DC Proposed Problems

1.1 An electron has the elementary charge $e = 1.6 \cdot 10^{-19} C$

$\Rightarrow$ 6400 electrons will have $6400 \cdot 1.6 \cdot 10^{-19}$

$= 10240 \cdot 10^{-19} C$

1.2 $I = 2A$

$\Delta t = 10 s$

$W = 4.6 kJ = 4.6 \cdot 10^3 J$

$U = ?$

$W = P \cdot \Delta t = U \cdot I \cdot \Delta t \Rightarrow U = \frac{W}{I \Delta t} = \frac{4.6 \cdot 10^3}{2 \cdot 10}$

$\Rightarrow U = 230 V$

1.3 $P = 80 W$

$\Delta t = 3h = 3 \cdot 3600 s$

$W = ?$

$W = P \Delta t = 80 \cdot 3 \cdot 3600 \Rightarrow W = 864 kJ$

1.4 $I = 10A$

$U = 230V$

$W = 80 kJ = 80 \cdot 10^3 J$

$\Delta t = ?$

$W = UI\Delta t \Rightarrow \Delta t = \frac{W}{U \cdot I} = \frac{80 \cdot 10^3}{230 \cdot 10}$

$\Rightarrow \Delta t = 34.78 s$

1.5 $q(t) = 10 \sin 4\pi t \ mC$

$u(t) = 2\cos 4\pi t \ V$

a) $p(0.3) = ?$

$i(t) = \frac{dq}{dt} = 40\pi \cos(4\pi t) \ mA = 40 \cdot 10^{-3} \pi \cos(4\pi t) \ A$

$p(t) = u(t)\, i(t) = 2\cos(4\pi t) 40 \cdot 10^{-3} \pi \cos(4\pi t) =$

$= 80 \cdot 10^{-3} \pi \cos(4\pi t)^2 \ W$

$p(0.3) = 0.1645 \ W$

b) $W = \int_{to}^{t} p\, dt = 80 \cdot 10^{-3} \pi \int_0^{0.8} \cos^2(4\pi t)\, dt$

$= 80 \cdot 10^{-3} \pi \int_0^{0.8} \frac{1+\cos(8\pi t)}{2} dt = 80 \cdot 10^{-3} \pi \left[\int_0^{0.8} \frac{1}{2} dt + \right.$

$\left. + \int_0^{0.8} \frac{\cos 8\pi t}{2} dt \right] = 80 \cdot 10^{-3} \pi \frac{1}{2} t \Big|_0^{0.8} + \frac{80 \cdot 10^{-3} \pi}{2} \cdot \frac{\sin 8\pi t}{8\pi} \Big|_0^{0.8}$

$= 32 \cdot 10^{-3} \pi + 5 \cdot 10^{-3} \sin 8\pi t \Big|_0^{0.8} = 32 \cdot 10^{-3} \pi + 5 \cdot 10^{-3} \sin(8\pi \cdot 0.8)$

$= 32 \cdot 10^{-3} \pi + 5 \cdot 10^{-3} \cdot 0.95 = (32\pi + 5 \cdot 0.95) \cdot 10^{-3}$

$= 105.28 \cdot 10^{-3} \ J$

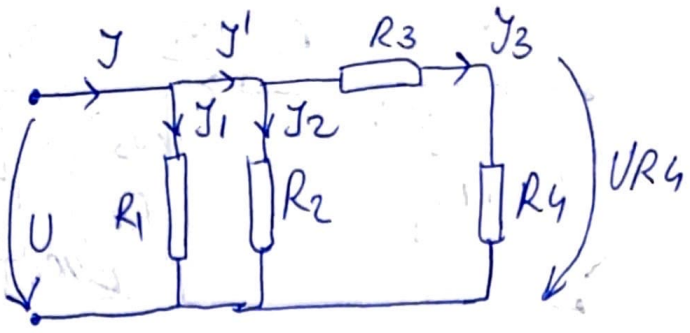
1.6 $U = 12 \ V$

$I = 150 \ mA = 150 \cdot 10^{-3} A$

a) $P = U \cdot I = 12 \cdot 150 \cdot 10^{-3} = 1.8 \ W$

b) $\Delta t = 30 \ min = 1800 \ s \qquad W = P \Delta t = 3240 \ J$

1.7



$R_1 = 15\,\Omega$

$R_2 = 10\,\Omega$

$R_3 = 7\,\Omega$

$R_4 = 5\,\Omega$

$U = 24\,V$

a) $R_S = R_3 + R_4 \Rightarrow R_S = 12\,\Omega$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_S} \Rightarrow \frac{1}{R_e} = \frac{1}{15} + \frac{1}{10} + \frac{1}{12}$$

a) $R_e = ?$

b) $J = ?$

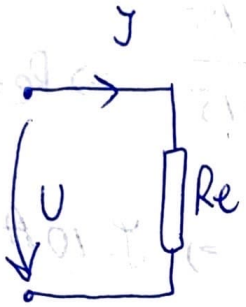
c) $J_3 = ?$

d) $U_{R_4} = ?$

$P_{R_2} = ?$

$$\frac{1}{R_e} = \frac{15}{60} \Rightarrow R_e = 4\,\Omega$$

b)



$U = J \cdot R_e$

$$\Rightarrow J = \frac{U}{R_e} = \frac{24}{4}$$

$\Rightarrow J = 6A$

c) $0 = J_1 R_1 - U \Rightarrow J_1 = \frac{U}{R_1} = \frac{24}{15} \Rightarrow J_1 = 1.6\,A$

$J = J_1 + J' \Rightarrow J' = 4.4\,A$

$J' = J_2 + J_3$

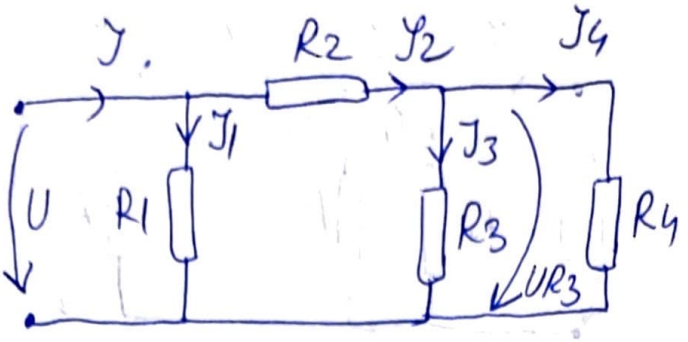
$0 = J_2 R_2 - J_1 R_1 \Rightarrow J_2 = \frac{J_1 R_1}{R_2} = \frac{1.6 \cdot 15}{10} \Rightarrow J_2 = 2.4\,A$

$J_3 = J' - J_2 \Rightarrow J_3 = 2\,A$

d) $U_{R_4} = J_3 R_4 \Rightarrow U_4 = 10\,V$

$P_{R_2} = J_2^2 R_2 \Rightarrow P_{R_2} = 57{,}6\,W$

1.8



$R_1 = 10\Omega$

$R_2 = 9\Omega$

$R_3 = 15\Omega$

$R_4 = 10\Omega$

$U = 60V$

a) $R_e = ?$

b) $I = ?$

c) $I_2 = ?$

d) $U_{R_3} = ?$

$P_{R_4} = ?$

a) $R_p = \frac{R_3 R_4}{R_3 + R_4} = \frac{15 \cdot 10}{15+10} = \frac{150}{25}$

$\Rightarrow R_p = 6\Omega$

$R_D = R_p + R_2 \Rightarrow R_D = 15\Omega$

$R_e = \frac{R_1 R_D}{R_1 + R_D} = \frac{10 \cdot 15}{10+15} \Rightarrow R_e = 6\Omega$

b) $U = IR_e \Rightarrow I = \frac{U}{R_e} \Rightarrow I = 10A$

c) $I = I_1 + I_2$

$0 = I_1 R_1 - U \Rightarrow I_1 = \frac{U}{R_1} \Rightarrow I_1 = 6A$

$I_2 = I - I_1 \Rightarrow I_2 = 4A$

d) $0 = I_2 R_2 + I_3 R_3 - I_1 R_1 \Rightarrow I_3 = \frac{I_1 R_1 - I_2 R_2}{R_3}$

$I_3 = 1.6A$

$U_{R_3} = I_3 R_3 \Rightarrow U_{R_3} = 24V$

$P_{R_4} = I_4^2 R_4$

$I_2 = I_3 + I_4 \Rightarrow I_4 = 2,4A$

$P_{R_4} = 57,6W$

1.9

$R_1 = 4\,\Omega$

$R_2 = 10\,\Omega$

$R_3 = 15\,\Omega$

$R_4 = 5\,\Omega$

$U = 75\,V$

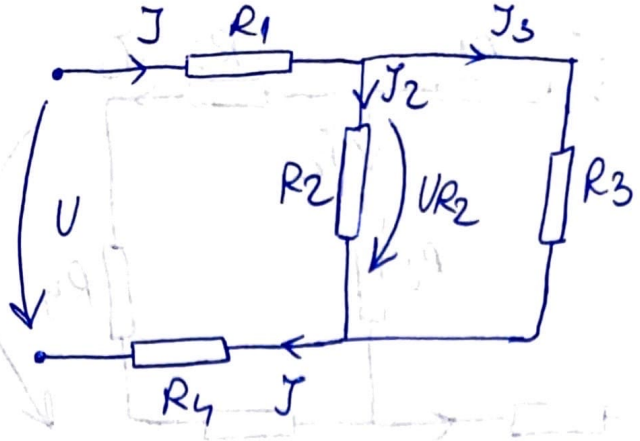
a) $R_e = ?$

b) $I = ?$

c) $I_3 = ?$

d) $U_{R_2} = ?$

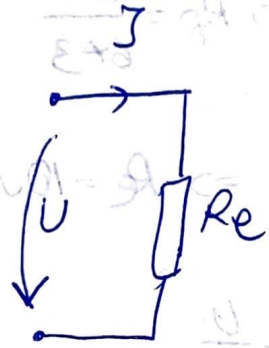
$P_{R_4} = ?$



a) $R_p = \frac{R_2 R_3}{R_2 + R_3} \Rightarrow R_p = \frac{10 \cdot 15}{10+15} \Rightarrow R_p = 6\,\Omega$

$R_e = R_1 + R_p + R_4 \Rightarrow R_e = 15\,\Omega$

b)



$U = IR_e$

$\Rightarrow I = \frac{U}{R_e}$

$I = \frac{75}{15} = 5A$

c) $IR_1 + I_2 R_2 + IR_4 = U$

$\Rightarrow I_2 = \frac{U - I(R_1 + R_4)}{R_2}$

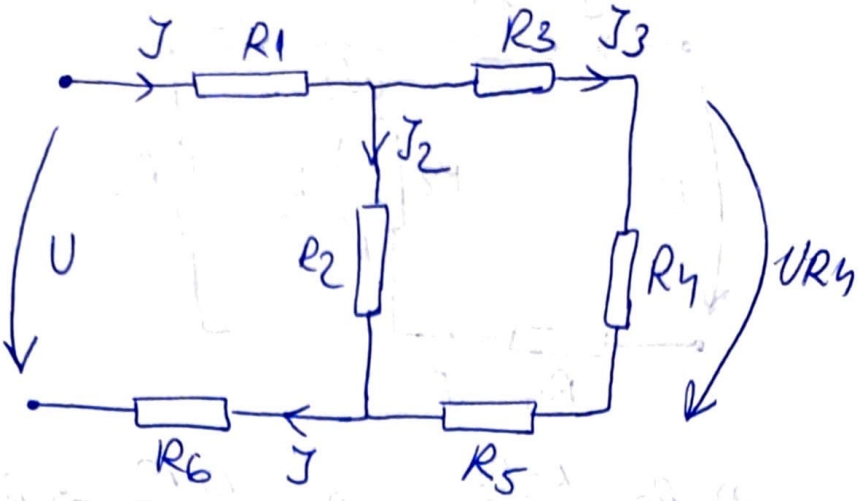
$I_2 = \frac{75 - 5(4+5)}{10} \Rightarrow I_2 = 3A$

$I = I_2 + I_3 \Rightarrow I_3 = I - I_2 \Rightarrow I_3 = 5 - 3 = 2A$

d) $U_{R_2} = I_2 R_2 \Rightarrow U_{R_2} = 30V$

$P_{R_4} = I^2 R_4 \Rightarrow P_{R_4} = 5 \cdot 5 \cdot 5 \Rightarrow P_{R_4} = 125W$

1.10



$R_1 = 5\Omega$

$R_2 = 3\Omega$

$R_3 = 2\Omega$

$R_4 = 1\Omega$

$R_5 = 3\Omega$

$R_6 = 3\Omega$

a) $R_e = ?$

b) $J = ?$

c) $J_3 = ?$

d) $U_{R_4} = ?$

$P_{R_5} = ?$

a) $R_S = R_3 + R_4 + R_5 \Rightarrow R_S = 6\Omega$

$$R_p = \frac{R_S R_2}{R_S + R_2} \Rightarrow R_p = \frac{6 \cdot 3}{6+3} \Rightarrow R_p = 2\Omega$$

$$R_e = R_1 + R_p + R_6 \Rightarrow R_e = 10\Omega$$

b) $U = JR_e \Rightarrow J = \frac{U}{R_e}$

$$J = \frac{30}{10} \Rightarrow J = 3A$$

c) $0 = JR_1 + J_2R_2 + JR_6 - U \Rightarrow J_2R_2 = U - JR_1 - JR_6$

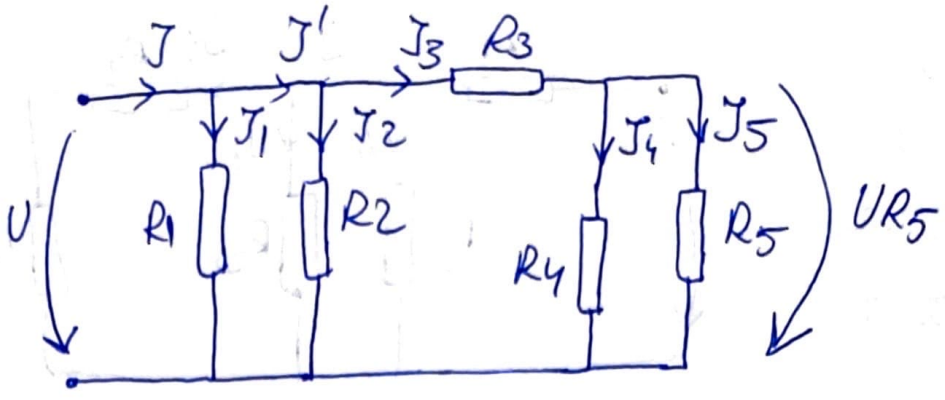
$$J_2 = \frac{U - J(R_1 + R_6)}{R_2} \Rightarrow J_2 = \frac{30 - 3(5+3)}{3} \Rightarrow J_2 = 2A$$

$J = J_2 + J_3 \Rightarrow J_3 = J - J_2 \Rightarrow J_3 = 1A$

d) $U_{R_4} = J_3 R_4 \Rightarrow U_{R_4} = 1V$

$P_{R_5} = J_3^2 \cdot R_5 \Rightarrow P_{R_5} = 3W$

1.11



$R_1 = 60\Omega$

$R_2 = 20\Omega$

$R_3 = 8\Omega$

$R_4 = 3\Omega$

$R_5 = 6\Omega$

$U = 120V$

a) $R_e = ?$

b) $J = ?$

c) $J_4 = ?$

d) $U_{R_5} = ?$

$P_{R_2} = ?$

a) $R_{p1} = \frac{R_4 R_5}{R_4 + R_5} \Rightarrow R_{p1} = \frac{3 \cdot 6}{3+6} \Rightarrow R_{p1} = 2\Omega$

$R_{D1} = R_3 + R_{p1} \Rightarrow R_{D1} = 10\Omega$

$R_{p2} = \frac{R_2 \cdot R_{D1}}{R_2 + R_{D1}} \Rightarrow R_{p2} = \frac{20 \cdot 10}{20+10}$

$\Rightarrow R_{p2} = \frac{20}{3}\Omega$

$R_e = \frac{R_1 R_{p2}}{R_1 + R_{p2}} \Rightarrow R_e = \frac{60 \cdot \frac{20}{3}}{60 + \frac{20}{3}} = \frac{20 \cdot 20}{\frac{200}{3}}$

$\Rightarrow R_e = 6\Omega$

b) $U = JR_e \Rightarrow J = \frac{U}{R_e} \Rightarrow J = \frac{120}{6} \Rightarrow J = 20A$

c) $U = J_1 R_1 \Rightarrow J_1 = \frac{120}{60} \Rightarrow J_1 = 2A$

$J = J_1 + J' \Rightarrow J' = J - J_1 \Rightarrow J' = 18A$

$U = J_3(R_3 + R_{p1}) \Rightarrow J_3 = \frac{U}{R_3 + R_{p1}} \Rightarrow J_3 = \frac{120}{10} \Rightarrow J_3 = 12A$

$J' = J_3 + J_2 \Rightarrow J_2 = J' - J_3 \Rightarrow J_2 = 6A$

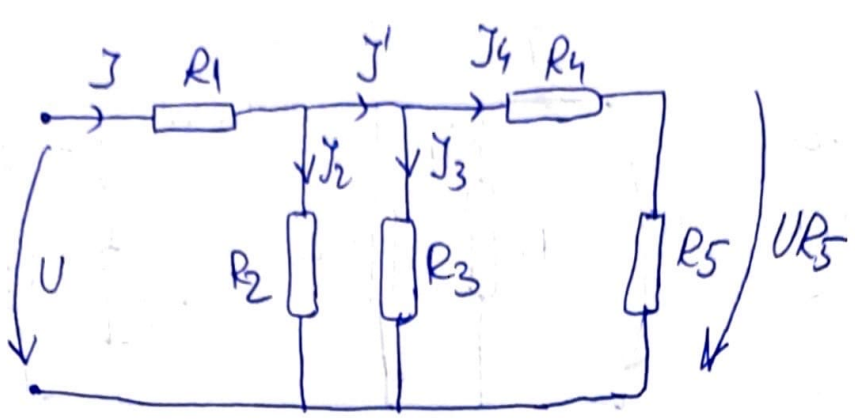
$J_4 R_4 = J_2 R_2 - J_3 R_3 \Rightarrow J_4 = \frac{J_2 R_2 - J_3 R_3}{R_4} \Rightarrow J_4 = \frac{6 \cdot 20 - 12 \cdot 8}{3}$

$\Rightarrow J_4 = 8A$

$J_3 = J_4 + J_5 \Rightarrow J_5 = 4A$

$U_{R_5} = J_5 R_5 \Rightarrow U_{R_5} = 24V$; $P_{R_2} = J_2^2 R_2 \Rightarrow P_{R_2} = 420W$

1.12



$R_1 = 9\Omega$

$R_2 = 60\Omega$

$R_3 = 20\Omega$

$R_4 = 6\Omega$

$R_5 = 4\Omega$

$U = 150V$

a) $R_e = ?$

b) $I = ?$

c) $I_3 = ?$

d) $U_{R_5} = ?$

$P_{R_4} = ?$

a) $R_5 = R_4 + R_5 \Rightarrow R_5 = 10\Omega$

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}$$

$$\frac{1}{R_p} = \frac{1}{60} + \frac{1}{20} + \frac{1}{10} = \frac{10}{60} \Rightarrow R_p = 6\Omega$$

$$R_e = R_1 + R_p \Rightarrow R_e = 15\Omega$$

b) $U = IR_e \Rightarrow I = \frac{U}{R_e} \Rightarrow I = \frac{150}{15} \Rightarrow I = 10A$

c) $IR_1 + I_2R_2 - U = 0 \Rightarrow I_2 = \frac{U - IR_1}{R_2} \Rightarrow I_2 = \frac{150 - 10 \cdot 9}{60}$

$$\Rightarrow I_2 = 1A$$

$$I = I_2 + I' \Rightarrow I' = I - I_2 \Rightarrow I' = 9A$$

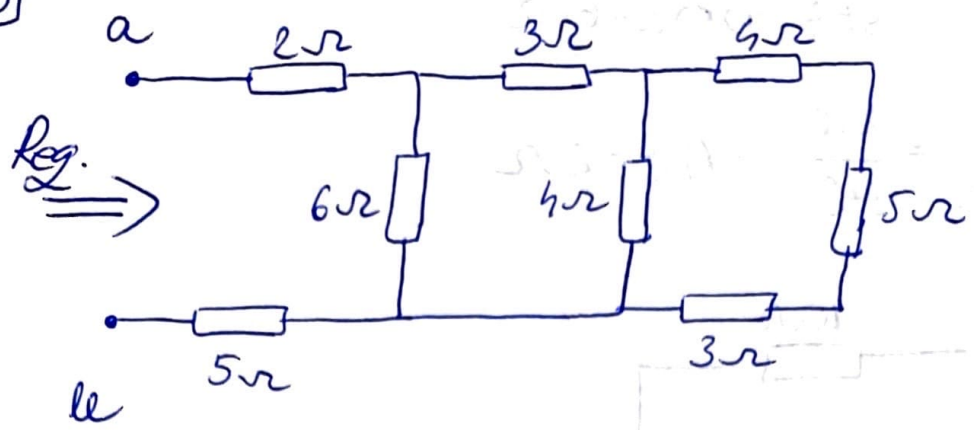
$$I_2R_2 = I_3R_3 \Rightarrow I_3 = \frac{I_2R_2}{R_3} \Rightarrow I_3 = \frac{60}{20} \Rightarrow I_3 = 3A$$

d) $I' = I_3 + I_4 \Rightarrow I_4 = I' - I_3 \Rightarrow I_4 = 6A$

$$U_{R_5} = I_4R_5 \Rightarrow U_{R_5} = 24V$$

$$P_{R_4} = I_4^2R_4 \Rightarrow P_{R_4} = 6 \cdot 6 \cdot 6 \Rightarrow P_{R_4} = 216W$$

1.13



$R_{S1} = 4+5+3 \Rightarrow R_{S1} = 12\,\Omega$

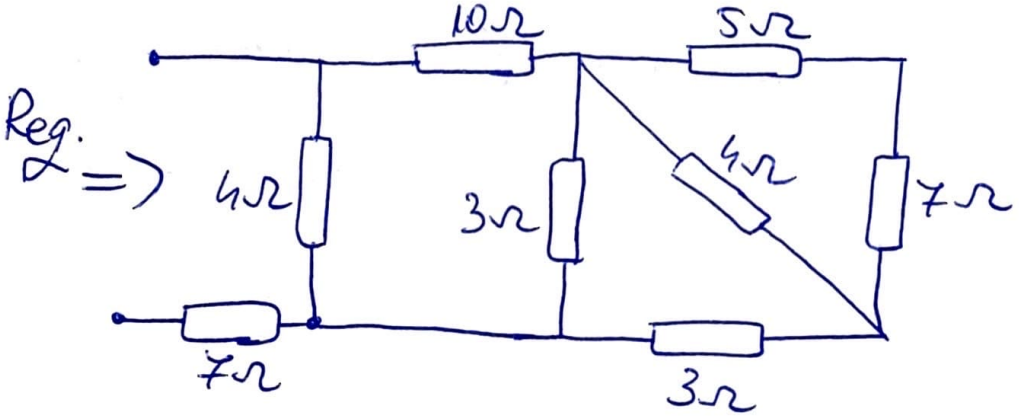
$R_{p1} = \frac{4 \cdot R_{S1}}{4+R_{S1}} = \frac{4 \cdot 12}{4+12} = \frac{4 \cdot 12}{16} \Rightarrow R_{p1} = 3\,\Omega$

$R_{S2} = 3+R_{p1} = 3+3 \Rightarrow R_{S2} = 6\,\Omega$

$R_{p2} = \frac{6 \cdot R_{S2}}{6+R_{S2}} = \frac{6 \cdot 6}{6+6} \Rightarrow R_{p2} = 3\,\Omega$

$R_{eq} = 2+R_{p2}+5 = 2+3+5 \Rightarrow R_{eq} = 10\,\Omega$

1.14

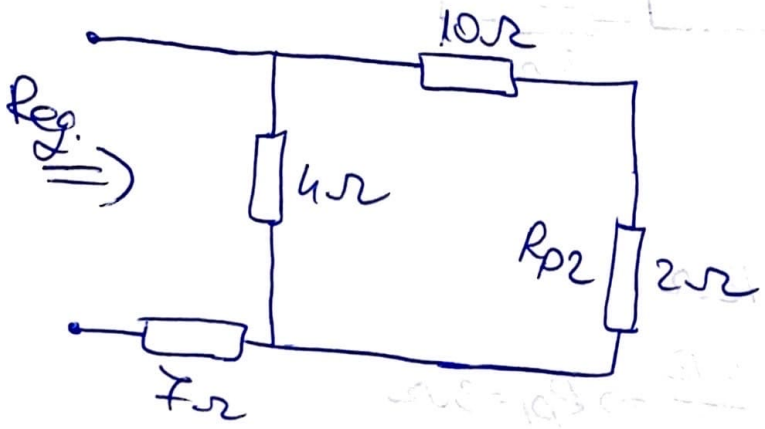


$R_{S1} = 5+4 \Rightarrow R_{S1} = 12\,\Omega$

$R_{p1} = \frac{4R_{S1}}{4+R_{S1}} = \frac{4 \cdot 12}{4+12} \Rightarrow R_{p1} = 3\,\Omega$

$R_{D2} = R_{p1+3} = 3+3 \Rightarrow R_{D2} = 6\Omega$

$R_{p2} = \frac{3R_{D2}}{3+R_{D2}} = \frac{3 \cdot 6}{3+6} \Rightarrow R_{p2} = 2\Omega$

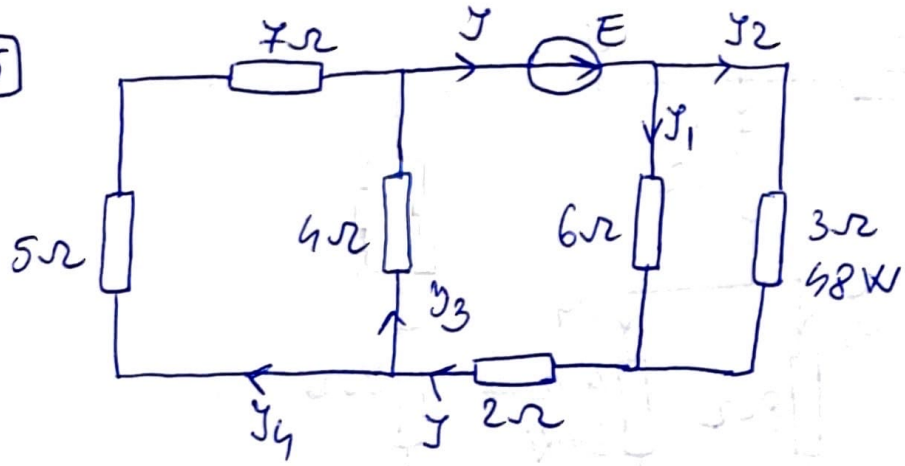


$R_{D3} = R_{p2+10} \Rightarrow R_{D3} = 12\Omega$

$R_{p3} = \frac{4R_{D3}}{4+R_{D3}} = \frac{4 \cdot 12}{4+12} \Rightarrow R_{p3} = 3\Omega$

$R_e = R_{p3+7} \Rightarrow R_e = 10\Omega$

1.15



$48 = J_2^2 \cdot 3 \Rightarrow J_2 = \sqrt{\frac{48}{3}} \Rightarrow J_2 = 4A$

$3J_2 = 6J_1 \Rightarrow J_1 = \frac{3J_2}{6} \Rightarrow J_1 = 2A$

$J = J_1 + J_2 \Rightarrow J = 6A$

$$\begin{cases} J = J_3 + J_4 \\ 4J_3 = 12J_4 \\ \Rightarrow J_3 = 3J_4 \end{cases} \Rightarrow J = J_3 + \frac{J_3}{3} \quad | \cdot 3$$

$3J = 3J_3 + J_3 \Rightarrow 3J = 4J_3$

$\Rightarrow J_3 = \frac{3J}{4} = \frac{3 \cdot 6}{4}$

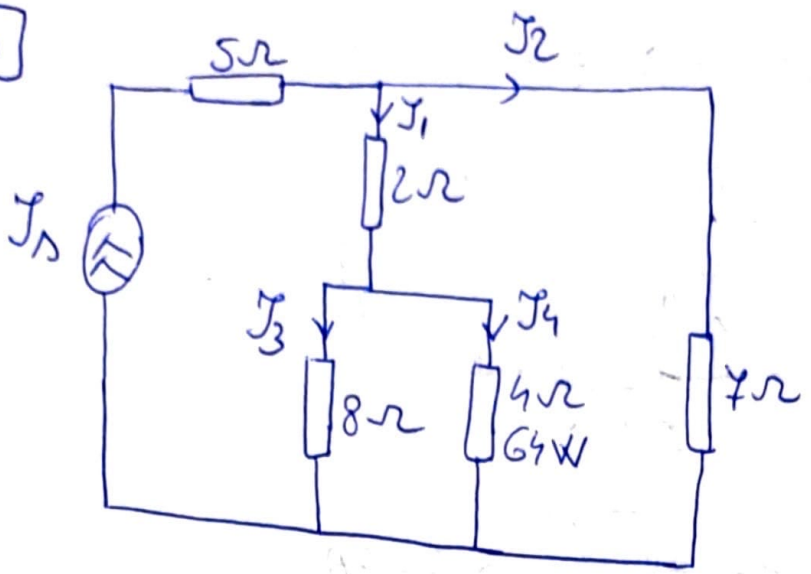
$J_3 = \frac{9}{2} A$

$E = 6J_1 + 2J + 4J_3$

$E = 6 \cdot 2 + 2 \cdot 6 + 4 \cdot \frac{9}{2} = 12 + 12 + 18$

$\Rightarrow E = 42V$

1.16



$64 = J_4^2 \cdot 4 \Rightarrow J_4 = \sqrt{\frac{64}{4}} \Rightarrow J_4 = 4A$

$4J_4 = 8J_3 \Rightarrow J_3 = \frac{4J_4}{8} \Rightarrow J_3 = \frac{4 \cdot 4}{8} \Rightarrow J_3 = 2A$

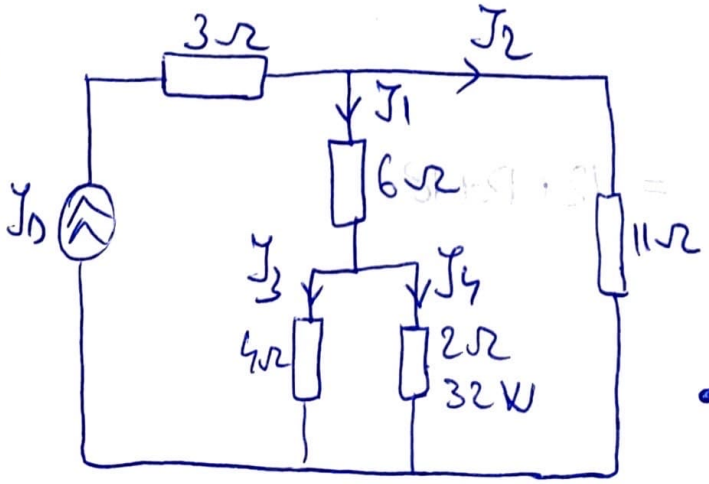
$J_1 = J_3 + J_4 \Rightarrow J = 6A$

$7J_2 = 4J_4 + 2J_1 \Rightarrow J_2 = \frac{4J_4 + 2J_1}{7}$

$J_2 = \frac{4 \cdot 4 + 2 \cdot 6}{7} = \frac{16+12}{7} \Rightarrow J_2 = 4A$

$J_5 = J_1 + J_2 \Rightarrow J_5 = 10A$

1.17



- $32 = J_4^2 \cdot 2$

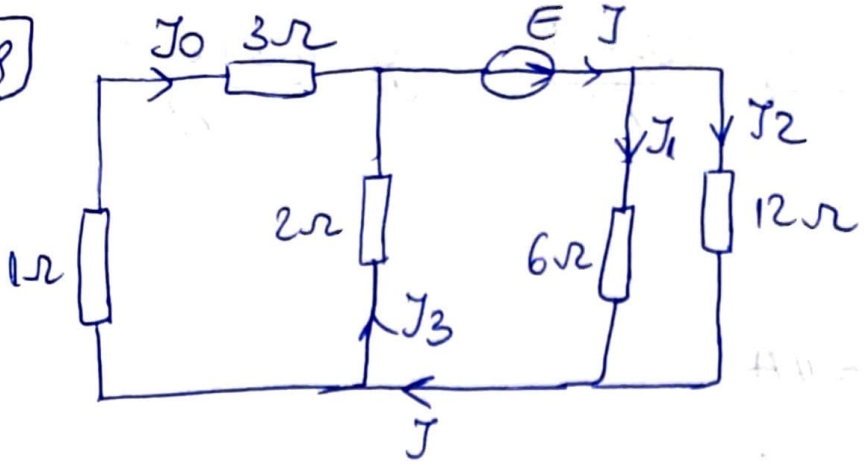
$\Rightarrow J_4 = \sqrt{\frac{32}{2}} \Rightarrow J_4 = 4A$

- $2J_4 = 4J_3 \Rightarrow J_3 = \frac{2J_4}{4}$

$J_3 = \frac{2 \cdot 4}{4} \Rightarrow J_3 = 2A$

- $J_1 = J_3 + J_4 \Rightarrow J_1 = 6A$
- $11J_2 = 2J_4 + 6J_1 \Rightarrow J_2 = \frac{2J_4 + 6J_1}{11} \Rightarrow J_2 = 4A$
- $J_5 = J_1 + J_2 \Rightarrow J_5 = 10A$

1.18

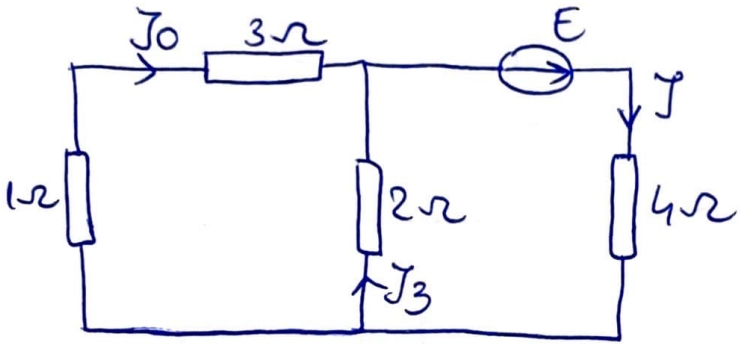


$J_0 = 4A$

$4J_0 = 2J_3 \Rightarrow J_3 = \frac{4J_0}{2} \Rightarrow J_3 = 8A$

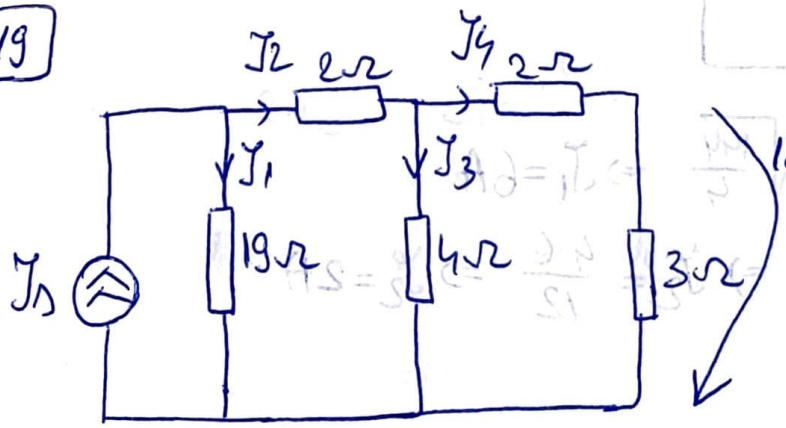
$J = J_3 + J_0 \Rightarrow J = 12A$

$R_p = \frac{6 \cdot 12}{6+12} = \frac{6 \cdot 12}{18} \Rightarrow R_p = 4\Omega$



$E = 4J + 2J_3 \Rightarrow E = 4 \cdot 12 + 2 \cdot 8 \Rightarrow E = 64V$

1.19



$12 = 3 \cdot J_4 \Rightarrow J_4 = 4A$

$4J_3 = 5J_4 \Rightarrow J_3 = \frac{5J_4}{4} \Rightarrow J_3 = 5A$

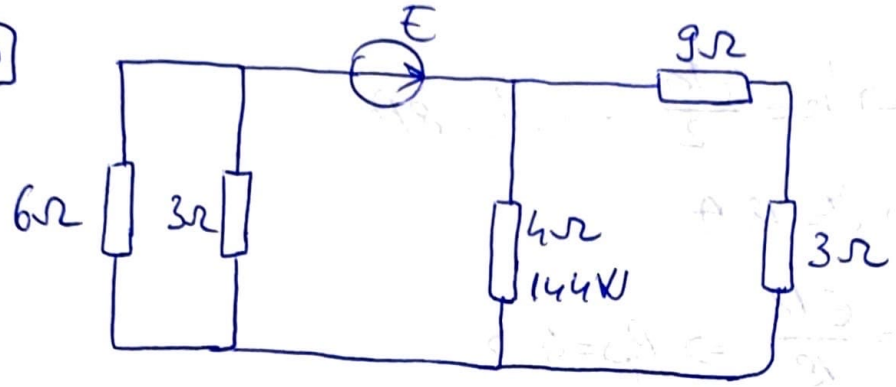
$J_2 = J_3 + J_4 \Rightarrow J_2 = 9A$

$$19J_1 = 4J_3 + 2J_2 \Rightarrow J_1 = \frac{4J_3 + 2J_2}{19} \Rightarrow J_1 = \frac{4 \cdot 5 + 2 \cdot 9}{19}$$

$$\Rightarrow J_1 = 2A$$

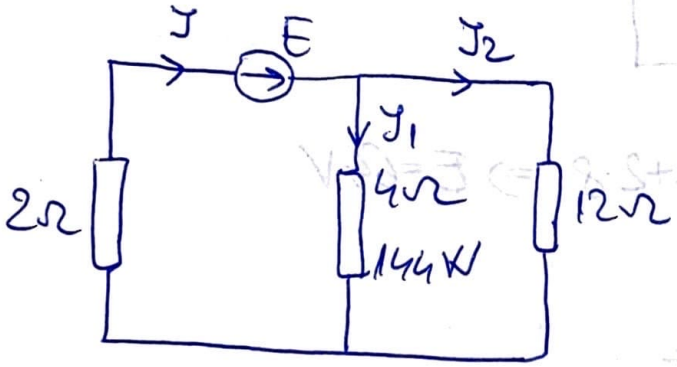
$$J_5 = J_1 + J_2 \Rightarrow J_5 = 11A$$

1.20



$$R_p = \frac{6 \cdot 3}{6+3} \Rightarrow R_p = 2\Omega$$

$$R_5 = 9 + 3 \Rightarrow R_5 = 12\Omega$$



$$144 = J_1^2 \cdot 4 \Rightarrow J_1 = \sqrt{\frac{144}{4}} \Rightarrow J_1 = 6A$$

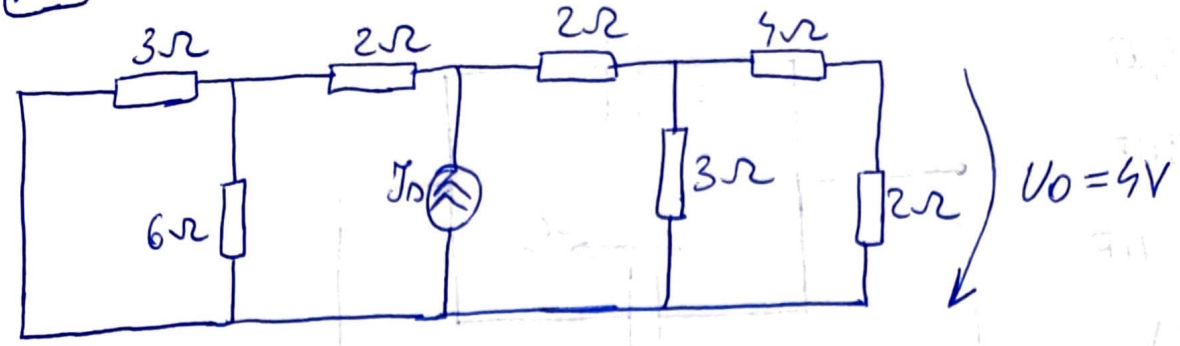
$$12J_2 = 4J_1 \Rightarrow J_2 = \frac{4J_1}{12} \Rightarrow J_2 = \frac{4 \cdot 6}{12} \Rightarrow J_2 = 2A$$

$$J = J_1 + J_2 \Rightarrow J = 8A$$

$$E = 2J + 4J_1 = 2 \cdot 8 + 4 \cdot 6$$

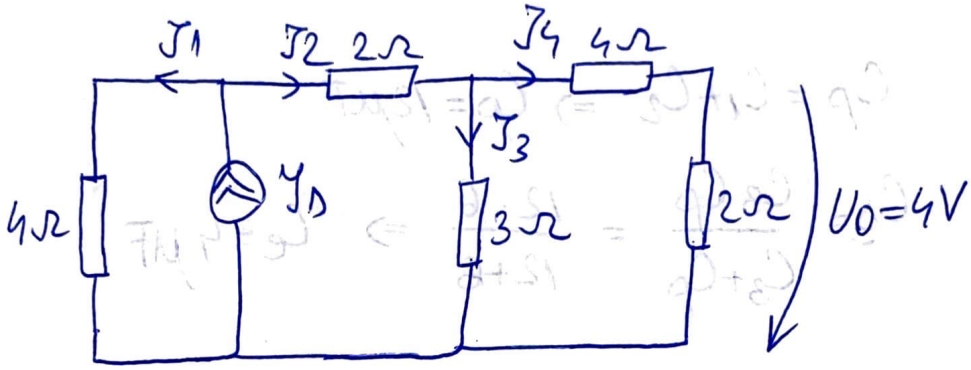
$$\Rightarrow E = 40V$$

1.21



$$R_p = \frac{3 \cdot 6}{3+6} \Rightarrow R_p = 2\Omega$$

$$R_D = R_p + 2 \Rightarrow R_D = 4\Omega$$



$$4 = 2J_4 \Rightarrow J_4 = 2A$$

$$3J_3 = 6J_4 \Rightarrow J_3 = \frac{6J_4}{3} \Rightarrow J_3 = 4A$$

$$J_2 = J_3 + J_4 \Rightarrow J_2 = 6A$$

$$4J_1 = 3J_3 + 2J_2 \Rightarrow J_1 = \frac{3J_3 + 2J_2}{4}$$

$$J_1 = \frac{3 \cdot 4 + 2 \cdot 6}{4} \Rightarrow J_1 = 6A$$

$$J_D = J_1 + J_2 \Rightarrow J_D = 12A$$

1.22

$C_1 = 3\mu F$

$C_2 = 9\mu F$

$C_3 = 6\mu F$

$U = 60V$

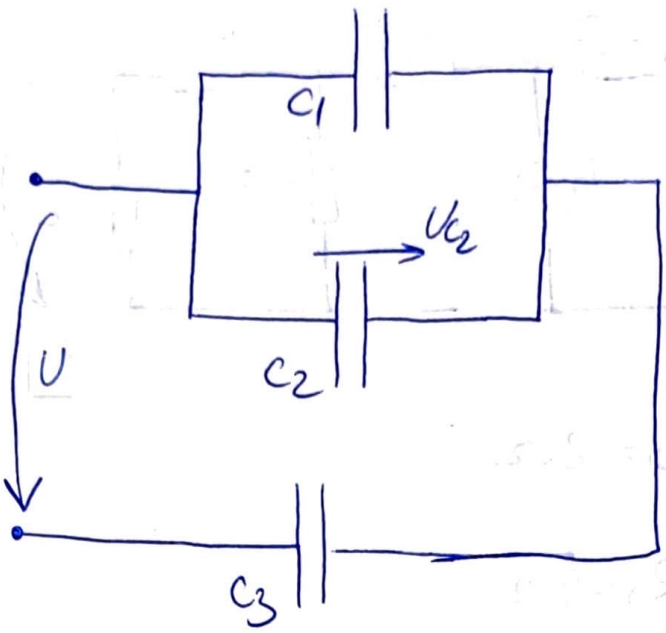
a) $C_e = ?$

b) $Q_e = ?$

c) $U_{C_2} = ?$

d) $Q_1 = ?$

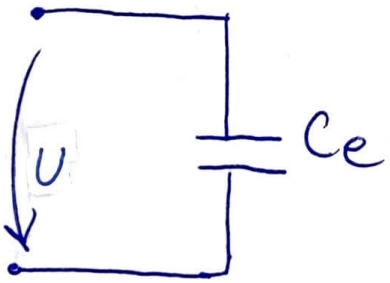
e) $W_{C_3} = ?$



$C_p = C_1 + C_2 \Rightarrow C_p = 12\mu F$

$C_e = \frac{C_3 C_p}{C_3 + C_p} = \frac{12 \cdot 6}{12+6} \Rightarrow C_e = 4\mu F$

b)



