DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
SSC106: MATHEMATICS FOR SOCIAL SCIENCES II
RAIN SEMESTER EXAMINATION (2001/2002 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A;**
- Answer any two questions from **Section B.**
- Show all workings clearly

Time allowed: 2 hours

SECTION A

1. What type of functions are the following:

   (i) $y = 2x - 5$

   (ii) $y = e^{x+1}$

   (iii) $y = \frac{2x^2+5}{2x+1}$

   (iv) $x^2 + y^2 = 64$

   (v) $y = \log_{10} X$

2. If $a_{12} = a_{21} = 5$; $a_{32} = 3a_{13}$
$a_{31} = 3a_{12} - 10$ and $a_{13} = (a_{31})^2$;
Determine completely the matrix A specified as:

$$A = \begin{pmatrix} 6 & - & - \\ - & 7 & 3 \\ - & - & 10 \end{pmatrix}$$

3. Differentiate the following functions:

(i) $y = 2n^2 + 5n + 6$
(ii) $y = 4\log 2x$
(iii) $y = e^{x^2+3}$

4. Integrate the following functions:
(i) $\int b\,dx$
(ii) $\int 4n^{-1}\,dn$
(iii) $\int (3x^3 - x + 1)dx$

5. *(a)* State Young's theorem.
(b) Verify in accordance with Young's theorem the following:
(i) $Z = 3x^2y^3$
(ii) $Z = 9x^2y^3$

6. Evaluate:

(i) $\int_1^5 \frac{dx}{x-2}$

(ii) $\int_0^1 e^{2t}\, dt$

SECTION B

1. *(a)* Using the Lagrangean multiplier method, optimize the objective function: $Z = f(x, y) = 4x^2 - 3x + 5xy - 8y + 2y^2$;
Subject to the constraint
$x + 2y = 10.$

(b) Establish whether the solutions established in *(a)* is a minimum or a maximum.

2. *(a)* Using Crammer's rule, solve the following system of linear equations;

$$3x + 4y + 5z = 4$$
$$2x - 3y + 3z = 8$$
$$2x + 2y - 4z = 4$$

(b) Given that $X = \begin{pmatrix} 4 & 0 & 3 \\ 1 & 5 & 7 \\ 3 & 3 & 9 \end{pmatrix}$

(i) Find $|X|$

(ii) Show that interchanging rows 1 and 2 will change the sign but not the absolute value of the determinant of X.

3. *(a)* Find $\int \frac{X+1}{X^2 + 5X + 6}\, dX$

(b) Given the marginal revenue $MR = 60 - 2Q - 2Q^2$; find

(i) the total revenue; TR function;

(ii) the value of TR function when $Q = 20$.

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2001/2002 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and two questions from Section B, we'll be answering everything in both sections in this book though; keep calm and move with me;

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

Question 1

What type of functions are the following:

(i) $y = 2x - 5$

(ii) $y = e^{x+1}$

(iii) $y = \frac{2x^2+5}{2x+1}$

(iv) $x^2 + y^2 = 64$

(v) $y = \log_{10} X$

Question 1

We're told to state the types of functions in the listed functions; we know all the different types of functions from the first major topic in this book; **FUNCTIONS;** for these questions; here are the solutions;

(i) $y = 2x - 5$

This is a linear function, of course it's an explicit function as well, but more notably, it's **a linear function**.

(ii) $y = e^{x+1}$

This is an **exponential function**. Of course it's also an implicit function but of course the most notable classification it belongs is **the exponential function.**

(iii) $y = \frac{2x^2+5}{2x+1}$

This is notably **a rational function**. It's an expression of two polynomials dividing each other.

(iv) $x^2 + y^2 = 64$

The most notable characteristic of this function is that it is **an implicit function**. It's not an explicit function and falls into none of the other classifications.

(v) $y = \log_{10} X$

This without any story is a logarithm function, it is very obvious; although it is also an explicit function, it's most characteristically **a logarithm function.**

Notice we have refused to choose explicit function in any of the above classifications; that is because it is a more general classification and the more specific types are needed; explicit function isn't a wrong answer, they're actually explicit but you'll be scored wrong if you place explicit as the answer, the more specific answers are needed here.

