DEPARTMENT OF ECONOMICS
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SSC106: MATHEMATICS FOR SOCIAL SCIENCES II

RAIN SEMESTER EXAMINATION
(2006/2007 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A;**
- Answer two questions in **Section B.**
- Show all workings clearly

Time allowed: 2½ hours

SECTION A

Provide short answers to the following questions

1. What is a matrix?
2. What precisely is meant by the size of a matrix?
3. What condition must be met before two matrices can be added?
4. What is the conformability condition for matrix multiplication?

5. A square matrix with scalar element which are not necessary in the leading matrix is called …………… matrix.
6. What is an orthogonal matrix?
7. If a matrix has two equal rows (or columns), then the value of the determinant is ………..
8. The open-scissors technique of determinant evaluation is applicable for what size of matrix?
9. How are the simultaneous equations $ax + by = c; dx + ey = f$ written in their equivalent matrix forms?
10. …………… is a function which specifies a particular type of relation between two or more variables.
11. The classes of increasing and decreasing function are together called …………….
12. Which of the following is an even function?

    (i) $f(x) = \frac{1}{x}$

    (ii) $f(x) = 4$

    (iii) $f(x) = x^2 - 2x$

13. ……………. is a function which expresses the relation of two distances as a function of an angle.
14. What is differential calculus?
15. The exponential function of differentiation states that ………………
16. The derivative of $y = \log(x^4 - 2x + 7)$ is …………………..
17. If the revenue from sales of x terms is given by $R(x) = 25x + \left(\frac{x^2}{10}\right)$, the marginal revenue when $x = 5$ is ……………
18. The function which satisfies Laplace equation is generally called ………………….
19. Young's theorem stipulates that …………..
20. If $z = f(x, y)$ is a continuous function and function of degree n, then $x\frac{df}{dx} + y\frac{df}{dy} = nf(x, y)$ is a statement of ………………
21. Calculate $\frac{d^2y}{dx^2}$ for the function: $y = 5x^2 - 2x + 4$.

22. If $x = r\sin\theta$ and $y = r\cos\theta$, then $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 =$ ……………

23. ……………… entails the determination of a function when its derivative is known.

24. If $\frac{dP}{dx} = 11 - 2x$, find P as a function of x.

25. $\displaystyle\int \frac{6x}{x^2 + 5x + 7}\, dx =$ … … … … …

26. $\displaystyle\int x^n\, dx =$ … … … … …

27. A definite integral has a ………… value when the two limits of integral are identical.

28. Define the order of a differential equation.

29. The degree of an ordinary differential equation refers to …………………

30. ………….. and ………….. are two positive numbers whose sum is 20 and whose product is as large as possible.

SECTION B

1. Explain any five reasons for the increasing use of mathematics as a tool and language of analysis in the social sciences.

2. *(a)* Determine x and y if $(x,\ x + y) = (y - 2,\ 6)$.

   (b) If A and B are square matrices of the same order, show that $\mathrm{tr}(A + B) = \mathrm{tr}A + \mathrm{tr}B$.

   (c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$,

   (i) Obtain A', AA'

   (ii) What nature of matrix is AA'?

3. *(a)* Distinguish between differentiation and integration.

   (b) Consider a consumer who buys commodities x and y at per unit prices: $Px = \#2$ and $Py = \#5$ respectively. If the money incomes of the consumer is $\#100$ for the period and her utility is $U = xy$; find the consumer's:

(i) budget constraint;

(ii) optimization problem;

(iii) determine her optimal purchases for a given desire to maximize total utility.

4. *(a)* Distinguish between definite and indefinite integral.

(b) Evaluate the following integrals:

(i) $$\int \frac{2x-5}{x^2-5x+6}\,dx$$

(ii) $$\int \frac{\cos x}{1+\sin x}\,dx$$

(iii) $$\int \cos x\, e^{\sin x}\,dx$$

(iv) $$\int_1^3 x^2\,dx$$

(v) $$\int_0^4 e^x\,dx$$

5. *(a)* What is a differential equation?

