1. Rotational Dynamics

Important Formulae and Shortcut methods

1. For uniformly accelerated motion, i.e. for constant 'α'

(a) $\omega = \omega_0 + \alpha t$

(b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \left(\frac{\omega + \omega_0}{2}\right)t$

(c) $\omega^2 = \omega_0^2 + 2\alpha\theta$

2. When a cyclist goes along a horizontal curved road of radius of curvature, 'r' :

(a) inclination to the vertical is 'θ',

$\therefore \tan\theta = \frac{v^2}{rg}$, where 'v' is its speed.

(b) the maximum velocity 'v', with which it can go so that there is no skidding, is

$v = \sqrt{\mu rg}$

where μ = coefficient of limiting friction between the wheels and the road.

3. When a vehicle goes along a horizontal curved road or level road of radius of curvature 'r'.

(a) the maximum velocity with which it can go without toppling, is given by

$$v = \sqrt{rg\frac{d}{2h}} = \sqrt{rg\tan\theta}$$

where, $\tan\theta = \frac{d}{2h}$, d = distance between the wheels.

h = height of centre of gravity from the road

g = acceleration due to gravity

(b) the maximum velocity so that there is no skidding, is given by

$v = \sqrt{\mu rg}$

where, μ = coefficient of friction between the wheels and road.

4. Banking of Roads

(a) The proper velocity or optimum 'v' on a road banked by an angle 'θ' with the horizontal is given by,

$v = \sqrt{rg\tan\theta}$

where, r = radius of curvature of road, g = acceleration due to gravity

(b) $v_{max} = \sqrt{\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}}$ $\quad v_{min} = \sqrt{\frac{rg(\mu_s - \tan\theta)}{1 + \mu_s\tan\theta}}$

5. Vertical circle

(i) v_1 = velocity at highest point $\geq \sqrt{rg}$

Minimum velocity at highest point = $\sqrt{rg}$

(ii) v_2 = Velocity at the lowest point = $\sqrt{v_1^2 + 4rg} \geq \sqrt{5rg}$

Minimum velocity at the lowest point = $\sqrt{5rg}$

(iii) Tension along the string

(a) at highest point, $T_1 = \frac{mv_1^2}{r} - mg \geq 0$

(b) at lowest point, $T_2 = \frac{mv_2^2}{r} + mg \geq 6mg$

(c) at a position where the string makes an angle 'θ' with the lower vertical line,

$$\frac{mv^2}{r} + mg\cos\theta$$

For highest point, $\theta = \pi$. For lowest point, $\theta = 0°$

6. Conical pendulum

(a) Angular velocity $= \omega = \sqrt{\frac{g}{L\cos\theta}}$

(b) Periodic time $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

(c) F = tension $\quad \therefore$ $F\cos\theta = mg \qquad F\sin\theta = \frac{mv^2}{r}$

(d) $r = L\sin\theta$

7. $I = \sum_{i=1}^{N} m_i r_i^2$ (system of particles) and $I = \int r^2 dm$ (rigid body)

8. $I = Mk^2 \qquad k = \sqrt{I/M}$

9. $I = I_{CM} + Mh^2, I_z = I_x + I_y$

10. MI and radius of gyration of some regular bodies of uniform density :

Body	Rotational axis	Moment of inertia	Radius a gyration
Rod	Transverse, through CM	$\frac{1}{12}ML^2$	$\frac{1}{2\sqrt{3}}L$
	Transverse, through an end	$\frac{1}{3}ML^2$	$\frac{1}{\sqrt{3}}L$
Ring	Transverse, through CM	MR^2	R
	Transverse, through an end	$2MR^2$	$\sqrt{2}R$
	Diameter	$\frac{1}{2}MR^2$	$\frac{1}{\sqrt{2}}R$
	Tangent in its plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
Disc	Transverse, through CM	$\frac{1}{2}MR^2$	$\frac{1}{\sqrt{2}}R$
	Transverse, tangent	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
	Diameter	$\frac{1}{4}MR^2$	$\frac{1}{2}R$
	Tangent in its plane	$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$

Body	Rotational axis	Moment of inertia	Radius a gyration
Solid cylinder	Transverse, through CM	$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{12}}$
	Transverse, through an end	$M\left(\frac{R^2}{4}+\frac{L^2}{3}\right)$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{3}}$
	Cylinder axis	$\frac{1}{2}MR^2$	$\frac{1}{\sqrt{2}}R$
Solid sphere	Diameter	$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
	Tangent	$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$

11. $\tau = I\omega$

12. $E_{rot} = \frac{1}{2}I\omega^2 = 2\pi^2 If^2 = 2\pi^2 \frac{I}{T^2} = \frac{1}{2}L\omega$

13. Work done by a constant external torque,

$$W = \tau\theta = \Delta KE_{rotational} = \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$$

Power, $P = \tau\omega$

14. $\vec{L} = I\vec{\omega}$

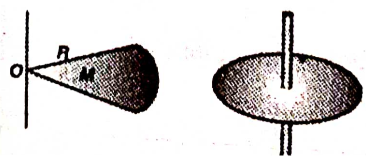
15. $\vec{\tau}_{external} = \frac{d\vec{L}}{dt} = \vec{L}$ is conserved if $\tau_{external} = 0 : I_1\vec{\omega}_1 = I_2\vec{\omega}_2$

Rotational Motion

- Theorem of parallel axes is application for any type of rigid body whether it is a two dimensional or three dimensional, while the theorem of perpendicular axes is applicable for laminar type or two dimensional bodies only.
- The point of intersection of three (X, Y and Z) axes, in theorem of perpendicular axes, may be any point on the plane of body (it may even lie outside the body). This point may or may not be the centre of mass.
- Moment of inertia of a symmetrically cut part of a rigid body has same form as that of the whole body. Example in figure (a) moment of inertia of a sector of circular disc shown about an axis perpendicular to its axis plane and passing through point 'O' is $\frac{1}{2}M_1R^2$ as the moment of inertia of the complete disc is also $\frac{1}{2}M_2R^2$
  (where M_1 is mass of the sector and M_2 is mass of the complete disc). This can be shown as in figure. If 'M' is the mass of $\frac{1}{n}^{th}$ part of the disc then mass of the disc = nM

$$I_{disc} = \frac{1}{2}(nM)R^2$$

$$I_{sector} = \frac{1}{n}I_{disc} = \frac{1}{2}MR^2$$



- Suppose a rod is suspended from a support at 'O' and particle strikes the rod at any point. The angular momentum of the 'rod + particle' system remains conserved only about point of suspension or point 'O'. Because in this case τ_{ext} on the system is zero only about 'O'.

- Work done by friction in pure rolling on a stationary ground is zero as the point of application of the force is at rest.

Multiple Choice Questions

MHT-CET 2004

1. A disc of moment of inertia $\frac{9.8}{\pi^2}$ kg-m^2 is rotating at 600 rpm. If the frequency of rotation changes from 600 rpm to 300 rpm, then what is the work done?
   (A) 1467 J (B) 1452 J (C) 1567 J (D) 1632 J

MHT-CET 2005

2. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body.
   (A) remains constant (B) becomes half
   (C) doubles (D) quadruples

3. Moment of inertia of a disc about an axis which is tangent and parallel to its plane is I Then, the moment of inertia of disc about a tangent, but perpendicular to its plane will be
   (A) $\frac{3I}{4}$ (B) $\frac{5I}{6}$ (C) $\frac{3I}{2}$ (D) $\frac{6I}{5}$

4. The angle of banking is independent of
   (A) speed of vehicle (B) radius of curvature of road
   (C) height of inclination (D) None of these

MHT-CET 2006

5. A body is just being revolved in a vertical circle of radius R with a uniform speed. The string breaks when the body is at the highest point. The horizontal distance covered by the body after the string breaks is
   (A) 2R (B) R (C) $R\sqrt{2}$ (D) 4R

6. The moment of inertia of a solid sphere about an axis passing through centre of gravity is 2/5 MR^2, then its radius of gyration about a parallel axis at a distance 2 R from first axis is
   (A) 5 R (B) $\sqrt{\frac{22}{5}}R$ (C) $\frac{5}{2}R$ (D) $\sqrt{\frac{12}{5}}R$

MHT-CET 2007

7. The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at its diameter and normal to the disc is
   (A) MR^2 (B) $\frac{2}{5}MR^2$ (C) $\frac{3}{2}MR^2$ (D) $\frac{1}{2}MR^2$

MHT-CET 2008

8. When a ceiling fan is switched OFF, its angular velocity falls to half while it makes 36 rotations. How many more rotations will it make before coming to rest?
(A) 24 (B) 36 (C) 18 (D) 12

9. Two spheres of equal masses, one of which is a thin spherical shell and the other a solid, have the same moment of inertia about their respective diameters. The ratio of their radii will be.
(A) 5:7 (B) 3:5 (C) $\sqrt{3}:\sqrt{5}$ (D) $\sqrt{3}:\sqrt{7}$

MHT-CET 2009

10. From a disc of radius R, a concentric circular portion of radius r is cut out, so as to leave an annular disc of mass M. The moment of inertia of this annular disc about the axis perpendicular to its plane and passing through its centre of gravity is
(A) $\frac{1}{2}M\left(R^2+r^2\right)$ (B) $\frac{1}{2}M\left(R^2-r^2\right)$ (C) (D) $\frac{1}{2}M\left(R^4-r^4\right)$

11. Moment of inertia of a rod of mass M and length L about an axis passing through a point midway between centre and end is
(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{12}$ (C) $\frac{7ML^2}{24}$ (D) $\frac{7ML^2}{48}$

12. Moment of inertia of big drop is I. If 8 droplets are formed from big drop, then moment of inertia of small droplet is
(A) $\frac{I}{32}$ (B) $\frac{I}{16}$ (C) $\frac{I}{8}$ (D) $\frac{I}{4}$

13. The moments of inertia of two freely rotating bodies A and B are I_A and I_B, respectively. $I_A > I_B$ and their angular momenta are equal. If K_A and K_B are their kinetic energies, then
(A) $K_A = K_B$ (B) $K_A \neq K_B$ (C) $K_A < K_B$ (D) $K_A = 2K_B$

14. A particle of mass m is rotating in a plane in circular path of radius r. Its angular momentum is L. The centripetal force acting on the particle is
(A) $\frac{L^2}{mr}$ (B) $\frac{L^2m}{r}$ (C) $\frac{L^2}{m^2r^2}$ (D) $\frac{L^2}{mr^3}$

15. A car is moving with speed 30 ms^{-1} on a circular path of radius 500 m. Its speed is increasing at a rate of 2 ms^{-2}, what is the acceleration of the car?
(A) 2 ms^{-2} (B) 2.7 ms^{-2} (C) 1.82 ms^{-2} (D) 9.82 ms^{-2}

MHT-CET 2010

16. If α is angular acceleration, ω is angular velocity and a is the centripetal acceleration then, which of the following is true?
(A) $\alpha = \frac{\omega a}{v}$ (B) $\alpha = \frac{v}{\omega a}$ (C) $\alpha = \frac{av}{\omega}$ (D) $\alpha = \frac{a}{\omega v}$

17. If KE of the particle of mass m performing UCM in a circle of radius r is E. Find the acceleration of the particle.
(A) $\frac{2E}{mr}$ (B) $\left(\frac{2E}{mr}\right)^2$ (C) 2Emr (D) $\frac{4E}{mr}$

18. A wheel has a speed of 1200 revolutions per minute and is made to slow down at a rate of 4 rad s^{-2}. The number of revolutions it makes before coming to rest is
(A) 143 (B) 272 (C) 314 (D) 722

19. When a disc is rotating with angular velocity ω, a particle situated at a distance of 4 cm just begins to slip. If the angular velocity is doubled, at what distance will the particle start to slip?
(A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm

20. Moment of inertia of a disc about a diameter is I. Find the moment of inertia of disc about an axis perpendicular to its plane and passing through its rim?
(A) 6 I (B) 4 I (C) 2 I (D) 8 I

21. The moment of inertia of a thin uniform rod of length L and mass M about an axis passing through a point at a distance of 1/3 from one of its ends and perpendicular to the rod is
(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{9}$ (C) $\frac{7ML^2}{48}$ (D) $\frac{ML^2}{48}$

22. Which relation is not correct of the following?
(A) Torque = Moment of inertia × angular acceleration
(B) Torque = Dipole moment × magnetic induction
(C) Moment of inertia = Torque × angular acceleration
(D) Angular momentum = Moment of inertia × angular velocity

MHT-CET 2011

23. A circular disc is to be made by using iron and aluminium, so that it acquires greater moment of inertia about its geometrical axis. It is possible with
(A) iron and aluminium layers in alternate order
(B) aluminium at interior and iron surrounding it
(C) iron at interior and aluminium surrounding it
(D) Either (a) or (c)

24. A sphere is suspended by a thread of length l. What minimum horizontal velocity has to be imparted the ball for it to reach the height of the suspension?
(A) gl (B) 2gl (C) $\sqrt{gl}$ (D) $\sqrt{2gl}$

25. If the body is moving in a circle of radius r with a constant speed v. Its angular velocity is
(A) v^2/r (B) vr (C) v/r (D) r/v

26. A car of mass 1500 kg is moving with a speed of 12.5 ms^{-1} on a circular path of radius 20 m on a level road. What should be the coefficient of friction between the car and the road, so that the car does not slip?
(A) 0.2 (B) 0.4 (C) 0.6 (D) 0.8

MHT-CET 2013

*27. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is
(A) ring (B) solid sphere (C) hollow sphere (D) disc

28. A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in the figure. When string is cut, the initial angular acceleration of the rod is

(A) $\frac{3g}{2L}$ (B) $\frac{g}{L}$ (C) $\frac{2g}{L}$ (D) $\frac{2g}{3L}$

MHT-CET 2014

29. Three identical spheres each of mass 1 kg are placed touching one another with their centres in a straight line. Their centres are marked as A, B, C, respectively. The distance of centre of mass of the system from A is

(A) $\frac{AB+AC}{2}$ (B) $\frac{AB+BC}{2}$ (C) $\frac{AC-AB}{3}$ (D) $\frac{AB+AC}{3}$

*30. An object of radius 'R' and mass 'M' is rolling horizontally without slipping with speed 'V'. It then rolls up the hill to a maximum height $h = \frac{3v^2}{4g}$. The moment of inertia of the object is (g = acceleration due to gravity)

(A) $\frac{2}{5}MR^2$ (B) $\frac{MR^2}{2}$ (C) MR^2 (D) $\frac{3}{2}MR^2$

31. The moment of inertia of a thin uniform rod rotating about the perpendicular axis passing through one end is 'I'. The same rod is bent into a ring and its moment of inertia about the diameter is 'I_1'. The ratio $\frac{I}{I_1}$ is

(A) $\frac{4\pi}{3}$ (B) $\frac{8\pi^2}{3}$ (C) $\frac{5\pi}{3}$ (D) $\frac{8\pi^2}{5}$

[MHT-CET 2015

32. A toy cart is tied to the end of an unstretched string of length 'λ'. When revolved, the toy cart moves in horizontal circle with radius '2λ' and time period T. If it is speeded until it moves in horizontal circle of radius '3λ' with period T_1, relation between T and T_1 is (Hooke's law is obeyed)

(A) $T_1 = \frac{2}{\sqrt{3}}T$ (B) $T_1 = \sqrt{\frac{3}{2}}T$ (C) $T_1 = \sqrt{\frac{2}{3}}T$ (D) $T_1 = \frac{\sqrt{3}}{2}T$

33. A solid cylinder has mass 'M' radius 'R' and length 'ℓ'. Its moment of inertia about an axis passing through its centre and perpendicular to its own axis is

(A) $\frac{2MR^2}{3}+\frac{M\ell^2}{12}$ (B) $\frac{MR^2}{3}+\frac{M\ell^2}{12}$ (C) $\frac{3MR^2}{4}+\frac{M\ell^2}{12}$ (D) $\frac{MR^2}{4}+\frac{M\ell^2}{12}$

34. A cord is wound around the circumference of wheel of radius 'r'. The axis of the wheel is horizontal and moment of inertia about it is 'I'. The weight 'mg' is attached to the end of the cord and falls from rest. After falling through a distance 'h', the angular velocity of the wheel will be

(A) $[mgh]^{1/2}$ (B) $\left[\frac{2\,mgh}{I+2\,mr^2}\right]^{1/2}$ (C) $\left[\frac{2\,mgh}{I+mr^2}\right]^{1/2}$ (D) $\left[\frac{mgh}{I+mr^2}\right]^{1/2}$

35. A particle of mass 'm' is moving in circular path of constant radius 'r' such that centripetal acceleration is varying with time 't' as $K^2 r t^2$ where K is a constant. The power delivered to the particle by the force acting on it is
(A) $m^2 K^2 r^2 t^2$ (B) $mK^2 r^2 t$ (C) $m K^2 r t^2$ (D) $m K r^2 t$

36. A hollow sphere of mass 'M' and radius 'R' is rotating with angular frequency 'ω'. It suddenly stops rotating and 75% of kinetic energy is converted to heat. If 'S' is the specific heat of the material in $\frac{J}{kg}K^{-1}$ then rise in temperature of the sphere is (M.I. of hollow sphere = $\frac{2}{3}MR^2$)
(A) $\frac{R\omega}{4S}$ (B) $\frac{R^2\omega^2}{4S}$ (C) $\frac{R\omega}{2S}$ (D) $\frac{R^2\omega^2}{2S}$

MHT-CET 2016

*37. A ring and a disc roll on the horizontal surface without slipping with same linear velocity. If both have same mass and total kinetic energy of the ring is 4 J then total kinetic energy of the disc is
(A) 3 J (B) 4 J (C) 5 J (D) 6 J

38. A disc of radius 'R' and thickness $\frac{R}{6}$ has moment of inertia 'I' about an axis passing through its centre and perpendicular to its plane. Disc is melted and recast into a solid sphere. The moment of inertia of a sphere about its diameter is
(A) $\frac{I}{5}$ (B) $\frac{I}{6}$ (C) $\frac{I}{32}$ (D) $\frac{I}{64}$

39. Let 'M' be the mass and 'L' be the length of a thin uniform rod. In first case, axis of rotation is passing through centre and perpendicular to the length of the rod. In second case axis of rotation is passing through one end and perpendicular to the length of the rod. The ratio of radius of gyration in first case to second case is
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

40. In vertical circular motion, the ratio of kinetic energy of a particle at highest point to that at lowest point is
(A) 5 (B) 2 (C) 0.5 (D) 0.2

41. A particle moves along a circle of radius 'r' with constant tangential acceleration. If the velocity of the particle is 'v' at the end of second revolution, after the revolution has started then the tangential acceleration is
(A) $\frac{v^2}{8\pi r}$ (B) $\frac{v^2}{6\pi r}$ (C) $\frac{v^2}{4\pi r}$ (D) $\frac{v^2}{2\pi r}$

42. A simple pendulum of length 'ℓ' has maximum angular displacement 'θ'. The maximum kinetic energy of the bob of mass 'm' is (g = acceleration due to gravity)
(A) $mg\ell(1 + \cos\theta)$ (B) $mg\ell(1 + \cos^2\theta)$
(C) $mg\ell(1 - \cos\theta)$ (D) $mg\ell(\cos\theta - 1)$

MHT-CET 2017

43. A wheel of moment of inertia 2 Kg m^2 is rotating about an axis passing through centre and perpendicular to its plane at a speed 60 rad/s. Due to friction, it comes to rest in 5 minutes. The angular momentum of the wheel three minutes before it stops rotating is
(A) 24 Kg m^2/s (B) 48 Kg m^2/s (C) 72 Kg m^2/s (D) 96 Kg m^2/s

***44.** A solid sphere of mass 2 kg is rolling on a frictionless horizontal surface with velocity 6 m/s. It collides on the free end of an ideal spring whose other end is fixed. The maximum compression produced in the spring will be (Force constant of the spring = 36 N/m).
(A) $\sqrt{14}$ m (B) $\sqrt{2.8}$ m (C) $\sqrt{1.4}$ m (D) $\sqrt{0.7}$ m