$Q_e = C_e \cdot U$

$Q_e = 4 \cdot 60 \Rightarrow Q_e = 240\mu C$

c) $U_{C_2} = U_{C_1} = U_{C_p} = \frac{Q_e}{C_p} = \frac{240}{12} = 20V$

d) $Q_1 = U_{C_1} \cdot C_1 = 20 \cdot 3 = 60\mu C$

e) $W_{C_3} = \frac{1}{2} \cdot Q_3 U_{C_3}$

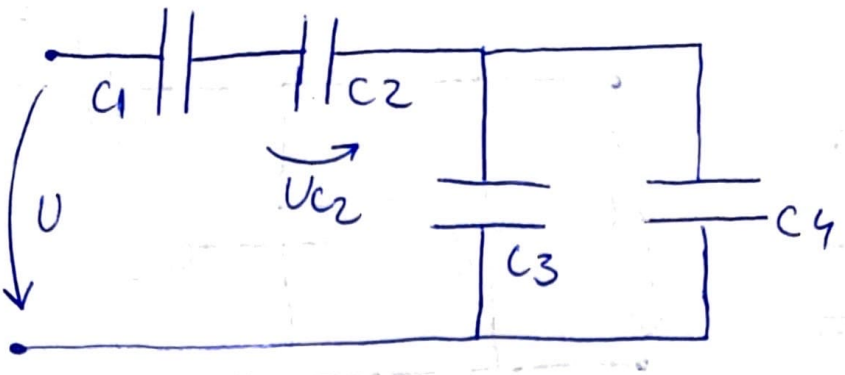
$Q_3 = Q_e = 240\mu C$

$U_{C_3} = \frac{Q_3}{C_3} = \frac{240}{6} = 40V$

$\Rightarrow W_{C_3} = \frac{1}{2} \cdot 240 \cdot 40$

$W_{C_3} = 4800\mu J$

1.23



$C_1 = 12\mu F$

$C_2 = 4\mu F$

$C_3 = 1\mu F$

$C_4 = 5\mu F$

$U = 24\,V$

a) $C_e = ?$

b) $Q_e = ?$

c) $U_{C_2} = ?$

d) $Q_4 = ?$

e) $W_{C_3} = ?$

a) $C_p = C_3 + C_4 \Rightarrow C_p = 6\mu F$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_p}$$

$$\frac{1}{C_e} = \frac{1}{12} + \frac{1}{4} + \frac{1}{6} \Rightarrow C_e = 2\mu F$$

b) $Q_e = C_e \cdot U \Rightarrow Q_e = 2 \cdot 24 \Rightarrow Q_e = 48\mu C$

c) $Q_1 = Q_2 = Q_s = U\left(\frac{C_1 C_2}{C_1 + C_2}\right) = 24 \cdot 3 = 72\mu C$

$$U_{C_2} = \frac{Q_2}{C_2} = \frac{72}{4} = 18V$$

d) $Q_4 = C_4 \cdot U \Rightarrow Q_4 = 5 \cdot 24 \Rightarrow Q_4 = 120\mu C$

e) $W_{C_3} = \frac{1}{2} \cdot Q_3 \cdot U_{C_3}$

$U_{C_3} = U_{C_4} \Rightarrow \frac{Q_3}{C_3} = \frac{Q_4}{C_4} \Rightarrow Q_3 = \frac{1 \cdot 120}{5} \Rightarrow Q_3 = 24\mu C$

$U_{C_3} = 24\mu C$

$W_{C_3} = 288\,\mu J$

1.24

$C_1 = 5\mu F$

$C_2 = 7\mu F$

$C_3 = 12\mu F$

$C_4 = 4\mu F$

$U = 20V$

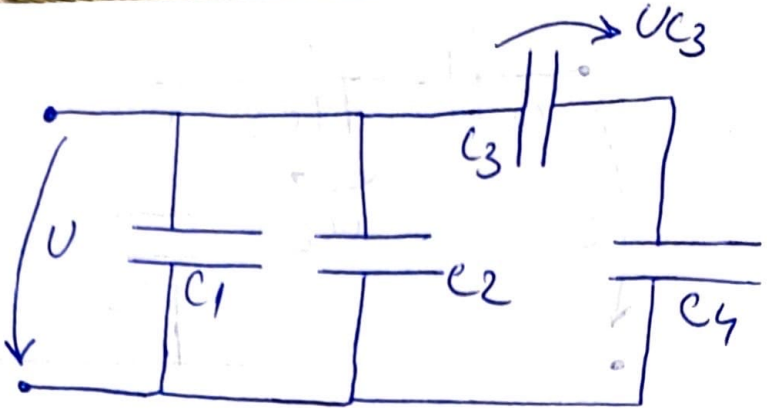
a) $C_e = ?$

b) $Q_e = ?$

c) $U_{C_3} = ?$

d) $Q_1 = ?$

e) $W_{C_4} = ?$



a) $C_s = \frac{C_3 C_4}{C_3 + C_4} \Rightarrow C_s = \frac{12 \cdot 4}{12+4} \Rightarrow C_s = 3\mu F$

$C_{p1} = C_s + C_2 \Rightarrow C_{p1} = 10\mu F$

$C_e = C_{p1} + C_1 \Rightarrow C_e = 15\mu F$

b) $Q_e = C_e \cdot U = 15 \cdot 20 \Rightarrow Q_e = 300\mu C$

c) $Q_3 = Q_4 = U \cdot C_s = 60\mu C$

$U_{C_3} = \frac{Q_3}{C_3} = \frac{60}{12} = 5V$

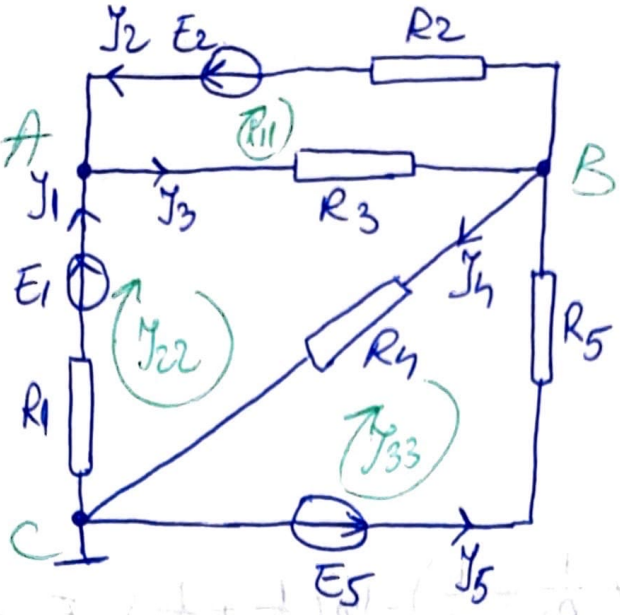
d) $U_{C_1} = U_{C_2} = \frac{Q_e}{C_1 + C_2} = 25V$

$U_{C_1} = \frac{Q_1}{C_1} \Rightarrow Q_1 = U_{C_1} \cdot C_1 \Rightarrow Q_1 = 125\mu C$

e) $W_{C_4} = \frac{1}{2} \cdot Q_4 \cdot U_{C_4} = \frac{1}{2} \cdot Q_4 \cdot \frac{Q_4}{C_4} = \frac{1}{2} \cdot 60 \cdot \frac{60}{4}$

$W_{C_4} = 450\mu J$

2.1



nodes: 3
branches: 5

$R_1 = 5\Omega$
$R_2 = 10\Omega$
$R_3 = 6\Omega$
$R_4 = 3\Omega$
$R_5 = 2\Omega$
$E_1 = 10V$
$E_2 = 5V$
$E_5 = 5V$

a) currents using KL
b) currents using nodal analysis
c) currents using mesh analysis
d) verify power conservation theorem

a) $\begin{cases} J_3 = J_1 + J_2 \\ J_4 = J_1 + J_5 \end{cases}$

$\begin{cases} E_1 = J_1R_1 + J_3R_3 + J_4R_4 \\ E_2 = J_2R_2 + J_3R_3 \\ E_5 = J_5R_5 + J_4R_4 \end{cases} \Rightarrow \begin{cases} 10 = 5J_1 + 6J_3 + 3J_4 \\ 5 = 10J_2 + 6J_3 \\ 5 = 2J_5 + 3J_4 \end{cases}$

$\begin{cases} J_3 = J_1 + J_2 \\ J_4 = J_1 + J_5 \\ 10 = 5J_1 + 6J_3 + 3J_4 \\ 5 = 10J_2 + 6J_3 \\ 5 = 2J_5 + 3J_4 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} 10 = 5J_1 + 6(J_1 + J_2) + 3(J_1 + J_5) \\ 5 = 10J_2 + 6(J_1 + J_2) \\ 5 = 2J_5 + 3(J_1 + J_5) \end{cases} \Rightarrow \begin{cases} 10 = 14J_1 + 6J_2 + 3J_5 \\ 5 = 16J_2 + 6J_1 \\ 5 = 5J_5 + 3J_1 \end{cases}$

$J_1 = \frac{5(1 - J_5)}{3}$; $J_2 = \frac{5 - 10(1 - J_5)}{16}$; $\boxed{J_5 = 0{,}69A}$

$\boxed{I_1 = 0{,}51\,A}$

$\boxed{I_2 = 0{,}11\,A}$

$\boxed{I_3 = 0{,}62\,A}$

$\boxed{I_4 = 1{,}2\,A}$

b) $A: V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - V_B\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = E_1 \cdot \frac{1}{R_1} + E_2 \cdot \frac{1}{R_2}$

$B: V_B\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) - V_A\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = -E_2\frac{1}{R_2} + E_5\frac{1}{R_5}$

$$\begin{cases} V_A\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{6}\right) - V_B\left(\frac{1}{10}+\frac{1}{6}\right) = 10\frac{1}{5} + 5\frac{1}{10} \\ V_B\left(\frac{1}{10}+\frac{1}{6}+\frac{1}{3}+\frac{1}{2}\right) - V_A\left(\frac{1}{10}+\frac{1}{6}\right) = -5\cdot\frac{1}{10} + 5\cdot\frac{1}{2} \end{cases}$$

$$V_A = \frac{2955}{398} = 7.42\,V$$

$$V_B = \frac{720}{199} = 3.61\,V$$

$V_A - V_C = E_1 - I_1 R_1 \iff V_A = E_1 - I_1 R_1 \Rightarrow \boxed{I_1 = 0.51\,A}$

$V_B - V_C = E_5 - I_5 R_5 \iff V_B = E_5 - I_5 R_5 \Rightarrow \boxed{I_5 = 0.69\,A}$

$V_B - V_C = I_4 R_4 \iff V_B = I_4 R_4 \Rightarrow \boxed{I_4 = 1.2\,A}$

$V_B - V_A = -E_2 + I_2 R_2 \Rightarrow \boxed{I_2 = 0.11\,A}$

$V_A - V_B = I_3 R_3 \Rightarrow \boxed{I_3 = 0.62\,A}$

c) $I_1 = I_{22}$

$I_2 = -I_{11}$

$I_3 = I_{22} - I_{11}$

$I_4 = I_{22} - I_{33}$

$I_5 = -I_{33}$

mesh 1: $(R_2+R_3)I_{11} - R_3 \cdot I_{22} = -E_2$

mesh 2: $(R_1+R_3+R_4)I_{22} - R_3 I_{11} - R_4 I_{33} = E_1$

mesh 3: $(R_4+R_5)I_{33} - R_4 I_{22} = -E_5$

$$\begin{cases} 16 I_{11} - 6 I_{22} = -5 \\ 14 I_{22} - 6 I_{11} - 3 I_{33} = 10 \\ 5 I_{33} - 3 I_{22} = -5 \end{cases} \Rightarrow$$

$$I_{11} = -\frac{95}{796} = -0.11\ A$$

$$I_{22} = \frac{205}{398} = 0.51\ A$$

$$I_{33} = -\frac{275}{398} = -0.69\ A$$

$\boxed{I_1 = 0.51\ A}$

$\boxed{I_2 = 0.11\ A}$

$\boxed{I_3 = 0.62\ A}$

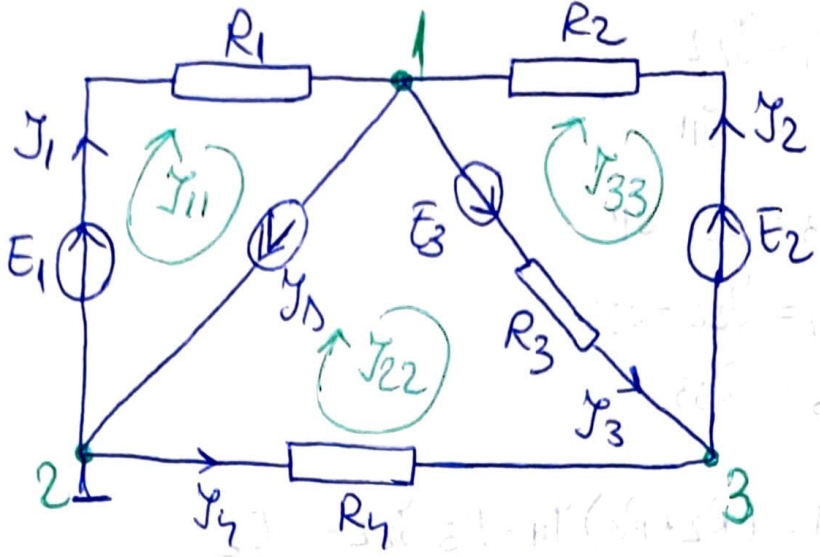
$\boxed{I_4 = 1.2\ A}$

$\boxed{I_5 = 0.69\ A}$

d) $P_g = E_1 \cdot I_1 + E_2 \cdot I_2 + E_5 I_5 = 9.1\ W$

$P_r = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5$

2.2



$R_1 = 2\Omega$

$R_2 = 2\Omega$

$R_3 = 6\Omega$

$R_4 = 2\Omega$

$E_1 = 12V$

$E_3 = 6V$

$J_5 = 4A$

currents:

a) KL

b) nodal analysis

c) mesh analysis

d) verify power cons. theorem

a) $\begin{cases} J_1 + J_2 = 4 + J_3 \\ J_1 + J_4 = 4 \\ J_2 = J_3 + J_4 \end{cases}$

$\begin{cases} E_1 = J_1 R_1 \Rightarrow \boxed{J_1 = 6A} \\ E_3 = J_3 R_3 - J_4 R_4 \\ E_2 + E_3 = J_2 R_2 + J_3 R_3 \end{cases}$

$\begin{cases} 6 + J_2 = 4 + J_3 \\ 6 + J_4 = 4 \Rightarrow \boxed{J_4 = -2A} \\ J_2 = J_3 + J_4 \end{cases}$

$\boxed{J_2 = -1.7A}$

$\boxed{J_3 = 0.3A}$

$\boxed{E_2 = -7,6V}$

b) Node 2 is the ground $\rightarrow V_2 = 0V$

node 1: $V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_3\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = E_1 \cdot \frac{1}{R_1} - E_3 \frac{1}{R_3} + E_2 \frac{1}{R_2} - J_1$

node 3: $V_3\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) - V_1\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = E_3 \cdot \frac{1}{R_3} - E_2 \cdot \frac{1}{R_2}$

$$\begin{cases} V_1\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{6}\right) - V_3\left(\frac{1}{2}+\frac{1}{6}\right) = 12\cdot\frac{1}{2} - 6\cdot\frac{1}{6} - 4{,}6\cdot\frac{1}{2} - 4 \\ V_3\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{2}\right) - V_1\left(\frac{1}{2}+\frac{1}{6}\right) = 6\cdot\frac{1}{6} + 4{,}6\cdot\frac{1}{2} \end{cases}$$

$$V_1 = \frac{-36}{47} = -0.76\ V$$

$$V_2 = \frac{672}{235} = 2.85\ V$$

$V_1 = E_1 - J_1 R_1 \Rightarrow \boxed{J_1 = 6A}$

$V_3 - V_1 = J_2 R_2 - E_2 \Rightarrow \boxed{J_2 = -1.7A}$

$V_1 - V_3 = J_3 R_3 - E_3 \Rightarrow \boxed{J_3 = 0.3A}$

$V_3 - V_2 = -J_4 R_4 \Rightarrow \boxed{J_4 = -2A}$

c) $J_1 = J_{11}$

$J_2 = J_{33}$

$J_4 = J_{22}$

$J_3 = J_{33} - J_{22}$

$J_1 R_1 = E_1 \Rightarrow J_{11} R_1 = E_1 \Rightarrow J_{11} = 6A \Rightarrow \boxed{J_1 = 6A}$

$J_4 R_4 - J_3 R_3 = -E_3 \Rightarrow J_{22}\cdot 2 - (J_{33} - J_{22})\cdot 6 = -6$

$\Rightarrow 8J_{22} - 6J_{33} = -6$

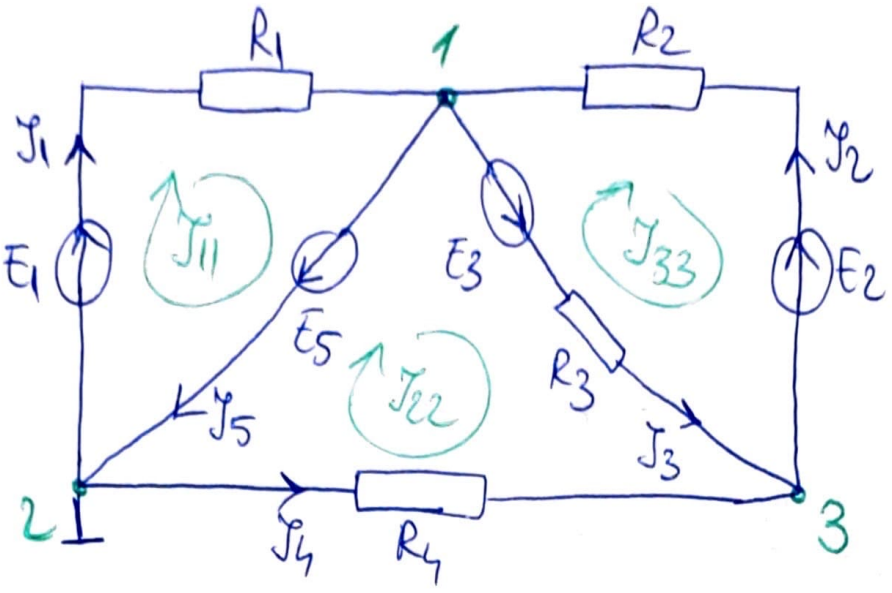
$J_3 R_3 + J_2 R_2 = E_2 + E_3 \Rightarrow (J_{33} - J_{22})\cdot 6 + J_{33}\cdot 2 = 6 - 4{,}6$

$\Rightarrow 8J_{33} - 6J_{22} = 1{,}6$

$\boxed{J_2 = -1.7A}$ $\boxed{J_3 = 0.3A}$ $\boxed{J_4 = -2A}$

2.3

$R_1 = 2\,\Omega$
$R_2 = 2\,\Omega$
$R_3 = 6\,\Omega$
$R_4 = 2\,\Omega$
$E_1 = 12\,V$
$E_3 = 6\,V$
$E_5 = 10\,V$



currents:

a) KL

b) nodal analysis

c) mesh analysis

d) verify power conservation theorem

$$\begin{cases} J_1 + J_2 = J_3 + J_5 \\ J_5 = J_1 + J_4 \\ J_2 = J_3 + J_4 \end{cases}$$

- $E_1 + E_5 = J_1 R_1 \Rightarrow \boxed{J_1 = 11\,A}$
- $E_3 - E_5 = J_3 R_3 - J_4 R_4 \Rightarrow$

$$\Rightarrow -4 = 6J_3 - 2J_4 \Rightarrow J_3 = \frac{2J_4 - 4}{6}$$

- $E_3 + E_2 = J_2 R_2 + J_3 R_3$
- $E_1 - E_2 = J_1 R_1 - J_2 R_2 - J_4 R_4$
- $E_1 + E_3 = J_1 R_1 + J_3 R_3 - J_4 R_4$

$$\begin{cases} 11 + J_2 = \frac{2J_4 - 4}{6} + J_5 \\ J_5 = 11 + J_4 \\ J_2 = \frac{2J_4 - 4}{6} + J_4 \end{cases}$$

$$11 + J_2 = \frac{2J_4 - 4}{6} + 11 + J_4$$

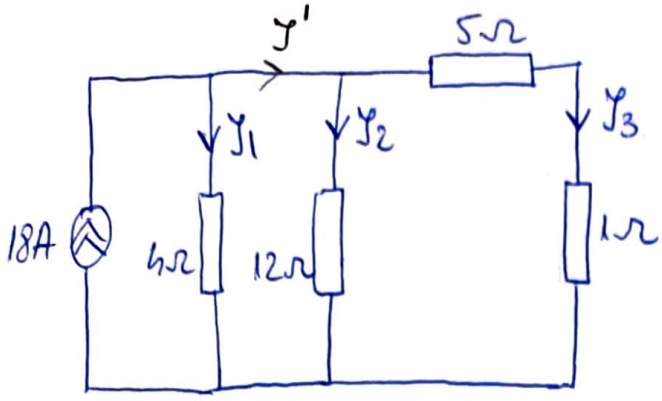
$$J_2 = \frac{2J_4 - 4}{6} + J_4$$

Not enough equations.

2.4

$I_1 = ?\quad I_2 = ?\quad I_3 = ?$

verify power cons. theorem



$$I_3 = I_1 + I' \Rightarrow \begin{cases} I_1 + I' = 18 \\ I' = I_2 + I_3 \end{cases}$$

$$12 I_2 = 6 I_3 \Rightarrow I_3 = 2 I_2$$

$$\begin{cases} I_1 + I' = 18 \\ I' = 3 I_2 \end{cases}$$

$$4 I_1 = 12 I_2 \Rightarrow I_1 = 3 I_2$$

$$\begin{cases} 3 I_2 + I' = 18 \\ I' = 3 I_2 \end{cases} \Rightarrow 6 I_2 = 18 \Rightarrow \boxed{I_2 = 3A}$$

$$I' = 9A$$

$$I_1 + 9 = 18 \Rightarrow \boxed{I_1 = 9A}$$

$$\boxed{I_3 = 6A}$$

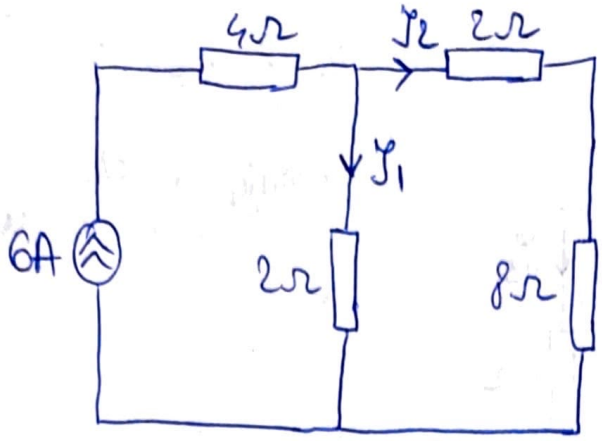
$$\frac{1}{Re} = \frac{1}{4} + \frac{1}{12} + \frac{1}{5+1} \Rightarrow Re = 2\Omega$$

$$P_g = I_s^2 \cdot Re = 18^2 \cdot 2 \Rightarrow P_g = 648W$$

$$P_R = I_1^2 R_1 + I_2^2 R_2 + I_3^2 (R_3 + R_4) \Rightarrow P_R = 648W$$

$$\Rightarrow P_g = P_R$$

2.5



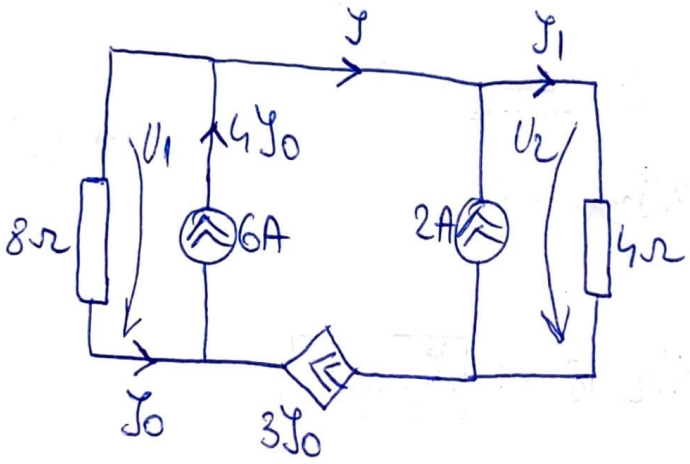
$I_1 + I_2 = 6$

$2I_1 = 10I_2 \Rightarrow I_1 = 5I_2$

$5I_2 + I_2 = 6 \Rightarrow I_2 = 1A$

$P_{8\Omega} = I_2^2 \cdot 8 \Rightarrow P_{8\Omega} = 8W$

2.6



$I_0 + 3I_0 = 4I_0$

$4I_0 = 6A \Rightarrow \boxed{I_0 = 1.5A}$

$4I_0 = I_0 + I \Rightarrow 3I_0 = I \Rightarrow \boxed{I = 4.5A}$

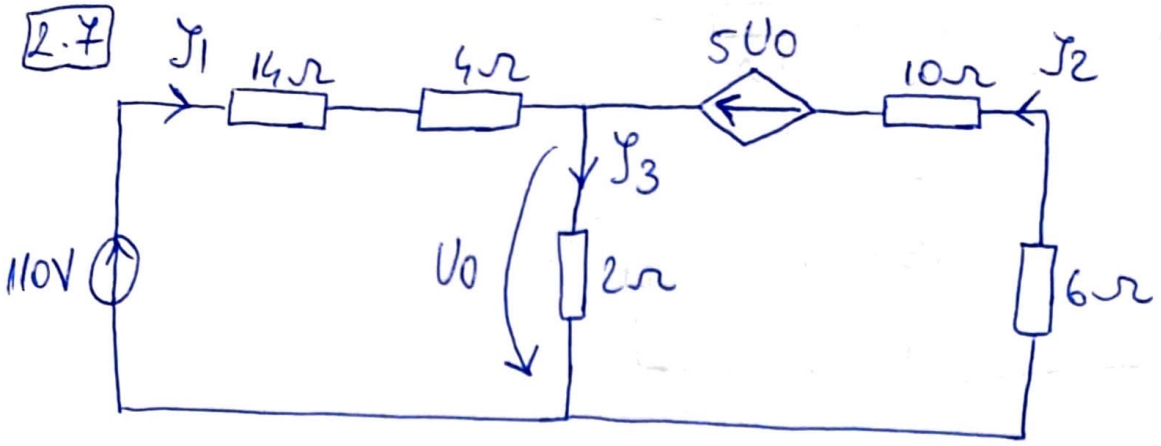
$I_1 = I + 2 \Rightarrow \boxed{I_1 = 6.5A}$

$P_R = I_0^2 \cdot 8 + I_1^2 \cdot 4 = 187W$

$P_g = 6U_1 + 2U_2 + 4.5(U_2 - U_1) = 187W$ $\Rightarrow P_R = P_g$

$U_1 = I_0 \cdot 8 = 12V$; $U_2 = I_1 \cdot 4 = 26V$

2.7



$J_3 = J_1 + J_2$

$110 = 18J_1 + 2J_3$ sau $110 = 18J_1 + U_0$

$5U_0 = 16J_2 + 2J_3$ sau $5U_0 = 16J_2 + U_0 \Rightarrow 4U_0 = 16J_2 \Rightarrow \boxed{U_0 = 4J_2}$

$110 = 18J_1 + 4J_2$

$J_3 = J_1 + J_2$

$110 = 18J_1 + 2J_3$

$18J_1 + 4J_2 = 18J_1 + 2J_3 \Rightarrow \boxed{J_3 = 2J_2}$

$2J_2 = J_1 + J_2 \Rightarrow \boxed{J_1 = J_2}$

$110 = 18J_2 + 4J_2 \Rightarrow 110 = 22J_2 \Rightarrow \boxed{J_2 = 5A}$

$\boxed{J_1 = 5A}$

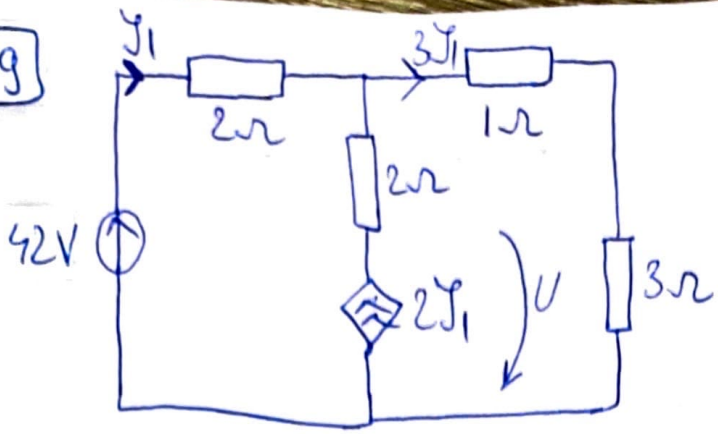
$\boxed{J_3 = 10A}$

$U_0 = 20V$

$P_g = 110J_1 + 5 \cdot U_0 \cdot J_2 = 1050W$

$P_r = J_1^2 \cdot 18 + J_3^2 \cdot 2 + J_2^2 \cdot 16 = 1050W$ $\Rightarrow P_g = P_r$

29



$I_1 + 2I_1 = 3I_1$

$42 = 2I_1 + 3I_1 + 3I_1 \cdot 3$

$42 = 14 I_1 \Rightarrow \boxed{I_1 = 3A}$

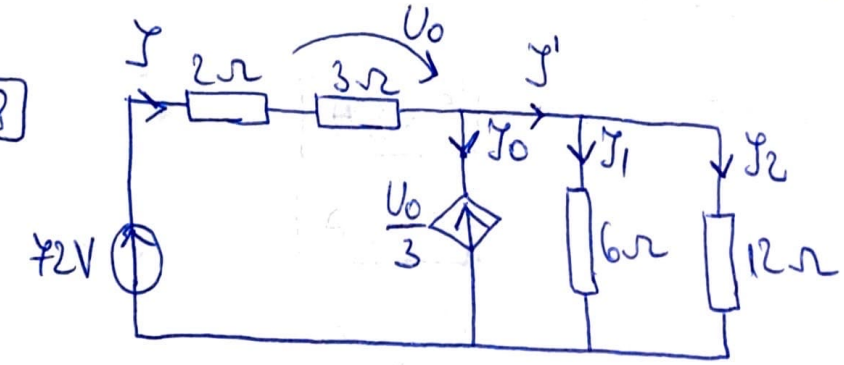
$U = 3I_1 + 3I_1 \cdot 3 + 4I_1$

$U = 16 I_1 \Rightarrow U = 48V$

$P_g = 42 \cdot I_1 + U \cdot 2I_1 = 414 W$

$P_R = I_1^2 \cdot 2 + (2I_1)^2 \cdot 2 + (3I_1)^2 \cdot 4 = 414 W$ $\Big\} \Rightarrow P_g = P_R$

2.8



$I = I_0 + I'$
$I' = I_1 + I_2$ $\Big\} \Rightarrow I = I_0 + I_1 + I_2$

$U_0 = 3I$

$72 = 2I + U_0 + \frac{U_0}{3} \Rightarrow 72 = 2I + 3I + \frac{3I}{3} \Rightarrow 72 = 6I$

$\Rightarrow \boxed{I = 12A}$

$6J_1 = 12J_2 \Rightarrow J_1 = 2J_2$

$\frac{U_0}{3} = 6J_1 \Rightarrow U = 6J_1 \Rightarrow \boxed{J_1 = 2A}$

$\boxed{J_2 = 1A}$

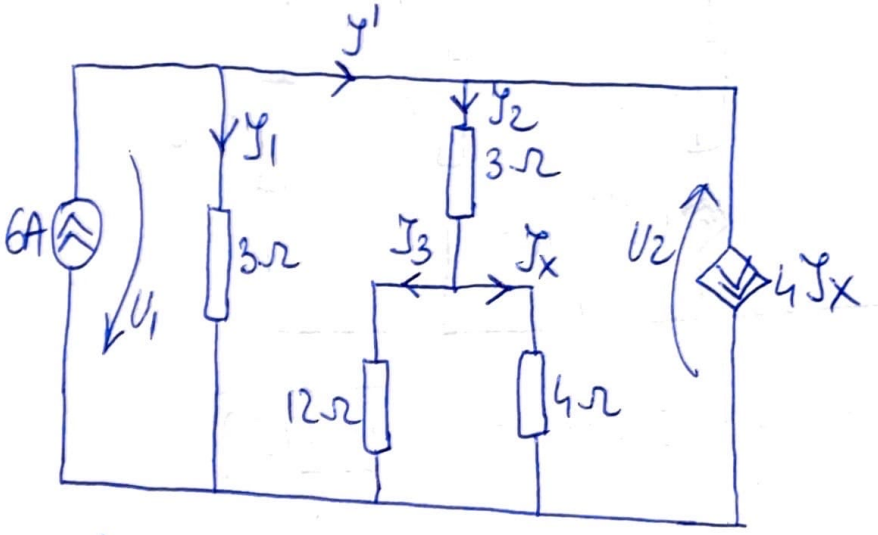
$\boxed{J_0 = 9A}$

$P_g = 72J + \frac{U_0}{3} \cdot J_0 = 72 \cdot 12 + 12 \cdot 9 = 972W$

$P_r = J^2 \cdot 5 + J_1 \cdot 6 + J_2 \cdot 12 = 756W$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} P_g \neq P_r$

2.10



$$\begin{cases} 6 = J_1 + J' \\ J' = J_2 + 4J_x \\ J_2 = J_3 + J_x \end{cases} \Rightarrow \begin{cases} 6 = J_1 + J_2 + 4J_x \\ J_2 = J_3 + J_x \end{cases}$$

$3J_1 - 12J_3 - 3J_2 = 0 \Rightarrow J_1 = J_2 + 4J_3$

$12J_3 = 4J_x \Rightarrow J_x = 3J_3$

$$\begin{cases} J_1 + J_2 + 4J_x = 6 \\ J_2 = J_3 + J_x \\ J_1 = J_2 + 4J_3 \\ J_x = 3J_3 \end{cases}$$

$J_1 = 2A$

$J_2 = 1A$

$J_3 = 0{,}25A$

$\boxed{J_x = 0{,}75A}$

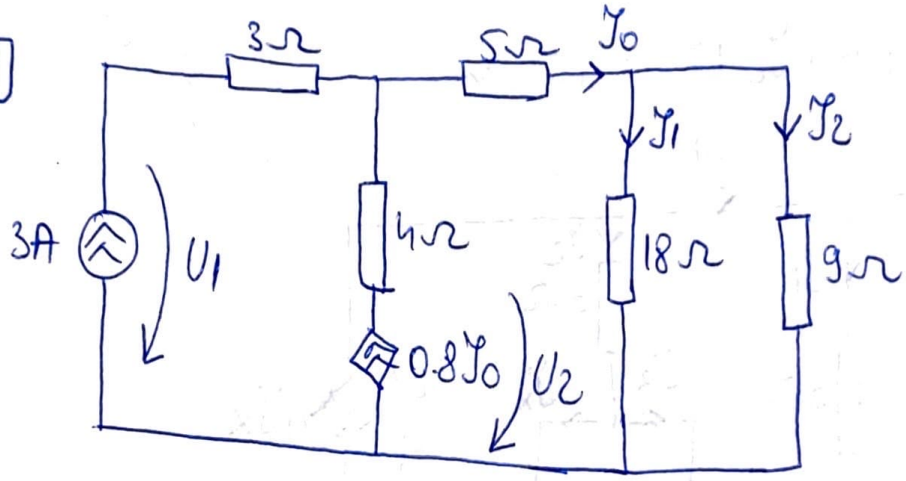
$U_1 = 3J_1 = 6V$

$U_2 = 3J_2 + 4J_x = 6V$

$P_g = 6U_1 + 4J_x U_2 = 54\,W$

$P_R = 3J_1 + 3J_2 + 12J_3 + 4J_x = 15\,W$ $\Big\} \Rightarrow P_g \neq P_R$

2.11



$3A + 0.8J_0 = J_0 \Rightarrow 0.2J_0 = 3 \Rightarrow \boxed{J_0 = 15A}$

$\begin{cases} J_0 = J_1 + J_2 \\ 18J_1 = 9J_2 \Rightarrow J_2 = 2J_1 \end{cases}$

$J_0 = J_1 + 2J_1 \Rightarrow J_0 = 3J_1 \Rightarrow \boxed{J_1 = 5A}$

$\boxed{J_2 = 10A}$

$U_1 = 3 \cdot 3 + 5J_0 + 18J_1 = 174V$

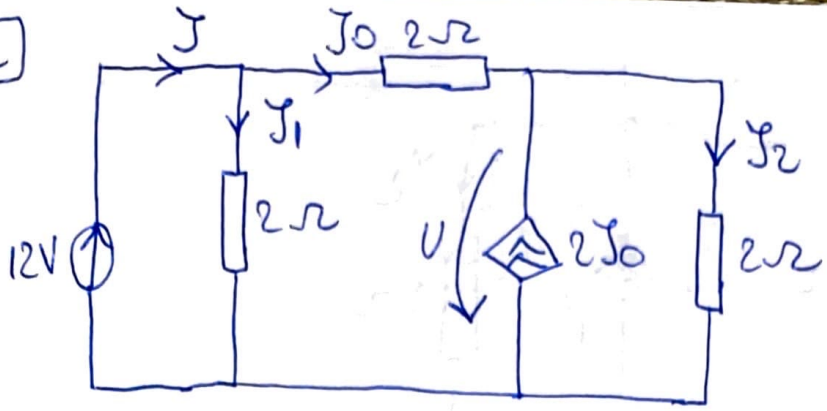
$U_2 = 12 \cdot 4 + 5J_0 + 18J_1 = 213V$

$P_g = 3U_1 + 0.8J_0 U_2 = 3078\,W$

$P_R = 3^2 \cdot 3 + (0{,}8 \cdot 15)^2 \cdot 4 + 15^2 \cdot 5 + 5^2 \cdot 18 + 10^2 \cdot 9 = 3078W$ $\Big\} \Rightarrow$

$\Rightarrow P_g = P_R$

2.12



$$\begin{cases} J = J_0 + J_1 \\ J_0 + 2J_0 = J_2 \end{cases} \Rightarrow \begin{cases} J = J_0 + J_1 \\ J_2 = 3J_0 \end{cases}$$

$12 = 2J_0 + 2J_2$

$12 = 2J_0 + 6J_0 \Rightarrow 12 = 8J_0 \Rightarrow \boxed{J_0 = 1.5A}$

$\boxed{J_2 = 4.5\,A}$

$2J_1 - 2J_0 = 0 \Rightarrow J_1 = J_0 \Rightarrow \boxed{J_1 = 1.5A}$

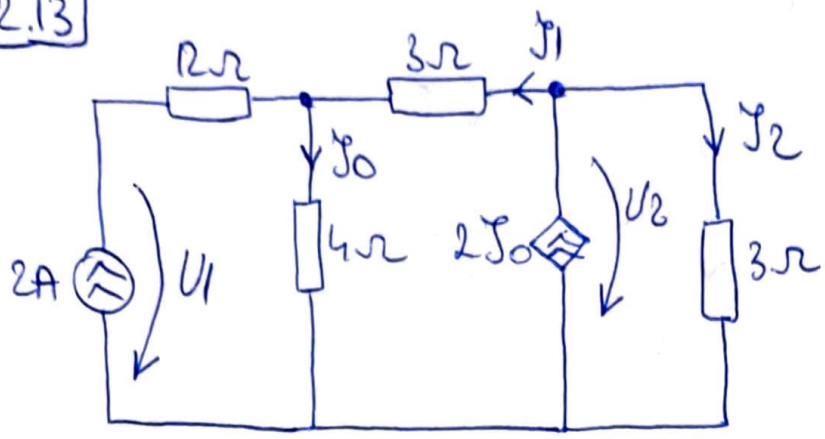
$\boxed{J = 3\,A}$

$U = -2J_0 + 2J_1 \Rightarrow U = 0$

$P_g = 12J \Rightarrow P_g = 36W$

$P_R = 2J_1 + 2J_0 + 2J_2 = 2(J_1 + J_0 + J_2) = 15W$ $\Big\} \Rightarrow P_g \neq P_R$

