DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
SSC106: MATHEMATICS FOR SOCIAL SCIENCES II
RAIN SEMESTER EXAMINATION (2002/2003 SESSION)

INSTRUCTIONS:

- Answer **Question 1** and any other two questions;
- Show all workings clearly

Time allowed: 2 hours

1. *(a)* Given matrices A and B with respective dimensions $m \times n$ and $p \times q$;
   - (i) State the condition for the product AB to exist;
   - (ii) Write out the dimension of the product AB if it exists.
   - (iii) Suppose $A = \begin{pmatrix} 10 & 3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}$, find the product AB and comment on your answer.

(b) Determine whether or not the following are increasing or decreasing at $x = 4$;

(i) $y = 5 - 14x + 3x^2$

(ii) $y = x^3 - 7x^2 + 6x - 2$

(c) Given the total cost, $TC = C(Q)$ and that total revenue, $TR = R(Q)$;

(i) Write out an expression for the profit function;

(ii) Show that at the profit maximizing level of output, marginal revenue equals marginal cost.

(d) A firm produces two products x and y and joint total cost function, TC, is given by $TC = 6x^2 + 3x + 1.5xy + 3y^2 + 2y + 50$

(i) Determine the marginal cost function for each of the products.

(ii) Evaluate the marginal cost when $y = 3$ and $x = 5$

(e) (i) "Integration can be defined as the reverse of differentiation." Illustrate this statement with the primitive function; $Y = 2x^2 + 5x + 6$

(ii) Given the marginal revenue function; $MR = 84 - 2Q - Q^2$, find the total revenue function.

2. A firm producing two goods x and y has the total cost function given as: $TC = 8x^2 - xy + 12y^2$. If the firm is bound by contract to produce a combination of two goods totaling 42;
   (i) Write out the objective and the constraint functions;
   (ii) Derive the equations of the first order conditions for the firm's constrained optimization problems.
   (iii) Use Crammer's rule to solve for the optimal values of x and y.

3. *(a)* Distinguish between definite and indefinite integrals.
   (b) Find:
   (i) $\int x^2(x^3 - 5)^3\,dx$

   (ii) $\int 4e^{2x}\,dx$

(c) Find:

(i) $$\int_1^2 x^2(x^3-5)^3\,dx$$

(ii) $$\int_0^2 4e^{2x}\,dx$$

(d) Given the marginal cost, $MC = 12e^{0.2Q}$, the fixed cost equals 72, completely determine the cost function.

4. *(a)* Distinguish between an inverse matrix and an identity matrix.

(b) Given the matrix equation;

$$A_{n\times 1}X_{n\times 1} = B_{n\times 1}$$

If the inverse A^{-1} exists, show that;

$$X_{n\times 1} = (A^{-1}B)_{n\times 1}$$

(c) Find the inverse of $\begin{pmatrix} 1 & 4 & -5 \\ 3 & -2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2002/2003 ACADEMIC SESSION

The instruction is you answer Questions 1 and any other two; we'll be answering everything though; keep calm and move with me;

Expect Question 1 to be one tough question, it's compulsory;

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

Question 1

This question one is indeed lengthy, lets tackle it one-by-one;

(a) Given matrices A and B with respective dimensions $m \times n$ and $p \times q$;

(i) State the condition for the product AB to exist;

(ii) Write out the dimension of the product AB if it exists.

(iii) Suppose $A = (10 \quad 3 \quad -2)$ and $= \begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}$, find the product AB and comment on your answer.