45. A ceiling fan rotates about its own axis with some angular velocity. When the fan is switched off, the angular velocity becomes $\left(\frac{1}{4}\right)^{th}$ of the original in time 't' and 'n' revolutions are made in that time. The number of revolutions made by the fan during the time interval between switch off and rest are (Angular retardation is uniform)
(A) $\frac{4n}{15}$ (B) $\frac{8n}{15}$ (C) $\frac{16n}{15}$ (D) $\frac{32n}{15}$

46. A disc of moment of inertia 'I_1' is rotating in horizontal plane about an axis passing through a centre and perpendicular to its plane with constant angular speed 'ω_1'. Another disc of moment of inertia 'I_2' having zero angular speed is placed coaxially on a rotating disc. Now both the discs are rotating with constant angular speed 'ω_2'. The energy lost by the initial rotating disc is
(A) $\frac{1}{2}\left[\frac{I_1+I_2}{I_1 I_2}\right]\omega_1^2$ (B) $\frac{1}{2}\left[\frac{I_1 I_2}{I_1-I_2}\right]\omega_1^2$ (C) $\frac{1}{2}\left[\frac{I_1-I_2}{I_1 I_2}\right]\omega_1^2$ (D) $\frac{1}{2}\left[\frac{I_1 I_2}{I_1+I_2}\right]\omega_1^2$

47. For a particle moving in vertical circle, the total energy at different positions along the path
(A) is conserved (B) increases
(C) decreases (D) may increase or decrease

48. A flywheel at rest is to reach an angular velocity of 24 rad/s in 8 second with constant angular acceleration. The total angle turned through during this interval is
(A) 24 rad (B) 48 rad (C) 72 rad (D) 96 rad

MHT-CET 2018

49. A mass attached to one end of a string crosses top–most point on a vertical circle with critical speed. Its centripetal acceleration when string becomes horizontal will be (g = gravitational acceleration)
(A) g (B) 3g (C) 4g (D) 6g

50. In non uniform circular motion, the ratio of tangential to radial acceleration is (r = radius of circle, v = speed of the particle, α = angular acceleration)
(A) $\frac{\alpha^2 r^2}{v}$ (B) $\frac{\alpha^2 r}{v^2}$ (C) $\frac{\alpha r^2}{v^2}$ (D) $\frac{v^2}{r^2\alpha}$

51. A disc has mass 'M' and radius 'R'. How much tangential force should be applied to the rim of the disc so as to rotate with angular velocity 'ω' in time 't'?
(A) $\frac{MR\omega}{4t}$ (B) $\frac{MR\omega}{2t}$ (C) $\frac{MR\omega}{t}$ (D) $MR\omega t$

52. A square frame ABCD is formed by four identical rods each of mass 'm' and length 'l' This frame is in X–Y plane such that side AB coincides with X-axis and side AD along Y-axis. The moment of inertia of the frame about X-axis is

(A) $\frac{5ml^2}{3}$ (B) $\frac{2ml^2}{3}$ (C) $\frac{4ml^2}{3}$ (D) $\frac{ml^2}{12}$

53. The moment of inertia of a ring about an axis passing through the centre and perpendicular to its plane is 'I'. It is rotating with angular velocity 'ω'. Another identical ring is gently placed on it so that their centres coincide. If both the rings are rotating about the same axis then loss in kinetic energy is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{4}$ (C) $\frac{I\omega^2}{6}$ (D) $\frac{I\omega^2}{8}$

MHT-CET 2019

54. The real force 'F' acting on a particle of mass 'm' performing circular motion acts along the radius of circle 'r' and is directed towards the centre of circle. The square root of magnitude of such force is (τ = periodic time)

(A) $\frac{2\pi}{T}\sqrt{mr}$ (B) $\frac{T^2mr}{4\pi}$ (C) $\frac{2\pi T}{\sqrt{mr}}$ (D) $\frac{Tmr}{4\pi}$

55. A stone of mass 1 kg is tied to a string 2 m long and is rotated at constant speed of 40 ms^{-1} in a vertical circle. The ratio of the tension at the top and the bottom is [Take g = 10 ms^{-2}]

(A) $\frac{12}{19}$ (B) $\frac{79}{81}$ (C) $\frac{19}{12}$ (D) $\frac{81}{79}$

56. A stone of mass 'm' tied to a string of length 'ℓ' is whirled in a circle of radius 'r' under the effect of gravity. If its radial acceleration is 'p' times the acceleration due to gravity (g) then its linear acceleration at a point on the circle, where the string becomes horizontal (P is +ve)

(A) $g\sqrt{(p^2-1)}$ (B) $g\sqrt{(p-1)}$ (C) $g\sqrt{(p+1)}$ (D) $g\sqrt{(p^2+1)}$

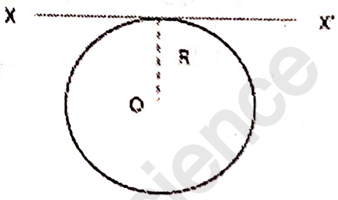
57. A thin metal wire of length 'L' and uniform linear mass density 'ρ' is bent into a circular coil with 'O' as centre. The moment of inertia of a coil about the axis XX' is

(A) $3\rho L^3/8\pi^2$

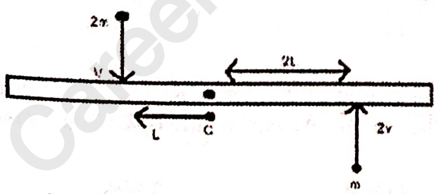
(B) $3\rho L^2/4\pi^2$

(C) $\rho L^3/8\pi^2$

(D) $\rho L^3/4\pi^2$



58. A uniform rod of length '6L' and mass '8m' is pivoted at its centre 'C'. Two masses 'm' and '2m' with speed 2v and v (as shown) strike the rod and stick to the rod. Initially the rod is at rest. Due to impact, if it rotates with angular velocity 'ω' then 'ω' will be



(A) $\frac{8v}{6L}$ (B) $\frac{11v}{3L}$ (C) $\frac{v}{5L}$ (D) zero

59. If radius of the solid sphere is doubled by keeping its mass constant, the ratio of their moment of inertia about any of its diameter is

(A) 1 : 4 (B) 2 : 3 (C) 2 : 5 (D) 1 : 8

60. Three point masses each of mass 'm' are kept at the corners of an equilateral triangle of side 'L'. The system rotates about the center of the triangle without any change in the separation of masses during rotation. The period of rotation is directly proportional to

($\cos 30° = \sin 60° = \sqrt{3}/2$)

(A) $L^{\frac{3}{2}}$ (B) $\sqrt{L}$ (C) L^{-2} (D) L

61. When a 12000 joule of work is done on a flywheel, its frequency of rotation increases from 10Hz to 20Hz. The moment of inertia of flywheel about its axis of rotation is ($\pi^2 = 10$)

(A) 1.688 kgm^2 (B) 2kgm^2 (C) 1.5 kgm^2 (D) 1 kgm^2

62. Three identical rods each of mass 'M' and length 'L' are joined to form a symbol 'H'. The moment of inertia of the system about one of the sides of 'H' is

(A) $ML^2/6$ (B) 4 $ML^2/3$ (C) $2ML^2/3$ (D) $ML^2/2$

***63.** A solid sphere rolls down from top of inclined plane, 7 m high, without slipping. Its linear speed at the foot of plane is ($g = 10$ m/s^2)

(A) $\sqrt{100}$ m/s (B) $\sqrt{\frac{280}{3}}$ m/s (C) $\sqrt{70}$ m/s (D) $\sqrt{\frac{140}{3}}$ m/s

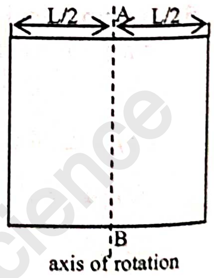
64. A molecule consists of two atoms each of mass 'm' and separated by a distance 'd'. At room temperature the average rotational kinetic energy is 'E', then its angular frequency is

(A) $\sqrt{\frac{m}{Ed}}$ (B) $\frac{d}{2}\sqrt{\frac{m}{E}}$ (C) $\frac{2}{d}\sqrt{\frac{E}{m}}$ (D) $\sqrt{\frac{Ed}{m}}$

65. Four metal rods each of mass 'M' and length 'L' are welded to form a square as shown. What is M.I. of the system about axis 'AB'?

(A) $\frac{2}{3}ML^2$ (B) $\frac{ML^2}{6}$

(C) $\frac{ML^2}{3}$ (D) $\frac{ML^2}{2}$



***66.** A disc and a solid sphere having same mass and radius roll down on the same inclined plane. The ratio of their linear speeds is

(A) $\sqrt{\frac{15}{14}}$ (B) $\sqrt{\frac{14}{15}}$ (C) $\frac{15}{14}$ (D) $\frac{14}{15}$

67. From a disc of mass 'M' and radius 'R', a circular hole of diameter 'R' is cut whose rim passes through the centre. The moment of inertia of the remaining part of the disc about perpendicular axis passing through the centre is

(A) $\frac{9MR^2}{32}$ (B) $\frac{13MR^2}{32}$ (C) $\frac{11MR^2}{32}$ (D) $\frac{7MR^2}{32}$

68. A metal sphere of radius 'r' and specific heat 's' is rotated about an axis passing through its centre with speed 'n' rotations per second. It is suddenly stopped and 50% of its energy is used in increasing the temperature of the sphere. The rise in temperature of the sphere is
(A) $\frac{5\pi^2n^2r^2}{14s}$ (B) $\frac{2\pi^2n^2r^2}{5s}$ (C) $\frac{7}{8}\pi n^2r^2s$ (D) $\frac{\pi^2n^2}{10r^2s}$

69. A wheel of moment of inertia 2 kg m^2 is rotating at a speed of 25 rad/s. Due to friction on the axis, it comes to rest in 10 minutes. Total work done by friction is
(A) 25 J (B) 625 J (C) 50 J (D) 600 J

*70. A ring, a disc and a solid sphere have same mass and radius. All of them are rolled down on an inclined plane from same height, simultaneously. The body that will reach at the bottom, last, amongst is
(A) ring (B) disc (C) ring and disc (D) solid sphere

*71. A circular coil and a disc having same mass roll without slipping on the horizontal with same linear velocity. If the total K.E. of the coil is 12 J then total K.E. of the disc is
(A) 6 J (B) 15 J (C) 9 J (D) 12 J

72. A child is swinging on a swing in sitting position. If he stands up on the swing, then periodic time of the swing will
(A) remains the same
(B) increase if the child is tall and decrease if the child is short
(C) increase
(D) decrease

73. A thin ring having mass 100 g and radius 10 cm is rotating about its axis with frequency 1 Hz. Four objects each of mass 12.5 g are kept gently to the opposite ends of two perpendicular diameters of the ring. The new frequency of rotation of the ring will be
(A) $\frac{4}{3}$Hz (B) $\frac{2}{3}$Hz (C) $\frac{3}{2}$Hz (D) $\frac{3}{4}$Hz

74. Earth revolves round the sun in a circular orbit of radius 'R'. The angular momentum of the revolving earth is directly proportional to
(A) R^2 (B) $\sqrt{R}$ (C) R^3 (D) R

75. A mass is whirled in a circular path constant angular velocity and its linear velocity is 'V'. If the string is now halved keeping the angular momentum same, the linear velocity is
(A) $\frac{V}{2}$ (B) 2V (C) V (D) $V\sqrt{2}$

76. A coin kept at a distance 'r_1' cm from the axis of rotation of a turn table, just begins to slip when the turntable rotates at an angular speed of 'ω_1' rad/s. If this distance is tripled, then at what angular speed of the turntable, will the coin begin to slip?
(A) $3\omega_1$ rad/s (B) $\sqrt{3}\omega_1$ rad/s (C) $\frac{\omega_1}{\sqrt{3}}$ rad/s (D) $\frac{\omega_1}{3}$ rad/s

77. By considering frictional force for a vehicle of mass 'm' moving along rough curved road, banked at an angle 'θ' the maximum safety speed of a vehicle is (R = radius of circular path, g = acceleration due to gravity)
(A) $V_m = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right]}$ (B) $V_m = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1+\mu_s\tan\theta}\right]}$
(C) $V_m = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1+\tan\theta}\right]}$ (D) $V_m = \sqrt{\frac{1}{Rg}\left[\frac{1+\mu_s\tan\theta}{\mu_s + \tan\theta}\right]}$

MHT-CET 2020

78. A particle rotates in a horizontal circle of radius 'R' in a conical funnel with speed 'v'. The inner surface of funnel is smooth. The height of the plane of the circle from the vertex of the funnel is (g = acceleration due to gravity)

(A) $\frac{v^2}{g}$ (B) $\frac{v}{2g}$ (C) $\frac{v}{g}$ (D) $\frac{v^2}{2g}$

79. From a disc of mass 'M' and radius 'R', a circular hole of diameter 'R' is cut whose rim passes through the centre. The moment of inertia of the remaining part of the disc about perpendicular axis passing through the centre is

(A) $\frac{7MR^2}{32}$ (B) $\frac{11MR^2}{32}$ (C) $\frac{9MR^2}{32}$ (D) $\frac{13MR^2}{32}$

80. A body is suspended from a rigid support by an inextensible string of length 'L' on which another identical body of mass 'm' struck inelastically moving with horizontal velocity $\sqrt{2gL}$. The increase in the tension in the string just after it is struck by the body is

(A) 4 mg (B) 3 mg (C) mg (D) 2 mg

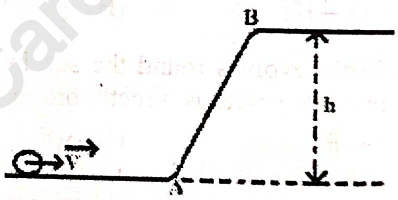
81. A molecule consists of two atoms each of mass 'm' and separated by a distance 'd'. At room temperature, the average rotational kinetic energy is 'E, then its angular frequency is

(A) $\frac{d}{2}\sqrt{\frac{m}{E}}$ (B) $\sqrt{\frac{m}{Ed}}$ (C) $\sqrt{\frac{Ed}{m}}$ (D) $\frac{2}{d}\sqrt{\frac{E}{m}}$

***82.** A solid sphere is rolling on a frictionless surface with translational velocity 'V'. It climbs the inclined plane from 'A' to 'B' and then moves away from B on the smooth horizontal surface. The value of 'V' should be

(A) $\geq\left[\frac{10\,gh}{7}\right]^{\frac{1}{2}}$ (B) $\sqrt{2gh}$

(C) $\frac{10gh}{7}$ (D) $\sqrt{gh}$



83. Two rings of radii R and nR made from the same wire have the ratio of moments of inertia about an axis passing through their centre and perpendicular to the plane of the rings is 1 : 8. The value of n is

(A) $2\sqrt{2}$ (B) 2 (C) 4 (D) $\frac{1}{2}$

84. A circular disc 'X' of radius 'R' made from iron plate of thickness 't' has moment of inertia 'I_x' about an axis passing through the centre of disc and perpendicular to its plane. Another disc 'Y' of radius '3R' made from an iron plate of thickness $\left(\frac{t}{3}\right)$ has moment of inertia 'I_y' about the axis same as that of disc X. The relation between I_x and I_y is

(A) $I_y = 9\,I_x$ (B) $I_y = I_x$ (C) $I_y = 3I_x$ (D) $I_y = 27I_x$

85. If 'I' is the moment of inertia and 'L' is angular momentum of a rotating body, then $\frac{L^2}{2I}$ is its

(A) translational kinetic energy (B) rotational kinetic energy
(C) linear momentum (D) torque

86. A particle of mass 4 gram moves along a circle of radius $\frac{10^2}{2\pi}$ cm with constant tangential acceleration. After beginning of the motion, by the end of second revolution, the kinetic energy of the particle becomes 18×10^{-5} J. Magnitude of tangential acceleration is
(A) 2.25×10^{-6} m/s^2
(B) 2.25×10^{-5} m/s^2
(C) 2.25×10^{-4} m/s^2
(D) 2.25×10^{-3} m/s^2

87. A particle of mass 'm' is rotating in a horizontal circle of radius 'r' with uniform velocity $\vec{V}$. The change in its momentum at two diametrically opposite points will be
(A) $-m\vec{V}$ (B) $-2m\vec{V}$ (C) $m\vec{V}$ (D) $3m\vec{V}$

*88. A body slides down a smooth inclined plane of inclination 'θ' and reaches the bottom with velocity 'V'. If the same body is a ring which rolls down the same inclined plane the liner velocity at the bottom of plane is
(A) 2V (B) V (C) $\frac{V}{\sqrt{2}}$ (D) $\frac{V}{2}$

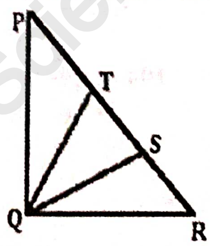
89. A particle is moving along the circular path with constant speed and centripetal acceleration 'a'. If the speed is doubled, the ratio of its acceleration after and before the change is
(A) 4 : 1 (B) 2 : 1 (C) 3 : 1 (D) 1: 4

90. A rod of length 'L' is hung from its one end and a mass 'm' is attached to its free end. What tangential velocity must be imparted to 'm', so that it reaches the top of the vertical circle? (g = acceleration due to gravity)
(A) $4\sqrt{gL}$ (B) $2\sqrt{gL}$ (C) $5\sqrt{gL}$ (D) $3\sqrt{gL}$

91. The overbridge of a canal is in the form of a concave circular arc of radius 'r'. The thrust at the lowest point is (m = mass of the vehicle, v = velocity of the vehicle, g = acceleration due to gravity.)
(A) $mg \div mv^2/r$ (B) $\left(mg+\frac{mv^2}{r}\right)$ (C) $\left(mg-\frac{mv^2}{r}\right)$ (D) $mg\times\frac{mv^2}{r}$

92. A ring of mass 'M' and radius 'R' is rotating about an axis passing through centre and perpendicular to its plane. Two particles of mass 'm' are placed gently on the opposite ends of a diameter of the ring. Now the angular speed of the ring is (ω = initial angular speed of ring)
(A) $M\omega/M - m$
(B) $M\omega/M + m$
(C) $M\omega/M + 2m$
(D) $M\omega^2/M - 2m$

93. Figure shows triangular lamina which can rotate about different axis of rotation. Moment of inertia is maximum about the axis



(A) QR
(B) PR
(C) QS
(D) PQ

94. A particle is performing vertical circular motion. The difference in tension at lowest and highest point is
(A) 8 mg (B) 2 mg (C) 6 mg (D) 4 mg

95. A uniform disc of mass 4 kg has radius of 0.4 m. Its moment of inertia about an axis passing through a point on its circumference and perpendicular to its plane is
(A) 0.32 kg – m^2 (B) 0.96 kg–m^2 (C) 0.16 kg–m^2 (D) 0.64 kg–m^2

96. What is the least radius of curve on a horizontal road, at which a vehicle can travel with a speed of 36 km/hr at an angle of inclination 45°?
[$g = 10 m/s^2$, tan45° = 1]
(A) 15 m (B) 20 m (C) 10 m (D) 25 m

97. Two bodies rotate with kinetic energies 'E_1' and 'E_2'. Moment of inertia about their axis of rotation is 'I_1' and 'I_2'. If $I_1 = \frac{I_2}{3}$ and $E_1 = 27 E_2$ then the ratio of the angular moment L_1 to L_2 is
(A) 1 : 9 (B) 9 : 1 (C) 3 : 1 (D) 1 : 3

98. Two loops of radii 'R' and 'nR' are made from same wire of same linear mass density. The ratio of moment of inertia about an axis through centre and perpendicular to plane is 1:27. The value of n is
(A) 27 (B) 3 (C) 9 (D) 81

99. A disc of mass 100 kg and radius 1m is rotating at 300 rpm. The torque required to rotate the disc in opposite direction with same speed in time 50 second is
(A) 30π Nm (B) 40π Nm (C) 20π Nm (D) 10π Nm

100. The angular velocity of minute hand of a clock in degree per second is
(A) 1.5 (B) 1 (C) 0.5 (D) 0.1

101. The maximum velocity with which vehicle can safely travel along banked road does NOT depend upon
(A) mass of the vehicle.
(B) acceleration due to gravity at a place.
(C) radius of the curved road.
(D) angle of banking.

