DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
SSC106: MATHEMATICS FOR SOCIAL SCIENCES II
RAIN SEMESTER EXAMINATION (2012/2013 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A;**
- Answer only one question from **Section B.**
- Show all workings clearly

Time allowed: 2 hours

SECTION A

1. Outline any three reasons why some knowledge of mathematics is useful in social sciences.

2. *(a)* Differentiate between the functions: $y = x^b, b \neq 0$ and $y = b^x, b > 1$

   (b) Give any four economic examples of monotonic functions.

3. What type of matrices have the following features:

   (a) $AA' = A'A = I$;

   (b) $AA^{-1} = -A$;

   (c) $A' = -A$.

4. Find $\frac{dy}{dx}$ if:

   (i) $y = e^{\sin x - \cos x}$;

   (ii) $y = \sin x \cos x$;

   (iii) $y = \log(e^{x} - e^{-x})$.

5. *(a)* What is partial differentiation?

   (b) If $Z = x^3 + 2x^2y + 3y^3$, compute Z_{xx}, Z_{yx}, Z_{xy}, Z_{yy}

6. State the following rules of integration.

   (a) Power function rule;

   (b) Product function rule;

   (c) Exponential function rule.

7. *(a)* What is a differential equation?

   (b) Distinguish between the order and degree of a differential equation.

8. What are the critical elements of a Lagrangean function?

9. If at various levels of Labour input (L) of a farm, the average output is described by the following average production equation:

$$AP = \frac{200}{L} + 12L - \frac{L^2}{3},$$

Determine the farm's production function for various levels of output.

10. Find y in terms of x when it is given that the elasticity of y with respect to x is a constant b.

SECTION B

1. *(a)* Let $U = \int \frac{\sin x}{a \sin x + b \cos x} dx$ and $V = \int \frac{\cos x}{a \sin x + b \cos x} dx$, find:

   (i) $aU + bV$;

   (ii) $aV - bU$.

   (b) Evaluate:

   (i) $\int \frac{-\tan x}{\log(\cos x)} \, dx$

   (ii) $\int \left(x^{\alpha} + x^{\beta} + x^{\varphi}\right) \, dx$, where α, β, φ are constants.

(c) Find

(i) $\int (3x+5)^{10}\, dx$

(ii) $\int_{-a}^{a} f(x)\; dx$

2. *(a)* State the following theorems:

(i) Young's theorem;

(ii) Euler's theorem;

(iii) Jacobian theorem.

(b) Maximize $Z = xy + 2x$;

x, y subject to $4x + 2y = 60$.

3. *(a)* Distinguish between ordinary and partial differential equations.

(b) Outline any three types of first order and first degree differential equations.

(c) Solve:

$$\frac{dy}{dx} = \frac{y+1}{x-1}$$

given the initial

condition $y = 4$ when $x = 2$.

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2011/2012 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and only one question from Section B. We'll be solving everything though, just sit tight as we solve.

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

Question 1

Outline any three reasons why some knowledge of mathematics is useful in social sciences.

- With mathematics, we can find desired equilibrium points for economical situations for market equilibrium such as when demand is equal to supply.

- With the concept of partial differentiation, we can find optimal points for functions of multiple situations.
- Mathematical optimization helps consumers to maximize their utilities; this helps consumers in making decisions.

Question 2

(a) Differentiate between the functions: $y = x^b, b \neq 0$ and $y = b^x, b > 1$

(b) Give any four economic examples of monotonic functions.

(a)

Of course, we even emphasized this when in the study of functions.

The first function: $y = b^x$ $(b > 1)$ is an exponential function; since x, the independent variable is an exponent, while the second one;

$y = x^b$ $(b \neq 0)$ is a power function since x, the independent variable is being raised to a constant power.