(b) State the order and degree of the following differential equations:

(i) $$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 2y = 0$$

(ii) $$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

(iii) $$\left(\frac{d^4y}{dx^4}\right)^2 - 4\frac{d^2y}{dx^2} + 4y = 0$$

(c) Separate the variable and solve the following differential equation:

$$\frac{dy}{dx} = e^{x-y}$$

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2006/2007 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and only two questions from Section B, we'll be answering everything in both sections in this book though; keep calm and move with me;

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

Pretty short structured questions in this section. As usual, some could be confusing so you really have to watch it!

Question 1

What is a matrix?

Simple, define it as you can see it.

A matrix (matrices for plural) is a rectangular array of numbers, symbols or expressions usually arranged in grid (rows and columns).

Question 2

What precisely is meant by the size of a matrix?

The size of a matrix is defined by the number of rows and columns that it contains.

Question 3

What condition must be met before two matrices can be added?

Two matrices can be added if and only if the two matrices are of the same size. That it, the two matrices must have same number of rows and columns.

Question 4

What is the conformability condition for matrix multiplication?

For two matrices to be multiplied; the number of columns in the pre-multiplier must be equal to the number of rows in the post-multiplier;

Question 5

A square matrix with scalar element which is not necessary in the leading matrix is called …………… matrix.

I don’t seem to understand where this question is towered to. Perhaps more information is needed.

Question 6

What is an orthogonal matrix?

An orthogonal matrix is a square matrix whose transpose is equal to its inverse; it is a special type of matrix related to both the transpose of a matrix and the inverse of a matrix.

Question 7

If a matrix has two equal rows (or columns), then the value of the determinant is ………..

So, I’m not sure I mentioned this in the course of the book; however,

When any rows or columns in a matrix are identical (or equal), the determinant of that question is zero.

Question 8

The open-scissors technique of determinant evaluation is applicable for what size of matrix?

As explained in the note, ***the open-scissors technique of evaluating determinants is used for the 2×2 matrix.***

Question 9

How are the simultaneous equations $ax + by = c$; $dx + ey = f$ written in their equivalent matrix forms?

Simple! Thoroughly explained in the notes! We have to equations.

$$ax + by = c$$
$$dx + ey = f$$

They are already arranged according to their corresponding variables; hence we just write the matrix equation as shown below.

$$\begin{pmatrix} a & b \\ d & e \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c \\ f \end{pmatrix}$$

Question 10

…………… is a function which specifies a particular type of relation between two or more variables.

This refers to an **explicit function**, the term ***"particular"*** says it all that the direction of the relationship is certain like we discussed in implicit and explicit functions.

Question 11

The classes of increasing and decreasing function are together called …………….

Straightforward! Functions that are either strictly increasing or strictly decreasing are called **monotonic functions.**

Question 12

Which of the following is an even function?

(i) $f(x) = \frac{1}{x}$

(ii) $f(x) = 4$

(iii) $f(x) = x^2 - 2x$

We need to remember what an even function is at first; an even function is such that the negative equivalent of the independent variable for a function still gives the same value;

For an even function:

$$f(-x) = f(x)$$

So; for the (i)

$$f(x) = \frac{1}{x}$$

$$f(-x) = \frac{1}{-x} = -\frac{1}{x}$$

Hence,

$$f(-x) \neq f(x)$$

For (ii)

$$f(x) = 4$$

This is a constant function; it has the same value for all the independent variables;

Hence,

$$f(-x) = 4$$

Hence,

$$f(-x) = f(x)$$

For (iii)

$$f(x) = x^2 - 2x$$

$$f(-x) = (-x)^2 - 2(-x)$$

$$f(-x) = x^2 + 2x$$

Hence,

$$f(-x) \neq f(x)$$

Hence, from our analysis, only the second one satisfies the condition and it is the only even function.

Question 13

……………. is a function which expresses the relation of two distances as a function of an angle.

Trigonometric functions, also called angle functions are functions of an angle. They relate the angles of a triangle to the length of its side, hence, the ratio of two distances are expressed as the function of an angle.