102. A solid sphere of mass 1 kg and radius 10 cm rolls without slipping on a horizontal surface, with velocity of 10 cm/s. The total kinetic energy of sphere is
(A) 0.007 J (B) 0.05 J (C) 0.01 J (D) 0.07 J

103. A uniform rod of length '2L' has constant mass per unit length 'm'. Moment of inertia of the rod about an axis passing through its centre and perpendicular to length is
(A) $\frac{mL^3}{3}$ (B) $\frac{mL^2}{4}$ (C) $\frac{2mL^3}{3}$ (D) $\frac{mL^2}{12}$

104. A sphere of mass 'M' is attached to one end of a metal wire having length 'L' and diameter 'D'. It is whirled in a vertical circle of radius R with angular velocity 'ω'. When the sphere is at lowest point of its path, the elongation of the wire is (Y = Young's modulus of the material of the wire, g = acceleration due to gravity)
(A) $\frac{4ML(R\omega^2+g)}{\pi D^2 Y}$ (B) $\frac{ML(R\omega^2+g)}{2\pi D^2 Y}$ (C) $\frac{6ML(R^2\omega^2+g)}{\pi D^2 Y}$ (D) $\frac{2ML(R^2\omega^2+g)}{\pi D^2 Y}$

105. A particle moves along a circular path of radius 'r' with uniform speed 'V'. The angle described by the particle in one second is

(A) V^2r (B) $\frac{r}{V}$ (C) Vr (D) $\frac{V}{r}$

106. A disc has mass 'M' and radius 'R'. How much tangential force should be applied to the rim of the disc so as to rotate with angular velocity 'ω' in time t?

(A) $\frac{MR^2\omega}{t}$ (B) $\frac{MR\omega}{t}$ (C) $\frac{MR^2\omega}{2t}$ (D) $\frac{MR\omega}{2t}$

107. In non-uniform circular motion, the ratio of tangential acceleration to radial acceleration is (r = radius of circle, V = speed and α = angular acceleration)

(A) $\frac{r\alpha}{V}$ (B) $\left(\frac{r}{V}\right)^2\alpha$ (C) $\left(\frac{V}{r}\right)^2\frac{1}{2}$ (D) $\left(\frac{V}{r}\right)^2\alpha$

108. Four spheres each of mass 'M' and radius 'R' are placed with their centres on the corners of a square of side 'L'. The moment of inertia of the system about any side of square is

(A) $\frac{6}{5}MR^2 + ML^2$ (B) $\frac{3}{5}MR^2 + 2ML^2$ (C) $\frac{8}{5}MR^2 + 2ML^2$ (D) $\frac{4}{3}MR^2 + ML^2$

109. The power (P) is supplied to rotating body having moment of inertia 'I' and angular acceleration 'α'. Its instantaneous angular velocity is

(A) $\frac{I}{P\alpha}$ (B) $\frac{P}{I\alpha}$ (C) PIα (D) $\frac{\alpha}{PI}$

110. A disc of mass 'M' and radius 'R' is rotating about its own axis. If one quarter part of the disc is removed then new moment of inertia of the disc about the same axis is

(A) $\frac{2MR^2}{15}$ (B) $\frac{MR^2}{8}$ (C) $\frac{2MR^2}{13}$ (D) $\frac{3MR^2}{8}$

111. A horizontal circular platform of mass 100 kg is rotating at 5 r.p.m. about vertical axis passing through its centre. A child of mass 20 kg is standing on the edge of platform. If the child comes to the centre of platform then frequency of rotation will become

(A) 7 r.p.m. (B) 9 r.p.m. (C) 5 r.p.m. (D) 12 r.p.m.

112. A flat curved road on highway has radius of curvature 400 m. A car rounds the curve at a speed of 24 m/s. The minimum value of coefficient of friction to prevent car from sliding is (take g = 10 m/s^2)

(A) 0.144 (B) 0.376 (C) 0.544 (D) 0.100

113. What is the ratio of the angular speeds of second hand and the minute hand of a clock ?

(A) 1 : 60 (B) 1 : 2 (C) 12 : 1 (D) 60 : 1

114. If the earth suddenly contracts to $\left(\frac{1}{3}\right)^{rd}$ of its present size without change in its mass, the ratio of kinetic energy of the earth after and before contraction will be (Earth is assumed to be a sphere)

(A) 3 (B) 9 (C) 7 (D) 5

115.Moment of inertia of a uniform solid sphere of mass 'M' and radius 'R' about an axis at a distance $\left(\frac{R}{2}\right)$ from the centre is

(A) $\frac{2}{5}MR^2$ (B) $\frac{9}{10}MR^2$ (C) $\frac{13}{20}MR^2$ (D) $\frac{7}{5}MR^2$

116.A body of mass 10 kg is attached to a wire 0.3 m long. Its breaking stress is 4.8×10^7 N/m². The area of cross-section of the wire is 10^{-6}m². The maximum angular velocity with which it can be rotated in a horizontal circle is

(A) 6 rad/s (B) 5 rad/s (C) 7 rad/s (D) 4 rad/s

117.In the case of conical pendulum, if 'T' is the tension in the string and 'θ' is the semi-vertical angle of cone, then the component which provides necessary centripetal force is

(A) T sin θ (B) T tan θ (C) T cos θ (D) (T sin θ)/2

118.A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature to a certain value, its speed of rotation

(A) remains constant.
(B) may increase or decrease depending on density of metal.
(C) increases.
(D) decreases.

119.Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal force is

(A) $m_1 : m_2$ (B) $r_1 : r_2$ (C) 1 : 1 (D) $m_1 r_1 : m_2 r_2$

120.From a disc of mass 'M' and radius 'R' a circular hole of diameter R is cut whose rim passes through the centre. The moment of inertia of the remaining part of the disc about perpendicular axis passing through the centre is

(A) $\frac{9MR^2}{32}$ (B) $\frac{7MR^2}{32}$ (C) $\frac{11MR^2}{32}$ (D) $\frac{13MR^2}{32}$

121.In the case of conical pendulum, if T is the tension in the string and θ is the semivertical angle of cone, then the component of tension which balances the centrifugal force in equilibrium position is

(A) T cos θ (B) $\frac{(T\sin\theta)}{2}$ (C) T sin θ (D) T tan θ

122.A torque of 50 Nm acts on a body for 8 second which is initially at rest. The change in its angular momentum is

(A) 400 kgm²/s (B) 800 kgm²/s (C) 1000 kgm²/s (D) 600 kgm²/s

123.A thin, uniform metal rod of mass 'M' and length 'L' is swinging about a horizontal axis passing through its end. Its maximum angular velocity is 'ω'. Its centre of mass rises to a maximum height of (g = acceleration due to gravity)

(A) $\frac{L^2\omega^2}{3g}$ (B) $\frac{L^2\omega^2}{g}$ (C) $\frac{L^2\omega^2}{2g}$ (D) $\frac{L^2\omega^2}{6g}$

124.A train has to negotiate a curve of radius 'r' m, the distance between the rails is 'ℓ' m and outer rail is raised above inner rail by distance of 'h' m. If the angle of banking is small, the safety speed limit on this banked road is

(A) $rg\frac{h}{\ell}$ (B) $\frac{\left(\frac{h}{\ell}\right)^2}{rg}$ (C) $\sqrt{rg\left(\frac{h}{\ell}\right)}$ (D) $\left(rg\frac{h}{\ell}\right)^2$

125. The relative angular speed of hour hand and second hand of a clock is

(A) $\frac{359\pi}{21600}$ (B) $\frac{719\pi}{21600}$ (C) $\frac{9\pi}{21600}$ (D) $\frac{11\pi}{21600}$

126. A particle of mass 'm' is performing U.C.M. along a circle of radius 'r'. The relation between centripetal acceleration 'a' and kinetic energy 'E' is given by

(A) $a=\left(\frac{2E}{mr}\right)^2$ (B) $a=\frac{E}{mr}$ (C) $a=\frac{2E}{mr}$ (D) $a=2Em$

127. A thin uniform rod has mass 'M' and length 'L'. The moment of inertia about an axis perpendicular to it and passing through the point at a distance $\frac{L}{3}$ from one of its ends, will be

(A) $\frac{ML^2}{3}$ (B) $\frac{ML^2}{9}$ (C) $\frac{ML^2}{12}$ (D) $\frac{7}{8}ML^2$

128. The ratio of radii of gyration of a ring to a disc (both circular) of same radii and mass, about a tangential axis perpendicular to the plane is

(A) $\frac{2}{\sqrt{5}}$ (B) $\frac{\sqrt{3}}{\sqrt{2}}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{2}}{1}$

129. A particle starting from rest moves along the circumference of a circle of radius 'r' with angular acceleration 'α'. The magnitude of the average velocity, in the time it completes the small angular displacement 'θ' is

(A) $r\left(\frac{\alpha\theta}{2}\right)^{\frac{1}{2}}$ (B) $r\left(\frac{\alpha\theta}{2}\right)^2$ (C) $r\left(\frac{\alpha\theta}{2}\right)$ (D) $r\left(\frac{2}{\alpha\theta}\right)^2$

*130. A solid cylinder of radius 'R' and mas 'M' rolls down an inclined plane of height 'h'. When it reaches the bottom of the plane, its rotational kinetic energy is
(g = acceleration due to gravity)

(A) $\frac{Mgh}{2}$ (B) $\frac{Mgh}{4}$ (C) $\frac{Mgh}{3}$ (D) Mgh

131. A particle is moving in a circle of radius 'R' with constant speed 'V'. The magnitude of average speed after half revolution is

(A) $\frac{2V^2}{\pi R}$ (B) $\frac{2\pi}{RV^2}$ (C) $\frac{2V}{\pi R^2}$ (D) $\frac{2R}{\pi V}$

132. If there is a change of angular momentum from 1J-s to 4J-s in 4 second, then the torque is

(A) $\left(\frac{3}{4}\right)$J (B) 1 J (C) $\left(\frac{4}{3}\right)$J (D) $\left(\frac{5}{4}\right)$J

133. In non-uniform circular motion, the ratio of tangential to radial acceleration is r = radius, α = angular acceleration, V = linear velocity)

(A) $\frac{r\alpha^2}{V^2}$ (B) $\frac{r^2\alpha}{V^2}$ (C) $\frac{r\alpha}{V}$ (D) $\frac{V^2}{r\alpha}$

134. Two rings of same mass 'M' and radius 'R' are so placed that their centre is common and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to any one ring is

(A) $\frac{3MR^2}{2}$ (B) MR^2 (C) $\frac{2MR^2}{3}$ (D) $\frac{MR^2}{2}$

135. If the spherical planet of mass 'M' and radius 'R' suddenly shrinks to half its size, its mass reduces to half. The new moment of inertia of the plant about its diameter is

(A) $\frac{MR^2}{20}$ (B) $\frac{2}{5}MR^2$ (C) $\frac{MR^2}{10}$ (D) $\frac{2}{3}MR^2$

136. The moment of inertia of a uniform square plate about an axis perpendicular to its plane and passing through the centre is $\frac{Ma^2}{6}$ where M is the mass and 'a' is the side of square plate. Moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corner is

(A) $\frac{2Ma^2}{3}$ (B) $\frac{Ma^2}{3}$ (C) $\frac{3}{Ma^2}$ (D) $\frac{3Ma^2}{2}$

137. A particle is revolving in anticlockwise sense along the circumference of a circle of radius 'r' with linear velocity 'v', then the angle between 'v' and angular velocity 'ω' will be

(A) 180° (B) 90° (C) 45° (D) 0°

138. An engine is moving on a circular path of radius 200 m with speed of 15 m/s. What will be the frequency heard by an observer who is at rest at the centre of the circular path, when engine blows the whistle with frequency 250 Hz?

(A) Greater than 250 Hz (B) 250 Hz
(C) zero (D) Less than 250 Hz

139. A particle of mass 'm' moves along a circle of radius 'r' with constant tangential acceleration. If kinetic energy 'E' of the particle becomes three times by the end of third revolution after beginning of acceleration the magnitude of tangential acceleration is

(A) $\frac{E}{6\pi rm}$ (B) $\frac{E}{12\pi rm}$ (C) $\frac{E}{24\pi rm}$ (D) $\frac{E}{3\pi rm}$

140. Three point masses, each of mass 'm' are placed at the corners of an equilateral triangle of side 'ℓ'. The moment of inertia of the system about an axis along any one side of the triangle is

(A) $m\ell^2$ (B) $\frac{1}{3}m\ell^2$ (C) $\frac{3}{2}m\ell^2$ (D) $\frac{3}{4}m\ell^2$

141. Two discs having moment of inertia I_1 and I_2 are made from same material have same mass. Their thickness and radii are t_1, t_2 and R_1, R_2 respectively. The relation between moment of inertia of each disc about an axis passing through its centre and perpendicular to its plane and its thickness is

(A) $I_1t_2^2 = I_2t_1^2$ (B) $I_1\,t_1 = I_2\,t_2$ (C) $I_1\,t_2 = I_2\,t_1$ (D) $I_1t_1^2 = I_2t_2^2$

142. Two circular rings 'A' and 'B' of radii 'nR' and 'R' are made from the same wire. The moment of inertia of 'A' about an axis passing through the centre and perpendicular to the plane of 'A' is 64 tiems that of the ring 'B'. The value of 'n' is

(A) 3 (B) 4 (C) 8 (D) 6

143. Four particles each of mass 'M' are placed at corners of a square of side 'L'. The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is

(A) $\frac{L}{\sqrt{2}}$ (B) $\frac{L}{\sqrt{8}}$ (C) $\frac{L}{\sqrt{5}}$ (D) $\frac{L}{\sqrt{3}}$

144. A same torque is applied to a disc and a ring of equal mass and radii then

(A) both will rotate with same angular acceleration.
(B) the ring will rotate with greater angular acceleration.
(C) both will rotate with same angular velocity.
(D) the disc will rotate with greater angular frequency.

145. Force $\vec{F}$ is acting on a particle having position vector $\vec{r}$. Let $\vec{\tau}$ be the torque of this force about the origin. The correct equation is

(A) $\vec{r}.\vec{\tau} \neq 0$ and $\vec{F}\cdot\vec{\tau} = 0$
(B) $\vec{r}.\vec{\tau} = 0$ and $\vec{F}\cdot\vec{\tau} = 0$
(C) $\vec{r}.\vec{\tau} \neq 0$ and $\vec{F}\cdot\vec{\tau} \neq 0$
(D) $\vec{r}.\vec{\tau} = 0$ and $\vec{F}\cdot\vec{\tau} \neq 0$

***146.** A car of mass 'm' is crossing the convex bridge of radius of curvature 'R' with a speed 'v'. At the highest point the thrust is (g = acceleration due to gravity)

(A) $mg + \frac{mv^2}{R}$ (B) mg (C) $mg - \frac{mv^2}{R}$ (D) $\frac{mv^2}{R}$

147. A disc of moment of inertia 'I_1' is rotating with angular velocity 'ω_1' about an axis perpendicular to its plane, passing through its centre. If another disc of moment of inertia 'I_2' about the same axis is gently placed over it, then the new angular velocity of the combined disc will be

(A) $\frac{(I_1 + I_2)\omega_1}{I_1}$ (B) $\frac{I_2\omega_1}{I_1 + I_2}$ (C) $\frac{I_1\omega_1}{I_1 + I_2}$ (D) ω_1

148. Moment of inertia of the rod about an axis passing through the centre and perpendicular to its length is 'I_1'. The same rod is bent into a ring and its moment of inertia about the diameter is 'I_2', then $\frac{I_2}{I_1}$ is

(A) $\frac{3}{2\pi^2}$ (B) $\frac{3}{4\pi^2}$ (C) $\frac{2\pi^2}{3}$ (D) $\frac{4\pi^2}{3}$

149. A stone is tied at the end of a rope of length 1 m and whirled in a vertical circle. The ratio of velocity at highest point to lowest point will be

(A) $\sqrt{3}:1$ (B) $1:\sqrt{5}$ (C) $\sqrt{3}:\sqrt{5}$ (D) $\sqrt{5}:1$

***150.** A disc rolls down a smooth inclined plane without slipping. An inclined plane makes an angle of 60° with the vertical. The linear acceleration of the disc along the inclined plane is

(g = acceleration due to gravity, $\sin 30° = \cos 60° = \frac{1}{2}$, $\sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$)

(A) $\frac{g}{9}$ (B) $\frac{g}{6}$ (C) $\frac{g}{3}$ (D) $\frac{g}{18}$

151. Two discs P and Q having equal masses and thickness but densities 'ρ_1' and 'ρ_2' respectively, are such that $\rho_1 < \rho_2$. The moment of inertia of the disc P and Q are related as

(A) $I_p = I_Q$ (B) $I_p << I_Q$ (C) $I_p < I_Q$ (D) $I_P > I_Q$

***152.** A hollow sphere rolls down from the top of inclined plane. Its velocity on reaching the bottom of plane is V_1. When the same sphere slides down from the top of the plane, its velocity on reaching the bottom is V_2. The ratio $V_2 : V_1$ is [Take M.I. of hollow sphere = $\frac{2}{3}MR^2$]

(A) $\sqrt{\frac{5}{3}}$ (B) $\sqrt{\frac{7}{5}}$ (C) $\sqrt{\frac{3}{5}}$ (D) $\sqrt{\frac{5}{7}}$

153. A particle at rest starts moving with constant angular acceleration 'α' in circular path. At what time the magnitude of centripetal acceleration is half the tangential acceleration?

