DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES
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SSC106: MATHEMATICS FOR SOCIAL SCIENCES II

RAIN SEMESTER EXAMINATION (2010/2011 SESSION)

INSTRUCTIONS:

- Attempt all questions in **Section A;**
- Answer one question in **Section B.**
- Show all workings clearly

Time allowed: 2 hours

SECTION A

(a) Outline any two reasons some knowledge of mathematics is required in Economics Science.

(b) A matrix can be classified using:

(i) the relationship between its rows and its columns;

(ii) the structure of its element;

(iii) some derived relationship with itself.

With appropriate examples;

list three types of matrices under classification (i), four types under classification (ii), and three types under classification (iii).

(c) Evaluate: $\begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}$

(d) The class governor of SSC106 claims that:

$A = \begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix}$ is an idempotent matrix and

$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is an identity matrix.

Are his claims correct?

(e) Enumerate three techniques of evaluating determinants of matrices in increasing order of generality.

(f) State the conformability condition for matrix multiplication.

(g) Show that:

$$\frac{1 + \sin x}{\cos x} - \frac{\cos x}{1 - \sin x} = 0$$

(h) Use Euler's theorem to determine the degree of homogeneity of the functions:

(i) $f(L, K) = AL^{\alpha}K^{\beta}$

(ii) $f(x_1, x_2, x_3) = 12x_1^{0.2}x_2^{0.3}x_3^{0.5}$

(i) Enumerate the salient features of orthogonal matrices using relevant examples.

(j) State the Young's theorem.

(k) If $Z = e^{\log x \log y}$, evaluate:

$$\frac{\partial^2 x}{\partial x \partial y} \text{ and } \frac{\partial^2 x}{\partial y \partial x}$$

SECTION B

1. *(a)* Given the following estimate of the Marginal revenue (MR) function: $MR = 8 - 0.8q$, where q is the output. Find the Total Revenue (TR) when sales are:

(i) 4; and (ii) 9.

(b) Find:

(i) $\int \frac{x^4}{(x^5+6)^7}\, dx$

(ii) $\int \frac{dx}{x^2-x-2}$

2. *(a)* Use Lagrangean method to find the stationary value of $Z = xy + 2x$ subject to $4x + 2y = 60$.

(b) Differentiate with respect to x:

(i) $\log_e(\log_e x)$

(ii) $\log_e(x^2 + 3x)$

(iii) $e^{\sin x}$

(iv) $(x^2 - 2x)e^x$

3. What do you understand by the term function? Discuss with relevant examples any six types of functions you know.

SOLUTION TO THE PAST QUESTIONS

All topics needed here have been thoroughly treated through and through, all you need to do is to sit down and answer all these questions like a joke, you are not expected to start reading from here but from the topics, you make a joke of yourself when you read the concept of any subject starting from the side of the past question; more so, this text isn't a past question but a complete guide to lead you to a straight A in your SSC106 exam.

Hence, keep calm and check the solutions to these past questions one-by-one;

GOOD LUCK AND GOD'S BEST!

SOLUTION TO THE SSC106 EXAMINATION 2010/2011 ACADEMIC SESSION

The instruction is you answer all questions in the **Section A** and only one question from Section B. We'll be solving everything though, just sit tight as we solve.

Remember every singular solution here, you can get the full concept by reading this book and not the past question section exclusively;

SECTION A

(a)

Outline any two reasons some knowledge of mathematics is required in Economic Sciences.

- Mathematics provide considerable insight into the way by which numerical information can be generated and presented to aid decision making in the social and management science.
- It helps in identifying and quantifying the relationship between variables that determine the decisions' outcome and their alternatives in the social and management science.

(b)

A matrix can be classified using:

(i) the relationship between its rows and its columns;

(ii) the structure of its element;

(iii) some derived relationship with itself.

With appropriate examples;
list three types of matrices under classification (i), four types under classification (ii), and three types under classification (iii).

We discussed all the above methods of classifications in the notes, and hence, this shouldn't be any problem to tackle.