Question 2

If $a_{12} = a_{21} = 5$; $a_{32} = 3a_{13}$
$a_{31} = 3a_{12} - 10$ and $a_{13} = (a_{31})^2$;

Determine completely the matrix A specified as:

$$A = \begin{pmatrix} 6 & - & - \\ - & 7 & 3 \\ - & - & 10 \end{pmatrix}$$

Here, we know how we deal with matrices positions, a_{mn}, with m the row position and n the column position, hence, this is what we have;

$$a_{12} = a_{21} = 5$$

The a_{12} and a_{21} positions are given thus as simple as that;

$$a_{32} = 3a_{13}$$

From our matrix, the a_{32} and a_{13} are still unknown;

$$a_{31} = 3a_{12} - 10$$

From our matrix that was shown to us, a_{12} was unknown but from the first information we were given; $a_{12} = a_{21} = 5$, hence, $a_{12} = 5$

Therefore;

$$a_{31} = 3(5) - 10 = 15 - 10 = 5$$

Also;

$$a_{13} = (a_{31})^2$$

We just evaluated a_{31}; hence;

$$a_{13} = (5)^2 = 25$$

Back here;

$$a_{32} = 3a_{13}$$

Hence,

$$a_{32} = 3(25) = 75$$

The complete matrix therefore is;

$$\begin{pmatrix} 6 & 5 & 25 \\ 5 & 7 & 3 \\ 5 & 75 & 10 \end{pmatrix}$$

Question 3

Differentiate the following functions:

(i) $y = 2n^2 + 5n + 6$

(ii) $y = 4\log 2x$

(iii) $y = e^{x^2+3}$

I do not expect you to have a single problem in this; this we will go straight; even though the questioner has omitted what we are to be differentiating with respect to, we'll help ourselves;
(i)
$y = 2n^2 + 5n + 6$ Here, we should be differentiating with respect to n here; hence, we have; $\frac{dy}{dn} = 2 \times 2 \times n^{2-1} + 1 \times 5 \times n^{1-1} + 0$ $\frac{dy}{dn} = 4n + 5$
(ii)
$y = 4 \log 2x$ Here, we'll be differentiating with respect to x;   4 is a constant and hence, we face $\log 2x$ squarely; $u = 2x$ $\frac{du}{dx} = 1 \times 2 \times x^{1-1} = 2$

Hence,

$$y = 4\log u$$

Without base implies base 10;
Hence,

$$y = 4\log_{10} u$$

$$\frac{dy}{du} = 4\left(\frac{1}{u\ln 10}\right)$$

Hence, from chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{4}{u\ln 10} \times 2 = \frac{8}{u\ln 10}$$

Return $u = 2x$

$$\frac{dy}{dx} = \frac{8}{2x\ln 10}$$

2 cancels out;

$$\frac{dy}{dx} = \frac{4}{x\ln 10}$$

(iii)

$$y = e^{x^2+3}$$

Straight, we need a substitution;

$$u = x^2 + 3$$

$$\frac{du}{dx} = 2 \times x^{2-1} + 0 = 2x$$

Hence,

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

Hence, from chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \times 2x = 2xe^u$$

Return $u = x^2 + 3$

$$\frac{dy}{dx} = 2xe^{x^2+3}$$

Question 4.

Integrate the following functions:

(i)	$\int b \, dx$

(ii)	$\int 4n^{-1}\,dn$
(iii)	$\int (3x^3 - x + 1)dx$

We have integrals here; let's take them and tackle them;
(i)
$$\int b\,dx$$ Pretty simple, since we're integrating with respect to x (from the sign dx), then, we'll have that b is nothing but a constant, we hence have; $$\int bx^0\,dx = b\int x^0 dx$$ $$b\left[\frac{x^{0+1}}{0+1}\right] = bx + C$$
(ii)
$$\int 4n^{-1}\,dn$$

Also pretty simple; differentiating with respect to n and hence, we're doing it thus; $$\int 4n^{-1}\,dn = 4\int n^{-1}\,dn$$ Integral of power of -1 does not follow other rules but is simply the natural log; hence; we have; $$4\ln n + C$$ Our arbitrary constant must be added;
(iii)
$$\int (3x^3 - x + 1)dx$$ Integral of sums, we'll split; $$\int 3x^3 dx - \int x dx + \int 1 dx$$ We take each separately; $$3\int x^3 dx - \int x dx + 1\int x^0 dx$$ $$3\left[\frac{x^{3+1}}{3+1}\right] - \left[\frac{x^{1+1}}{1+1}\right] + \left[\frac{x^{0+1}}{0+1}\right]$$

$$\frac{3x^4}{4} - \frac{x^2}{2} + x + C$$
Our arbitrary constant is added in all cases of integration;
Question 5. (a) State the Young's theorem (b) Verify in accordance with Young's theorem the following: (i) $Z = 3x^2y^3$ (ii) $Z = 9x^2y^3$
(a)
State the Young's theorem, that's English language; state it as required, the theory of mixed partials! It states that two complementary mixed partials of a continuous and twice differentiable function are equal;

