BASIC OPERATIONS IN MATHEMATICS

This may sound as a ridiculous topic; maybe irrelevant but trust me, you don't know how much an understanding of some concepts costs you the privilege of solving even questions you can solve correctly, some of these basic operations stand in your way even in the midst of moving on in a singular process of solving a question; therefore, stand here with me for the next few minutes and get yourself grounded in little little concepts that may cause you to fall in course of working with the math in this.

Let's begin from the rudiments;

ALGEBRA RULES

As far as dealings are made with natural algebra which includes variables and all; these are the most excellent rules;

- $a + b = b + a$; addition is commutative; changing the order of operation doesn't change it;

- $a - b \neq b - a$; subtraction isn't commutative; changing the order of operations changes the answer, not totally but by negating it, turning it to a negative number. This will be explained during factorization;

- $-a + b = b - a$; it's desirable to always keep the positive number first; to make math work neater;

- $a + (-b) = a - b$; expanding an addition and a subtraction ends up in subtraction.

- $a - (-b) = a + b$; expanding a subtraction and a subtraction ends up in addition.

Expansion;

- Expansion only occurs when brackets are involved to grouped sums and differences of terms together;

Terms are expanded when one term multiplies terms in a bracket;

Here: $a(b - c + d) = ab - ac + ad$; the term outside the bracket multiplies each term in the bracket for successful expansion while the signs remain the same;

In this case:

$$-a(b - c + d) = -ab + ac - ad$$

Rearranging to place positive terms first;

$$-a(b - c + d) = ac - ab - ad$$

Here, $-a$ which is a negative number multiplies the rest as usual but changes the signs in the expansion process since the negative signs reverse the operation.

In the case of two brackets:

$(a + b)(c + d)$; here, each term in the first bracket multiplies the bracket one by one with the sign maintained. We have:

$$a(c + d) + b(c + d) = ac + ad + bc + bd$$

Other cases include:

$$(a - b)(c + d)$$

$$a(c + d) - b(c + d) = ac + ad - bc - bd$$

$$(a + b)^2 = (a + b)(a + b)$$
$$a^2 + ab + ba + b^2 = a^2 + ab + ab + b^2$$

$$a^2 + 2ab + b^2$$

NOTE HERE THAT: $ab + ab = 2ab$

The above is the same case that makes:

$$a + a = 2a$$

In essence;
When two same terms come together by addition (or subtraction), they reinforce each other (in addition) or cause a reduction (in subtraction) and need not be written twice;

$$(a + 7b) + (a + 7b) = 2(a + 7b)$$

$$8(x - y)^3 - 5(x - y)^3 = 3(x - y)^3$$

Other cases of expansion;

$$(-a)b = a(-b) = ab$$

This introduces the concept of double or triple equality;

The equality:

$$a = b = c$$

Means that;

$$a = b$$
$$b = c$$
$$a = c$$

In essence, when equality forms a chain, it means all the terms are equal to each other.

The difference of two squares;

$$(a + b)(a - b) = a^2 - ab + ba - b^2$$
$$a^2 - ab + ab - b^2 = a^2 - b^2$$

Note that, in natural numbers, the order which multiplication takes doesn't matter, hence;

$ab = ba$; hence, $-ab + ba = 0$ means the same thing subtracted from each other, in essence, equal to zero.

Addition, subtraction, multiplication and division in fractions

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Those are different ways division of two terms can be expressed when they're negative, the three mean the same thing;

$\frac{a}{b}$ can be expressed as $a \times \frac{1}{b}$ in the case of need for convenience such as times when both a and b are ambiguous and relatively large terms.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

When two fractions are multiplied; it is equal to the product of their numerators divided by the product of their denominators.