(i) the relationship between its rows and its columns;

- A row matrix:
- A column matrix:
- A square matrix

(ii) the structure of its element;

- A diagonal matrix
- A triangular matrix
- An identity matrix

- A null matrix

(iii) some derived relationship with itself.

- A symmetric matrix
- A skew-symmetric matrix
- An idempotent matrix

(c)

Evaluate: $\begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}$

Only square matrices have determinants, the above matrix is a 3×4 matrix and hence, it doesn't have a determinant.

(d)

The class governor of SSC106 claims that:

$A = \begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix}$ is an idempotent matrix and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is an identity matrix.

Are his claims correct?

This claims stuff again; this time it's your class governor that is doing the claiming something, well, it's like this;

Just check if those two matrices claims are correct and conclude if the class governor is correct or not;

$$A = \begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix}$$

If it is idempotent, its square is equal to itself;

$$A^2 = \begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -2(-2) + (-2)(3) & -2(-2) + (-2)(3) \\ 3(-2) + (3)(3) & 3(-2) + (3)(3) \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -2 & -2 \\ 3 & 3 \end{pmatrix}$$

Hence, A is idempotent as you can see, A^2 is equal to A;

However, your class governor is not totally correct as you can see that B isn't an identity matrix, the main diagonal isn't all through 1 and hence, it is not an identity matrix; so you can conclude the answer yourself.

So maybe we'll conclude that the class governor is partially correct or what? I don't know.
Most importantly, show that A is an idempotent matrix and B is not an identity matrix.

(e)

Enumerate three techniques of evaluating determinants of matrices in increasing order of generality.

Simple Stuff! How many times has this question been asked in past questions?

It's a simple question, you know all the methods but this one you're been asked in increasing order of generality, that may prove difficult. So, this is it; the open scissors technique is used at virtually all times since it is even used when finding the determinants of 3×3 matrices and higher, hence, it is the most general, the Laplace expansion technique is next since it is used for 3×3 matrices and higher, the Sarrus method is the least common, it is only limited to 3×3 matrices and as a matter of fact, is mostly learnt for the sake of formalities. Hence; we have the list in increasing order of generality as:

- Open scissors technique
- The Laplace expansion method
- The Sarrus method.

(f)

State the conformability condition for matrix multiplication.

Softest question ever!

The conformability condition for matrix multiplication states that for two matrices to be multiplied; the number of columns in the pre-multiplier must be equal to the number of rows in the post-multiplier.

(g)

Show that:

$$\frac{1+\sin x}{\cos x}-\frac{\cos x}{1-\sin x}=0$$

I really don't know why trigonometry was brought into SSC106 (I guess it's in SSC105), let's do it though, the common denominator is $(\cos x)(1-\sin x)$

$$\frac{(1+\sin x)(1-\sin x)-(\cos x)(\cos x)}{(\cos x)(1-\sin x)}$$

Expanding;

$$\frac{1-\sin x+\sin x-\sin^2 x-\cos^2 x}{(\cos x)(1-\sin x)}$$

$$\frac{1-\sin^2 x-\cos^2 x}{(\cos x)(1-\sin x)}$$

$$\frac{1-(\sin^2 x+\cos^2 x)}{(\cos x)(1-\sin x)}$$

$$\sin^2 x + \cos^2 x = 1$$

We have;

$$\frac{1-1}{(\cos x)(1-\sin x)} = \frac{0}{(\cos x)(1-\sin x)}$$

Hence, since the numerator is zero; the whole fraction is zero!

(h)

Use Euler's theorem to determine the degree of homogeneity of the functions:

(i) $f(L, K) = AL^{\alpha}K^{\beta}$

(ii) $f(x_1, x_2, x_3) = 12x_1^{0.2}x_2^{0.3}x_3^{0.5}$

(i)

$$f(L, K) = AL^{\alpha}K^{\beta}$$

It's well stated, we have a multivariate function of L and K with A, α and β being constants.

Hence, Euler's theorem here will be;

$$Lf_L + Kf_K = nf(L, K)$$

Where f_L and f_K are the derivatives of $f(L, K)$ with respect to L and K respectively, I expect you to know that!