Hence, the answer is ***trigonometric functions.***

Question 14

What is differential calculus?

Differentiation basically means the process of finding the derivative or differential coefficient of a function. This is differential calculus.

Differentiation can also be defined as the branch of calculus that deals with the rate of change in one quantity with respect to another.

Question 15

The exponential function of differentiation states that ………………

This question is simply asking you for the rule of differentiation of the exponential function.

The rule, as you should know, is:

If: $y = a^x$

$$\frac{dy}{dx} = a^x \ln a$$

If: $y = e^x$

$$\frac{dy}{dx} = e^x$$

AND:

If: $y = a^{f(x)}$

$$\frac{dy}{dx} = \ln a\, f'(x)a^{f(x)}$$

If: $y = e^{f(x)}$

$$\frac{dy}{dx} = f'(x)e^{f(x)}$$

Question 16

The derivative of $y = \log(x^4 - 2x + 7)$ is …………………..

Differentiation problem!

$$y = \log(x^4 - 2x + 7)$$

No log base indicates log to base 10, hence;

$$y = \log_{10}(x^4 - 2x + 7)$$

Chain rule; Substitution;

$$u = x^4 - 2x + 7$$

$$\frac{du}{dx} = 4x^3 - 2$$

Hence,

$$y = \log_{10} u$$

From log rules;

$$\frac{dy}{du} = \frac{1}{u \ln 10}$$

Hence,

Chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence,

$$\frac{dy}{dx} = \frac{1}{u \ln 10} \times (4x^3 - 2)$$

Substitute u;

$$\frac{dy}{dx} = \frac{4x^3 - 2}{(x^4 - 2x + 7)\ln 10}$$

Question 17

If the revenue from sales of x terms is given by $R(x) = 25x + \left(\frac{x^2}{10}\right)$, the marginal revenue when $x = 5$ is ……………

Marginal functions, differentiate the original function. Hence;

$$R(x) = 25x + \left(\frac{x^2}{10}\right)$$

$$R'(x) = 1 \times 25x^{1-1} + 2 \times \frac{x^{2-1}}{10}$$

$$R'(x) = 25 + \frac{x}{5}$$

At $x = 5$;

$$R'(5) = 25 + \frac{5}{5} = 26$$

Question 18

The function which satisfies Laplace equation is generally called ……………………

The functions obeying Laplace equation are generally called **Harmonic functions.**

Question 19

Young's theorem stipulates that ……………..

Young's theorem states that two complementary second order mixed partials of a continuous and twice differentiable function are equal; for a multivariate function dependent on x and y;

Question 20

If $z = f(x, y)$ is a continuous function and function of degree n, then:

$$x\frac{df}{dx} + y\frac{df}{dy} = nf(x, y)$$

is a statement of …………………

The above statement is the statement of the homogeneity of a function **which is the statement of Euler's theorem.**

Question 21

Calculate $\frac{d^2y}{dx^2}$ for the function:

$$y = 5x^2 - 2x + 4$$

Solution straight!

$$y = 5x^2 - 2x + 4$$

$$\frac{dy}{dx} = 2 \times 5x^{2-1} - 1 \times 2x^{1-1} + 0$$

$$\frac{dy}{dx} = 10x - 2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = 1 \times 10x^{1-1} - 0$$

$$\frac{d^2y}{dx^2} = 10$$

Question 22

If $x = r \sin \theta$ and $y = r \cos \theta$, then:

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \ldots\ldots\ldots\ldots\ldots\ldots$$

It's actually in your note so you should've trashed it already.