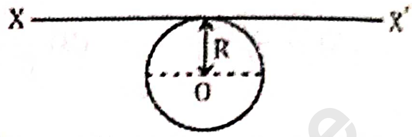
(A) $\frac{1}{2\sqrt{\alpha}}$ (B) $\frac{1}{\sqrt{\alpha}}$ (C) $\sqrt{\alpha}$ (D) $\frac{1}{\sqrt{2\alpha}}$

154. The moment of inertia of a thin unfirom rod of mass 'M' and length 'L' about an axis passing through a point at a distance $\frac{L}{4}$ from one of its ends and perpendicular to the length of rod is

(A) $\frac{3ML^2}{16}$ (B) $\frac{5ML^2}{48}$ (C) $\frac{7ML^2}{48}$ (D) $\frac{9ML^2}{16}$

155. A thin wire of length 'L' and uniform linear mass density 'ρ' is bent into a circular coil with 'O' as centre. The moment of inertia of a coil about the axis XX' is

(A) $3\rho L^3 / 8\pi^2$
(B) $\rho L^3 / 4\pi^2$
(C) $3\rho L^2 / 4\pi^2$
(D) $\rho L^3 / 8\pi^2$



156. A body of mass 'm' is suspended from the rigid support by an inextensible string of length 'L' on which another identical body of mass 'm' struck inelastically moving with horizontal velocity $\sqrt{2gL}$. The increase in the tension just after it is struck by the body is (g = acceleration due to gravity)

(A) mg (B) 4 mg (C) 2 mg (D) 3 mg

157. A disc of radius 0.4 m and mass 1 kg rotates about an axis passing through its centre and perpendicular to its plane. The angular acceleration is 10 rad/s^2. The tangential force applied to the rim of the disc is

(A) 2 N (B) 4 N (C) 3 N (D) 1 N

***158.** A solid sphere of radius is r is rolling without sliding. The ratio of rotational kinetic energy and total kinetic energy associated with the sphere is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{7}$

159. The moment of inertia of a ring about an axis passing through its centre and perpendicular to its plane is 'I'. It is rotating with angular velocity 'ω'. Another identical ring is gently placed on it so that their centres coincide. If both the ring are rotating about the same axis, then loss in kinetic energy is

(A) $\frac{I\omega^2}{3}$ (B) $\frac{I\omega^2}{4}$ (C) $I\omega^2$ (D) $\frac{I\omega^2}{2}$

160. A particle of mass 'm' is rotating in a circle of radius 'r' having angular momentum 'L'. Then the centripetal force will be

(A) $\frac{L^2}{mr^2}$ (B) $\frac{L^2}{mr^2}$ (C) $\frac{L^2}{mr}$ (D) $\frac{L^2}{mr^3}$

***161.** A ring and a disc roll on horizontal surface without slipping with same linear velocity. If both have same mass and total kinetic energy of the ring is 4 J then total kinetic energy of the disc is

(A) 3 J (B) 8 J (C) 6 J (D) 2 J

162. Three-point masses each of mass 'M' are placed at the corners of an equilateral triangle of side 'a'. The moment of inertia of this system about an axis passing through one side of a triangle is

(A) $\frac{Ma^2}{4}$ (B) $\frac{2Ma^2}{3}$ (C) $\frac{Ma^2}{3}$ (D) $\frac{3Ma^2}{4}$

163. A grindstone acquires angular speed of 90 rpm after 6 second, starting from rest. The angular acceleration of the grindstone in rad/s^2, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

164. A solid sphere has mass 'M' and radius 'R'. Its moment of inertia about a parallel axis passing through a point at a distance $\frac{R}{2}$ from its centre is

(A) $\frac{13}{20}MR^2$ (B) $\frac{8}{11}MR^2$ (C) $\frac{11}{15}MR^2$ (D) $\frac{6}{10}MR^2$

165. Two discs A and B of equal mass and thickness have densities 6800 kg/m^3 and 8500 kg/m^3 respectively. The ratio of their moments of inertia (A to B) is

(A) $\frac{1}{6.8\times8.5}$ (B) $\frac{4}{5}$ (C) $\frac{5}{4}$ (D) $\frac{5}{9}$

166. A cord is wound round the circumference of a wheel of radius 'r'. The axis of the wheel is horizontal and moment of inertia about it is I. A block of mass 'm' is attached to free end of the cord, initially at rest. When the wheel rotates and the block moves vertically downwards through distance 'h', the angular velocity of the wheel will be
(Neglect the mass of cord, g = acceleration due to gravity)

(A) $\left(\frac{2mgh}{I+mr^2}\right)^{1/2}$ (B) $\left(\frac{2gh}{I+mr}\right)^{1/2}$ (C) $(2gh)^{1/2}$ (D) $\left(\frac{2mgh}{I+2m}\right)^{1/2}$

167. A flywheel of mass 2 kg has radius of gyration 0.5m. If it makes 10 r.p.s. then its rotational kinetic energy will be

(A) $100\pi^2$ erg (B) $50\pi^2$ J (C) $100\pi^2$ J (D) $50\pi^2$ erg

168. A motor cycle racer takes a round with speed 20 m/s on a curved road of radius 40 m. The leaning angle of motor cycle with vertical for safe turn is ($g = 10\ m/s^2$, $\tan 45° = 1$)

(A) 30° (B) 75° (C) 60° (D) 45°

169. A thin wire of length 'L' and uniform linear mass density 'm' is bent into a circular loop. The moment of inertia of this loop about the tangential axis and in the plane of the coil is

(A) $\frac{3mL^3}{4\pi^2}$ (B) $\frac{3mL^3}{8\pi^2}$ (C) $\frac{3mL^3}{16\pi^2}$ (D) $\frac{3mL^3}{2\pi^2}$

170. A body situated on earth's surface at its equator becomes weightless when the rotational kinetic energy of the earth reaches a critical value which is given by (M and R be the mass and radius of earth respectively)

(A) $\frac{MgR}{2}$ (B) $\frac{MgR}{3}$ (C) $\frac{MgR}{4}$ (D) $\frac{MgR}{5}$

171. A thin circular ring of mass 'M' and radius 'r' is rotating about its axis with an angular speed 'ω'. Two particles each of mass 'm' are now attached at diametrically opposite points. The angular speed of the ring will become

(A) $\frac{\omega M}{M+2m}$ (B) $\frac{\omega(M-2m)}{M}$ (C) $\frac{\omega(M-2m)}{M+2m}$ (D) $\frac{\omega M}{M+m}$

172. Moment of inertia of a solid sphere about its diameter is 'I'. It is then casted into 27 small spheres of same diameter. The moment of inertia of each new sphere is

(A) $\frac{I}{31}$ (B) $\frac{I}{122}$ (C) $\frac{I}{243}$ (D) $\frac{I}{62}$

173. A wheel of radius 2 cm is at rest on the horizontal surface. A point P on the circumference of the wheel is in contact with the horizontal surface. When the wheel rolls without slipping on the surface, the displacement of point P after half rotation of wheel is

(A) $2(\pi^2+2)^{\frac{1}{2}}$ cm (B) $(\pi^2+2)^{\frac{1}{2}}$ cm (C) $(\pi^2+4)^{\frac{1}{2}}$ cm (D) $2(\pi^2+4)^{\frac{1}{2}}$ cm

174. A pendulum has length of 0.4 m and maximum speed 4 m/s. When the length makes an angle 30° with the horizontal, its speed will be

$\left[\sin\frac{\pi}{6}=\cos\frac{\pi}{3}=0.5 \text{ and } g=10\,m/s^2\right]$

(A) $2\sqrt{3}$ m/s (B) $2\sqrt{2}$ m/s (C) $\sqrt{3}$ m/s (D) $2\sqrt{5}$ m/s

175. A heavy mass is attached at one end of a thin wire and whirled in a vertical circle. The changes of breaking the wire are maximum when

(A) The wire makes an angle of 60° with the horizontal
(B) The mass is at the highest point of the circle
(C) The mass is at the lowest point of the circle
(D) The wire is horizontal

176. A wheel is at rest horizontal position. Its M.I. about vertical axis passing through its centre is 'I'. A constant torque 'τ' acts on it for 't' second. The change in rotational kinetic energy is

(A) $\frac{\tau^2 t^2}{2I}$ (B) $\left[\frac{\tau t}{2I}\right]^2$ (C) $\left[\frac{\tau t}{2I}\right]$ (D) $\left[\frac{\tau t}{2I}\right]h\frac{1}{2}$

***177.** A solid cylinder of mass 'M' and radius 'R' rolls down a smooth inclined plane about its own axis and reaches the bottom with velocity 'v'. The height of the inclined plane is (g = acceleration due to gravity)

(A) $\frac{2v^2}{3g}$ (B) $\frac{7v^2}{9g}$ (C) $\frac{4v^2}{5g}$ (D) $\frac{3v^2}{4g}$

178. A disc of mass 10 kg and radius 0.1 m is rotating at 120 r.p.m. A retarding torque brings it to rest in 10s. If the same torque is due to force applied tangentially on the rim of the disc then magnitude of force is

(A) 0.1 πN (B) 0.2 πN (C) 0.4 πN (D) 0.8 πN

179. A solid sphere of mass 'M' and radius 'R' is rotating about its diameter. A disc of same mass and radius is also rotating about an axis passing through its centre and perpendicular to the plane but angular speed is twice that of the sphere. The ratio of kinetic energy of disc to that of sphere is

(A) 4 : 1 (B) 6 : 1 (C) 3 : 1 (D) 5 : 1

180. A bucket containg water is revolved in a vertical circle of radius 'r'. To prevent the water from falling down, the minimum frequency of revolution required is [g = acceleration due to gravity]

(A) $\frac{1}{2\pi}\sqrt{\frac{r}{g}}$ (B) $2\pi\sqrt{\frac{g}{r}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{g}{r}}$ (D) $\frac{2\pi g}{r}$

181. A rotating body has angular momentum 'L'. If its frequency of rotation is halved and rotational kinetic energy is doubled, its angular momentum becomes

(A) $\frac{L}{2}$ (B) 4 L (C) 2L (D) $\frac{L}{4}$

182. From a uniform circular thin disc of mass 9M and radius R, a small disc of radius $\frac{R}{3}$ is removed. The centre of the small disc is at a distance $\frac{2R}{3}$ from the centre of original disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through the centre of the disc of radius R is

(A) $\frac{MR^2}{2}$ (B) $4MR^2$ (C) MR^2 (D) $3MR^2$

183. Two bodies have their moments of inertia I and 2I respectively about their axes of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

(A) 1 : 2 (B) $1 : 2\sqrt{2}$ (C) 2 : 1 (D) $1 : \sqrt{2}$

184. A ring and a disc have same mass and same radius. The ratio of moment of inertia of a ring about a tangent in its plane to that of the disc about its diameter is

(A) 6 : 1 (B) 4 : 1 (C) 2 : 1 (D) 8 : 1

185. A thin circular ring of mas 'M' and radius 'R' is rotating about a transverse axis passing through its centre with constant angular velocity 'ω'. Two objects each of mass 'm' are attached gently to the opposite ends of a diameter of the ring. What is the new angular velocity?

(A) $\frac{M\omega}{M+m}$ (B) $\frac{(M+2m)\omega}{M}$ (C) $\frac{(M-2m)\omega}{M+2m}$ (D) $\frac{M\omega}{M+2m}$

186. A liquid kept in a cylindrical vessel is rotated about vertical axis through the centre of circular base. The difference in the heights of the liquid at the centre of vessel and its edge is (R = radius of vessel, ω = angular velocity of rotation, g = acceleration due to gravity)

(A) $\frac{R\omega}{g}$ (B) $\frac{R^2\omega^2}{g}$ (C) $\frac{R\omega}{2g}$ (D) $\frac{R^2\omega^2}{2g}$

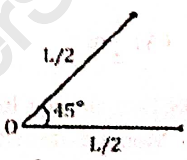
187. A solid sphere of mass 'M' and radius 'R' has moment of inertia 'I' about its diameter. It is recast into a disc of thickness 't' whose moment of inertia about an axis passing through its edge and perpendicular to its plane, remains 'I'. Radius of the disc will be

(A) $R/\sqrt{19}$ (B) $2R/\sqrt{15}$ (C) $2R/\sqrt{19}$ (D) $R/\sqrt{15}$

188. A particle moves along a circular path with decreasing speed. Hence

(A) its angular momentum remains constant
(B) the direction of angular momentum remains constant
(C) its resultant acceleration is towards the centre
(D) it moves in a spiral path with decreasing radius

189. A thin uniform rod of length 'L' and mass 'M' is bent at the middle point 'O' at an angle of 45° as shown in the figure. The moment of inertia of the system about an axis passing through 'O' and perpendicular to the plane of the bent rod, is



(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{24}$ (C) $\frac{ML^2}{3}$ (D) $\frac{ML^2}{12}$

190. A particle rotates in horizontal circle of radius 'R' in a conical funnel, with speed 'V'. The inner surface of the funnel is smooth. The height of the plane of the circle from the vertex of the funnel is (g = acceleration due to gravity)

(A) $\frac{V^2}{2g}$ (B) $\frac{V}{g}$ (C) $\frac{V^2}{g}$ (D) $\frac{V}{2g}$

191. The moment of inertia of a thin uniform rod about a perpendicular axis passing through one of its ends is 'I'. Now, the rod is bent in a ring and its moment of inertia about diameter is 'I_1', Then I/I_1 is

(A) $\frac{8\pi^2}{3}$ (B) $\frac{\pi^2}{3}$ (C) $\frac{11\pi^2}{3}$ (D) $\frac{4\pi^2}{3}$

192. Two rings of radius 'R' and 'nR' made of same material have the ratio of moment of inertia about an axis passing through its centre and perpendicular to the plane as 1:8. The value of 'n' is (mass per unit length is constant)

(A) 2 (B) 4 (C) 1 (D) 3

193. Let M and L be the mass and length of thin uniform rod respectively. In 1st case, axis of rotation is passing through centre and perpendicular to its length. In 2nd case, axis of rotation is passing through one end and perpendicular to its length. The ratio of radius of gyration in first case to second case is

(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

194. A body is mvoing along a circular track of radius 100 m with velocity 20 m/s. Its tangential acceleration is $3 m/s^2$, then its resutlant accelaration will be

(A) $5 m/s^2$ (B) $3 m/s^2$ (C) $4 m/s^2$ (D) $2 m/s^2$

195. A rope is wound around a solid cylinder of mass 1 kg and radius 0.4 m. What is the angular acceleration of cylinder, if the rope is pulled with a force of 25 N? (cylinder is rotating about its own axis)

(A) $125 rad/s^2$ (B) $50 rad/s^2$ (C) $10 rad/s^2$ (D) $1 rad/s^2$

196. A child starts running from rest along a circular track of radius 'r' with constant tangential acceleration 'a'. After time 't' he feels that slipping of shoes on the ground has started. The coefficient of friction between shoes and the ground is [g = acceleration due to gravity]

(A) $\frac{\left[a^4t^4 + a^2r^2\right]}{rg}$ (B) $\frac{\left[a^2t^2 + a^2r^4\right]}{rg}$ (C) $\frac{\left[a^4t^4 - a^2r^2\right]^{\frac{1}{2}}}{rg}$ (D) $\frac{\left[a^4t^4 + a^2r^2\right]^{\frac{1}{2}}}{gr}$

197. Three points masses, each of mass 'm' are placed at the corners of an equilateral triangel of side 'ℓ'. The moment of inertia of the system about an axis pasing through one of the vertices and parallel to the side joining other two vertices, will be

(A) $\frac{1}{4}m\ell^2$ (B) $\frac{1}{2}m\ell^2$ (C) $\frac{3}{4}m\ell^2$ (D) $\frac{3}{2}m\ell^2$

198. Two circular loops A and B of radii 'R' and 'NR' respectively are made from a uniform wire. Moment of inertia of B about its axis is 3 times that of A and about its axis. The value of N is

(A) $[2]^{\frac{1}{3}}$ (B) $[4]^{\frac{1}{3}}$ (C) $[5]^{\frac{1}{3}}$ (D) $[3]^{\frac{1}{3}}$

199. A constant torque of 200 N turns a flywheel, which is at rest, about an axis through its centre and perpendicular to its plane. If its moment of inertia is 50 kg-m^2, then in 4 second, what will be change in its angular momentum?

(A) 200 kg-m^2/s (B) 800 kg-m^2/s (C) 20 kg-m^2/s (D) 40 kg-m^2/s

200. A particle executes uniform circular motion with angular momentum 'L'. Its rotational kinetic energy becomes half, when the angular frequency is doubled. Its new angular momentum is

(A) 4L (B) $\frac{L}{4}$ (C) 2L (D) $\frac{L}{2}$

201. A motorcyclist rides in a horizontal circle about central vertical axis inside a cylindrical chamber of radius 'r'. If the coefficient of friction between the tyres and the inner surface of chamber is 'μ', the minimum speed of motorcyclist to prevent him from skidding is ('g' = acceleration due to gravity)

(A) $\sqrt{\frac{r\mu}{g}}$ (B) $\sqrt{\frac{\mu g}{r}}$ (C) $\sqrt{\frac{rg}{\mu}}$ (D) $\sqrt{\frac{g}{r\mu}}$

202. A thin metal rod of mass 'M' and length 'L' is cut into 4 equal parts by cutting it perpendicular to its length. If moment of inertia of the rod about an axis passing through its centre and perpendicular to its axis is 'I' then moment of inertia of each part about the similar axis is

(A) $\frac{I}{16}$ (B) $\frac{I}{32}$ (C) $\frac{I}{64}$ (D) $\frac{I}{128}$

203. A uniform rod AB of mass 'm' and length 'ℓ' is at rest on a smooth horizontal surface. An impulse 'P' is applied to the end B. The time taken by the rod to turn through a right angle is

(A) $\frac{\pi P}{m\ell}$ (B) $\frac{\pi}{12}\frac{m\ell}{P}$ (C) $2\frac{\pi P}{m\ell}$ (D) $2\pi\frac{m\ell}{P}$

SOLUTIONS

1. (A)

According to work-energy theorem,

Work done = Change in rotational kinetic energy

$W = (\Delta KE_r)_1 - (\Delta KE_r)_2$ ...(i)

But rotational kinetic energy, $K = \frac{1}{2}I\omega^2$

From Eq. (i), we get

$$W = \frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2 = \frac{1}{2}I(\omega_1^2 - \omega_2^2)$$

As $\omega = 2\pi n$

Hence, we get

$$W = \frac{1}{2}I[2\pi n_1)^2 - (2\pi n_2)^2] \quad = \frac{1}{2}I \times 4\pi^2(n_1^2 - n_2^2) \quad ...(ii)$$

Given, $I = \frac{9.8}{\pi^2}$ kg – m^2

$n_1 = 600$ rpm = 10 rps

$n_2 = 300$ rpm = 5 rps

From Eq.(ii), we get

$$W = \frac{1}{2} \times \frac{9.8}{\pi^2} \times 4\pi^2(10^2 - 5^2) = 1467\text{ J}$$

2. (B)

Angular momentum of the body is given by

$$L = I\omega$$

or $\quad L = I \times \frac{2\pi}{T} \quad$ or $\quad L \propto \frac{I}{T}$

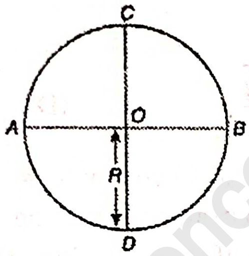
$\Rightarrow \quad \frac{L_1}{L_2} = \frac{T_2}{T_1} \quad$ or $\quad \frac{L}{L_2} = \frac{2T}{T} \quad$ (as $T_2 = 2T$)

So, $\quad L_2 = \frac{L}{2}$

Thus, on doubling the time period, angular momentum of body becomes half.

3. (D)

The moment of inertia of the disc about an axis parallel to its plane is, $I_t = I_d + MR^2$



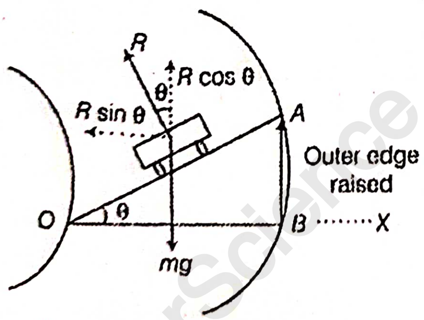
$\Rightarrow \quad I = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2 \quad$ or $\quad MR^2 = \frac{4I}{5}$

Now, moment of inertia about a tangent perpendicular to its plane is

$$r = \frac{3}{2}MR^2 = \frac{3}{2} \times \frac{4}{5}I = \frac{6}{5}I$$

4. (D)

In figure, OX is a horizontal line. OA is the level of banked curved road whose outer edge has been raised. $\angle XOA = \theta$ = angle of banking



According to the figure $R\cos\theta = mg$ ...(i)

and $R\sin\theta$ provides the necessary centripetal force.

$$R\sin\theta = \frac{mv^2}{r}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{rmg} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

So, we can say that option (d) is correct.

5. (A)
Time taken by the body to reach the ground when string breaks,

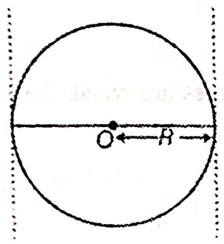
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2R}{g}}$$

Hence, horizontal distance covered by the body

$$= v_H \times t = \sqrt{gR} \times \sqrt{\frac{4R}{g}} = 2R$$

6. (B)
As $I = MK^2 = \Sigma MR^2$
where, M is the total mass of the body.