Hence,

$$f_L = \alpha \times AL^{\alpha-1}K^{\beta}$$

$$f_L = \alpha AL^{\alpha-1}K^{\beta}$$

$$f_K = \beta AL^{\alpha}K^{\beta-1}$$

$$f_K = \beta AL^{\alpha}K^{\beta-1}$$

Hence,

$$L\left(\alpha AL^{\alpha-1}K^{\beta}\right) + K\left(\beta AL^{\alpha}K^{\beta-1}\right) = n[f(L, K)]$$

Hence,

$$L\left(\alpha AL^{\alpha-1}K^{\beta}\right) + K\left(\beta AL^{\alpha}K^{\beta-1}\right) = n\left(AL^{\alpha}K^{\beta}\right)$$

Indices!

$$\alpha AL^{\alpha-1+1}K^{\beta} + \beta AL^{\alpha}K^{\beta-1+1} = n\left(AL^{\alpha}K^{\beta}\right)$$

Simplify;

$$\alpha AL^{\alpha}K^{\beta} + \beta AL^{\alpha}K^{\beta} = n\left(AL^{\alpha}K^{\beta}\right)$$

Hence, $AL^{\alpha}K^{\beta}$ is common, factorize!

$$AL^{\alpha}K^{\beta}(\alpha + \beta) = n\left(AL^{\alpha}K^{\beta}\right)$$

$$(\alpha + \beta)AL^{\alpha}K^{\beta} = n\left(AL^{\alpha}K^{\beta}\right)$$

By comparison;

$$n = \alpha + \beta$$

(ii)

$$f(x_1, x_2, x_3) = 12x_1^{0.2}x_2^{0.3}x_3^{0.5}$$

We have a multivariate function of x_1, x_2 and x_3.

Hence, Euler's theorem here will be;

$$x_1 f_{x_1} + x_2 f_{x_2} + x_3 f_{x_3} = nf(x_1, x_2, x_3)$$

Where f_{x_1}, f_{x_2} and f_{x_3} are the derivatives of $f(x_1, x_2, x_3)$ with respect to x_1, x_2 and x_3 respectively.

Hence,

$$f_{x_1} = 0.2 \times 12x_1^{0.2-1}x_2^{0.3}x_3^{0.5}$$

$$f_{x_1} = 2.4x_1^{-0.8}x_2^{0.3}x_3^{0.5}$$

$$f_{x_2} = 0.3 \times 12x_1^{0.2}x_2^{0.3-1}x_3^{0.5}$$

$$f_{x_2} = 3.6x_1^{0.2}x_2^{-0.7}x_3^{0.5}$$

$$f_{x_3} = 0.5 \times 12x_1^{0.2}x_2^{0.3}x_3^{0.5-1}$$

$$f_{x_3} = 6x_1^{0.2}x_2^{0.3}x_3^{-0.5}$$

Hence,

$$x_1(2.4x_1^{-0.8}x_2^{0.3}x_3^{0.5}) + x_2(3.6x_1^{0.2}x_2^{-0.7}x_3^{0.5}) + x_3(6x_1^{0.2}x_2^{0.3}x_3^{-0.5}) = n[f(x_1, x_2, x_3)]$$

Hence,

$$x_1(2.4x_1^{-0.8}x_2^{0.3}x_3^{0.5}) + x_2(3.6x_1^{0.2}x_2^{-0.7}x_3^{0.5}) + x_3(6x_1^{0.2}x_2^{0.3}x_3^{-0.5}) = n(12x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

Indices for like bases!

$$(2.4x_1^{-0.8+1}x_2^{0.3}x_3^{0.5}) + (3.6x_1^{0.2}x_2^{-0.7+1}x_3^{0.5}) + (6x_1^{0.2}x_2^{0.3}x_3^{-0.5+1}) = n(12x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

Simplify;

$$(2.4x_1^{0.2}x_2^{0.3}x_3^{0.5}) + (3.6x_1^{0.2}x_2^{0.3}x_3^{0.5}) + (6x_1^{0.2}x_2^{0.3}x_3^{0.5}) = n(12x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