(b)
To verify for each functions; take the mixed partials; i.e. differentiate partially with respect to x first and then with y and then differentiate partially with respect to y first and then with x
(i) $Z = 3x^2y^3$ $$Z_x = 2 \times 3x^{2-1} \times y^3 = 6xy^3$$ $$Z_y = 3 \times 3 \times x^2 \times y^{3-1} = 9x^2y^2$$ $$Z_{xy} = \frac{\partial}{\partial y}(6xy^3) = 3 \times 6x \times y^{3-1} = 18xy^2$$ $$Z_{yx} = \frac{\partial}{\partial x}(9x^2y^2) = 2 \times 9 \times x^{2-1} \times y^2 = 18xy^2$$ Obviously; $Z_{xy} = Z_{yx}$ and hence, Young's theorem is true in (i);
(ii) $Z = 9x^2y^3$ $$Z_x = 2 \times 9x^{2-1} \times y^3 = 18xy^3$$ $$Z_y = 3 \times 9 \times x^2 \times y^{3-1} = 27x^2y^2$$

$$Z_{xy} = \frac{\partial}{\partial y}(18xy^3) = 3 \times 18x \times y^{3-1} = 54xy^2$$

$$Z_{yx} = \frac{\partial}{\partial x}(27x^2y^2) = 2 \times 27 \times x^{2-1} \times y^2 = 18xy^2$$

Obviously; $Z_{xy} = Z_{yx}$ and hence, Young's theorem is also true in (ii);

Recall I told you that Young's theorem is true for all functions that can be partially differentiated at least twice.

Question 6

Evaluate;

(i)	$\int_1^5 \frac{dx}{x-2}$
(ii)	$\int_0^1 e^{2t}\,dt$

Integrals again;

(i)

$$\int_1^5 \frac{dx}{x-2}$$

We evaluate the indefinite integral first;

$$\int \frac{dx}{x-2} = \int \frac{1}{x-2}\,dx$$

Put $u = x - 2$

$$\frac{du}{dx} = 1$$

Hence,

$$dx = du$$

We have;

$$\int \frac{1}{u}\,du = \ln u$$

Replace u;

We have;

$$\ln(x-2)$$

Evaluate with the limits;

$$[\ln(x-2)]_1^5 = \ln(5-2) - \ln(1-2)$$

We have:

$$\ln 3 - \ln(-1)$$

Logarithms do not exist for negative numbers;

We'll simply keep the answer as above as an undefined input is part of the answer;

(ii)

$$\int_0^1 e^{2t}\, dt$$

Again, let's evaluate the indefinite integral first;

$$\int e^{2t}\, dt$$

Put $u = 2t$

$$\frac{du}{dt} = 2$$

Hence,

$$dt = \frac{du}{2}$$

We have;

$$\int e^u \frac{du}{2} = \frac{1}{2}\int e^u\, du$$

$$\frac{1}{2}(e^u) = \frac{1}{2}e^u = \frac{1}{2}e^{2t}$$

Evaluate with the limits:

$$\left[\frac{1}{2}e^{2t}\right]_0^1 = \frac{1}{2}e^{2(1)} - \frac{1}{2}e^{2(0)}$$

$$\frac{1}{2}e^2 - \frac{1}{2}e^0 = \frac{1}{2}(7.3891) - \frac{1}{2}(1)$$

$$3.6945 - 0.5 = 3.1945$$
We evaluated e^2 from our calculator; e^0 is 1, anything raised to power zero;

SECTION B

Question 1

(a) Using the Langragean multiplier method, optimize the objective function: $Z = f(x, y) = 4x^2 - 3x + 5xy - 8y + 2y^2$; Subject to the constraint $x + 2y = 10$.

(b) Establish whether the solutions established in *(a)* is a minimum or a maximum.