(a) (i)

For two matrices to be conformable for multiplication, then the number of columns in the pre-multiplier must be equal to the number of rows in the post-multiplier;

In AB, A is the pre-multiplier and B is the post-multiplier;

Dimension of *A*: $m \times n$

Dimension of *B*: $p \times q$

Columns in $A = n$
Rows in $B = p$

Therefore, the condition that they are conformable is that;

$$n = p$$

(ii)

From what we learnt in matrix multiplication, if they're conformable, then the product takes its rows from the pre-multiplier and its columns from the post-multiplier; hence, the dimension of AB will be;

$$m \times q$$

(iii)

$$A = \begin{pmatrix} 10 & 3 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 10 & 3 & -2 \end{pmatrix}\begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}$$

$$AB = (10(7) + 3(5) - 2(2))$$

$$AB = (81)$$

Well, we're told to comment on our answer, I'll suggest a wild comment; since the matrix has just one element, the determinant of AB is equal to the only element in the matrix.

(b)

(b) Determine whether or not the following are increasing or decreasing at $x = 4$;

(iii) $y = 5 - 14x + 3x^2$

(iv) $y = x^3 - 7x^2 + 6x - 2$

This is *kinda* straightforward, remember the rule right?

(i)

$$y = 5 - 14x + 3x^2$$

$$\frac{dy}{dx} = 0 - 1 \times 14 \times x^{1-1} + 2 \times 3 \times x^{2-1}$$

$$\frac{dy}{dx} = -14 + 6x$$

At $x = 4$;

$$\frac{dy}{dx} = -14 + 6(4) = -14 + 24 = 10$$

It is positive, greater than zero, hence, it is increasing at $x = 4$

(ii)

$$y = x^3 - 7x^2 + 6x - 2$$

$$\frac{dy}{dx} = 3x^{3-1} - 14x^{2-1} + 6 \times x^{1-1} - 0$$

$$\frac{dy}{dx} = 3x^2 - 14x + 6$$

At $x = 4$;

$$\frac{dy}{dx} = 3(4)^2 - 14(4) + 6$$

$$\frac{dy}{dx} = 48 - 56 + 6 = 54 - 56 = -2$$

It is negative, less than zero, hence, it is decreasing at $x = 4$

(c)

Given the total cost, $TC = C(Q)$ and that total revenue, $TR = R(Q)$;

(iii) Write out an expression for the profit function;

(iv) Show that at the profit maximizing level of output, marginal revenue equals marginal cost.

We have something similar to this in the note, this was proved in this book;

(i)

The profit; $P(Q) = R(Q) - C(Q)$

Where $P(Q)$ is the profit function;

(ii)

$$P(Q) = R(Q) - C(Q)$$

From the rule of derivative of sums; we have that:

$P'(Q) = R'(Q) - C'(Q)$;

$P'(Q)$ is the marginal profit;
Now, $R'(Q)$ is the marginal revenue;
Also, $C'(Q)$ is the marginal cost;

Hence, marginal profit, MP is given by:

$MP =$ marginal revenue $-$ marginal cost;

Now, we know that at maximum profit, the marginal profit is zero;

$$MP = 0$$

Hence, it follows that:

$$MR - MC = 0$$

At maximum profit.

Hence,

$$MR = MC$$

PROVED!

(d)

A firm produces two products x and y and joint total cost function, TC, is given by:

$$TC = 6x^2 + 3x + 1.5xy + 3y^2 + 2y + 50$$

(i) Determine the marginal cost function for each of the products.

(ii) Evaluate the marginal cost when $y = 3$ and $x = 5$

(i)

To find the marginal cost function for each product; we will be taking the partial derivative of the cost function with respect to x and y;

$$MC(x) = \frac{\partial}{\partial x}(6x^2 + 3x + 1.5xy + 3y^2 + 2y + 50)$$

$$MC(x) = 2 \times 6x^{2-1} + 1 \times 3x^{1-1} + 1 \times 1.5x^{1-1} \times y + 0 + 0 + 0$$

$$MC(x) = 12x + 3 + 1.5y$$

$$MC(y) = \frac{\partial}{\partial y}(6x^2 + 3x + 1.5xy + 3y^2 + 2y + 50)$$

$$MC(y) = 0 + 0 + 1 \times 1.5x \times y^{1-1} + 2 \times 3 \times y^{2-1} + 1 \times 2y^{1-1} + 0$$

$$MC(y) = 1.5x + 6y + 2$$

Those are the separate marginal cost functions for each product;