So we have two functions here:

$$x = r\sin\theta$$
$$y = r\cos\theta$$

We're told to evaluate both $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ and work out some things on them.

$$x = r\sin\theta$$

To find $\frac{dx}{d\theta}$, we'll know that r is a constant since we're differentiating with respect to θ; hence,

$$\frac{dx}{d\theta} = r \times \frac{d}{d\theta}(\sin\theta)$$

Of course from our trigonometric rules, we know the derivative of $\sin\theta$ is $\cos\theta$ just like for $\sin x$ is $\cos x$, just a change of variable;

$$\frac{dx}{d\theta} = r \times \cos\theta = r\cos\theta$$

$$y = r\cos\theta$$

To find $\frac{dy}{d\theta}$, same way, we'll know that r is a constant since we're differentiating with respect to θ; hence,

$$\frac{dy}{d\theta} = r \times \frac{d}{d\theta}(\cos\theta)$$

From our trigonometric rules, we know the derivative of $\cos\theta$ is $-\sin\theta$ just like for $\cos x$ is $-\sin x$, just a change of variable;

$$\frac{dy}{d\theta} = r \times -\sin\theta = -r\sin\theta$$

Now, we're told to solve for this:

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$$

To solve, we take the squares of our derivatives just like it is in the question;

$$\left(\frac{dx}{d\theta}\right)^2 = (r\cos\theta)^2 = r^2(\cos\theta)^2 = r^2(\cos^2\theta)$$

$$\left(\frac{dy}{d\theta}\right)^2 = (-r\sin\theta)^2 = r^2(\sin\theta)^2 = r^2(\sin^2\theta)$$

Taking their sum, we have:

$$r^2(\cos^2\theta) + r^2(\sin^2\theta)$$

r^2 is common between both, factorize it:

$$r^2[\cos^2\theta + \sin^2\theta]$$

Now, this is a solid trigonometric identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Hence, we have that this reduces to:

$$r^2(1) = r^2$$

Hence, the answer is r^2

Question 23

……………… entails the determination of a function when its derivative is known.

Of course, when the derivative is known and you want to find the function, it is called ***integration.***

Question 24

If $\frac{dP}{dx} = 11 - 2x$, find P as a function of x.

This question is in your notes!

$$\frac{dP}{dx} = 11 - 2x$$

To find, P in terms of x; we evaluate it thus;

$$dP = (11 - 2x)dx$$

Take integral of both sides; of course you already know the LHS will simply cancel out the differential coefficient, we now have:

$$\int dP = \int (11 - 2x)dx$$

We'll split out integral on the RHS and continue as usual:

$$P = \int 11\,dx - \int 2x\,dx$$

We bring out our constants as usual, you know all these already!

$$P = 11\int dx - 2\int x\,dx$$

$$P = 11[x] - 2\left[\frac{x^{1+1}}{1+1}\right]$$

$$P = 11x - 2\left[\frac{x^2}{2}\right]$$

$$P = 11x - 2x^2 + C$$

We have added our arbitrary constant as it must be added after every integration exercise!

Question 25

$$\int \frac{6x}{x^2 + 5x + 7}\, dx = \dots\dots\dots\dots\dots$$

The question is completely unclear, the derivative of the denominator is $2x + 5$ and hence, cannot be expressed as a case of substitution. The denominator cannot be factorized, and hence, it cannot be broken down into partial fractions. The other cases which aren't covered in this book will involve completing the square of the denominator plus other manipulations **way out of the scope of SSC106**, hence, it is much doubtful that this is supposed to be here;

Question 26

$$\int x^n \, dx = \ldots\ldots\ldots\ldots\ldots$$

This is even an integral rule;

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

Question 27

A definite integral has a ………… value when the two limits of integral are identical.

If the limits of a definite integral are equal, we have something like this;

$$\int_a^a f(x)$$

When we substitute in the integral, we have the same thing subtracting each other in the upper and lower limit, yielding a zero value; hence, when limits are identical, **the definite integral is zero;**

Question 28

Define the order of a differential equation.

The order of a differential equation is the highest derivative involved in the differential equation.

Question 29

The degree of an ordinary differential equation refers to …………………

The degree of a differential equation is the power the highest derivative has been raised to.

Question 30

………….. and ………….. are two positive numbers whose sum is 20 and whose product is as large as possible.

This question is similar to the question on Page 43 of Differentiation Applications and is the same as asking:

Find the greatest product of two numbers whose sum is 20.

