5. Oscillations

Important Formulae and Shortcut Methods

1. Periodic Motion : It is the motion which repeats itself in equal interval of time.

2. Linear S.H.M. : Linear S.H.M. is the linear periodic motion of a body, in which the force (or acceleration) is always directed towards the mean position and its magnitude is directly proportional to the displacement from the mean position.
All simple harmonic motions are periodic but all periodic motions are not simple harmonic.

3. Differential equation of S.H.M. is, $m\frac{d^2x}{dt^2}+kx=0$

4. Displacement of S.H.M. is $x = a \sin(\omega t+\alpha)$.
   (i) If particle starts from mean position then displacement $x = a \sin \omega t$
   (ii) If particle starts from extreme position then displacement $x = a \cos \omega t$

5. Velocity of particle performing S.H.M. is, $v=\pm\omega\sqrt{a^2-x^2}$
   (i) At mean position, velocity is maximum $V_{max} = \pm a\omega$
   (ii) At extreme position, velocity is minimum $V_{min} = 0$

6. Acceleration of particle performing S.H.M. is acceleration $= -\omega^2 x$ and its magnitude is $\omega^2 x$.
   (i) At mean position acceleration is minimum which equals to zero.
   (ii) At extreme position acceleration is maximum which equal to $a\omega^2$.

7. Amplitude of S.H.M. : The magnitude of maximum displacement of the particle performing S.H.M. from its mean position is known as amplitude of S.H.M.

8. Period of S.H.M. : The time taken by the particle to complete one oscillation is called period of S.H.M.

9. Oscillation : In S.H.M. the particle performs the same set of movement again and again. Such one set of movement is called oscillation.

10. Frequency of S.H.M. : The number of oscillations performed by particle performing S.H.M. per unit time is called frequency of S.H.M.

11. Phase of S.H.M. : The physical quantity which describes the state of oscillation of particle performing S.H.M. is called phase of S.H.M.

12. Epoch of S.H.M. : The physical quantity which describes the state of oscillation of particle performing S.H.M. at the start of motion is called epoch of S.H.M.

13. Kinetic energy of particle performing S.H.M. is $K.E.=\frac{1}{2}m\omega^2\left(a^2-x^2\right)=\frac{1}{2}k\left(a^2-x^2\right)$

14. Potential energy of particle performing S.H.M. is $P.E.=\frac{1}{2}m\omega^2x^2=\frac{1}{2}kx^2$

15. Total energy of particle performing S.H.M. is $=\frac{1}{2}m\omega^2a^2=\frac{1}{2}ka^2=2m\pi^2f^2a^2$
Energy of S.H.M. is directly proportional to square of frequency of oscillation and square of amplitude of oscillation.

16. Composition of S.H.M. : Resultant amplitude of the resultant S.H.M. is

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_1 - \alpha_2)}$$

Resultant phase of the resultant S.H.M. is

$$\delta = \tan^{-1}\left(\frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}\right)$$

17. Period of simple pendulum is

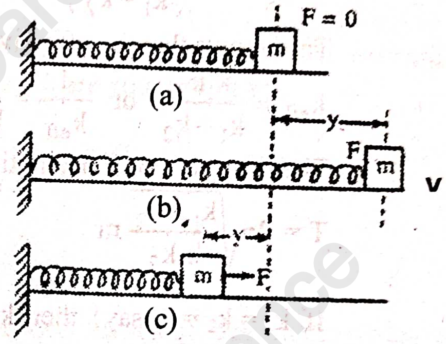
$$T = 2\pi\sqrt{\frac{L}{g}}$$

18. Seconds pendulum is a simple pendulum whose period is two seconds.

19. Periodic oscillations of gradually decreasing amplitude are called damped harmonic oscillations and the oscillator is called a damped oscillator.

