MISCELLANEOUS PAST QUESTIONS (COMPILED FROM TESTS)

Here, some past questions from tests have been picked, mostly picked specifically, especially those that haven't featured in the examination in any of the years.

1. Evaluate the determinant;

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{vmatrix}$$

2. Distinguish between the exponential function rule and logarithm function rule of differentiation.

3. Find $\frac{dy}{dx}$ if:

$$y = \frac{a^2 + x^2}{a^2 - x^2}$$

4. If $y = e^x + e^{-x}$; show that:

(i) $y = y^{11}$

(ii) $y^1 = y^{111}$

5. If $y = e^{ax}$, find $\frac{d^n y}{dx^n}$

6. Functions that can be expressed in polynomial are otherwise called ……………

7. Show that the matrix;

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Is orthogonal.

8. A function of the form:

$$y = \frac{x^2 - 1}{x + 1}, x \neq -1$$

Is a ……………………..

9. A function whose value stays the same when the explanatory variable of the function changes sign (from positive to negative and vice-versa) is called ………………

10. Find Z_x and Z_y if:

$$Z = e^{\sin x \sin y}$$

11. What type of matrix have the feature:

(i) $A.A = A$

(ii) $A = A^1$

12. A monopolist's demand function is given by:

$$p = 200 - 4q$$

Find the marginal and average revenue functions.

13. The two basic components of calculus are ……………… and ………………

14. What is the "function of function" rule of differentiation?

15. Evaluate:

$$\int_2^4 \frac{dt}{t^3 - 4t}$$

16. Show that $Z_{xy} = Z_{yx}$, given:

$$Z = \log(\sin x - \cos y)$$

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE MISCELLANEOUS PAST QUESTIONS (COMPILED MAINLY FROM PAST TEST QUESTIONS)

Question 1

Evaluate the determinant;

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{vmatrix}$$

Hello from the other side, this is quite basic. OPEN YOUR EYES!!!

When we treated matrices determinants, we understood that determinants exists only for square matrices, the matrix above is a 3×4 matrix and hence, **the matrix does not have a determinant.**

Question 2

Distinguish between the exponential function rule and logarithm function rule of differentiation.

Well, for all I know, there is no difference as it were correspondingly between the two rules of differentiation, hence, the way to answer this question is to simply write both rules out.

The exponential function rule of differentiation is that:
If:

$$y = e^x$$

Then;

$$\frac{dy}{dx} = e^x$$

And if;

$$y = a^x$$

Then;

$$\frac{dy}{dx} = a^x \ln a$$

While the logarithm rule of differentiation is that:
If:

$$y = \log_e x$$

Then;

$$\frac{dy}{dx} = \frac{1}{x}$$

And if;

$$y = \log_a x$$

Then;

$$\frac{dy}{dx} = \frac{1}{x \ln a}$$

Question 3

Find $\frac{dy}{dx}$ if:

$$y = \frac{a^2 + x^2}{a^2 - x^2}$$

This is simply a case of quotient rule of differentiation!

Here; we have:

$$u = a^2 + x^2$$

$$v = a^2 - x^2$$

Here; we know that a is a constant since we seek $\frac{dy}{dx}$ which makes us aware that y is the dependent variable and x is the independent variable.

$$\frac{du}{dx} = 0 + 2 \times x^{2-1} = 2x$$

$$\frac{dv}{dx} = 0 - 2 \times x^{2-1} = -2x$$

Quotient rule;

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Hence;

$$\frac{dy}{dx} = \frac{(a^2 - x^2)(2x) - (a^2 + x^2)(-2x)}{(a^2 - x^2)^2}$$

Simplifying;

$$\frac{dy}{dx} = \frac{2xa^2 - 2x^3 + 2xa^2 + 2x^3}{(a^2 - x^2)^2}$$

$$\frac{dy}{dx} = \frac{4xa^2}{(a^2 - x^2)^2}$$

Question 4

If $y = e^x + e^{-x}$; show that:

(i) $y = y^{11}$

(ii) $y^1 = y^{111}$

Well, this is a quite straightforward question, we only need to solve for the different things we're asked to solve for and establish if they are equal.

