DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
SSC106: MATHEMATICS FOR SOCIAL SCIENCES II
RAIN SEMESTER EXAMINATION (2011/2012 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A;**
- Answer only one question in **Section B.**
- Show all workings clearly

Time allowed: 2 hours

SECTION A

1. If:

$$y = \frac{U}{V}$$

where U and V are functions of x, then

$$\frac{dy}{dx} = .\ \ldots \ldots \ldots \ldots \ldots \ldots$$

2. Differentiate:

(i) $y = \log(ax^2 + bx + c)$

(ii) $y = \log(\log x)$

3. What is partial differentiation?

4. Verify $Z_{xy} = Z_{yx}$ when $Z = x^2e^{-y}$

5. If $Z = \frac{xy}{x+y}$, show that $xZ_x + yZ_y = Z$

6. Distinguish between differential calculus and integral calculus.

7. Show that the function:
$Z = e^x \sin y + e^y \sin x$ satisfies the Laplace equation $Z_{xx} + Z_{yy} = 0$

8. Compute the elasticity of the following demand function $q = kp^c$ where k and c are constants.

9. Define the following functions and give appropriate examples:
   (i) Monotonic function;
   (ii) Multivariate function.

10. Distinguish between $|M|$ and $Spur\ M$.

$$M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 8 & 8 & 8 & 0 \\ 16 & 16 & 16 & 16 \end{bmatrix}$$

SECTION B

1. *(a)* Why is some knowledge of mathematics necessary in Social Sciences?

   (b) Obtain the integral of:

   (i) $\dfrac{2ax + b}{ax^2+bx+c}$

   (ii) $x^a + x^b + x^c$, where a, b and c are constants.

2. *(a)* Let $Q = \alpha p^{-n}$ where $n, \alpha, p > 0$, be the demand function for a commodity. Evaluate the marginal function, the average function and the elasticity of the demand for the commodity.

   (b) Suppose that the sales revenue (S) depends upon the quality of advertising (A) in a relationship estimated as $S = 14 + 16A - 2A^2$. Find the value of A which maximizes S.

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2011/2012 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and only one question from Section B. We'll be solving everything though, just sit tight as we solve.

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

Question 1

If:

$$y = \frac{U}{V}$$

where U and V are functions of x, then

$$\frac{dy}{dx} = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$y = \frac{U}{V}$$

The above obviously is a situation of quotient rule; hence;

$$\frac{dy}{dx} = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$$

Question 2

Differentiate:

(i) $y = \log(ax^2 + bx + c)$

(ii) $y = \log(\log x)$

(i)

$$y = \log(ax^2 + bx + c)$$

We'll make the assumption we're differentiating with respect to x since its not stated in the question;

A substitution situation;

$$u = ax^2 + bx + c$$

$$\frac{du}{dx} = 2ax + b$$

Hence;

$$y = \log u = \log_{10} u$$

$$\frac{dy}{du} = \frac{1}{u \ln 10}$$

Hence;

Chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u \ln 10} \times (2ax + b)$$

Return u

$$\frac{dy}{dx} = \frac{2ax + b}{\ln 10 \ (ax^2 + bx + c)}$$

(ii)

$$y = \log(\log x)$$

Another substitution situation;

$$u = \log x = \log_{10} x$$

$$\frac{du}{dx} = \frac{1}{x \ln 10}$$

Hence;

$$y = \log u = \log_{10} u$$

$$\frac{dy}{du} = \frac{1}{u \ln 10}$$

Hence;
Chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u \ln 10} \times \frac{1}{x \ln 10}$$

Return u

$$\frac{dy}{dx} = \frac{1}{x(\log x)(\ln 10)^2}$$

(c)

What is partial differentiation?

Partial differentiation is the differentiation of multivariable functions which yields several partial derivatives (differential coefficients) where a partial derivative of a function of several variables (the multivariate functions) is its derivative with respect to one of the variables that constitute the function with other variables temporarily kept constant.