(a)

So, this Lagrangean something; let's do it by the rule; we write the Lagrangean function;

$$\mathcal{L}(x, y, \lambda) = f(x, y) - \lambda[g(x, y)]$$

All symbols have their usual meanings;

Express the constraint equal to zero;

$$x + 2y = 10$$
$$x + 2y - 10 = 0$$

Hence, we have the Lagrangean equation for this question as:

$$\mathcal{L}(x, y, \lambda) = 4x^2 - 3x + 5xy - 8y + 2y^2 - \lambda(x + 2y - 10)$$

Let's take the partials with respect to x, y and λ

$$\mathcal{L}_x = 2 \times 4 \times x^{2-1} - 1 \times 3 \times x^{1-1} + 1 \times 5 \times x^{1-1} \times y - 0 + 0 - \lambda(1 \times x^{1-1} + 0 - 0)$$

$$\mathcal{L}_x = 8x - 3 + 5y - \lambda$$

$$\mathcal{L}_y = 0 - 0 + 1 \times 5x \times y^{1-1} - 1 \times 8 \times y^{1-1} + 2 \times 2 \times y^{2-1} - \lambda(0 + 1 \times 2 \times y^{1-1} - 0)$$

$$\mathcal{L}_y = 5x - 8 + 4y - 2\lambda$$

$$\mathcal{L}_\lambda = 0 - 0 + 0 - 0 + 0 - 1 \times \lambda^{1-1}(x + 2y - 10)$$

$$\mathcal{L}_\lambda = -x - 2y + 10$$

Then, we equate each to zero;

$$\mathcal{L}_x = 0$$
$$\mathcal{L}_y = 0$$
$$\mathcal{L}_\lambda = 0$$

$$8x - 3 + 5y - \lambda = 0 \dots \dots \dots (1)$$

$$5x - 8 + 4y - 2\lambda = 0 \dots \dots \dots (2)$$

$$-x - 2y + 10 = 0 \dots \dots \dots (3)$$

Multiplying equation (1) by 2;

$$\mathbf{2} \times (\mathbf{1}):\ 2(8x - 3 + 5y - \lambda = 0)$$

$$16x - 6 + 10y - 2\lambda = 0 \dots \dots \dots (4)$$

Subtract eq(2) from eq(4);

$$16x - 6 + 10y - 2\lambda = 0$$
$$5x - 8 + 4y - 2\lambda = 0$$

-2λ cancels out;

$$11x + 2 - 6y = 0 \dots \dots \dots (5)$$

Combine this (5) with (3);

Multiplying (3) by 3;

$$\mathbf{3} \times (\mathbf{3}):\ 3(-x - 2y + 10 = 0)$$

$$-3x - 6y + 30 = 0 \dots\dots\dots (6)$$

Subtract eq(6) from eq(5);

$$14x - 28 = 0$$
$$14x = 28$$
$$x = 2$$

From (5);

$$11x + 2 - 6y = 0$$
$$11(2) + 2 - 6y = 0$$
$$22 + 2 - 6y = 0$$
$$24 - 6y = 0$$
$$6y = 24$$
$$y = 4$$

From (4);

$$16x - 6 + 10y - 2\lambda = 0$$
$$16(2) - 6 + 10(4) - 2\lambda = 0$$
$$32 - 6 + 40 - 2\lambda = 0$$
$$66 - 2\lambda = 0$$
$$\lambda = 33$$

(b)

To establish the nature of this optimal points;

Take the direct second order partial derivatives;

$$\mathcal{L}_{xx} = \frac{\partial}{\partial x}(8x - 3 + 5y - \lambda)$$

$$\mathcal{L}_{xx} = 1 \times 8 \times x^{1-1} - 0 + 0 - 0 = 8$$

$$\mathcal{L}_{yy} = \frac{\partial}{\partial y}(5x - 8 + 4y - 2\lambda)$$

$$\mathcal{L}_{yy} = 0 - 0 + 1 \times 4 \times y^{1-1} - 0 = 4$$

Since both $\mathcal{L}_{xx}$ and $\mathcal{L}_{yy}$ are positive, i.e. greater than zero; from the second order conditions; they're minimum points;