As I said in the textbook, y^1 is the same thing as y' and hence, this denotes the first derivative of y, same thing for y^{11} which is the same as y'' which denotes the second derivative of y and so on.

So, let's get the ball rolling!

$$y = e^x + e^{-x}$$

$$y^1 = \frac{dy}{dx} = \frac{d}{dx}(e^x + e^{-x})$$

$$y^1 = e^x + (-e^{-x}) = e^x - e^{-x}$$

Note that the derivative of e^x is e^x and that for e^{-x}, substituting for $u = -x$ and using chain rule, the derivative is $-e^{-x}$; going further;

$$y^{11} = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(e^x - e^{-x})$$

$$y^{11} = e^x - (-e^{-x}) = e^x + e^{-x}$$

Finally;

$$y^{111} = \frac{d^3y}{dx^3} = \frac{d}{dx}(y^{11}) = \frac{d}{dx}(e^x + e^{-x})$$

$$y^{111} = e^x + (-e^{-x}) = e^x - e^{-x}$$

Obviously, from our results, we have:

$$y = e^x + e^{-x}$$
$$y^1 = e^x - e^{-x}$$
$$y^{11} = e^x + e^{-x}$$
$$y^{111} = e^x - e^{-x}$$

And;

(i)

$$y = y^{11}$$

(ii)

$$y^1 = y^{111}$$

Question 5

If $y = e^{ax}$, find $\dfrac{d^ny}{dx^n}$

This is a quite technical question. However, by observance of a certain series of derivatives, we'll discover something quickly together;

$$y = e^{ax}$$

Since we are seeking $\frac{d^n y}{dx^n}$, we know that a is a constant here;

Obviously, $\frac{d^n y}{dx^n}$ signifies the higher derivatives of y, only that we are looking for the higher derivative to any random degree, n. For this to be so, it means all the higher derivatives of the function must follow a particular pattern. This pattern we want to discover here and now.

Let's go first, for $\frac{dy}{dx}$ where $n = 1$

$$y = e^{ax}$$

Chain rule;

$$u = ax$$

$$\frac{du}{dx} = a \times 1 \times x^{1-1} = a$$

Hence;

$$y = e^u$$

Straight;

$$\frac{dy}{du} = e^u$$

Chain rule;

$$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^u \times a = ae^u$$

Return u;

$$\frac{dy}{dx} = ae^{ax}$$

Going for $\frac{d^2y}{dx^2}$ where $n = 2$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(ae^{ax})$$

Again, here, a is a constant, hence, we have;

$$\frac{d^2y}{dx^2} = a \times \frac{d}{dx}(e^{ax})$$

Since, e^{ax} is the same thing differentiated in the first one, we have:

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

And hence;

$$\frac{d^2y}{dx^2} = a \times ae^{ax} = a^2e^{ax}$$

Let's go the last time and check if we'll discover any pattern;

Going for $\frac{d^3y}{dx^3}$ where $n = 3$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}(a^2e^{ax})$$

Again, here, a^2 is a constant, hence, we have;

$$\frac{d^3y}{dx^3} = a^2 \times \frac{d}{dx}(e^{ax})$$

Since, e^{ax} is the same thing differentiated in the first one, we have:

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

And hence;

$$\frac{d^3y}{dx^3} = a^2 \times ae^{ax} = a^3e^{ax}$$

Hence, collating;

$$\frac{dy}{dx} = ae^{ax}$$

$$\frac{d^2y}{dx^2} = a^2e^{ax}$$

$$\frac{d^3y}{dx^3} = a^3e^{ax}$$

Looking at the above well, it is obvious that e^{ax} is common to all of them, and also, a to a certain power is common to all of them. Looking well too, the power of a corresponds to the order of the differentiation and hence, obviously,

$$\frac{d^ny}{dx^n} = a^ne^{ax}$$

Question 6

Functions that can be expressed in polynomial are otherwise called ……………

This is quite basic, just a little bit different from how it was stated in the textbook.