We have:

$$2.4x_1^{0.2}x_2^{0.3}x_3^{0.5} + 3.6x_1^{0.2}x_2^{0.3}x_3^{0.5} + 6x_1^{0.2}x_2^{0.3}x_3^{0.5} = n(12x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

Hence, $(x_1^{0.2}x_2^{0.3}x_3^{0.5})$ is common, factorize!

$$x_1^{0.2}x_2^{0.3}x_3^{0.5}(2.4 + 3.6 + 6) = n(12x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

$$x_1^{0.2}x_2^{0.3}x_3^{0.5}(12) = n(12x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

Dividing through by 12;

$$x_1^{0.2}x_2^{0.3}x_3^{0.5} = n(x_1^{0.2}x_2^{0.3}x_3^{0.5})$$

Hence, by comparison;

$$n = 1$$

(i)

Enumerate the salient features of orthogonal matrices using relevant examples.

Well, the orthogonal matrix has two major features;

- The first is that the transpose of the matrix is equal to its inverse;
- And of course, the determinant of the matrix is -1 or 1.

These two above properties were mentioned in the notes.

A perfect example of an orthogonal matrix is the identity matrix; hence;

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is an example of an orthogonal matrix;

(j)

State the Young's theorem.

It states that two complementary second order mixed partials of a continuous and twice differentiable function are equal; for a multivariate function dependent on x and y;

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

In function notation;

$$f_{xy} = f_{yx}$$

(k)

If: $Z = e^{\log x \log y}$, evaluate:

$$\frac{\partial^2 Z}{\partial x \partial y} \quad \text{and} \quad \frac{\partial^2 Z}{\partial y \partial x}$$

This is an exponential function of function situation;

For the derivative with respect to x, we'll be taking y as a constant; we have;

$$\frac{\partial Z}{\partial x} = \frac{\partial}{\partial x}\left(e^{\log x \log y}\right)$$

As usual in functions of functions (chain rule), we need a substitution;

$$u = \log x \log y$$

$\log y$ is a constant in this case, hence, we differentiate $\log x$;

$$\frac{\partial u}{\partial x} = \frac{1}{x \ln 10} \log y = \frac{\log y}{x \ln 10}$$

Hence,

$$Z = e^u$$

$$\frac{\partial Z}{\partial u} = e^u$$

From chain rule;

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial u} \times \frac{\partial u}{\partial x}$$

$$\frac{\partial Z}{\partial x} = e^u \times \frac{\log y}{x \ln 10}$$

$$\frac{\partial Z}{\partial x} = \frac{e^u \log y}{x \ln 10}$$

Returning the value of u; we have

$$\frac{\partial Z}{\partial x} = \frac{e^{\log x \log y} \log y}{x \ln 10}$$

Using basically the same process but this time, with respect to y and with x taken as a constant, we have;

$$\frac{\partial Z}{\partial y} = \frac{\partial}{\partial y}\left(e^{\log x \log y}\right)$$

We need the same substitution;

$$u = \log x \log y$$

$\log x$ is a constant in this case, hence, we differentiate $\log y$;

$$\frac{\partial u}{\partial y} = \log x \frac{1}{y \ln 10} = \frac{\log x}{y \ln 10}$$

Hence,

$$Z = e^u$$

$$\frac{\partial Z}{\partial u} = e^u$$

From chain rule;

$$\frac{\partial Z}{\partial y} = \frac{\partial Z}{\partial u} \times \frac{\partial u}{\partial y}$$

$$\frac{\partial Z}{\partial y} = e^u \times \frac{\log x}{y \ln 10}$$

$$\frac{\partial Z}{\partial y} = \frac{e^u \log x}{y \ln 10}$$

Returning the value of u; we have

$$\frac{\partial Z}{\partial y} = \frac{e^{\log x \log y} \log x}{y \ln 10}$$

It'll be quite tedious to find:

$$\frac{\partial^2 Z}{\partial x \partial y} \quad \text{and} \quad \frac{\partial^2 Z}{\partial y \partial x}$$

However,
Nothing is skipped in **THE SSC106 WAY;** hence, we'll do it.