Hence, $x = 2$, $y = 4$ is a minimum point

Question 2

This is the use of Crammer's rule; you know the basic rules; read the matrices topic in case you have forgotten the rules;

$$3x + 4y + 5z = 4$$
$$2x - 3y + 3z = 8$$
$$2x + 2y - 4z = 4$$

We make our first determinant; Δ

$$\Delta = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -3 & 3 \\ 2 & 2 & -4 \end{vmatrix}$$

$$3\begin{vmatrix} -3 & 3 \\ 2 & -4 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 2 & -4 \end{vmatrix} + 5\begin{vmatrix} 2 & -3 \\ 2 & 2 \end{vmatrix}$$

$$3[(-3)(-4) - (3)(2)] - 4[(2)(-4) - (3)(2)] + 5[(2)(2) - (2)(-3)]$$

$$3[6] - 4[-14] + 5[10] = 124$$

For Δ_x, replace the column of x with the column matrix of the answers;

$$\Delta_x = \begin{vmatrix} 4 & 4 & 5 \\ 8 & -3 & 3 \\ 4 & 2 & -4 \end{vmatrix}$$

$$4[(-3)(-4) - (3)(2)] - 4[(8)(-4) - (3)(4)] + 5[(8)(2) - (4)(-3)]$$

$$4[6] - 4[-44] + 5[28] = 340$$

For Δ_y, replace the column of y with the column matrix of the answers;

$$\Delta_y = \begin{vmatrix} 3 & 4 & 5 \\ 2 & 8 & 3 \\ 2 & 4 & -4 \end{vmatrix}$$

$$3[(8)(-4)-(3)(4)]-4[(2)(-4)-(3)(2)] \\ +5[(2)(4)-(8)(2)]$$

$$3[-44]-4[-14]+5[-8]=-116$$

For Δ_z, replace the column of z with the column matrix of the answers;

$$\Delta_y = \begin{vmatrix} 3 & 4 & 4 \\ 2 & -3 & 8 \\ 2 & 2 & 4 \end{vmatrix}$$

$$3[(-3)(4)-(8)(2)]-4[(2)(4)-(8)(2)] \\ +4[(2)(2)-(2)(-3)]$$

$$3[-28]-4[-8]+4[10]=-12$$

Hence,

$$x=\frac{\Delta_x}{\Delta}=\frac{340}{124}=\frac{85}{31}$$

$$y=\frac{\Delta_y}{\Delta}=\frac{-116}{124}=-\frac{29}{31}$$

$$z=\frac{\Delta_z}{\Delta}=\frac{-12}{124}=-\frac{3}{31}$$

That's it about that simultaneous equation; the next part;

(b)

To prove what we're told to do; we simply evaluate the determinant as we're told to in the first part and interchange the required rows in the second part;

(i)

$$X = \begin{pmatrix} 4 & 0 & 3 \\ 1 & 5 & 7 \\ 3 & 3 & 9 \end{pmatrix}$$

$$|X| = \begin{vmatrix} 4 & 0 & 3 \\ 1 & 5 & 7 \\ 3 & 3 & 9 \end{vmatrix}$$

$$4[(5)(9) - (7)(3)] - 0[(1)(9) - (3)(7)] + 3[(1)(3) - (5)(3)]$$

$$|X| = 4[24] - 0[-12] + 3[-12] = 60$$

(ii)

The second part is very interesting;

This is X;

$$X = \begin{pmatrix} 4 & 0 & 3 \\ 1 & 5 & 7 \\ 3 & 3 & 9 \end{pmatrix}$$

Let Y be another matrix such that row 1 and row 2 are interchanged; hence;

$$Y = \begin{pmatrix} 1 & 5 & 7 \\ 4 & 0 & 3 \\ 3 & 3 & 9 \end{pmatrix}$$

Then, the determinant;

$$|Y| = \begin{vmatrix} 1 & 5 & 7 \\ 4 & 0 & 3 \\ 3 & 3 & 9 \end{vmatrix}$$

$$1[(0)(9) - (3)(3)] - 5[(4)(9) - (3)(3)] + 7[(4)(3) - (0)(3)]$$

$$1[-9] - 5[27] + 7[12] = -60$$

Hence, we can see that only the sign is changed; the absolute value is 60 in both cases; absolute value means the positive value of any number, negative or positive, it was mentioned in the note; check ***Page 36 of Calculus Application to Economics***

So we move to the last question

Question 3

(a) Find $\int \frac{X+1}{X^2+5X+6}\, dX$

(b) Given the marginal revenue $MR = 60 - 2Q - 2Q^2$; find

(iii) the total revenue; TR function;

(iv) the value of TR function when $Q = 20$.