Algebraic functions are functions that can be expressed as a root of a polynomial, in essence, they have a finite number of terms. Hence, algebraic functions are the functions been talked about here.

Question 7

Show that the matrix;

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Is orthogonal.

What is an orthogonal matrix?

For an orthogonal matrix, A;

$$A' = A^{-1}$$

Hence;

For the matrix shown;

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

We have:

$$A^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Also; for the inverse, it is much easier since it is a 2×2 matrix;

Recall; for a 2×2 matrix,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The inverse is:

$$\frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Where;

$$\frac{1}{ad-bc} \text{ is the determinant of the matrix}$$

Hence, for A;

$$|A| = \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)$$

$$|A| = \frac{1}{2} + \frac{1}{2} = 1$$

Hence; the inverse, as stated above is:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Obviously;

$$A^T = A^{-1}$$

Question 8

A function of the form:

$$y = \frac{x^2 - 1}{x + 1}, x \neq -1$$

Is a ……………………….

The function is an algebraic fraction;

$x^2 - 1$ is a polynomial
$x + 1$ is a polynomial

Since both numerator and denominator are polynomial, we have that it is **a rational function.**

Question 9

A function whose value stays the same when the explanatory variable of the function changes sign (from positive to negative and vice-versa) is called ………………..

Too much stories; since the function remains the same regardless the value of the variable, it is **a constant function.**

Question 10

Find Z_{xx}, Z_{yy} and Z_{xy} if:

$$Z = e^{\sin x \sin y}$$

Partial differentiation!

$$Z_x = \frac{\partial}{\partial x}\left(e^{\sin x \sin y}\right)$$

Chain rule;

$$u = \sin x \sin y$$

Hence;

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(\sin x \sin y)$$

y is a constant in differentiating partially with respect to x and hence, $\sin y$ is also a constant.

$$\frac{\partial u}{\partial x} = \sin y \times \frac{\partial}{\partial x}(\sin x) = \sin y \cos x$$

Hence;

$$Z = e^u$$

Straight!

$$\frac{\partial Z}{\partial u} = e^u$$

Chain rule;

$$Z_x = \frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial u} \times \frac{\partial u}{\partial x}$$

$$Z_x = e^u \times (\sin y \cos x)$$

Return u;

$$Z_x = e^{\sin x \sin y}(\cos x \sin y)$$

$$Z_y = \frac{\partial}{\partial y}\left(e^{\sin x \sin y}\right)$$

Chain rule;

$$u = \sin x \sin y$$

Hence;

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(\sin x \sin y)$$

x is a constant in differentiating partially with respect to y and hence, $\sin x$ is also a constant.

$$\frac{\partial u}{\partial y} = \sin x \times \frac{\partial}{\partial y}(\sin y) = \sin x \cos y$$

Hence;

$$Z = e^u$$

Straight!

$$\frac{\partial Z}{\partial u} = e^u$$

Chain rule;

$$Z_y = \frac{\partial Z}{\partial y} = \frac{\partial Z}{\partial u} \times \frac{\partial u}{\partial y}$$

$$Z_y = e^u \times (\sin x \cos y)$$

Return u;

$$Z_y = e^{\sin x \sin y}(\sin x \cos y)$$

Question 11

What type of matrix have the feature:

(i) $A.A = A$

(ii) $A = A^1$

(i)

$A.A$ means $A \times A$ and hence, A^2

In essence,

$$A^2 = A$$

That, you should know, is an **idempotent matrix.**

(ii)

$$A = A^1$$

Well, just to reinstate that A^1 is same as A', just a dispersion is symbol usage.

Of course, a matrix equal to its transpose is **a symmetric matrix.**

Question 12

A monopolist's demand function is given by:

$$p = 200 - 4q$$

Find the marginal and average revenue functions.

Of course, we know the relationship between the price and the quantity and we know that the price function is the same as the inverse demand function. And hence, the demand function mentioned makes us know that it is the price that p means here.