This means that $K = \sqrt{\left(\frac{I}{M}\right)}$



According to theorem of parallel axes, $I = I_{CG} + M(2R)^2$
where, I_{CG} is moment of inertia about an axis through centre of gravity.

$$\therefore \quad I = \frac{2}{5}MR^2 + 4MR^2 = \frac{22}{5}MR^2$$

$$\Rightarrow \quad MK^2 = \frac{22}{5}MR^2$$

$$\therefore \quad K = \sqrt{\frac{22}{5}}R$$

7. (C)
The moment of inertia about an axis passing through centre of mass of disc and perpendicular to its plane is

$$I_{CM} = \frac{1}{2}MR^2$$

where, M is the mass of disc and R its radius. According to theorem of parallel-axes, moment of inertia of circular disc about an axis touching the disc at its diameter and normal to the disc is

$$= I_{CM} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

8. (D)
From third equation of angular motion, $\omega^2 = \omega_0^2 - 2\alpha\theta$

[Here, $\omega = \frac{\omega_0}{2}, \theta = 36 \times 2\pi$]

$$\therefore \quad \left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha \times 36 \times 2\pi$$

or $4 \times 36\pi\alpha = \frac{3\omega_0^2}{4}$ or $\alpha = \frac{\omega_0^2}{16 \times 12\pi}$

According to the question again applying the third equation of angular motion

$$\omega^2 = \omega_0^2 - 2\alpha\theta \quad [\text{Here, } \omega = 0]$$

$$\therefore \quad 0 = \left(\frac{\omega_0}{2}\right)^2 - 2\times\frac{\omega_0^2\cdot\theta}{16\times 12\pi}$$

or $\theta = 24\pi$ or $\theta = 12\times 2\pi$

But $2\pi = 1$ cycle So, $\theta = 12$ cycle

9. (C)

Let the radii of the thin spherical shell and the solid sphere are R_1 and R_2, respectively. Then, the moment of inertia of the spheres about their diameter

$$I = \frac{2}{3}MR_1^2 \quad \ldots(i)$$

And the moment of inertia of the solid sphere is given by

$$I = \frac{2}{5}MR_2^2 \quad \ldots(ii)$$

Given that, the masses and moment of inertia for both the bodies are equal, then from Eqs. (i) and (ii)

$$\frac{2}{3}MR_1^2 = \frac{2}{5}MR_2^2 \quad \Rightarrow \frac{R_1^2}{R_2^2} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \sqrt{\frac{3}{5}} \Rightarrow R_1 : R_2 = \sqrt{3} : \sqrt{5}$$

10. (A)

The moment of inertia of this annular disc about the axis perpendicular to its plane will be $\frac{1}{2}M(R^2+r^2)$.

$$I = \frac{M'R^2}{2} \qquad I_s = \frac{mr^2}{2}$$

$$m = PV = Pta = Pt\cdot\pi r^2$$

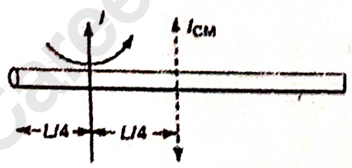
$$M' = PV = Pt\cdot\pi R^2$$

$$I_{net} = \frac{M'R^2}{2} - \frac{mr^2}{2} = \frac{Pt\pi R^2R^2 - Pt\pi r^2r^2}{2}$$

$$= \frac{Pt\pi\left(R^4 - r^4\right)}{2} = \frac{Pt\pi\left(R^2-r^2\right)\left(R^2+r^2\right)}{2} = \frac{1}{2}M\left(R^2+r^2\right)$$

11. (D)

$$\ell I = \ell_{CM} + Mx^2$$



$$= \frac{ML^2}{12} + M\left[\frac{L}{4}\right]^2 = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$

12. (A)

Moment of inertia of big drop is $I = \frac{2}{5}MR^2$. When small droplets are formed from big drop volume of liquid remain same

$$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow n^{1/3} r = R \text{ as } n = 8 \Rightarrow r = \frac{R}{2}$$

Mass of each small droplet $= \frac{M}{8}$

$\therefore$ Moment of inertia of each small droplet $= \frac{2}{5}\left[\frac{M}{8}\right]\left[\frac{R}{2}\right]^2 = \frac{1}{32}\left[\frac{2}{5}MR^2\right] = \frac{I}{32}$

13. (C)

Kinetic energy $E = \frac{L^2}{2I}$.

If angular momenta are equal, then $E \propto \frac{1}{I}$

14. (D)

Centripetal force, $F = \frac{mv^2}{r} = \frac{m}{r}\cdot\frac{L^2}{m^2r^2} = \frac{L^2}{mr^3}$ $\quad\left[\text{as } L = mvr \therefore v = \frac{L}{mr}\right]$

15. (B)

Net acceleration in non-uniform circular motion,

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left[\frac{900}{500}\right]^2} = 2.7\ \text{ms}^{-2}$$

16. (A)

Centripetal acceleration, $\alpha = \frac{\omega}{t}$, $\alpha = \frac{\omega v}{vt}$ and $\alpha = \frac{\omega a}{v}$

17. (A)

Kinetic energy, $\frac{1}{2}mv^2 = E = \frac{1}{2}mr\frac{v^2}{r} = E$

$$\frac{1}{2}mra = E \Rightarrow a = \frac{2E}{mr}$$

18. (C)

$\omega^2 = \omega_0^2 - 2a\theta$ or $0 = 4\pi^2n^2 - 2a\theta$

$$\theta = \frac{4\pi^2\left(\frac{1200}{60}\right)^2}{2\times 4} = 200\pi^2 \text{ rad}$$

$$\therefore 2\pi n = 200\pi^2 \Rightarrow n = 100\pi = 314 \text{ revolution}$$

19. (A)

$F = r_1\omega_1^2 = r_2\omega_2^2$

$$\therefore r_2 = r_1\frac{\omega_1^2}{\omega_2^2} = 4\times\left(\frac{1}{2}\right)^2 = 4\times\frac{1}{4} = 1 \text{ cm}$$

20. (A)

Moment of inertia of a disc about a diameter is

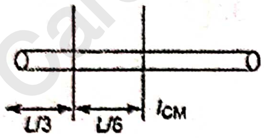
$\frac{1}{4}MR^2 = I$ (given)

$\therefore\ MR^2 = 4I$

Now, required moment of inertia $= \frac{3}{2}MR^2 = \frac{3}{2}(4I) = 6I$

21. (B)

$I_{CM} = \frac{ML^2}{12}$ (about middle point)



$\therefore\ I = I_{CM} + Mx^2 = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$

22. (A)

As we know that, $T = I\alpha$

23. (B)

By doing, so the distribution of mass can be made away from the axis of rotation.

24. (D)

KE given to a sphere at lowest point $=$ PE at the height of suspension

$\frac{1}{2}mv^2 = mgl \Rightarrow v = \sqrt{2gl}$

25. (C)

Linear velocity $v = r\omega \Rightarrow \omega = \frac{v}{r}$

26. (D)

In this case centripetal force provides by friction

$\therefore\ \frac{mv^2}{r} = \mu mg$ and $\mu = \frac{v^2}{rg} = \mu = \frac{12.5 \times 12.5}{20 \times 9.8} = 0.8$

27. (D)

As $v = \sqrt{\frac{2gh}{1+\frac{k^2}{r^2}}}$ Given, $h = \frac{3v^2}{4g}$

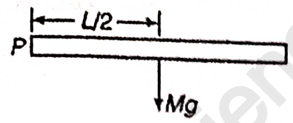
$$v^2 = \frac{2gh}{1+\frac{k^2}{r^2}} = \frac{2g3v^2}{4g\left(1+\frac{k^2}{r^2}\right)} = \frac{6gv^2}{4g\left(1+\frac{u^2}{v^2}\right)}$$

$$\Rightarrow\ 1 = \frac{3}{2\left(1+\frac{k^2}{v^2}\right)}$$

or $1+\frac{k^2}{r^2} = \frac{3}{2} \Rightarrow \frac{k^2}{r^2} = \frac{3}{2} - 1 = \frac{1}{2}$ $k^2 = \frac{1}{2}r^2$ (equation of disc)

Hence, the object is disc.

28. (A)



Torque on the rod = Moment of weight of the rod about P

$$\tau = mg\frac{L}{2} \qquad \ldots(i)$$

$\because$ Moment of inertia of rod about,

$$P = \frac{ML^2}{3} \qquad \ldots(ii)$$

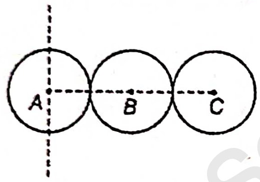
As $\quad \tau = I\alpha$

From Eqs. (i) and (ii), we get

$$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha \quad \Rightarrow \quad \alpha = \frac{3g}{2L}$$

29. (D)

The distance is to be measured from A



$\therefore$ Origin will be at A

Now, for centre of mass $= \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

For the above figure, Centre of mass $= \dfrac{1\times 0 + 1\times AB + 1\times AC}{1+1+1} = \dfrac{AB+AC}{3}$

30. (B)

Applying conservation of energy,

Initial KE = Final PE $\Rightarrow \dfrac{1}{2}Mv^2 + \dfrac{1}{2}I\omega^2 = Mgh$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v^2}{R^2}\right) = Mg\left(\frac{3v^2}{4g}\right)$$

$$\Rightarrow \quad \frac{1}{2}M + \frac{1}{2}\frac{I}{R^2} = \frac{3M}{4} \quad \Rightarrow \quad \frac{I}{2R^2} = \left(\frac{3}{4} - \frac{1}{2}\right)M \Rightarrow I = 2\left(\frac{1}{4}\right)MR^2 = \frac{1}{2}MR^2$$

31. (B)

Moment of inertia of a thin uniform rod about perpendicular axis through it one end

$$I = \frac{1}{3}ML^2 \qquad \ldots(i)$$

Same rod is bent into a ring, $L = 2\pi R \Rightarrow \dfrac{L}{R} = 2\pi \qquad \ldots(ii)$

Moment of inertia of ring about diameter, $I_1 = \dfrac{MR^2}{2}$

Dividing Eq. (i) by (iii), we get

$$\frac{I}{I_1} = \frac{2}{3}\frac{ML^2}{MR^2} = \frac{2}{3}\frac{L^2}{R^2}$$

$$\frac{I}{I_1} = \frac{2}{3}(2\pi)^2 = \frac{8\pi^2}{3} \quad \text{[Using Eq. (ii)]}$$

32. (D)

Initial centripetal force $F = 2\lambda \cdot \omega^2$

Final centripetal force $F_1 = 3\lambda \cdot \omega_1^2$

$F_1 = 2F$ ($\because$ the extension is double and Hooke's law is obeyed)

$\therefore \; 3\lambda \cdot \omega_1^2 = 2(2\lambda \cdot \omega^2)$

$$\omega_1 = \frac{2}{\sqrt{3}} \cdot \omega \quad \text{or} \quad T_1 = \frac{\sqrt{3}}{2} T$$

33. (D)

34. (C)

By law of conservation of energy

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$v = r\omega$$

$$\therefore \; mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mr^2\omega^2 = \frac{1}{2}\omega^2(I + mr^2)$$

$$\therefore \; \omega = \sqrt{\frac{2\,mg\,h}{I + mr^2}}$$

35. (B)

$$\frac{v^2}{r} = K^2rt^2 \quad \therefore \frac{v^2}{r^2} = K^2t^2 \quad \therefore \frac{v}{r} = Kt \quad \therefore \omega = Kt$$

$$\therefore \frac{d\omega}{dt} = \alpha = K$$

Tangential acceleration $a_t = r\alpha = Kr$

$\therefore \; F_t = mKr$

Power $P = F_t \cdot v = (m\,K\,r)\,(r\,K\,t)$ ($\because v = r\,K\,t$)

$= m\,K^2\,r^2\,t$

36. (B)

$$K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{2}{3}\times MR^2\times\omega^2 = \frac{1}{3}MR^2\omega^2$$

$$\therefore \; \frac{3}{4}\times\frac{1}{3}MR^2\omega^2 = MS\Delta\theta$$

$$\therefore \; \Delta\theta = \frac{R^2\omega^2}{4S}$$

37. (A)

Total K.E of the rolling disc or ring is given by.

$$K.E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For ring and disk, translational kinetic energy $\frac{1}{2}mv^2$ is constant.

Rolling K.E. of disc is $\frac{1}{4}mR^2\omega^2$

Rolling K.E of ring is $\frac{1}{2}mR^2\omega^2$

As for ring, $4J = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2$

$\therefore\ mR^2\omega^2 = 4J$

For disc $\frac{1}{2}mR^2\omega^2 + \frac{1}{4}mR^2\omega^2 = \left(\frac{4}{2}+\frac{4}{2}\right)J = (2+1)\,J = 3J$

38. (A)

Volume of disc is $A\cdot d = \pi\cdot R^2 \times \frac{R}{6} = \frac{R^3\pi}{6}$

Moment of inertia of disc is $I = \frac{1}{2}MR^2$

When the disk is remolded in solid sphere of volume V having radius r, then

$\frac{\pi R^3}{6} = \frac{4}{3}\times\pi r^3 \Rightarrow \frac{R^3}{6}\times\frac{3}{4} = r^3$

$\therefore\ r^3 = \frac{R^3}{8} \quad \Rightarrow r = \frac{R}{2}$

Moment of inertia of sphere is given by

$\frac{2}{5}m\cdot r^2 = \frac{2}{5}\times m\cdot\frac{R^2}{4} = \frac{MR^2}{10} = \frac{MR^2}{2}\times\frac{1}{5} = \frac{I}{5}$

39. (B)

M.I of rod whose axis of rotation is passing through center and perpendicular to the plane of rod is

$I = \frac{ML^2}{12}$ and $I = MK_1^2$ (where K_1 is radius of gyration)

$\therefore\ MK_1^2 \Rightarrow K_1 = \frac{L}{2\sqrt{3}}$ ...(1)

When axis of rotation of rod is passing through one and of rod, then

$I = MK_2^2 = \frac{ML^2}{3} \Rightarrow K_2 = \frac{L}{\sqrt{3}}$ ...(2)

Taking ratio of (1) and (2) we get

$\frac{K_1}{K_2} = \frac{L}{2\sqrt{3}}\times\frac{\sqrt{3}}{L} = \frac{1}{2} \Rightarrow \frac{K_1}{K_2} = \frac{1}{2}$

40. (D)

$K.E = \frac{1}{2}mv^2$

At lowest point in vertical circular motion. $V_L = \sqrt{5rg}$ and at highest point $V_h = \sqrt{rg}$

$\therefore\ \frac{(K.E)_h}{(K.E)_L} = \frac{1}{5} = 0.2$

41. (A)

Using $v^2 - u^2 = 2as$ and $u = 0$ and $s = 4\pi r$

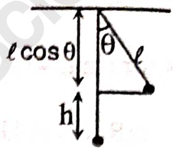
$\therefore \quad 2as = v^2 \qquad \Rightarrow \dfrac{v^2}{2s} = a = \dfrac{v^2}{2\times 4\pi r} = \dfrac{v^2}{8\pi r}$

42. (C)

When bob is at rest at extreme position, the pendulum has only potential energy which is given by

P.E. = $mgh = mg(\ell - \ell\cos\theta) = mg\,\ell(1-\cos\theta)$.

When bob comes to the mean position, the pendulum loses P.E. of bob and it gets converted to K.E.



43. (C)

$I = 2$ kg m^2, $\omega_0 = 60$ rad/s

$t = 5$ min $= 5 \times 60 = 300$ s

$\alpha = \dfrac{0-60}{300} = \dfrac{-60}{300} = \dfrac{-1}{5}$ rad/s^2

for 2 min (from starting) (2 min = 120 sec)

$\omega \; = \omega_0 + \alpha t \; = 60 - \dfrac{1}{5} \times 120 = 60 - 24$

$\omega \; = 36$ rad/s & $L = I\omega = 2 \times 36 = 72$ kg m^2/s

44. (B)

Kinetic energy of rolling solid sphere

$= \dfrac{1}{2}mV^2 + \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}mV^2 + \dfrac{1}{2}\times\dfrac{2}{5}mr^2\omega^2 = \dfrac{1}{2}mV^2 + \dfrac{1}{5}mV^2 = \dfrac{7}{10}mV^2$

The potential energy of the spring on maximum compression x is

$\therefore \quad \dfrac{1}{2}kx^2 = \dfrac{7}{10}mV^2$

$x^2 = \dfrac{14}{10}\dfrac{mV^2}{k} = \dfrac{14}{10}\times\dfrac{2\times(6)^2}{36} = 2.8$

$\therefore \quad x = \sqrt{2.8}$ m

45. (C)

$\left(\dfrac{\omega}{4}\right)^2 \; = \; \omega^2 - 2\alpha n(2\pi)$

$\therefore \; 2\alpha n\,(2\pi) \; = \; \omega^2 - \dfrac{\omega^2}{16}$

$\therefore \; 2\pi n \; = \; \dfrac{15}{16}\left(\dfrac{\omega^2}{2\alpha}\right)$

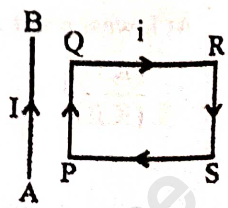
$0 \; = \; \omega^2 - 2\alpha n'$

$\therefore \; 2\pi n' \; = \; \dfrac{\omega^2}{2\alpha} \qquad \therefore \qquad n' \; = \; \dfrac{16}{15}n$

46. (D)

$I_1\omega_1 = (I_1 + I_2)\,\omega_2$

$\dfrac{\omega_2}{\omega_1} = \dfrac{I_1}{I_1 + I_2}$



$$E_1 - E_2 = \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}(I_1+I_2)\,\omega_2^2 = \frac{1}{2}\omega_1^2\left[I_1 - (I_1+I_2)\frac{\omega_2^2}{\omega_1^2}\right] = \frac{1}{2}\omega_1^2\left[I_1 - (I_1+I_2)\frac{I_1^2}{(I_1+I_2)^2}\right]$$

$$= \frac{1}{2}\omega_1^2\left[\frac{I_1^2 + I_1I_2 - I_1^2}{I_1+I_2}\right] = \frac{1}{2}\left[\frac{I_1I_2}{I_1+I_2}\right]\omega_1^2$$

47. (A)

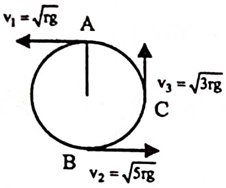
48. (D)

$\omega_0 = 0, \omega = 24$ rad/s, $t = 8$s

$$\therefore \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{24}{8} = 3\text{rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\times 3\times(8)^2 = \frac{3\times 64}{2} = 96 \text{ rad.}$$

49. (B)



Centripetal acceleration at C $= \dfrac{v_3^2}{r} = \dfrac{3rg}{r} = 3g$

50. (C)

$a_t = r\alpha$

$$a_r = \frac{\upsilon^2}{r}$$

$$\therefore \quad \frac{a_t}{a_r} = \frac{r\alpha}{\left(\frac{\upsilon^2}{r}\right)} = \frac{\alpha r^2}{\upsilon^2}$$

51. (B)

$$\alpha = \frac{\omega}{t}$$

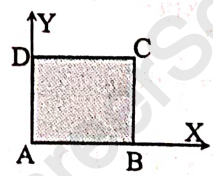
$$\text{Torque } \tau = I\alpha = \frac{MR^2}{2}\cdot\frac{\omega}{t} = \frac{MR^2\omega}{2t}$$

$$\text{Force} = F = \frac{\tau}{R} = \frac{MR\omega}{2t}$$

52. (A)

$$I_{AB} = 0;\ I_{AD} = I_{BC} = \frac{m\ell^2}{3};\ I_{DC} = m\ell^2$$

$$\therefore \text{ Total moment of inertia} = I = 2\frac{m\ell^2}{3} + m\ell^2 = \frac{5}{3}m\ell^2$$