The stuffs in the bracket are just formalities that describe the values for which the functions are valid;

(b)

Economic types; WOW!
We saw some economic examples of monotonic functions when treating monotonic functions under the topic functions, however, we only saw two. And as far as the research of this book can go, we only have two economic examples of monotonic functions, monotonic functions are not common functions in real life; functions which go only one way (all increasing or all decreasing) are not common in economics. The two economic examples of monotonic functions we have are:

- Supply functions (increasing, as prices increases, supply always increase)
- Demand function (decreasing, as prices increases, demand always decrease)

Question 2

What type of matrices have the following features:

(a) $AA' = A'A = I$;

(b) $AA^{-1} = -A$;

(c) $A' = -A$.

This is quite simple, from our studies;

(a) is a matrix that the product of its transpose and the matrix either way is equal to an identity matrix, hence, A is an **orthogonal matrix,** since the transpose of an orthogonal matrix serves as its inverse.

As for (b), normally, a matrix when multiplying its inverse will be an identity matrix, hence, we can replace this equation as:

Normally;

$$AA^{-1} = I$$

Hence, we can conclude that from (b):

$$I = -A$$

Dividing through by -1, we have:

$$A = -I$$

Hence, A is the **negative identity matrix**

(c) is a matrix that its transpose is equal to its negative and hence, A is a **skew-symmetric matrix.**

Question 4

Find $\frac{dy}{dx}$ if:

(i) $y = e^{\sin x - \cos x}$
(ii) $y = \sin x \cos x$
(iii) $y = \log(e^{x} - e^{-x})$

(i)

$$y = e^{\sin x - \cos x}$$

Substitution;
Chain rule;

$$u = \sin x - \cos x$$

Straight!

$$\frac{du}{dx} = \cos x - (-\sin x)$$

$$\frac{du}{dx} = \cos x + \sin x$$

Hence;

$$y = e^u$$

Straight!

$$\frac{dy}{dx} = e^u$$

Hence;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \times (\cos x + \sin x)$$

Return u;

$$\frac{dy}{dx} = e^{\sin x - \cos x}(\cos x + \sin x)$$

(ii)

$$y = \sin x \cos x$$

Simple!

Product rule!

$$u = \sin x$$

$$v = \cos x$$

Hence;

$$\frac{du}{dx} = \cos x$$

$$\frac{dv}{dx} = -\sin x$$

We have;

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{dy}{dx} = (\cos x)(\cos x) + (\sin x)(-\sin x)$$

Expanding;

$$\frac{dy}{dx} = \cos^2 x - \sin^2 x$$

(iii)

$$y = \log(e^x - e^{-x})$$

Substitution;

Chain rule!

$$u = e^x - e^{-x}$$

Straight!

$$\frac{du}{dx} = e^x - (-e^{-x})$$

Chain rule in differentiating e^{-x} makes it $-e^{-x}$

$$\frac{du}{dx} = e^x + e^{-x}$$

Hence;

$$y = \log u$$

Straight!

$$\frac{dy}{dx} = \frac{1}{u \ln 10}$$

Hence;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u \ln 10} \times (e^{x} + e^{-x})$$

Return u;

$$\frac{dy}{dx} = \frac{e^{x} + e^{-x}}{\ln 10 \ (e^{x} - e^{-x})}$$

Question 5

(a) What is partial differentiation?

(b) If $Z = x^3 + 2x^2y + 3y^3$, compute Z_{xx}, Z_{yx}, Z_{xy}, Z_{yy}

(a)

Partial differentiation is the differentiation of multivariable functions which yields several partial derivatives (differential coefficients) where a partial derivative of a function of several variables (the multivariate functions) is its derivative with respect to one of the variables that constitute the function with other variables temporarily kept constant.

(b)

$$f(x,y) = x^3 + 2x^2y + 3y^2$$

Little error in the question, we are told to calculate Z_{xx}, Z_{xy}, Z_{yx} and Z_{yy}; however, the function isn't even stated in terms of Z; hence, here, we'll just assume the question is:

$$Z = x^3 + 2x^2y + 3y^2$$

Hence,

$$Z_x = 3 \times x^{3-1} + 4 \times x^{2-1}y + 0$$

$$Z_x = 3x^2 + 4xy$$

And

$$Z_y = 0 + 1 \times 2x^2y^{1-1} + 2 \times 3y^{2-1}$$

$$Z_y = 2x^2 + 6y$$

Hence, going for the second order partial derivatives;