The reciprocal of a number a, is $\frac{1}{a}$

When two fractions are linked by division as shown below; it is equal to the product of the first and the reciprocal of the second;

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} = \frac{ad}{bc}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} = \frac{ad}{bc}$$

$$a \times \frac{b}{c} = \frac{a}{1} \times \frac{b}{c} = \frac{a \times b}{1 \times c} = \frac{ab}{c}$$

Adding fractions;

Here;

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

bd above is the common denominator, it is taken as the product of the two denominators **in most cases**; each denominator divides the common denominator and the result multiplies the numerator of each fraction to form the sums of the terms at the numerator; example;

$$\frac{2}{3}+\frac{3}{4}=\frac{4\times 2+3\times 3}{12}=\frac{8+9}{12}=\frac{17}{12}$$

$$\frac{3}{5}-\frac{2}{7}=\frac{7\times 3-5\times 2}{35}=\frac{21-10}{35}=\frac{11}{35}$$

Here;

$$a+\frac{b}{c}=\frac{a}{1}+\frac{b}{c}$$

Here, the common denominator is automatically the only available denominator;

$$\frac{a}{1}+\frac{b}{c}=\frac{ac+b}{c}$$

More operations on fractions;

When a fraction has a single denominator as shown below;

$$\frac{a+b+c}{d}$$

The denominator can be split into the numerators one-by-one, in case where simplification is needed. In the above; we have;

$$\frac{a}{d}+\frac{b}{d}+\frac{c}{d}$$

For example, it helps simplification via indices in cases like:

$$\frac{a^6+5a^4+2a^2}{a^2}$$

Splitting;

$$\frac{a^6}{a^2}+\frac{5a^4}{a^2}+\frac{2a^2}{a^2}$$

After indicial simplification, we have;

$$a^4+5a^2+2$$

The above process needs a bit of indices; we'll see indices below;

Let's move to indices;

Indices as you did in SSC105 last year is the study of number bases and powers; an indicial term is given as below:

$$\text{base} \quad a^n \quad \text{power}$$

Indicial terms are very common terms in everyday mathematics;

Here are some quick rules;

When equal bases of different powers are multiplied, one base is taken and the powers added;

$$a^m \times a^n = a^{m+n}$$

When equal bases of different powers are divided, one base is taken and the powers subtracted, the second one (or denominator) subtracted from the first one (or numerator);

$$a^m \div a^n = a^{m-n}$$

OR

$$\frac{a^m}{a^n} = a^{m-n}$$

Whichever ways the same base occur, the rule ends, the same base means an entity; an entity could be united in a bracket;

$$(a+b)^m \times (a+b)^n = (a+b)^{m+n}$$

$$(a+b)^m \div (a+b)^n = (a+b)^{m-n}$$

$$\frac{(a+b)^m}{(a+b)^n} = (a+b)^{m-n}$$

In the above, we see $(a+b)$ as a common base!

Now, this same rule can be reversed; when we have a base with added powers; the operation can be reversed; example;

$$(2x+3y)^{\frac{1}{2}-\frac{2}{3}+\frac{4}{5}}$$

$$= (2x+3y)^{\frac{1}{2}} \div (2x+3y)^{\frac{2}{3}}$$

$$\times (2x+3y)^{\frac{4}{5}}$$

$$(2x+3y)^{\frac{1}{2}-\frac{2}{3}+\frac{4}{5}} = \frac{(2x+3y)^{\frac{1}{2}}(2x+3y)^{\frac{4}{5}}}{(2x+3y)^{\frac{2}{3}}}$$

So, powers can be reversed and broken down; on a simpler note;

$$e^{x+c} = e^{x} \times e^{c}$$

Where e is the base

Converting fractional powers to roots

$a^{\frac{m}{n}}$ is the nth root of a raised to the power of m; the denominator constitute the root and the numerator constitute the power;

In root form, this is equivalent to:

$$a^{\frac{m}{n}} = \sqrt[n]{a^{m}}$$

Hence, when you are told that square root is the same as the power of $\frac{1}{2}$, it is obvious here that the numerator is 1 and the denominator is 2, here, we'll be having a square root, square root being the most basic isn't written in the root and hence,

$$a^{\frac{1}{2}} = \sqrt{a^1} = \sqrt{a}$$

So, in other situations,

$$a^{\frac{2}{3}} = \sqrt[3]{a^2}$$

$$a^{\frac{4}{7}} = \sqrt[7]{a^4}$$

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$a^{\frac{6}{5}} = \sqrt[5]{a^6}$$

