DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
SSC106: MATHEMATICS FOR SOCIAL SCIENCES II
RAIN SEMESTER EXAMINATION (2013/2014 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A;**
- Answer any two questions from **Section B.**
- Show all workings clearly

Time allowed: 2 hours

SECTION A

1. *(a)* What is a function?

   (b) Distinguish between a power function and an exponential function.

2. What types of matrices are A and B if:

   (a) $AA' = A'A = I$

   (b) $BB^{-1} = B^{-1}B = I$

3. *(a)* What is integration?

   (b) Distinguish between definite and indefinite integral.

4. Find the relative optima of the function:

$$y = 4x - x^2$$

5. Differentiate with respect to x;

   (a) $y = \log(\sin x + \cos x)$

   (b) $y = ax^n + bx^m$

6. Use Euler's theorem to determine the degree of homogeneity of the functions;

   (a) $f(x, y) = x^3 + 2x^2y + y^3$

   (b) $f(L, K) = AL^{\alpha}K^{\beta}$

7. Compute the second order partial derivatives of the function:

$$f(x, y) = x^2e^{-y}$$

   and verify Young's theorem.

8. Learning of most skills starts at a rapid rate and then slows down. A psychologist measures the learning performance of a laboratory rat by a numerical score on a

standardized test. Assume the rat's score $p(t)$ after t weeks of learning is:

$$p(t) = 15t^2 - t^3$$

At what point in time does the rat's rate of learning begin to decline?

9. Distinguish between a differential equation and a difference equation.

10. The aggregate consumption function for a community is given by:

$$C(x) = 210 + 4\sqrt{x}$$

Where $C(x) =$ total consumption; and

$x =$ disposable income of the community.

Find the marginal propensity to consume (MPC) when x = 16, 64, 100.

SECTION B

1. *(a)* Outline the allied calculus conditions for the optimization of the function:

$$y = f(x)$$

(b) $\int \frac{f'(x)}{f(x)} dx = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

(c) If $y = e^{f(x)}$; then:

$$\frac{dy}{dx} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(d) Verify that:

$$y = A_1 \log_e x + A_2$$

is the solution of the differential equation:

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

2. *(a)* A matrix can be classified using:

(i) The relationship between its rows and columns;

(ii) The structure of its elements;

(iii) A derived relationship with itself.

With appropriate examples; list:

(i) three types of matrices under classification (i);

(ii) four types of matrices under classification (ii);

(iii) three types of matrices under classification (iii);

(b) Let:

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 8 & 16 \\ 7 & 6 & 1 \end{pmatrix} \text{ and } E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

Show that $|EA| = |E||A|$.

3. *(a)* State the following rules of integration;

(i) Power function rule;

(ii) Exponential function rule;

(iii) Logarithm function rule.

(b) Evaluate:

(i) $\int (x^{\alpha} + x^{\beta} + x^{\gamma})dx$

(ii) $\int (3x + 5)^{10} dx$

(iii) $\int_{-a}^{a} f(x)dx$

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2013/2014 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and only two questions from Section B, we'll be answering everything in both sections in this book though; keep calm and move with me;

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

Question 1

(a) What is a function?

(b) Distinguish between a power function and an exponential function.

(a)

A function is a mathematical relationship between sets of inputs and a set of permissible outputs with each input related to one output.

(b)

A power function is of the form:

$$y = ax^b$$

While an exponential function is of the form:

$$y = ab^x$$

The main difference is that in the power function, x the independent variable is raised to a constant power while in the exponential function, a constant is raised to a power in the independent variable, x.

Question 2

What types of matrices are A and B if:

(a) $AA' = A'A = I$

(b) $BB^{-1} = B^{-1}B = I$

(a)

From our study of matrices, something that multiplies a matrix to give the identity matrix is the inverse. A' is the transpose, hence, the transpose is here is the inverse and hence, that is the property of an orthogonal matrix, hence, A is an orthogonal matrix.

(b)

From our study of matrices, a matrix that multiplies another matrix to give the identity matrix is the inverse of the matrix. Hence, B doesn't look like a special matrix since it follows the rule of every matrix. Note that even though in general matrix multiplication, $AB \neq BA$, for inverse multiplication, the product either way yields an identity matrix and hence, B is a normal matrix. However, to be specific, only square matrices have determinants and hence, only square matrices have inverses. Hence, a better conclusion is that ***B* is a square matrix,** since it involves inverses.