We know that;

$$R(q) = p \times q$$

Where $R(q)$ is the revenue function!
Hence;

$$R(q) = (200 - 4q)q$$

$$R(q) = 200q - 4q^2$$

We need the marginal and average revenue functions, since we now have the revenue function, getting the two of them are quite easy!

For the marginal revenue;

$$R'(q) = \frac{d}{dq}(200q - 4q^2)$$

$$R'(q) = 200q^{1-1} - 2 \times 4q^{2-1}$$

$$R'(q) = 200 - 8q$$

And for the average revenue function;

$$\overline{R(q)} = \frac{R(q)}{q} = \frac{200q - 4q^2}{q}$$

$$\overline{R(q)} = 200 - 4q$$

Of course, we established it that the average revenue function and the price function are the same thing.

Question 13

The two basic components of calculus are ……………… and ………………

Very very simple!

The two basic components of calculus are differentiation (differential calculus) and integration (integral calculus)

Question 14

What is the “function of function” rule of differentiation?

The function of function rule is explicitly stated thus;

If:

$$u = f(x)$$

And;

$$y = f(u)$$

Then;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

That's the summary of chain rule (the function of function rule).

Question 15

Evaluate:

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t}$$

Fine! Here is a short question on integration! As usual, we need to find the indefinite integral first!

$$\int \frac{dt}{t^3 - 4t}$$

Well, what method do you think will resolve this? Look at it well and write it down before we reach the next page!

Well, what did you write? No other method can solve this asides partial fractions actually, no substitution can do this, hence, we have:

$$\frac{1}{t^3 - 4t} = \frac{1}{t(t^2 - 4)}$$

$t^2 - 4$ is the difference of two squares;

$$t^2 - 2^2 = (t + 2)(t - 2)$$

$$\frac{1}{t(t^2 - 4)} = \frac{1}{t(t + 2)(t - 2)}$$

$$\frac{1}{t(t + 2)(t - 2)} = \frac{A}{t} + \frac{B}{t + 2} + \frac{C}{t - 2}$$

Clear fractions;

$$1 = A(t + 2)(t - 2) + Bt(t - 2) + Ct(t + 2)$$

Put $t = -2$

$$1 = A(-2 + 2)(-2 - 2) + B(-2)(-2 - 2) \\ + C(-2)(-2 + 2)$$

Hence;

$$1 = 8B$$

$$B = \frac{1}{8}$$

Put $t = 2$

$$1 = A(2+2)(2-2) + B(2)(2-2) + C(2)(2+2)$$

Hence;

$$1 = 8C$$

$$C = \frac{1}{8}$$

Put $t = 0$

$$1 = A(0+2)(0-2) + B(0)(0-2) + C(0)(0+2)$$

Hence;

$$1 = -4A$$

$$A = -\frac{1}{4}$$

Hence;

$$\frac{1}{t(t+2)(t-2)} = -\frac{1}{4t} + \frac{1}{8(t+2)} + \frac{1}{8(t-2)}$$

And hence;

$$\int \frac{dt}{t^3 - 4t}$$
$$= \int -\frac{1}{4t}dt + \int \frac{1}{8(t+2)}dt$$
$$+ \int \frac{1}{8(t-2)}dt$$

Taking integrals separately;

$$\int -\frac{1}{4t}dt = -\frac{1}{4}\int \frac{1}{t}dt$$

Standard integral;

$$-\frac{1}{4}\ln t$$

$$\int \frac{1}{8(t+2)}dt$$

Linear substitution;

$$u = t + 2$$
$$\frac{du}{dt} = 1$$

And hence;

$$dt = du$$

We have:

$$\int \frac{1}{8u} du$$

Which is;

$$\frac{1}{8} \int \frac{1}{u} du = \frac{1}{8} \ln u$$

Return u;

$$\frac{1}{8} \ln(t + 2)$$

$$\int \frac{1}{8(t - 2)} dt$$

Linear substitution;

$$z = t - 2$$

$$\frac{dz}{dt} = 1$$

And hence;

$$dt = dz$$

We have:

$$\int \frac{1}{8z} dz$$

Which is;

$$\frac{1}{8} \int \frac{1}{z} dz = \frac{1}{8} \ln z$$

Return z;

$$\frac{1}{8}\ln(t-2)$$

And finally, the total integral is:

$$\int \frac{dt}{t^3 - 4t} = -\frac{1}{4}\ln t + \frac{1}{8}\ln(t+2) + \frac{1}{8}\ln(t-2)$$