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial Z}{\partial y}\right)$$

Hence;

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{e^{\log x \log y} \log x}{y \ln 10}\right)$$

This is a product, and not a quotient actually, when we are differentiating with respect to x, we'll have the following (outside the bracket) as constants;

$$\frac{1}{y\ln 10}\left(e^{\log x\log y}\log x\right)$$

Hence, only the terms in the bracket are functions which form a product; we have:

$$u = e^{\log x\log y}$$
$$v = \log x$$

Hence,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(e^{\log x\log y}\right)$$

The derivative above is the same derivative for $\frac{\partial z}{\partial x}$

Hence;

$$\frac{\partial u}{\partial x} = \frac{e^{\log x\log y}\log y}{x\ln 10}$$

$$v = \log x$$

Straight!

$$\frac{\partial v}{\partial x} = \frac{1}{x\ln 10}$$

Hence;

Product rule;

$$\frac{\partial^2 Z}{\partial x\partial y} = \frac{1}{y\ln 10}\left(v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x}\right)$$

Notice that we have brought the constants along with it to multiply the derivative of the product; hence;

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{1}{y \ln 10}\left(\log x \frac{e^{\log x \log y} \log y}{x \ln 10} + e^{\log x \log y}\left(\frac{1}{x \ln 10}\right)\right)$$

Simplifying!

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{1}{y \ln 10}\left(\frac{(\log x \log y)e^{\log x \log y}}{x \ln 10} + \frac{e^{\log x \log y}}{x \ln 10}\right)$$

Factorize $e^{\log x \log y}$;

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{e^{\log x \log y}}{y \ln 10}\left(\frac{(\log x \log y)}{x \ln 10} + \frac{1}{x \ln 10}\right)$$

Add the terms in bracket, they have the same denominator;

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{e^{\log x \log y}}{y \ln 10}\left(\frac{\log x \log y + 1}{x \ln 10}\right)$$

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{e^{\log x \log y}(\log x \log y + 1)}{(y \ln 10)(x \ln 10)}$$

It's as usual no big deal; just try your possible best to follow it one by one!

Next one;

$$\frac{\partial^2 Z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial Z}{\partial x}\right)$$

Hence;

$$\frac{\partial^2 Z}{\partial y \partial x} = \frac{\partial}{\partial x}\left(\frac{e^{\log x \log y} \log y}{x \ln 10}\right)$$

This is a product, and not a quotient again, when we are differentiating with respect to y, we'll have the following (outside the bracket) as constants;

$$\frac{1}{x \ln 10}\left(e^{\log x \log y} \log y\right)$$

Hence, only the terms in the bracket are functions which form a product; we have:

$$u = e^{\log x \log y}$$

$$v = \log y$$

Hence,

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(e^{\log x \log y}\right)$$

The derivative above is the same derivative for $\frac{\partial Z}{\partial y}$

Hence;

$$\frac{\partial u}{\partial y} = \frac{e^{\log x \log y} \log x}{y \ln 10}$$

$$v = \log y$$

Straight!

$$\frac{\partial v}{\partial y} = \frac{1}{y \ln 10}$$

Hence;

Product rule;

$$\frac{\partial^2 Z}{\partial y \partial x} = \frac{1}{x \ln 10}\left(v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x}\right)$$

Notice that we have brought the constants along with it to multiply the derivative of the product; hence;

$$\frac{\partial^2 Z}{\partial y \partial x} = \frac{1}{x \ln 10}\left(\log y\left(\frac{e^{\log x \log y} \log x}{y \ln 10}\right) + e^{\log x \log y}\left(\frac{1}{y \ln 10}\right)\right)$$

Simplifying!

$$\frac{\partial^2 Z}{\partial y \partial x} = \frac{1}{x \ln 10}\left(\frac{(\log x \log y) e^{\log x \log y}}{y \ln 10} + \frac{e^{\log x \log y}}{y \ln 10}\right)$$

Factorize $e^{\log x \log y}$;

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{e^{\log x \log y}}{x \ln 10}\left(\frac{(\log x \log y)}{y \ln 10} + \frac{1}{y \ln 10}\right)$$

Add the terms in bracket, they have the same denominator;

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{e^{\log x \log y}}{x \ln 10}\left(\frac{\log x \log y + 1}{y \ln 10}\right)$$

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{e^{\log x \log y}(\log x \log y + 1)}{(x \ln 10)(y \ln 10)}$$

As you can confirm, the two indirect partial derivatives we've just gotten are equal. They actually must be equal, if the work is correct!