$$Z_{xx} = (Z_x)_x$$

$$Z_{xx} = \frac{\partial Z}{\partial x}(3x^2 + 4xy)$$

$$Z_{xx} = 2 \times 3x^{2-1} + 1 \times 4x^{1-1}y$$

$$Z_{xx} = 6x + 4y$$

Also

$$Z_{xy} = (Z_x)_y$$

$$Z_{xy} = \frac{\partial Z}{\partial y}(3x^2 + 4xy)$$

$$Z_{xy} = 0 + 1 \times 4xy^{1-1}$$

$$Z_{xy} = 4x$$

Then;

$$Z_{yx} = \left(Z_y\right)_x$$

$$Z_{yx} = \frac{\partial Z}{\partial x}(2x^2 + 6y)$$

$$Z_{yx} = 2 \times 2x^{2-1} + 0$$

$$Z_{yx} = 4x$$

Lastly;

$$Z_{yy} = \left(Z_y\right)_y$$

$$Z_{yy} = \frac{\partial Z}{\partial y}(2x^2 + 6y)$$

$$Z_{yy} = 0 + 1 \times 6y^{1-1}$$

$$Z_{yy} = 6$$

Question 6

State the following rules of integration.

(a) Power function rule;

(b) Product function rule;

(c) Exponential function rule.

Some integration rules here;

(a) Power function rule;

The power rule in integration? It's stated below:

$$\int x^n = \frac{x^{n+1}}{n+1}$$

(b) Product function rule;

The power rule in integration? That should be the integration by parts; stated below:

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

(c) Exponential function rule;

The exponential rule in integration? It's stated below:

$$\int e^x\,dx = e^x + C$$

Question 7

(a) What is a differential equation?

(b) Distinguish between the order and degree of a differential equation.

(a)

A differential equation is simply a mathematical relationship that describes the relationship between functions and their various derivatives (their various differential coefficients). In simple terms, a differential equation consists of derivative(s), could be just one derivative or more, identifying a differential equation is very easy; an equation where derivatives are shown (with mathematical operations such as adding, subtraction and etc) is simply a differential equation.

(b)

The order of a differential equation is the highest derivative involved in the differential equation while the degree of a differential equation is determined after the order of the differential equation has been determined

Question 8

What are the critical elements of a Lagrangean function?

We know the Lagrangean function;

The critical elements are just three;

- The objective function;
- The constraint function;
- The Lagrangean multiplier.

Question 9

If at various levels of Labour input (L) of a farm, the average output is described by the following average production equation:

$AP = \frac{200}{L} + 12L - \frac{L^2}{3}$,

Determine the farm's production function for various levels of output.

We know that the average function for any function is gotten by dividing the function by the unit it depends on, hence, the average production function is given by;

$$AP = \frac{P(L)}{L}$$

Hence;

$$P(L) = AP \times L$$

$$P(L) = \left(\frac{200}{L} + 12L - \frac{L^2}{3}\right) \times L$$

Expanding;

$$P(L) = 200 + 12L^2 - \frac{L^3}{3}$$

Hence, we have gotten the firm's production form, $P(L)$; the general form for various output levels.

Question 10

Find y in terms of x when it is given that the elasticity of y with respect to x is a constant b.

This question has been treated in the notes already!

So, the elasticity of y with respect to x is a constant $b\dots$

Now, the point elasticity of y with respect to x will be given by:

$$PEy = \left(\frac{dy}{dx}\right) \times \left(\frac{x}{y}\right)$$

We’re told $PEy = b$; ***since the elasticity of y with respect to x is a constant b***

Hence,

$$b = \left(\frac{dy}{dx}\right) \times \left(\frac{x}{y}\right)$$

Above is a differential equation.
Since b is a constant, this is a simple first order differential equation.

Separating variables;

$$\frac{bdx}{x} = \frac{dy}{y}$$

Evaluate their separate integrals;

$$b\int \frac{dx}{x} = \int \frac{dy}{y}$$

I’m sure you know b is constant and can be left out of the integral.