2.13



$$\begin{cases} J_0 = 2 + J_1 \\ 2J_0 = J_1 + J_2 \\ 4J_0 + 3J_1 = 3J_2 \end{cases} \Rightarrow \begin{cases} 2(2+J_1) = J_1 + J_2 \\ 4(2+J_1) + 3J_1 = 3J_2 \end{cases}$$

$$\Rightarrow \begin{cases} 4 + 2J_1 = J_1 + J_2 \\ 8 + 4J_1 + 3J_1 = 3J_2 \end{cases} \Rightarrow \begin{cases} J_2 - J_1 = 4 \Rightarrow J_2 = 4 + J_1 \\ 3J_2 - 7J_1 = 8 \end{cases}$$

$$3(4+J_1) - 7J_1 = 8$$

$$12 + 3J_1 - 7J_1 = 8 \Rightarrow -4J_1 = 8 - 12 \Rightarrow \boxed{J_1 = 1\,A}$$

$$\boxed{J_2 = 5\,A}$$

$$\boxed{J_0 = 3\,A}$$

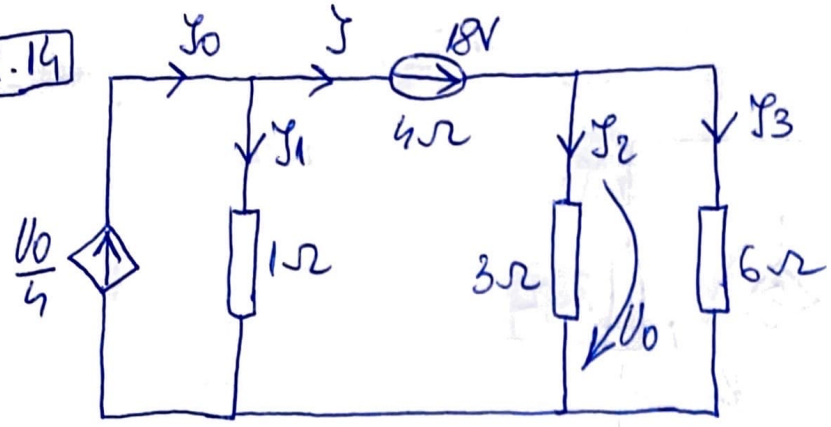
$$U_1 = 2 \cdot 12 + 4J_0 = 36\,V$$

$$U_2 = 3J_2 = 15\,V$$

$$P_g = 2U_1 + 2J_0U_2 = 162\,W$$

$$P_r = 2^2 \cdot 12 + J_0^2 \cdot 4 + J_1^2 \cdot 3 + J_2^2 \cdot 3 = 162\,W \Rightarrow P_g = P_r$$

2.14



$$\begin{cases} I_0 = I + I_1 \\ I = I_2 + I_3 \end{cases}$$

$$U_0 = 3I_2$$

$$3I_2 = 6I_3 \Rightarrow I_3 = \frac{I_2}{2}$$

$$I = I_2 + \frac{I_2}{2} \Rightarrow I = \frac{3I_2}{2}$$

$$\frac{U_0}{4} + 18 = 3I_2 \Rightarrow \frac{U_0}{4} + 18 = U_0 \mid \cdot 4 \Rightarrow U_0 + I_2 = 4U_0$$

$$3U_0 = I_2 \Rightarrow U_0 = 24\,V$$

$\boxed{I_2 = 8A}$

$\boxed{I_3 = 4A}$

$\boxed{I = 12\,A}$

$$\frac{U_0}{4} = 1 \cdot I_1 \Rightarrow \boxed{I_1 = 6A}$$

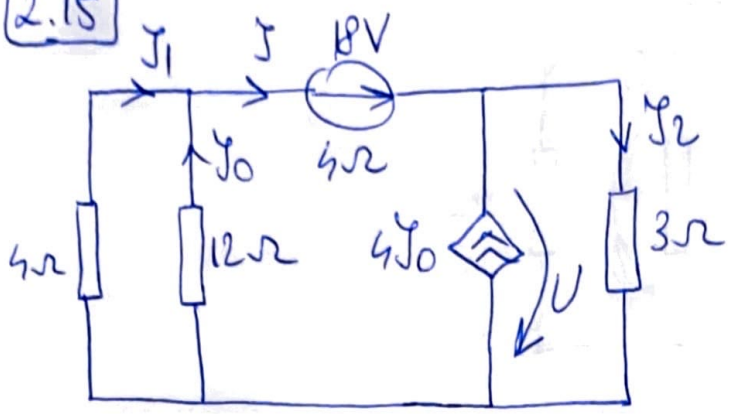
$\boxed{I_0 = 18\,A}$

$$P_g = \frac{U_0}{4} \cdot I_0 + 18I = 324\,W$$

$$P_r = I_1^2 \cdot 1 + I_2^2 \cdot 3 + I_3^2 \cdot 6 = 324\,W$$

$\Rightarrow P_g = P_r$

2.15



$$\begin{cases} J = J_1 + J_0 \\ J_2 = J + 4J_0 \end{cases}$$

$$18 = 12J_0 \Rightarrow \boxed{J_0 = 1.5\,A}$$

$$\begin{cases} J = J_1 + 1.5 \\ J_2 = J + 6 \end{cases}$$

$$18 = 4J_1 \Rightarrow \boxed{J_1 = 4.5\,A}$$

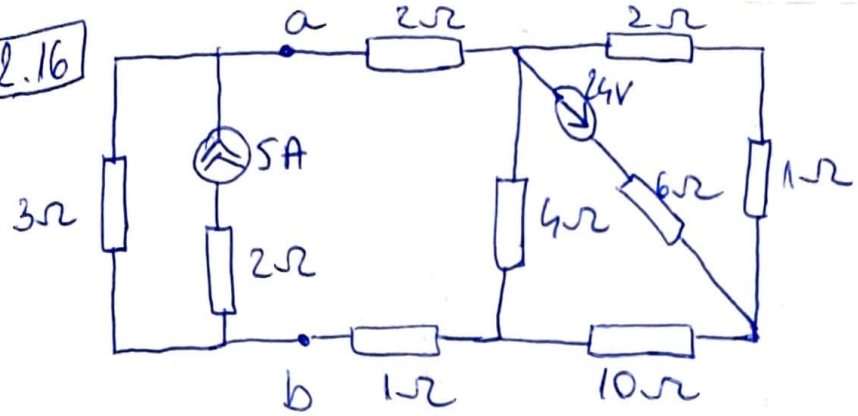
$$\boxed{J = 6\,A}$$

$$\boxed{J_2 = 12\,A}$$

$$U = 3J_2 = 36\,V$$

$$P_g = 18J + 4J_0U = 324\,W$$
$$P_r = J_1^2 \cdot 4 + J_0^2 \cdot 12 + J_2^2 \cdot 3 = 540\,W$$
$$\Rightarrow P_g \neq P_r$$

2.16



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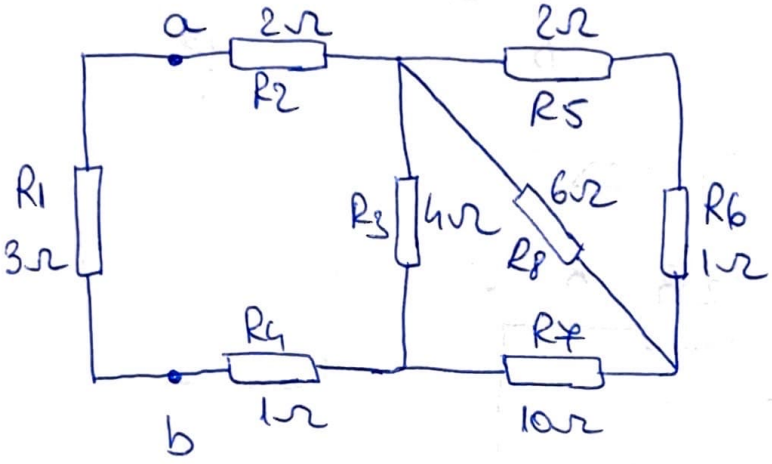


$$R_{D1} = R_5 + R_6 = 3\Omega$$

$$R_{p1} = \frac{R_{D1} R_8}{R_{D1} + R_8} = \frac{3 \cdot 6}{3+6} = 2\Omega$$

$$R_{D2} = R_{p1} + R_7 = 12\Omega$$

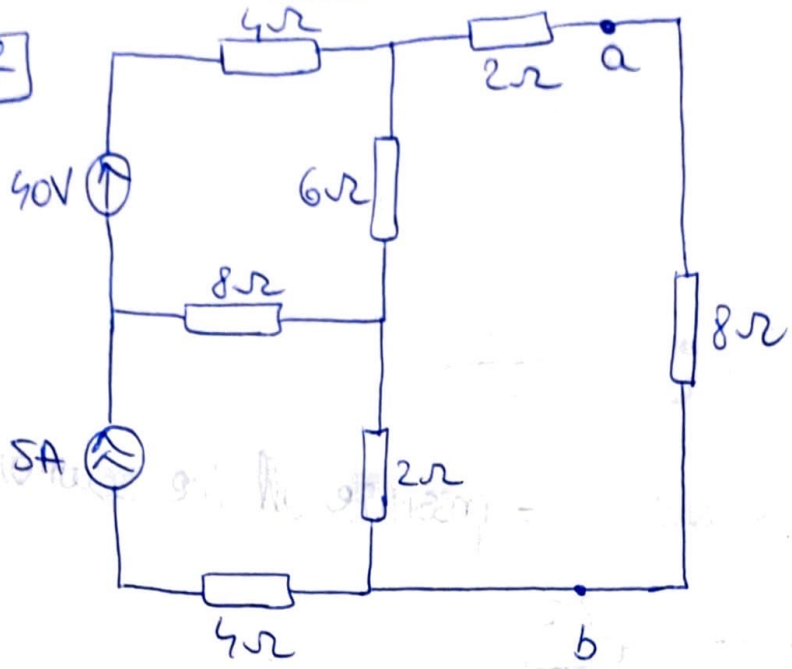
$$R_{p2} = \frac{R_3 R_{D2}}{R_3 + R_{D2}} = \frac{4 \cdot 12}{4+12} = 3\Omega$$

$$R_{D3} = R_2 + R_{p2} + R_4 = 6\Omega$$

$$R_e = \frac{R_1 R_{D3}}{R_1 + R_{D3}} = \frac{3 \cdot 6}{3+6} = 2\Omega$$

$$\boxed{R_e = 2\Omega}$$

2.17



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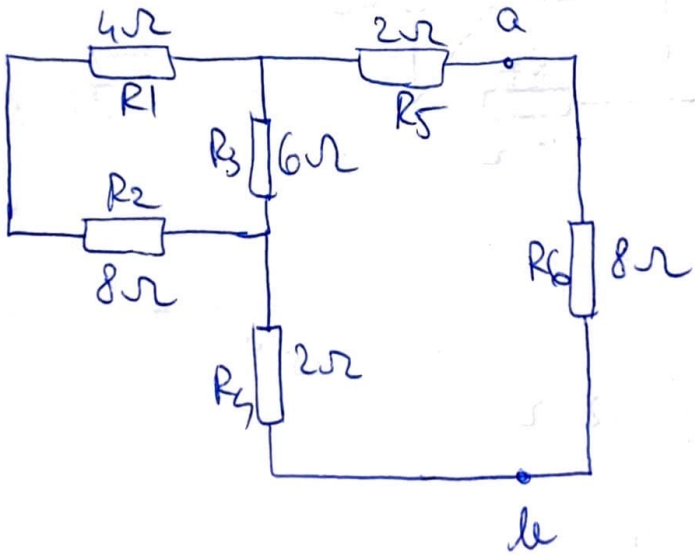


$$R_{S1} = R_1 + R_2 = 12\Omega$$

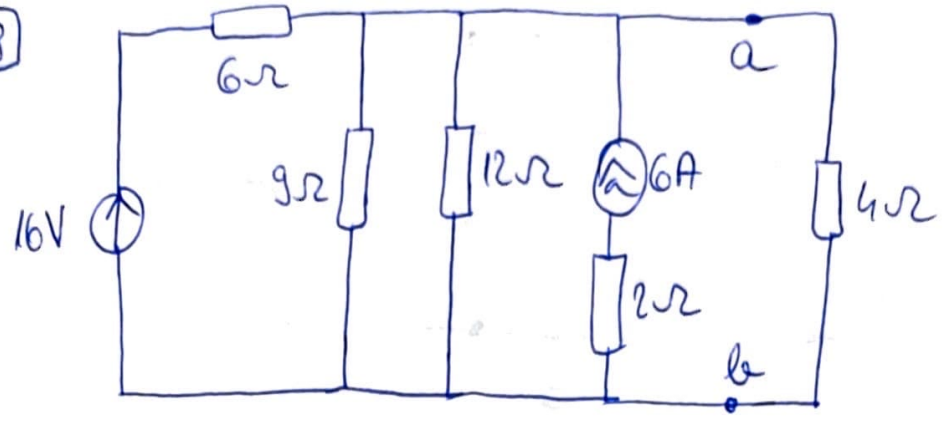
$$R_{p1} = \frac{R_{S1} R_3}{R_{S1} + R_3} = \frac{12 \cdot 6}{12+6} = 4\Omega$$

$$R_{S2} = R_5 + R_{p1} + R_4 = 8\Omega$$

$$R_e = \frac{R_{S2} \cdot R_6}{R_{S2} + R_6} = \frac{8 \cdot 8}{8+8} = 4\Omega$$

$$\boxed{R_e = 4\Omega}$$

2.18



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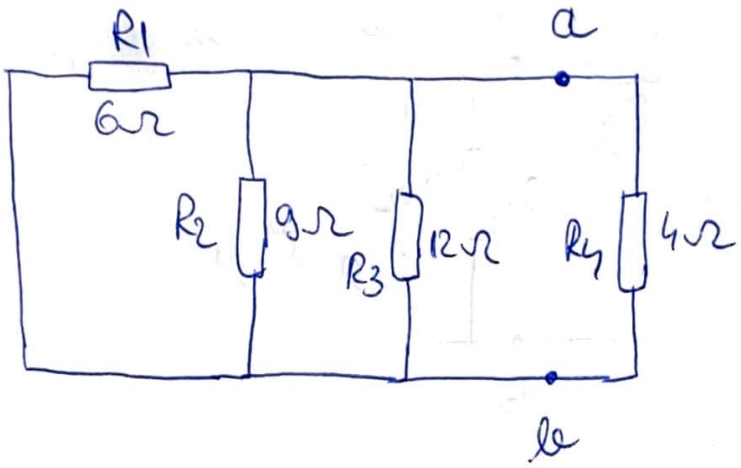
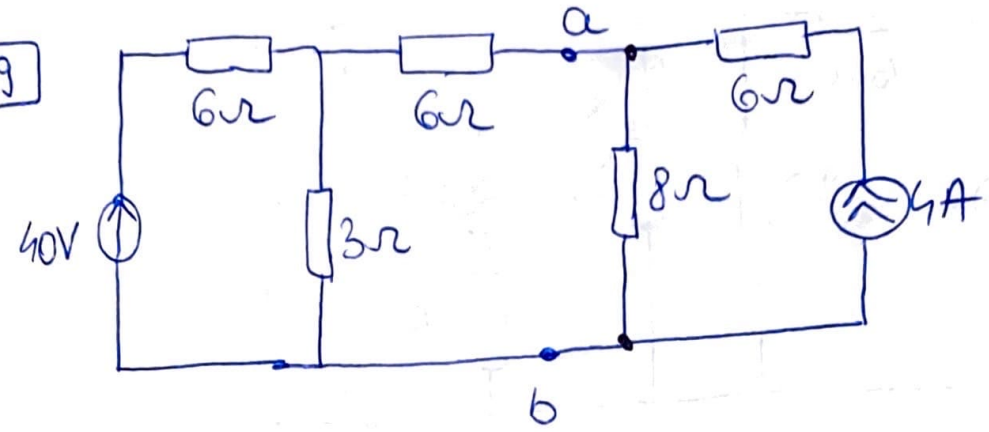


$$\frac{1}{Re} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{Re} = \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{4}$$

$$Re = 1.63\ \Omega$$

2.19



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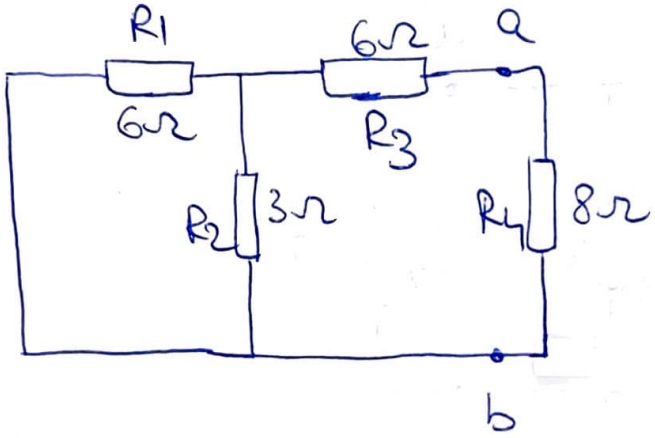


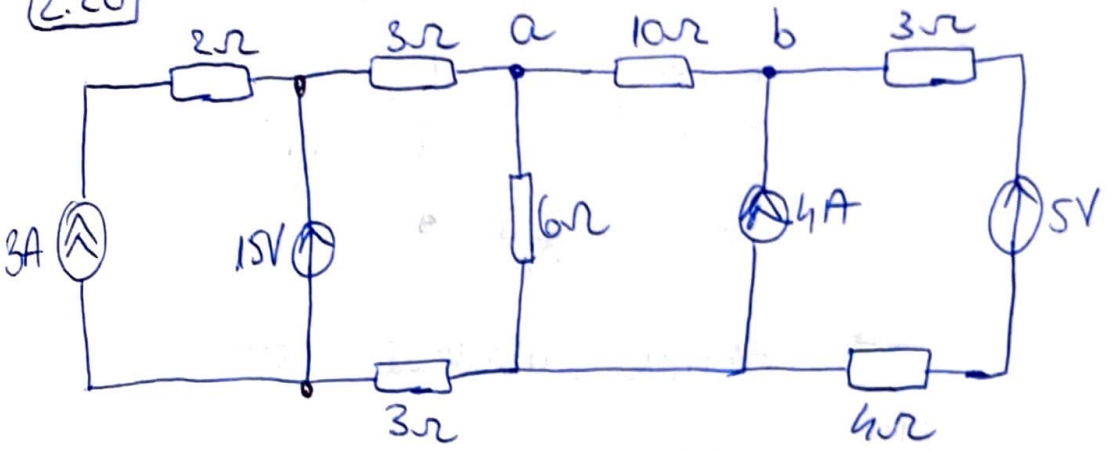
$$R_{p1} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \cdot 3}{6+3} = 2\Omega$$

$$R_{s1} = R_{p1} + R_3 = 2 + 6 = 8\Omega$$

$$R_e = \frac{R_{s1} R_4}{R_{s1} + R_4} = \frac{8 \cdot 8}{8+8} = 4\Omega$$

$R_e = 4\Omega$

2.20



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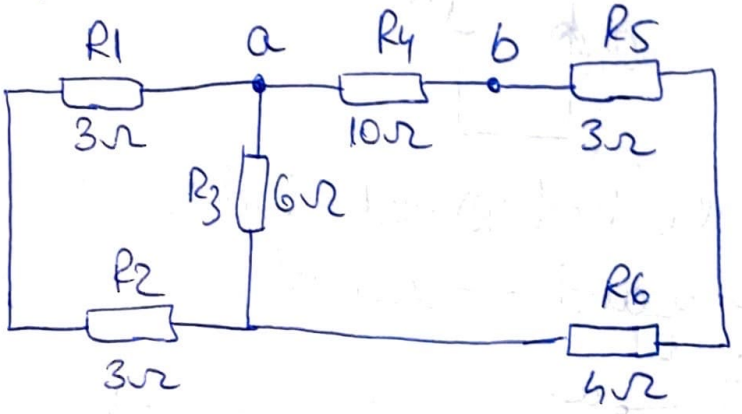


$$R_{S1} = R_1 + R_2 = 6\Omega$$

$$R_{P1} = \frac{R_{S1} R_3}{R_{S1} + R_3} = \frac{6 \cdot 6}{6+6} = 3\Omega$$

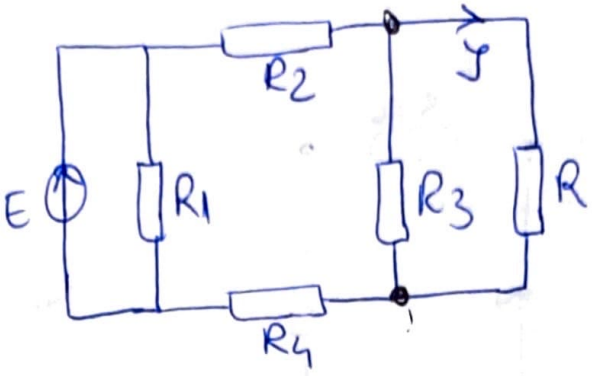
$$R_{S2} = R_5 + R_6 = 7\Omega$$

$$R_{S3} = R_{P1} + R_{S2} = 10\Omega$$

$$R_e = \frac{R_{S3} R_4}{R_{S3} + R_4} = \frac{10 \cdot 10}{10+10} = 5\Omega$$

$$\boxed{R_e = 5\Omega}$$

2.21



$R_{eq}=?$ $U_0=?$ $J=?$

load is R

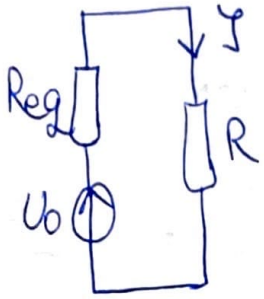
$E=15V$

$R_1=220\Omega$

$R_2=150\Omega$

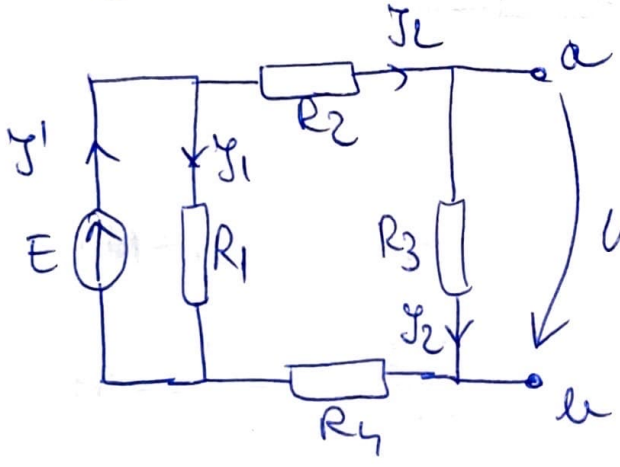
$R_3=R_4=R=100\Omega$

Thevenin equivalent circuit:



*

Firstly, we remove the load (R) and calculate the voltage between the a and b terminals.



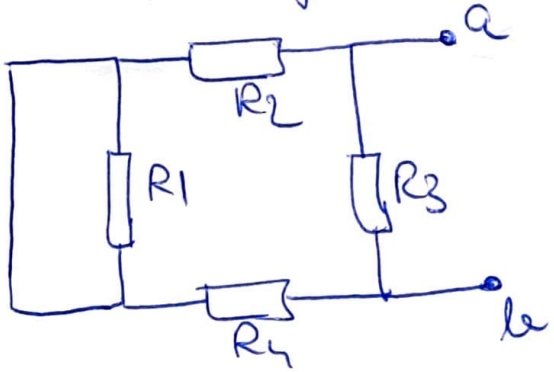
$$E=J_2(R_2+R_3+R_4)$$

$$\Rightarrow J_2=\frac{E}{R_2+R_3+R_4}=\frac{3}{70}A$$

$$U_0=J_2R_3=\frac{3}{70}\cdot 100$$

$$\Rightarrow \boxed{U_0=4.28V}$$

Next, to find R_{eq}, we have to suppress all sources.



$$R_1=R_1+R_2+R_4=470\Omega$$

$$R_{eq}=\frac{R_1R_3}{R_1+R_3}=\frac{470\cdot 100}{470+100}$$

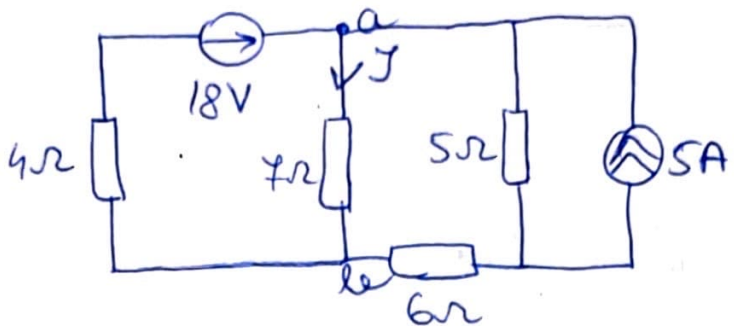
$$\boxed{R_{eq}=82.45\Omega}$$

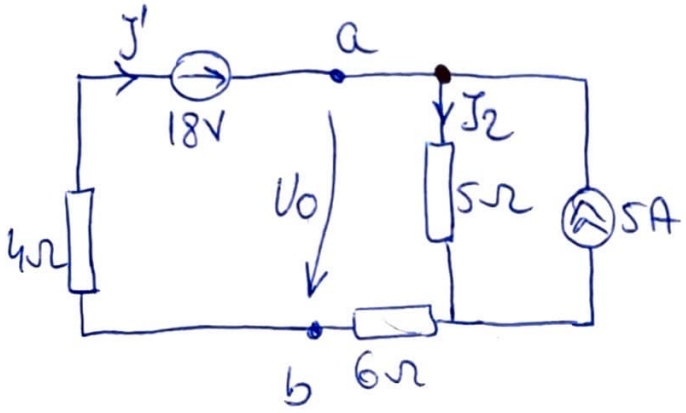
From * $U_0=J(R_{eq}+R) \Rightarrow J=\frac{U_0}{R_{eq}+R} \Rightarrow \boxed{J=0,023A}$

2.22

$R_{eq}=?\ U_0=?\ I=?$

load is 7Ω resistance





$I_2 = I' + 5$

$18 = (4+6)I' + 5I_2$

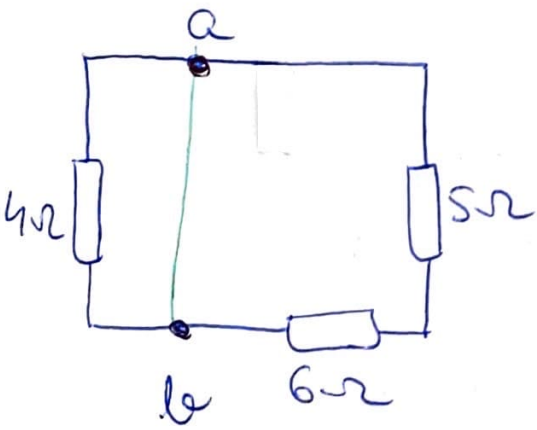
$18 = 10I' + 5(I' + 5)$

$18 = 15I' + 25$

$15I' = -7 \Rightarrow I' = -\frac{7}{15} A$

$18 = 4I' + U_0 \Rightarrow U_0 = 18 - 4I' = 19.86V$

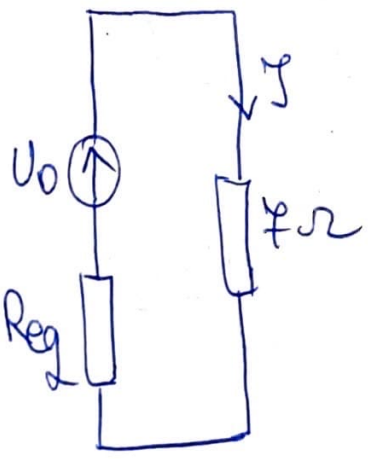
$\Rightarrow \boxed{U_0 = 19.86\ V}$



$R_1 = 5+6 = 11\Omega$

$R_{eq} = \frac{R_1 \cdot 4}{R_1 + 4} = \frac{11 \cdot 4}{11+4}$

$\boxed{R_{eq} = 2.93\Omega}$



$U_0 = I(7 + R_{eq})$

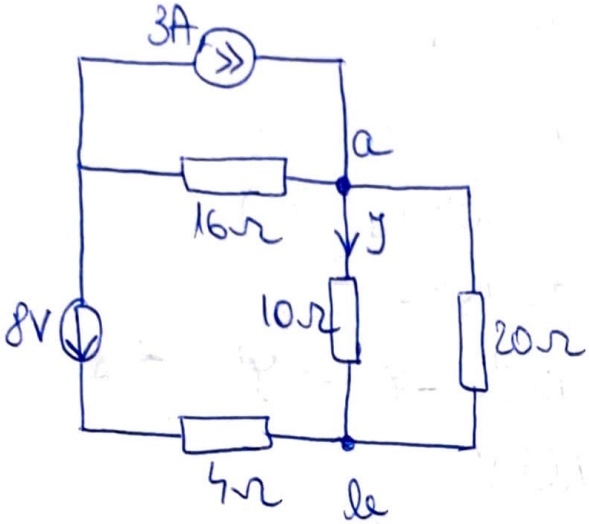
$I = \frac{19.86}{9.93}$

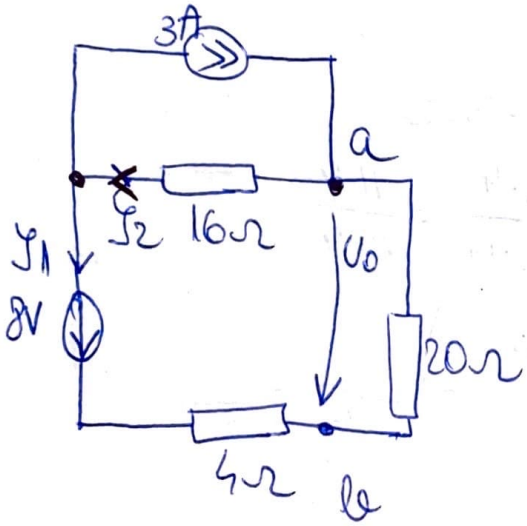
$\boxed{I = 2A}$

2.23

$R_{eq}=?\ U_0=?\ I=?$

load: 10Ω resist.





$I_2 = I_1 + 3$

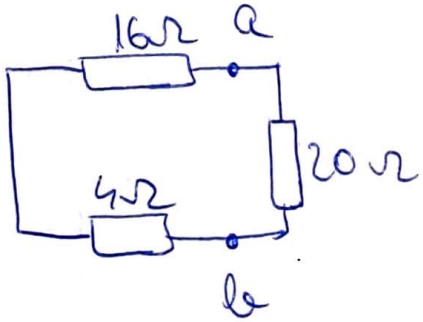
$8 = (4+20)I_1 - 16I_2$

$8 = 24I_1 - 16(I_1+3)$

$8 = 24I_1 - 16I_1 - 48$

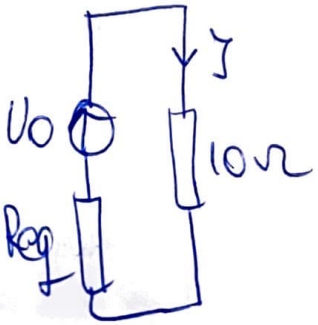
$56 = 8I_1 \Rightarrow I_1 = 7A$

$U_0 = 20I_1 \Rightarrow \boxed{U_0 = 140V}$



$R_3 = 16+4 = 20\Omega$

$R_{eq} = \frac{R_3 \cdot 20}{R_3+20} = \frac{20 \cdot 20}{20+20} \Rightarrow \boxed{R_{eq}=10\Omega}$



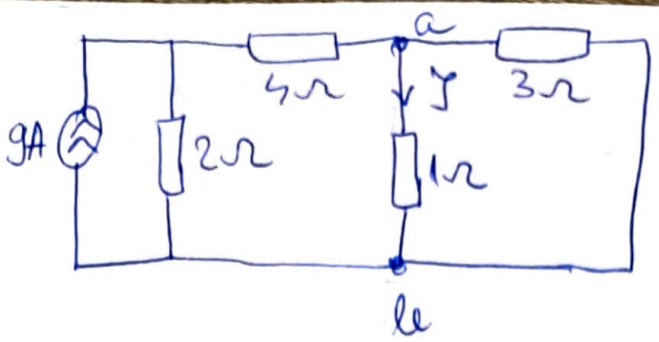
$U_0 = I(10+R_{eq})$

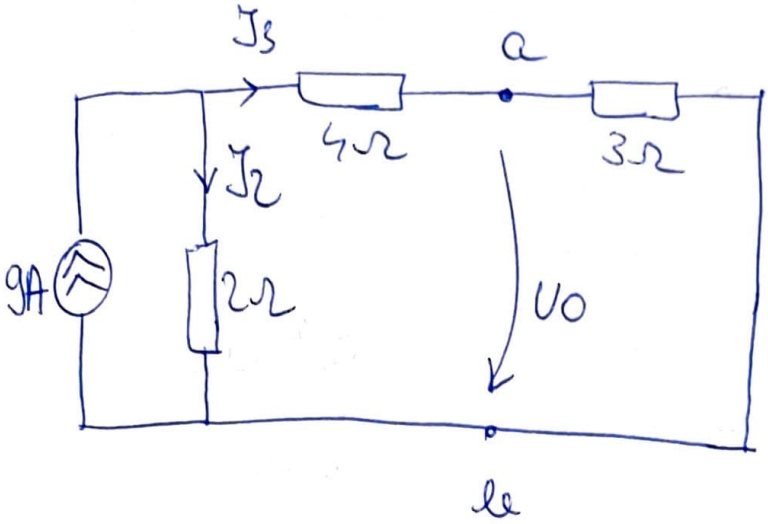
$\boxed{I = 7A}$

2.24

$R_{eq}=?\ U_0=?\ J=?$

by 1rs restrict.





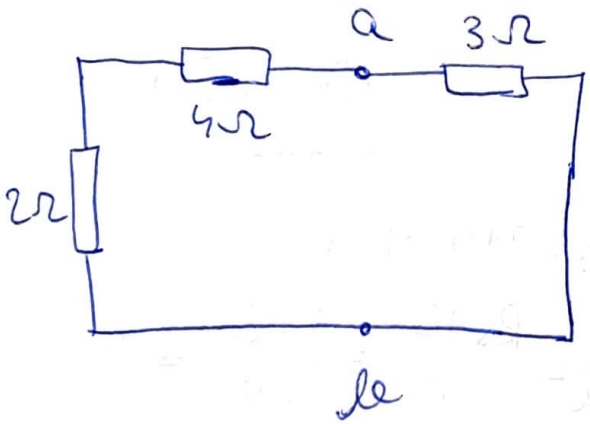
$J_2+J_3=9$

$0=(4+3)J_3-2J_2$

$\begin{cases} 2J_2=7J_3 \\ J_2+J_3=9 \end{cases}$

$J_2=7A\ ;\ J_3=2A$

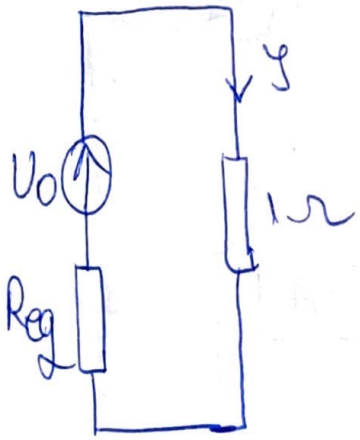
$U_0=3J_3 \Rightarrow \boxed{U_0=6V}$



$R_1=2+4=6\Omega$

$R_{eq}=\dfrac{R_1\cdot 3}{R_1+3}=\dfrac{6\cdot 3}{6+3}=2\Omega$

$\boxed{R_{eq}=2\Omega}$



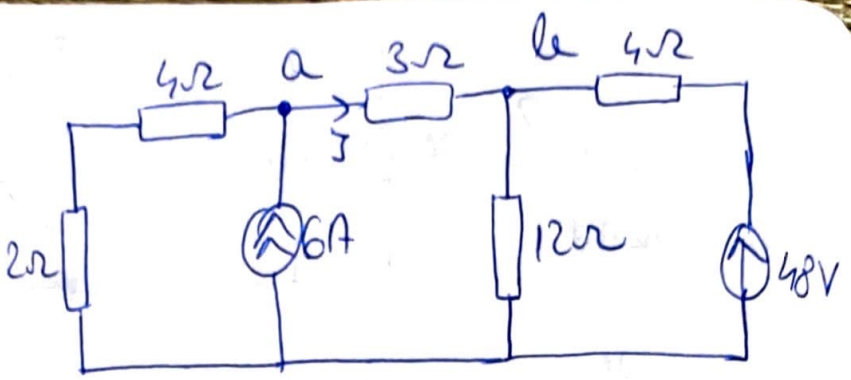
$U_0=J(1+R_{eq})$

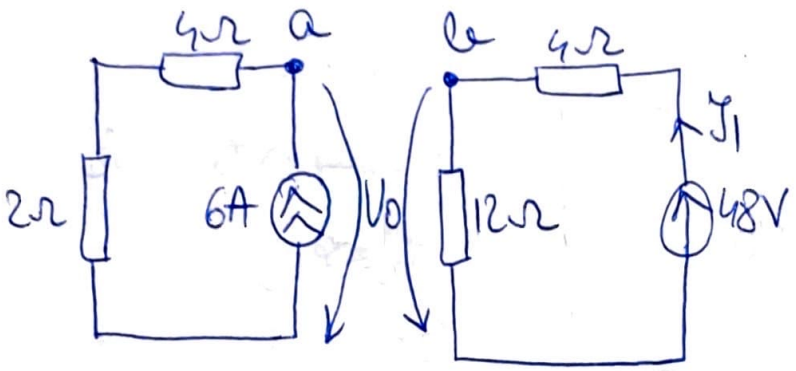
$\boxed{J=2A}$

2.25

R_{eq}=? U_0=? J=?

load: 3Ω resist.





$48 = (12+4) J_1 \Rightarrow J_1 = 3A$

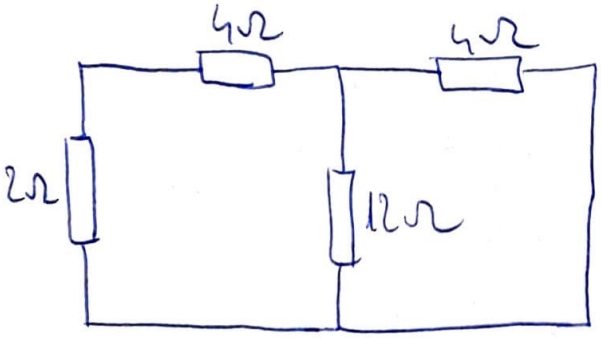
$U_0 = 12 J_1 \Rightarrow U_0 = 36V$

↳ right circuit

$U_0 = (2+4) \cdot 6 = 36V$

↳ left circuit

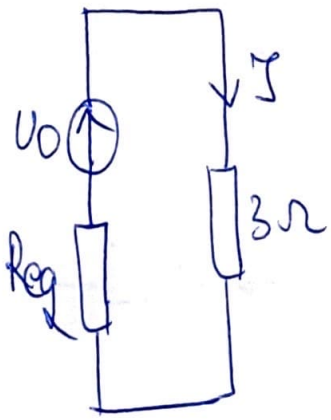
$\boxed{U_0 = 36V}$



$R_S = 2+4 = 6\Omega$

$R_p = \frac{R_S 12}{R_S + 12} = \frac{6 \cdot 12}{6+12} = 4\Omega$

$R_{eq} = R_p + 4 \Rightarrow \boxed{R_{eq} = 8\Omega}$



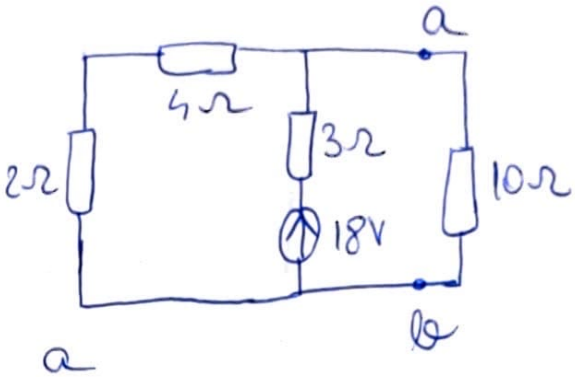
$U_0 = J(3 + R_{eq})$

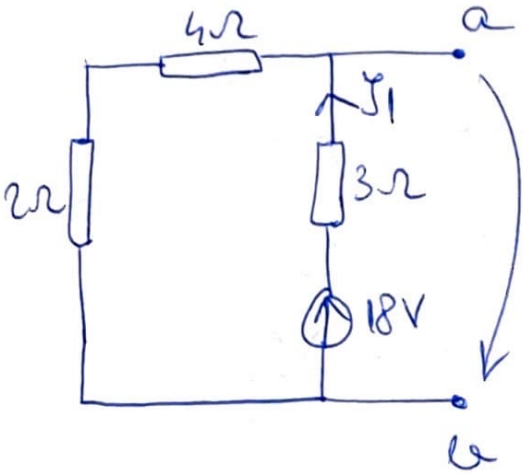
$J = \frac{36}{3+8} \Rightarrow \boxed{J = 3A}$

2.26

$R_{eq}=?\ \ U_0=?\ \ I=?$

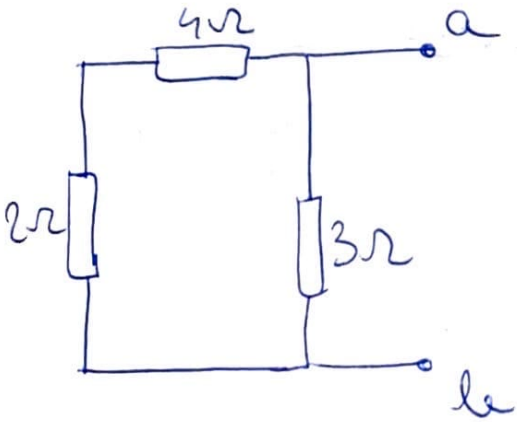
by 10Ω resist.