(d)

Verify $Z_{xy} = Z_{yx}$ when $Z = x^2 e^{-y}$

$$Z = x^2 e^{-y}$$

Differentiating with respect to x; y is constant.

$$Z_x = 2 \times x^{2-1} e^{-y}$$

$$Z_x = 2xe^{-y}$$

Differentiating with respect to y; x is constant.

$$Z_y = x^2 \frac{\partial}{\partial y}(e^{-y})$$

Derivative with respect to$y$ of e^{-y} is $-e^{-y}$ (using chain rule, as done several times in this text)

$$Z_y = x^2(-e^{-y}) = -x^2 e^{-y}$$

$$Z_{xy} = \frac{\partial}{\partial y}(Z_x) = \frac{\partial}{\partial y}(2xe^{-y})$$

Here;

x will be a constant;

$$Z_{xy} = 2x \frac{\partial}{\partial y}(e^{-y})$$

Here;

$$Z_{xy} = 2x(-e^{-y}) = -2xe^{-y}$$

$$Z_{yx} = \frac{\partial}{\partial x}(Z_y) = \frac{\partial}{\partial x}(-x^2e^{-y})$$

y will now be a constant;

$$Z_{yx} = -2 \times x^{2-1}e^{-y}$$

$$Z_{yx} = -2xe^{-y}$$

Hence;
Quite obviously;

$$Z_{xy} = Z_{yx}$$

Question 5

If $Z = \frac{xy}{x+y}$, show that $xZ_x + yZ_y = Z$

Simple stuff! Quotient in partial derivatives;

As for Z_x;

$$u = xy$$
$$v = x + y$$

$$\frac{\partial u}{\partial x} = 1 \times x^{1-1}y = y$$

$$\frac{\partial v}{\partial x} = 1 \times x^{1-1} + 0 = 1$$

Hence;
Quotient rule;

$$Z_x = \frac{v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x}}{v^2}$$

$$Z_x = \frac{(x+y)(y) - (xy)(1)}{(x+y)^2}$$

$$Z_x = \frac{y^2}{(x+y)^2}$$

As for Z_y;
Same substitutions;

$$u = xy$$
$$v = x + y$$

$$\frac{\partial u}{\partial y} = 1 \times xy^{1-1} = x$$

$$\frac{\partial v}{\partial y} = 0 + 1 \times y^{1-1} = 1$$

Hence;

Quotient rule;

$$Z_y = \frac{v\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial y}}{v^2}$$

$$Z_y = \frac{(x+y)(x) - (xy)(1)}{(x+y)^2}$$

$$Z_y = \frac{x^2}{(x+y)^2}$$

To prove;

$$xZ_x + yZ_y = Z$$

Simply substitute and show that the LHS is equal to the RHS;

$$x\left(\frac{y^2}{(x+y)^2}\right) + y\left(\frac{x^2}{(x+y)^2}\right)$$

Expand;

$$\frac{xy^2}{(x+y)^2} + \frac{x^2y}{(x+y)^2}$$

Common denominator, add;

$$\frac{xy^2 + x^2y}{(x+y)^2}$$

Factorize the numerator;

$$\frac{xy(y+x)}{(x+y)^2} = \frac{xy(x+y)}{(x+y)^2}$$

$(x + y)$ cancels off; we're left with;

$$\frac{xy}{x+y}$$

From the beginning;

$$Z = \frac{xy}{x+y}$$

Hence; it is proved that:

$$xZ_x + yZ_y = Z$$

Question 6

Distinguish between differential calculus and integral calculus.

Differential calculus is the process of finding the derivative of a function while integral calculus is the process of finding a function from its derivative.

Differentiation deals with the rate of change of a quantity with respect to another, while integration deals with the area under the curve of a function. The two processes are reverse processes with respect to each other.

Question 7

Show that the function:
$Z = e^x \sin y + e^y \sin x$ satisfies the Laplace equation $Z_{xx} + Z_{yy} = 0$