Question 3

(a) What is integration?

(b) Distinguish between definite and indefinite integral.

(a)

Integration is the process of finding a function from its derivative. It is the process of finding the area under a curve for any given function.

(b)

The difference between the indefinite integral and the definite integral is that the indefinite integral gives the general form of the anti-derivative of a function while the definite integral gives the area under a curve between two given points and is the value gotten by evaluating the integral from the two limits. The indefinite integral contains an arbitrary constant while the definite integral doesn't contain an arbitrary constant.

Question 4

Find the relative optima of the function:

$$y = 4x - x^2$$

Finding the relative optima, we're to completely optimize the function and determine the real nature.

$$y = 4x - x^2$$

$$\frac{dy}{dx} = 1 \times 4x^{1-1} - 2x^{2-1}$$

$$\frac{dy}{dx} = 4 - 2x$$

At stationary point;

$$\frac{dy}{dx} = 0$$

Hence,

$$4 - 2x = 0$$

$$x = 2$$

To determine the nature;

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = 0 - 1 \times 2x^{1-1}$$

$$\frac{d^2y}{dx^2} = -2$$

Hence, since the second derivative is negative, the point $x = 2$ is a maximum point.

As for the maximum value, substitute $x = 2$ into the main function.

$$y = 4x - x^2$$

$$y = 4(2) - (2)^2$$

$$y = 4$$

Hence, the relative optima is $y = 4$, corresponding to $x = 2$ (a maximum value)

Question 5

Differentiate with respect to x;

(a) $y = \log(\sin x + \cos x)$

(b) $y = ax^n + bx^m$

(a)

A substitution is needed;

$$u = \sin x + \cos x$$

$$\frac{du}{dx} = \cos x + (-\sin x)$$

$$\frac{du}{dx} = \cos x - \sin x$$

Hence,

$$y = \log u$$

Log without base is log to base 10;

$$y = \log_{10} u$$

By rules;

$$\frac{dy}{du} = \frac{1}{u \ln 10}$$

Hence,

Chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence,

$$\frac{dy}{dx} = \frac{1}{u \ln 10} \times (\cos x - \sin x)$$

Return u;

$$\frac{dy}{dx} = \frac{\cos x - \sin x}{(\sin x + \cos x)\ln 10}$$

(b)

$$y = ax^n + bx^m$$

Since we're differentiating with respect to x, both n and m will be nothing but constant, hence, this is a case of constant power.

Here;

$$\frac{dy}{dx} = n \times ax^{n-1} + m \times bx^{m-1}$$

$$\frac{dy}{dx} = nax^{n-1} + mbx^{m-1}$$

Question 6

Use Euler's theorem to determine the degree of homogeneity of the functions;

(a) $f(x, y) = x^3 + 2x^2y + y^3$

(b) $f(L, K) = AL^\alpha K^\beta$

What is Euler's theorem?

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$$

Where n is the degree of homogeneity; Hence;

(a)

$$f(x, y) = x^3 + 2x^2y + y^3$$

$$\frac{\partial f}{\partial x} = 3x^{3-1} + 2 \times 2x^{2-1}y + 0$$

$$\frac{\partial f}{\partial x} = 3x^2 + 4xy$$

$$\frac{\partial f}{\partial y} = 0 + 1 \times 2x^2y^{1-1} + 3y^{3-1}$$

$$\frac{\partial f}{\partial y} = 2x^2 + 3y^2$$

Hence,

$$x(3x^2 + 4xy) + y(2x^2 + 3y^2) = nf(x, y)$$

Expand!

$$3x^3 + 4x^2y + 2x^2y + 3y^3$$
$$= n(x^3 + 2x^2y + y^3)$$

Simplify;

$$3x^3 + 6x^2y + 3y^3 = n(x^3 + 2x^2y + y^3)$$

Factorization can be done on the left hand side;

$$3(x^3 + 2x^2y + y^3) = n(x^3 + 2x^2y + y^3)$$

Hence, by comparison;

$$n = 3$$

(b)

$$f(L, K) = AL^{\alpha}K^{\beta}$$

Here,

We have f, a multivariate function of L and K with A, α and β being constants.