Integrating... ... ... on each side.

$$b \ln x = \ln y$$

Our focus is making y the subject so let's settle down and do some manipulations, we did similar manipulations while treating differential equations.

$$\ln y = b \ln x \; ;$$

***Take natural exponents of both sides;

$$e^{\ln y} = e^{b \ln x}$$

Natural exponent of a natural log cancels out, given:::

$$y = e^{b \ln x}$$

We can also simplify the right hand side by separating the powers into double powers as shown below;

$$y = \left(e^{\ln x}\right)^{b}$$

The exponent and the log once again cancels out; thus:

$$y = x^{b}$$

SECTION B

Question 1

(a) Let $U = \int \frac{\sin x}{a \sin x + b \cos x} dx$ and

$V = \int \frac{\cos x}{a \sin x + b \cos x} dx$, find:

(i) $aU + bV$;

(ii) $aV - bU$.

(b) Evaluate:

(i) $\int \frac{-\tan x}{\log(\cos x)} \, dx$

(ii) $\int \left(x^{\alpha} + x^{\beta} + x^{\varphi}\right) \, dx$,
where α, β, φ are constants.

(c) Find

(i) $\int (3x + 5)^{10} \, dx$

(ii) $\int_{-a}^{a} f(x) \, dx$

(a)

This has been thrashed in the notes, kindly see the explanation for details; you should have read it before coming to past questions though;

$$U = \int \frac{\sin x}{a \sin x + b \cos x} dx$$

$$V = \int \frac{\cos x}{a \sin x + b \cos x} dx$$

$$aU + bV$$

Go ahead and multiply them;

$$aU = a \int \frac{\sin x}{a \sin x + b \cos x} dx$$

Take the a inside the integral, if it was inside, we could take it outside, so now, let's do the reverse;

$$aU = \int \frac{a(\sin x)}{a \sin x + b \cos x} dx$$

In same way;

$$bV = b \int \frac{\cos x}{a \sin x + b \cos x} dx$$

Take the b inside the integral, if it was inside, we could take it outside, so now, let's do the reverse;

$$bV = \int \frac{b(\cos x)}{a\sin x + b\cos x}dx$$

$aU + bV$ will be given by:

$$\int \frac{a(\sin x)}{a\sin x + b\cos x}dx + \int \frac{b(\cos x)}{a\sin x + b\cos x}dx$$

So, another most basic integral rule; these are split integral; they could be together before they are split, let's bring them together as if we were there before;

$$\int \left(\frac{a(\sin x)}{a\sin x + b\cos x} + \frac{b(\cos x)}{a\sin x + b\cos x}\right)dx$$

Let's add that fraction within; the denominators are the same so we can add them straight with one common denominator;

$$\int \left(\frac{a(\sin x) + b(\cos x)}{a\sin x + b\cos x}\right)dx$$

$$\int \left(\frac{a\sin x + b\cos x}{a\sin x + b\cos x}\right)dx$$

Cancel off!

$$\int (1)\,dx = \int 1x^0 dx = 1\int x^0 dx$$

$$1\left[\frac{x^{0+1}}{0+1}\right] = x + C$$

Hence;

$$aU + bV = x + C$$

The question was actually little of asking you about integration laws but the properties of integration; we'll be treating the second part just like this;

$$aV - bU$$

$$aV = a\int \frac{\cos x}{a\sin x + b\cos x}dx$$

Take the a inside the integral, if it was inside, we could take it outside, so now, let's do the reverse;

$$aV = \int \frac{a\cos x}{a\sin x + b\cos x}dx$$

$$bU = b\int \frac{\sin x}{a\sin x + b\cos x}dx$$

Take the b inside the integral, if it was inside, we could take it outside, so now, let's do the reverse;

$$bV = \int \frac{b\sin x}{a\sin x + b\cos x}dx$$

$aV - bU$ will be given by:

$$\int \frac{a\cos x}{a\sin x + b\cos x}dx - \int \frac{b\sin x}{a\sin x + b\cos x}dx$$