Differentiating partially, keeping terms temporarily constant;

$$Z = e^x \sin y + e^y \sin x$$

$$Z_x = (e^x) \sin y + e^y (\cos x)$$

$$Z_x = e^x \sin y + e^y \cos x$$

$$Z_y = e^x (\cos y) + (e^y) \sin x$$

$$Z_y = e^x \cos y + e^y \sin x$$

Going further;

$$Z_{xx} = \frac{\partial}{\partial x}(Z_x) = \frac{\partial}{\partial x}(e^x \sin y + e^y \cos x)$$

$$Z_{xx} = (e^x \sin y + e^y(-\sin x)$$

$$Z_{xx} = e^x \sin y - e^y \sin x$$

$$Z_{yy} = \frac{\partial}{\partial y}\left(Z_y\right) = \frac{\partial}{\partial y}(e^x \cos y + e^y \sin x)$$

$$Z_{yy} = e^x(-\sin y) + (e^y)\sin x$$

Rearranging;

$$Z_{yy} = e^y \sin x - e^x \sin y$$

Test for:

$$Z_{xx} + Z_{yy}$$

$$(e^x \sin y - e^y \sin x) + (e^y \sin x - e^x \sin y)$$

Everything cancels off to ZERO! It's been proved!

Question 8

Compute the elasticity of the following demand function $q = kp^c$ where k and c are constants.

Here, since k and c are constants, the demand function is q and it is dependent on p; quite normal variables;

The elasticity is given by:

$$P.E = \left(\frac{dq}{dp}\right) \times \left(\frac{p}{q}\right)$$

Here;

$$q = kp^c$$

Power rule;

$$\frac{dq}{dp} = c \times kp^{c-1}$$

$$\frac{dp}{dq} = kcp^{c-1}$$

Hence;

$$P.E = kcp^{c-1} \times \frac{p}{q}$$

As we know; $\left(\frac{p}{q}\right)$ are corresponding values of p and q which aren't given here; hence; we leave it as $\left(\frac{p}{q}\right)$ and simplify using indices;

$$P.E = \frac{kc}{q} \times p^{c-1} \times p$$

$$P.E = \frac{kc}{q} \times p^{c-1+1}$$

$$P.E = \frac{kc}{q} \times p^{c}$$

Hence;

$$P.E = \frac{kcp^{c}}{q}$$

Question 9

Define the following functions and give appropriate examples:

(i) Monotonic function;

(ii) Multivariate function.

Direct definition questions from the notes;

(i)

They're the types of functions which are either strictly increasing or decreasing, unlike other functions that are increasing and decreasing over separate intervals; the monotonic functions are either always increasing or always decreasing. **They're the classes of increasing or decreasing functions.**

Examples are:

$$y = 3x + 7$$

$$y = 8 - 3x$$

$$y = e^{x}$$

$$y = \left(\frac{3}{5}\right)^{x}$$

(ii)

The multivariable (multivariate) functions are functions that have more than one independent variable, they also usually also have one dependent variable in situations where the dependent variable is shown; they're in the form;

$$Z = f(x, y)$$

Where Z is a variable (the dependent variable) and x and y are the independent variables; Examples are:

$$Z = x^2y - y^2x$$

$$y = u^3 - v^3 + 3$$

$$f(x, y) = x^5 + y^3$$

Question 10

Distinguish between $|M|$ and $Spur\ M$.

$$M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 8 & 8 & 8 & 0 \\ 16 & 16 & 16 & 16 \end{bmatrix}$$

We have a lower triangular matrix and hence, we have a triangular matrix to work on.

The spur of the matrix, which is equal to its trace, is equal to the sum of the elements on its main diagonal, hence, we have:

$$\text{spur}(M^T) = 2 + 4 + 8 + 16 = 30$$

While since it's a triangular matrix, the determinant is the product of the elements on the main diagonal.

$$|M^T| = 2 \times 4 \times 8 \times 16 = 1024$$

Hence, to distinguish;

$\text{spur}(M^T) =$ sum of elements on the main diagonal

$|M^T| =$ product of elements on the main diagonal

SECTION B

Question 1

(a) Why is some knowledge of mathematics necessary in Social Sciences?

(b) Obtain the integral of:

(i) $\dfrac{2ax + b}{ax^2+bx+c}$

(ii) $x^a + x^b + x^c$, where a, b and c are constants.

(a)

The knowledge of mathematics is very useful in the field of Social Sciences because:

- It helps social scientists to state their research problems in specific and clear terms.
- Mathematics provide considerable insight into the way by which numerical information can be generated and presented to aid decision making in the social and management science.
- It helps in Identifying and quantifying the relationship between variables that determine the outcome of the decisions and their alternatives in the social and management science.
- It minimizes subjectivity and enhances the chance of making objective decision.
- It assists in the prediction or forecasting of future events.