Hence, Euler's theorem here will be;

$$Lf_L + Kf_K = nf(L, K)$$

Where f_L and f_K are the derivatives of Q with respect to L and K respectively.

Hence,

$$f_L = \alpha \times AL^{\alpha-1}K^{\beta}$$

$$f_L = \alpha AL^{\alpha-1}K^{\beta}$$

$$f_K = \beta AL^{\alpha}K^{\beta-1}$$

$$f_K = \beta AL^{\alpha}K^{\beta-1}$$

Hence,

$$L\left(\alpha AL^{\alpha-1}K^{\beta}\right) + K\left(\beta AL^{\alpha}K^{\beta-1}\right) = nf(L, K)$$

Hence,

$$L\left(\alpha AL^{\alpha-1}K^{\beta}\right) + K\left(\beta AL^{\alpha}K^{\beta-1}\right) = n\left(AL^{\alpha}K^{\beta}\right)$$

Indices!

$$\alpha AL^{\alpha-1+1}K^{\beta} + \beta AL^{\alpha}K^{\beta-1+1} = n\left(AL^{\alpha}K^{\beta}\right)$$

Simplify;

$$\alpha AL^{\alpha}K^{\beta} + \beta AL^{\alpha}K^{\beta} = n\left(AL^{\alpha}K^{\beta}\right)$$

Hence, $AL^{\alpha}K^{\beta}$ is common, factorize!

$$AL^{\alpha}K^{\beta}(\alpha + \beta) = n\left(AL^{\alpha}K^{\beta}\right)$$

$$(\alpha + \beta)AL^{\alpha}K^{\beta} = n\left(AL^{\alpha}K^{\beta}\right)$$

By comparison;

$$n = \alpha + \beta$$

Question 7

Compute the second order partial derivatives of the function:

$$f(x, y) = x^2 e^{-y}$$

and verify Young's theorem.

Looks like product rule but if you've thoroughly studies partial differentiation from the textbook, you'll then know that it isn't.

$$f(x, y) = x^2 e^{-y}$$

For f_x; y is a constant and in essence, e^{-y} is a constant as well.

$$f_x = e^{-y}(2 \times x^{2-1}) = 2xe^{-y}$$

Also; for f_y; x is a constant and in essence, x^2 is a constant as well.

$$f_y = x^2(-e^{-y}) = -x^2e^{-y}$$

Note that $\frac{\partial}{\partial y}(e^{-y}) = -e^{-y}$, you can verify that using chain rule, substituting for $u = -1$;

For the second order partials; we have:

f_{xx} is the partial derivative with respect to x of f_x

$$f_{xx} = (f_x)_x = \frac{\partial}{\partial x}(2xe^{-y})$$

Hence;

$$f_{xx} = e^{-y}(1 \times 2x^{1-1}) = 2e^{-y}$$

f_{yy} is the partial derivative with respect to y of f_y

$$f_{yy} = \left(f_y\right)_y = \frac{\partial}{\partial y}(-x^2e^{-y})$$

Hence;

$$f_{yy} = -x^2(-e^{-y}) = x^2e^{-y}$$

f_{xy} is the partial derivative with respect to y of f_x

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}(2xe^{-y})$$

Hence;

$$f_{xy} = 2x(-e^{-y}) = -2xe^{-y}$$

f_{yx} is the partial derivative with respect to x of f_y

$$f_{yx} = \left(f_y\right)_x = \frac{\partial}{\partial x}(-x^2e^{-y})$$

Hence;

$$f_{yx} = -e^{-y}(2 \times x^{2-1}) = -2xe^{-y}$$

Hence, as obvious enough; we have that:

$$f_{xy} = f_{yx}$$

The above verifies Young's theorem!

N.B.: as you know already, if they aren't equal, then, your solution is not correct.