53. (B)

$$I_1\omega_1 = I_2\omega_2 = 2I_1\omega_2$$

$$\therefore \quad \omega_2 = \frac{I_1\omega_1}{2I_1} = \frac{\omega_1}{2}$$

$\therefore\ KE_1 = \frac{1}{2}I\omega^2$

$KE_2 = \frac{1}{2}(2I)\frac{\omega^2}{4} = \frac{I\omega^2}{4}$

$\therefore\ KE_1 - KE_2 = \frac{1}{2}I\omega^2\left[1-\frac{1}{2}\right] = \frac{1}{2}I\omega^2 \times \frac{1}{2} = \frac{I\omega^2}{4}$

54. (A)

$F = \frac{mv^2}{r} = mr\omega^2$

$\sqrt{F} = \sqrt{mr}\,\omega = \sqrt{mr}\,\frac{2\pi}{T}$

55. (B)

$T_1 = \frac{mv^2}{L} - mg = \frac{40\times40}{2} - 10 = 790$

$T_2 = \frac{mv^2}{L} + mg = \frac{40\times40}{2} + 10 = 810$

$\frac{T_1}{T_2} = \frac{79}{81}$

56. (D)

$a = a_T + a_r$

$a^2 = a_r^2 + a_T^2 = \frac{v^2}{r} + g^2 = p^2g^2 + g^2 = g^2(1+p^2)$

$a = \sqrt{1+p^2}\cdot g$

57. (A)

$I = \frac{3}{2}mR^2 = \frac{3}{2}L\rho R^2 \qquad L = 2\pi R$

$= \frac{3}{2}L\frac{\rho L^2}{4\pi^2} = \frac{3\rho L^3}{8\pi^2}$

58. (C)

$2mLv + m\,2v\,2L = I\omega$

$6mLv = 30mL^2\omega \Rightarrow \omega = \frac{v}{5L}$

59. (A)

$I_{sphere} = \frac{2}{5}MR^2$

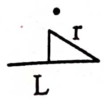
$\frac{4}{3}\pi R_1^3\rho_1 = \frac{4}{3}\pi R_2^3\rho_2$

$= \frac{4}{3}\pi(2R_1)^3\rho_2$

$\therefore\ \rho_1 = 8\rho_2$

$\frac{I_1}{I_2} = \frac{\rho_1}{\rho_2}\left(\frac{R_1}{R_2}\right)^5 = 8\times\frac{1}{32} = \frac{1}{4}$

60. (B)



$$\cos 30° = \frac{L/2}{r} = \frac{\sqrt{3}}{2}$$

$$\frac{L}{2r} = \frac{\sqrt{3}}{2}$$

$$r \propto L$$

$$F = \frac{mv^2}{r} = mrw^2 = mr\frac{4\pi^2}{T^2}$$

$$\therefore \quad T^2 \propto r \propto L$$

$$T \propto \sqrt{L}$$

61. (B)

Work done = change in KE

$$\therefore \quad W = \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$$

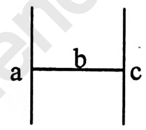
$$12000 = \frac{1}{2}I(400-100)4\pi^2 \qquad \text{or} \quad I = 2$$

62. (B)

$I_a = 0$

$I_b = \frac{ML^2}{3}$

$I_c = ML^2$



$$I = I_a + I_b + I_c = \frac{ML^2}{3} + ML^2 = \frac{4ML^2}{3}$$

63. (A)

$$V = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}\times 10\times 7} = \sqrt{100}$$

64. (C)

$$E = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = I\omega^2 = \frac{1}{2}m\left(\frac{d}{2}\right)^2\omega^2 + \frac{1}{2}m\left(\frac{d}{2}\right)^2\omega^2$$

$$= \frac{d^2}{8}m\omega^2 + \frac{d^2}{8}m\omega^2 = \frac{d^2}{4}m\omega^2$$

$$\omega = \sqrt{\frac{E}{\frac{md^2}{4}}} = \frac{2}{d}\sqrt{\frac{E}{m}}$$

65. (A)

$$2\times\frac{ML^2}{12} + 2\times\frac{ML^2}{4} = \frac{4}{6}ML^2 = \frac{2}{3}ML^2$$

66. (A)

$$\frac{V_1^2}{V_2^2} = \frac{1+\frac{K_1^2}{R^2}}{1+\frac{K_2^2}{R^2}} = \frac{1+\frac{R^2}{2R^2}}{1+\frac{\frac{2}{5}R^2}{R^2}} = \frac{1+\frac{1}{2}}{1+\frac{2}{5}} = \frac{3/2}{7/5} = \frac{3}{2}\times\frac{5}{7} = \frac{15}{14}$$

67. (B)

$$I_1 = \frac{MR^2}{2} \quad M' = \frac{M}{4}$$

$$I_2 = M'\left(\frac{R}{2}\right)^2 + \frac{1}{2}M'\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$$

$$I = I_1 - I_2 = \frac{13MR^2}{32}$$

68. (B)

$$ms\,\Delta T = \frac{1}{2}\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}\frac{2}{5}mr^2\right)(2\pi n)^2$$

$$\Delta T = \frac{2\pi^2 n^2 r^2}{5s}$$

69. (B)

$$W = \tau\theta = I\,\alpha\,\theta = 2\alpha\theta$$

$$\omega_2^2 = \omega_1^2 - 2\alpha\theta$$

$$0 = (25)^2 - 2\alpha\theta$$

$\therefore\ 2\alpha\theta = 625 \qquad \therefore\ W = 625\ J$

70. (A)

$$a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$$

For ring $K = R$

For disc $K = \frac{R}{\sqrt{2}}$

For sphere $K = \sqrt{\frac{2}{5}}R$

Putting these

$a_{ring} = 0.5\ g\sin\theta$

$a_{disc} = 0.6\ g\sin\theta$

$a_{sphere} = 0.7\ g\sin\theta$

$\therefore$ ring will reach the bottom, last.

71. (C)

$I_{coil} = mr^2$

$I_{disc} = \frac{1}{2}mr^2$

$$KE_{coil} = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}mr^2\omega^2 = mr^2\omega^2$$

$$KE_{disc} = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}\cdot\frac{1}{2}m\omega^2 r^2 = \frac{3}{4}m\omega^2 r^2$$

$$\frac{KE_{coil}}{KE_{disc}} = \frac{4}{3} \Rightarrow KE_{disc} = KE_{coil} \times \frac{3}{4} = 9\ J$$

72. (D)

73. (B)

$$m = 100\text{ gm} = \frac{1}{10}\text{kg}$$

$$r = 10\text{ cm} = \frac{1}{10}\text{m}$$

$$f = 1\text{ Hz} \quad \therefore \quad \omega_1 = 2\pi$$

$$I_1\omega_1 = 2\pi\times10^{-3}$$

$$I_2 = 10^{-3} + 4\times\frac{12.5}{1000}\times\frac{1}{100} = 1.5\times10^{-3}$$

$$I_2\omega_2 = 1.5\times10^{-3}\times2\pi f_2$$

$$\therefore \quad 2\pi\times10^{-3} = 1.5\times10^{-3}\times2\pi f_2$$

$$f_2 = \frac{1}{1.5} = \frac{2}{3}\text{Hz}$$

74. (D)

$$L = mvR$$

$$L \propto R$$

75. (B)

$$L = mvr = m(2v)\left(\frac{r}{2}\right)$$

76. (C)

$$\frac{mv_1^2}{r_1} = \mu mg = \frac{mv^2}{r_2}$$

$$m_1r_1\omega_1^2 = m\,r_2\omega_2^2$$

$$\therefore \quad m_1r_1\omega_1^2 = m\,3r_1\omega_2^2$$

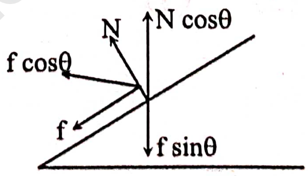
$$\omega_2^2 = \frac{\omega_1^2}{3} \Rightarrow \omega = \frac{\omega_1}{\sqrt{3}}$$

77. (A)

$$N\cos\theta = mg + f\sin\theta$$

$$mg = N\cos\theta - f\sin\theta$$

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$



$$\frac{mv^2}{R} = N\sin\theta + f\cos\theta$$

$$\frac{v^2}{Rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

$$V_m = \sqrt{Rg\left[\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}\right]} = \sqrt{Rg\left[\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right]}$$

78. (A)

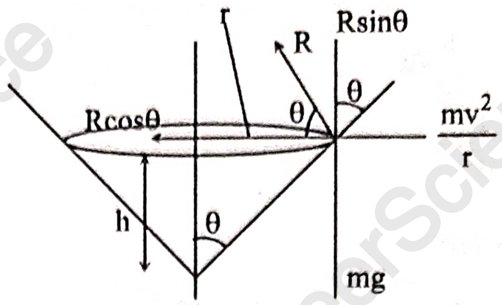
$mg = R\sin\theta$

$\frac{mv^2}{r} = R\cos\theta$

$\tan\theta = \frac{rg}{v^2}$

$\tan\theta = \frac{r}{h}$

$h = \frac{v^2}{g}$



79. (D)

$I_{total} = \frac{MR^2}{2}$

$I_{removed} = \frac{M}{4}\frac{(R/2)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$ $\left[\because \frac{mr^2}{2} + mh^2\right]$

$I_{remain} = I_{total} - I_{removed}$

$= \frac{MR^2}{2} - \frac{3}{32}MR^2 = \frac{13}{32}MR^2$

80. (D)

$mv = (m+m)v_1$

$v_1 = \frac{v}{2}$

$T = \frac{2mv_1^2}{L} + 2mg = \frac{2mv^2}{4L} + 2mg$

$= \frac{2m(2gL)}{4L} + 2mg = 3mg$

Initial tension = mg

$\therefore$ Increased tension = 2mg

81. (D)

MI of the molecule = $m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2$

Rotational KE = $\frac{1}{2}I\omega^2$

$E = \frac{1}{2} \times \frac{md^2}{2} \times \omega^2$

$\therefore \omega^2 = \frac{4E}{md^2}$ $\qquad \omega = \frac{2}{d}\sqrt{\frac{E}{m}}$

82. (A)

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2 \times \frac{v^2}{r^2} = mgh$

$$\frac{v^2}{2}+\frac{2v^2}{10}=gh$$

$$\frac{5v^2+2v^2}{10}=gh$$

$$v^2=\frac{10}{7}gh$$

$$v=\sqrt{\frac{10}{7}gh}$$

83. (B)

$I_1 = m_1R^2$ $\qquad$ $I_2 = m_2n^2R^2$

$$\frac{I_1}{I_2}=\frac{m_1}{m_2}\frac{R^2}{n^2R^2}=\frac{2\pi R}{2\pi nR}\times\frac{R^2}{n^2R^2}=\frac{1}{n^3}=\frac{1}{8}$$

$\therefore n = 2$

84. (D)

$$I_x=\frac{M_xR^2}{2} \qquad M_x=(\pi R^2t)\rho$$

$$=\frac{\pi R^4t\rho}{2}$$

$$I_y=\frac{3\pi R^2t\rho R^2\times 9}{2} \qquad M_y=\left(\pi 9R^2\frac{t}{3}\right)\rho$$

$$=\frac{27\pi t\rho R^4}{2} \qquad \therefore\ 27I_x=I_y$$

85. (B)

86. (C)

$m = 4\times10^{-3}$ kg $\qquad r=\frac{1}{2\pi}$ m $\qquad E = 18\times10^{-5}$ J

$$\frac{1}{2}mV^2=18\times10^{-5}\text{ J}$$

$$V^2=\frac{2\times18\times10^{-5}}{4\times10^{-3}}=9\times10^{-2}\ (\text{m/s})^2$$

$V = 3\times10^{-1}$ m/s

$V^2 = 2a_t s = 2a_t\times(4\pi r)$

$$a_t=\frac{V^2}{8\pi r}=\frac{9\times10^{-2}}{8\pi\times\frac{1}{2\pi}}=\frac{9}{8}\times10^{-2}=1.1\times10^{-2}\text{ m/s}^2$$

87. (B)

$mv-(-mv)=2mv$

88. (C)

Case 1 : $\frac{1}{2}mV^2=mgh$ $\qquad$ ...(1)

Case 2 : $\frac{1}{2}mv'^2+\frac{1}{2}I\omega^2=mgh$

$$\frac{1}{2}mv'^2 + \frac{1}{2}(mR^2)\frac{v'^2}{R^2} = mgh$$

$$mv'^2 = mgh \qquad \ldots(2)$$

From (1) and (2)

$$\frac{1}{2}mV^2 = mv'^2$$

$$v' = \frac{1}{\sqrt{2}}V$$

89. (A)

$$a_1 = \frac{v_1^2}{R} \qquad a_2 = \frac{(2v_1)^2}{R}$$

$$\therefore \frac{a_2}{a_1} = 4$$

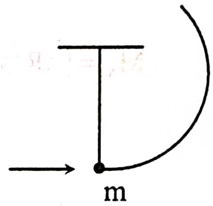
90.(B)

$h = 2L$

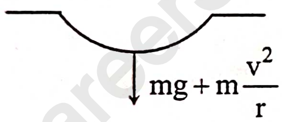
$$\frac{1}{2}mv^2 = mg2L$$

$$v^2 = 4gL$$

$$v = 2\sqrt{gL}$$



91. (B)



Both centrifugal force and weight of the vehicle will be experienced downward.

92. (C)

$I = I_1 + I_2$

$$= MR^2 + 2\left(mR^2\right) = MR^2 + 2mR^2 = (M+2m)R^2$$

$$I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{MR^2\omega}{(M+2m)R^2} = \frac{M\omega}{M+2m}$$

93. (A)

94. (C)

$$T_1 = \frac{mv_1^2}{r} - mg \qquad T_2 = \frac{mv_2^2}{r} + mg$$

$$\therefore\ T_2 - T_1 = \left(V_2^2 - V_1^2\right) + 2mg = 4mg + 2mg = 6mg$$

95. (B)

$M = 4$ kg, $R = 0.4$ m

$MI = I_0 + Mh^2$

$$= \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$

$$= \frac{3}{2}\times 4\times\frac{4\times 4}{100} = \frac{6\times 16}{100} = \frac{96}{100}$$

$$= 0.96 \text{ kg m}^2$$

96. (C)

$$\frac{mv^2}{R} = \mu mg$$

$$R = \frac{v^2}{\mu g} \qquad (\tan 45° = \mu \quad \therefore \quad \mu = 1)$$

$$= \frac{10\times 10}{10} = 10 \text{ m} \qquad [v = 36 \text{ km/hr} = \frac{36000}{60\times 60} = 10 \text{ m/s}]$$

97. (C)

$$I_1 = \frac{I_2}{3} \qquad \frac{L_1}{L_2} = ?$$

$$\therefore \; E_1 = \frac{1}{2}I_1\omega_1^2 \qquad E_1 = 27E_2$$

$$E_2 = \frac{1}{2}I_2\omega_2^2$$

$$\frac{1}{2}I_1\omega_1^2 = 27\times\frac{1}{2}I_2\omega_2^2$$

$$\frac{\omega_1^2}{\omega_2^2} = \frac{27I_2}{I_1} = 27\times 3$$

$$\frac{\omega_1}{\omega_2} = 9$$

$$\frac{L_1}{L_2} = \frac{I_1\omega_1}{I_2\omega_2} = \frac{1}{3}\times 9 = \frac{3}{1}$$

98. (B)

$$I_1 = 2\pi R\times R^2$$

$$I_2 = 2\pi\,(nR)\,(nR)^2$$

$$\therefore \; \frac{I_1}{I_2} = \frac{1}{n^3} = \frac{1}{27}$$

$$\therefore \; n = 3$$

99. (C)

$M = 100$ kg $\quad R = 1$ m

$f = 300$ rpm $\quad = \frac{300}{60}$ rps $= 5$ rps

$\omega = 2\pi\times 5 = 10\pi$ rad/s

$t = 50$ sec

$$\tau = I\,\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{\omega}{t}\right) = \frac{1}{2}\times 100\times 1^2\times\frac{10\pi}{50} = 10\pi \text{ Nm}$$

To rotate the disc in opposite direction torque required = 20π Nm

100.(D)

Angular velocity of minute hand $= \dfrac{360^\circ}{60\times 60} = \dfrac{1}{10} = 0.1$

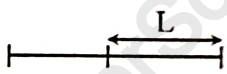
101.(A)

$$\frac{mv^2}{r} = \mu m g$$
$$v = \sqrt{\mu g r}$$

102.(A)

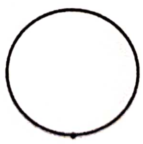
$$KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2} + \frac{1}{2}Mv^2$$
$$= \frac{1}{5}Mv^2 + \frac{1}{2}Mv^2$$
$$= \frac{7}{10}Mv^2 = \frac{7}{10}\times 1\times 10\times 10^{-2} = 7\times 10^{-2}\,J$$
$$= 0.007\ J$$

103.(C)



$$\ell = 2L$$
$$M = m\ell$$
$$M = m\,2L$$
$$\ell^2 = 4L^2$$
$$I = \frac{M\ell^2}{12}$$
$$I = \frac{2mL\times 4L^2}{12} = \frac{2}{3}mL^3$$

104.(A)



Force at the lowest point

$$F = Mg + \frac{mv^2}{R}$$
$$F = Mg + MR\omega^2$$
$$Y = \frac{FL}{A\Delta L} = \frac{M(g+R\omega^2)L}{\frac{\pi D^2}{4}\Delta\ell}$$
$$\Delta\ell = \frac{4M(g+R\omega^2)L}{\pi D^2 Y}$$

105.(D)

$$V = r\omega = r\frac{\theta}{t}$$
$$\therefore\ \frac{\theta}{t} = \frac{V}{r}$$

106.(D)

$$\tau = I\alpha$$

$$\therefore \alpha = \frac{\tau}{I} = \frac{F \times R}{\frac{MR^2}{2}} = \frac{2F}{MR} = \text{constant}$$

$$\omega = \omega_0 + \alpha t = \frac{2F}{MR} \cdot t$$

$$\therefore F = \frac{MR\omega}{2t}$$

107.(B)

Tangential acceleration = $r\alpha$

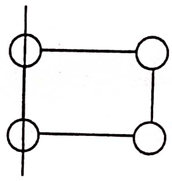
Radial acceleration = $\frac{v^2}{r}$

$$\therefore \frac{\text{tangential acceleration}}{\text{radial acceleration}} = \frac{r\alpha}{v^2/r} = \frac{r^2}{v^2}\alpha = \left(\frac{r}{v}\right)^2 \alpha$$

108.(C)

$$I = \left(\frac{2}{5}MR^2\right)4 + 2ML^2$$

$$= \frac{8}{5}MR^2 + 2ML^2$$



109.(B)

We know $P = F \cdot v$

$= \tau \cdot \omega$ $\quad$ [$W = F.S$

$= I\alpha \cdot \omega$ $\quad$ $P = F.V$]

$$\therefore \omega = \frac{P}{I\alpha}$$

110.(D)

MI of the disc is $\frac{1}{2}MR^2$

$$I_{Total} = I_{removed} + I_{remain}$$

$$\frac{1}{2}MR^2 = \left(\frac{M}{4}\right)\frac{R^2}{2} + I_{remain} \quad = \frac{MR^2}{8}$$

$$I_{remain} = \frac{1}{2}MR^2 - \frac{1}{8}MR^2 \quad = \frac{(4-1)}{8}MR^2 = \frac{3}{8}MR^2$$

111.(A)

$$\left(\frac{1}{2}Mr^2 + mr^2\right)\omega_1 = \frac{1}{2}Mr^2\omega_2$$

$$\therefore \omega_2 = \frac{\left(\frac{1}{2}Mr^2 + mr^2\right)\omega_1}{\frac{1}{2}Mr^2}$$

$$= \frac{\left(\frac{1}{2}M+m\right)\omega_1 r^2}{\frac{1}{2}Mr^2} = \frac{50+20}{50}\times 5$$

$$= \frac{70\times 5}{50} = 7 \text{ r.p.m.}$$

112.(A)

$R = 400$ m, $\quad v = 24$ m/s

$$\frac{v^2}{R} = \mu g$$

$$\mu = \frac{v^2}{Rg} = \frac{24\times 24}{400\times 10} = \frac{144}{1000} = 0.144$$