(ii)

$$MC(x) = 12x + 3 + 1.5y$$

At $y = 3$ and $x = 5$

$$MC(x) = 12(5) + 3 + 1.5(3)$$

$$MC(x) = 60 + 3 + 4.5 = 67.5$$

$$MC(y) = 1.5x + 6y + 2$$

At $y = 3$ and $x = 5$

$$MC(y) = 1.5(5) + 6(3) + 2$$

$$MC(y) = 7.5 + 18 + 2 = 27.5$$

Hence, the marginal cost of product x is 67.5 and the marginal cost for y is 27.5 at $y = 3$ and $x = 5$

(e)

(iii) "Integration can be defined as the reverse of differentiation." Illustrate this statement with the primitive function;

$$Y = 2x^2 + 5x + 6$$

(iv) Given the marginal revenue function; $MR = 84 - 2Q - Q^2$, find the total revenue function.

(i)

This is a very tricky but simple question; we're given a function to prove that integration and differentiation are two reverse processes with respect to each other, it's simple;

Differentiate the function and integrate the result you get and you must get the initial function back!

$$Y = 2x^2 + 5x + 6$$

Differentiating Y with respect to x;

$$\frac{dY}{dx} = 2 \times 2x^{2-1} + 1 \times 5x^{1-1} + 0$$

$$\frac{dY}{dx} = 4x + 5$$

Now taking the integral;

$$\frac{dY}{dx} = 4x + 5$$

$$dY = (4x + 5)dx$$

$$\int dY = \int (4x + 5)dx$$

$$Y = \int 4xdx + \int 5dx$$

$$Y = 4\int xdx + 5\int x^0dx$$

$$Y = 4\left[\frac{x^{1+1}}{1+1}\right] + 5\left[\frac{x^{0+1}}{0+1}\right]$$

$$Y = \frac{4x^2}{2} + \frac{5x}{1}$$

$$Y = 2x^2 + 5x + C$$

Since 6 is also a constant, then, we have confirmed that integration is the reverse process of differentiation!

(ii)

$$MR = 84 - 2Q - Q^2$$

To get the total revenue function, integrate the marginal revenue;

$$TR = \int MR = \int (84 - 2Q - Q^2)dQ$$

$$TR = \int 84dQ - \int 2QdQ - \int Q^2dQ$$

$$TR = 84\int Q^0dQ - 2\int QdQ - \int Q^2dQ$$

$$TR = 84\left[\frac{Q^{0+1}}{0+1}\right] - 2\left[\frac{Q^{1+1}}{1+1}\right] - \left[\frac{Q^{2+1}}{2+1}\right]$$

$$TR = 84Q - Q^2 - \frac{Q^3}{3} + C$$

In every revenue function, the constant is zero since zero sales yields zero revenues;

$$TR = 84Q - Q^2 - \frac{Q^3}{3}$$

Question 2

A firm producing two goods x and y has the total cost function given as:

$$TC = 8x^2 - xy + 12y^2$$

If the firm is bound by contract to produce a combination of two goods totaling 42;

(iv) Write out the objective and the constraint functions;

(v) Derive the equations of the first order conditions for the firm's constrained optimization problems.

(vi) Use Crammer's rule to solve for the optimal values of x and y.