Question 8

Learning of most skills starts at a rapid rate and then slows down. A psychologist measures the learning performance of a laboratory rat by a numerical score on a standardized test. Assume the rat's score $p(t)$ after t weeks of learning is:

$$p(t) = 15t^2 - t^3$$

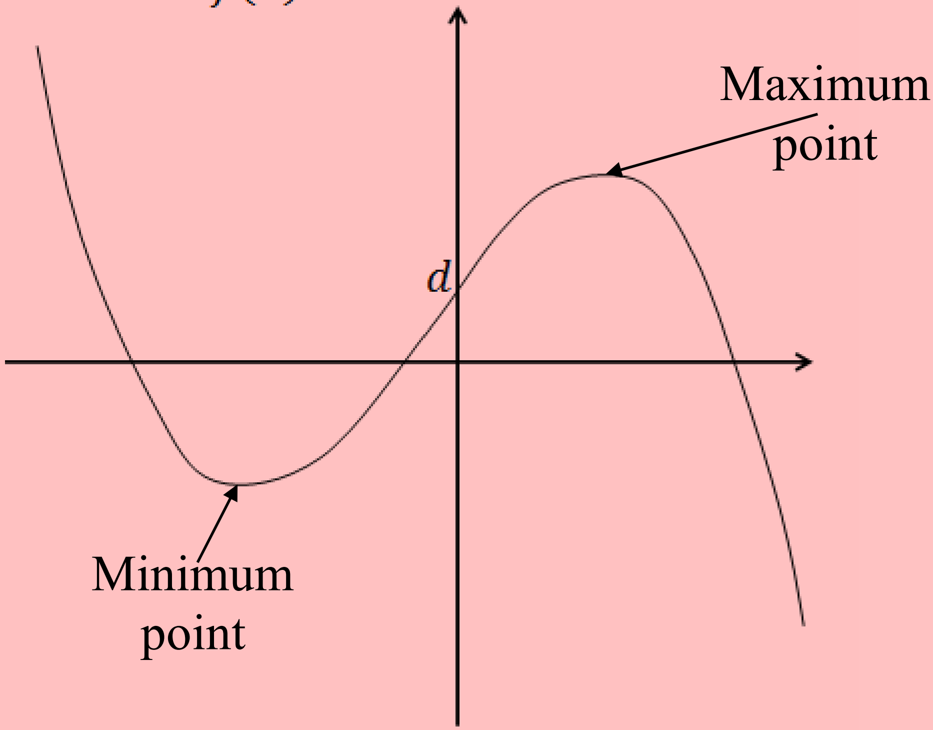
At what point in time does the rat's rate of learning begin to decline?

Lol. Don't lie. You're scared and probably think you can't answer this. However, we'll be done soon, it is something you know. I'll explain some things briefly now.

When we discussed function optimization, we discussed both the minimum and the maximum values, we'll get some light into it now using a sketch of the cubic function that has both minimum and maximum values.

The sketch of the curve is shown below, as an example of a cubic function.

$$f(x) = ax^3 + bx^2 + cx + d$$



The above picture represents a cubic function with its maximum and minimum points well represented. Looking at the curve's minimum point, it is quite obvious that from that minimum point, the curve starts going up (i.e. increasing) and that from the maximum point onwards, the curve starts going down. Hence, it's quite agreeable with normal life, when you are at the minimum (and it's truly the minimum), the only possible thing is to start going higher. And when you have reached your maximum, if you keep going onward, you'll definitely start going down. **Hence, to for the rat's rate of learning to start declining, we need the maximum point of the equation of the rat's rate of learning.**

$$p(t) = 15t^2 - t^3$$

Finding the first derivative;

$$p'(t) = 2 \times 15t^{2-1} - 3 \times t^{3-1}$$

$$p'(t) = 30t - 3t^2$$

At turning point; $p'(t) = 0$

$$30t - 3t^2 = 0$$

Solving;

$$3t(10 - t) = 0$$

Hence,

$$t = 0$$

or

$$t = 10$$

Solving for the second derivatives to find the nature of the turning points;

$$p''(t) = \frac{d}{dt}\big(p'(t)\big) = 1 \times 30t^{1-1} - 2 \times 3t^{2-1}$$

$$p''(t) = 30 - 6t$$

Testing for the turning points;

At $t = 0$;

$$p''(0) = 30 - 6(0) = 30 > 0$$

At $t = 10$;

$$p''(10) = 30 - 6(10) = -30 < 0$$

Hence, since we need the maximum point, the maximum point here is $t = 10$ since it is the point where the second derivative is less than zero.

