ADDENDUM NOTES ON MATRICES

No much clarifications needed as it were, **the notes on matrices need no clarifications.** However, somewhere in the past questions for certain questions on matrices, a matrix, **positive semi-indefinite matrix** was made mentioned and I'll like to make some clarifications here;

A positive semi-indefinite matrix is no different from a symmetric matrix, it's just a mere difference in name. By the definition in the past questions, it is a matrix whose row elements are equal to its corresponding column elements. Such a matrix, when transposed will yield itself, and hence, it is similarly a matrix whose transpose is equal to itself and hence, **the positive semi-indefinite matrix is a symmetric matrix.** Since the symmetric matrix is the commonest matrix type, the symmetric matrix is used most often. Refer to Exam **2016/2017 (Q1(a)), 2003/2004 (Q2(b)), 2006/2007 (Q2(c))** for reference to this.

ADDENDUM NOTES ON DIFFERENTIATION

For the sake of few clarifications, here are some added notes on differential calculus.

The logarithm rule of differentiation;

The logarithm rules of differentiation can be really confusing many times. Especially for the difference in the natural log and log to a certain base. As clearly stated in the notes;

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Hence, many times, in instances like this:

$$\frac{d}{dx}(\log x)$$

We know that $\log x$ is same as $\log_{10} x$ since log without base is the base 10; hence;

$$\frac{d}{dx}(\log_{10} x) = \frac{1}{x \ln 10}$$

Also, for the definition of the logarithm rule of differentiation; some common texts state the logarithm rule of differentiation thus;

$$\frac{d}{dx}[\log_e f(x)] = \frac{f'(x)}{f(x)}$$

Well, the above is simply derived from chain rule since we can easily explain thus; consider;

$$y = \log_e(x^2 + 3)$$

You know as it is done to differentiate y with respect to x;

$$u = x^2 + 3$$

$$\frac{du}{dx} = 2x$$

Hence;

$$y = \log_e u$$

$$\frac{dy}{du} = \frac{1}{u}$$

From chain rule, we have that:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u} \times 2x = \frac{2x}{u}$$

Return u;

$$\frac{dy}{dx} = \frac{2x}{x^2 + 3}$$

Now compare $\log_e(x^2 + 3)$ to $\log_e f(x)$, we can see that:

$$f(x) = x^2 + 3$$

And hence;

$$f'(x) = 2x$$

And by the definition;

$$\frac{d}{dx}[\log_e f(x)] = \frac{f'(x)}{f(x)}$$

Obviously;

$$\frac{d}{dx}[\log_e(x^2 + 3)] = \frac{2x}{x^2 + 3}$$

And hence, the definition of the logarithm rule of differentiation was derived from the chain rule you know well.

Hence, for the logarithm function; $\log_e f(x)$

$$\frac{d}{dx}[\log_e f(x)] = \frac{f'(x)}{f(x)}$$

Similarly;

$$\frac{d}{dx}[\log_a f(x)] = \frac{f'(x)}{f(x)\ \ln a}$$

I hope that was quite clear right? It was derived from chain rule and can be taken as a definition for the logarithm rule of differentiation. However, as carefully explained in the notes, proper use of chain rule is the safest way to go about differentiation without entering into conflicts. We took enough examples on using chain rule to resolve derivatives of logarithm functions and the examples are sufficient enough. The above proof is just for a definitional purpose.

The exponential rule of differentiation;

The exponential rules of differentiation can be really confused many times. Especially for the difference in the natural exponent and other exponent. As clearly stated in the notes;

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Hence, in instances like this:

$$\frac{d}{dx}(3^x)$$

We have:

$$\frac{d}{dx}(3^x) = 3^x \ln 3$$

Also, for the definition of the exponential rule of differentiation; some common texts state the exponential rule of differentiation thus;

$$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)e^x$$

Well, the above is again, simply derived from chain rule which we can easily explain thus; consider;

$$y = e^{x^2+3}$$

You know as it is done to differentiate y with respect to x;

$$u = x^2 + 3$$

$$\frac{du}{dx} = 2x$$

Hence;

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

From chain rule, we have that:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \times 2x = 2xe^u$$

Return u;

$$\frac{dy}{dx} = 2xe^{x^2+3}$$

Now compare e^{x^2+3} to $e^{f(x)}$, we can see that:

$$f(x) = x^2 + 3$$

And hence;

$$f'(x) = 2x$$

And by the definition;

$$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)e^{x}$$

Obviously;

$$\frac{d}{dx}\left[e^{x^2+3}\right] = 2xe^{x^2+3}$$

And hence, the definition of the exponential rule of differentiation was derived from the chain rule you know well.

Hence, for the exponential function; $e^{f(x)}$

$$\boldsymbol{\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)e^{x}}$$

Similarly;

$$\boldsymbol{\frac{d}{dx}\left[a^{f(x)}\right] = f'(x)a^{x}\ln a}$$

That again should be quite clear? It was again derived from chain rule and can be taken as a definition for the exponential rule of differentiation. However, as carefully explained in the notes, proper use of chain rule is the safest way to go about differentiation without entering into conflicts. We took enough examples on using chain rule to resolve derivatives of exponential functions and the examples are sufficient enough. The above proof is just for a definitional purpose.

By using the above method, it is obvious that in other rules such as trigonometric rules:

By substitution of:

$$u = f(x)$$

Applying chain rule, it applies to all functions that:

$$\frac{d}{dx}[\sin f(x)] = f'(x) \cos f(x)$$

$$\frac{d}{dx}[\cos f(x)] = -f'(x) \sin f(x)$$

$$\frac{d}{dx}[\tan f(x)] = f'(x) \sec^2 f(x)$$

$$\frac{d}{dx}[\sec f(x)] = f'(x)\sec f(x)\tan f(x)$$

$$\frac{d}{dx}[\operatorname{cosec} f(x)] = -f'(x)\cot f(x)\operatorname{cosec} f(x)$$

$$\frac{d}{dx}[\cot f(x)] = -f'(x)\operatorname{cosec}^2 f(x)$$

All the above is by knowing fully well that:

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x\tan x$$

$$\frac{d}{dx}[\operatorname{cosec} x] = -\cot x\operatorname{cosec} x$$

$$\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$$

Hence, we simply understood all the following without overbearing formulas but by applying chain rule gradually. The formulas above were gotten through chain rule using the very most basic formulas **and are only for definition sake.** For real problem solving, the broken down method of chain rule is the best.