So, another most basic integral rule; these are split integral; they could be together before they are split, let's bring them together as if we were there before;

$$\int \left(\frac{a\cos x}{a\sin x + b\cos x} - \int \frac{b\sin x}{a\sin x + b\cos x}\right)dx$$

Let's subtract that fraction within; the denominators are the same so we can add them straight with one common denominator;

$$\int \left(\frac{a\cos x - b\sin x}{a\sin x + b\cos x}\right)dx$$

It's not looking to cancel each other as the previous part; however, checking the denominator, the numerator could be its derivative;

Let's see;

$$z = a \sin x + b \cos x$$

$$\frac{dz}{dx} = a \cos x + b(-\sin x)$$

$$\frac{dz}{dx} = a \cos x - b \sin x$$

That obviously is the numerator; hence; here;

$$dx = \frac{dz}{a \cos x - b \sin x}$$

Hence, we have;

$$\int \left(\frac{a \cos x - b \sin x}{a \sin x + b \cos x}\right) \times \frac{dz}{a \cos x - b \sin x}$$

$a \cos x - b \sin x$ cancels out;

$$\int \left(\frac{a \cos x - b \sin x}{a \sin x + b \cos x}\right) \times \frac{dz}{a \cos x - b \sin x}$$

$$\int \left(\frac{1}{z}\right) \times dz = \ln z$$

Return the substitution;

We have;

$$\ln(a \sin x + b \cos x) + C$$

Of course the arbitrary constant cannot be forgotten, we have that;

$$aV - bU = \ln(a \sin x + b \cos x) + C$$

(b)

(i)

$$\int \frac{-\tan x}{\log(\cos x)}\, dx$$

This question is really thick! Nonetheless, it's an integral of the form:

$$\frac{f'(x)}{f(x)}$$

However, as I said in the textbook section, once you are not clear, attempt differentiating the denominator and see if it'll yield the numerator.

$$\frac{d}{dx}(\log(\cos x))$$

$$z = \cos x$$

$$\frac{dz}{dx} = -\sin x$$

We have:

$$\log z$$

$$\frac{d}{dz}(\log z) = \frac{1}{z \ln 10}$$

Hence;

$$\frac{d}{dx}(\log(\cos x)) = \frac{d}{dz}(\log z) \times \frac{dz}{dx}$$

$$\frac{d}{dx}(\log(\cos x)) = \frac{1}{z \ln 10} \times (-\sin x)$$

Return z;

$$\frac{d}{dx}(\log(\cos x)) = -\frac{\sin x}{\cos x \ln 10}$$

From trigonometry identity;

$$\tan x = \frac{\sin x}{\cos x}$$

Hence;

$$\frac{d}{dx}(\log(\cos x)) = -\frac{\tan x}{\ln 10}$$

Hence; we can see that the derivative of the denominator and the numerator just differ by $\ln 10$, hence, we have confirmed that:

Hence; we make the integration substitution;

$$u = \log(\cos x)$$

$\frac{du}{dx}$ is what we got above, hence

$$\frac{du}{dx} = -\frac{\tan x}{\ln 10}$$

Hence;

$$dx = \frac{-du \ln 10}{\tan x}$$

Hence;

The integral becomes;

$$\int \frac{-\tan x}{u} \times \frac{-du \ln 10}{\tan x}$$

$\tan x$ cancels off and $- \times -$ cancels off

$$\int \frac{1}{u} du \ln 10$$

Bring the constant out:

$$\ln 10 \int \frac{1}{u} du$$

Standard integral;

$$\ln 10\,(\ln u) + C$$

Return u;

$$\ln 10\,(\ln(\log(\cos x))) + C$$

(ii)

$$\int \left(x^{\alpha} + x^{\beta} + x^{\varphi}\right)\, dx$$

where α, β, φ are constants.

$x^{\alpha} + x^{\beta} + x^{\varphi}$, where α, β and φ are constants.