Now, we'll extract our objective and constraint functions from here;

Let the two numbers be x and y

The sum must be 20; hence, our constraint is;

$$x + y = 20$$

We want to find the greatest product; it in essence means we want to **maximize the product of the two numbers** subject to the constraint:

Hence, the question can be summarized as:

Maximize $P = xy$
Subject to $x + y = 20$

Here, P is the product and we want to maximize it, the product of x and y is xy and hence, we want to maximize P, the product;

Hence, let's go through the short process;

The constraint function here is:

$$x + y = 20;$$

We're making use of the method of direct substitution;
Let's make y the subject;

$$y = 20 - x$$

Now, take this into the objective function!

The objective function is;

$$P = xy$$

Substitute for y; from our constraint function

$$P = x(20 - x)$$

Expand and make it a one-variable function.

$$P = 20x - x^2$$

Then, to optimize this (for a maximization in this process), we know allied calculus conditions here, we'll simply treat this as a situation to maximize P with respect to x only.

$$\frac{dP}{dx} = 1 \times 12 \times x^{1-1} - 2 \times x^{2-1}$$

$$\frac{dP}{dx} = 20 - 2x$$

At stationary point;

$$\frac{dP}{dx} = 0$$

$$20 - 2x = 0$$

$$2x = 20$$

$$x = 10$$

We have just one stationary point, let's test for the second derivative; it should be a maximum point though; since that's what the question requires;

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left(\frac{dP}{dx}\right) = \frac{d}{dx}(12 - 2x)$$

$$\frac{d^2P}{dx^2} = 0 - 1 \times 2 \times x^{1-1} = -2$$

So as expected, the second derivative is a constant negative and hence a maximum value;

So, now, we have our $x = 10$ as our value for the maximizing of the objective function whilst still obeying the constraint;

Hence, let's bring back our y;

$$y = 20 - x$$

$$y = 12 - 10 = 10$$

Hence, **the maximum values occur when $\boldsymbol{x = 10}$ and $\boldsymbol{y = 10}$;** while the constraint that their sum is 20 is still; the two numbers for the optimum situation is **10 and 10.**

I'm still a bit confused as to how some of these questions will be answered with short answers; examples are this last question, question 16 and so on. Well, once you know how to solve it, the constraint of space is super story!

SECTION B

Some heavier questions though!

Question 1

Explain any five reasons for the increasing use of mathematics as a tool and language of analysis in the social sciences.

We'd pick the reasons we listed in the notes and then select the longest ones here;

- It helps social scientists to state their research problems in specific and clear terms.
- Mathematics provide considerable insight into the way by which numerical information can be generated and presented to aid decision making in the social and management science.

- It helps in Identifying and quantifying the relationship between variables that determine the outcome of the decisions and their alternatives in the social and management science.
- It minimizes subjectivity and enhances the chance of making objective decision.
- It assists in the prediction or forecasting of future events.
- The language of mathematics is very easy to understand.
- Mathematics makes problems that could take lengthy periods to be resolved in minutes;
- With calculus, we can find the relative optima of different economical functions to easily find desired optimum results;

Question 2

(a) Determine x and y if
$(x,\ x+y) = (y-2,\ 6)$.

(b) If A and B are square matrices of the same order, show that
$\mathrm{tr}(A+B) = \mathrm{tr}A + \mathrm{tr}B$.

(c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$,

(i) Obtain A', AA'

(ii) What nature of matrix is AA'?

(a)

A case of matrix equality! We know that for two matrices to be equal, all corresponding elements must be equal. All these we treated!

$$(x, \quad x+y) = (y-2, \quad 6)$$

Hence,

$$x = y - 2 \ldots\ldots\ldots(1)$$
$$x + y = 6 \ldots\ldots\ldots(2)$$

Put (1) into (2) by substituting for x;

$$(y-2) + y = 6$$

Hence,

$$y = 4$$

From (1);

$$x = 4 - 2$$
$$x = 2$$

Hence,

$$x = 2$$
$$y = 4$$

(b)

To show this, the only way is to represent primitive (sample) matrices;

Let;