$18=(4+2+3)I_1 \Rightarrow I_1=2A$

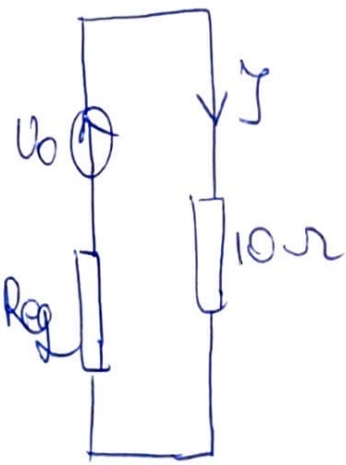
$18=3I_1+U_0 \Rightarrow \boxed{U_0=12V}$



$R_S=2+4=6\Omega$

$R_{eq}=\frac{3R_S}{3+R_S}=\frac{3\cdot 6}{3+6}=2\Omega$

$\Rightarrow \boxed{R_{eq}=2\Omega}$



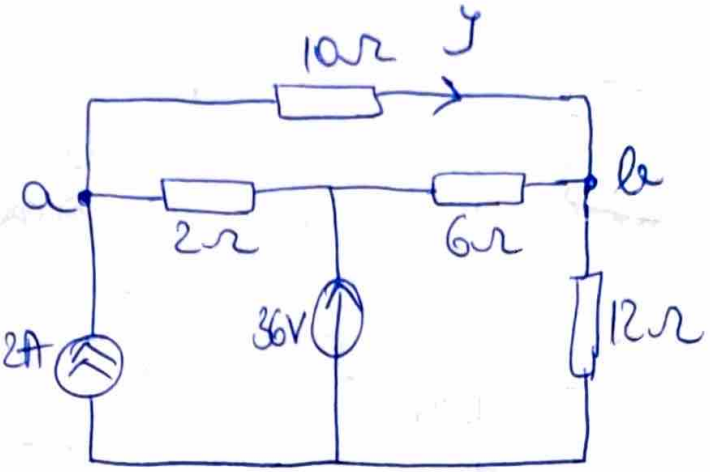
$U_0=I(10+R_{eq})$

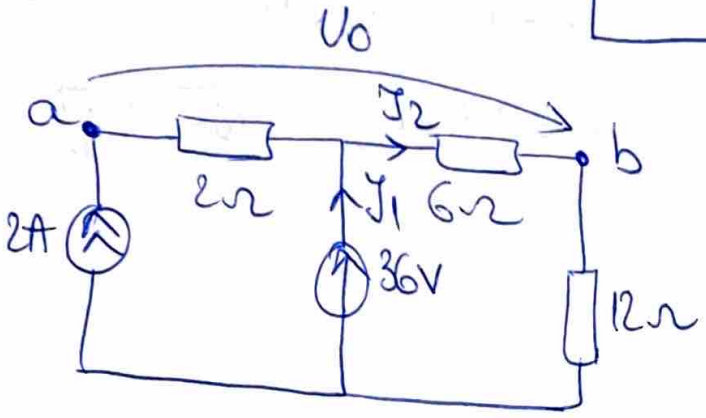
$\boxed{I=1A}$

2.27

Req=? U_0=? J=?

by ions resist.





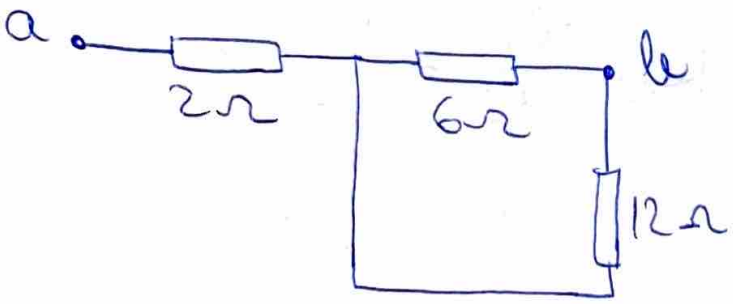
$J_2 = J_1 + 2$

$36 = (6+12) J_2$

$\Rightarrow J_2 = 2A$

$0 = 2 \cdot 2 + 6 J_2 - U_0$

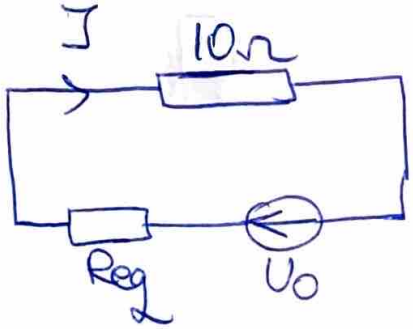
$\Rightarrow \boxed{U_0 = 16V}$



$R_p = \frac{6 \cdot 12}{6+12} = 4\Omega$

$R_{eq} = R_p + 2$

$\boxed{R_{eq} = 6\Omega}$



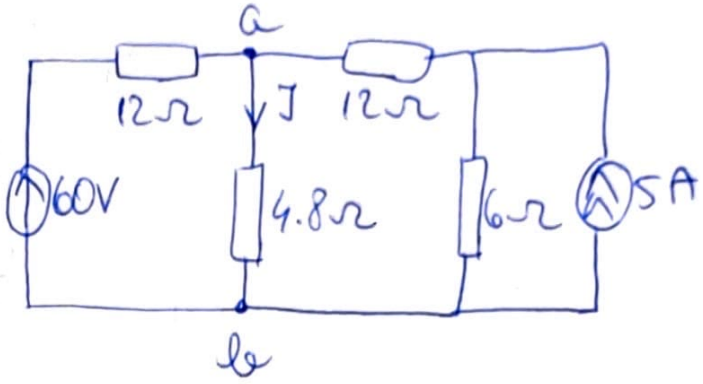
$U_0 = J(10 + R_{eq})$

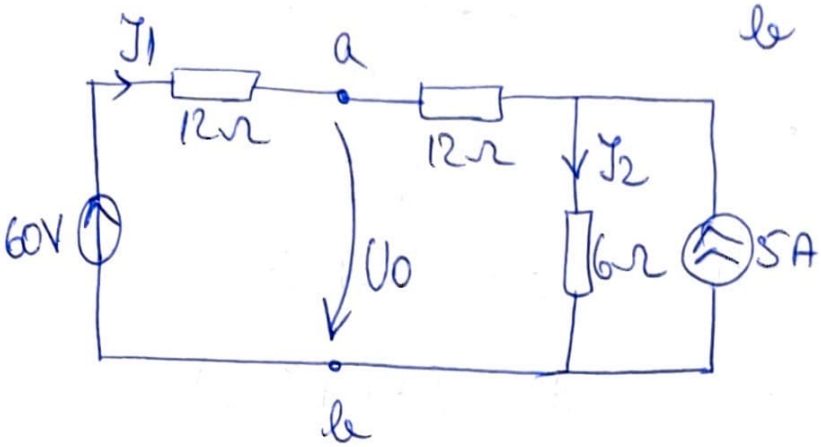
$J = \frac{16}{16} \Rightarrow \boxed{J = 1A}$

2.28

$R_{eq}=?\ U_0=?\ J=?$

load: 4.8Ω resist.





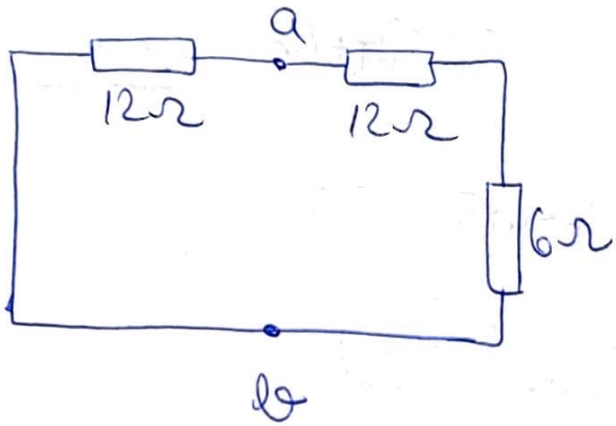
$J_2 = J_1 + 5$

$60 = 24J_1 + 6J_2$

$60 = 24J_1 + 6(J_1 + 5)$

$60 = 30J_1 + 30 \Rightarrow J_1 = 1A$

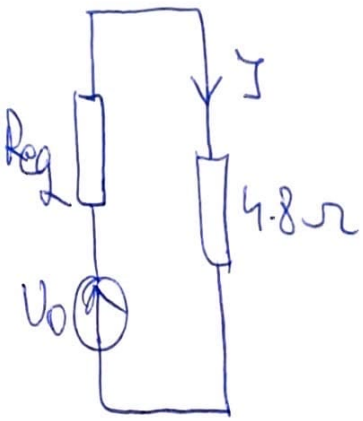
$60 = 12J_1 + U_0 \Rightarrow \boxed{U_0 = 48V}$



$R_S = 12 + 6 = 18\Omega$

$R_{eq} = \frac{12R_S}{12+R_S} = \frac{12 \cdot 18}{12+18}$

$\boxed{R_{eq} = 7.2\Omega}$



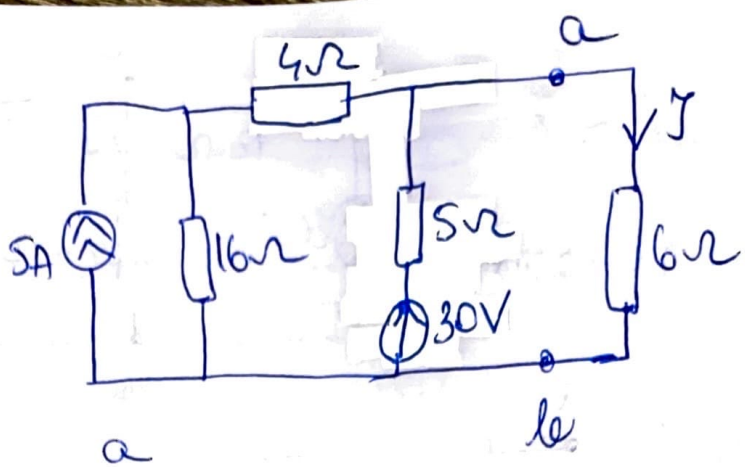
$U_0 = J(4.8 + R_{eq})$

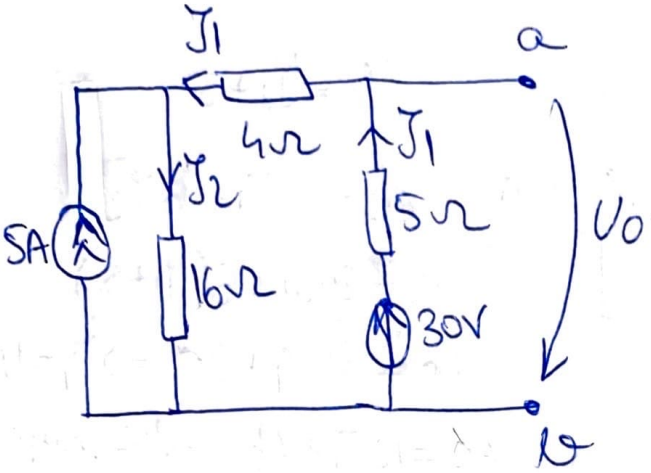
$\boxed{J = 4A}$

2.29

Req=? U_0=? J=?

by 6Ω resist.





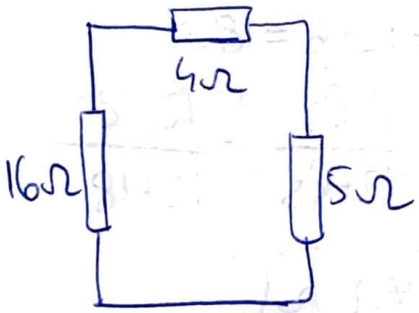
$J_2 = J_1 + 5$

$30 = 9J_1 + 16J_2$

$30 = 9J_1 + 16(J_1 + 5)$

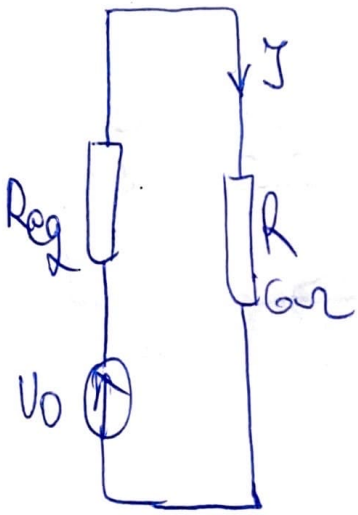
$30 = 25J_1 + 80 \Rightarrow J_1 = -2A$

$30 = 5J_1 + U_0 \Rightarrow$ **$U_0 = 40V$**



$Req = 16 + 4 + 5 = 25\Omega$

$Req = 25\Omega$



$U_0 = J(6 + Req)$

$J = 1.3A$

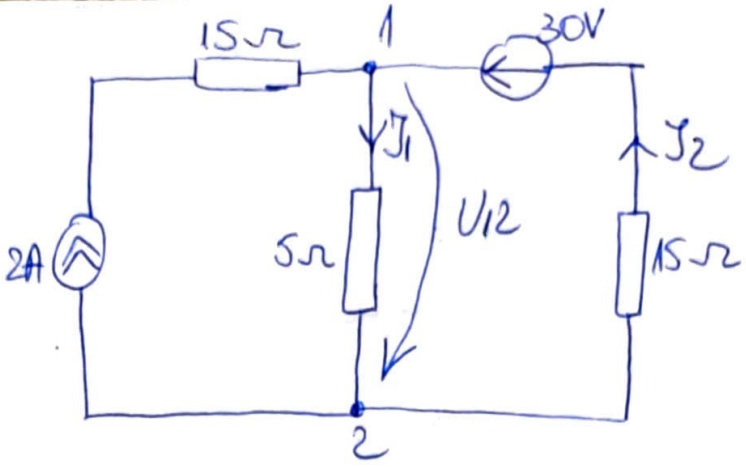
2.30

a) currents

b) $\mathcal{I}_2 = ?$ using Thevenin

$U_{12} = ?$

c) verify power cons. theorem



a) $\mathcal{I}_1 = \mathcal{I}_2 + 2$

$30 = 15\mathcal{I}_2 + 5\mathcal{I}_1$

$30 = 15\mathcal{I}_2 + 5(\mathcal{I}_2 + 2)$

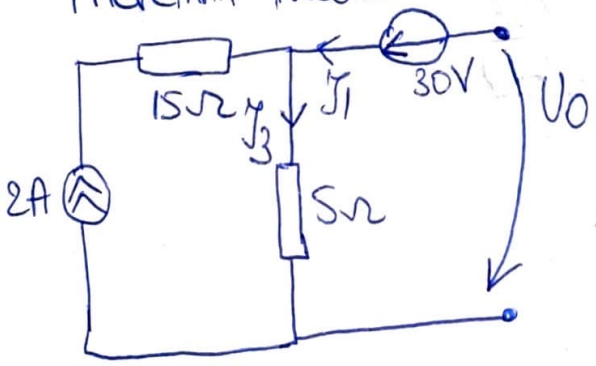
$30 = 20\mathcal{I}_2 + 10$

$20 = 20\mathcal{I}_2 \Rightarrow \boxed{\mathcal{I}_2 = 1A}$

$\boxed{\mathcal{I}_1 = 3A}$

b) $U_{12} = 5\mathcal{I}_1 \Rightarrow \boxed{U_{12} = 15V}$

Thevenin theorem:



$\mathcal{I}_3 = \mathcal{I}_1 + 2$

$5\mathcal{I}_3 = 30 \Rightarrow \mathcal{I}_3 = 6A$

$\mathcal{I}_1 = 4A$

$U_0 = 0$

$R_{eq} = 5\Omega$

2.31

$R_1 = R_4 = 4\Omega$

$R_2 = 2\Omega$

$R_3 = 6\Omega$

$R_5 = 3\Omega$

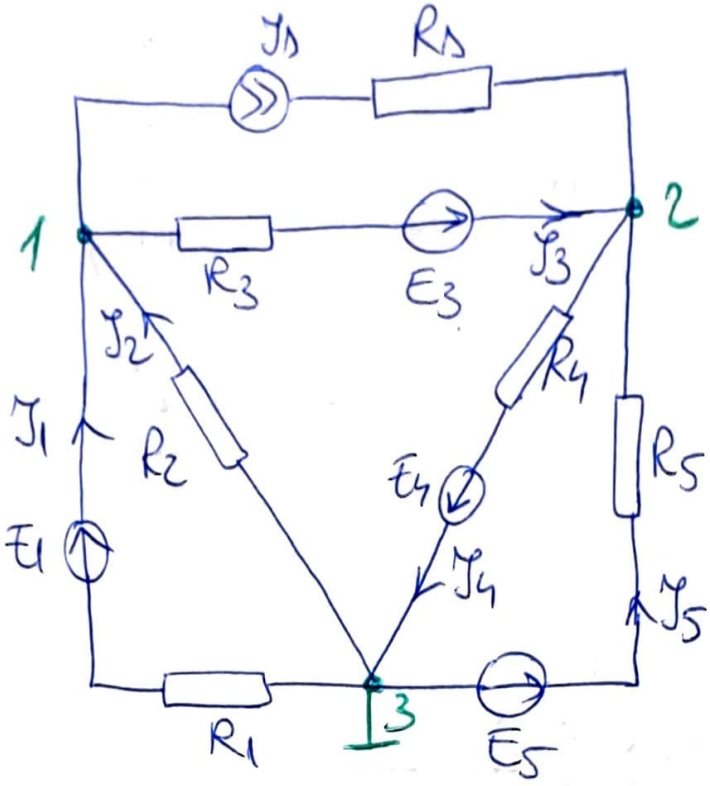
$R_J = 10\Omega$

$E_1 = E_3 = 6V$

$E_4 = 3V$

$E_5 = 4V$

$J_J = 2A$



node 3 = reference node $\Rightarrow \boxed{V_3 = 0V}$

node 1: $V_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - V_2 \cdot \frac{1}{R_3} = E_1\frac{1}{R_1} - E_3\frac{1}{R_3} - J_J$

node 2: $V_2\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) - V_1 \cdot \frac{1}{R_3} = E_3\frac{1}{R_3} + E_5\frac{1}{R_5} - E_4\frac{1}{R_4}$

$$\begin{cases} 11V_1 - 2V_2 = -18 \\ 9V_2 - 2V_1 = 43 \end{cases} \Rightarrow \boxed{\begin{array}{l} V_1 = -0.8V \\ V_2 = 4.6V \end{array}}$$

$V_3 - V_1 = J_1R_1 - E_1 \Rightarrow \boxed{J_1 = 1.7A}$

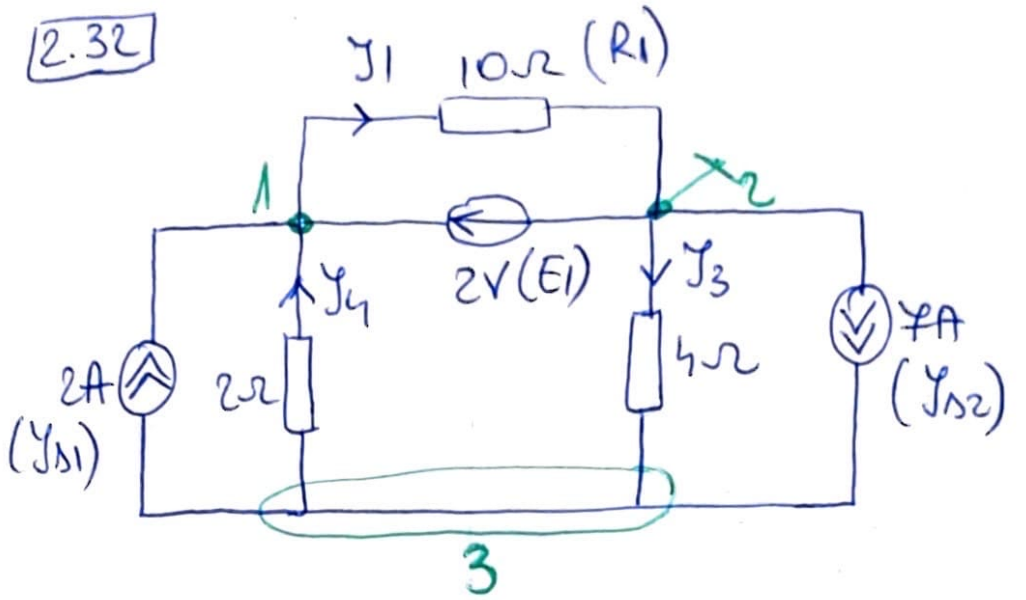
$V_1 - V_2 = J_3R_3 - E_3 \Rightarrow \boxed{J_3 = 0.1A}$

$V_2 - V_3 = J_4R_4 - E_4 \Rightarrow \boxed{J_4 = 1.9A}$

$V_3 - V_2 = J_5R_5 - E_5 \Rightarrow \boxed{J_5 = -0.2A}$

$V_3 - V_1 = J_2R_2 \Rightarrow \boxed{J_2 = 0.4A}$

2.32



Because between 1 and 2 we only have a voltage source, we have a supernode.

Reference node: 2 $\Rightarrow \boxed{V_2 = 0V}$

$V_1 - V_2 = E_1 \Rightarrow \boxed{V_1 = 2V}$

node 3: $V_3\left(\frac{1}{R_2}+\frac{1}{R_3}\right) - V_1 \cdot \frac{1}{R_2} = J_{s2} - J_{s1} \Rightarrow \boxed{V_3 = 8V}$

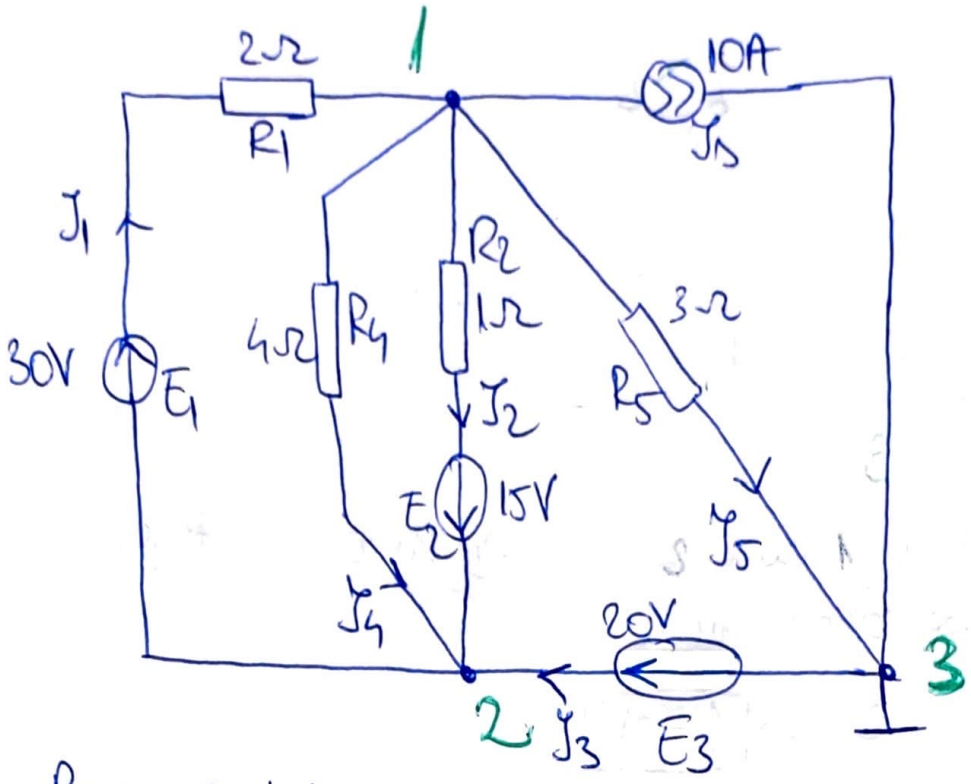
$V_1 - V_2 = J_1 R_1 \Rightarrow \boxed{J_1 = 0.2A}$

$V_3 - V_1 = J_4 R_2 \Rightarrow \boxed{J_4 = 3A}$

$J_{s1} + J_4 = J_1 + J_2 \Rightarrow \boxed{J_2 = 4.8A}$

$J_1 + J_2 = J_3 + J_{s2} \Rightarrow \boxed{J_3 = -2A}$

2.33



Because between 2 and 3 there is no resistance (only a voltage source), we have a supernode.

Reference = node 3 $\Rightarrow \boxed{V_3 = 0V}$

$V_2 - V_3 = E_3 \Rightarrow \boxed{V_2 = 20V}$

node 1: $V_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right) - V_2\left(\frac{1}{R_2}+\frac{1}{R_4}\right)$

$= E_1\frac{1}{R_1} - E_2\cdot\frac{1}{R_2} - J_1$

$\Rightarrow \boxed{V_1 = 7.2V}$

$V_1 - V_2 = J_2R_2 - E_2 \Rightarrow \boxed{J_2 = 2.2A}$

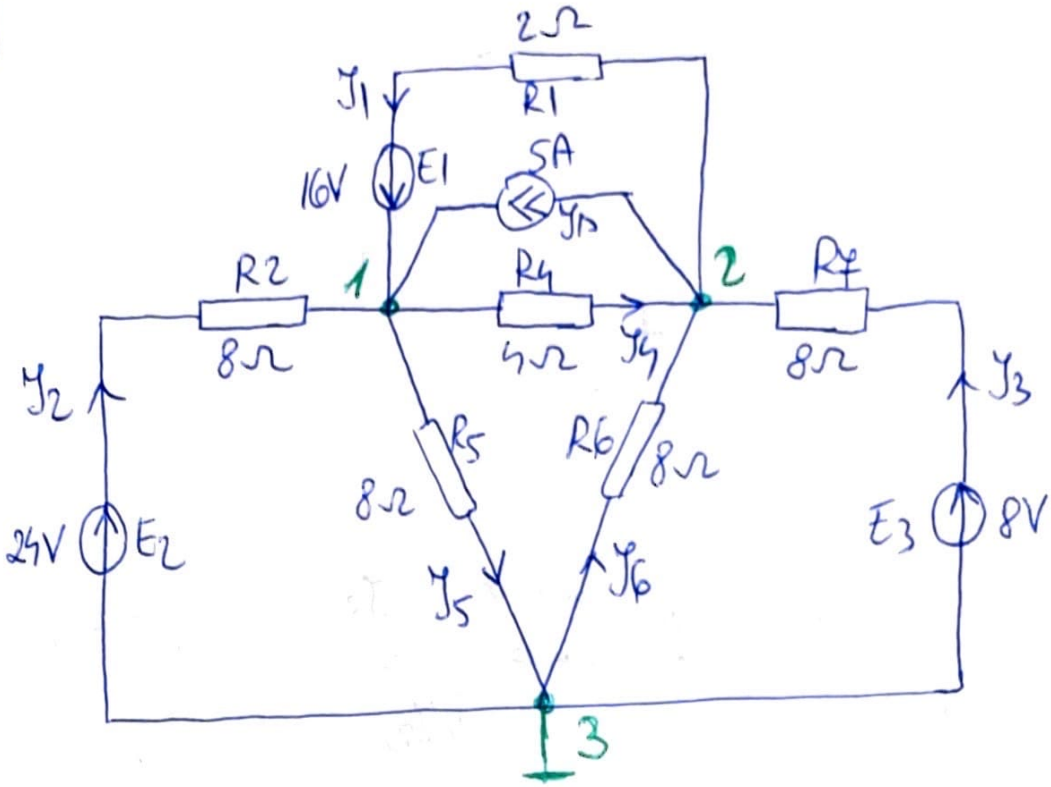
$V_2 - V_1 = J_1R_1 - E_1 \Rightarrow \boxed{J_1 = 21.4A}$

$V_1 - V_3 = J_5R_5 \Rightarrow \boxed{J_5 = 2.4A}$

$J_3 = J_5 + 10 \Rightarrow \boxed{J_3 = 12.4A}$

$J_1 = J_4 + J_2 + J_3 \Rightarrow \boxed{J_4 = 6.8A}$

2.34



Reference node = 3 $\boxed{V_3 = 0V}$

node 1: $V_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right) - V_2\left(\frac{1}{R_1}+\frac{1}{R_4}\right) = E_1\cdot\frac{1}{R_1} + E_2\frac{1}{R_2} + J_5$

node 2: $V_2\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_6}\right) - V_1\left(\frac{1}{R_1}+\frac{1}{R_4}\right) = -E_1\frac{1}{R_1} + E_3\frac{1}{R_3} - J_5$

$$\begin{cases} 12V_1 - 9V_2 = 192 \\ -12V_1 + 16V_2 = -192 \end{cases} \Rightarrow \boxed{V_2 = 0V} \quad \boxed{V_1 = 16V}$$

$V_3 - V_1 = J_2R_2 - E_2 \Rightarrow \boxed{J_2 = 1A}$

$V_2 - V_1 = J_1R_1 - E_1 \Rightarrow \boxed{J_1 = 0A}$

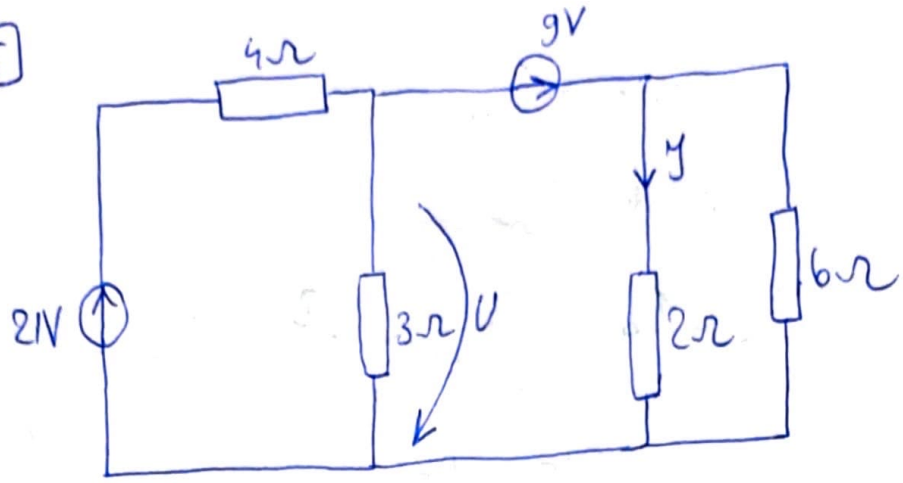
$V_3 - V_2 = J_3R_3 - E_3 \Rightarrow \boxed{J_3 = 1A}$

$V_1 - V_2 = J_4R_4 \Rightarrow \boxed{J_4 = 4A}$

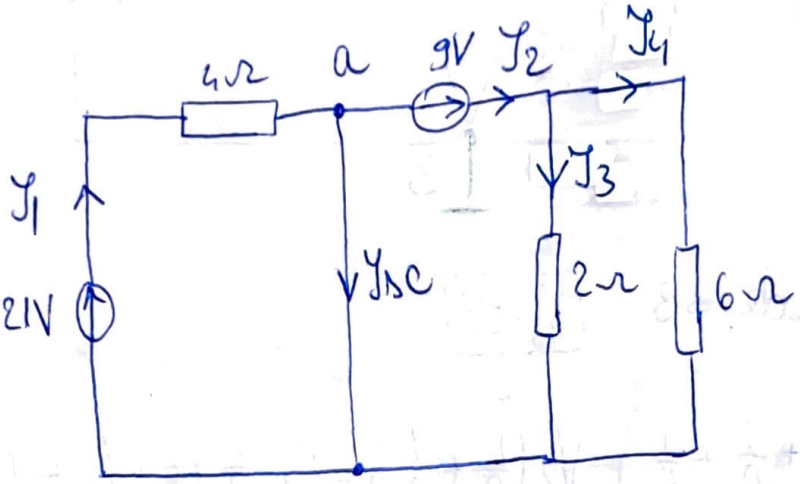
$J_1 + J_2 + J_5 = J_5 + J_4 \Rightarrow \boxed{J_5 = 2A}$

$J_3 + J_4 + J_6 = J_1 + J_5 \Rightarrow \boxed{J_6 = 0A}$

2.35



a)



b

$$\begin{cases} J_2 = J_1 + J_{SC} \\ J_2 = J_3 + J_4 \end{cases}$$

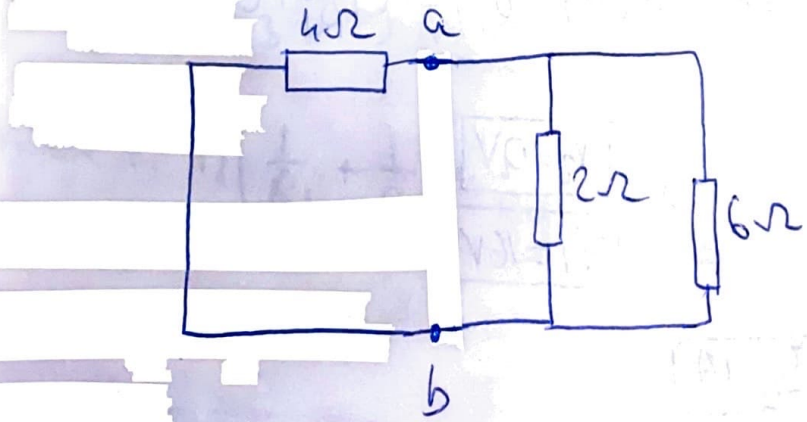
$21 = 4J_1 \Rightarrow J_1 = 5.25A$

$9 = 2J_3 \Rightarrow J_3 = 4.5A$

$2J_3 = 6J_4 \Rightarrow J_4 = 1.5A$

$J_2 = 6A$

$J_{SC} = 0.75A$



$$G_0 = \frac{1}{6} + \frac{1}{2} + \frac{1}{4}$$

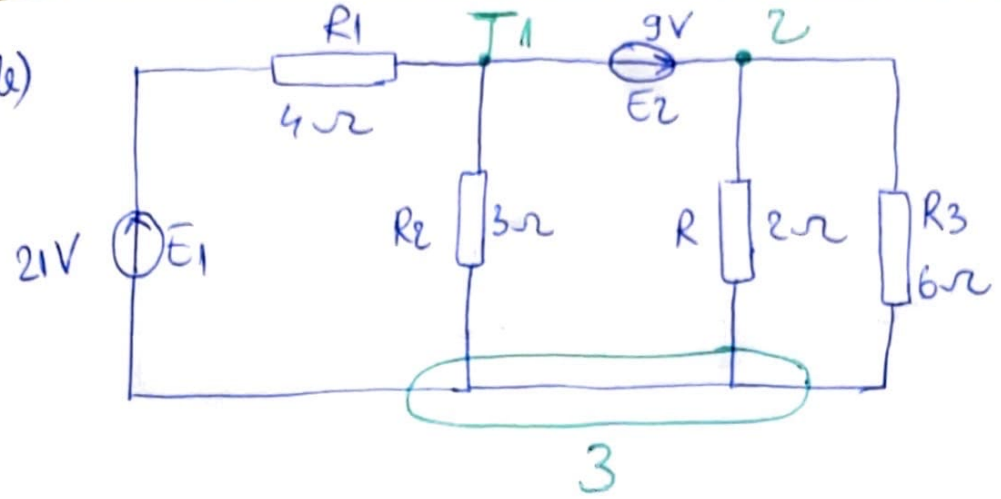
$G_0 = 0.91\,\mho$

$$6 = \frac{1}{R} = \frac{1}{3}$$

$6 = 0{,}33\,\mho$

$$U = \frac{J_{SC}}{6 + 6_0} \Rightarrow U = 0.60V$$

b)



Reference node = 1 $\Rightarrow V_1 = 0V$

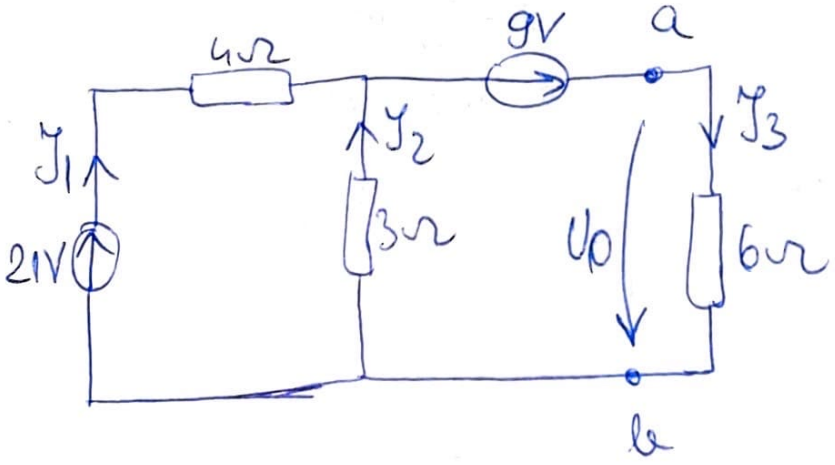
node 3 : $V_3\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R}+\frac{1}{R_3}\right) - V_2\left(\frac{1}{R}+\frac{1}{R_3}\right) = -E_1\frac{1}{R_1}$

$\Rightarrow \boxed{V_3 = 7V}$

$V_2 - V_1 = E \Rightarrow \boxed{V_2 = 21V}$

$U = V_1 - V_3 = -7V \Rightarrow \boxed{U = -7V}$

c) Thèvenin



$J_3 = J_1 + J_2$

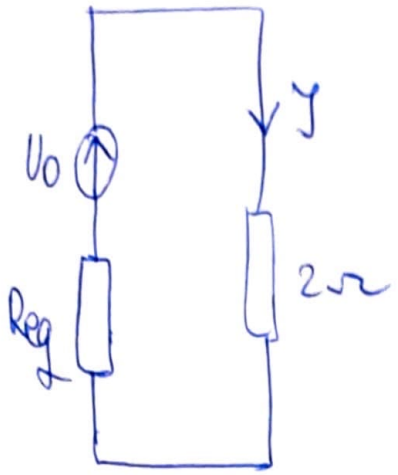
$21 = 4J_1 - 3J_2$

$9 = 6J_3 + 3J_2$

$J_1 = 4A$; $J_2 = -1.66A$; $J_3 = 2.33A$

$U_0 = J_3 R_3 \Rightarrow \boxed{U_0 = 14V}$

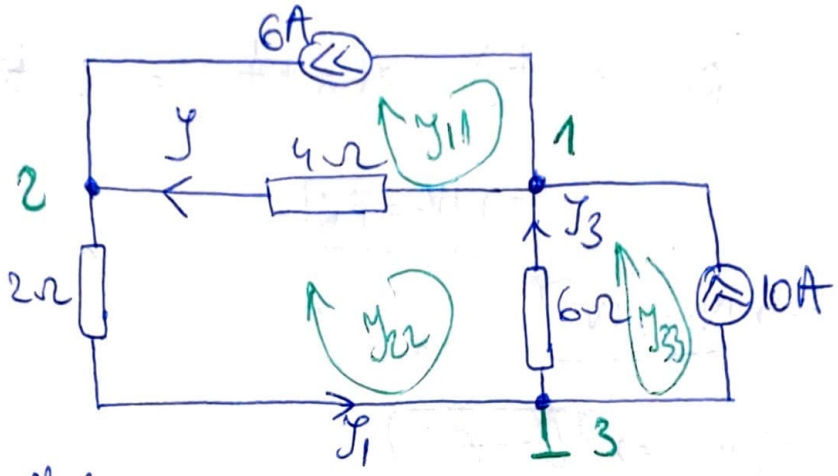
from Norton $\Rightarrow R_{eq} = \frac{1}{60} = \frac{12}{9}\Omega$



$$J = \frac{U_0}{2+R_{eq}}$$

$$\Rightarrow \boxed{J = 4.2A}$$

2.36



a) Kirchhoff laws
b) nodal analysis
c) mesh analysis

verify J by Thevenin
verify power cons. theorem

a) $J+6 = J_3+10$

$J+6 = J_1$

$J_1 = J_3+10$

$J_1R_1 + J_3R_3 + JR_2 = 0$

$\Rightarrow J_1 + 3J_3 + 2J = 0$

$\boxed{J_1 = -4A}$

$\boxed{J = 1A}$

$\boxed{J_3 = -3A}$

b) Node 3 is the ground $\Rightarrow \boxed{V_3 = 0V}$

$$\left.\begin{aligned} V_1\left(\tfrac{1}{4}+\tfrac{1}{6}\right) - V_2 \cdot \tfrac{1}{4} &= 10-6 \\ V_2\left(\tfrac{1}{4}+\tfrac{1}{6}\right) - V_1 \cdot \tfrac{1}{4} &= 6 \end{aligned}\right\} \Rightarrow \boxed{V_1 = 18V}, \ \boxed{V_2 = 14V}$$

$V_1 - V_2 = J R_2 \Rightarrow \boxed{J = 1A}$

$V_1 - V_3 = -J_3 R_3 \Rightarrow \boxed{J_3 = -3A}$

$V_2 - V_3 = J_1 R_1 \Rightarrow \boxed{J_1 = 7A}$

c) $J_{11} = 6A$

$J_{33} = 10A$

$J = J_{11} - J_{22} = J_{22} - 6$

$J_1 = J_{22}$

$J_3 = J_{22} - J_{33} = J_{22} - 10$

$J_1 R_1 + J_3 R_3 + J = 0 \Rightarrow 2J_{22} + 6(J_{22} - 10) + 4(J_{22} - 6) = 0$

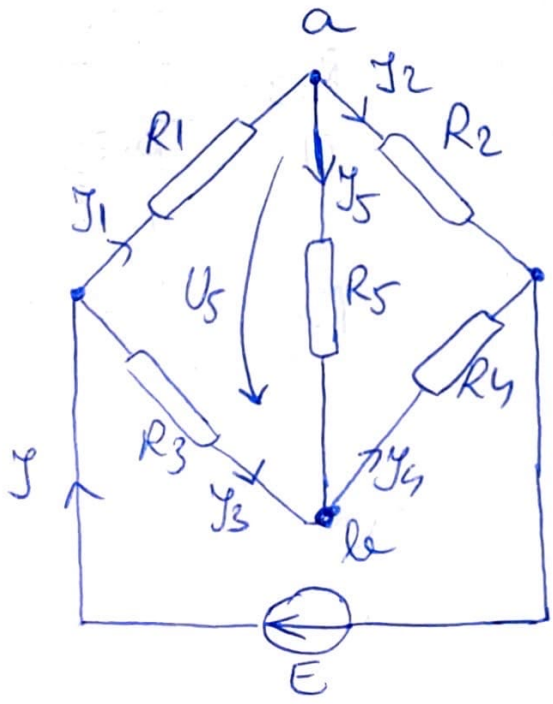
$\Rightarrow J_{22} = 7A$

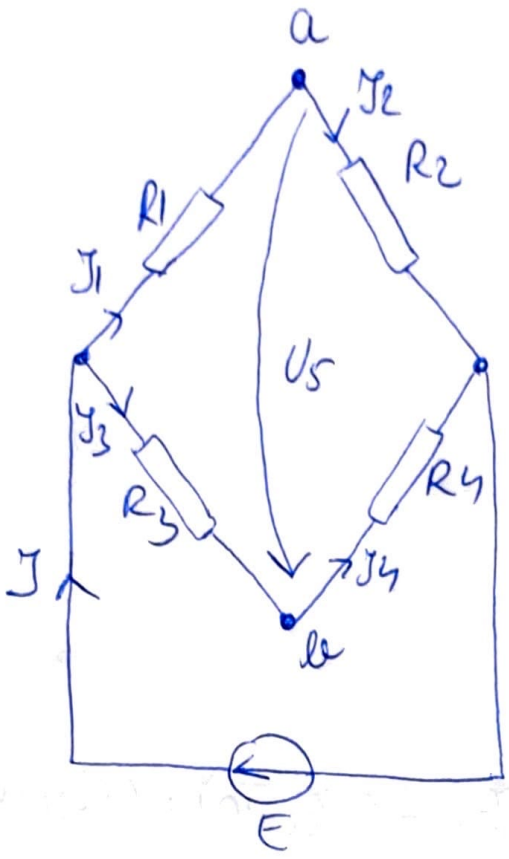
$J_1 = 7A$

$J = 1A$

$J_3 = -3A$

2.37





$$J = J_1 + J_3 \quad \left.\begin{matrix} \\ \end{matrix}\right\} \Rightarrow J = J_1 + J_2 + J_3 + J_4$$
$$J = J_2 + J_4$$