Quite some question though! I guess it 'mistakenly' sneaked into the Section A part. It has added to your knowledge though!

SECTION B

Question 1

(a) Given the following estimate of the Marginal revenue (MR) function: $MR = 8 - 0.8q$, where q is the output. Find the Total Revenue (TR) when sales are:

(i) 4; and (ii) 9.

(b) Find:

(i) $$\int \frac{x^4}{(x^5 + 6)^7}\,dx$$

(ii) $$\int \frac{dx}{x^2 - x - 2}$$

(a)

This quite basic. When you have the marginal function, integrate it to get the main function.

$$MR = 8 - 0.8q$$

Note that TR, total revenue is same as the revenue function. I expect you to know that.

$$TR = \int MR \, dq = \int (8 - 0.8q) dq$$

Hence;

$$TR = \int 8 \, dq - \int 0.8q \, dq$$

Power rule;

$$TR = 8\left[\frac{q^{0+1}}{0+1}\right] - 0.8\left[\frac{q^{1+1}}{1+1}\right]$$

$$TR = 8q - 0.4q^2 + C$$

In revenue function, C is always zero, by common sense, no revenue can be made whilst nothing has been sold. Hence;

$$TR = 8q - 0.4q^2$$

(i); when sales are 4;

$$TR = 8(4) - 0.4(4)^2$$

$$TR = 32 - 6.4 = 25.6$$

(ii); when sales are 9;

$$TR = 8(9) - 0.4(9)^2$$

$$TR = 72 - 32.4 = 39.6$$

(b)

$$\int \frac{x^4}{(x^5+6)^7}\,dx$$

This is equivalent to, by indices;

$$\int x^4(x^5+6)^{-7}\,dx$$

A substitution case of: $f(x)g[f(x)]$

$$u = x^5 + 6$$

$$\frac{du}{dx} = 5x^4$$

The difference between the derivative of the denominator and the numerator is just the constant, 5 and hence, it is a substitution case;

$$dx = \frac{du}{5x^4}$$

We have:

$$\int x^4(u)^{-7}\frac{du}{5x^4}$$

x^4 cancels off;

$$\int u^{-7}\frac{du}{5}=\frac{1}{5}\int u^{-7}du$$

Power rule of integration;

$$\frac{1}{5}\left[\frac{u^{-7+1}}{-7+1}\right]=\frac{1}{5}\left[\frac{u^{-6}}{-6}\right]$$

Hence;
The integral is, after returning u;

$$-\frac{1}{30(x^5+6)^6}$$

(ii)

$$\int\frac{dx}{x^2-x-2}$$

This is a case of partial fraction; we have the integral;

$$\int\frac{1}{x^2-x-2}dx$$

We’ll break the fraction into partial fractions, firstly, we work on the denominator.

$$x^2 - x - 2$$

$$x^2 - 2x + x - 2$$

$$x(x-2) + 1(x-2)$$

$$(x-2)(x+1)$$

Hence;
We have:

$$\frac{1}{(x-2)(x+1)} \equiv \frac{A}{x-2} + \frac{B}{x+1}$$

Clear;

$$1 = A(x+1) + B(x-2)$$

$$x + 1 = 0$$
$$x = -1$$

Put $x = -1$;

$$1 = A(-1+1) + B(-1-2)$$

Hence;

$$B = -\frac{1}{3}$$

Again;

$$x - 2 = 0$$
$$x = 2$$

Put $x = 2$;

$$1 = A(2 + 1) + B(2 - 2)$$

Hence;

$$A = \frac{1}{3}$$

Hence;

$$\frac{1}{(x-2)(x+1)} \equiv \frac{1}{3(x-2)} - \frac{1}{3(x+1)}$$

Hence; the integral becomes;

$$\int \frac{1}{x^2 - x - 2}\,dx$$
$$= \int \frac{1}{3(x-2)}\,dx - \int \frac{1}{3(x+1)}\,dx$$