Here, we seek to optimize the cost while we still obey the constraint; but we must first of all extract the constraint;

The total number of both goods must be 42; Hence,

$$x + y = 42$$

(i)

The objective function which we want to optimize is the cost function;

Objective function: $8x^2 - xy + 12y^2$

The constraint function, we have seen it above; but the only thing else is to express it equated to zero;

$$x + y = 42$$
$$x + y - 42 = 0$$

The constraint function: $x + y - 42$

(ii)

To derive the first order conditions, we'll write our Lagrangean function; sure you know how to fix all that;

$$\mathcal{L}(x, y, \lambda) = 8x^2 - xy + 12y^2 - \lambda(x + y - 42)$$

Take the first order partials;

$$\mathcal{L}_x = 2 \times 8x^{2-1} - 1 \times x^{1-1} \times y + 0 - \lambda(1 \times x^{1-1} + 0 - 0)$$

$$\mathcal{L}_x = 16x - y - \lambda$$

$$\mathcal{L}_y = 0 - 1 \times x \times y^{1-1} + 2 \times 12y^{2-1} - \lambda(0 + 1 \times y^{1-1} - 0)$$

$$\mathcal{L}_y = -x + 24y - \lambda$$

$$\mathcal{L}_\lambda = 0 - 0 + 0 - 1 \times \lambda^{1-1}(x + y - 42)$$

$$\mathcal{L}_\lambda = -x - y + 42$$

For the first order conditions, equate each to zero;

$$16x - y - \lambda = 0 \dots \dots \dots (1)$$

$$-x + 24y - \lambda = 0 \dots \dots \dots (2)$$

$$-x - y + 42 = 0$$

$$-x - y = -42 \dots \dots \dots (3)$$

(iii)

We're told to resolve this using the Crammer's rule, like I've said severally, you can always use Crammer's rule even when not specified in case you can't cope with simultaneous equations in three variables;

Here, we have three equations;

$$16x - y - \lambda = 0 \dots \dots \dots (1)$$

$$-x + 24y - \lambda = 0 \dots \dots \dots (2)$$

$$-x - y = -42 \dots \dots \dots (3)$$

The equivalent matrix determinant needed here is :

$$\Delta = \begin{vmatrix} 16 & -1 & -1 \\ -1 & 24 & -1 \\ -1 & -1 & 0 \end{vmatrix}$$

Evaluating this determinant;

$$\Delta = 16[(24)(0) - (-1)(-1)] \\ - (-1)[(-1)(0) - (-1)(-1)] \\ + (-1)[(-1)(-1) - (24)(-1)]$$

$$\Delta = 16(-1) + 1(-1) - 1(25)$$

$$\Delta = -16 - 1 - 25 = -42$$

To evaluate Δ_x, we replace the column of x with the column matrix of answers;

$$\Delta_x = \begin{vmatrix} 0 & -1 & -1 \\ 0 & 24 & -1 \\ -42 & -1 & 0 \end{vmatrix}$$

Evaluating this determinant;

$$\begin{aligned} \Delta_x = 0[(24)(0) - (-1)(-1)] \\ - (-1)[(0)(0) - (-1)(-42)] \\ + (-1)[(0)(-1) - (24)(-42)] \end{aligned}$$

$$\Delta_x = 0(-1) + 1(-42) - 1(1008)$$

$$\Delta_x = 0 - 42 - 1008 = -1050$$

To evaluate Δ_y, we replace the column of y with the column matrix of answers;

$$\Delta_y = \begin{vmatrix} 16 & 0 & -1 \\ -1 & 0 & -1 \\ -1 & -42 & 0 \end{vmatrix}$$

Evaluating this determinant;

$$\Delta_y = 16[(0)(0) - (-1)(-42)] \\ - 0[(-1)(0) - (-1)(-1)] \\ + (-1)[(-1)(-42) - (0)(-1)]$$

$$\Delta_y = 16(-42) + 0(-1) - 1(42)$$

$$\Delta_y = -714$$

To evaluate Δ_λ, we replace the column of λ with the column matrix of answers;

$$\Delta_z = \begin{vmatrix} 16 & -1 & 0 \\ -1 & 24 & 0 \\ -1 & -1 & -42 \end{vmatrix}$$