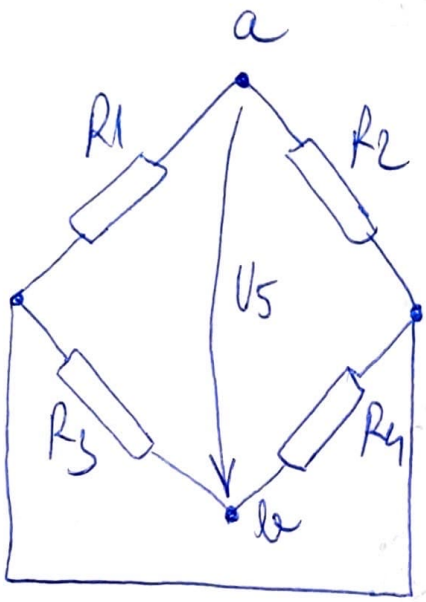
$$E = J_1 R_1 + J_2 R_2$$

$$E = J_3 R_3 + J_4 R_4$$

$$J_1 R_1 + J_2 R_2 = J_3 R_3 + J_4 R_4$$

$$U_5 = J_2 R_2 - J_4 R_4$$

$$U_5 = J_3 R_3 - J_1 R_1$$

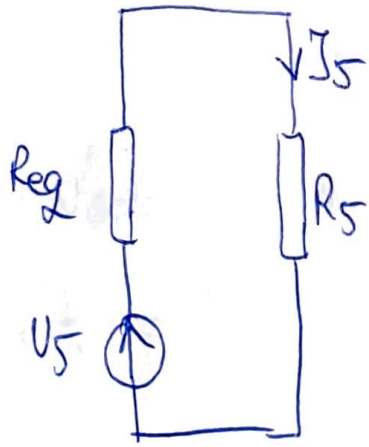


$$R_{p1} = \frac{R_1 R_3}{R_1 + R_3}$$

$$R_{p2} = \frac{R_2 R_4}{R_2 + R_4}$$

$$R_{eq} = \frac{\frac{R_1 R_3}{R_1 + R_3} \cdot \frac{R_2 R_4}{R_2 + R_4}}{\frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}}$$

$$R_{eq} = \frac{R_1 R_2 R_3 R_4}{R_1 R_3 (R_2 + R_4) + R_2 R_4 (R_1 + R_3)}$$

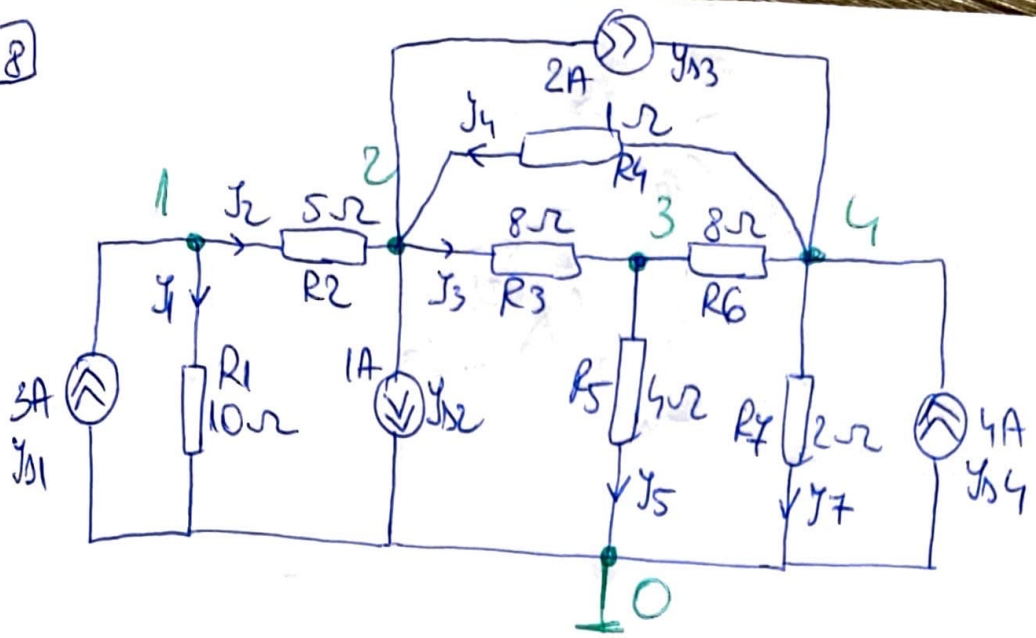


$$U_5 = J_5 (R_{eq} + R_5)$$

$$J_5 = \frac{U_5}{R_{eq} + R_5}$$

$$J_5 = \frac{J_2 R_2 - J_4 R_4}{\frac{R_1 R_2 R_3 R_4}{R_1 R_3 (R_2 + R_4) + R_2 R_4 (R_1 + R_3)} + R_5}$$

2.38



Let node 0 be the ground $\Rightarrow \boxed{V_0 = 0V}$

$$\begin{cases} V_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - V_2 \cdot \frac{1}{R_2} = J_{S1} \\ V_2\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) - V_1 \cdot \frac{1}{R_2} - V_3 \cdot \frac{1}{R_3} - V_4 \cdot \frac{1}{R_4} = -J_{S2} - J_{S3} \\ V_3\left(\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{R_6}\right) - V_2 \cdot \frac{1}{R_3} - V_4 \cdot \frac{1}{R_6} = 0 \\ V_4\left(\frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_7}\right) - V_3 \cdot \frac{1}{R_6} - V_2 \cdot \frac{1}{R_4} = J_{S4} + J_{S3} \end{cases}$$

$V_1 = 12.4\,V$

$V_2 = 3.69\,V$

$V_3 = 2.46\,V$

$V_4 = 6.15\,V$

$V_1 - V_0 = J_1 R_1 \Rightarrow J_1 = 1.24\,A$

$V_1 - V_2 = J_2 R_2 \Rightarrow J_2 = 1.55\,A$

$V_2 - V_3 = J_3 R_3 \Rightarrow J_3 = 0.15\,A$

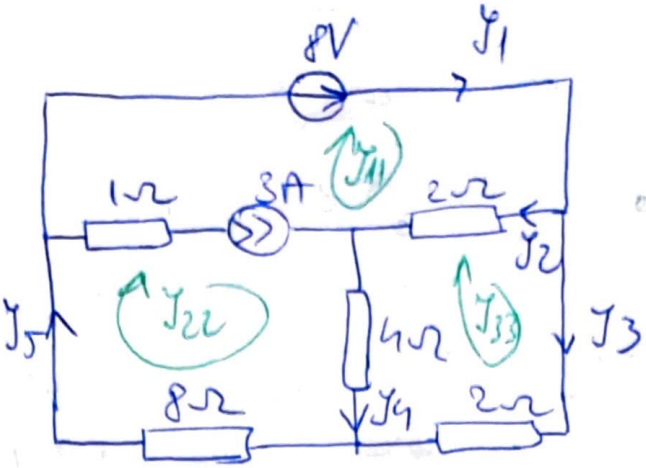
$V_4 - V_2 = J_4 R_4 \Rightarrow J_4 = 2.46\,A$

$V_3 - V_0 = J_5 R_5 \Rightarrow J_5 = 0.61\,A$

$V_4 - V_3 = J_6 R_6 \Rightarrow J_6 = 0.61\,A$

$V_4 - V_0 = J_7 R_7 \Rightarrow J_7 = 3.07\,A$

L.89



$J_1 = J_{11}$

$J_2 = J_{11} - J_{33}$

$J_3 = J_{33}$

$J_4 = J_{22} - J_{33}$

$J_5 = J_{22}$

$8 = 2J_2 - 3 \Rightarrow \boxed{J_2 = 5.5\,A} \Rightarrow J_{11} - J_{33} = 5.5$

$0 = 3 + 4J_4 + 8J_5 \Rightarrow 0 = 3 + 4(J_{22} - J_{33}) + 8J_{22}$

$\Rightarrow 12J_{22} - 4J_{33} = -3$

$0 = 2J_3 - 4J_4 - 2J_2 \Rightarrow 0 = 2J_{33} - 4(J_{22} - J_{33}) - 2\underbrace{(J_{11} - J_{33})}_{5.5}$

$\Rightarrow 6J_{33} - 4J_{22} = 11$

$$\begin{cases} 12J_{22} - 4J_{33} = -3 \\ 6J_{33} - 4J_{22} = 11 \end{cases}$$

$\Rightarrow J_{22} = 0.46\,A$

$J_{33} = 2.14$

$J_{11} = 7.64\,A$

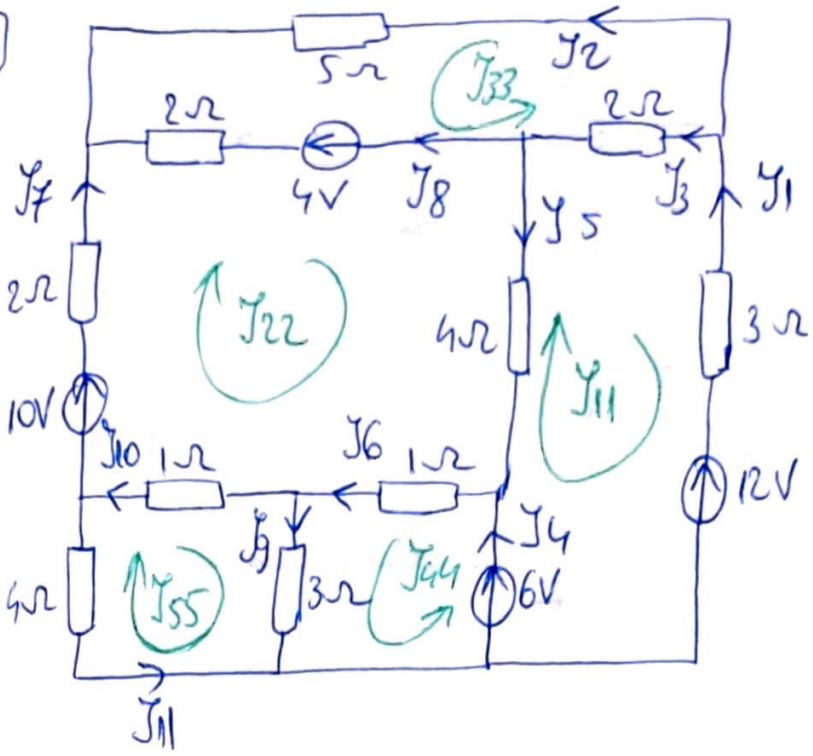
$\boxed{J_3 = 2.14\,A}$

$\boxed{J_4 = -1.67\,A}$

$\boxed{J_5 = 0.46\,A}$

$\boxed{J_1 = 7.64\,A}$

2.40



$J_1 = -J_{11}$
$J_2 = J_{33}$
$J_3 = -J_{11} - J_{33}$
$J_4 = J_{11} + J_{44}$
$J_5 = J_{11} - J_{22}$
$J_6 = J_{22} + J_{44}$
$J_7 = J_{22}$
$J_8 = -J_{22} - J_{33}$
$J_9 = J_{44} + J_{55}$
$J_{10} = J_{22} - J_{55}$
$J_{11} = -J_{55}$

mesh 1: $(4+2+3)J_{11} - 4J_{22} + 2J_{33} = 6 - 12$

$9J_{11} - 4J_{22} + 2J_{33} = -6$

mesh 2: $(1+1+2+2+4)J_{22} - 4J_{11} + 2J_{33} + J_{44} - J_{55} = 10 - 4$

$10J_{22} - 4J_{11} + 2J_{33} + J_{44} - J_{55} = 6$

mesh 3: $(2+2+5)J_{33} + 2J_{11} + 2J_{22} = -4$

$9J_{33} + 2J_{11} + 2J_{22} = -4$

mesh 4: $(3+1)J_{44} + J_{22} + 3J_{55} = 6$

$4J_{44} + J_{22} + 3J_{55} = 6$

mesh 5: $(4+1+3)J_{55} - J_{22} + 3J_{44} = 0$

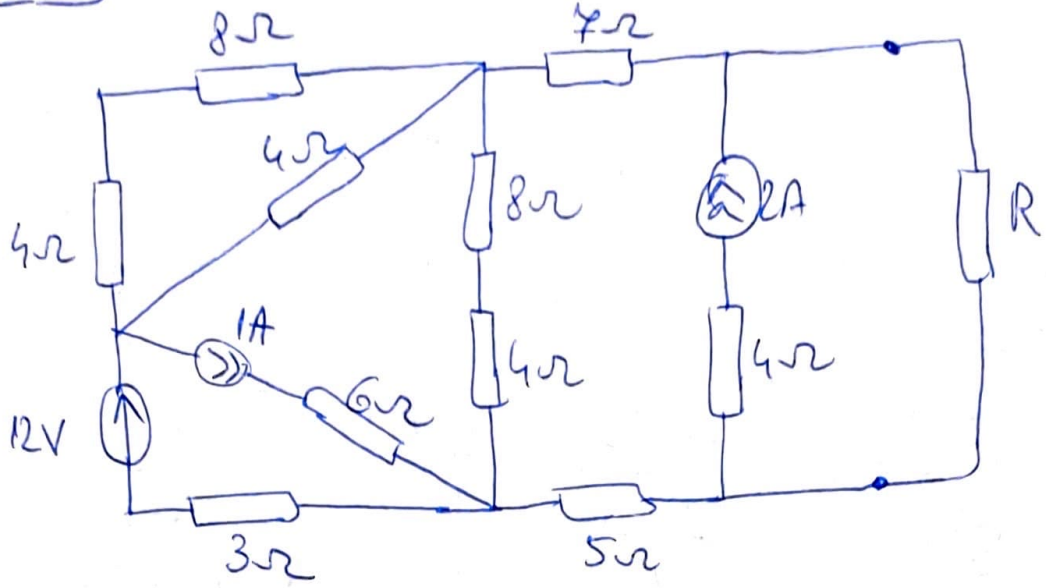
$8J_{55} - J_{22} + 3J_{44} = 0$

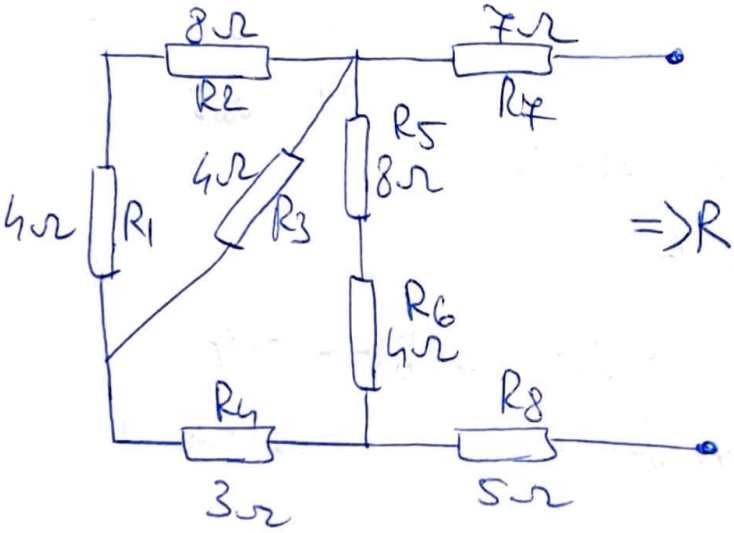
$J_{11} = -0.4\,A$
$J_{22} = 0.2\,A$
$J_{33} = -0.3\,A$
$J_{44} = 1.9\,A$
$J_{55} = -0.7\,A$

$\Rightarrow$

$J_1 = 0.4\,A$
$J_2 = -0.3\,A$
$J_3 = 0.8\,A$
$J_4 = 1.4\,A$
$J_5 = -0.6\,A$
$J_6 = 2.1\,A$
$J_7 = 0.2\,A$
$J_8 = 0.1\,A$
$J_9 = 1.2\,A$
$J_{10} = 0.9\,A$
$J_{11} = 0.7\,A$.

2.41





$\Rightarrow R$

$R_{S1} = R_1 + R_2 = 12\Omega$

$R_{p1} = \frac{R_{S1} R_3}{R_{S1} + R_3} = \frac{12 \cdot 4}{12+4} = 3\Omega$

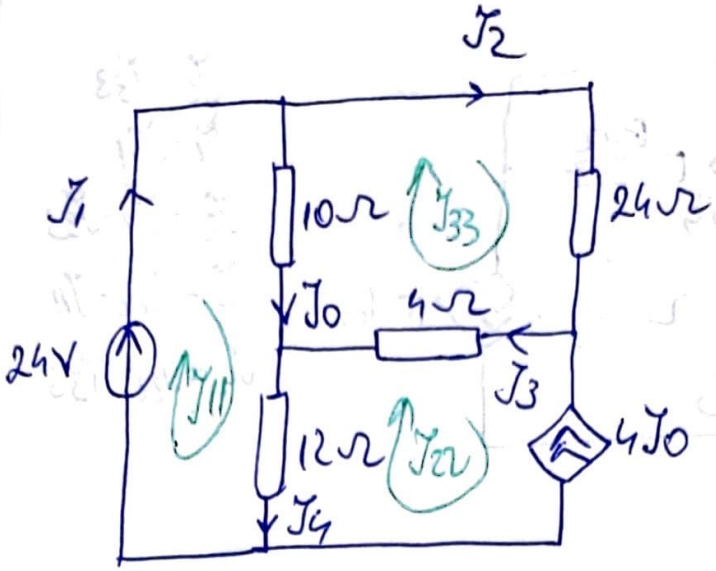
$R_{S2} = R_{p1} + R_4 = 6\Omega$

$R_{S3} = R_5 + R_6 = 12\Omega$

$R_{p2} = \frac{R_{S2} R_{S3}}{R_{S2} + R_{S3}} = \frac{6 \cdot 12}{6+12} = 4\Omega$

$R = R_7 + R_{p2} + R_8 \Rightarrow R = 16\Omega$

2.42



$J_1 = J_{11}$

$J_0 = J_{11} - J_{33}$

$J_2 = J_{33}$

$J_3 = J_{33} + J_{22}$

$J_4 = J_{11} + J_{22}$

1: $24 = 10J_0 + 12J_4$

$24 = 10J_{11} - 10J_{33} + 12J_{11} + 12J_{22}$

$24 = 22J_{11} + 12J_{22} - 10J_{33}$

2: $J_{22} = 4J_{11} - 4J_{33}$

$4J_{11} - J_{22} - J_{33} = 0$

3: $-10J_{11} + 4J_{22} + 38J_{33} = 0$

$J_{11} = 0.2A$

$J_{22} = 1.4A$

$J_{33} = -0.07A$

$J_1 = 0.2A$

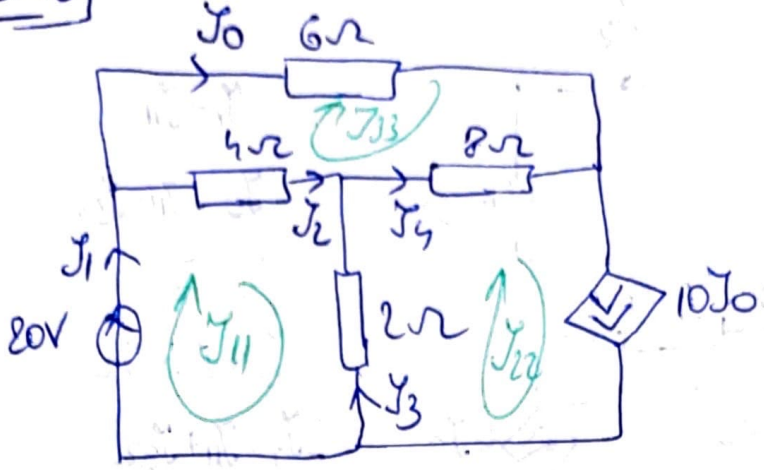
$J_0 = 0.3A$

$J_2 = -0.07A$

$J_3 = 1.3A$

$J_4 = 1.27A$

2.43



$J_0 = J_{33}$

$J_1 = J_{11}$

$J_2 = J_{11} - J_{33}$

$J_3 = J_{22} - J_{11}$

$J_4 = J_{22} - J_{33}$

1: $4J_2 - 2J_3 = 20$

$4(J_{11} - J_{33}) - 2(J_{22} - J_{11}) = 20$

$6J_{11} - 2J_{22} - 4J_{33} = 0$

2: $2J_3 + 8J_4 = 0$

$2(J_{22} - J_{11}) + 8(J_{22} - J_{33}) = 0$

$-2J_{11} + 10J_{22} - 8J_{33} = 0$

3: $6J_0 - 8J_4 - 4J_2 = 0$

$6J_{33} - 8(J_{22} - J_{33}) - 4(J_{11} - J_{33}) = 0$

$-4J_{11} - 8J_{22} + 18J_{33} = 0$

$J_{11} = 6.9\,A$

$J_{22} = 4\,A$

$J_{33} = 3.3\,A$

$\Rightarrow$

$J_0 = 3.3\,A$

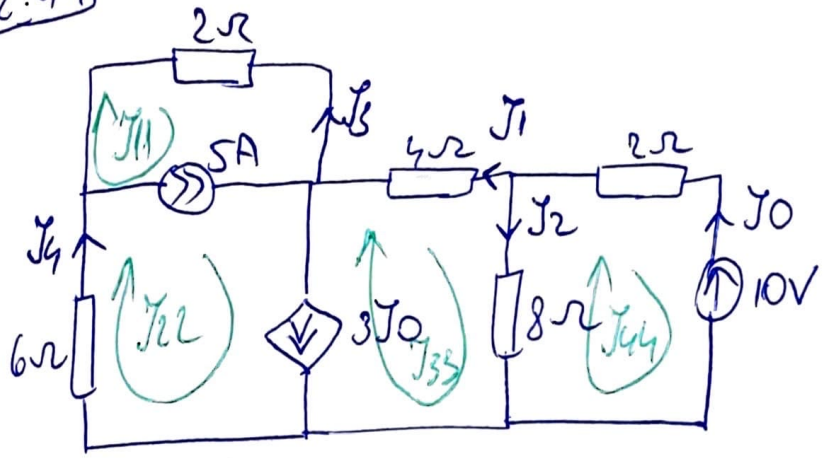
$J_1 = 6.9\,A$

$J_2 = 3.5\,A$

$J_3 = -2.8\,A$

$J_4 = 0.7\,A$

2.44



$J_0 = -J_{44}$

$J_1 = -J_{33}$

$J_2 = J_{33} - J_{44}$

$J_3 = -J_{11}$

$J_4 = J_{22}$

4: $2J_0 + 8J_2 = 10$

$8J_{33} - 10J_{44} = 10$

3: $8J_2 - 4J_1 = 0$

$12J_{33} - 8J_{44} = 0$

2: $J_4 = 0A$

1: $J_3 = 0A$

$J_{22} = 0A$

$J_{11} = 0A$

$J_{33} = 0.4A$

$J_{44} = 0.6A$

$J_1 = -0.4A$

$J_0 = -0.6A$

$J_2 = -0.2A$

AC Proposed Problems

3.1 $A(t) = 10\sin(10\pi t - 45°)$ $\qquad A(t) = A_m \sin(\omega t - \varphi)$

$= 10\sin(10\pi t - \frac{\pi}{4})$

$A_m = 10\ V$ → amplitude

$A = \frac{A_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07\ V$ → rms value

$\omega = 10\pi\ \frac{rad}{s}$ → angular frecquency

$\omega = 2\pi f \Rightarrow 10\pi = 2\pi f \Rightarrow f = 5\ Hz$ → frequency

$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} \Rightarrow T = 0.2\ s$ → period

$\varphi = -45°$ → phase angle

3.2 $u_1(t) = 200\sqrt{2}\sin(5t + 30°)$

$u_2(t) = 50\cos(5t)$

we use: $-\cos\alpha = \sin(\alpha - 90°)$

$\cos\alpha = \sin(90° - \alpha)$

$u_1(t) = 200\sqrt{2}\sin(5t + 30°)$

$u_2(t) = 50\sin(+5t + 90°)$

$\varphi_1 = 30°\ ;\ \varphi_2 = 90°$

$\varphi_{12} = \varphi_1 - \varphi_2 = 30° - 90° = -60°$

$u_1(t)$ lags $u_2(t)$ by the angle of 60°

$u_2(t)$ leads $u_1(t)$ by the angle of 60°

3.3

$i_1(t) = -4\sin(4\pi t + 30°)$

$i_2(t) = -7\cos(4\pi t - 35°)$

$i_1(t) = -4\sin(4\pi t + 30°) = 4\sin(4\pi t + 30° - 180°) = 4\sin(4\pi t - 150°)$

$i_2(t) = 7\sin(4\pi t - 35° - 90°) = 7\sin(4\pi t - 125°)$

$\varphi_1 = 150°$, $\varphi_2 = 125°$

$\varphi_{12} = \varphi_1 - \varphi_2 = 25° > 0 \Rightarrow i_1(t)$ leads $i_2(t)$

3.4

$i_1(t) = 2\sqrt{2}\sin(\omega t + 45°)\,A$

$i_2(t) = 2\sqrt{2}\cos(\omega t - 45°)\,A$

$i(t) = i_1(t) + i_2(t) = ??$

$i_1(t) = 2\sqrt{2}\sin(\omega t + 45°)\,A$

$i_2(t) = 2\sqrt{2}\sin(\omega t - 45° + 90°)\,A$

$i(t) = 4\sqrt{2}\sin(\omega t + 45°)\,A$

3.5

$u_1(t) = 20\sqrt{2}\sin(\omega t + 45°)\,V$

$u_2(t) = -10\sqrt{2}\cos(\omega t - 30°)\,V = 10\sqrt{2}\sin(\omega t - 120°)\,V$

$u(t) = u_1(t) + u_2(t) = ??$

$U_1 = 20\frac{\sqrt{2}}{2} + 20\frac{\sqrt{2}}{2}j = 10\sqrt{2} + 10\sqrt{2}j$

$U_2 = 10\cdot\left(-\frac{1}{2}\right) + 10\cdot\left(-\frac{\sqrt{3}}{2}\right)j = -5 - 5\sqrt{3}j$

$$U = U_1 + U_2 = 5(2\sqrt{2}-1) + 5j(2\sqrt{2}-\sqrt{3})$$

3.6

a) $\underline{U} = -10\, e^{j30^\circ}\ V$

b) $\underline{I} = 2\sqrt{2}(\cos 45^\circ - j\sin 45^\circ)\ A$

c) $\underline{Y} = j(5 - j12)\ A$

a) $u(t) = -10\sqrt{2}\sin(\omega t + 30^\circ)$
$= 10\sqrt{2}\sin(\omega t - 150^\circ)$

b) $i(t) = 2\sin(\omega t - 45^\circ)$

c) $\underline{Y} = 12 + 5j \Rightarrow i(t) = 13\sqrt{2}\sin(\omega t + \operatorname{arctg}\frac{5}{13})$

3.7

$$4i(t) + 8\int i(t)\,dt - 3\frac{di(t)}{dt} = 50\sqrt{2}\sin(2t + 75^\circ)$$

$$\frac{4}{\sqrt{2}}\underline{I} + \frac{8}{\sqrt{2}}\cdot\frac{1}{j}\underline{I} - \frac{3}{\sqrt{2}}2j\cdot\underline{I} = 50\, e^{j75^\circ}$$

$$\underline{I}\left(\frac{5}{\sqrt{2}} - \frac{10}{\sqrt{2}}j\right) = 50\, e^{j75^\circ}$$

$$\underline{I} = \frac{50\, e^{j75^\circ}}{\sqrt{\frac{125}{2}}\, e^{\operatorname{arctg}2}} = \frac{10\sqrt{2}}{\sqrt{5}}\, e^{j(75^\circ + \operatorname{arctg}2)}$$

$$\underline{I} = 5e^{j120^\circ} \Rightarrow i(t) = 5\sqrt{2}\sin(2t + 120^\circ)$$

3.8 $2\sqrt{2}\frac{du(t)}{dt} + 5\sqrt{2}\,u(t) + 10\sqrt{2}\int u(t)\,dt = 50\sqrt{2}\sin(5t-30^\circ)$

$2.5j\,\underline{U} + 5\,\underline{U} + 10\cdot\frac{1}{5j}\,\underline{U} = 50e^{-j30^\circ}$

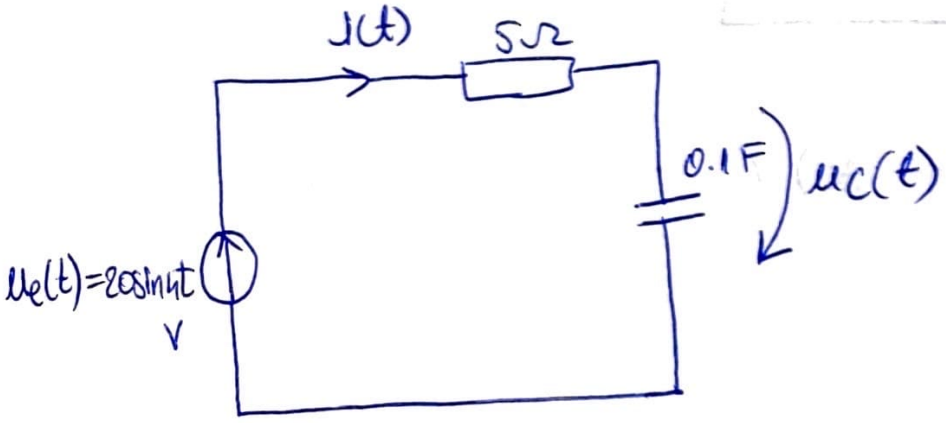
$\underline{U}(5+8j) = 50e^{-j30^\circ}$

$\underline{U} = \frac{50e^{-j30^\circ}}{\sqrt{89}\cdot e^{j\,\mathrm{arctg}\frac{8}{5}}} \Rightarrow u(t) = \frac{50\sqrt{2}}{\sqrt{80}}\sin(5t-30^\circ+\mathrm{arctg}\frac{8}{5})$

3.9

$J(t) = ?$

$u_C(t) = ?$



$u_e(t) = 20\sin 4t \Rightarrow \omega = 4\,\frac{rad}{s}$

$X_C = \frac{1}{\omega C} = \frac{1}{4\cdot 0.1} = 2.5\,\Omega$

$z = \sqrt{R^2 + (-X_C)^2} = \sqrt{5^2 + (-2.5)^2} = \frac{5\sqrt{5}}{2}\,\Omega = 5.6\,\Omega$

$tg\,\varphi = -\frac{X_C}{R} = -\frac{1}{2} \Rightarrow \varphi = \mathrm{arctg}(-\frac{1}{2})$

$$i(t) = \frac{8\sqrt{5}}{5} \sin\left(4t + \operatorname{arctg}\left(-\frac{1}{2}\right)\right) A$$

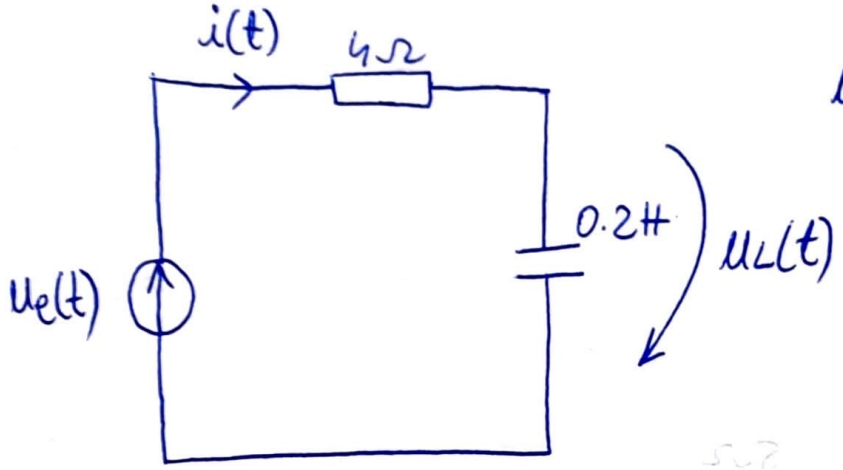
$$u_{Cm} = i_m \cdot X_C = \frac{8\sqrt{5}}{5} \cdot \frac{5}{2} = 4\sqrt{5} \cong 9\,V$$

$$u_C(t) = 4\sqrt{5} \sin\left(4t + \operatorname{arctg}\left(-\frac{1}{2}\right) - \frac{\pi}{2}\right)$$

3.10

$i(t) = ?$

$u_L(t) = ?$



$$u_e(t) = 20\cos(10t - 30)\,V$$

$$\omega = 10\,\frac{rad}{s}$$

$$X_L = \omega L = 10 \cdot 0.2 \Rightarrow X_L = 2\Omega$$

$$z = \sqrt{R^2 + X_L^2} = \sqrt{4^2 + 2^2} = \sqrt{20} \Rightarrow z = \sqrt{20}\Omega = 2\sqrt{5}\Omega$$

$$tg\,\varphi = \frac{X_L}{R} = \frac{1}{2} \Rightarrow \varphi = \operatorname{arctg}\left(\frac{1}{2}\right)$$

$$u_m = z \cdot i_m \Rightarrow i_m = \frac{20}{2\sqrt{5}} = \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}\,A$$

$$i(t) = 2\sqrt{5} \sin\left(10t + \operatorname{arctg}\left(\frac{1}{2}\right) - 90^\circ\right) A$$

$$u_{Lm} = i_m \cdot X_L = 2\sqrt{5} \cdot 2 = 4\sqrt{5}$$

$u_L(t) = 4\sqrt{5}\sin(10t + 123°)\ V$

3.11

sinusoidal current

at $t=0$, $i(0) = 5\ A$

$t_1 = 2.5\ ms$, peak value

$T = 20\ ms$

- $\omega = \frac{2\pi}{T} = \frac{2\pi}{20} = \frac{\pi}{10}$

a) $\varphi = ?$

b) $i(t) = ?$

- we will consider a sinusoidal current in the form:

$i(t) = I_m \sin(\omega t + \varphi)$

- peak value means $\sin(\omega t_1 + \varphi) = 1$, at $t_1 = 2.5\ ms$

$\omega t_1 + \varphi = \frac{\pi}{2} \Rightarrow \varphi = \frac{\pi}{2} - \omega t_1 = \frac{\pi}{2} - \frac{\pi}{10} t_1 = \frac{5}{2}\pi - \frac{2.5\pi}{10}$

$\Rightarrow \varphi = \frac{2.5\pi}{10} = \frac{\pi}{4} \Rightarrow \boxed{\varphi = 45°}$

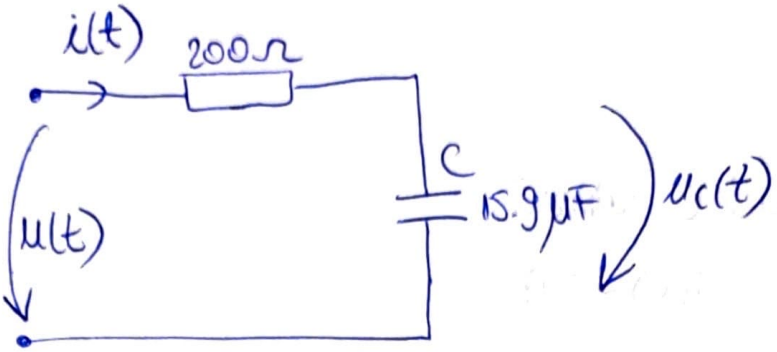
- at $t=0$, $i(0) = 5A$

$\Rightarrow 5 = I_m \sin\varphi \Rightarrow 5 = I_m \cdot \frac{\sqrt{2}}{2} \Rightarrow I_m = \frac{5 \cdot 2}{\sqrt{2}} = \frac{5 \cdot 2 \cdot \sqrt{2}}{2}$

$I_m = 5\sqrt{2}\ A$

$\boxed{i(t) = 5\sqrt{2}\sin(314t + 45°)\ A}$

3.12



$f = 50\ Hz$

$I_m = 2.82\ A$

$R = 200\ \Omega$

$C = 15.9\ \mu F$

$\varphi_i = 0$

a) $u(t) = ?$

b) $u_C(t) = ?$

circuit impedance:

$$Z = \sqrt{R^2 + \left(-\frac{1}{\omega C}\right)^2}$$

$\omega = 2\pi f \Rightarrow \omega = 100\pi \frac{rad}{s}$

$$Z = \sqrt{200^2 + \left(-\frac{1}{100\pi \cdot 15.9 \cdot 10^{-6}}\right)^2} = 283\ \Omega$$

$U_m = Z \cdot I_m \Rightarrow U_m = 283 \cdot 2{,}82 = 798\ V$

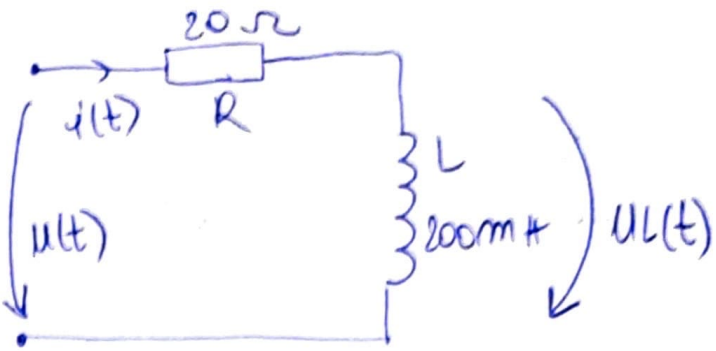
$$tg\varphi = \frac{-X_C}{R} = \frac{-200}{200} \Rightarrow tg\varphi = -1 \Rightarrow \varphi = -45°$$

$$u(t) = 798 \sin\left(100\pi t - \frac{\pi}{4}\right) V$$

$$U_{Cm} = I_m \cdot \frac{1}{\omega C} = I_m \cdot X_C = 2{,}82 \cdot 200 = 564\ V$$

$$u_C(t) = 564 \sin\left(100\pi t - \frac{\pi}{2}\right)$$

3.13



$R = 20\,\Omega$

$L = 200\,mH$

$u_L(t) = 200\sin(314t - 60^\circ)$

a) $i(t) = ?$

b) $u(t) = ?$

$U_{Lm} = 200V$

$\omega = 100\pi \frac{rad}{s} = 314 \frac{rad}{s}$

$U_{Lm} = I_m \cdot X_L$

$X_L = \omega L = 100\pi \cdot 200 \cdot 10^{-3} \Rightarrow X_L = 20\pi\,\Omega$

$I_m = \frac{U_{Lm}}{X_L} = \frac{200}{20\pi} = \frac{10}{\pi} = 3.2\,A$

$i(t) = 3.2\sin(100\pi t - 60^\circ - 90^\circ)\,A$

$= 3.2\sin(100\pi t - 150^\circ)\,A$

$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + (20\pi)^2} = 66\,\Omega$

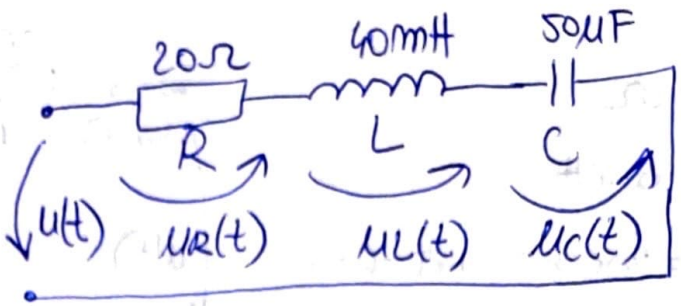
$U_m = Z \cdot I_m = 66 \cdot 3{,}2 = 211.2\,V$

$tg\,\varphi = \frac{X_L}{R} = \frac{20\pi}{20} \Rightarrow tg\,\varphi = \pi \Rightarrow \varphi = arctg(\pi) = 72^\circ$

$u(t) = 211.2\sin(100\pi t - 60^\circ + 72^\circ)\,V$

$= 211.2\sin(100\pi t + 12^\circ)\,V$

3.14



$R=20\Omega$

$L=40mH$

$C=50\mu F$

$u(t)=200\sin(1000t-30°)V$

a) $X_L, X_C, Z = ?$

b) $I=?\ i(t)=?$

c) $U_R=?\ u_R(t)=?$

d) $U_L=?\ u_L(t)=?$

e) $U_C=?\ u_C(t)=?$

a) $\omega = 1000 \frac{rad}{s}$

$X_L = \omega L = 1000 \cdot 40 \cdot 10^{-3} = 40\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{1000 \cdot 50 \cdot 10^{-6}} = 20\Omega$

$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{20^2+20^2}$

$= 20\sqrt{2}\,\Omega = 28.3\Omega$

b) $I = \frac{\frac{U_m}{\sqrt{2}}}{Z} = \frac{\frac{200}{\sqrt{2}}}{20\sqrt{2}} \Rightarrow I = 5A$

$tg\,\varphi = \frac{X_L - X_C}{R} = 1 \Rightarrow \varphi = arctg(1) = \frac{\pi}{4}$

because $X_L > X_C$ circ. behaves inductively, so the current lags the voltage by $\varphi = \frac{\pi}{4}$

$i(t) = 5\sqrt{2}\sin(1000t - 30° - 45°) = 5\sqrt{2}\sin(1000t - 75°)\ A$

c) $U_R = I \cdot R = 100\ V$

$u_R(t) = 100\sqrt{2}\sin(1000t - 75°)\ V$

d) $U_L = X_L \cdot I = 200\ V$

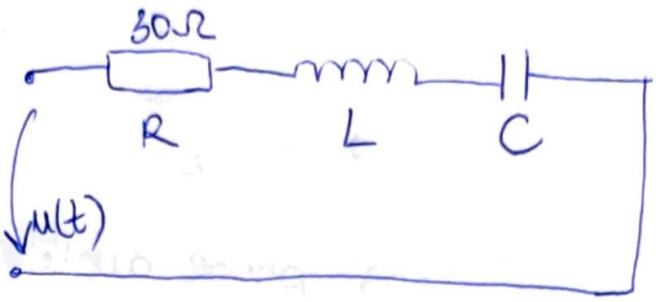
$u_L(t) = 200\sqrt{2}\sin(1000t - 75° + 90°)\ V$