- The language of mathematics is very easy to understand.
- Mathematics makes problems that could take lengthy periods to be resolved in minutes;
- With calculus, we can find the relative optima of different economical functions to easily find desired optimum results;
- Mathematical optimization helps consumers to maximize their utilities; this helps consumers in making decisions.
- Mathematics is a fast method of solving critical problem that are relevant to the study of economics and management science for example, the price of the quantity demand and supply of a commodity.
- The use of matrices is very useful in solving cases of multiple inputs;

(b)

Integrals;

(i)

We have:

$$\int \frac{2ax + b}{ax^2 + bx + c} dx$$

This is a case of:

$$\frac{f'(x)}{f(x)}$$

Hence;

We have:

$$u = ax^2 + bx + c$$

$$\frac{du}{dx} = 2ax + b$$

Hence;

$$dx = \frac{du}{2ax + b}$$

The integral becomes;

$$\int \frac{2ax + b}{u} \times \frac{du}{2ax + b}$$

Cancelling;

$$\int \frac{du}{u}$$

Standard integral; we have;

$$\ln u + C$$

Return u;

$$\ln(ax^2 + bx + c) + C$$

(ii)

$x^a + x^b + x^c$, where a, b and c are constants.

Very very simple! Power rule since the powers are constants;

$$\left[\frac{x^{a+1}}{a+1}\right] + \left[\frac{x^{b+1}}{b+1}\right] + \left[\frac{x^{c+1}}{c+1}\right]$$

Solution!

$$\frac{x^{a+1}}{a+1} + \frac{x^{b+1}}{b+1} + \frac{x^{c+1}}{c+1}$$

No serious simplification can be done!

Question 2

(a) Let $Q = \alpha p^{-n}$ where $n, \alpha, p > 0$, be the demand function for a commodity. Evaluate the marginal function, the average function and the elasticity of the demand for the commodity.

(b) Suppose that the sales revenue (S) depends upon the quality of advertising (A) in a relationship estimated as $S = 14 + 16A - 2A^2$. Find the value of A which maximizes S.

(a)

$$Q = \alpha p^{-n}$$

Since it's a demand function, the marginal function still remains the derivative. The derivative this time is power rule;

$$\frac{dQ}{dp} = -n \times \alpha p^{-n-1}$$

$$MQ = \frac{dQ}{dp} = -n\alpha p^{-n-1}$$

As for the average function, the same formula for average functions, dividing the function by the unit it depends on; this depends on p;

Hence; we have;

$$AQ = \frac{Q}{p} = \frac{\alpha p^{-n}}{p}$$

Simplifying p (indices);

$$AQ = \alpha p^{-n-1}$$

Elasticity; $P.E.D$;

$$P.E.D = \left(\frac{dQ}{dp}\right) \times \left(\frac{p}{Q}\right)$$

As done, when seeking the marginal function, we have $\left(\frac{dQ}{dp}\right)$ already:

$$P.E.D = -n\alpha p^{-n-1} \times \frac{p}{Q}$$

Here; we don't have any corresponding p and Q value hence, we simply simplify;

$$P.E.D = -\frac{n\alpha}{Q} \times p^{-n-1} \times p$$

$$P.E.D = -\frac{n\alpha}{Q} \times p^{-n-1+1}$$

$$P.E.D = -\frac{n\alpha}{Q} \times p^{-n}$$

Negative power in indices;

$$P.E.D = -\frac{n\alpha}{Qp^{n}}$$

(b)

$$S = 14 + 16A - 2A^2$$

To maximize S, the marginal sales revenue is zero!

$$MS = \frac{d}{dA}(S) = \frac{d}{dA}(14 + 16A - 2A^2)$$

Don't get worked up, MR, marginal revenue and MS, marginal sales revenue are same things, just change of words, revenues come from sales anyway!

Hence;

$$MS = 0 + 1 \times 16A^{1-1} - 2 \times 2A^{2-1}$$

$$MS = 16 - 4A$$

At maximum revenue;

$$MS = 0$$

Hence;

$$16 - 4A = 0$$

By solving;

$$A = 4$$

Hence, the value of A which maximizes S is 4.

DONE!