Evaluating this determinant;

$$\Delta_\lambda = 16[(24)(-42) - (0)(-1)] \\ - (-1)[(-1)(-42) - (0)(-1)] \\ + 0[(-1)(-1) - (-1)(24)]$$

$$\Delta_\lambda = 16(-1008) + 1(42) - 0(25)$$

$$\Delta_\lambda = -16086$$

Hence, we have our x, y and λ values as;

$$x = \frac{\Delta_x}{\Delta} = \frac{-1050}{-42} = 25$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-714}{-42} = 17$$

$$\lambda = \frac{\Delta_\lambda}{\Delta} = \frac{-16086}{-42} = 383$$

Question 3

(a) Distinguish between definite and indefinite integrals.

(b) Find:

(i) $$\int x^2(x^3 - 5)^3\, dx$$

(ii) $$\int 4e^{2x}\, dx$$

(c) Find:

(i) $$\int_1^2 x^2(x^3 - 5)^3\, dx$$

(ii) $$\int_0^2 4e^{2x}\, dx$$

Right, this is basically an integration question and hence can be tackled straight up; but there is a theoretical question in the *(a)* part which you can simply see in this book;

(a)

For the (a) part, we know the difference between definite and indefinite integral;

Definite integral gives the area under the curve of the graph of a given function while an indefinite integral gives the anti-derivative of a function. A very obvious difference also is that indefinite integrals contain an arbitrary constant while a definite integral doesn't contain an arbitrary constant.

(b)

(i)

$$\int x^2(x^3 - 5)^3\, dx$$

This is a substitution question;

$$u = x^3 - 5$$

$$\frac{du}{dx} = 3 \times x^{3-1} - 0 = 3x^2$$

Hence,

$$dx = \frac{du}{3x^2}$$

Hence, our integral becomes;

$$\int x^2(u)^3 \times \frac{du}{3x^2}$$

x^2 cancels out;

$$\int (u)^3 \times \frac{du}{3} = \frac{1}{3}\int u^3 du$$

We have;

$$\frac{1}{3}\left[\frac{u^{3+1}}{3+1}\right] = \frac{1}{3} \times \frac{u^4}{4} = \frac{u^4}{12}$$

Replace u and add your arbitrary constant;

$$\frac{(x^3-5)^4}{12} + C$$

(ii)

$$\int 4e^{2x}\,dx$$

This is also a substitution question;

$$u = 2x$$

$$\frac{du}{dx} = 1 \times 2 \times x^{1-1} = 2$$

Hence,

$$dx = \frac{du}{2}$$

Hence, our integral becomes;

$$\int 4e^{u} \times \frac{du}{2}$$

Bring out the constant;

$$\frac{4}{2}\int e^{u}du = 2\int e^{u}du$$

We have;

$$2(e^{u}) = 2e^{u}$$

Replace u and add your arbitrary constant;

$$2e^{2x} + C$$

(c)

This is a situation of definite integrals;

$$\int_1^2 x^2(x^3-5)^3\,dx$$

We have the indefinite integral from (b) above as:

$$\frac{(x^3-5)^4}{12}+C$$

Evaluate the limits skipping the arbitrary constant since it'll eventually cancel out;

$$\left[\frac{(x^3-5)^4}{12}\right]_1^2=\frac{((2)^3-5)^4}{12}-\frac{((1)^3-5)^4}{12}$$

$$\frac{3^4}{12}-\frac{(-4)^4}{12}=\frac{81}{12}-\frac{256}{12}=\frac{81-256}{12}$$

Hence;

$$\int_1^2 x^2(x^3-5)^3\,dx=-\frac{175}{12}$$

The second one;

(ii)

$$\int_0^2 4e^{2x}\,dx$$

We also have the indefinite integral from (b) above as:

$$2e^{2x} + C$$

Evaluate the limits skipping the arbitrary constant since it'll eventually cancel out;

$$[2e^{2x}]_1^2 = [2e^{2\times 2}] - [2e^{2\times 1}]$$

$$[2e^4] - [2e^2] = 2[e^4 - e^2]$$

From our calculator, we have;

$$2(54.59815 - 7.38906)$$

$$\int_0^2 4e^{2x}\,dx = 94.41818$$

(d)

Given the marginal cost, $MC = 12e^{0.2Q}$, the fixed cost equals 72, completely determine the cost function.