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \text{ and } B = \begin{pmatrix} m & n & o \\ p & q & r \\ s & t & u \end{pmatrix}$$

Then $A + B$ is;

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} + \begin{pmatrix} m & n & o \\ p & q & r \\ s & t & u \end{pmatrix}$$

$$A + B = \begin{pmatrix} a+m & b+n & c+o \\ d+p & e+q & f+r \\ g+s & h+t & i+u \end{pmatrix}$$

Then; $\text{tr}(A+B)$ is (the sum of the terms on the diagonal):

$$\text{tr}(A+B) = (a+m) + (e+q) + (i+u)$$
$$\text{tr}(A+B) = a+m+e+q+i+u$$

Then, let's find the separate traces and add;

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \text{ and } B = \begin{pmatrix} m & n & o \\ p & q & r \\ s & t & u \end{pmatrix}$$

$$\text{tr}(A) = a+e+i$$

$$\text{tr}(B) = m+q+u$$

Then,

$$\text{tr}(A) + \text{tr}(B) = a+e+i+m+q+u$$

If you rearrange the terms as you like (added terms can be arranged anyhow)

$$\text{tr}(A) + \text{tr}(B) = a+m+e+q+i+u$$

Hence, it is proved that: $\text{tr}(A+B) = \text{tr}A + \text{tr}B$

(c)

(i)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

We're told to obtain A' which from our matrix studies, we know is the transpose of A;

$$A' = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

We're to also obtain AA'; hence, we have matrix multiplication; this will be quite tedious;

$$AA' = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

$$\begin{pmatrix} (1)(1) + (2)(2) + (3)(3) & (1)(4) + (2)(5) + (3)(6) & (1)(7) + (2)(8) + (3)(9) \\ (4)(1) + (5)(2) + (6)(3) & (4)(4) + (5)(5) + (6)(6) & (4)(7) + (5)(8) + (6)(9) \\ (7)(1) + (8)(2) + (9)(3) & (7)(4) + (8)(5) + (9)(6) & (7)(7) + (8)(8) + (9)(9) \end{pmatrix}$$

Sorry for the small font in the expansion, kindly zoom it;

$$AA' = \begin{pmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{pmatrix}$$

(ii)

Of course, we treated this in the book, the product of a matrix and its transpose is a **symmetric matrix;** that is, a matrix whose row elements are equal to its corresponding column elements, hence, such a matrix is equal to its transpose.

Hence, AA' is such that:

$$AA' = (AA')'$$

Question 3

(a) Distinguish between differentiation and integration.

(b) Consider a consumer who buys commodities x and y at per unit prices: $Px = \#2$ and $Py = \#5$ respectively. If the money incomes of the consumer is $\#100$ for the period and her utility is $U = xy$; find the consumer's:

(iv) budget constraint;

(v) optimization problem;

(vi) determine her optimal purchases for a given desire to maximize total utility.

(a)

Distinguishing between differentiation and integration is quite simple.

Differentiation is the process of finding the derivative of a function while **integration** is the process of finding a function from its derivative.

Differentiation deals with the rate of change of a quantity with respect to another, while **integration** deals with the area under the curve of a function. The two processes are reverse processes with respect to each other.

(b)

(i)

To find the budget constraint of the consumer; x is the quantity of the first commodity and y is the quantity of the second commodity; we have the budget constraint as has been explained in the note of this since x costs ₦2 and y costs ₦5:

$$2x + 5y = 100$$

We'll now express it equated to zero as:

$$2x + 5y - 100 = 0$$

Hence, the budget constraint is as shown above!