$= 200\sqrt{2}\sin(1000t + 15°)\ V$

e) $U_C = X_C \cdot I = 100\ V$

$u_C(t) = 100\sqrt{2}\sin(1000t - 75° - 90°) = 100\sqrt{2}\sin(1000t - 165°)$ ✓

3.15



$R = 30\Omega$

$X_L = \omega L = 80\Omega$

$X_C = \frac{1}{\omega C} = 40\Omega$

$u(t) = 100\sqrt{2}\sin(100\pi t - 60°)\,V$

$\omega = 100\pi \frac{rad}{s}$

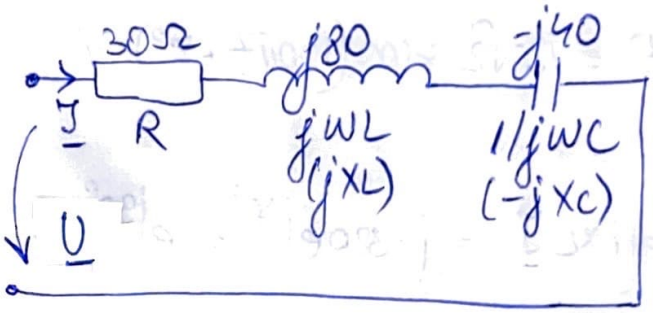
a) $\underline{Z} = ?$ $Z = ?$

b) $\underline{Y} = ?$ $i(t) = ?$

c) $P = ?$ $Q = ?$ $S = ?$

verify cons. of active, reactive, power

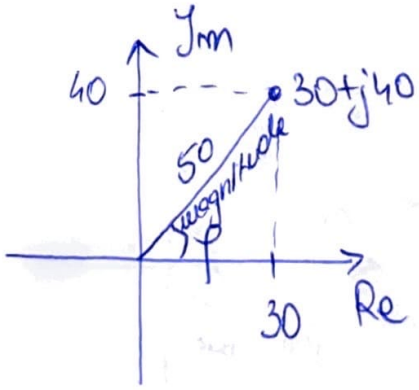
circuit in phasor form:



b) $\underline{U} = 100 e^{j60°}$

$\underline{U} = \underline{I}R + jX_L\underline{I} - jX_C\underline{I} \Rightarrow \underline{Y} = \frac{\underline{U}}{R + j(X_L - X_C)} = \frac{\underline{U}}{\underline{Z}}$

$\underline{Y} = \frac{100 e^{j60°}}{30 + j40}$



$tg\,\varphi = \frac{40}{30}$

$\varphi = arctg\left(\frac{4}{3}\right) = 53°$

so $30 + j40 = 50 e^{j53°}$

$\underline{Y} = \frac{100 e^{j60°}}{50 e^{j53°}} = 4 e^{j7°} \Rightarrow Y = 4\,A$

$i(t) = 4\sqrt{2}\sin(100\pi t + 7°)\,A$

a) $\underline{Z} = R + j(X_L - X_C) = 30 + j40 = 50 e^{j53^\circ}$

$\Rightarrow Z = 50\,\Omega$

$\varphi = 53^\circ$ phase angle introduced by the impedance

c) $\underline{U}_R = \underline{I}R = 120 e^{j7^\circ} \Rightarrow U_R = 120\,V$

$u_R(t) = 120\sqrt{2}\sin(100\pi t + 7^\circ)\ V$

$\underline{U}_L = jX_L\underline{I} = j \cdot 320 e^{j7^\circ} = e^{j90^\circ} \cdot 320 \cdot e^{j7^\circ} = 320 e^{j97^\circ}$

$j = e^{j90^\circ}$

$U_L = 320\,V$

$u_L(t) = 320\sqrt{2}\sin(100\pi t + 97^\circ)\ V$

$\underline{U}_C = -jX_C\underline{I} = -j160 e^{j7^\circ} = e^{-j90^\circ} \cdot 160 \cdot e^{j7^\circ}$

$= 160 e^{-j83^\circ}$

$-j = e^{-j90^\circ}$

$U_C = 160\,V$

$u_C(t) = 160\sqrt{2}\sin(100\pi t - 83^\circ)\ V$

d) $u(t) = 100\sqrt{2}\sin(100\pi t - 60^\circ)\ V$

$i(t) = 4\sqrt{2}\sin(100\pi t + 7^\circ)\ A$

Active power:

$$P = UI\cos(\varphi_u - \varphi_I) = 100 \cdot 4\cos(-60-7) = 400\cos(-67°)$$

$$P = 156.3\ W$$

Reactive power:

$$Q = UI\sin(\varphi_U - \varphi_I) = 400\sin(-67°)$$

$$Q = 368.2\ VAR$$

Apparent power:

$$S = UI = 400\ VA$$

3.16

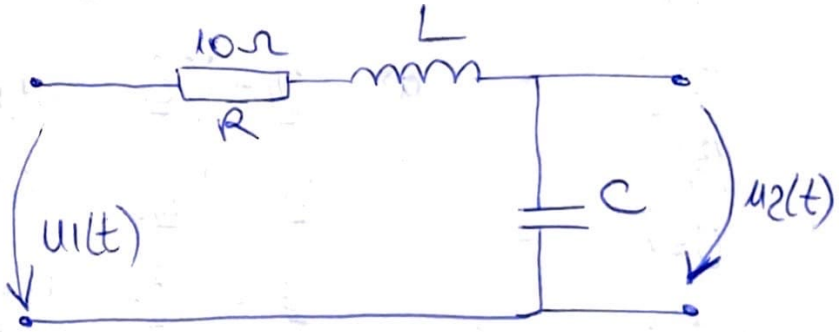
$R = 10\Omega$

$X_L = \omega L = 5\Omega$

$X_C = \frac{1}{\omega C} = 15\Omega$

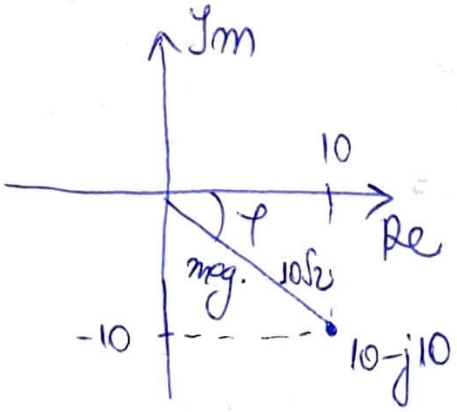
$u_1(t) = 100\sqrt{2}\sin 100\pi t\ V$

$u_2(t) = ?$



$$\underline{U_1} = 100\,e^{j\cdot 0°} = 100e^{0}$$

$$\underline{Z} = R + j(X_L - X_C) = 10 - j10$$



$$tg\,\varphi = -1 \Rightarrow \varphi = -45°$$

$$\underline{Z} = 10\sqrt{2}\,e^{-j45°}$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{100e^{0}}{10\sqrt{2}\,e^{-j45°}}$$

$$\underline{I} = 5\sqrt{2}\,e^{j45°}$$

$$\underline{U}_2 = \underline{I}(-jX_C) = 75\sqrt{2}\, e^{-j45^\circ}$$

$$\Rightarrow u_2(t) = 75\sqrt{2}\sqrt{2}\sin(100\pi t - 45^\circ)\ V$$
$$= 150\sin(100\pi t - 45^\circ)\ V$$

3.17

$\underline{Z}_1 = 3+j4$

$\underline{Z}_2 = -j6$

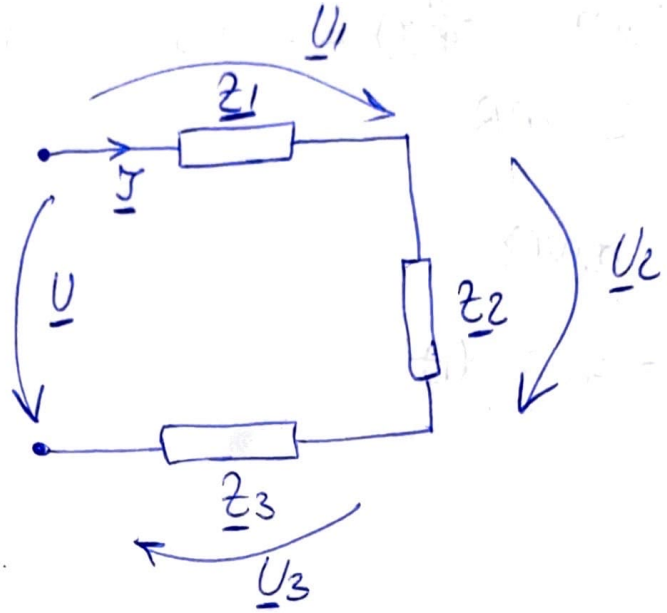
$\underline{Z}_3 = 6+j8$

$I = 2A$

a) $U_1 = ?$
$U_2 = ?$
$U_3 = ?$

b) $U = ?$
$u(t) = ?$

c) $P = ?$
$Q = ?$
$S = ?$



a) $\underline{U}_1 = I\underline{Z}_1 = 6+j8 \Rightarrow U_1 = 10V$

$\underline{U}_2 = I\underline{Z}_2 = -j12 \Rightarrow U_2 = 12V$

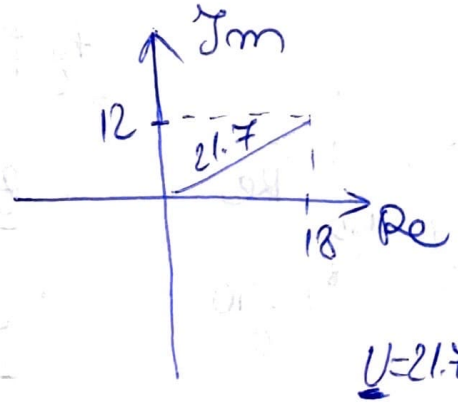
$\underline{U}_3 = I\underline{Z}_3 = 12+j16 \Rightarrow U_3 = 20V$

b) $\underline{U} = I(\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3)$

$= 2(3+j4-j6+6+j8) = 2(9+j6)$

$= 18+j12 \Rightarrow U = 21.7\ V$ ✓

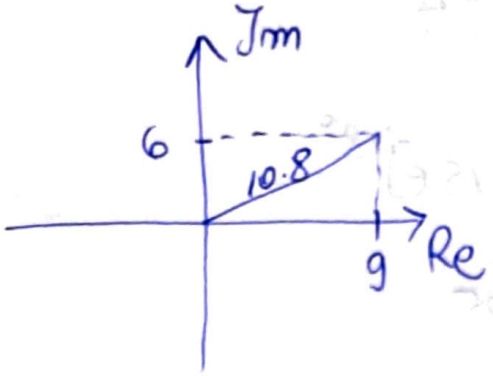
c)



$\operatorname{tg}\varphi = \frac{12}{18} = \frac{2}{3}$

$= 33^\circ$

$\underline{U} = 21.7\, e^{j33^\circ}$



$$\operatorname{tg}\varphi = \frac{6}{9} = \frac{2}{3}$$

$$\underline{Z} = 10.8\, e^{j34^\circ}$$

$$P = UJ\cos(\varphi_U - \varphi_J) = 43.4\cos 34^\circ$$

$$Q = UJ\sin(\varphi_U - \varphi_J) = 43.4\sin 34^\circ$$

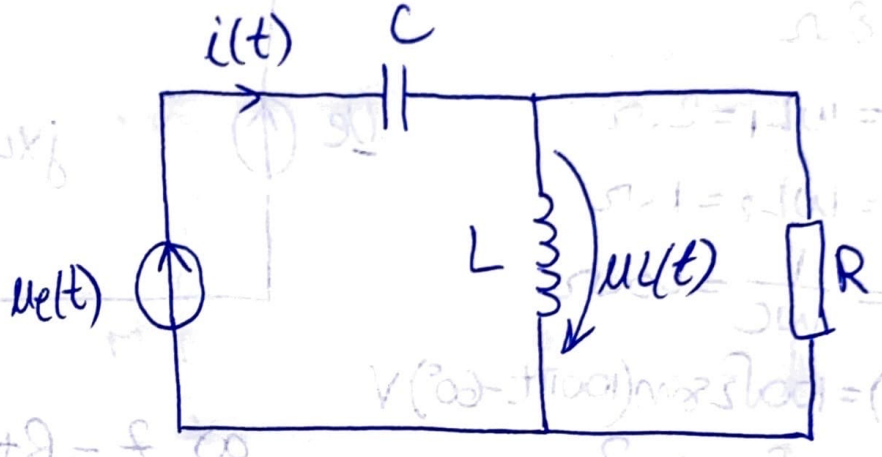
$$S = UJ$$

3.18

$R = 10\,\Omega$

$L = \frac{3}{10\pi}\,H$

$C = \frac{1}{2.1\pi}\,mF$



$u_e(t) = 100\sqrt{2}\sin(100\pi t - 60^\circ)\,V$

a) $Z = ?$ $\underline{Z} = ?$

b) $J = ?$

c) $u_L(t) = ?$

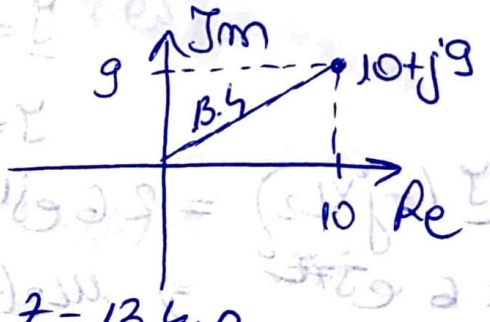
d) $P = ?$
$Q = ?$
$S = ?$

a) $\omega = 100\pi\ \frac{rad}{s}$

$$X_L = \omega L = 100\pi \cdot \frac{3}{10\pi} = 30\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \cdot \frac{1}{2.1\pi}} = 21\,\Omega$$

$$\underline{Z} = R + j(X_L - X_C) = 10 + j\cdot 9$$



$$\operatorname{tg}\varphi = \frac{9}{10} \Rightarrow \varphi = 42^\circ$$

$$\underline{Z} = 13.4\, e^{j42^\circ}$$

$Z = 13.4\,\Omega$

b) $I = \frac{U}{Z} = \frac{100}{13.4} \Rightarrow I = 7.5A$

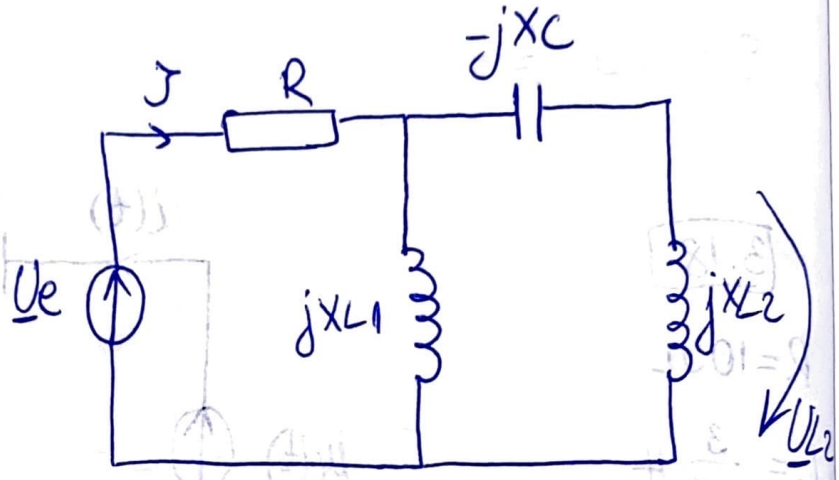
c) $\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{200\,e^{-j60°}}{13.4\,e^{j42°}} = 15e^{-j18°}$

$\underline{U_C} = \underline{I}(-jX_C) = 5.5 - j0.05$

$\underline{U_L} = \underline{U} - \underline{U_C} = 44.5 + j18$

$u_L(t) = 48\sqrt{2}\sin(100\pi t + 22°)$

3.19



$R = 8\Omega$

$X_{L1} = \omega L_1 = 2\Omega$

$X_{L2} = \omega L_2 = 1\Omega$

$X_C = \frac{1}{\omega C} = 40\Omega$

$u_e(t) = 100\sqrt{2}\sin(100\pi t - 60°)\,V$

a) $\underline{Z} = ?$ $Z = ?$

b) $I = ?$

c) $u_{L2}(t) = ?$

d) $P = ?$
$Q = ?$
$S = ?$

a) $\underline{Z} = R + j(X_{L1} + X_{L2} - X_C)$

$\underline{Z} = 8 + j(2 + 1 - 40)$

$= 8 - j37 = 37.9\,e^{-j78°}$

$Z = 37.9\Omega$

b) $\underline{I} = \frac{100\,e^{-j60°}}{37.9\,e^{-j78}}$

$\underline{I} = 2.6\,e^{j18°}$

$I = 2.6A$

c) $\underline{U_{L2}} = \underline{I}(-jX_{L2}) = 2.6e^{j18°} - j\cdot 1 = 2.6e^{j18°} + e^{-j90°}$

$= 2.6\,e^{-j72°} \Rightarrow u_{L2}(t) = 2.6\sqrt{2}\sin(100\pi t - 72°)\,V$

3.20

$R = 15\Omega$

$X_L = \omega L = 30\Omega$

$X_M = \omega M = 20\Omega$

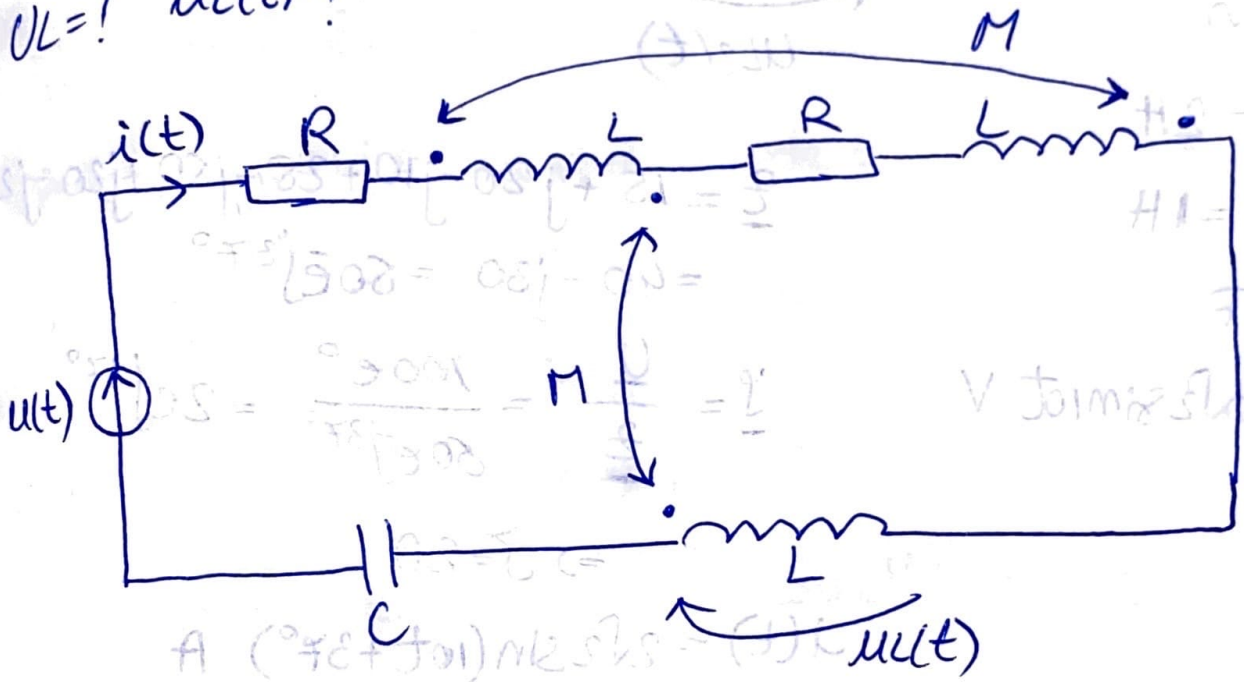
$X_C = \frac{1}{\omega C} = 50\Omega$

$u(t) = 100\sqrt{2}\sin(100\pi t + 60°)\,V$

a) $\underline{Z} = ?$ $Z = ?$

b) $\underline{I} = ?$ $i(t) = ?$

c) $\underline{U_L} = ?$ $u_L(t) = ?$



a) $\underline{Z} = 15 + j30 + j20 + 15 + j30 - j20 + j30 - j20 - j50$

$= 30 - j20$

$Z = 36\Omega$

b) $\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{100e^{j60°}}{36e^{j34°}} = 2.7e^{j26°}$

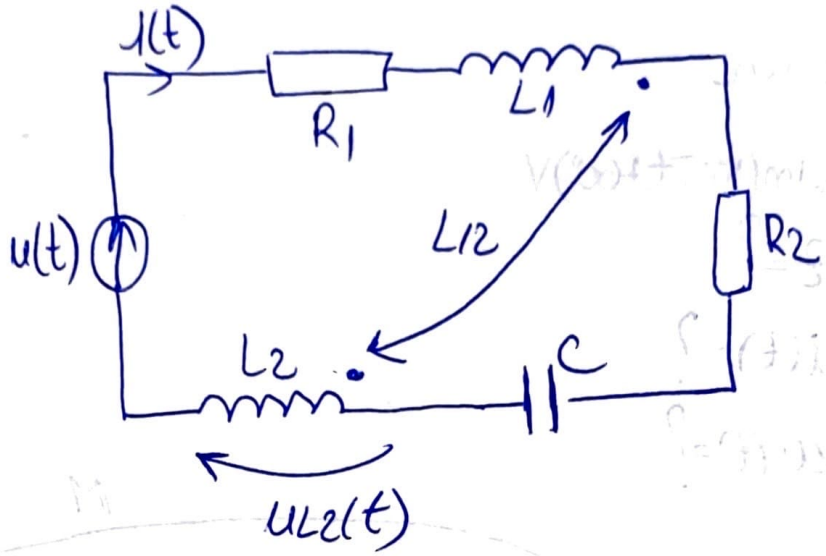
$I = 2.7\,A$

$i(t) = 2.7\sqrt{2}\sin(100\pi t + 26°)\,A$

c) $\underline{U_L} = \underline{I}(jX_L) = 2.7\, e^{j26^\circ} \cdot j30 = 83.2\, e^{j116^\circ}$

$\Rightarrow u_L(t) = 83.2\sqrt{2}\sin(100\pi t + 116^\circ)\ V$

3.21



$I = ?$

$u_{L2}(t) = ?$

$R_1 = 15\Omega$

$R_2 = 25\Omega$

$L_1 = L_2 = 2H$

$L_{12} = M = 1H$

$C = 2\mu F$

$u(t) = 100\sqrt{2}\sin 10t\ V$

$\underline{Z} = 15 + j20 - j10 + 25 - j50 + j20 - j20$

$= 40 - j30 = 50 e^{-j37^\circ}$

$\underline{I} = \dfrac{\underline{U}}{\underline{Z}} = \dfrac{100 e^{0^\circ}}{50 e^{-j37^\circ}} = 2 e^{j37^\circ}$

$\Rightarrow I = 2A$

$i(t) = 2\sqrt{2}\sin(10t + 37^\circ)\ A$

$\underline{U_{L2}} = \underline{I}(-X_{L2} - j\omega M) = 2e^{j37^\circ}(20e^{j90^\circ} - 10e^{j90^\circ})$

$= 20 e^{j127^\circ}$

$u_{L2}(t) = 20\sqrt{2}\sin(10t + 127^\circ)A$

3.22

$i_1(t) = ?$

$U_2 = ?$

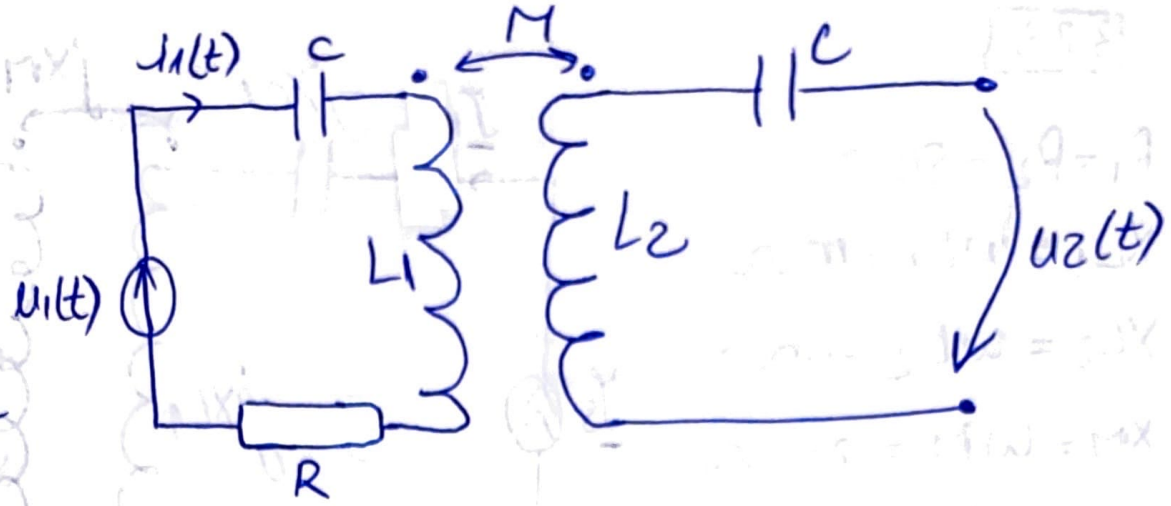
$R = 40\,\Omega$

$X_{L1} = X_{L2} = 60\,\Omega$

$X_M = 30\,\Omega$

$X_C = 20\,\Omega$

$u_1(t) = 400\sin(500t - 30°)\,V$



$$\underline{U}_1 = 400\sqrt{2}\,e^{j30°}$$

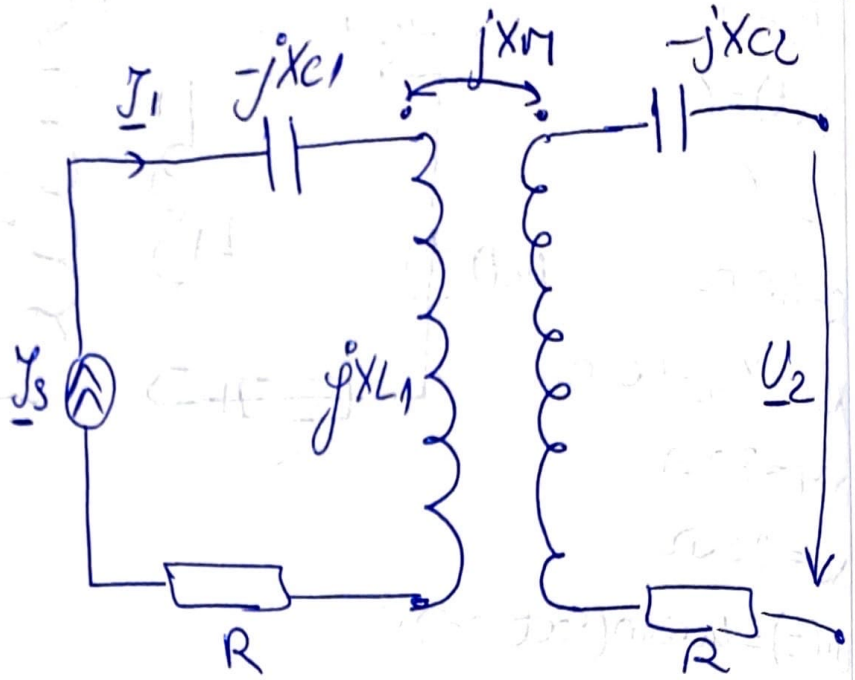
$$\underline{Z} = 40 - j20 + j60 = 40 + j40 = 40\sqrt{2}\,e^{j45°}$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{40\sqrt{2}\,e^{-j30°}}{40\sqrt{2}\,e^{j45°}} = 10e^{-j75°}$$

$$i_1(t) = 10\sqrt{2}\sin(500t - 75°)\ A$$

$$\underline{U}_2 = \underline{U}_{L2} = 30e^{j90°} \cdot 10e^{j-75°} = 300e^{j15°} \Rightarrow U_2 = 300\,V$$

3.23



$R_1 = R_2 = 50\,\Omega$

$X_{L1} = \omega L_1 = 10\,\Omega$

$X_{L2} = \omega L_2 = 40\,\Omega$

$X_M = \omega M = 20\,\Omega$

$X_{C1} = \frac{1}{\omega C} = 30\,\Omega$

$X_{C2} = \frac{1}{\omega C} = 10\,\Omega$

$i_S(t) = 3\sqrt{2}\sin(100\pi t + 30°)\,V$

a) $U_{L1} = ?$

b) $u_2(t) = ?$

a) $\underline{J}_S = 3e^{j30°}$

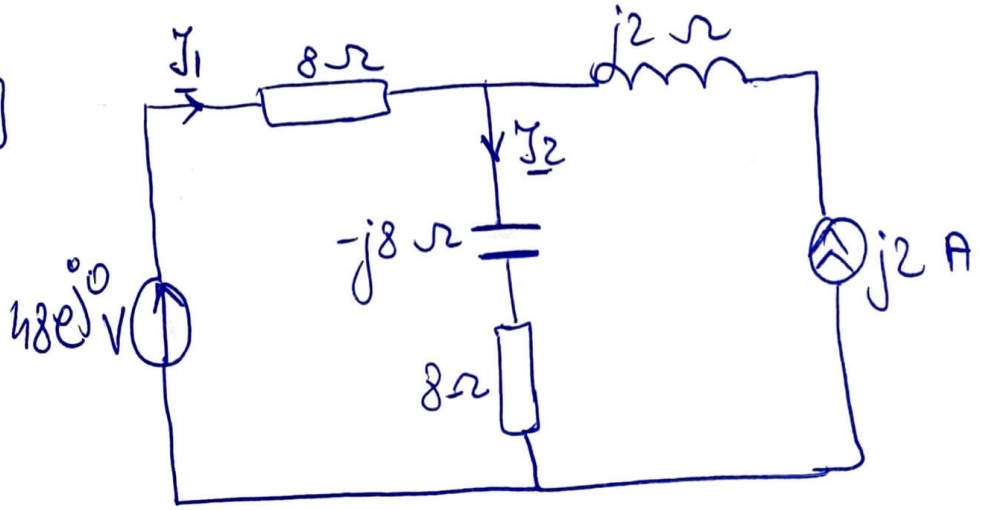
$\underline{U}_{L2} = 3e^{j30°} \cdot 10e^{j90°}$

$= 30e^{j120°} \Rightarrow \underline{U}_{L1} = \underline{J}_S$

b) $\underline{U}_{L2} = \underline{U}_2 = \underline{J}_S \cdot -jX_{C2} = 90e^{j120°}$

$u_2(t) = 90\sqrt{2}\sin(100\pi t + 120°)$

3.24



$$\begin{cases} \underline{I}_2 = \underline{I}_1 + j2 \\ 8\underline{I}_1 + (-j8+8)\underline{I}_2 = 48 \end{cases} \Rightarrow \begin{matrix} \underline{I}_1 = 2 \\ \underline{I}_2 = 2+2i \end{matrix}$$

3.25

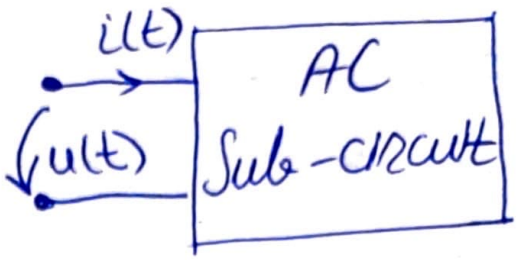
$u(t) = 200\sqrt{2}\sin(500t - 30°)\ V$

$i(t) = 5\sin(500t + 60°)\ A$

$\underline{V} = ?\quad \underline{I} = ?$

$\underline{Z} = ?\quad Z = ?$

$P = ?\quad Q = ?$



$\underline{V} = 200\,e^{-j30°}\ V$

$\underline{I} = \frac{5}{\sqrt{2}}\,e^{j60°}\ A$

$$\underline{Z} = \frac{\underline{V}}{\underline{I}} = \frac{200\,e^{-j30°}}{\frac{5}{\sqrt{2}}\,e^{j60°}} = 40\sqrt{2}\,e^{-j90°} \Rightarrow Z = 40\sqrt{2}\ \Omega$$

$$P = V \cdot I \cos(\varphi_u - \varphi_I) = 200 \cdot \frac{5}{\sqrt{2}} \cos(-30° - 60°)$$

$$= 200 \cdot \frac{5}{\sqrt{2}} \cdot 0 \Rightarrow P = 0\ W$$

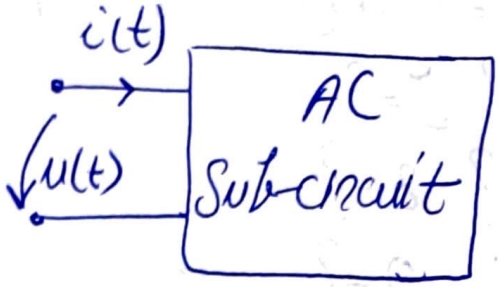
$$Q = V \cdot I \sin(\varphi_u - \varphi_I) = -200 \cdot \frac{5}{\sqrt{2}} \sin 90° = -200 \cdot \frac{5}{\sqrt{2}}$$

$$= -500\sqrt{2}\ VAR$$

3.26

$u(t) = 100\sin(100\pi t + 30°)\ V$

$i(t) = 2\cos 100\pi t\ A$



a) $\underline{U} = ?$ $\underline{I} = ?$

b) $\underline{Z} = ?$ $Z = ?$

c) $P = ?$ $Q = ?$ $S = ?$

$$\underline{U} = 50\sqrt{2}\, e^{j30°}$$

$$\underline{I} = \sqrt{2}\, e^{j0°}$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50\sqrt{2}\, e^{j30°}}{\sqrt{2}\, e^{j0°}} = 50\, e^{j30°}$$

$$\Rightarrow Z = 50\ \Omega$$

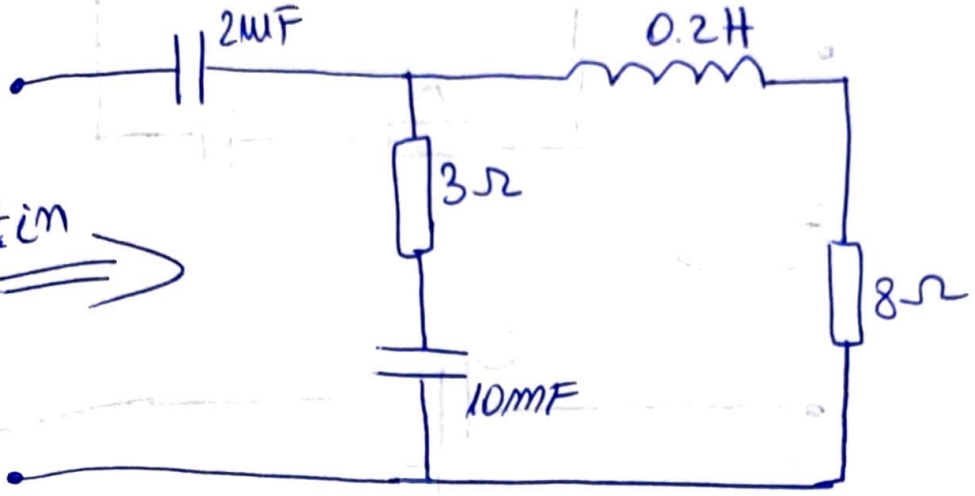
$$P = U \cdot I \cos 30° = 50\sqrt{2} \cdot \sqrt{2} \cdot \frac{\sqrt{3}}{2} = 50\sqrt{3}\ W$$

$$Q = U \cdot I \sin 30° = 50\sqrt{2} \cdot \sqrt{2} \cdot \frac{1}{2} = 50\ VAR$$

$$S = U \cdot I = 100\ VA$$

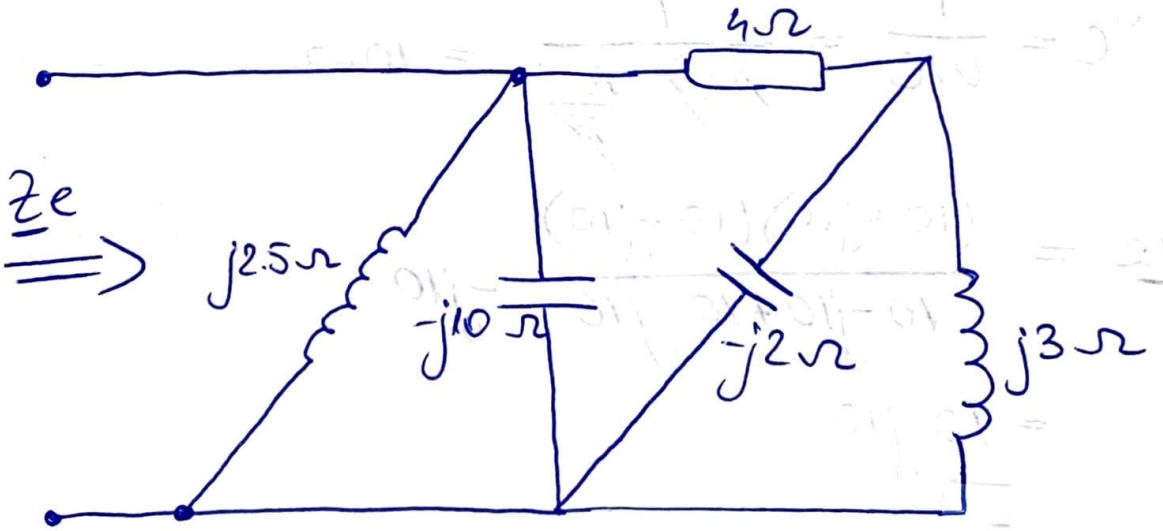
3.27

$\omega = 50 \frac{rad}{s}$



$$\underline{Z}_{in} = j20 + \frac{(3-j2)(8+j20)}{3-j2+8+j90} = 3.2+j12$$

3.28

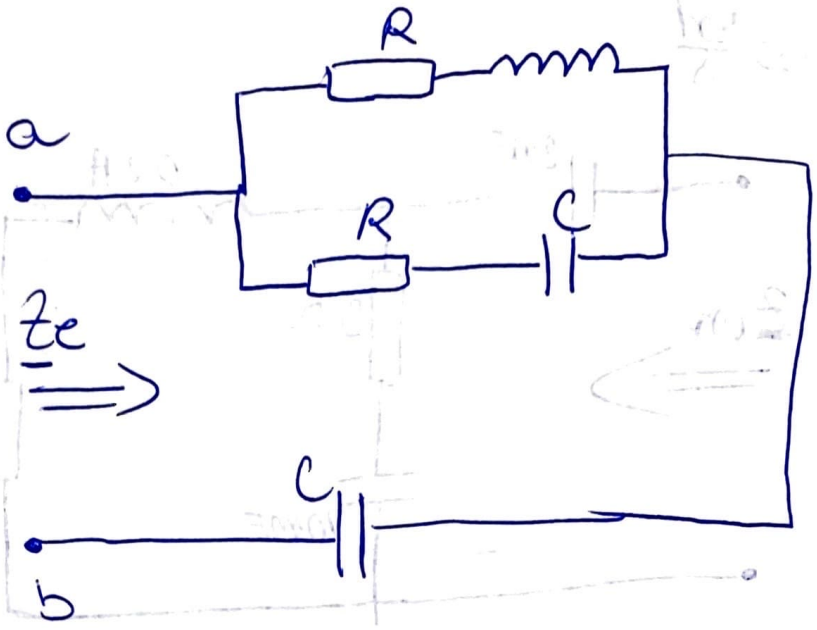


$$\frac{1}{\underline{Z}_e} = \frac{1}{j2.5} - \frac{1}{j20} + \frac{1}{4-j6}$$

$$\underline{Z}_e = 2+j4.6$$

3.29

$\underline{z}_e = ?$



$R = 10\,\Omega$

$L = \frac{100}{\pi}\,mH$

$C = \frac{1}{\pi}\,mF$

$f = 50\,Hz$

$$\omega = 2\pi f \Rightarrow \omega = 100\pi \frac{rad}{s}$$

$$X_L = \omega L = 100\pi \cdot \frac{100}{\pi} \cdot 10^{-3} = 10\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \cdot \frac{10^{-3}}{\pi}} = 10\,\Omega$$

$$\underline{z}_e = \frac{(10+j10)(10-j10)}{10-j10+10-j10} - j10$$

$$= 10 - j10$$

3.30

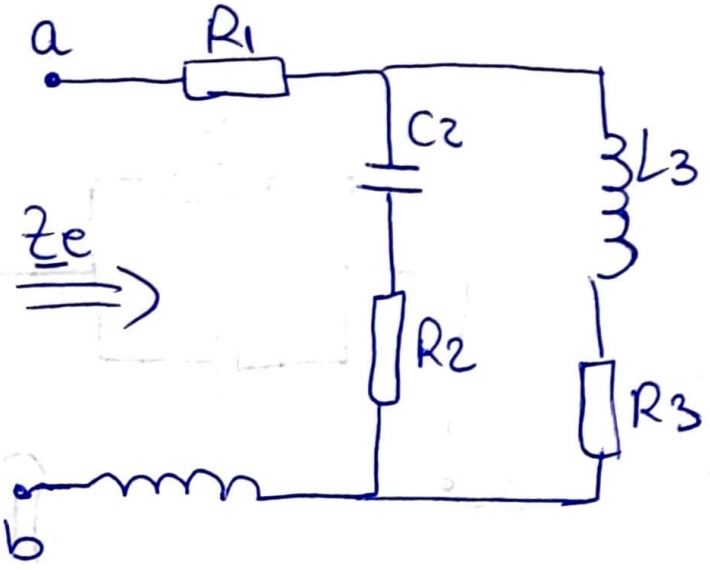
$\underline{z}e$

$R_1 = R_2 = R_3 = 10\,\Omega$

$L_3 = \frac{20}{\pi}\,mH$

$C_2 = \frac{5}{\pi}\,mF$

$f = 50\,Hz$



$w = 2\pi f \Rightarrow w = 100\pi \frac{rad}{s}$

$X_L = wL_3 = 100\pi \cdot \frac{20}{\pi} \cdot 10^{-3} = 2\,\Omega$

$X_C = \frac{1}{wC_2} = \frac{1}{100\pi \cdot \frac{5}{\pi} \cdot 10^{-3}} = 2\,\Omega$