In the case of double powers in indices;
Consider;

$$(a^m)^n$$

In the case of double powers, the powers are simply expanded and sorted out properly;

In this case; the power is expanded as in normal multiplication;

$$(a^m)^n = a^{mn}$$

In practical examples;

$$(\sqrt{x})^{\frac{2}{3}} = \left(x^{\frac{1}{2}}\right)^{\frac{2}{3}} = x^{\frac{1}{2}\times\frac{2}{3}} = x^{\frac{1}{3}}$$

Other cases are simply similar;

$$[(2x+3)^3]^{\frac{1}{5}} = (2x+3)^{3\times\frac{1}{5}} = (2x+3)^{\frac{3}{5}}$$

Negative powers in indices;

The implication of a negative power in indices means ***an equivalent positive power in the inverted form.***

$$a^{-n} = \frac{1}{a^n}$$

As seen above, the expression is inverted and the power turned to positive value;

In practical cases;

$$\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$$

Hence, the rule helps in converting fractional indices to plain terms not in fraction (as above) and help to convert final answers from containing negative powers (as shown below);

$$2x^{-\frac{3}{4}} = 2 \times x^{-\frac{3}{4}} = 2 \times \frac{1}{x^{\frac{3}{4}}}$$

$$\frac{2}{x^{\frac{3}{4}}}$$

Other cases;

$$\frac{2}{(x+4)^{\frac{3}{2}}} = 2(x+4)^{-\frac{3}{2}}$$

$$\frac{7}{4(x^2-4)^3} = \frac{7}{4}(x^2-4)^{-3}$$

Moving to logarithms;

The logarithm to base a of a number b is given by;

$$\log_a b$$

Here,

b is the log number
a is the log base

We have the most basic rule of logarithm given thus:

If:

$$\log_a b = x$$

Then;

$$b = a^x$$

In words, the log base raised to the power of the log value is equal to the log number.

Therefore; for:

$$\log_3 9 = 2$$

It means;

$$3^2 = 9$$

The above can obviously be confirmed by normal mathematics;

Operations in logarithm are given below;

Logarithm without shown base (the common logarithm);

When the log base isn't shown, it simply implies the log to base 10; meaning;

$$\log a = \log_{10} a$$

$$\log 3 = \log_3 10$$

And so on;

The natural logarithm;

The Euler's number is the number, $\boldsymbol{e = 2.718}$

The above is the approximate value of the Euler's number;

Hence, when a logarithm is expressed to the base, e, the Euler's number, it is called the natural logarithm of a number;

Examples include;

$$\log_e 10$$

$$\log_e a$$

And so on;

The natural logarithm of a number has a special way it is expressed; it is expressed as "ln"

Hence; the following are equivalent;

$$\log_e 10 = \ln 10$$

$$\log_e x = \ln x$$

$$\log_e a = \ln a$$

And so on!

Logarithm of equal log number and log base;
When the log number is equal to the log base, the value of the logarithm is 1;
Hence;

$$\log_a a = 1$$
$$\log_2 2 = 1$$
$$\log_{10} 10 = 1$$
$$\ln e = \log_e e = 1$$

The power of a log number;

In a logarithm, when the log number is raised to a power, it is equivalent to the power multiplying the log when the power has been moved away from the log number;

In essence;

$$\log_e 3^2$$

If the power of (3) the log number is moved away, it will be in the form;

$$2 \log_e 3$$

Hence, the power of 3 (2) is now multiplying the whole log; hence;

$$3 \log 7 = \log 3^7$$

$$\ln 16 = \ln 4^2 = 2 \ln 4$$

Again; it could be;

$$\ln 4 = \ln 2^2 = 2 \ln 2$$

And so on;

Addition of logarithm;