Very very simple! Power rule since the powers are constants;

$$\left[\frac{x^{\alpha+1}}{\alpha+1}\right] + \left[\frac{x^{\beta+1}}{\beta+1}\right] + \left[\frac{x^{\varphi+1}}{\varphi+1}\right]$$

Solution!

$$\frac{x^{a+1}}{a+1}+\frac{x^{b+1}}{b+1}+\frac{x^{c+1}}{c+1}$$

No serious simplification can be done!

(c)

(i)

$$\int (3x+5)^{10}\,dx$$

Linear substitution!

$$u = 3x+5$$

$$\frac{du}{dx}=3$$

Hence;

$$dx=\frac{du}{3}$$

We have:

$$\int (u)^{10}\frac{du}{3}$$

Bring constants out;

$$\frac{1}{3}\int u^{10}\,du$$

Power rule;

$$\frac{1}{3}\left[\frac{u^{10+1}}{10+1}\right] = \frac{1}{3} \times \frac{u^{11}}{11}$$

Return u;

$$\frac{(3x+5)^{11}}{33} + C$$

(ii)

$$\int_{-a}^{a} f(x)\ dx$$

Now, this tests your understanding on integral limits;

Let, the indefinite integral, yield another function of x, $g(x)$;

$$\int f(x)\,dx = g(x)$$

Then;

$$\int_{-a}^{a} f(x)\ dx = [g(x)]_{-a}^{a}$$

Then,

$$\int_{-a}^{a} f(x)\ dx = g(a) - g(-a)$$

Question 2

(a) State the following theorems:

(i) Young's theorem;
(ii) Euler's theorem;
(iii) Jacobian theorem.

(b) Maximize $Z = xy + 2x$;
x, y subject to $4x + 2y = 60$.

(a)

Check the textbook and state them as they are written there;

(i)

Young's theorem states that two complementary mixed partials of a continuous and twice differentiable function are equal;

(ii)

Euler's theorem states that the sum of the product of the arguments and their first order partial derivatives of a function is equal to the degree of homogeneity of a function multiplying the function.

(iii)

Jacobian theorem states that; The Jacobian determinant for a set of functions will be equal to zero for all values of the arguments of the functions if and only if the functions are (linearly or nonlinearly) dependent.

(b)

How many times will this same question be set?

Here, the objective function is:

$$xy + 2x$$

The constraint function is:

$$4x + 2y - 60 = 0$$

Note that it has been equated to zero;

Hence, using the method of Lagrangean multiplier; we have;

$$\mathcal{L}(x, y, \lambda) = xy + 2x - \lambda(4x + 2y - 60)$$

Take the first order partials;

$$\mathcal{L}_x = 1 \times x^{1-1}y + 1 \times 2x^{1-1} - \lambda(1 \times 4x^{1-1} + 0 - 0)$$

$$\mathcal{L}_x = y + 2 - 4\lambda$$

$$\mathcal{L}_y = 1 \times x \times y^{1-1} + 0 - \lambda(0 + 2 \times y^{1-1} - 0)$$

$$\mathcal{L}_y = x - 2\lambda$$

$$\mathcal{L}_\lambda = 0 + 0 - 1 \times \lambda^{1-1}(4x + 2y - 60)$$

$$\mathcal{L}_\lambda = -4x - 2y + 60$$

Equate each to zero;

$$y + 2 - 4\lambda = 0$$

$$y - 4\lambda = -2 \dots\dots\dots (1)$$

$$x - 2\lambda = 0 \dots\dots\dots (2)$$

$$-4x - 2y + 60 = 0$$

$$-2x - y = -30 \ldots\ldots\ldots (3)$$

(1) and (2) can be easily sorted out;

$$\mathbf{2} \times (\mathbf{2}): 2x - 4\lambda = 0$$

From (1);

$$y + 2 = 4\lambda$$

From (2);

$$2x = 4\lambda$$

From (1) and (2);

$$y + 2 = 2x$$

Since both are equal to 4λ

Hence,

$$-2x + y = -2 \ldots\ldots\ldots (4)$$

Solving simultaneously with (3);

$$-2x - y = -30 \ldots\ldots\ldots (3)$$

$$-2x + y = -2 \ldots\ldots\ldots (4)$$

Adding;

$$-4x = -32$$

$$x = 8$$

Hence, from (3);

$$-2x - y = -30$$

Hence,

$$-2(8) - y = -30$$

$$y = 14$$

Hence, the optimal point is $x = 8$ and $y = 14$

From (2);

$$2x = 4\lambda$$

$$2(8) = 4\lambda$$

$$\lambda = 4$$

Hence, to maximize Z, find its value at the optimal point;

$$Z = xy + 2x$$

Hence, maximum Z is:

$$Z = 8(14) + 2(8)$$

$$Z = 128$$

Question 3

(a) Distinguish between ordinary and partial differential equations.