20. **Mass–spring**

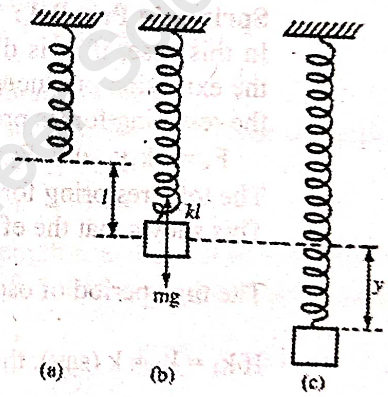
System – when a spring is compressed or stretched by a small amount, a restoring force is produced in it, which is proportional to the displacement y i.e., $F = -ky$. The constant 'k' is called the spring constant or the force constant of the spring.



21. **Horizontal Oscillations**

Consider a mass-less spring of constant 'k', one end of which is fixed rigidly to a wall and the other end is attached to a body of mass 'm' which is free to move on a frictionless horizontal surface. (see above figure (a)), shows the position of equilibrium. When the body is pulled to the right (figure above (b)), the restoring force exerted by the spring on the body is directed to the left. When the body is pushed to the left (fig.(c) above), the restoring force is directed to the right. When the body is released it executes S.H.M. with time period.

$$T = 2\pi\sqrt{\frac{m}{k}}$$



22. **Vertical Oscillations**

Consider a massless spring suspended vertically from a fixed support, having a mass 'm' connected to its lower end. In this case the equilibrium position of the spring is that position in which the spring is stretched by a length 'ℓ' (figure (b) above) such that the restoring force balances the weight mg, i.e.,

$$k\ell = mg \quad \text{or} \quad k = \frac{mg}{\ell} \quad \Rightarrow k \propto \frac{1}{\ell} \quad \ldots(1)$$

When the body is pulled further from this position through a distance 'y' (fig. (c) above), it executes S.H.M.

The time period is,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/\ell}} = 2\pi\sqrt{\frac{\ell}{g}}$$

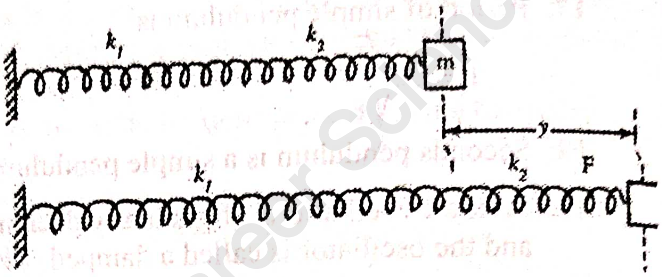
It should be noted that the time period in vertical oscillations is same as that in horizontal oscillations. It does not depend on g.

23. Coupled Spring Systems

Spring in Series :

Suppose two springs of force constants k_1 and k_2 are connected in series to a mass 'm'. Let 'm' be displaced to the right through a distance 'y'. If the extensions of the two springs are y_1 and y_2, respectively, then $y = y_1 + y_2$. If the restoring force is 'F', then

$$F = -k_1 y_1 = -k_2 y_2 \text{ or } y_1 = -\frac{F}{k_1},\ y_2 = -\frac{F}{k_2}$$



$$y = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$\text{or}\quad F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right) y$$

This shows that the effective force constant of the two springs is given by

$$k_{eff} = \frac{k_1 k_2}{k_1 + k_2} \quad \text{or} \quad \frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

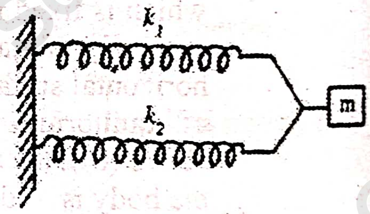
The time period of oscillation is

$$T = 2\pi\sqrt{\frac{k_1 + k_2}{k_1 k_2} m} \qquad \ldots(2)$$

If $k_1 = k_2 = k$ (say), then $k_{eff} = \frac{k}{2}$.