Integrate separately;

$$\int \frac{1}{3(x-2)}\,dx$$

Linear substitution;

$$z = x - 2$$

$$\frac{dz}{dx} = 1$$

Hence;

$$dx = dz$$

We have:

$$\int \frac{1}{3z} dz = \frac{1}{3} \int \frac{1}{z} dz$$

Standard integral;

$$\frac{1}{3} \ln z$$

Return z;

$$\frac{1}{3} \ln(x - 2)$$

Next!

$$\int \frac{1}{3(x + 1)} dx$$

Linear substitution;

$$u = x + 1$$

$$\frac{du}{dx} = 1$$

Hence;

$$dx = du$$

We have:

$$\int \frac{1}{3u} du = \frac{1}{3} \int \frac{1}{u} du$$

Standard integral;

$$\frac{1}{3} \ln u$$

Return z;

$$\frac{1}{3} \ln(x + 1)$$

Hence; the final integral is:

$$\int \frac{1}{x^2 - x - 2} dx$$
$$= \int \frac{1}{3(x - 2)} dx - \int \frac{1}{3(x + 1)} dx$$

$$\frac{1}{3} \ln(x - 2) - \frac{1}{3} \ln(x + 1) + C$$

The arbitrary constant must be added.

Question 2

(a) Use Lagrangean method to find the stationary value of $Z = xy + 2x$ subject to $4x + 2y = 60$.

(b) Differentiate with respect to x:

(i) $\log_e(\log_e x)$

(ii) $\log_e(x^2 + 3x)$

(iii) $e^{\sin x}$

(iv) $(x^2 - 2x)e^x$

(a)

This can be solved by both direct substitution and Lagrangean equation, however, we are told to use Lagrangean method; hence, we will do so;

So here; the objective function is: $Z = xy + 2x$

The constraint function is $4x + 2y = 60$

We'll express the constraint function equated to zero; Hence,

$$4x + 2y - 60 = 0$$

So, let's write the Lagrangean expression here, introducing the Lagrangean multiplier:

Following the rule below:

$$\mathcal{L}(x, y, \lambda) = f(x, y) - \lambda \times g(x, y)$$

We have:

$$\mathcal{L}(x, y, \lambda) = xy + 2x - \lambda(4x + 2y - 60)$$

$$\mathcal{L}(x, y, \lambda) = xy + 2x - 4x\lambda - 2y\lambda + 60\lambda$$

Take the first order partials of $\mathcal{L}(x, y, \lambda)$ with respect to x, y and λ. As you are differentiating partially for any variable in the function, the rest are taken as constants, as usual.

$$\mathcal{L}_x = 1 \times x^{1-1} \times y + 1 \times 2 \times x^{1-1} - 1 \times 4 \times x^{1-1} \times \lambda - 0 + 0$$

$$\mathcal{L}_x = y + 2 - 4\lambda$$

$$\mathcal{L}_y = 1 \times x \times y^{1-1} + 0 - 0 - 1 \times 2 \times y^{1-1} \times \lambda + 0$$

$$\mathcal{L}_y = x - 2\lambda$$

$$\mathcal{L}_\lambda = 0 + 0 - 1 \times 4 \times x \times \lambda^{1-1} - 1 \times 2 \times y \times \lambda^{1-1} + 1 \times 60 \times \lambda^{1-1}$$

$$\mathcal{L}_\lambda = -4x - 2y + 60$$

From the first order partials, equate everything to zero, I mean each of the first partials; that's similar to the first order conditions;

$\mathcal{L}_x = y + 2 - 4\lambda = 0 \dots \dots \dots (1)$

$\mathcal{L}_y = x - 2\lambda = 0 \dots \dots \dots (2)$

$\mathcal{L}_\lambda = -4x - 2y + 60 = 0 \dots \dots \dots (3)$

Solving simultaneously;