(ii)

The optimization problem in this case is a utility maximization problem. Here, the:

- objective function to be optimized is the utility:
$$U = xy$$
- the constraint function is the budget constraint:
$$2x + 5y - 100 = 0$$

(iii)

For her optimal purchase;
Our Lagrangean equation is written, hence, we'll form our Lagrangean equation thus:

$$L(x, y, \lambda) = xy - \lambda(2x + 5y - 100)$$

Now, evaluate the first order partials with respect to x, y and λ and equate them to zero; you surely remember partial differentiation, it's in this book!

$$L_X = y - 2\lambda = 0$$

$$L_Y = x - 5\lambda = 0$$

$$L_\lambda = -2x - 5y + 100 = 0$$

$y - 2\lambda = 0 \ldots \ldots \ldots \ldots$(1)

$x - 5\lambda = 0 \ldots \ldots \ldots \ldots$(2)

$-2x - 5y + 100 = 0 \ldots \ldots \ldots \ldots$(3)

Let's eliminate λ from (1) and (2);

$$\mathbf{5} \times (\mathbf{1}): 5y - 10\lambda = 0 \ldots \ldots \ldots (3)$$
$$\mathbf{2} \times (\mathbf{2}): 2x - 10\lambda = 0 \ldots \ldots \ldots (4)$$

From (3);

$$5y = 10\lambda$$

From (4);

$$2x = 10\lambda$$

Hence,

$$5y = 2x \ldots \ldots \ldots (5)$$

Since both are equal to 10λ;

Substitute (5) into (3);

$$-(5y) - 5y + 100 = 0$$
$$-10y + 100 = 0$$
Hence,
$$y = 10$$
From (5);
$$2x = 5(10)$$
Hence,
$$x = 25$$

Hence, the optimal purchase for maximum utility is purchasing 25 units of x and 10 units of y.

For the sake of full marks;
From (1);

$$y - 2\lambda = 0$$
Hence,
$$10 - 2\lambda = 0$$
Hence,
$$\lambda = 5$$

USE CRAMMER'S RULE IF YOU DON'T UNDERSTAND SIMULTANEOUS EQUATIONS WELL;

Question 4

(a) Distinguish between definite and indefinite integral.

(b) Evaluate the following integrals:

(i) $$\int \frac{2x - 5}{x^2 - 5x + 6}\, dx$$

(ii) $$\int \frac{\cos x}{1 + \sin x}\, dx$$

(iii) $$\int \cos x\, e^{\sin x} dx$$

(iv) $$\int_1^3 x^2\, dx$$

(v) $$\int_0^4 e^x\, dx$$

(a)

Definite integral gives the area under the curve of the graph of a given function while an **indefinite integral** gives the general form of the anti-derivative of a function. A very obvious difference also is that indefinite integrals contain an arbitrary constant while a definite integral doesn't contain an arbitrary constant.

$$\int f(x)\,dx \text{ is an indefinite integral}$$

$$\int_{b}^{a} f(x)\,dx \text{ is a definite integral}$$

(b)

(i)

$$\int \frac{2x-5}{x^2-5x+6}\,dx$$

This is looking very much like integration by partial fraction but it is purely a case of substitution.

This is a case of

$$\int \frac{f'(x)}{f(x)}\,dx$$

Hence, put $u = x^2 - 5x + 6$

$$\frac{du}{dx} = 2x^{2-1} - 1 \times 5x^{1-1} + 0 = 2x - 5$$

Hence,

$$dx = \frac{du}{2x - 5}$$

We have;

$$\int \frac{2x - 5}{u} \times \frac{du}{2x - 5}$$

Hence, $2x - 5$ cancels out;

$$\int \frac{1}{u} du$$

From integral rules; this is:

$$[\ln u] + C$$

Return $u = x^2 - 5x + 6$

$$\ln(x^2 - 5x + 6) + C$$

(ii)

$$\int \frac{\cos x}{1 + \sin x} \, dx$$

Another pure case of substitution! You may not see it firsthand but it is.

This is a case of

$$\int \frac{f'(x)}{f(x)} dx$$

Hence, put $u = 1 + \sin x$

$$\frac{du}{dx} = 0 + \cos x = \cos x$$

Hence,

$$dx = \frac{du}{\cos x}$$

We have;

$$\int \frac{\cos x}{u} \times \frac{du}{\cos x}$$

Hence, $\cos x$ cancels out;

$$\int \frac{1}{u} du$$

From integral rules; this is:

$$[\ln u] + C$$

Return $u = 1 + \sin x$

$$\ln(1 + \sin x) + C$$

(iii)

$$\int \cos x \, e^{\sin x} dx$$

Another pure case of substitution!