Springs in Parallel :

In this case, if m is displaced to the right through a distance 'y', the extension produced in each spring is the same. If F_1 and F_2 are the restoring forces produced in k_1 and k_2 respectively, then



$F_1 = -k_1 y$ and $F_2 = -k_2 y$

The total restoring force F is $F = F_1 + F_2 = -(k_1 + k_2)\, y$

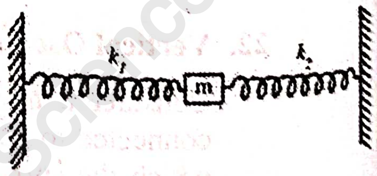
This shows that the effective force constant is $k_{eff} = k_1 + k_2$.

The time period of oscillation is $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$ ...(3)

If $k_1 = k_2 = k$ (say), then $k_{eff} = 2k$.

24. Mass connected between two springs :

If the body is displaced to one side, one of the springs gets extended and the other gets compressed. The restoring forces due to both say F_1 and F_2, are in the same direction. The total restoring force F is $F_1 + F_2$. Now, if y is the displacement of the body, then $F = -k_1 y$ and $F_2 = -k_2 y$



$F = -(k_1 + k_2)\, y$

This shows that the effective force constant is $k_{eff} = k_1 + k_2$.

25. Differential equation of angular S.H.M. ; $I\frac{d^2\theta}{dt^2} + c\theta = 0$, c is torsional constant

26. Period of angular S.H.M. $T = 2\pi\sqrt{\frac{I}{C}}$

27. Period of angular S.H.M. of magnet $= 2\pi\sqrt{\frac{I}{\mu B}}$

Multiple Choice Questions

MHT-CET 2004

1. The velocity of a particle performing simple harmonic motion, when it passes through its mean position is
(A) infinity (B) zero (C) minimum (D) maximum

2. The acceleration of particle executing SHM when it is at mean position is
(A) infinity (B) varies (C) maximum (D) zero

3. For a particle executing SHM having amplitude a, the speed of the particle is one-half of its maximum speed when its displacement from the mean position is
(A) 2a (B) $\sqrt{3}\frac{a}{2}$ (C) a (D) $\frac{a}{2}$

4. Two springs of spring constant k_1 and k_2 have equal maximum velocities, when executing simple harmonic motion. The ratio of their amplitudes (masses are equal) will be
(A) $\left(\frac{k_2}{k_1}\right)^{1/2}$ (B) $\left(\frac{k_1}{k_2}\right)^{1/2}$ (C) $\frac{k_1}{k_2}$ (D) k_1k_2

MHT-CET 2005

5. A simple pendulum of length ℓ and mass (bob) m is suspended vertically. The string makes an angle θ with the vertical. The restoring force acting on the pendulum is -
(A) mg tan θ (B) – mg sin θ (C) mg sin θ (D) –mg cos θ

6. The displacement of a particle performing simple harmonic motion is given by, $x = 8 \sin \omega t + 6 \cos \omega t$, where distance is in cm and time is in second. The amplitude of motion is
(A) 10 cm (B) 2 cm (C) 14 cm (D) 3.5 cm

7. A point mass m is suspended at the end of a massless wire of length L and cross-section area A. If Y is the Young's modulus for the wire, then the frequency of oscillations for the SHM along the vertical line is
(A) $\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (B) $2\pi\sqrt{\frac{mL}{YA}}$ (C) $\frac{1}{\pi}\sqrt{\frac{YA}{mL}}$ (D) $\pi\sqrt{\frac{mL}{YA}}$

8. The minimum phase difference between two simple harmonic oscillations,
$y_1 = \frac{1}{2}\sin \omega t + \frac{\sqrt{3}}{2}\cos \omega t$, $y_2 = \sin \omega t + \cos \omega t$ is
(A) $\frac{7\pi}{12}$ (B) $\frac{\pi}{12}$ (C) $-\frac{\pi}{6}$ (D) $\frac{\pi}{6}$

9. An SHM is represented by
$x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of the SHM is
(A) 10 cm (B) 20 cm (C) $5\sqrt{2}$ cm (D) 50 cm