(a)

$$\int \frac{X+1}{X^2+5X+6}\, dX$$

This is a clear case of partial fractions, so we'll resolve it into partial fractions;

Factorize the denominator;

$$X^2 + 5X + 6$$

$6 \times X^2 = 6X^2$
$2X + 3X = 5X$
$2X \times 3X = 6X^2$

$$X^2 + 2X + 3X + 6$$
$$X(X+2) + 3(X+2)$$
$$(X+2)(X+3)$$

Hence, we have;

$$\frac{X+1}{X^2+5X+6} \equiv \frac{A}{X+2} + \frac{B}{X+3}$$

Clear out every needed clearing, we're left with;

$$X + 1 = A(X + 3) + B(X + 2)$$

Here,

Let $X + 3 = 0$

Here,

$$X = -3$$

Then, substitute in the main equation

$$-3 + 1 = A(-3 + 3) + B(-3 + 2)$$

$$-2 = A(0) + B(-1)$$

$$-2 = -B$$

Here,

$$B = 2$$

Again,

Let $X + 2 = 0$

Hence,

$$X = -2$$

Then, substitute in the main equation

$$-2 + 1 = A(-2 + 3) + B(-2 + 2)$$

$$-1 = A(1) + B(0)$$

$$-1 = A$$

Here,

$$A = -1$$

We have found our two values for A and B and we're good!

$$\frac{X+1}{X^2+5X+6} \equiv \frac{-1}{X+2} + \frac{2}{X+3}$$

We can rearrange this;

$$\frac{X+1}{X^2+5X+6} \equiv \frac{2}{X+3} - \frac{1}{X+2}$$

$$\int \frac{X+1}{X^2+5X+6}\, dX \equiv \int \left(\frac{2}{X+3} - \frac{1}{X+2}\right) dX$$

From the integrals of sums and differences; we'll have:

$$\int \frac{X+1}{X^2+5X+6}\, dX \equiv \int \frac{2}{X+3}\, dX - \int \frac{1}{X+2}\, dX$$

Take the integrals separately;

$$\int \frac{2}{X+3}\,dX$$

Put $u = X + 3$;

$$\frac{du}{dX} = 1$$

Hence,

$$dX = du$$

We have;

$$\int \frac{2}{u}du = 2\int \frac{1}{u}du = 2(\ln u)$$

We have;

$$2\ln(X+3)$$

Next!

$$\int \frac{1}{X+2}\,dX$$

Put $z = X + 2$;

$$\frac{dz}{dX} = 1$$

Hence,

$$dX = dz$$

We have;

$$\int \frac{1}{z}dz = (\ln z)$$

We have;

$$\ln(X+2)$$

Finally, let's combine everything;

$$\int \frac{2}{X+3} dX - \int \frac{1}{X+2} dX$$

$$2\ln(X+3) - \ln(X+2) + C$$

Of course, our arbitrary constant is added;

(b)

From our marginal revenue; integrate to get the revenue function; this was a household statement in the chapter of the application of calculus to economics;

(i)

$$MR = 60 - 2Q - 2Q^2$$

Integrate;

$$TR = \int MR = \int 60 - 2Q - 2Q^2 \, dQ$$

$$TR = \int 60dQ - \int 2Q \, dQ - \int 2Q^2 \, dQ$$

$$TR = 60 \int Q^0 dQ - 2 \int Q \, dQ - 2 \int Q^2 \, dQ$$

$$TR = 60\left[\frac{Q^{0+1}}{0+1}\right] - 2\left[\frac{Q^{1+1}}{1+1}\right] - 2\left[\frac{Q^{2+1}}{2+1}\right]$$

$$TR = 60Q - Q^2 - \frac{2Q^3}{3} + C$$

Now, the arbitrary constant is added; but normally, zero revenue is made when zero products are sold, hence, C is always zero;;

$$TR = 60Q - Q^2 - \frac{2Q^3}{3}$$

(ii)

At $Q = 20$;

$$TR = 60(20) - (20)^2 - \frac{2(20)^3}{3}$$

$$TR = 1200 - 400 - 5333.333 = -4533.333$$

That's it about SSC106 2001/2002 SESSION. TRUST YOU CAN BEAR WITNESS THAT EVERYTHING HAS BEEN TAUGHT IN DETAILS IN THIS BOOK!!!