From the marginal cost function, integrate it to get the cost function;

$$MC = 12e^{0.2Q}$$

$$TC = \int MC = \int 12e^{0.2Q}\, dQ$$

Hence,

$$TC = 12 \int e^{(0.2Q)} dQ$$

This is a substitution situation;

$$u = 0.2Q$$

$$\frac{du}{dQ} = 1 \times 0.2Q^{1-1} = 0.2$$

$$dQ = \frac{du}{0.2}$$

Hence, we have;

$$TC = 12 \int e^{u} \frac{du}{0.2}$$

$$TC = \frac{12}{0.2} \int e^{u} du = 60 \int e^{u} du$$

$$60[e^{u}] = 60e^{u}$$

Replace u and add the arbitrary constant; hence;

$$TC = 60e^{0.2Q} + C$$

The fixed cost is 72; hence, at 0 quantity produced; the cost is 72

Hence,

$$72 = 60e^{0.2\times 0} + C$$
$$72 = 60e^{0} + C$$
$$72 = 60(1) + C$$
$$C = 12$$

Hence, the total cost function is;

$$TC = 60e^{0.2Q} + 12$$

Notice the strong possible error over here, to think that the fixed cost is always the arbitrary constant, that isn't the definition of the fixed cost. The fixed cost is the cost when even 0 units of the products have been produced! Mostly, the fixed cost is the arbitrary constant, but here is a fine exception!

Question 4

(a) Distinguish between an inverse matrix and an identity matrix.

(b) Given the matrix equation;

$$A_{n\times 1}X_{n\times 1} = B_{n\times 1}$$

If the inverse A^{-1} exists, show that;

$$X_{n\times 1} = (A^{-1}B)_{n\times 1}$$

(c) Find the inverse of $\begin{pmatrix} 1 & 4 & -5 \\ 3 & -2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$

(a)

Recalling our studies of matrix, the identity matrix and inverse matrix are both square matrix; however, their difference is that;

- The inverse of a matrix when multiplying its own matrix yields an identity matrix;

WHILE

- An identity matrix still yields the same matrix when multiplying a matrix, it doesn't affect the matrix;

(b)

Given the matrix equation;

$$A_{n\times 1}X_{n\times 1} = B_{n\times 1}$$

If the inverse A^{-1} exists, show that;

$$X_{n\times 1} = (A^{-1}B)_{n\times 1}$$

Well, I have some problem with this question; I guess we need some edit here; only square matrices have determinants and hence only square matrices have inverses; hence, I guess the order that should be written below the matrices in this question should $n \times n$; hence, let's reframe our questions as;

$$\boldsymbol{A_{n\times n}X_{n\times n} = B_{n\times n}}$$

If the inverse $\boldsymbol{A^{-1}}$ exists, show that;

$$\boldsymbol{X_{n\times n} = (A^{-1}B)_{n\times n}}$$

So, let's begin; if A^{-1} exists; it'll also be an $n \times n$ matrix as well; from the equation;

$$A_{n\times n}X_{n\times n} = B_{n\times n}$$

Pre-multiplying both sides by A^{-1}

$$A^{-1} \times A_{n\times n}X_{n\times n} = A^{-1} \times B_{n\times n}$$

From the law of matrix inverses, when an inverse matrix multiplies the original matrix, it yields an identity matrix, hence, we have;

$$A^{-1} \times A_{n\times n} = I_{n\times n}$$

$$I_{n\times n}X_{n\times n} = A^{-1} \times B_{n\times n}$$

From the rule of identity matrix, an identity matrix multiplying any matrix still yields the matrix, hence,

$$I_{n\times n}X_{n\times n} = X_{n\times n}$$

Hence,

$$X_{n\times n} = A^{-1} \times B_{n\times n}$$

Since, A^{-1} is also an $n \times n$ matrix, the product of A^{-1} and $B_{n\times n}$ will also yield an $n \times n$ matrix;