Since, we are told in the question that it is t weeks, we have that the rat's learning rate starts declining at **10 weeks.**

I guess that was understood. It should with this explanation given here.

Question 9

Distinguish between a differential equation and a difference equation.

We didn't see anything like **a difference equation throughout our textbook.** However, we will define it here, keep it for reference purposes.

A differential equation is simply a mathematical relationship that describes the relationship between functions and their various derivatives (their various differential coefficients) *while* **a difference equation** is a mathematical equation involving an ordered sequence of real numbers (or numbers generally) when the sequence involves a recurrent (repeated) relationship.

Question 10

The aggregate consumption function for a community is given by:

$$C(x) = 210 + 4\sqrt{x}$$

Where $C(x) =$ total consumption; and

$x =$ disposable income of the community.

Find the marginal propensity to consume (MPC) when: $x = 16, 64, 100$.

Fine question! Don't allow English stop you from answering a question you should be answering quite easily!

Propensity is the tendency or possibility of something happening.

The aggregate consumption function is given by:

$$C(x) = 210 + 4\sqrt{x}$$

The consumption function gives the amount of consumption by the community and hence, it is the tendency for the community to consume.

By study of economics, the aggregate consumption function gives the propensity to consume. *You can study further concerning that in the field of economics.*

Hence, the marginal propensity to consume is the marginal aggregate consumption function and hence, we just find the marginal function of the given function **which I believe you know is by finding the first derivative.**

$$C(x) = 210 + 4\sqrt{x}$$
$$C(x) = 210 + 4x^{\frac{1}{2}}$$

The marginal function is given by the derivative:

$$C'(x) = 0 + \frac{1}{2} \times 4x^{\frac{1}{2}-1}$$

$$C'(x) = 2x^{-\frac{1}{2}} = 2 \times x^{-\frac{1}{2}}$$

$$C'(x) = \frac{2}{x^{\frac{1}{2}}}$$

Hence,
To evaluate the MPC at the different points we are given, we have;

MPC at $x = 16$;

$$C'(16) = \frac{2}{(16)^{\frac{1}{2}}} = \frac{2}{\sqrt{16}}$$

$$C'(16) = \frac{2}{4} = \frac{1}{2}$$

MPC at $x = 64$;

$$C'(64) = \frac{2}{(64)^{\frac{1}{2}}} = \frac{2}{\sqrt{64}}$$

$$C'(64) = \frac{2}{8} = \frac{1}{4}$$

MPC at $x = 100$;

$$C'(100) = \frac{2}{(100)^{\frac{1}{2}}} = \frac{2}{\sqrt{100}}$$

$$C'(100) = \frac{2}{10} = \frac{1}{5}$$

SECTION B

Question 1

(a) Outline the allied calculus conditions for the optimization of the function:

$$y = f(x)$$

(b) $\int \frac{f'(x)}{f(x)} dx =$. . ...

(c) If $y = e^{f(x)}$; then:

$$\frac{dy}{dx} = \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

(d) Verify that:

$$y = A_1 \log_e x + A_2$$

is the solution of the differential equation:

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

(a)

The conditions are in your notes;

For stationary points on a given function, $y = f(x)$ where:

$$\frac{dy}{dx} = 0 \quad \text{or} \quad f'(x) = 0$$

If;

$$\frac{d^2y}{dx^2} > 0, f''(x) > 0$$

Then the stationary point is **a minimum point.**

If;

$$\frac{d^2y}{dx^2} < 0, f''(x) < 0$$

Then the stationary point is **a maximum point.**

If;

$$\frac{d^2y}{dx^2} = 0, f''(x) = 0$$

Then the stationary point is **a point of inflexion**.

(b)

$$\int \frac{f'(x)}{f(x)} dx = \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

The above form is part of the cases we discussed when discussing the integration by substitution. The above is the **case 1** we discussed.

