization helps producers help producers in production to find optimum quantity to minimize cost and maximize revenue.
- Mathematics help firms to be able to achieve equilibrium between all factors of production in the production function and help in using

the production functions to maximize the quantity of products produced while maintaining minimum cost.

- Mathematics makes problems that could take lengthy periods to be resolved in minutes;
- The language of mathematics is every easy to understand.

(b)

(i)

$$\frac{x^n}{1+n}$$

We have:

$$\int \frac{x^n}{1+n}dx$$

We are to integrate with respect to x as we'll just assume since we were not told;

It's extremely simple but it looks confusing as the expression itself even looks like the power rule of integration, but as it were, it isn't, just treat it like a normal fraction; the first thing is to bring out the constant since anything that is not x is a constant.

$$\frac{1}{1+n}\int x^n dx$$

Apply the integral;

$$\frac{1}{1+n}\left[\frac{x^{n+1}}{n+1}\right]+C$$

We have;

$$\frac{1}{n+1}\times\frac{x^{n+1}}{n+1}+C$$

$$\frac{x^{n+1}}{(n+1)^2}+C$$

(ii)

$$x^4+x^3+x^2+x+4$$

We have:

$$\int(x^4+x^3+x^2+x+4)dx$$

Integral of sums; we have straight power rules;

$$\int x^4\,dx+\int x^3\,dx+\int x^2\,dx+\int x\,dx+\int 4\,dx$$

$$\int x^4\,dx+\int x^3\,dx+\int x^2\,dx+\int x\,dx+4\int x^0\,dx$$

Integrate using the power rule;

$$\left[\frac{x^{4+1}}{4+1}\right]+\left[\frac{x^{3+1}}{3+1}\right]+\left[\frac{x^{2+1}}{2+1}\right]+\left[\frac{x^{1+1}}{1+1}\right]+4\left[\frac{x^{0+1}}{0+1}\right]$$

$$\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+4x$$

(iii)

$$\frac{\cos x}{1+\sin x}$$

We have:

$$\int \frac{\cos x}{1+\sin x}\,dx$$

This is a case of substation, the case of:

$$\frac{f'(x)}{f(x)}$$

This has even been solved in the notes;

The numerator is the derivative of the function the second function depends on, hence, our substitution is going on smoothly!

$$u = 1 + \sin x$$

$$\frac{du}{dx} = \cos x$$

Hence,

$$dx = \frac{du}{\cos x}$$

Substitute back into the integral, substitute for $(1 + \sin x)$ and for dx;

$$\int \frac{\cos x}{u} \times \frac{du}{\cos x}$$

$\cos x$ cancels out! Leaving:

$$\int \frac{1}{u} du$$

The integral, a standard integral;

$$\ln u + C$$

We won’t forget our arbitrary constant, return u, the integral is:

$$\ln(1 + \sin x) + C$$

THAT’S IT!