No need for the arbitrary constant since we are seeking a definite integral on the long run; and the answer can be simplified by factorization;

$$\int \frac{dt}{t^3 - 4t} = -\frac{1}{8}(2\ln t - \ln(t+2) - \ln(t-2))$$

By log rules; $2\ln t = \ln t^2$

$$\int \frac{dt}{t^3 - 4t} = -\frac{1}{8}(\ln t^2 - \ln(t+2) - \ln(t-2))$$

Hence, we can apply the rule of addition and subtraction of logarithms;

$$\int \frac{dt}{t^3 - 4t} = -\frac{1}{8}\left(\ln \frac{t^2}{(t+2)(t-2)}\right)$$

Now applying the definite integral;

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t} = \left[-\frac{1}{8}\left(\ln \frac{t^2}{(t+2)(t-2)}\right)\right]_2^4$$

Substitute appropriately;

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t}$$
$$= \left[-\frac{1}{8}\left(\ln \frac{4^2}{(4+2)(4-2)}\right)\right] - \left[-\frac{1}{8}\left(\ln \frac{2.5^2}{(2.5+2)(2.5-2)}\right)\right]$$

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t} = \left[-\frac{1}{8}\left(\ln \frac{16}{12}\right)\right] - \left[-\frac{1}{8}\left(\ln \frac{6.25}{2.25}\right)\right]$$

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t}$$
$$= \left[-\frac{1}{8}(0.287682)\right] - \left[-\frac{1}{8}(1.021651)\right]$$

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t}$$
$$= [-0.03596025] - [-0.127706375]$$

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t} = -0.03596025 + 0.127706375$$

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t} = -0.03596025 + 0.127706375$$

$$\int_{2.5}^{4} \frac{dt}{t^3 - 4t} = 0.091746125$$

Question 16

Show that $Z_{xy} = Z_{yx}$, given:

$$Z = \log(\sin x - \cos y)$$

Partial derivatives!

Differentiate each partially while taking the other as a constant;

It is a case of chain rule and hence, we need a substitution!

$$u = \sin x - \cos y$$

$$\frac{\partial u}{\partial x} = \cos x - 0 = \cos x$$

Of course, $\cos y$ in this case will be a constant;
Hence;

$$Z = \log u$$

$$\frac{\partial Z}{\partial u} = \frac{1}{u \ln 10}$$

Chain rule;

$$Z_x = \frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial u} \times \frac{\partial u}{\partial x}$$

$$Z_x = \frac{1}{u \ln 10} \times \cos x$$

Return u;

$$Z_x = \frac{\cos x}{\ln 10\,(\sin x - \cos y)}$$

For Z_y; same substitution is needed;

$$u = \sin x - \cos y$$

$$\frac{\partial u}{\partial y} = 0 - (-\sin y) = \sin y$$

Of course, $\sin x$ in this case will be a constant;
Hence;

$$Z = \log u$$

$$\frac{\partial Z}{\partial u} = \frac{1}{u \ln 10}$$

Chain rule;

$$Z_y = \frac{\partial Z}{\partial y} = \frac{\partial Z}{\partial u} \times \frac{\partial u}{\partial y}$$

$$Z_y = \frac{1}{u \ln 10} \times \sin y$$

Return u;

$$Z_y = \frac{\sin y}{\ln 10\,(\sin x - \cos y)}$$

For Z_{xy}, we have:

$$Z_{xy} = (Z_x)_y = \frac{\partial}{\partial y}(Z_x)$$
$$= \frac{\partial}{\partial y}\left(\frac{\cos x}{\ln 10\,(\sin x - \cos y)}\right)$$

The above isn't a quotient situation as we'll be taking x as a constant in this case since we are differentiating with respect to y.