10. Time period of a simple pendulum will be double, if we
(A) decrease the length 2 times (B) decrease the length 4 times
(C) increase the length 2 times (D) increase the length 4 times

MHT-CET 2006

11. Two particles execute SHM of the same amplitude and frequency along the same straight line. If they pass one another when going in opposite directions, each time their displacement is half their amplitude, the phase difference between them is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{2\pi}{3}$

MHT-CET 2007

12. The displacement equation of a simple harmonic oscillator is given by

$y = A \sin \omega t - B \cos \omega t$

The amplitude of the oscillator will be

(A) $A - B$ (B) $A + B$ (C) $\sqrt{A^2 + B^2}$ (D) $(A^2 + B^2)$

13. The potential energy of a simple harmonic oscillator, when the particle is half way to its end point is

(A) $\frac{1}{4}E$ (B) $\frac{1}{2}E$ (C) $\frac{2}{3}E$ (D) $\frac{1}{8}E$

(where, E is the total energy)

MHT-CET 2008

14. Two simple harmonic motions of angular frequency 100 rad/s and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum acceleration is

(A) 1 : 10 (B) 1 :10^2 (C) 1:10^3 (D) 1:10^4

15. The graph between the time period and the length of a simple pendulum is

(A) straight line (B) curve (C) ellipse (D) parabola

MHT-CET 2009

16. The periodic time of a particle doing simple harmonic motion is 4 s. The time taken by it to go from its mean position to half the maximum displacement (amplitude) is

(A) 2 s (B) 1 s (C) $\frac{2}{3}$s (D) $\frac{1}{3}$s

17. If a simple pendulum oscillates with an amplitude of 50 mm and time period of 2 s, then its maximum velocity is

(A) 0.10 ms^{-1} (B) 0.15 ms^{-1} (C) 0.8 ms^{-1} (D) 0.26 ms^{-1}

MHT-CET 2010

18. The average acceleration of a particle performing SHM over one complete oscillation is

(A) $\frac{\omega^2 A}{2}$ (B) $\frac{\omega^2 A}{\sqrt{2}}$ (C) zero (D) $A\omega^2$

19. U is the PE of an oscillating particle and F is the force acting on it at a given instant. Which of the following is correct?

(A) $\frac{U}{F} + x = 0$ (B) $\frac{2U}{F} + x = 0$ (C) $\frac{F}{U} + x = 0$ (D) $\frac{F}{2U} + x = 0$

MHT-CET 2011

20. A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to go directly from its mean position to half the amplitude
(A) T/2 (B) T/4 (C) T/8 (D) T/12

21. In SHM restoring force is F = -kx, where k is force constant, x is displacement and A is amplitude of motion, then total energy depends upon
(A) k, A and M (B) k, x, M (C) k, A (D) k, x

22. Total energy of a particle executing SHM is proportional to
(A) square of amplitude of motion (B) frequency of oscillation
(C) velocity in equilibrium position (D) displacement from equilibrium position

23. A particle executes simple harmonic motion of amplitude A. At what distance from the mean position is its kinetic energy equal to its potential energy?
(A) 0.81 A (B) 0.71 A (C) 0.41 A (D) 0.91 A

24. The maximum velocity and the maximum acceleration of a body moving in a simple harmonic oscillator are $2ms^{-1}$ and $4\ ms^{-2}$. Then, angular velocity will be
(A) 3 $rads^{-1}$ (B) 0.5 $rads^{-1}$ (C) 1 $rads^{-1}$ (D) 2 $rads^{-1}$

MHT-CET 2014

25. A block resting on the horizontal surface executes S.H.M. in horizontal plane with amplitude 'A'. The frequency of oscillation for which the block just starts to slip is (μ = coefficient of friction, g = gravitational acceleration)
(A) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$ (B) $\frac{1}{4\pi}\sqrt{\frac{\mu g}{A}}$ (C) $2\pi\sqrt{\frac{A}{\mu g}}$ (D) $4\pi\sqrt{\frac{A}{\mu g}}$