From (2);

$$x = 2\lambda \dots \dots \dots (4)$$

Put (4) in (3);

$$-4x - 2y + 60 = 0$$

$$-4(2\lambda) - 2y + 60 = 0$$

$$-8\lambda - 2y = -60 \dots \dots \dots (5)$$

Combine (5) with (1) since both are in y and λ

$-8\lambda - 2y = -60 \dots \dots \dots (5)$

$y + 2 - 4\lambda = 0 \dots \dots \dots (1)$

$\mathbf{2} \times (\mathbf{1})$: $2y - 8\lambda = -4 \dots \dots \dots (6)$

Subtract (6) from (5)

$-8\lambda - 2y = -60 \dots \dots \dots (5)$

$2y - 8\lambda = -4 \dots \dots \dots (6)$

$-4y = -56$

$$\text{Here, } y = 14$$

From (3)

$-4x - 2y + 60 = 0$

Hence,

$$-4x - 2(14) + 60 = 0$$
$$-4x - 28 + 60 = 0$$
$$4x = 32$$
$$x = 8$$

The value of λ is of no necessity of no use to the solution, but let's put it for full marks.

From (5);

$$-8\lambda - 2y = -60$$

$y = 14;$

$$-8\lambda - 2(14) = -60$$
$$-8\lambda = -32$$
$$\lambda = 4$$

(b)

(i)

$$\log_e(\log_e x)$$

Let;

$$y = \log_e(\log_e x)$$

Hence;

This is a function of function substitution case;

$$u = \log_e x$$

Straight!

$$\frac{du}{dx} = \frac{1}{x}$$

Hence;

$$y = \log_e u$$

Straight;

$$\frac{dy}{du} = \frac{1}{u}$$

From chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence;

$$\frac{dy}{dx} = \frac{1}{u} \times \frac{1}{x}$$

Return u;

$$\frac{dy}{dx} = \frac{1}{(\log_e x) \times x}$$

$$\frac{dy}{dx} = \frac{1}{x \log_e x}$$

(ii)

$$\log_e(x^2 + 3x)$$

Let;

$$y = \log_e(x^2 + 3x)$$

Hence;
This is a function of function substitution case;

$$u = x^2 + 3x$$

Power rule;

$$\frac{du}{dx} = 2x + 3$$

Hence;

$$y = \log_e u$$

Straight;

$$\frac{dy}{du} = \frac{1}{u}$$

From chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence;

$$\frac{dy}{dx} = \frac{1}{u} \times (2x + 3)$$

Return u;

$$\frac{dy}{dx} = \frac{2x + 3}{x^2 + 3x}$$

(iii)

$$e^{\sin x}$$

Let;

$$y = e^{\sin x}$$

Hence;

This is a function of function substitution case;

$$u = \sin x$$

Straight!

$$\frac{du}{dx} = \cos x$$

Hence;

$$y = e^u$$

Straight;

$$\frac{dy}{du} = e^u$$

From chain rule;

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence;

$$\frac{dy}{dx} = e^{u} \times (\cos x)$$

Return u;

$$\frac{dy}{dx} = e^{\sin x} \cos x$$

(iv)

$$(x^2 - 2x)e^{x}$$

Let;

$$y = (x^2 - 2x)e^{x}$$

Short product rule;

$$u = x^2 - 2x$$
$$v = e^{x}$$

Here;

$$\frac{du}{dx} = 2x - 2$$

$$\frac{dv}{dx} = e^{x}$$

Hence;

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Hence;

$$\frac{dy}{dx} = (e^x)(2x - 2) + (x^2 - 2x)(e^x)$$

Factorize e^x;

$$\frac{dy}{dx} = e^x\big(2x - 2 + (x^2 - 2x)\big)$$

$$\frac{dy}{dx} = e^x(x^2 - 2)$$

Question 3

What do you understand by the term function? Discuss with relevant examples any six types of functions you know.

This is complete story! It'd amount to recopying the notes back here. Simply find the topic functions for full details of this question. While giving examples; don't give implicit and explicit, single-variable and multivariable as types as they are general types of all functions. There are many other functions which you can pick six of them.