From what we well derived, we have that:

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$$

(c)

Fine;

$$y = e^{f(x)}$$

This is a case of chain rule;

$$u = f(x)$$

We have:

$$\frac{du}{dx} = f'(x)$$

I believe you are clear with the above, differentiating $f(x)$ gives $f'(x)$

Hence, we have:

$$y = e^{u}$$

Hence;

$$\frac{dy}{du} = e^{u}$$

By chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \times f'(x)$$

Hence;

$$\frac{dy}{dx} = f'(x)e^u$$

Return u;

$$\frac{dy}{dx} = f'(x)e^{f(x)}$$

(d)

$$y = A_1 \log_e x + A_2$$

To prove the identity below, don't be confused by English again to verify that it is the solution of the differential equation, we just need to solve for $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$, slot it into the equation and show that it is equal to zero.

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Hence,

$$y = A_1 \log_e x + A_2$$

We have:

$$\frac{dy}{dx} = A_1\left(\frac{1}{x}\right) + 0$$

We know that A_1 and A_2 are constants and that the derivative of $\log_e x$ is $\left(\frac{1}{x}\right)$

Hence;

$$\frac{dy}{dx} = \frac{A_1}{x}$$

Going further;

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{A_1}{x}\right)$$

We have:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(A_1 \times \frac{1}{x}\right) = \frac{d}{dx}(A_1 \times x^{-1})$$

$$\frac{d^2y}{dx^2} = A_1 \times -1 \times x^{-1-1}$$

$$\frac{d^2y}{dx^2} = -A_1 x^{-2} = -\frac{A_1}{x^2}$$

Hence, we now have the derivatives, let's slot into the equation.

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Hence; check;

$$x\left(-\frac{A_1}{x^2}\right) + \left(\frac{A_1}{x}\right)$$

Clear;

$$-\frac{A_1}{x} + \frac{A_1}{x}$$

We have:

$$0$$

It has been verified!

Question 2

(a) A matrix can be classified using:

(i) The relationship between its rows and columns;

(ii) The structure of its elements;

(iii) A derived relationship with itself.

With appropriate examples; list:

(i) three types of matrices under classification (i);

(ii) four types of matrices under classification (ii);

(iii) three types of matrices under classification (iii);

(b) Let:

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 8 & 16 \\ 7 & 6 & 1 \end{pmatrix} \text{ and } E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

Show that $|EA| = |E||A|$.

(a)

We had an exhaustive list in the textbook. Pick any you like;

(i) The relationship between its rows and columns;

✓ Rectangular matrix

✓ Square matrix

✓ Row matrix

(ii) The arrangement of its elements

- ✓ Null matrix
- ✓ Diagonal matrix
- ✓ Identity matrix
- ✓ Triangular matrix

(iii) A derived relationship with itself

- ✓ Orthogonal matrix
- ✓ Idempotent matrix
- ✓ Symmetric matrix

(b)

Given;

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 8 & 16 \\ 7 & 6 & 1 \end{pmatrix} \text{ and } E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

To show that $|EA| = |E||A|$.

This is the rule of the determinant of a product being equal to the product of their determinants, a similar question is in your note, it'll be equally lengthy; let's do this!

$$EA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 2 \\ 2 & 8 & 16 \\ 7 & 6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0(1) + (1)(2) + 0(7) & 0(4) + (1)(8) + 0(6) & 0(2) + (1)(16) + 0(1) \\ 1(1) + 0(2) + 0(7) & 1(4) + 0(8) + 0(6) & 1(2) + 0(16) + 0(1) \\ 0(1) + 0(2) + 1(7) & 0(4) + 0(8) + 1(6) & 0(2) + 0(16) + 1(1) \end{pmatrix}$$

Kindly zoom that!

$$EA = \begin{pmatrix} 2 & 8 & 16 \\ 1 & 4 & 2 \\ 7 & 6 & 1 \end{pmatrix}$$

Hence, we have EA;

$$|EA| = \begin{vmatrix} 2 & 8 & 16 \\ 1 & 4 & 2 \\ 7 & 6 & 1 \end{vmatrix}$$

$$|EA| = 2\begin{vmatrix} 4 & 2 \\ 6 & 1 \end{vmatrix} - (8)\begin{vmatrix} 1 & 2 \\ 7 & 1 \end{vmatrix} + 16\begin{vmatrix} 1 & 4 \\ 7 & 6 \end{vmatrix}$$