We have:

$$Z_{xy} = \frac{\partial}{\partial y}\left(\frac{\cos x}{\ln 10}\left(\frac{1}{(\sin x - \cos y)}\right)\right)$$

Since;

$$Z_x = \frac{\cos x}{\ln 10}\left(\frac{1}{(\sin x - \cos y)}\right)$$

$\frac{\cos x}{\ln 10}$ is a constant; hence, we are differentiating $\frac{1}{(\sin x - \cos y)}$ partially with respect to y

Substitution and chain rule;

$$u = \sin x - \cos y$$

$$\frac{\partial u}{\partial y} = 0 - (-\sin y) = \sin y$$

Hence;
We have:

$$Z_x = \frac{\cos x}{\ln 10}\left(\frac{1}{u}\right) = \frac{\cos x}{\ln 10}(u^{-1})$$

$$\frac{\partial Z_x}{\partial u} = \frac{\cos x}{\ln 10}(-1 \times u^{-1-1}) = \frac{\cos x}{\ln 10} \times -\frac{1}{u^2}$$

Hence;

$$Z_{xy} = \frac{\partial^2 Z}{\partial y \partial x} = \frac{\partial Z_x}{\partial u} \times \frac{\partial u}{\partial y}$$

$$Z_{xy} = \frac{\cos x}{\ln 10} \times -\frac{1}{u^2} \times \sin y$$

Return u; we have:

$$Z_{xy} = \frac{\cos x}{\ln 10} \times -\frac{1}{(\sin x - \cos y)^2} \times \sin y$$

$$Z_{xy} = -\frac{\cos x \sin y}{\ln 10\,(\sin x - \cos y)^2}$$

Finally, we are done! Kindly follow it gently, it's really no big deal, it's something we have learnt together.

Up next, $\boldsymbol{Z_{yx}}$;

$$Z_{yx} = \left(Z_y\right)_x = \frac{\partial}{\partial x}\left(Z_y\right)$$
$$= \frac{\partial}{\partial x}\left(\frac{\sin y}{\ln 10\,(\sin x - \cos y)}\right)$$

The above isn't a quotient situation as we'll be taking y as a constant in this case since we are differentiating with respect to x.

We have:

$$Z_{xy} = \frac{\partial}{\partial y}\left(\frac{\sin y}{\ln 10}\left(\frac{1}{(\sin x - \cos y)}\right)\right)$$

Since;

$$Z_y = \frac{\sin y}{\ln 10}\left(\frac{1}{(\sin x - \cos y)}\right)$$

$\frac{\sin y}{\ln 10}$ is a constant; hence, we are differentiating $\frac{1}{(\sin x - \cos y)}$ partially with respect to x

Substitution and chain rule;

$$u = \sin x - \cos y$$

$$\frac{\partial u}{\partial x} = \cos x - (0) = \cos x$$

Hence;
We have:

$$Z_y = \frac{\sin y}{\ln 10}\left(\frac{1}{u}\right) = \frac{\sin y}{\ln 10}(u^{-1})$$

$$\frac{\partial Z_y}{\partial u} = \frac{\sin y}{\ln 10}(-1 \times u^{-1-1}) = \frac{\sin y}{\ln 10} \times -\frac{1}{u^2}$$

Hence;

$$Z_{yx} = \frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial Z_y}{\partial u} \times \frac{\partial u}{\partial x}$$

$$Z_{yx} = \frac{\sin y}{\ln 10} \times -\frac{1}{u^2} \times \cos x$$

Return u; we have:

$$Z_{yx} = \frac{\sin y}{\ln 10} \times -\frac{1}{(\sin x - \cos y)^2} \times \cos x$$

$$Z_{xy} = -\frac{\cos x \sin y}{\ln 10\,(\sin x - \cos y)^2}$$

Evidently, we see that both Z_{xy} and Z_{yx} are equal, yeah, they must be equal as that is the statement of Young's theorem.