The plain rule of addition of logarithm is common to logarithm of equal bases; logarithm of different bases can be added but would digress a bit beyond the basics, this chapter is meant to set you up for basic knowledge and refreshing in these concepts;

When common bases are added, the log numbers are multiplied; hence;

$$\log a + \log b = \log(a \times b) = \log ab$$

$$\ln(a+2) + \ln(b-3) = \ln(a+2)(b-3)$$

$$\log_2(x^2+2) + \log_2(2x) = \log_2\left((x^2+2)(2x)\right)$$

On expansion, we have;

$$\log_2(x^2+2) + \log_2(2x) = \log_2(2x^3+4x)$$

Subtraction of logarithm;

Just like in logarithm addition, the plain rule of subtraction of logarithm is common to logarithm of equal bases; logarithm of different bases can be subtracted but is beyond basic knowledge.

The term on the left must be taken consciously; hence, here, the log number of the term on the left is divided by the log number of the term on the right (the term being subtracted from)

$$\log a - \log b = \log(a \div b) = \log\left(\frac{a}{b}\right)$$

$$\ln(a+2) - \ln(b-3) = \ln\left[\frac{a+2}{b-3}\right]$$

$$\log_2(x^2+2) - \log_2(2x) = \log_2\left(\frac{x^2+2}{2x}\right)$$

On indices simplification within the bracket, we have;

$$\log_2\left(\frac{x^2+2}{2x}\right) = \log_2\left(\frac{x^2}{2x} + \frac{2}{2x}\right) = \log_2\left(\frac{x}{2} + \frac{1}{x}\right)$$

$$\ln 5 - \ln 2 = \ln\left(\frac{5}{2}\right) = \ln(2.5)$$

Combination of addition and subtraction of logarithms;

When both occur together, the log numbers of added terms are multiplied and the log numbers of the subtracted terms are used as denominators correspondingly! Examples;

$$\log 2 + \log 3 - \log 5 = \log\left(\frac{2 \times 3}{5}\right) = \log\left(\frac{6}{5}\right)$$

Furthermore;

$$\log(2x-3)-\log(3x+4)+\log(x-2)$$

We bring the addition together and the subtraction together;

$$\log(2x-3)+\log(x-2)-\log(3x+4)$$

Hence, we have;

$$\log\left[\frac{(2x-3)(x-2)}{(3x+4)}\right]$$

And so on;

Factorization;

Factorization is a very key factor that occurs in everyday mathematics but could be a challenge as simple as it is;

Factorization is done when two terms that have a common term in the separate terms that are related by multiplication in each term.

$$a \times b - a \times c = ab - ac$$

Now, a is common in these two separate terms and hence, a **is factorized** in the two; it is factorized because it is the common term between the two.

$$ab - ac = a(b - c)$$

You notice that if the right hand side is expanded, we're back to the original expression;

Factorization and expansion are two directly opposite processes;

Now, let's see some more factorization processes;

$$(2x - 4)(3x + 7) + (2x + 5)(2x - 4)$$

$$(2x - 4)[(3x + 7) + (2x + 5)]$$

$$(2x - 4)(3x + 7 + 2x + 5)$$

$$(2x - 4)(5x + 12)$$

So, when one term is common in a sum, it can be factorized; here; $(2x - 4)$ is the common factor;

In:

$$a^3 - 2a^2 - 3a$$

a is common since $a^3 = a \times a \times a$

a is common since $2a^2 = 2 \times a \times a$

Hence, we can factorize this as;

$$a(a^2 - 2a - 3)$$

In:

$$a^4 - 3a^3 + 5a^2$$

Not only a is common, a^2 is common; we can factorize it:

$$a^4 = a \times a \times a \times a$$
$$a^3 = a \times a \times a$$

Hence, after factorization; we can have;

$$a^2(a^2 - 3a + 5)$$

In:

$$(3a-2)^3(2a-5)^2 - (3a-2)^2(2a-5)(a+3)$$

This is because;

$$(3a-2)^3(2a-5)^2$$
$$= (3a-2)(3a-2)(3a-2)(2a-5)(2a-5)$$

And;