This is a case of

$$\int f'(x) g[f(x)] dx$$

Hence, put $u = \sin x$

$$\frac{du}{dx} = \cos x$$

Hence,

$$dx = \frac{du}{\cos x}$$

We have;

$$\int \cos x\, e^{u} \times \frac{du}{\cos x}$$

Hence, $\cos x$ cancels out;

$$\int e^{u} du$$

From integral rules; this is:

$$e^{u} + C$$

Return $u = \sin x$

$$e^{\sin x} + C$$

(iv)

$$\int_{1}^{3} x^{2}\, dx$$

Simple algebra integration with limits!

$$\int x^{2}\, dx$$

$$\frac{x^{2+1}}{2+1} + C$$

$$\frac{x^3}{3} + C$$

For definite integral, no need for arbitrary constant;

$$\int_1^3 x^2\, dx = \left[\frac{x^3}{3}\right]_1^3 = \left(\frac{3^3}{3}\right) - \left(\frac{1^3}{3}\right)$$

$$9 - \frac{1}{3} = \frac{26}{3} = 8\frac{2}{3}$$

(v)

$$\int_0^4 e^x\, dx$$

Very very simple integration with limits!

$$\int e^x\, dx$$

$$e^x + C$$

For definite integral, no need for arbitrary constant;

$$\int_0^4 e^x \, dx = [e^x]_0^4 = (e^4) - (e^0)$$

$$54.598 - 1 = 53.598$$

e^4 was gotten on a calculator. e^0 is definitively 1.

Question 5

(a) What is a differential equation?

(b) State the order and degree of the following differential equations:

(i) $$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 2y = 0$$

(ii) $$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

(iii) $$\left(\frac{d^4y}{dx^4}\right)^2 - 4\frac{d^2y}{dx^2} + 4y = 0$$

(c) Separate the variable and solve the following differential equation:

$$\frac{dy}{dx} = e^{x-y}$$

(a)

A differential equation is simply a mathematical relationship that describes the relationship between functions and their various derivatives (their various differential coefficients).

(b)

Order and degree stuff, we've treated it extensively in the notes and hence, you shouldn't have issues with them.

(i)

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 2y = 0$$

The highest derivative here is $\frac{d^2y}{dx^2}$;

Hence, **the order is 2.**

The highest derivative here, $\frac{d^2y}{dx^2}$ is raised to a power of 1, hence,
The degree is 1.

(ii)

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

The highest derivative here is $\frac{d^2y}{dx^2}$;
Hence, **the order is 2.**

The highest derivative here, $\frac{d^2y}{dx^2}$ is raised to a power of 1, hence,
The degree is 1.

(iii)

$$\left(\frac{d^4y}{dx^4}\right)^2 - 4\frac{d^2y}{dx^2} + 4y = 0$$

The highest derivative here is $\frac{d^4y}{dx^4}$;
Hence, **the order is 4.**

The highest derivative here, $\frac{d^4y}{dx^4}$ is raised to a power of 2, hence,
The degree is 2.

(c)

Similar question solved in the notes already, so you have nothing to worry about!

$$\frac{dy}{dx} = e^{x-y}$$

From indicial rule;

$$a^{m-n} = \frac{a^m}{a^n}$$

Hence, here we have:

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

So relatively easy, we can separate our variables and solve;

$$e^y dy = e^x dx$$

Integrate both sides with respect to their respective variables;

$$\int e^y dy = \int e^x dx$$

LHS:

$$\int e^y dy$$

Straight and standard integral!

$$e^y$$

RHS:

$$\int e^x dx$$

Straight and standard integral;

$$e^x$$

Hence, $LHS = RHS$, our arbitrary constant is added just once since the sum of arbitrary constants is still an arbitrary constant; hence, we add one arbitrary constant;

$$e^y = e^x + C$$

$$e^y - e^x = C$$

Final answer………… Solution!

WOW!