$$|EA| = 2(-8) - 8(-13) + 16(-22)$$

$$|EA| = -16 + 104 - 352 = -264$$

Then, proving the second identity; $|E||A|$, we take the determinants separately and multiply them; **it is that basic!**

$$|A| = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 8 & 16 \\ 7 & 6 & 1 \end{vmatrix}$$

$$|A| = 1\begin{vmatrix} 8 & 16 \\ 6 & 1 \end{vmatrix} - (4)\begin{vmatrix} 2 & 16 \\ 7 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 8 \\ 7 & 6 \end{vmatrix}$$

$$|A| = 1[(8)(1) - (16)(6)] - 4[(2)(1) - (16)(7)] + 2[(2)(6) - (8)(7)]$$

$$|A| = 1(-88) - 4(-110) + 2(-44) = 264$$

$$|E| = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$|E| = 0\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} - (1)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$$

$$|E| = 0 - 1[(1)(1) - (0)(0)] + 0$$

$$|E| = -1(1) = -1$$

Hence;

$$|E||A| = -1 \times 264 = -264$$

Hence, we have confirmed what we need to confirm since both are equal to -264.

Question 3

(a) State the following rules of integration;

(i) Power function rule;
(ii) Exponential function rule;
(iii) Logarithm function rule.

(b) Evaluate:

(i) $$\int \left(x^{\alpha} + x^{\beta} + x^{\gamma}\right) dx$$

(ii) $$\int (3x + 5)^{10} dx$$

(iii) $$\int_{-a}^{a} f(x) dx$$

(a)

We are asked about rules of integration, some are direct rules we learnt, however, some are indirect, we'll see how we'll carefully answer them though!

(i)

The power function rule of integration refers to the integral of a function with a constant power;

Hence, we have the power function rule as:

$$\int x^n = \frac{x^{n+1}}{n+1}$$

(ii)

As for the exponential function rule, we know what an exponential function is, hence, we have the integrals of the two forms of exponents which are a^x and e^x; hence, we have:

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

And;

$$\int e^x \, dx = e^x + C$$

(iii)

As for the logarithm functions, there is no standard form as it were; however, throwback into the note, you'll remember that we were able to find the integral of $\log_e x$ using the integration by parts formula, hence, we will now copy the results we had there right in here;

For the integration of logarithm function, we have:

$$\int \log_e x \, dx = x \log_e x - x$$

(b)

Integrals! Integrals! Integrals!

(i)

$$\int (x^{\alpha} + x^{\beta} + x^{\gamma}) dx$$

This is the power rule since it's with respect to x we're integrating to;

$$\int (x^{\alpha} + x^{\beta} + x^{\gamma}) dx$$

$$\int x^{\alpha} dx + \int x^{\beta} dx + \int x^{\gamma} dx$$

$$\left[\frac{x^{\alpha+1}}{\alpha+1}\right]+\left[\frac{x^{\beta+1}}{\beta+1}\right]+\left[\frac{x^{\gamma+1}}{\gamma+1}\right]+C$$

Perfect!

(ii)

$$\int (3x+5)^{10}dx$$

Constant substitution!
We have:

$$u=3x+5$$

$$\frac{du}{dx}=3$$

Hence;

$$dx=\frac{du}{3}$$

We have:

$$\int u^{10}\frac{du}{3}$$

Bring constant out!

$$\frac{1}{3}\int u^{10}=\frac{1}{3}\left[\frac{u^{10+1}}{10+1}\right]$$

Power rule was used above!

$$\frac{1}{3} \times \frac{u^{11}}{11} = \frac{u^{11}}{33}$$

Return the substitution!

$$\frac{(3x+5)^{11}}{33} + C$$

(iii)

$$\int_{-a}^{a} f(x)dx$$

This tests your understanding on integral limits;

Let the indefinite integral of $f(x)$ yield another function of x, $g(x)$, we have:

$$\int f(x)dx = g(x)$$

Then;

$$\int_{-a}^{a} f(x)dx = [g(x)]_{-a}^{a}$$

And;
From integral limits, we have;

$$\int_{-a}^{a} f(x)dx = g(a) - g(-a)$$

THAT'S IT! Quite some a year with some really technical questions, as usual, they are never unsolved in the SSC106 way!