$$(3a-2)^2(2a-5)(a+3)$$
$$= (3a-2)(3a-2)(2a-5)(a+3)$$

It is obvious that after breaking the powers; we have that; $(3a-2)(3a-2)(2a-5)$ are common, we then factorize;

$$(3a-2)^2(2a-5)[(3a-2)(2a-5)-(a+3)]$$

$$(3a-2)^2(2a-5)[6a^2-15a-4a+10-(a+3)]$$

$$(3a-2)^2(2a-5)[6a^2-19a+10-a-3]$$

$$(3a-2)^2(2a-5)[6a^2-19a+10-a-3]$$

$$(3a-2)^2(2a-5)[6a^2-20a+7]$$

Notice that we always retain the sign in front of the terms while factorizing!

That's factorization in all, that's the major meaning of factorization;

We do not always show how terms are broken down first but see it clearly during factorization. So, let's see factorization in the operation of subtraction; In:

$$b - a$$

We can see $-a$ as $a(-1)$ and b as $-b(-1)$

Hence, when -1 is factorized; we have:

$$-1(-b + a) = -1(a - b)$$

1 has no basic significance; when multiplying anything, it's still the same;
We have:

$$-(a - b) = b - a$$

WORKING WITH EQUATIONS

We'll be looking at some general rules of equations; using the primitive equation below:

$$a + b = c + d$$

Here;

Let's see the process of **multiplying through;**

If multiplying through by n, we have n multiplying both sides;

$$n(a + b) = n(c + d)$$

$$an + bn = cn + dn$$

In essence, when terms in an equation are multiplied through, it affects all terms in the equation by multiplying all terms!

Now, all terms means terms that are added and subtracted, terms multiplied (and divided) are regarded as the same terms; example;

In:

$$\boldsymbol{a \times n + b}$$

There are only two terms in the above expression;

$a \times n$ which is equivalent to an is a term
b is the second term

Dividing through an equation;

In the same way, if dividing through by n, we have n multiplying dividing sides;

$$\frac{(a+b)}{n} = \frac{(c+d)}{n}$$

From what we learnt in fractions having a single denominator just few pages above, we know that the above is equivalent to;

$$\frac{a}{n} + \frac{b}{n} = \frac{c}{n} + \frac{d}{n}$$

In essence, when terms in an equation are divided through, it affects all terms in the equations by dividing each term!

Adding and subtracting to both sides of an equation;

$$a + b = c + d$$

If n is added to both sides of an equation, it is simply added once and joined to the terms already in the equation! We'll be having;

$$a + b + n = c + d + n$$

In essence, it is added once on each side of the equation and not added one by one to each terms;

The same goes for subtraction!

If n is subtracted from both sides of an equation, it is simply subtracted once and joined to the terms already in the equation! We'll be having;

$$a + b - n = c + d - n$$

Applying this to a real case of an equation, we make use of multiplying through to remove fractions from an equation; hence, in;

$$\frac{1}{3}(x-1) + \frac{3}{4}(x+3) = \frac{5}{6}(2x-3)$$

Since 12 can cancel off all the denominators, we can multiply through by 12, here, it'll affect all terms in the equation, hence, we have:

$$12 \times \frac{1}{3}(x-1) + 12 \times \frac{3}{4}(x+3)$$
$$= 12 \times \frac{5}{6}(2x-3)$$

We can now clear the fractions and we're left with;

$$4(x-1)+9(x+3)=10(2x-3)$$

Then, it is now expanded, we have:

$$4x-4+9x+3=20x-30$$

Collecting like terms;

$$13x-20x=-30+1$$

Hence;

$$-7x=-29$$

Dividing through by -7;

$$x=\frac{29}{7}$$

Those are simple bits;

In case of dividing through;

$$18x^3+6x+12=0$$

We can divide through the above equation by 6;

It’ll affect each term;

$$\frac{18x^3}{6} + \frac{6x}{6} + \frac{12}{6} = \frac{0}{6}$$

$$3x^3 + x + 2 = 0$$

SOLVING QUADRATIC EQUATIONS

For any given quadratic equation;

$$ax^2 + bx + c = 0$$

It can be solved mainly by two methods;

- Quadratic factorization
- Formula method;

Just two days before writing this, I saw a quote in a book that I think could be of help here; ***the method of factorization is learnt mainly by experience and hence, most average mathematicians use the quadratic formula in solving quadratic equations.***

Hence, for the above reason, we'll be taking only the method of the quadratic formula since it is easier to use, if you're already good in most basics of mathematics, this isn't really useful for you and you may already be good at quadratic factorization!