(b) Outline any three types of first order and first degree differential equations.

(c) Solve:

$$\frac{dy}{dx} = \frac{y+1}{x-1}$$

given the initial

condition $y = 4$ when $x = 2$.

(a)

There isn't so much difference between the two, they both contain differential coefficients, but they contain different types of differential coefficients.

An ordinary differential equation is a differential equation involving one independent and one dependent variable while a partial differential equation involves partial derivatives. It is a differential equation involving a dependent variable and more than one independent variable.

(b)

I'm not so clear as to what is being asked exactly! Whether to list examples of differential equations as there are no fixed types of first order differential equations, there are only methods in which we treated just two methods, while I however listed the five methods for first order differential equations. However, though; I'll succumb with examples, since the question also specified the degree as degree 1.

$$\frac{dy}{dx} + 5x = 5y$$

$$x\frac{dy}{dx} - y^2 = 0$$

$$\frac{dy}{dx} - \cos x = 0$$

And a host of other examples; there is no limits to examples possible. Ensure the differential coefficient is of the first order and it is raised to power 1.

(c)

$$\frac{dy}{dx} = \frac{y+1}{x-1}$$

given the initial condition $y = 4$ when $x = 2$

We played with differential equations in the textbook, this is a case of separation of variables;

Separate the variables gradually;

$$dy = \frac{y+1}{x-1}dx$$

Finally, we have:

$$\frac{dy}{y+1} = \frac{dx}{x-1}$$

Integrate each sides separately with respect to their respective variables;

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}$$

Let's integrate the left hand side and right hand side separately; it's better slow than wrong;

LHS:

$$\int \frac{dy}{y+1}$$

Linear substitution;

Put:

$$u = y + 1$$

$$\frac{du}{dy} = 1$$

Here,

$$dy = du$$

Yielding;

$$\int \frac{1}{u} du$$

We now have:

Standard integral;

$$\int \frac{du}{u} = (\ln u) = (\ln(y+1))$$

RHS:

$$\int \frac{dx}{x-1}$$

Linear substitution;

Put:

$$z = x - 1$$

$$\frac{dz}{dx} = 1$$

Here,

$$dx = dz$$

Yielding;

$$\int \frac{1}{z} dz = \int \frac{dz}{z}$$

We now have:

$$\int \frac{dz}{z}$$

Standard integral;

$$\int \frac{dz}{z} = (\ln z) = (\ln(x - 1))$$

Hence, $LHS = RHS$, our arbitrary constant is added just once since the sum of arbitrary constants is still an arbitrary constant; hence, we add one arbitrary constant;

$$\ln(y + 1) = \ln(x - 1) + C$$

Simplifying;

C is another constant $\ln A$
Hence; we have:

$$\ln(y+1) = \ln(x-1) + \ln A$$

Log rules;

$$\log a + \log b = \log(ab)$$

$$\ln(y+1) = \ln[(x-1) \times A]$$

$$\ln(y+1) = \ln A(x-1)$$

Hence, the log numbers can be equated;

$$y+1 = A(x-1)$$

Where A is a constant. That's the general solution of the differential equation

We have an initial condition to get the value of A;

$y = 4$ when $x = 2$; hence;

$$4+1 = A(2-1)$$

Solving;

$$A = 5$$

Hence;

$$y+1 = 5(x-1)$$

Expanding;

$$y+1 = 5x-5$$

Finally;

$$y - 5x + 6 = 0$$

Above is the particular solution of the differential equation.