Therefore;

$$X_{n\times n} = (A^{-1}B)_{n\times n}$$

We've proved what we need to prove! And hence, the correction we made to the question is perfect!

(c)

Find the inverse of $\begin{pmatrix} 1 & 4 & -5 \\ 3 & -2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$

Firstly, we need to find the matrix of minors of each element in this, please don't attempt this if you haven't read it yet; let's begin;

Let:

$$A = \begin{pmatrix} 1 & 4 & -5 \\ 3 & -2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$

Let's find the minor elements;

$$min(1) = \begin{vmatrix} -2 & 1 \\ 3 & 4 \end{vmatrix} \qquad min(4) = \begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix}$$

$$min(-5) = \begin{vmatrix} 3 & -2 \\ -1 & 3 \end{vmatrix} \quad min(3) = \begin{vmatrix} 4 & -5 \\ 3 & 4 \end{vmatrix}$$

$$min(-2) = \begin{vmatrix} 1 & -5 \\ -1 & 4 \end{vmatrix} \qquad min(1) = \begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix}$$

$$min(-1) = \begin{vmatrix} 4 & -5 \\ -2 & 1 \end{vmatrix} \qquad min(3) = \begin{vmatrix} 1 & -5 \\ 3 & 1 \end{vmatrix}$$

$$min(4) = \begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix}$$

Hence,

The matrix of minors, after evaluating the determinants above is;

$$\text{minor}(A) = \begin{pmatrix} -11 & 13 & 7 \\ 31 & -1 & 7 \\ -6 & 16 & -14 \end{pmatrix}$$

From the cofactor matrix rule below:

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

Hence,

The matrix of cofactors is;

$$\text{cofactor}(A) = \begin{pmatrix} -11 & -13 & 7 \\ -31 & -1 & -7 \\ -6 & -16 & -14 \end{pmatrix}$$

Then the adjoint matrix is given by the transpose of the cofactor matrix:

$$Adj(A) = [\text{cof}(A)]^T = \begin{pmatrix} -11 & -13 & 7 \\ -31 & -1 & -7 \\ -6 & -16 & -14 \end{pmatrix}^T$$

$$Adj(A) = \begin{pmatrix} -11 & -31 & -6 \\ -13 & -1 & -16 \\ 7 & -7 & -14 \end{pmatrix}$$

Lastly; find the determinant of A, the original matrix;

$$|A| = \begin{vmatrix} 1 & 4 & -5 \\ 3 & -2 & 1 \\ -1 & 3 & 4 \end{vmatrix}$$

$$1[(-2)(4) - (1)(3)] - 4[(3)(4) - (1)(-1)] + (-5)[(3)(3) - (-2)(-1)]$$

$$1[-11] - 4[13] - 5[7]$$

$$|A| = -98$$

The inverse of A, A^{-1}, hence is;

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$A^{-1} = \frac{1}{-98}\begin{pmatrix} -11 & -31 & -6 \\ -13 & -1 & -16 \\ 7 & -7 & -14 \end{pmatrix}$$

$$A^{-1} = -\frac{1}{98}\begin{pmatrix} -11 & -31 & -6 \\ -13 & -1 & -16 \\ 7 & -7 & -14 \end{pmatrix}$$

That's it about SSC106 2002/2003 SESSION. TRUST YOU CAN BEAR WITNESS THAT EVERYTHING HAS BEEN TAUGHT IN DETAILS IN THIS BOOK!!!