113.(D)

$$\frac{\omega_m}{\omega_s} = \frac{\frac{2\pi}{60\times 60}}{\frac{2\pi}{60}} = \frac{1}{60}$$

$$\therefore \quad \frac{\omega_s}{\omega_n} = 60$$

114.(B)

External torque = 0

$\therefore$ $L_1 = L_2$

$I_1\omega_1 = I_2\omega_2$

$$\frac{2}{5}MR_1^2\omega_1 = \frac{2}{5}MR_2^2\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{R_2^2}{R_1^2}$$

$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2}I_1\omega_1^2}{\frac{1}{2}I_2\omega_2^2} = \frac{R_1^2}{R_2^2}\times\frac{R_2^4}{R_1^4} = \frac{R_2^2}{R_1^2} = \frac{1}{9}$$

$$\therefore \quad \frac{KE_2}{KE_1} = 9$$

115.(C)

$$I = I_o + Mh^2$$

$$= \frac{2}{5}MR^2 + M\frac{R^2}{4}$$

$$= MR^2\left[\frac{2}{5}+\frac{1}{4}\right]$$

$$= MR^2\frac{8+5}{20} = \frac{13}{20}MR^2$$

116.(D)

$m = 10\text{ kg} \quad \ell = 0.3\text{ m}$

B.S. $= 4.8 \times 10^7\text{ N/m}^2 \qquad A = 10^{-6}\text{ m}^2$

$\therefore\ F = 4.8 \times 10^7 \times 10^{-6} = 48\text{ N}$

$$\frac{mv^2}{r} = 48$$

$$v^2 = \frac{48 \times 0.3}{10} = \frac{144}{100}$$

$$v = \frac{12}{10} = 1.2\text{ m/s}$$

$$\omega = \frac{1.2}{r} = \frac{1.2}{0.3} = 4\text{ rad/sec}$$

117.(A)

118.(D)

When temperature is increased, length will increase

$$I_1\omega_1 = I_2\omega_2$$

$$\frac{mL_1^2}{12}\omega_{12} = \frac{mL_2^2}{12}\omega_2$$

Since $L_2 > L_1 \qquad \therefore\ \omega_2 < \omega_1$

119.(D)

$$\frac{F_1}{F_2} = \frac{\frac{m_1v_1^2}{r_1}}{\frac{m_2v_2^2}{r_2}} = \frac{m_1}{m_2} \times \frac{v_1^2}{v_2^2} \times \frac{r_2}{r_1}$$

$$t = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

$$\therefore\ \frac{r_2}{r_1} = \frac{v_2}{v_1}$$

$$\therefore\ \frac{F_1}{F_2} = \frac{m_1}{m_2} \times \frac{v_1^2}{v_2^2} \times \frac{v_2}{v_1} = \frac{m_1v_1}{m_2v_2} = \frac{m_1r_1}{m_2r_2}$$

120.(D)

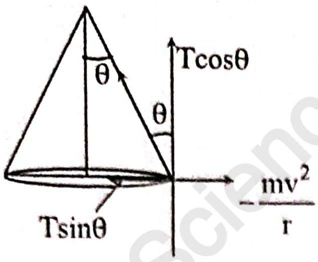
$$I_{total} = \frac{MR^2}{2} \qquad M_{removed} = \frac{M}{4}$$

$$I_{removed} = \frac{M}{4} \times \frac{(R/2)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$$

$$\therefore\ I_{remaining} = I_{Total} - I_{removed} = \frac{MR^2}{2} - \frac{3MR^2}{32}$$

$$= \frac{13MR^2}{32}$$

121.(C)



122.(A)

$\tau = I\alpha \qquad \therefore \alpha = \frac{50}{I}$

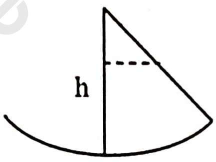
$\omega = \omega_0 + \alpha\, 8 = \alpha\, 8 = \frac{50}{I}\times 8$

$L = I\omega$

$= I\frac{50\times 8}{I}$

$= 400\ \text{kg m}^2/\text{s}$

123.(D)



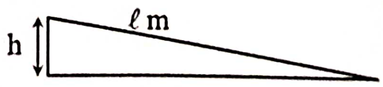
$mgh = \frac{1}{2}I\omega^2$

$I = \frac{mL^2}{3}$

$\therefore \quad mgh = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2$

$h = \frac{1}{6g}L^2\omega^2$

124.(C)



$\frac{mv^2}{r} = \mu R = \mu mg$

$v^2 = \mu gr$

$v = \sqrt{\mu gr} = \sqrt{\tan\theta\, gr} = \sqrt{\sin\theta\, g\, r}$ [For small angle of θ]

$= \sqrt{gr\frac{h}{\ell}}$

125.(B)

$\frac{\omega_h}{\omega_s} = \frac{\frac{2\pi}{12\times 60\times 60}}{\frac{2\pi}{60}} = \frac{1}{12\times 60}$

$\omega_s = 720\,\omega_h$

$\frac{\omega_s - \omega_h}{\omega_s} = \frac{720-1}{720}$

$\omega_s - \omega_h = \frac{719}{720}\omega_s = \frac{719}{720}\times\frac{2\pi}{60} = \frac{719\pi}{21600}$

126.(C)

$$E=\frac{1}{2}m\omega^2r^2$$

$$\therefore\ 2E=m\omega^2r^2$$

$$\therefore\ r\omega^2=\frac{2E}{mr}$$

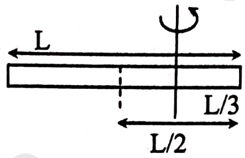
$$a=\frac{v^2}{r}=r\omega^2$$

$$a=\frac{2E}{mr}$$

127.(B)

$$I\ =\ I_o+Mh^2$$

$$h\ =\frac{L}{2}-\frac{L}{3}=\frac{L}{6}$$



$$I=\frac{ML^2}{12}+Mh^2$$

$$=\frac{ML^2}{12}+\frac{ML^2}{36}$$

$$=\frac{4ML^2}{36}=\frac{ML^2}{9}$$

128.(C)

$$I_{Ring}=MR^2+Mh^2=MR^2+MR^2=2MR^2$$

$$I_{Disc}=\frac{1}{2}MR^2+Mh^2=\frac{1}{2}MR^2+MR^2=\frac{3}{2}MR^2$$

$$\therefore\ k_{ring}=\sqrt{2}R$$

$$k_{disc}=\sqrt{\frac{3}{2}}R$$

$$\therefore\ \frac{k_{ring}}{k_{disc}}=\frac{\sqrt{2}R}{\frac{\sqrt{3}}{\sqrt{2}}R}=\frac{2}{\sqrt{3}}$$

129.(A)

$$S=r\theta$$

$$\theta=\omega_0t+\frac{1}{2}\alpha t^2$$

$$\therefore\ t=\sqrt{\frac{2\theta}{\alpha}}$$

$$\text{Average velocity}=\frac{\text{displacement}}{\text{time}}=\frac{r\theta}{\sqrt{\frac{2\theta}{\alpha}}}$$

$$=r\sqrt{\frac{\alpha}{2\theta}}\cdot\theta=r\sqrt{\frac{\alpha\theta}{2}}$$

130.(C)

$$\text{Total KE} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2 = Mgh$$

$$\text{But, Rotational KE} = \frac{1}{4}Mv^2 = \frac{Mgh}{3}$$

131.(A)

Change in speed after half revolution is 2V

$$\text{Time taken} = \frac{\pi R}{V}$$

$$\therefore \text{ average speed} = \frac{2V^2}{\pi R}$$

132.(A)

$L_1 = 1$ Js $\qquad L_2 = 4$ Js

$t = 4$

$$\tau = \frac{4-1}{4} = \frac{3}{4}\ \text{J}$$

133.(B)

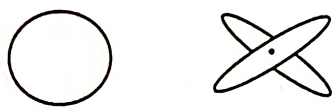
$$\text{Tangential acceleration} = a = \alpha r \qquad \ldots(1)$$

$$\text{Radial acceleration} = \frac{V^2}{r} \qquad \ldots(2)$$

$$\therefore \ (1) \div (2) \quad \frac{\alpha r \times r}{V^2} = \frac{\alpha r^2}{V^2}$$

134.(A)

$$MR^2 + \frac{MR^2}{2} = \frac{3MR^2}{2}$$

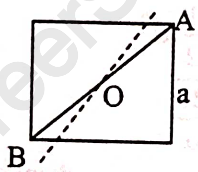


135.(A)

$$I_1 = \frac{2}{5}MR^2$$

$$I_2 = \frac{2}{5}\frac{M}{2}\times\left(\frac{R}{2}\right)^2 = \frac{2}{5}\times\frac{M}{2}\times\frac{R^2}{4} = \frac{MR^2}{20}$$

136.(A)



$$I_o = \frac{Ma^2}{6}$$

$$AB = \sqrt{2a^2} = \sqrt{2}a \qquad AO = \frac{a}{\sqrt{2}} = h$$

$$\therefore \ I_A = I_o + Mh^2$$

$$= \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{Ma^2 + 3Ma^2}{6} = \frac{4Ma^2}{6} = \frac{2Ma^2}{3}$$

137.(B) **138.**(B)

139.(A)

$$E_1 = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega_1^2$$

$$E_2 = \frac{3}{2}mr^2\omega_1^2$$

$$\frac{E_2}{E_1} = 3 \qquad \therefore \quad E_2 = 3E_1$$

$$\frac{1}{2}mr^2\omega^2 = 3\frac{1}{2}m\omega_0^2r^2$$

$$\omega^2 = 3\omega_0^2$$

$$\frac{1}{2}m\omega_0^2r^2 = E$$

$$\omega_0^2 = \frac{2E}{mr^2}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$3\omega_0^2 = \omega_0^2 + 2\alpha(3.2\pi)$$

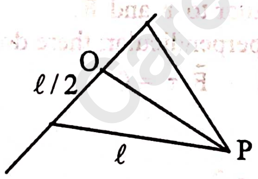
$$2\omega_0^2 = 12\alpha\pi$$

$$\alpha = \frac{\omega_0^2}{6\pi} = \frac{2E}{mr^2}\times\frac{1}{6\pi} = \frac{E}{3\pi mr^2}$$

$$\text{But } a = r\alpha = r\times\frac{E}{3\pi mr^2} = \frac{E}{3\pi mr}$$

140.(D)

$$OP = \sqrt{\ell^2 - \frac{\ell^2}{4}} = \frac{\sqrt{3}}{2}\ell$$



$$\therefore \quad MOI = m\left(\frac{\sqrt{3}\ell}{2}\right)^2 = \frac{3}{4}m\ell^2$$

141.(B)

$$I_1 = \frac{mR_1^2}{2}$$

$$I_2 = \frac{mR_2^2}{2}$$

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{t_2}{t_1}$$

$$\therefore \quad I_1t_1 = I_2t_2$$

$$m_1 = \pi R_1^2 t_1$$

$$m_2 = \pi R_2^2 t_2$$

$$m_1 = m_2$$

$$\therefore R_1^2 t_1 = R_2^2 t_2$$

142.(B)

$$\frac{I_A}{I_B} = \frac{64}{1}$$

$$64 = \frac{m_A r_A^2}{m_B r_B^2} = \frac{2\pi r_A}{2\pi r_B}\frac{r_A^2}{r_B^2} = \frac{r_A^3}{r_B^3} = n^3$$

$$\therefore \quad n = 4$$

143.(A)

The length of the diagonal of the square $= \sqrt{2}\,L$

Half of the diagonal $= \dfrac{\sqrt{2}L}{2} = \dfrac{L}{\sqrt{2}}$

Moment of inertia of the system about an axis perpendicular to the square and passing through its centre is

$$I = 4m\left(\frac{L}{\sqrt{2}}\right)^2 = 2\,ML^2$$

If k is the radius of gyration

then $Mk^2 = 2ML^2$

$\therefore\ k = \sqrt{2}\,L$

144.(D)

The angular acceleration $\alpha = \dfrac{\tau}{I}$

α is inversely proportional to the moment of inertia I.

The moment of inertia of the ring is MR^2 which is greater than moment of inertia of the disc $\left(\dfrac{MR^2}{2}\right)$.

Hence angular acceleration and therefore angular velocity will be greater for the disc.

145.(B)

Torque is given by $\vec{\tau} = \vec{r}\times\vec{f}$

$\therefore\ \vec{\tau}$ is perpendicular to $\vec{r}$ and $\vec{F}$.

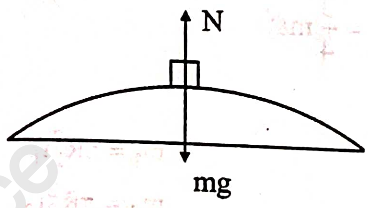
If two vectors are perpendicular, there dot product is zero.

$\therefore\ \vec{r}\cdot\vec{\tau} = 0$ and $\vec{F}\cdot\vec{\tau} = 0$

146.(C)

$$mg - N = \frac{mV^2}{R}$$

$$\therefore\ N = mg - \frac{mV^2}{R}$$



147.(C)

By law of conservation of angular momentum :

$I_1\omega_1 = (I_1 + I_2)\omega$

$$\therefore\ \omega = \frac{I_1\omega_1}{I_1 + I_2}$$

$$\therefore\ \sin r = \sin^2\theta = \frac{1}{n^2}$$

$$\therefore\ r = \sin^{-1}\left(\frac{1}{n^2}\right)$$

148.(A)

$$I_1 = \frac{ML^2}{12}$$

when the rod is bent into a ring,

$$L = 2\pi r \quad \text{or} \quad r = \frac{L}{2\pi}$$

Moment of inertia of a ring about a diameter is given by

$$I_2 = \frac{Mr^2}{2} = \frac{M}{2} \cdot \frac{L^2}{4\pi^2} = \frac{ML^2}{8\pi^2}$$

$$\therefore \frac{I_2}{I_1} = \frac{ML^2}{8\pi^2} \times \frac{12}{ML^2} = \frac{3}{2\pi^2}$$

149.(B)

Velocity at the highest point $v_1 = \sqrt{rg}$

Velocity at the lowest point $v_2 = \sqrt{5rg}$

$$\therefore \frac{v_1}{v_2} = \frac{1}{\sqrt{5}}$$

150.(C)

The acceleration of rolling disc is given by

$$a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$ where θ is the angle made by the inclined plane with the horizontal.

$$\therefore \theta = 30°, \text{ for a disc } \frac{k^2}{R^2} = \frac{1}{2}$$

$$\therefore a = \frac{g\sin 30°}{1+\frac{1}{2}} = \frac{g}{2\times\frac{3}{2}} = \frac{g}{3}$$

151.(D)

$M = \pi R^2 t \rho$

M and t are same for both the discs.

$\therefore R^2 \rho = \text{constant}$

$$\therefore R_2^2\rho_2 = R_1^2\rho_1 \quad \text{or} \quad \frac{R_2^2}{R_1^2} = \frac{\rho_1}{\rho_2} < 1$$

$$R_2^2 < R_1^2$$

Moment of inertia $I = \dfrac{MR^2}{2}$

For disc P radius is greater

$\therefore I_P > I_Q$

152.(A)

$$V_1 = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}, \quad V_2 = \sqrt{2gh}$$

$$\therefore \quad \frac{V_2}{V_1} = \sqrt{1+\frac{k^2}{R^2}}$$

For hallow sphere $\left(1+\frac{k^2}{R^2}\right) = 1+\frac{2}{3} = \frac{5}{3}$

$$\therefore \quad \frac{V_2}{V_1} = \sqrt{\frac{5}{3}}$$

153.(D)

If a is the linear or tangential acceleration then the velocity after time t will be

$v = at$

Radial or centripetal acceleration $= a_r = \frac{v^2}{r} = \frac{a^2t^2}{r}$

but $\quad a = r\alpha$

$$\therefore \quad a_r = \frac{r^2\alpha^2t^2}{r} = r\alpha^2t^2$$

when $a_r = \frac{a}{2}$, we have $\quad r\alpha^2t^2 = \frac{r\alpha}{2}$

$$\therefore \quad \alpha t^2 = \frac{1}{2}$$

$$\therefore \quad t = \frac{1}{\sqrt{2\alpha}}$$

154.(C)

$$I = I_o + Mh^2$$

$$= \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 \quad \text{[The distance from centre will also be L/4]}$$

$$= \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$

155.(A)

Mass of the wire $M = \rho L$

If r is the radius of the coil then

$$2\pi r = L \quad \text{or} \quad r = \frac{L}{2\pi}$$

The moment of inertia of a ring about a tangent in its plane is

$$I = \frac{3}{2}Mr^2 = \frac{3}{2}\rho L \times \frac{L^2}{4\pi^2} = \frac{3}{8}\frac{\rho L^3}{\pi^2}$$

156.(C)

The tension in the string initially is given by

$T_1 = mg$

Assuming elastic collision the velocities will be exchanged.