$\underline{z}e = 10 + \frac{(10+j2)(10-j2)}{10+j2+10-j2} + j2$

$\underline{z}e = 15.2 + j2$

3.31

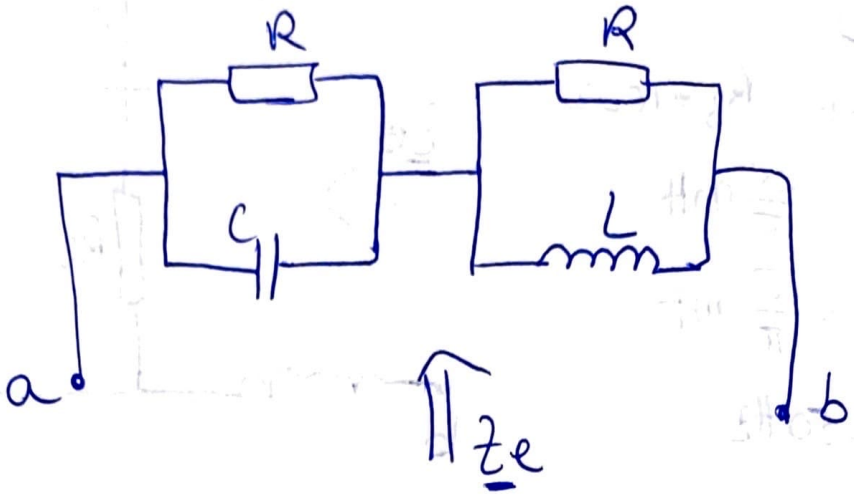
$\underline{z}_e$

$R = 10\,\Omega$

$L = \frac{100}{\pi}\,mH$

$C = \frac{1}{\pi}\,mF$

$f = 50Hz$



$$\omega = 2\pi f = 100\pi \frac{rad}{s}$$

$$X_L = \omega L = 100\pi \cdot \frac{100}{\pi} \cdot 10^{-3} = 10\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \cdot \frac{1}{\pi} \cdot 10^{-3}} = 10\,\Omega$$

$$\underline{z}_e = \frac{10 \cdot (-j20)}{10 - j10} + \frac{10 \cdot j10}{10 + j20}$$

$$z = 10\,\Omega$$

3.32

$\underline{Z}e$

$R_1 = 5\,\Omega$

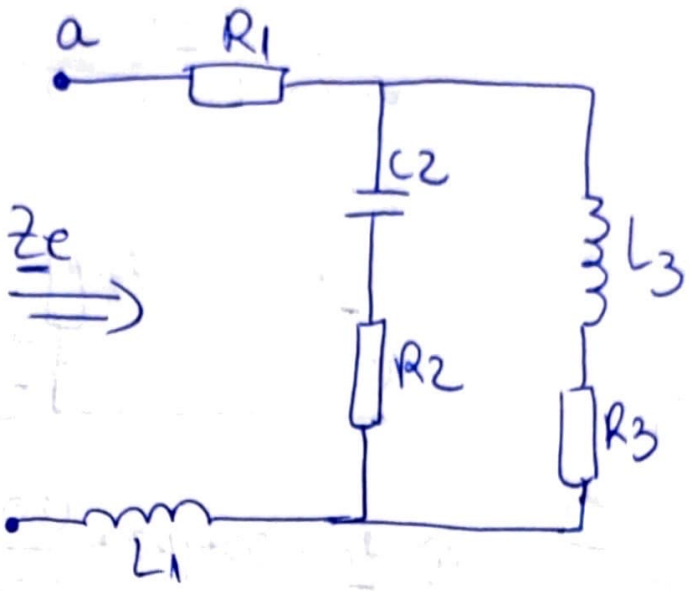
$R_2 = R_3 = 10\,\Omega$

$L_1 = \frac{150}{\pi}$ mH

$L_3 = \frac{100}{\pi}$ mH

$C_2 = \frac{10^3}{\pi}\,\mu F$

$f = 50$ Hz



$$\omega = 2\pi f = 100\pi \ \frac{rad}{s}$$

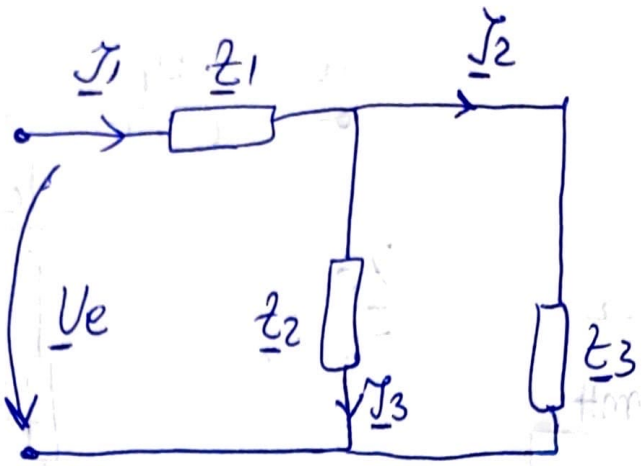
$$X_{L_1} = \omega L_1 = 100\pi \cdot \frac{150}{\pi} \cdot 10^{-3} = 15\,\Omega$$

$$X_{L_3} = \omega L_3 = 100\pi \cdot \frac{100}{\pi} \cdot 10^{-3} = 10\,\Omega$$

$$X_{C_2} = \frac{1}{\omega C} = \frac{1}{100\pi \cdot \frac{10^3}{\pi} \cdot 10^{-6}} = 10\,\Omega$$

$$\underline{Z}e = 5 + \frac{(10+j10)(10-j10)}{10+j10+10-j10} + j15 = 15 + j15$$

3.33



$\underline{z}_1 = 10+j40$

$\underline{z}_2 = 20-j20$

$\underline{z}_3 = 20+j20$

$J = 4A$

a) $\underline{z}_e = ?$

b) $U_e = ?$

$u_e(t) = ?$

c) $P = ?$

$Q = ?$

$S = ?$

a) $\underline{z}_e = 20+j40 + \dfrac{(20+j20)(20-j20)}{20+j20+20-j20}$

$= 30+j40$

b) $\underline{U}_e = 4e^{j\varphi_i} \cdot 50 e^{j53°}$

$= 200 e^{j(\varphi_i + 53°)}$

$U_e = 200V$

$u_e(t) = 200\sqrt{2}\sin(\omega t + \varphi_i + 53°)\ V$

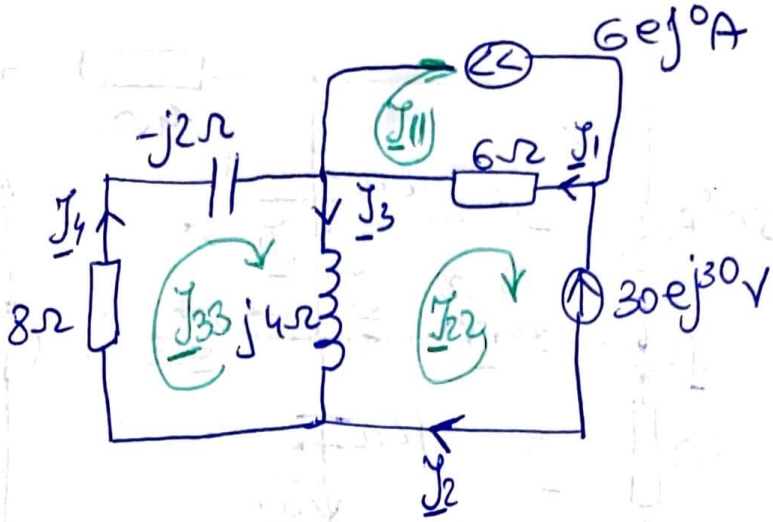
c) $P = U \cdot J \cos(\varphi_u - \varphi_J) = 482\ W$

$Q = 639\ VAR$

$S = 800\ VA$

3.34

$\underline{I}_0 = ?$



$$\begin{cases} \underline{I}_{11} = -6\,e^{j0^\circ} \\ \underline{I}_{22}(6+j4) - \underline{I}_{11}\,6 - \underline{I}_{33}\,j4 = -30e^{j30^\circ} \\ \underline{I}_{33}(8+j2) - \underline{I}_{22}\,j4 = 0 \end{cases}$$

$\underline{I}_{22} = -7.2 + j1.3 = \underline{I}_2$ A

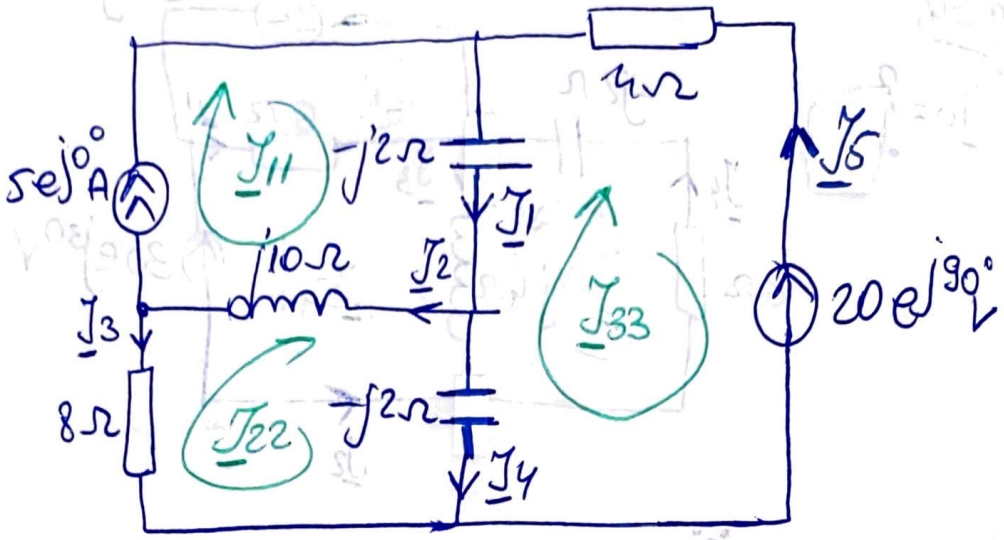
$\underline{I}_{33} = -1.4 - j3.2 = \underline{I}_4$

$\underline{I}_1 = \underline{I}_{11} - \underline{I}_{22} = 1.2 + j1.3$ A

$\underline{I}_3 = \underline{I}_{33} - \underline{I}_{22} = 5.7 - j4.5$ A

3.35

$\underline{I}_0 = ?$



$$\begin{cases} \underline{I}_{11} = 5A \\ \underline{I}_{22}(8-j2) - \underline{I}_{11} j10 - \underline{I}_{33}(-j2) = 0 \\ \underline{I}_{33}(4-j4) - j2(I_{11} + I_{22}) = -j20 \end{cases}$$

$$\underline{I}_{22} = 29 - 18 = -\underline{I}_3$$

$$\underline{I}_{33} = -4 + 18 = \underline{I}_2$$

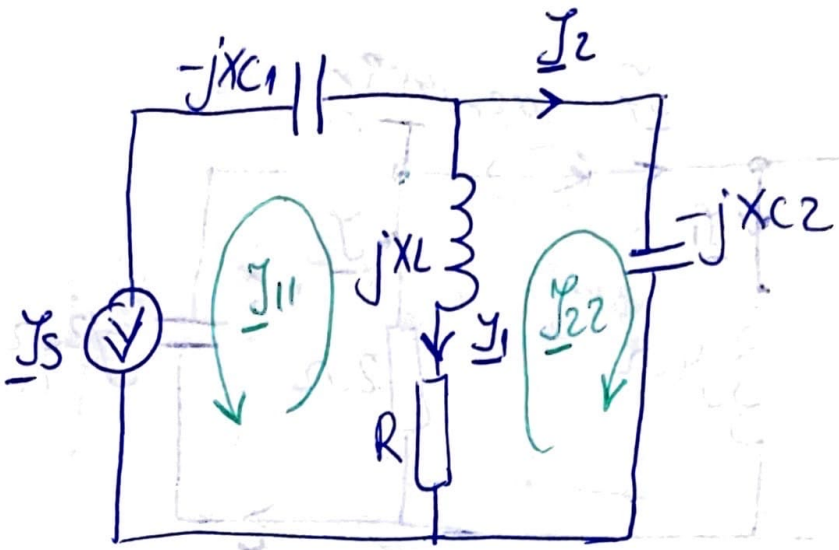
$$\underline{I}_1 = \underline{I}_{11} - \underline{I}_{33} = 5 - j8 \text{ A}$$

$$\underline{I}_2 = \underline{I}_{11} - \underline{I}_{22} = -24 + j8 \text{ A}$$

$$\underline{I}_4 = \underline{I}_{22} - \underline{I}_{33} = 33 - j16 \text{ A}$$

3.36

$R = 9.3\,\Omega$
$L = 100\,\mu H$
$C_1 = 1.2\,nF$
$C_2 = 820\,pF$
$f = 570\,kHz$
$\underline{J}_S = 3e^{-j30°}$
$\underline{J}_1 = ?\ \ \underline{J}_2 = ?$



$$\omega = 2\pi f = 2\pi \cdot 570 \cdot 10^3 = 1140\pi \cdot 10^4 \ \frac{rad}{s}$$

$$X_L = \omega L = 1140\pi \cdot 10^4 \cdot 100 \cdot 10^{-6} = 1140\pi\,\Omega \approx 359\,\Omega$$

$$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{1140\pi \cdot 10^4 \cdot 1.2 \cdot 10^{-9}} \approx 233\,\Omega$$

$$X_{C2} = \frac{1}{\omega C_2} = \frac{1}{1140\pi \cdot 10^4 \cdot 820 \cdot 10^{-12}} \approx 341\,\Omega$$

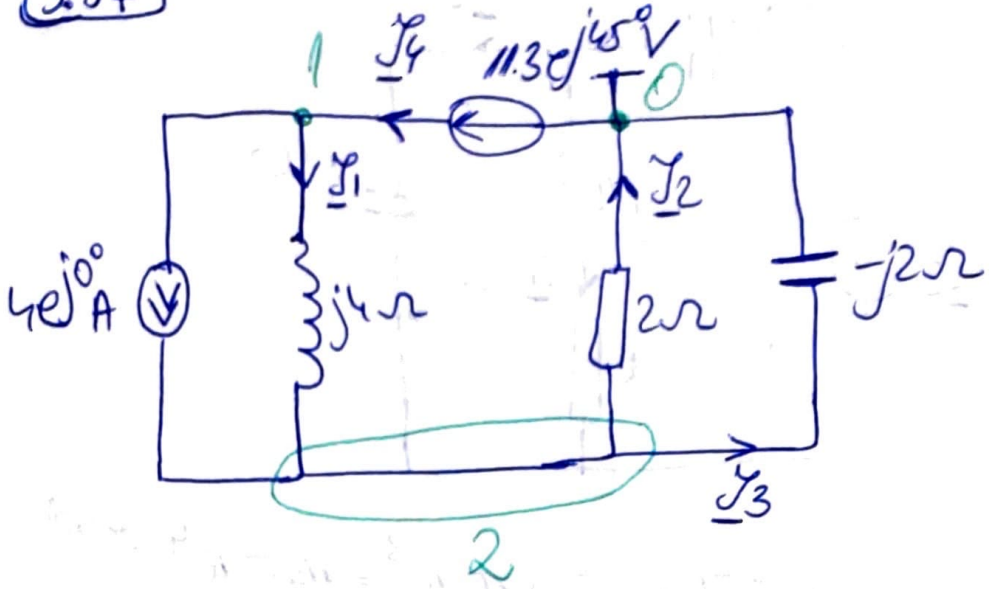
$$\begin{cases} \underline{J}_{11} = 3e^{-j30°}\,A \\ \underline{J}_{22}(9.3 + j125.5) + \underline{J}_{11}(9.3 + j358) = 0 \end{cases}$$

$$\underline{J}_{22} = -4.5 + j3.9 = \underline{J}_2$$

$$\underline{J}_1 = -\underline{J}_{11} - \underline{J}_{22} = 5 - j2.4$$

✓

3.37



$$V_0 = 0V$$

$$\begin{cases} V_1 = 8 + j4 \\ V_2 \cdot \left(\frac{1}{2} + \frac{1}{j4} - \frac{1}{j2}\right) - V_1 \cdot \frac{1}{j4} = 4 \end{cases}$$

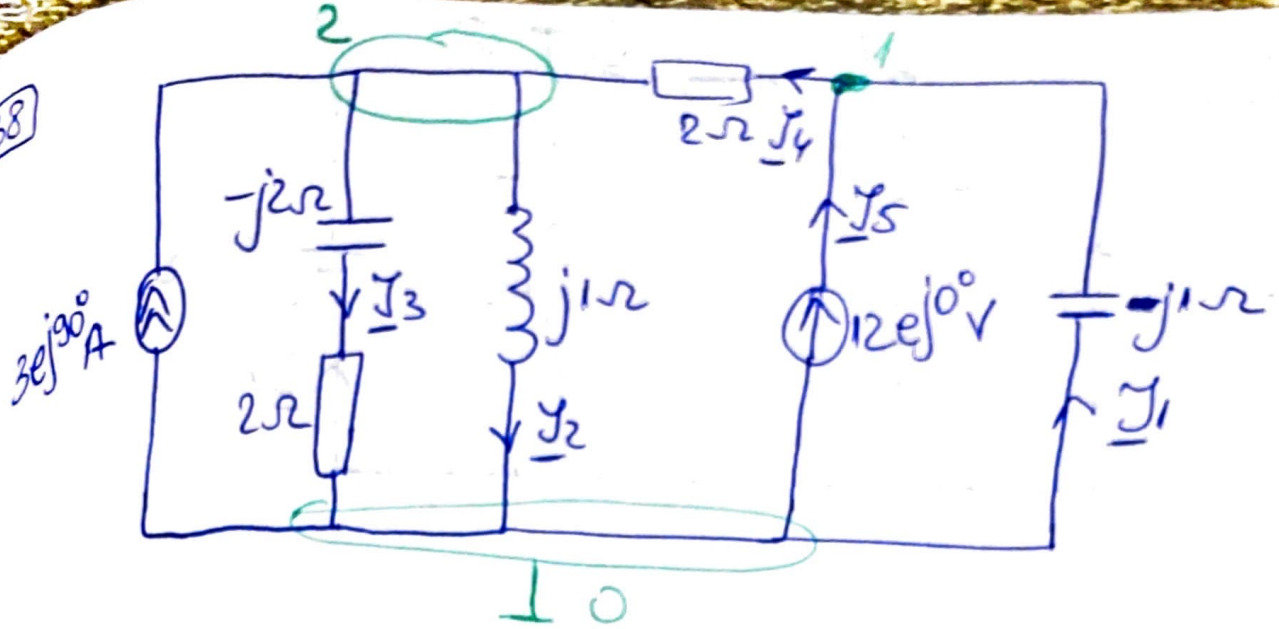
$$V_2 = 8 - j8$$

$$\underline{J}_1 = \frac{V_1 - V_2}{j4} = \frac{j26}{j4} = 4A \Rightarrow \underline{J}_4 = 8A$$

$$\underline{J}_2 = \frac{V_2}{2} = 4 - j4\ A$$

$$\underline{J}_3 = \frac{V_2}{j2} = 4 + j4\ A$$

3.38



$V_0 = 0$

$$\begin{cases} V_1 = 12\,V \\ V_2\left(\frac{1}{-j2+2} + \frac{1}{j} + \frac{1}{2}\right) - V_1 \cdot \frac{1}{2} = j3 \end{cases}$$

$V_2 = 0.3 + j4.5$

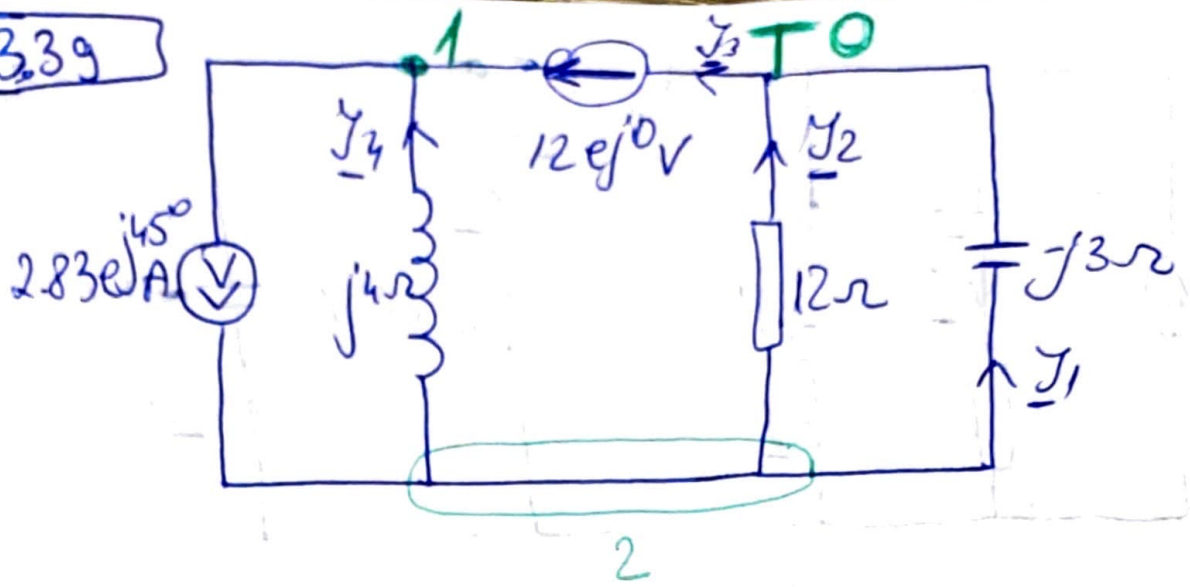
$$\underline{I}_1 = \frac{-V_1}{-j} = \frac{12}{j} = -j12\ A$$

$$\underline{I}_2 = \frac{V_2}{j} = 4.5 - 0.3\ A$$

$$\underline{I}_3 = \frac{V_2}{2-j2} = -1 + j1.2\ A$$

$$\underline{I}_4 = \frac{V_1 - V_2}{2} = 5.8 - j2.2 \Rightarrow \underline{I}_5 = 5.8 - j14.2$$

3.39



$V_0 = 0$

$$\begin{cases} V_1 = 12V \\ V_2\left(\frac{1}{j4} + \frac{1}{12} - \frac{1}{j3}\right) - V_1 \cdot \frac{1}{j4} = 2+j2 \end{cases}$$

$V_2 = 6 - j18$

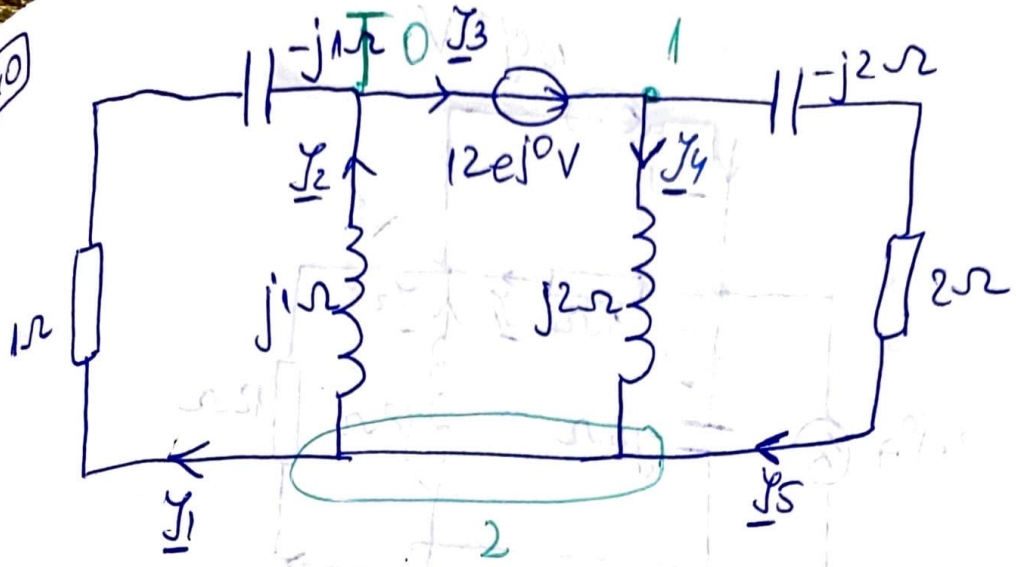
$\underline{J}_1 = \frac{V_2}{-j3} = 6 + j2\ A$

$\underline{J}_2 = \frac{V_2}{12} = 0.5 - j1.5\ A$

$\underline{J}_3 = 6.5 + j0.5\ A$

$\underline{J}_4 = \frac{V_2 - V_1}{j4} = \frac{-6 - j18}{j4} = -8.5 + j25$

3.40



$V_0 = 0$

$$\begin{cases} V_1 = 12\,V \\ V_2\left(\frac{1}{1} + \frac{1}{j} + \frac{1}{j2} + \frac{1}{2}\right) - V_1\frac{1}{j2} = 0 \end{cases}$$

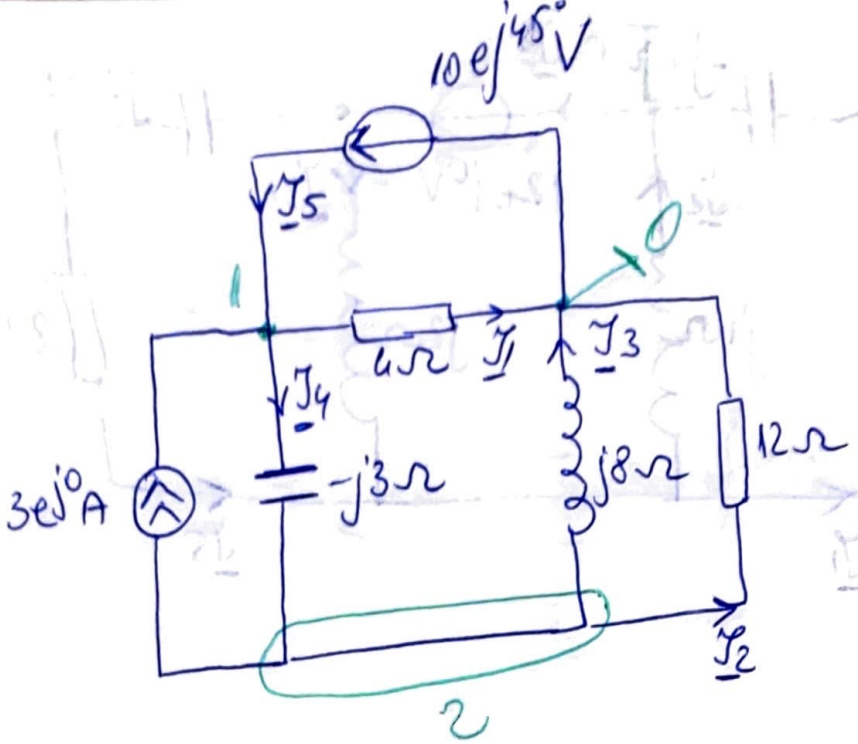
$V_2 = 3 - j3$

$$\underline{I}_1 = \frac{V_2}{2-j} = 3\,A$$

$$\underline{I}_2 = -\frac{V_2}{j} = -3 - j3\,A \Rightarrow \underline{I}_3 = -j3$$

$$\underline{I}_4 = \frac{V_1 - V_2}{j2} = 1.5 - j4.5 \Rightarrow \underline{I}_5 = -1.5 + j1.5$$

3.41



$V_0 = 0\,V$

$$\begin{cases} V_1 = 5\sqrt{2} + j5\sqrt{2} \\ V_2\left(\frac{1}{12} + \frac{1}{j8} + \frac{1}{j3}\right) + V_1 \cdot \frac{1}{j3} = -3 \end{cases}$$

$V_2 = 0.8 + j26$

$\underline{I}_1 = \frac{V_1}{4} = 1.25\sqrt{2} + j1.25\sqrt{2} = 1.7 + j1.7$

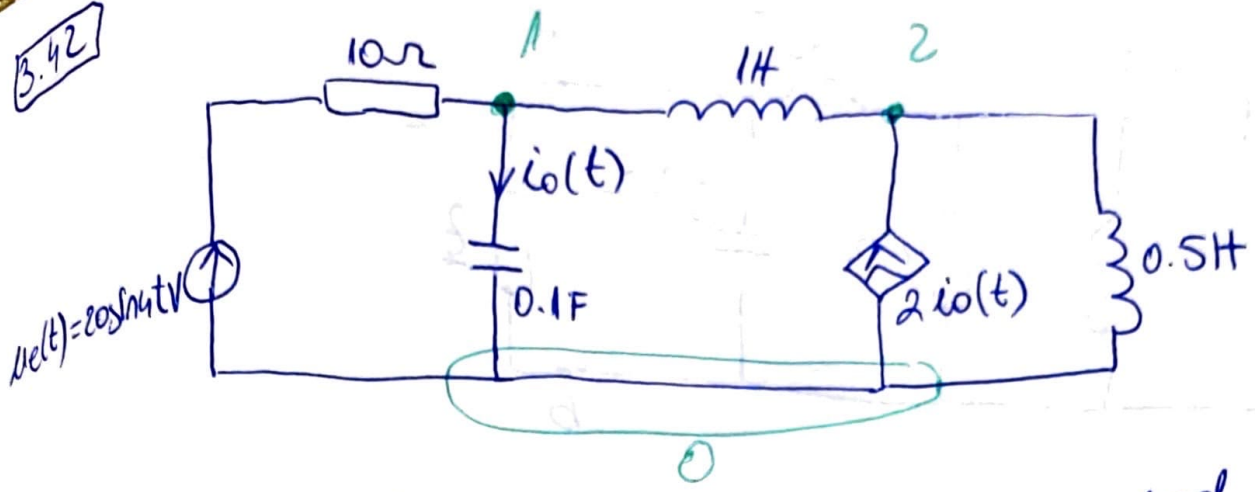
$\underline{I}_2 = \frac{V_2}{12} = 0.074 + j2$

$\underline{I}_3 = \frac{V_2}{j8} = 3.25 - j0.11$

$\underline{I}_4 = \frac{V_1 - V_2}{-j3} = 6.3 + j2$

$\underline{I}_5 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 22.4 + j5.9$

3.42



$$V_e = \frac{20}{\sqrt{2}} = \frac{20\sqrt{2}}{2} = 10\sqrt{2}$$

$\omega = 4\ \frac{rad}{s}$

$$X_{L1} = \omega L_1 = 4\,\Omega$$

$$X_{L2} = \omega L_2 = 2\,\Omega$$

$$X_C = \frac{1}{\omega C} = 2.5\,\Omega$$

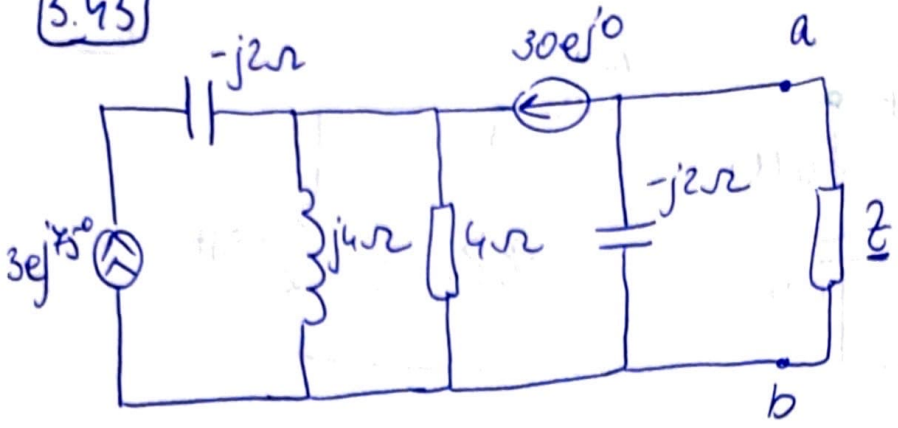
$$\begin{cases} V_1\left(\frac{1}{10} - \frac{1}{j2.5} + \frac{1}{j4}\right) - V_2 \cdot \frac{1}{j4} = \sqrt{2} \\ V_2\left(\frac{1}{j4} + \frac{1}{j2}\right) - V_1 \cdot \frac{1}{j4} = \frac{4V_1}{5} \end{cases}$$

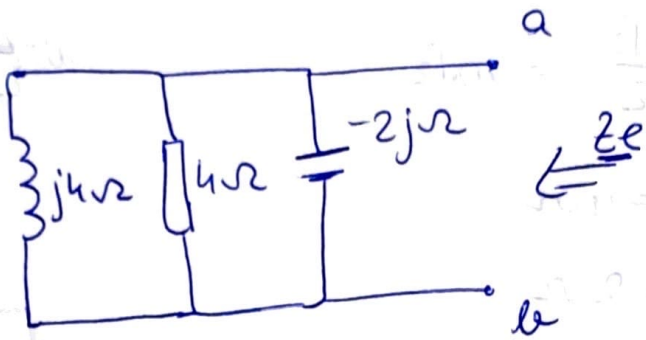
$$V_1 = -2.8 - j4$$

$$\underline{I}_o = -1.1 - j1.6 = 2e^{-j125^\circ}$$

$$i_o(t) = 2\sqrt{2}\sin(4t - 125^\circ)\ A$$

3.43

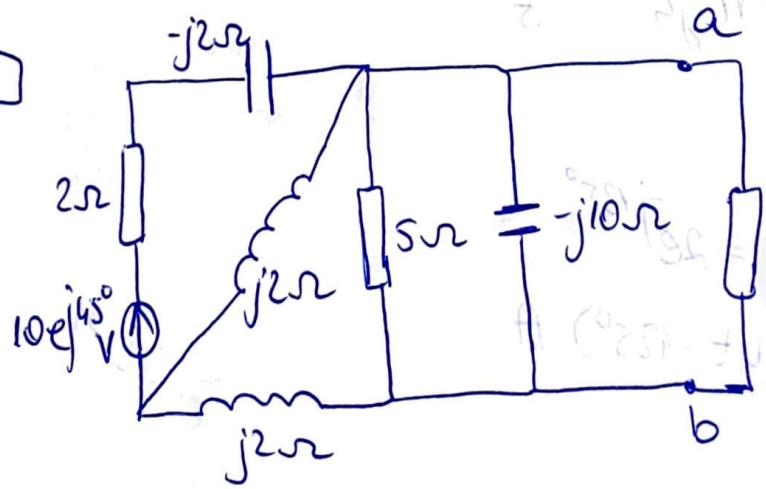




$$\frac{1}{\underline{Z}e} = \frac{1}{j4} + \frac{1}{4} + \frac{1}{-j2} \Rightarrow \underline{Z}e = 2 - j2\ \Omega$$

$$\underline{Z} = \underline{Z}e^{*} = 2 + j2\ \Omega$$

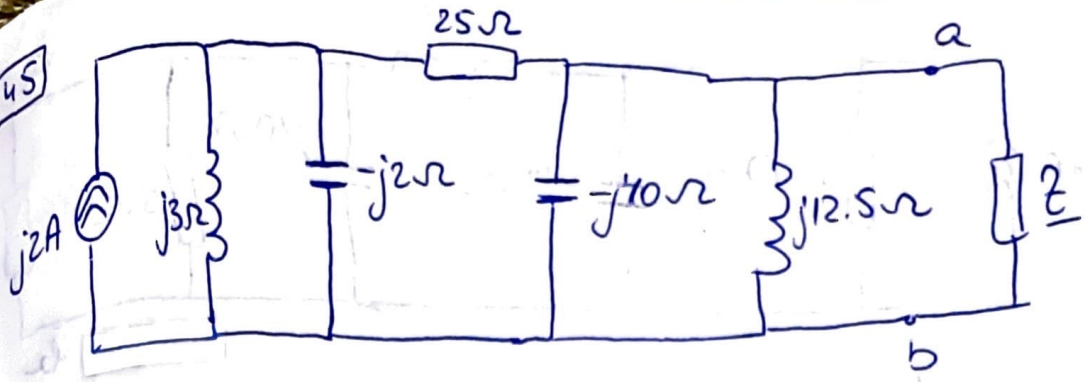
3.44



$$\frac{1}{\underline{Z}e} = \frac{1}{\frac{j2(2-j2)}{j2+2-j2} + j2} + \frac{1}{5} + \frac{1}{-j10} \Rightarrow \underline{Z}e = 3 + j$$

$$\Rightarrow \underline{Z} = 3 - j\ \Omega$$

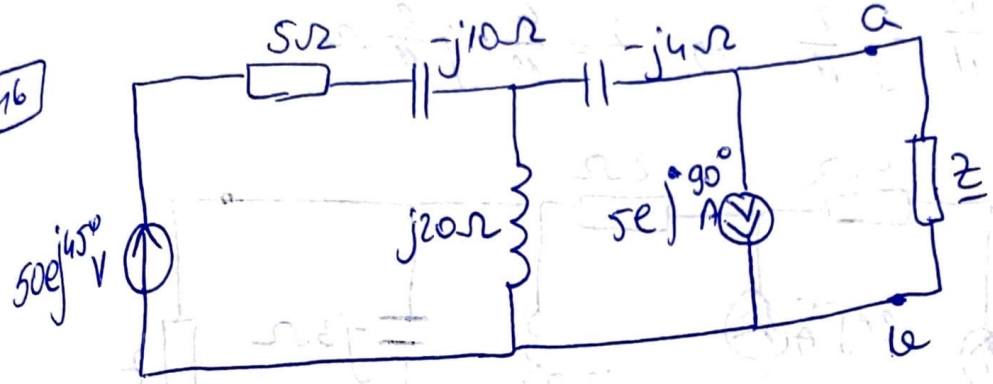
3.45



$$\frac{1}{\underline{Z}e} = \frac{1}{25+\frac{6}{i}} + \frac{1}{-10i} + \frac{1}{125i} \Rightarrow \underline{Z}e = 2-j5.5$$

$$\Rightarrow \underline{Z} = 2+j5.5\,\Omega$$

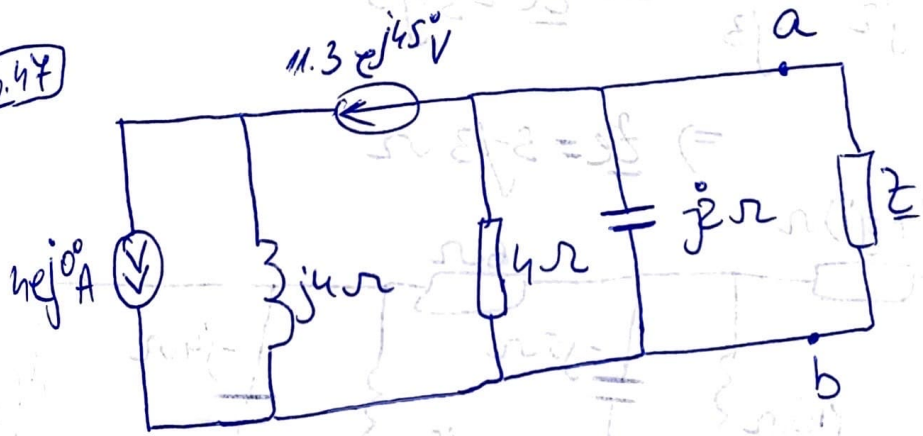
3.46



$$\underline{Z}e = \frac{j20(5-j10)}{j20+5-j10} + (-j4) = 16-j16$$

$$\Rightarrow \underline{Z} = 16+j16\,\Omega$$

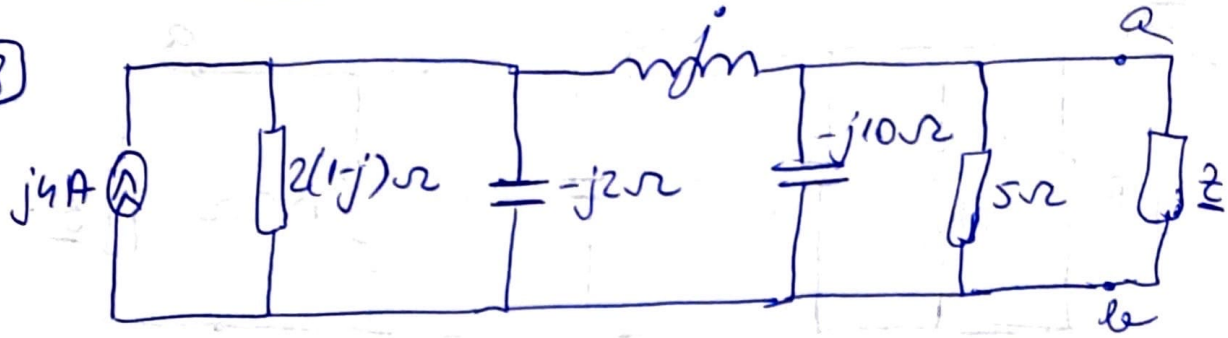
3.47



$$\frac{1}{\underline{Z}e} = \frac{1}{j4} + \frac{1}{4} + \frac{1}{-j2} \Rightarrow \underline{Z}e = 2-2j$$

$$\Rightarrow \underline{Z} = 2+2j\,\Omega$$

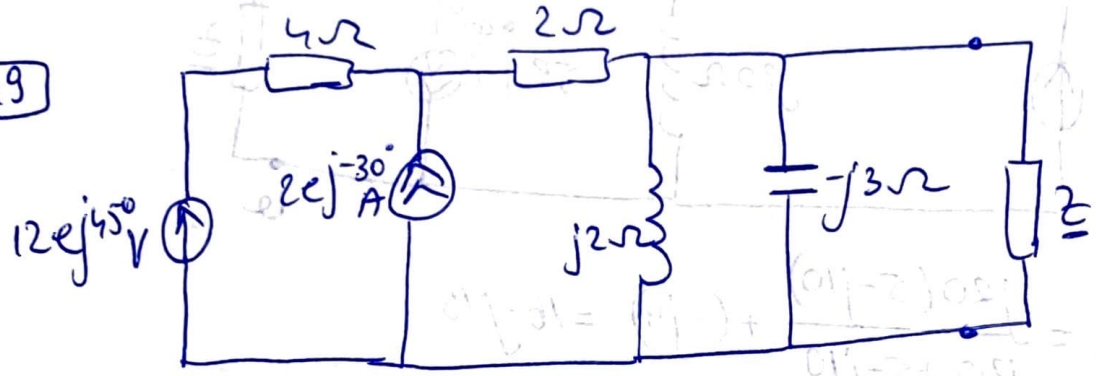
3.48



$$\frac{1}{\underline{Z}_e} = \frac{1}{\frac{2(1-j)(-j2)}{2(1-j)-j2}+j} + \frac{1}{-j10} + \frac{1}{5}$$

$$\underline{Z}_e = \frac{4}{11} - j\frac{2}{11} \Rightarrow \underline{Z} = \frac{4}{11} + j\frac{2}{11}\ \Omega$$

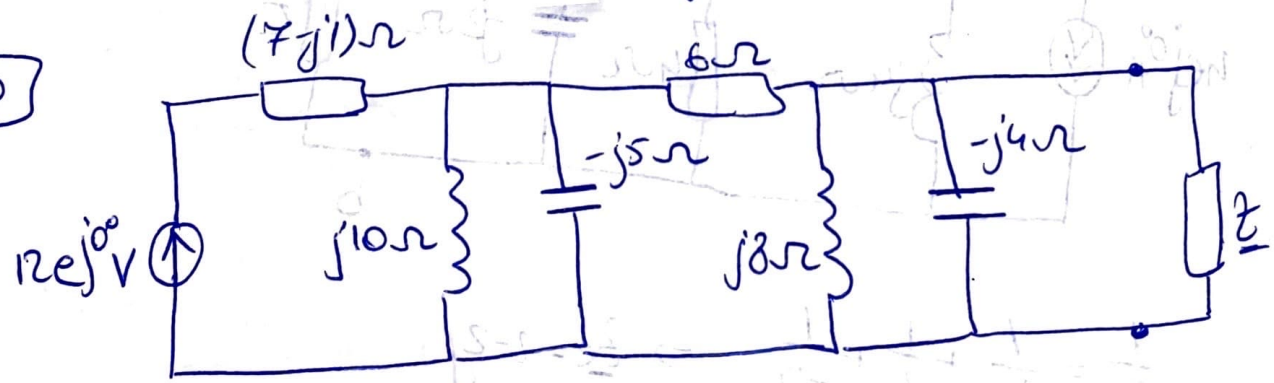
3.49



$$\frac{1}{\underline{Z}_e} = \frac{1}{4+2} + \frac{1}{j2} + \frac{1}{-j3} \Rightarrow \underline{Z}_e = 3+j3$$

$$\Rightarrow \underline{Z} = 3-j3\ \Omega$$

3.50

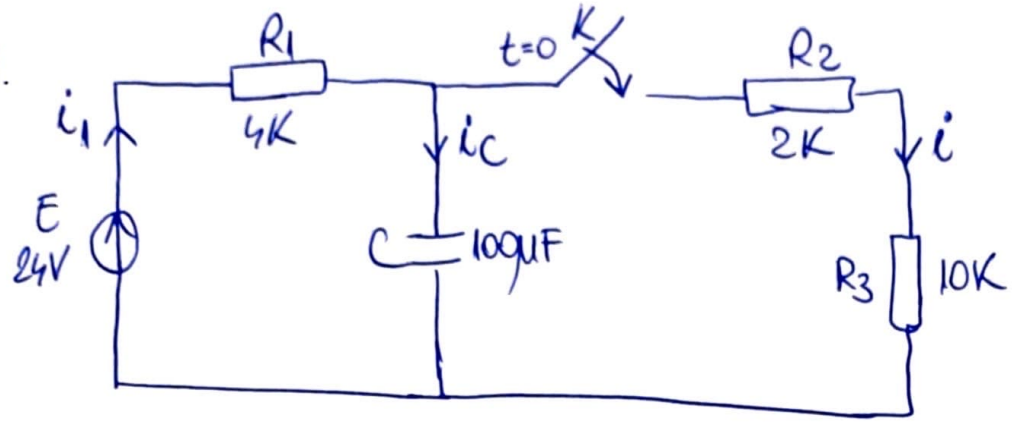


$$\frac{1}{\underline{Z}_e} = \frac{1}{\frac{1}{7-j1}+\frac{1}{j10}+\frac{1}{-j5}+6} + \frac{1}{j8} + \frac{1}{j4} \Rightarrow \underline{Z}_e = 2.7 - j4.8$$

$$\Rightarrow \underline{Z} = 2.7 + j4.8\ \Omega$$

Transient problems

1.