Hence;

The quadratic formula is this: for:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ops, before I continue, this sign, $\pm$ means, plus or minus, it means a dual (two different) values for the same variable;

For example;

$\pm a$ means plus or minus a

The above means we have two values (or answers) which are:

$$-a \text{ and } +a$$

That’s a bit of information you should know!

Back to the quadratic equation, we’ll take three examples and we’re done with this basic chapter, remember it isn’t SSC106 but a basic ladder to helping you understand SSC106 well.

- $5 - 5x - 2x^2 = 0$
- $3x + 2x^2 = x^2 + 7$
- $x^2 + 9x = -8$

I’ve put very compounded examples to help you understand this wholly!

$$5 - 5x - 2x^2 = 0$$

It is expedient you know that the quadratic formula is for the equation;

$$ax^2 + bx + c = 0$$

Hence, we must be conscious that a is the coefficient of x^2, b is the coefficient of x and c is the constant. Hence;

In:

$$5 - 5x - 2x^2 = 0$$

a is -2;
b is -5
c is 5;

You should see that quite clearly now!

Hence; we'll simply substitute in the equation and make use of the knowledge we already have:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-2)(5)}}{2(-2)}$$

$$x = \frac{5 \pm \sqrt{25 + 40}}{-4}$$

$$x = \frac{5 \pm \sqrt{65}}{-4}$$

$\sqrt{65}$ can be gotten from a calculator! We have:

$$x = \frac{5 \pm 8.062}{-4}$$

Finally, splitting our plus or minus into what it actually means, we have;

$$x = \frac{5 + 8.062}{-4} \quad \text{or} \quad x = \frac{5 - 8.062}{-4}$$

$$x = \frac{13.062}{-4} \quad \text{or} \quad x = -\frac{3.062}{-4}$$

Finishing this off from a calculator;

$$x = -3.2655 \quad \text{or} \quad x = 0.7655$$

NEXT!

- $3x + 2x^2 = x^2 + 7$

Here, we have to bring all terms to the left and see what happens next! **You should know that terms change sign as they cross the equality sign!**

$$3x + 2x^2 - x^2 - 7 = 0$$

$$x^2 + 3x - 7 = 0$$

Hence,

We now have our quadratic situation! We'll simply substitute in the equation and make use of the knowledge we already have:

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 28}}{2}$$

$$x = \frac{-3 \pm \sqrt{37}}{2}$$

$\sqrt{37}$ can be gotten from a calculator! We have:

$$x = \frac{-3 \pm 6.082}{2}$$

Finally, splitting our plus or minus into what it actually means, we have;

$$x = \frac{-3 + 6.082}{2} \quad \text{or} \quad x = \frac{-3 - 6.082}{2}$$

$$x = \frac{3.082}{2} \quad \text{or} \quad x = -\frac{9.092}{2}$$

Finishing this off from a calculator;

$$x = 1.541 \quad \text{or} \quad x = -4.541$$

You should be able to solve the last one!

- $x^2 + 9x = -8$

Bring all terms to the left!

$$x^2 + 9x + 8 = 0$$

Slotting into the quadratic equation, you should arrive at:

$$x = -1 \quad \text{or} \quad x = -8$$

Those are basic mathematical tips! Ensure you get all of them right as you now step into the main course of THE SSC106 WAY! I'm sure you'll get an A, I sure trust you. Enjoy reading and remember to check back here in case you get hooked on any basic step.