If the mass m is given velocity $\sqrt{2gL}$, then the body moves in a circular path and the tension is given by

$$T_2 = mg + \frac{mv^2}{L} = mg + \frac{m(2gL)}{L} = mg + 2mg$$

$$\therefore \quad T_2 - T_1 = 2mg$$

157.(A)

Moment of inertia $= I = \dfrac{MR^2}{2} = \dfrac{1\times(0.4)^2}{2} = 0.08\ \text{kg-m}^2$

Torque $\tau = I\alpha = 0.08 \times 10 = 0.8$ N–m

$\tau = rF$

$\therefore\quad F = \dfrac{\tau}{r} = \dfrac{0.8}{0.4} = 2$ N

158.(D)

$$K_{rot} = \frac{1}{2}mv^2\frac{k^2}{R^2} = \frac{1}{2}mv^2\left(\frac{2}{5}\right)$$

$$K_{tot} = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right) = \frac{1}{2}mv^2\left(\frac{7}{5}\right)$$

$$\therefore\quad \frac{K_{rot}}{K_{tot}} = \frac{2/5}{7/5} = \frac{2}{7}$$

159.(B)

By law of conservation of angular momentum

$I_1\omega_1 = I_2\omega_2 \quad \therefore\ I\omega = 2I\omega$

$\therefore\quad \omega' = \dfrac{\omega}{2}$

Initial kinetic energy $K_1 = \dfrac{1}{2}I\omega^2$

Final kinetic energy $K_2 = \dfrac{1}{2}(2I)\omega'^2$

$$= \frac{1}{2}\times 2I\times\frac{\omega^2}{4} = \frac{I\omega^2}{4}$$

$$K_1 - K_2 = \frac{1}{2}I\omega^2 - \frac{1}{4}I\omega^2 = \frac{I\omega^2}{4}$$

160.(D)

$L = mvr$

$\therefore\quad v = \dfrac{L}{mr}$

$$F = \frac{mv^2}{r} = \frac{m}{r}\cdot\frac{L^2}{m^2r^2} = \frac{L^2}{mr^3}$$

161.(A)

Kinetic energy of rolling body is given by

$$K.E. = \frac{1}{2}mv^2\left(1+\frac{k^2}{r^2}\right)$$

For a ring $\dfrac{k^2}{r^2} = 1$

$$\therefore\quad K.E._R = \frac{1}{2}mv^2(1+1) = mv^2$$

For a disc $\frac{k^2}{r^2} = \frac{1}{2}$

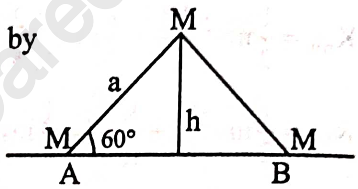
$\therefore$ $K.E._D = \frac{1}{2}mv^2\left(1+\frac{1}{2}\right) = \frac{3}{4}mv^2$

$K.E._R = mv^2 = 4\,J$

$K.E._D = \frac{3}{4}mv^2 = \frac{3}{4}\times 4 = 3\,J$

162.(D)

Moment of inertia of the system about axis AB is given by



$I = Mh^2$

$h = a\sin 60° = a\times\frac{\sqrt{3}}{2}$

$\therefore$ $I = \frac{3Ma^2}{4}$

163.(D)

Frequency $f = 90\text{ rpm} = \frac{90}{60} = 1.5\text{ rps}$

$\omega = 2\pi f = 2\pi\times 1.5 = 3\pi\text{ rad/s}$

Angular acceleration $\alpha = \frac{\omega_2-\omega_1}{t} = \frac{3\pi-0}{6}$

$= \frac{\pi}{2}\text{ rad/s}^2$

164.(A)

By parallel axis theorem

$I = \frac{2}{5}MR^2 + M\left(\frac{R}{2}\right)^2$

$= \frac{2}{5}MR^2 + \frac{MR^2}{4} = \frac{13}{20}MR^2$

165.(C)

Mass M and thickness t are equal

Densities are $\rho_1 = 6800\text{ kg/m}^3$ and $\rho_2 = 8500\text{ kg/m}^3$

$\therefore$ $M = \pi R_1^2 t\rho_1 = \pi R_2^2 t\rho_2$

$\therefore$ $R_1^2\rho_1 = R_2^2\rho_2$

$\therefore$ $\frac{R_1^2}{R_2^2} = \frac{\rho_2}{\rho_1} = \frac{8500}{6800} = \frac{5}{4}$

Moment of inertia of disc $I = \frac{MR^2}{2}$

$\therefore$ $\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{5}{4}$

166.(A)
Loss of potential energy of the mass 'm' is equal to the gain in kinetic energy of the wheel and the mass.

$$\therefore \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\therefore \frac{1}{2}I\omega^2 + \frac{1}{2}mr^2\omega^2 = mgh$$

$$\therefore \omega^2 = \frac{2mgh}{I + mr^2}$$

167.(C)

$$K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}mk^2(2\pi f)^2$$

$$= \frac{1}{2}mk^2 \times 4\pi^2 f^2 = \frac{1}{2} \times 2 \times (0.5)^2 \times 4\pi^2 \times (10)^2$$

$$= 100\,\pi^2\ J$$

168.(D)

$$\tan\theta = \frac{v^2}{rg} = \frac{(20)^2}{40 \times 10} = \frac{400}{400} = 1$$

$$\therefore \theta = 45°$$

169.(B)
The mass of the wire = M = mL

The radius of the circular loop = $r = \frac{L}{2\pi}$

The moment of inertia of this loop about the tangential axis in the plane of the coil is

$$I = \frac{3}{2}mr^2 = \frac{3}{2} \times mL \times \frac{h^2}{4\pi^2}$$

$$= \frac{3mL^2}{8\pi^2}$$

170.(D)
A body becomes weightless when the centrifugal force on it is equal to its weight.

$$\therefore mR\omega^2 = \frac{GMm}{R^2}$$

$$\therefore \omega^2 = \frac{GM}{R^3} = \frac{gR^2}{R^3} = \frac{g}{R} \qquad (GM = gR^2)$$

Kinetic energy of earth = $\frac{1}{2}I\omega^2$

$$= \frac{1}{2} \times \frac{2}{5}MR^2 \times \frac{g}{R} = \frac{MgR}{5}$$

171.(A)
By law of conservation of momentum

$I_1\omega_1 = I_2 w_2$ $\quad I_1 = Mr^2 \quad I_2 = Mr^2 + 2mr^2 \quad \omega_1 = \omega$

$\therefore Mr^2\omega = (Mr^2 + 2mr^2)\,\omega_2$

$\therefore M\omega = (M + 2m)\,\omega_2$

$$\omega_2 = \frac{M\omega}{M + 2m}$$

172.(C)

$$I = \frac{2}{5}MR^2$$

$$27\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$27r^3 = R^3$$

$$3r = R \quad \text{or} \quad r = \frac{R}{3}$$

$$I' = \frac{2}{5}mr^2$$

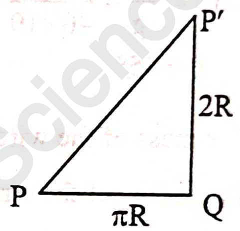
$$\frac{I'}{I} = \frac{m}{M}\frac{r^2}{R^2} = \frac{r^3}{R^3}\cdot\frac{r^2}{R^2} \qquad [M \propto V \propto R^3]$$

$$= \left(\frac{r}{R}\right)^5 = \left(\frac{1}{3}\right)^5 = \frac{1}{243}$$

$$I' = \frac{I}{243}$$

173.(D)

After half the rotation, the horizontal distance travelled by point P will be half the circumference of the wheel i.e. πR and the vertical distance will be 2R.



$$\therefore \text{ Displacement } PP' = \sqrt{\pi^2R^2 + 4R^2} = R\sqrt{\pi^2+4}$$

$$= 2\sqrt{\pi^2+4} \text{ cm}$$

174.(A)

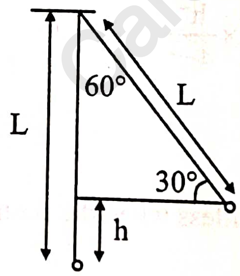
$$h = L - L\cos\theta = L(1-\cos\theta)$$

$$= 0.4\,(1-\cos 60°)$$

$$= 0.4\left(1-\frac{1}{2}\right)$$

$$= 0.4 \times \frac{1}{2}$$

$$= 0.2$$



$$\frac{1}{2}mv_1^2 = \frac{1}{2}mV_2^2 + mgh$$

$$V_1^2 = V_2^2 + 2gh$$

$$\therefore \quad V_2^2 = V_1^2 - 2gh = (4)^2 - 2\times 10\times 0.2$$

$$= 16 - 4 = 12$$

$$\therefore \quad V_2 = \sqrt{12} = 2\sqrt{3} \text{ m/s}$$

175.(C)

The tension in the wire is maximum when the mass is at the lowest position.

$$T = \frac{m\upsilon^2}{r} + mg\cos\theta$$

At the lowest position $\theta = 0°$ and $\cos\theta = 1$

176.(A)

If α is the angular acceleration, then the angular velocity after time t is given by $\omega = \alpha t$

but $\alpha = \frac{\tau}{I}$ $\therefore$ $\omega = \frac{\tau}{I} \cdot t$

kinetic energy $K = \frac{1}{2} I\omega^2 = \frac{1}{2} I \cdot \frac{\tau^2}{I^2} \cdot t^2 = \frac{\tau^2 t^2}{2I}$

177.(D)

Gain in kinetic energy = loss in potential energy

$$\frac{1}{2} m\left(1+\frac{k^2}{R^2}\right)V^2 = mgh$$

$$\frac{1}{2}\left(1+\frac{1}{2}\right)V^2 = gh \qquad \left[\text{for solid cylinder } \frac{K^2}{R^2} = \frac{1}{2}\right]$$

$$\therefore \frac{3}{4}V^2 = gh$$

$$\therefore h = \frac{3V^2}{4g}$$

178.(B)

frequency $f = 120 \text{ rpm} = \frac{120}{60} = 2\text{rps}$

$\omega = 2\pi f = 2\pi \times 2 = 4\pi$ rad/s

Angular acceleration $= \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0-4\pi}{10} = \frac{-2\pi}{5}$ rad / s^2

Moment of inertia $I = \frac{MR^2}{2} = \frac{10\times(0.1)^2}{2}$

$= 0.05$ kg m^2

Torque $\tau = I\alpha = 0.05\times\frac{2}{5}\pi = 0.02\,\pi$ Nm.

$\tau = Fr$

$\therefore F = \frac{\tau}{r} = \frac{0.02\,\pi}{0.1} = 0.2\,\pi$ N

179.(D)

Kinetic energy is given by :

For sphere $k_s = \frac{1}{2} I_s\, \omega_s^2$

For disc $k_d = \frac{1}{2} I_d\, \omega_d^2$

$$\frac{k_d}{k_s} = \frac{I_d\omega_d^2}{I_s\omega_s^2} = \frac{\frac{1}{2}MR^2(2\omega)^2}{\frac{2}{5}MR^2\omega^2} = 5$$

180.(C)

For minimum velocity at the highest point we should have

$mr\omega^2 = mg$

$\therefore \omega^2 = \frac{g}{r}$ or $\omega = \sqrt{\frac{g}{r}}$

$$2\pi f = \sqrt{\frac{g}{r}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

181.(B)

Kinetic energy $k = \frac{1}{2}I\omega^2$

$$\therefore \frac{k_2}{k_1} = \frac{I_2 w_2^2}{I_1\omega_1^2}$$

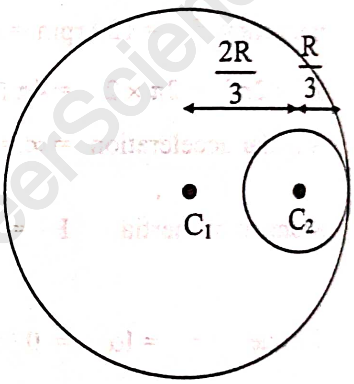
$$\therefore 2 = \frac{I_2}{I_1}\left(\frac{1}{2}\right)^2 \qquad \therefore \frac{I_2}{I_1} = 8$$

Angular momentum $L = I\omega$

$$\frac{L_2}{L_1} = \frac{I_2\omega_2}{I_1\omega_1} = 8\times\frac{1}{2} = 4$$

$$\therefore L_2 = 4L_1 = 4L$$

182.(B)



The moment of inertia of the complete disc $I_1 = \frac{9MR^2}{2}$

The moment of inertia of the removed disc =

$$I' = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{MR^2}{18} + \frac{4MR^2}{9}$$

$$= \frac{MR^2}{2}$$

[Since mass is proportional to area, it is proportional to square of the radius. Since the radius of the removed disc is $\frac{R}{3}$, its mass will be M]

$\therefore$ Moment of inertia of the remaining disc will be

$$I_2 = I_1 - I' = \frac{9mR^2}{2} - \frac{MR^2}{2} = 4MR^2$$

183.(D)

$$(kE)_1 = (kE)_2$$

$$\therefore \frac{1}{2}I\omega_1^2 = \frac{1}{2}(2I)\omega_2^2$$

$$\therefore \frac{\omega_1^2}{\omega_2^2} = 2$$

$$\therefore \frac{\omega_1}{\omega_2} = \sqrt{2}$$

Ratio of angular momenta$= \frac{I\omega_1}{2I\,\omega_2} = \frac{1}{2}\frac{\omega_1}{\omega_2} = \frac{1}{\sqrt{2}}$

184.(A)

Moment of inertia of a ring about a tangent in its plane is given by $I = \frac{3}{2}MR^2$

Moment of inertia of a disc about its diameter is given by $I' = \frac{1}{4}MR^2$

$\therefore \ \frac{I}{I'} = 6$

185.(D)

Initial angular momentum of the ring

$= I\omega = MR^2\omega$

If the new angular velocity is ω' then the final angular momentum $= I'\omega'$

where $I' = MR^2 + 2mR^2 = (M + 2m)R^2$.

By law of conservation of momentum

$I'\omega' = I\omega$

$(M + 2m)R^2\omega' = MR^2\omega$

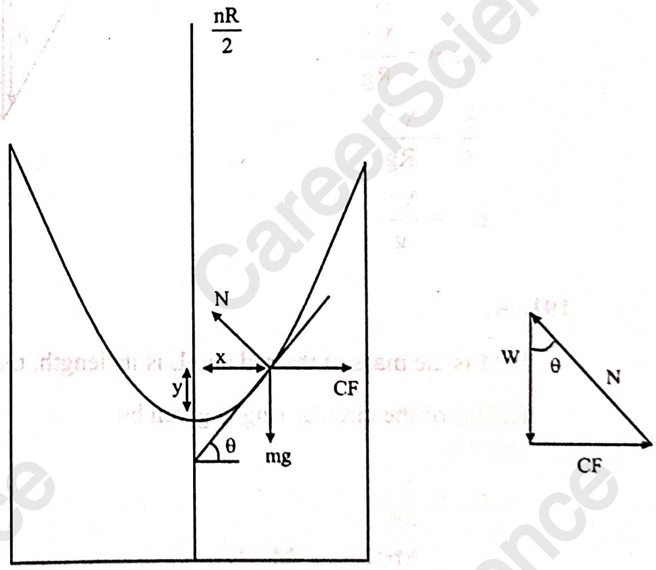
$\therefore \ \omega' = \frac{M\omega}{M + 2m}$

186.(D)

$\tan\theta = \frac{CF}{W}$

$\therefore \ \tan\theta = \frac{m\omega^2 x}{mg} = \frac{\omega^2 x}{g}$

where $\tan\theta$ is the slope of the surface of the paraboloid at any distance x from the axis of rotation.



slope $= \frac{dy}{dx} = \tan\theta = \frac{\omega^2 x}{g}$

$\therefore \ dy = \frac{\omega^2 x}{g}dx$

$\therefore \ \int dy = \frac{\omega^2}{g}\int x dx$

$\therefore \ y = \frac{\omega^2 x^2}{2g} \qquad = \frac{\omega^2 R^2}{2g}$

187.(B)

Moment of inertia of solid sphere is

$I_s = \frac{2}{5}MR^2$

Moment of inertia of the disc is given by about the given axis is

$I_d = \frac{3}{2}Mr^2 \qquad \therefore \ \frac{3}{2}Mr^2 = \frac{2}{5}MR^2$

$\therefore \ r^2 = \frac{4}{15}R^2 \qquad \therefore \ r = \frac{2R}{\sqrt{15}}$

188.(B)

The angular momentum is given by

$$\vec{L} = \vec{r}\times\vec{p} = m\vec{r}\times\vec{V}$$

If speed decreases, then the magnitude of the angular momentum decreases but direction does not change.

Hence option 1 is wrong but option 2 is correct.

The resultant acceleration is towards the centre only if the speed remains constant. Hence option 3 is wrong. It is given that the particle is moving in a circular path. Hence option 4 is wrong.

189.(D)

Moment of inertia of each half of the rod about the mid point is given by

$$I_1 = \frac{\frac{M}{2}\left(\frac{L}{2}\right)^2}{3} = \frac{ML^2}{24}$$

Total moment of inertia $= I = 2I_1 = \dfrac{ML^2}{12}$

190.(C)

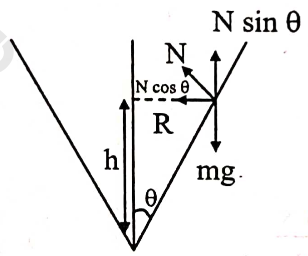
$N\sin\theta = mg$

$N\cos\theta = \dfrac{mV^2}{R}$

$\therefore\ \cos\theta = \dfrac{V^2}{Rg}$

$\therefore\ \dfrac{h}{R} = \dfrac{V^2}{Rg}$

$\therefore\ h = \dfrac{V^2}{g}$



191.(A)

If M is the mass of the rod and L is its length, then $I = \dfrac{ML^2}{3}$

Radius of the circular ring is given by

$2\pi r = L$

$\therefore\ r = \dfrac{L}{2\pi}$

$I_1 = \dfrac{Mr^2}{2} = \dfrac{M}{2}\cdot\dfrac{L^2}{4\pi^2}$

$\therefore\ \dfrac{I}{I_1} = \dfrac{8\pi^2}{3}$

192.(A)

$I_1 = MR^2,\ I_2 = (nM)(nR)^2 = n^3 mR^2$

$\therefore\ \dfrac{I_1}{I_2} = \dfrac{1}{n^3} = \dfrac{1}{8}$

$\therefore\ n = 2$

193.(A)

$$I_1 = \frac{ML^2}{12} = MK_1^2 \qquad \therefore\ K_1 = \frac{L}{\sqrt{12}} = \frac{L}{2\sqrt{3}}$$

$$I_2 = \frac{ML^2}{3} = MK_2^2$$

$$k_2 = \frac{L}{\sqrt{3}}$$

$$\frac{k_1}{k_2} = \frac{1}{2}$$

194.(A)

Tangential acceleration $a_t = 3\ m/s^2$

radial acceleration $a_r = \frac{V^2}{r} = \frac{(20)^2}{100} = 4\ m/s^2$

Total acceleration $= \sqrt{a_r^2 + a_C^2} = \sqrt{(4)^2 + (3)^2} = 5\ m/s^2$

195.(A)

Torque $\tau = Fr = 25 \times 0.4 = 10$ Nm

$$I = \frac{MR^2}{2} = \frac{1 \times (0.4)^2}{2} = 0.08\ \text{kg-m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{10}{0.08} = 125\ \text{rad/s}^2$$

196.(D)

After time t, velocity $V = at$

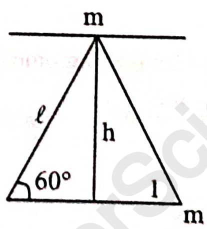
$\therefore$ radial acceleration $a_r = \frac{V^2}{r} = \frac{a^2t^2}{r}$

Total acceleration $= \sqrt{\frac{a^4t^4}{r^2} + a^2}$

$$\therefore\ \mu g = \sqrt{\frac{a^4t^4 + a^2r^2}{r^2}}$$

$$\therefore\ \mu = \frac{\left[a^4t^4 + a^2r^2\right]^{\frac{1}{2}}}{gr}$$

197.(D)



$$I = 2mh^2$$

$$h = \ell \sin 60° = \ell \times \frac{\sqrt{3}}{2}$$

$$I = 2m \times \frac{3}{4} \cdot \ell^2 = \frac{3}{2} m\ell^2$$

198.(D)

If m is the mass per unit length of the wire, then

mass of loop A $= M_A = 2\pi Rm$

mass of loop B $= M_B = 2\pi NRm$

$$\therefore\ \frac{M_B}{M_A} = N$$

$I_A = M_A R^2$, $I_B = M_B(NR)^2 = M_B N^2 R^2$

$\therefore \dfrac{I_B}{I_A} = \dfrac{M_B N^2}{M_A} = N^3$

$\therefore 3 = N^3$

$\therefore N = (3)^{\frac{1}{3}}$

199.(B)

Change in angular momentum $= \tau . t$

$= 200 \times 4 = 800$ kg m^2/s

200.(B)

$K = \dfrac{1}{2} I\omega^2$, $\quad L = I\omega$

$K' = \dfrac{1}{2} I'\omega'^2$, $\quad \omega' = 2\omega$

$\therefore K' = \dfrac{1}{2} I'(2\omega)^2$, $\quad K' = \dfrac{K}{2}$

$\therefore \dfrac{1}{2}\left(\dfrac{1}{2} I\omega^2\right) = \dfrac{1}{2} I'(2\omega)^2$

$\therefore I' = \dfrac{I}{8}$

$L' = I'\omega' = \dfrac{I}{8} \times 2\omega = \dfrac{I\omega}{4} = \dfrac{L}{4}$

201.(C)

Theory question

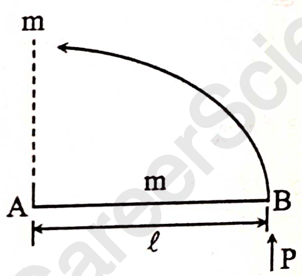
202.(C)

$I = \dfrac{ML^2}{12}$

$\therefore \dfrac{I'}{I} = \dfrac{\dfrac{M}{4} \cdot \left(\dfrac{L}{4}\right)^2}{ML^2} = \dfrac{1}{64}$

203.(B)

Impulse = Change in momentum



$P\dfrac{\ell}{2} = I\omega = \dfrac{m\ell^2}{12}\omega$

$\omega = \dfrac{6P}{m\ell}$

Now $\theta = \dfrac{\pi^c}{2}$ $\quad \theta = \omega t + \dfrac{1}{2}\alpha t^2$

$\dfrac{\pi}{2} = \dfrac{6P}{m\ell} t$

$\therefore t = \dfrac{\pi}{2} \times \dfrac{m\ell}{6P} = \dfrac{\pi m\ell}{12P}$