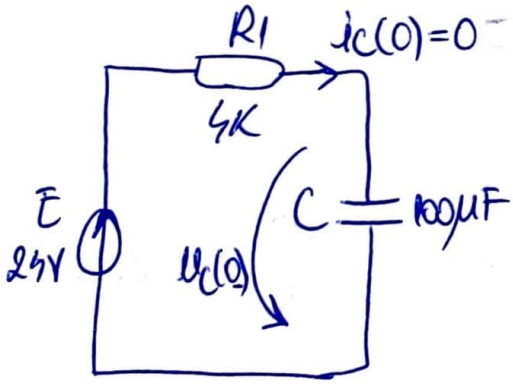


$u_C(0_-) = ?$

$u_C(t) = ?$

$i_C(t) = ?$

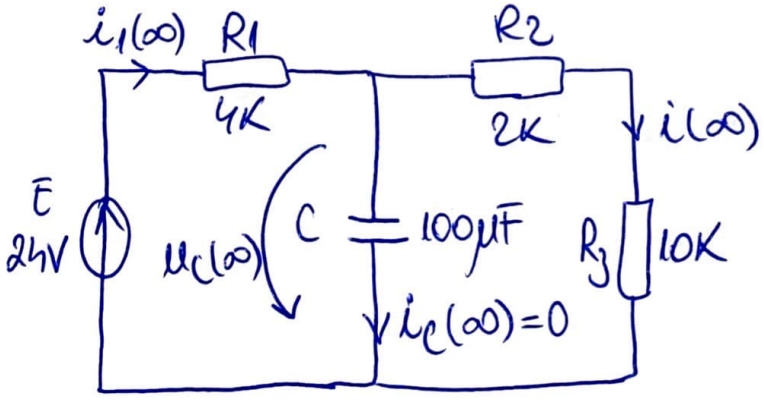
$t < 0$



$u_C(0_-) = 24V$

$i_C(0) = 0$

$t \to \infty$



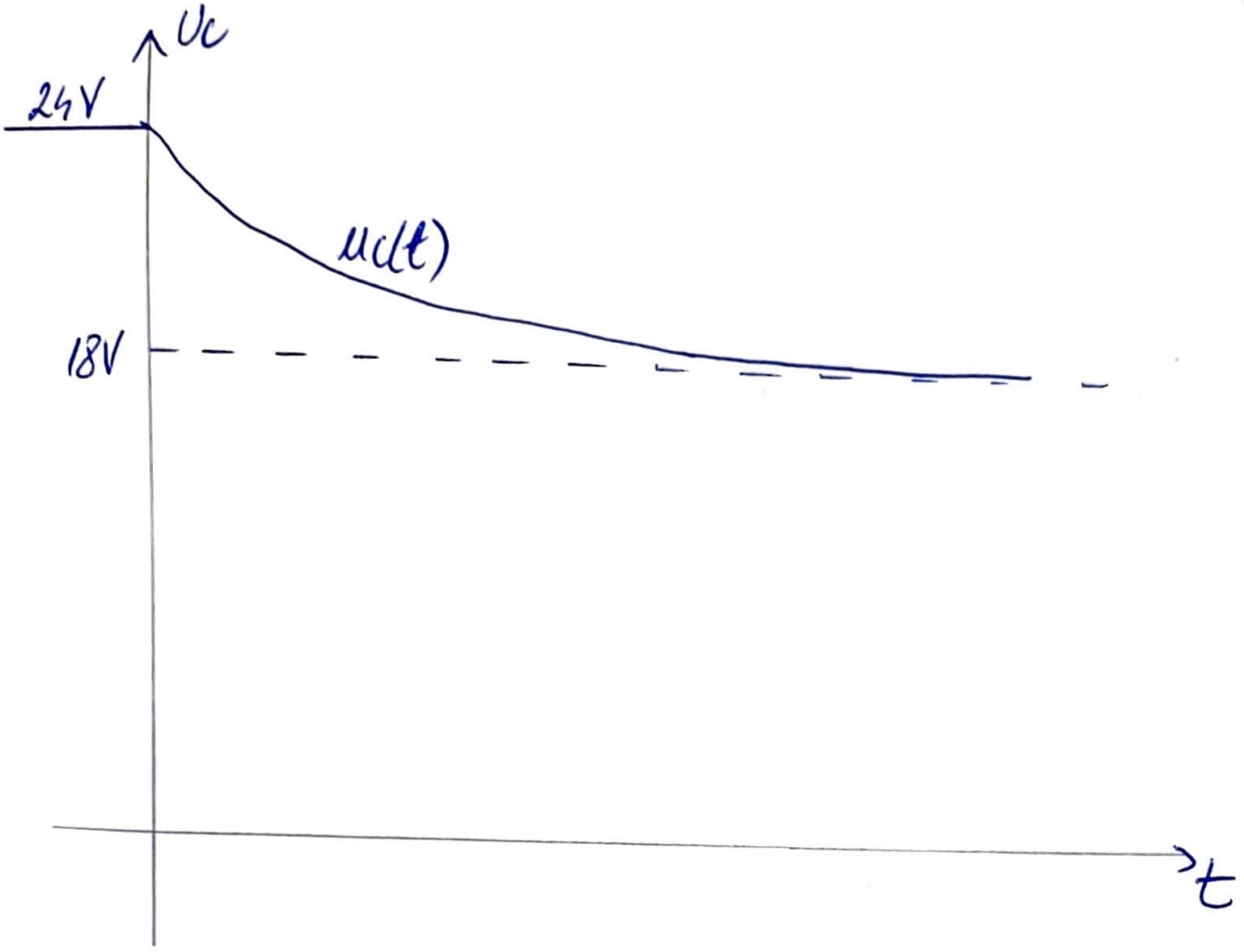
$i_C(\infty) = 0$

$i_1(\infty) = i(\infty) = \frac{E}{R_1+R_2+R_3}$

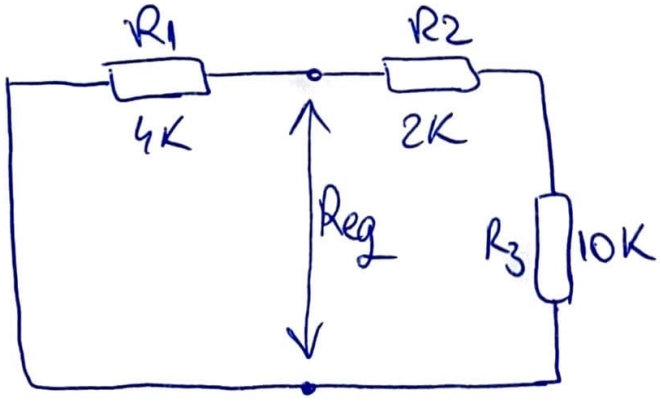
$= \frac{24}{16} = 1.5\,mA$

$u_C(\infty) = i_1(\infty)(R_2+R_3)$

$= 1.5(2+10) = 18V$



$$u_C(t) = u_C(\infty) - (u_C(\infty) - u_C(0))\, e^{-\frac{t}{\tau}}$$



$$R_{eq} = \frac{4 \cdot 12}{4+12} = 3\ K\Omega$$

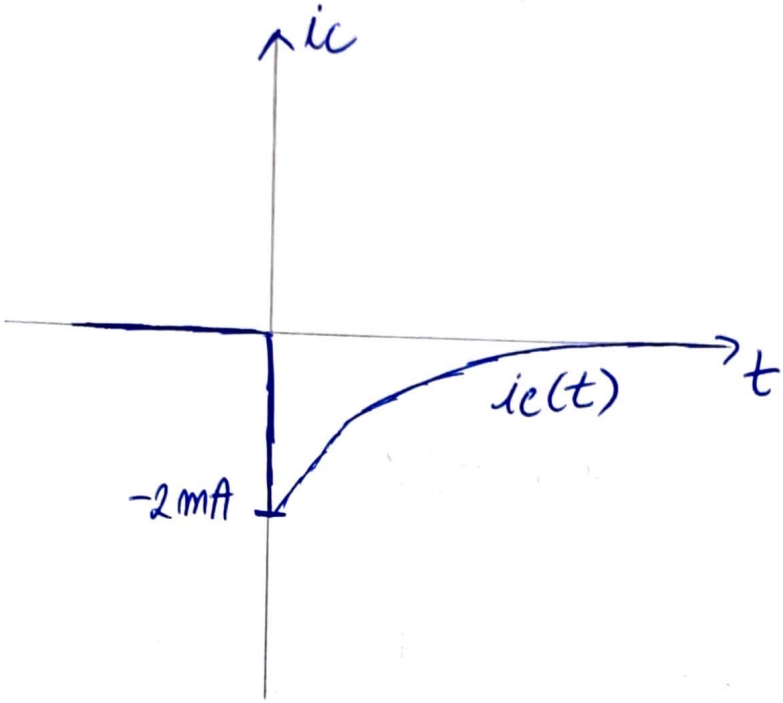
$$\tau = R_{eq} C = 3 \cdot 10^{3} \cdot 0.1 \cdot 10^{-3} = 0.3\ s$$

$$u_C(t) = 18 - (18-24)\, e^{-\frac{t}{0.3}}$$

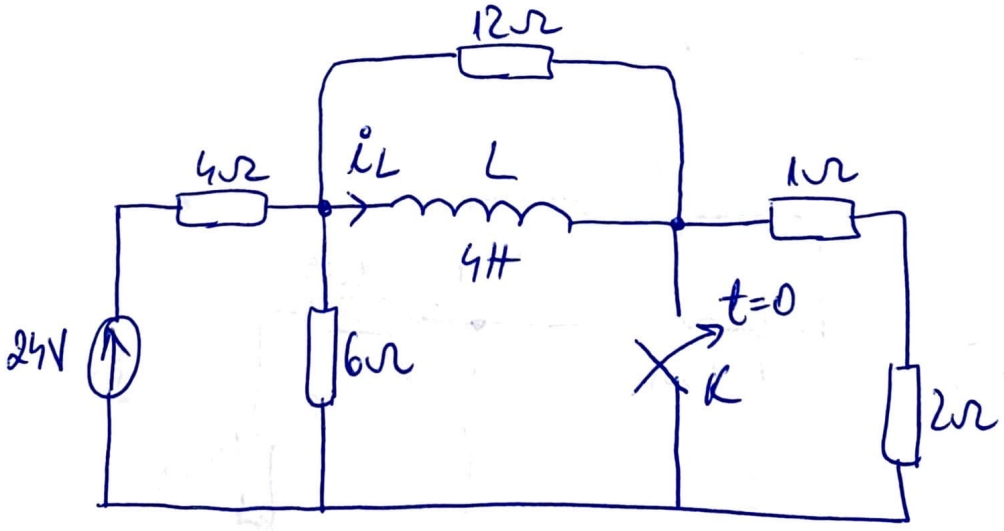
$$u_C(t) = 18 + 6\, e^{-\frac{t}{0.3}}$$

$$i_C(t) = C\frac{du_C}{dt} = 0.1 \cdot 10^{-3} \cdot 6\left(-\frac{1}{0.3}\right)e^{-\frac{t}{0.3}}$$

$$= -2e^{-\frac{t}{0.3}} \quad mA$$

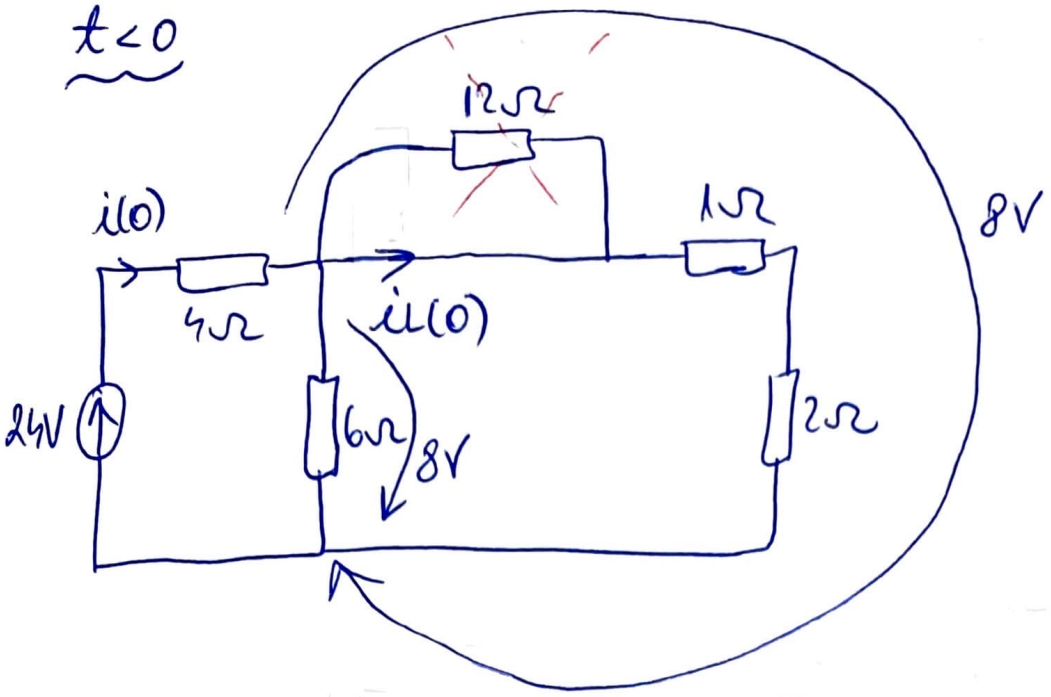


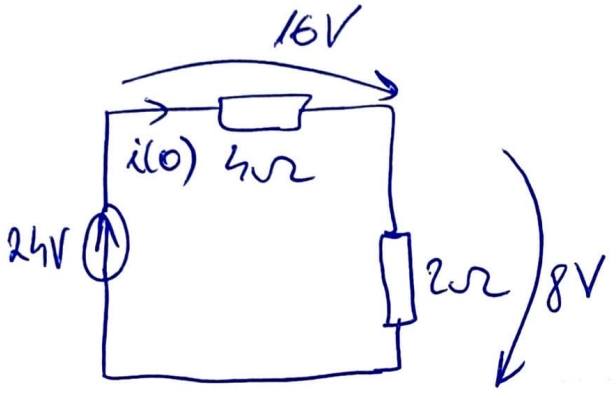
2.



$$i_L(t) = i_L(\infty) - \left(i_L(\infty) - i_L(0)\right)e^{-\frac{t}{\tau}} \quad , \quad \tau = \frac{L}{R_{eq}}$$

$t<0$

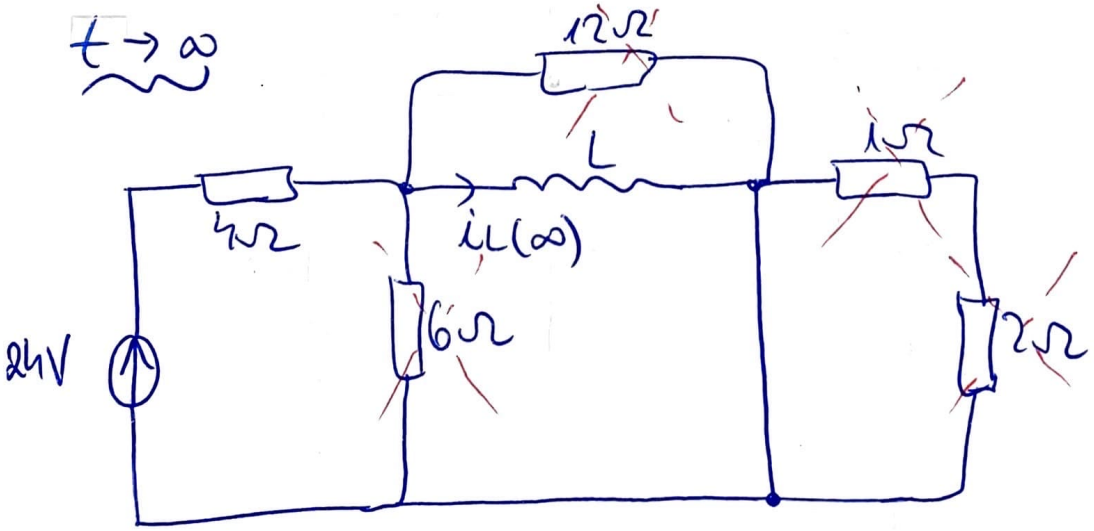




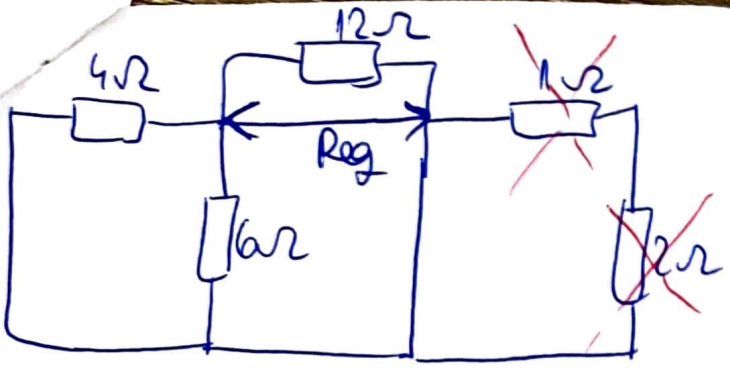
$$i(0) = \frac{24}{6} = 4\,A$$

$$i_L(0) = \frac{8}{3}\,A$$

$t \to \infty$



$$i_L(\infty) = \frac{24}{4} = 6A$$



$$\frac{1}{Req} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}$$

$$Req = 2\Omega$$

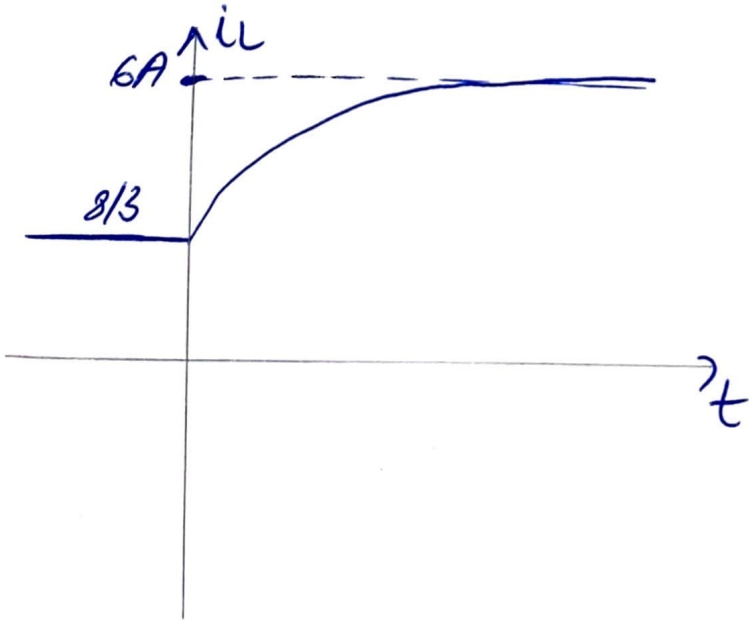
$$\tau = \frac{4}{2} = 2s$$

$$i_L(t) = 6 - \left(6 - \frac{8}{3}\right) e^{-\frac{t}{2}}$$

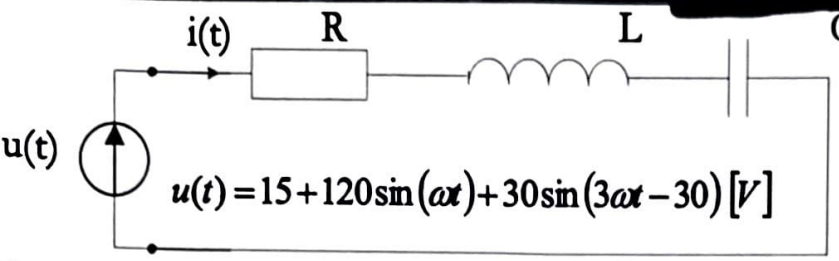
$$= 6 - \frac{10}{3} e^{-\frac{t}{2}} \; A$$

$$i_L(0) = \frac{8}{3} A$$

$$i_L(\infty) = 6A$$



Example

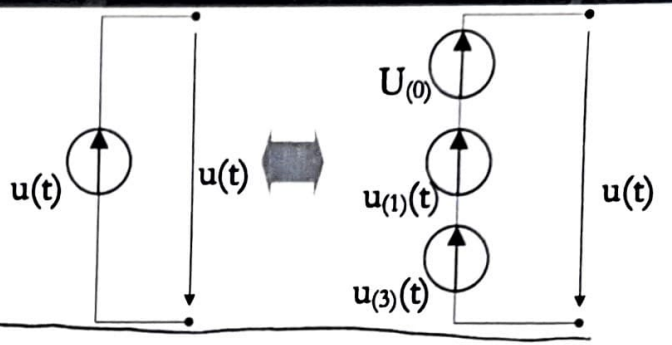


$$u(t)=15+120\sin(\omega t)+30\sin(3\omega t-30)\,[V]$$

$R=6\Omega$

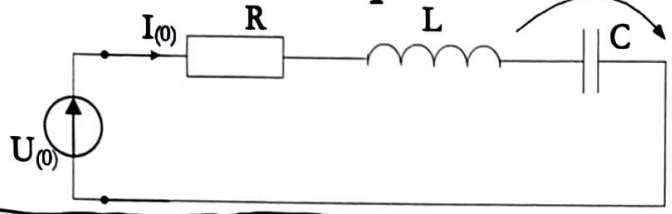
$\omega L=1\Omega$

$1/\omega C=9\Omega$



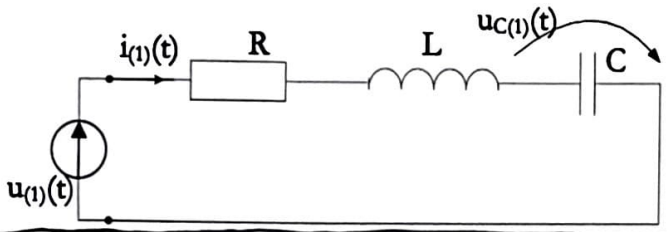
$I=?\qquad u_C(t)=?$

For the DC component:



$$U_{(0)}=15V,\qquad I_{(0)}=0\,A,\qquad U_{C(0)}=U_{(0)}=15V$$

For the fundamental component:

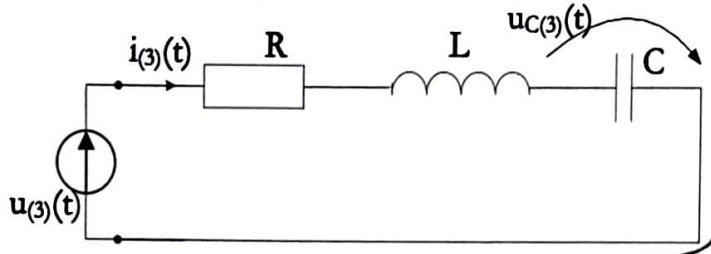


$$u_1(t)=120\sin(\omega t)\;\rightarrow\;\underline{U}_{(1)}=\frac{120}{\sqrt{2}}$$

$$\underline{I}_{(1)}=\frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}}=\frac{120}{\sqrt{2}\left[R+j\left(\omega L-\frac{1}{\omega C}\right)\right]}=\frac{120}{\sqrt{2}\,[6+j(1-9)]}=8.485e^{j53}$$

$$\underline{U}_{C(1)}=\frac{-j}{\omega C}\underline{I}_{(1)}=-9j\,8.485e^{j53}=76.365e^{-j37}$$

For the 3th order harmonic:



$$u_3(t)=30\sin(\omega t-30^0)\;\rightarrow\;\underline{U}_{(3)}=\frac{30}{\sqrt{2}}e^{-j30}$$

$$\underline{I}_{(3)}=\frac{\underline{U}_{(3)}}{\underline{Z}_{(3)}}=\frac{30e^{-j30}}{\sqrt{2}\left[R+j\left(3\omega L-\frac{1}{3\omega C}\right)\right]}=\frac{30e^{-j30}}{\sqrt{2}\,[6+j(3-3)]}=3.536e^{-j30}$$

$$\underline{U}_{C(3)}=\frac{-j}{3\omega C}\underline{I}_{(3)}=-3j\,3.536e^{-j30}=10.608e^{-j120}$$

$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(3)}^2}=\sqrt{0^2+8.485^2+3.536^2}=9.192\;(A)$$

$$u_C(t)=U_{(0)}+u_{C(1)}(t)+u_{C(3)}(t)=15+76.365\sqrt{2}\sin(\omega t-37^0)+10.608\sqrt{2}\sin(3\omega t-120^0)\,[V]$$

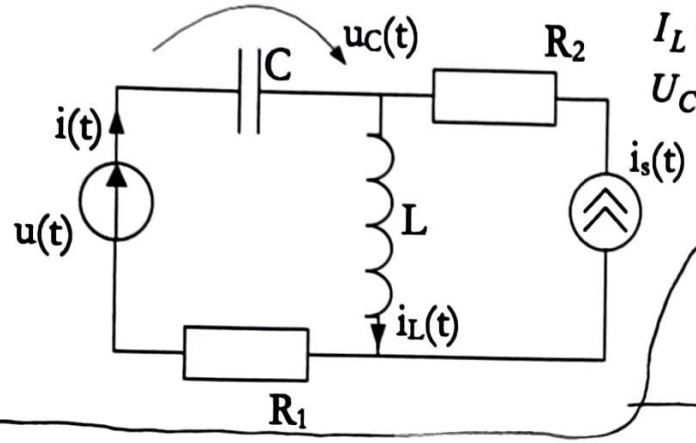
FOURIER

DC: - prin condensator nu trece curent

- bobina e considerată ca fir (nu are rezist.)

$\varphi_1 = \varphi_{U1} - \varphi_{I1}$ - for the fundamental

- same for 3rd, 4th, ... armonics.



$I_L = ?,\ i_L(t) = ?$

$U_C = ?,\ u_C(t) = ?$

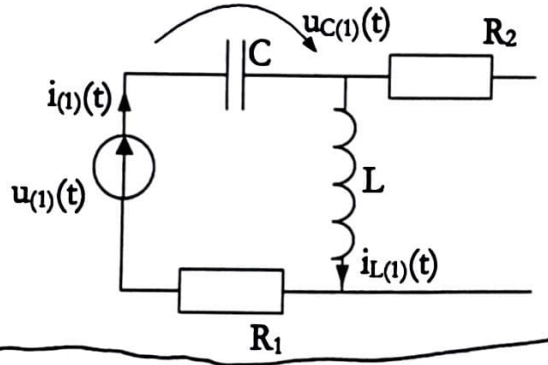
$R1=5\Omega,\ R2=\omega L=1/\omega C=10\Omega$

$u(t) = 100 + 50\sin(\omega t - 30)\,[V] \qquad i_s(t) = 10 + 5\sin(2\omega t)\,[A]$

For the DC component: $U_{(0)} = 100V, \quad I_{s(0)} = 10A$

$$U_{C(0)} = U_{(0)} = 100V, \quad I_{L(0)} = I_{s(0)} = 10A$$

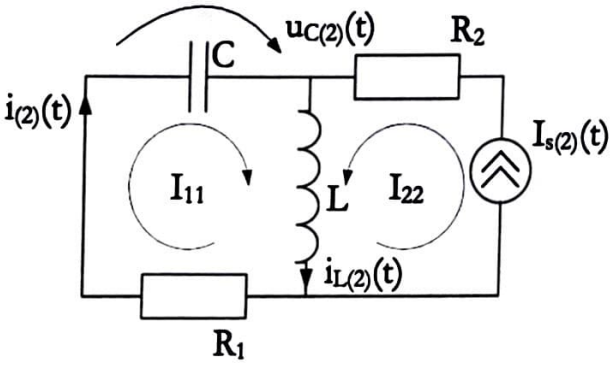
For the fundamental component: $u_1(t) = 50\sin(\omega t - 30^0) \rightarrow \underline{U}_{(1)} = \frac{50e^{-j30}}{\sqrt{2}}$



$$\underline{I}_{(1)} = \underline{I}_{L(1)} = \frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}} = \frac{50e^{-j30}}{\sqrt{2}\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]} = \frac{50e^{-j30}}{\sqrt{2}\left[5 + j(10-10)\right]} = \frac{10}{\sqrt{2}}e^{-j30}$$

$$\underline{U}_{C(1)} = \frac{-j}{\omega C}\underline{I}_{(1)} = -10j\frac{10}{\sqrt{2}}e^{-j30} = \frac{100}{\sqrt{2}}e^{-j120}$$

For the 2th order harmonic:



$$i_{s(2)}(t) = 5\sin(2\omega t) \rightarrow \underline{I}_{s(2)} = \frac{5}{\sqrt{2}}$$

$$\begin{cases} \underline{I}_{22} = \underline{I}_{s(2)} = \frac{5}{\sqrt{2}} \\ \underline{I}_{11}\left[R_1 + j2\omega L - \frac{j}{2\omega C}\right] + j2\omega L\,\underline{I}_{22} = 0 \end{cases}$$

$$\underline{I}_{(2)} = \underline{I}_{11} = -4.243 - j1.414 = 4.472e^{-j162}$$

$$\underline{I}_{L(2)} = \underline{I}_{11} + \underline{I}_{22} = -0.707 - j1.414 = 1.581e^{-j117}$$

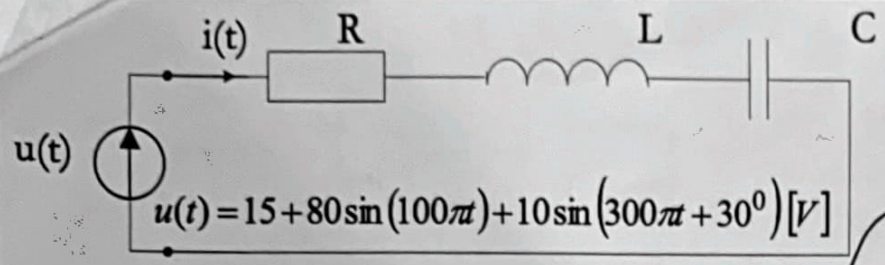
$$\underline{U}_{C(2)} = \frac{-j}{2\omega C}\underline{I}_{(2)} = -7.071 + j21.213 = 108.435e^{j108}$$

$$I = \sqrt{I_{(0)}^2 + I_{(1)}^2 + I_{(2)}^2} = \sqrt{10^2 + \left(\frac{10}{\sqrt{2}}\right)^2 + 1.581^2} = 12.349\ [A]$$

$$i_L(t) = I_{L(0)} + i_{L(1)}(t) + i_{L(2)}(t) = 10 + 10\sin\left(\omega t - 30^0\right) + 1.581\sqrt{2}\sin\left(2\omega t - 117^0\right)[A]$$

$$U_C = \sqrt{U_{C(0)}^2 + U_{C(1)}^2 + U_{C(2)}^2} = \sqrt{100^2 + \left(\frac{100}{\sqrt{2}}\right)^2 + 108.435^2} = 163.579\ (V)$$

$$u_C(t) = U_{C(0)} + u_{C(1)}(t) + u_{C(2)}(t) = 100 + 100\sin\left(\omega t - 120^0\right) + 108.435\sqrt{2}\sin\left(2\omega t - 108^0\right)[V]$$



$R=6\Omega$ $\quad U=?$

$L=100mH$ $\quad u_C(t)=?$

$C=200\mu F$ $\quad P=?,\ Q=?,\ D=?$

$$U=\sqrt{U_{(0)}^2+U_{(1)}^2+U_{(3)}^2}=\sqrt{15^2+\left(\frac{80}{\sqrt{2}}\right)^2+\left(\frac{10}{\sqrt{2}}\right)^2}=59\ (V)$$

For the DC component: $U_{(0)}=15V,\quad I_{(0)}=0\,A,\quad U_{C(0)}=U_{(0)}=15V$

For the fundamental component: $u_1(t)=80\sin(100\pi t)\ \rightarrow\ \underline{U}_{(1)}=\frac{80}{\sqrt{2}}$

$$X_{L(1)}=\omega L=100\pi\,0.1=31.42(\Omega)$$

$$X_{C(1)}=\frac{1}{\omega C}=\frac{1}{100\pi\,200\cdot10^{-6}}=15.92(\Omega)$$

$$\underline{I}_{(1)}=\frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}}=\frac{\underline{U}_{(1)}}{R+j\left(X_{L(1)}-X_{C(1)}\right)}=3.07\,e^{-j57}$$

$$\underline{U}_{C(1)}=-jX_{C(1)}\underline{I}_{(1)}=48.81e^{-j147}$$

For the 3th order harmonic: $u_3(t)=10\sin(\omega t+30^0)\ \rightarrow\ \underline{U}_{(3)}=\frac{10}{\sqrt{2}}e^{j30}$

$$X_{L(3)}=3\omega L=3\cdot100\pi\,0.1=94.25(\Omega)$$

$$X_{C(3)}=\frac{1}{3\omega C}=\frac{1}{3\cdot100\pi\,200\cdot10^{-6}}=5.31(\Omega)$$

$$\underline{I}_{(3)}=\frac{\underline{U}_{(3)}}{\underline{Z}_{(3)}}=\frac{\underline{U}_{(3)}}{R+j\left(X_{L(3)}-X_{C(3)}\right)}=0.08\,e^{-j53}$$

$$\underline{U}_{C(3)}=-jX_{C(3)}\underline{I}_{(3)}=0.42\,e^{-j143}$$

$$u_C(t)=U_{(0)}+u_{C(1)}(t)+u_{C(3)}(t)=15+48.81\sqrt{2}\sin\left(100\pi t-147^0\right)+0.42\sqrt{2}\sin\left(300\pi t-143^0\right)[V]$$

$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(3)}^2}=\sqrt{0^2+3.07^2+0.08^2}\cong3.07\ (A)$$

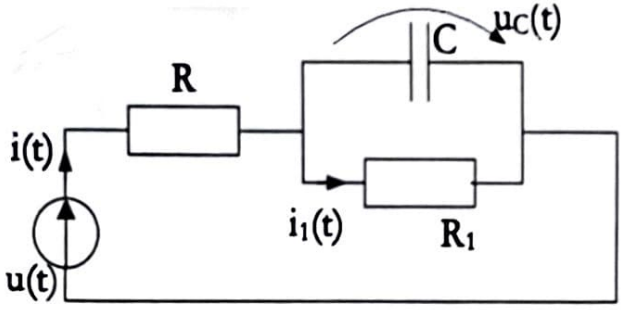
$$S=UI=59\cdot3.07=180.83[VA]$$

$$P=U_{(0)}I_{(0)}+U_{(1)}I_{(1)}\cos\varphi_1+U_{(3)}I_{(3)}\cos\varphi_3=\frac{80}{\sqrt{2}}3.07\cos(0-(-57))+\frac{10}{\sqrt{2}}0.08\cos(30-(-53))=94.11[W]$$

$$P=I^2R=94.11[W]$$

$$Q=U_{(1)}I_{(1)}\sin\varphi_1+U_{(3)}I_{(3)}\sin\varphi_3=\frac{80}{\sqrt{2}}3.07\sin(0-(-57))+\frac{10}{\sqrt{2}}0.08\sin(30-(-53))=146.33[VAR]$$

$$D=\sqrt{S^2-P^2-Q^2}=49.34\ [VAD]$$



$$u(t)=50+100\sin\left(400t+\frac{\pi}{6}\right)+50\sin\left(1200t-\frac{\pi}{4}\right)[V]$$

$U=?$

$I=?,\ I_1=?$

$u_C(t)=?$

$P=?,\ Q=?,\ D=?$

$R=30\Omega,\ R_1=50\Omega,\ C=50\mu F$

For the DC component:

$$U_{(0)}=50V,\quad I_{(0)}=I_{1(0)}=\frac{U_{(0)}}{R+R_1}=0.625\,A\qquad U_{C(0)}=I_{1(0)}R_1=31.25V$$

For the fundamental component:

$$u_1(t)=100\sin\left(400t+\frac{\pi}{6}\right)\ \rightarrow\ \underline{U}_{(1)}=\frac{100}{\sqrt{2}}e^{j\frac{\pi}{6}}$$

$$X_{C(1)}=\frac{1}{\omega C}=\frac{1}{400\cdot50\cdot10^{-6}}=50(\Omega)\qquad \underline{Z}_{p(1)}=\frac{R_1\left(-jX_{C(1)}\right)}{R_1-jX_{C(1)}}=25-25j\ (\Omega),\quad \underline{Z}_{e(1)}=R+\underline{Z}_{p(1)}=55-25j\ (\Omega)$$

$$\underline{I}_{(1)}=\frac{\underline{U}_{(1)}}{\underline{Z}_{e(1)}}=0.681+j0.952=1.17e^{j54}\qquad \underline{U}_{C(1)}=\underline{I}_{(1)}\underline{Z}_{p(1)}=40.819+j6.79=41.38e^{j9}\qquad \underline{I}_{1(1)}=\frac{\underline{U}_{C(1)}}{R_1}=0.816+j0.136=0.828e^{j9}$$

For the 3th order harmonic:

$$u_3(t)=50\sin\left(1200t-\frac{\pi}{4}\right)\ \rightarrow\ \underline{U}_{(3)}=\frac{50}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

$$X_{C(3)}=\frac{1}{3\omega C}=\frac{1}{1200\cdot50\cdot10^{-6}}=16.667(\Omega)\qquad \underline{Z}_{p(3)}=\frac{R_1\left(-jX_{C(3)}\right)}{R_1-jX_{C(3)}}=5-15j\ (\Omega),\quad \underline{Z}_{e(1)}=R+\underline{Z}_{p(1)}=35-15j\ (\Omega)$$

$$\underline{I}_{(3)}=\frac{\underline{U}_{(3)}}{\underline{Z}_{e(3)}}=0.862-j0.345=0.928e^{-j22}\quad \underline{U}_{C(3)}=\underline{I}_{(3)}\underline{Z}_{p(3)}=-0.862-j14.655=14.68e^{-j93}\qquad \underline{I}_{1(3)}=\frac{\underline{U}_{C(3)}}{R_1}=-0.017-j0.293=0.294e^{-j93}$$

$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(3)}^2}=\sqrt{0.625^2+1.17^2+0.928^2}\cong1.619\ (A)\qquad I_1=\sqrt{I_{1(0)}^2+I_{1(1)}^2+I_{1(3)}^2}=\sqrt{0.625^2+0.828^2+0.294^2}\cong1.078\ (A)$$

$$u_C(t)=U_{(0)}+u_{C(1)}(t)+u_{C(3)}(t)=31.25+41.38\sqrt{2}\sin\left(400t+9^0\right)+14.68\sqrt{2}\sin\left(300\pi t-93^0\right)[V]$$

$$P=U_{(0)}I_{(0)}+U_{(1)}I_{(1)}\cos\varphi_1+U_{(3)}I_{(3)}\cos\varphi_3=50\cdot0.625+\frac{100}{\sqrt{2}}1.17\cos\left(-24^0\right)+\frac{50}{\sqrt{2}}0.928\cos\left(-23^0\right)=136.765[W]$$

$$P=I^2R+I_1^2R_1=136.765W$$

$$Q=U_{(1)}I_{(1)}\sin\varphi_1+U_{(3)}I_{(3)}\sin\varphi_3=\frac{100}{\sqrt{2}}1.17\sin\left(-24^0\right)+\frac{50}{\sqrt{2}}0.928\sin\left(-23^0\right)=-47.178[VAR]$$

$$U=\sqrt{U_{(0)}^2+U_{(1)}^2+U_{(3)}^2}=\sqrt{50^2+\left(\frac{100}{\sqrt{2}}\right)^2+\left(\frac{50}{\sqrt{2}}\right)^2}=93.541\ (V)$$

$$S=UI=93.541\cdot1.619=151.484[VA]$$

$$D=\sqrt{S^2-P^2-Q^2}=45\ [VAD]$$