MHT-CET 2015

26. A particle performs S.H.M. with amplitude 25 cm and period 3 s. The minimum time required for it to move between two points 12.5 cm on either side of the mean position is
(A) 0.6 s (B) 0.5 s (C) 0.4 s (D) 0.2 s

27. A particle is executing S.H.M. of periodic time 'T'. The time taken by a particle in moving from mean position to half the maximum displacement is (sin30° = 0.5)
(A) $\frac{T}{2}$ (B) $\frac{T}{4}$ (C) $\frac{T}{8}$ (D) $\frac{T}{12}$

28. A mass is suspended from a spring having spring constant 'K' is displaced vertically and released, it oscillates with period 'T'. The weight of the mass suspended is (g = gravitational acceleration)
(A) $\frac{KTg}{4\pi^2}$ (B) $\frac{KT^2g}{4\pi^2}$ (C) $\frac{KTg}{2\pi^2}$ (D) $\frac{KT^2g}{2\pi^2}$

29. A simple pendulum is oscillating with amplitude 'A' and angular frequency 'ω'. At displacement 'x' from mean position, the ratio of kinetic energy to potential energy is
(A) $\frac{x^2}{A^2-x^2}$ (B) $\frac{x^2-A^2}{x^2}$ (C) $\frac{A^2-x^2}{x^2}$ (D) $\frac{A-x}{x}$

MHT-CET 2016

30. A mass 'm_1' connected to a horizontal spring performs S.H.M. with amplitude 'A'. While mass 'm_1' is passing through mean position another mass 'm_2' is placed on it so that both the masses move together with amplitude 'A_1'. The ratio of $\frac{A_1}{A}$ is ($m_2 < m_1$)

(A) $\left[\frac{m_1}{m_1+m_2}\right]^{\frac{1}{2}}$ (B) $\left[\frac{m_1+m_2}{m_1}\right]^{\frac{1}{2}}$ (C) $\frac{v_0}{2L}$ (D) $\left[\frac{m_1+m_2}{m_2}\right]^{\frac{1}{2}}$

31. The bob of a simple pendulum performs S.H.M. with period 'T' in air and with period 'T_1' in water. Relation between 'T' and 'T_1' is (neglect friction due to water, density of the material of the bob is $= \frac{9}{8}\times10^3$ kg/m^3, density of water = 1 g/cc)

(A) $T_1 = 3T$ (B) $T_1 = 2T$ (C) $T_1 = T$ (D) $T_1 = T/2$

*32. Which of the following quantity does NOT change due to damping of oscillations?

(A) Angular frequency (B) Time period (C) Initial phase (D) Amplitude

MHT-CET 2017

33. A particle performing S.H.M. starts from equilibrium position and its time period is 16 seconds. After 2 seconds its velocity is π m/s. Amplitude of oscillation is $\left(\cos 45° = \frac{1}{\sqrt{2}}\right)$

(A) $2\sqrt{2}$ m (B) $4\sqrt{2}$m (C) $6\sqrt{2}$ m (D) $8\sqrt{2}$ m

34. A particle is performing S.H.M. starting from extreme position. Graphical representation shows that, between displacement and acceleration, there is a phase difference of

(A) 0 rad (B) $\frac{\pi}{4}$ rad (C) $\frac{\pi}{2}$ rad (D) π rad

35. A simple pendulum of length 'L' has mass 'M' and it oscillates freely with amplitude 'A'. At extreme position, its potential energy is (g = acceleration due to gravity)

(A) $\frac{MgA^2}{2L}$ (B) $\frac{MgA}{2L}$ (C) $\frac{MgA^2}{L}$ (D) $\frac{2MgA^2}{L}$

36. A particle performs linear S.H.M. At a particular instant, velocity of the particle is 'u' and acceleration is 'α' while at another instant velocity is 'v' and acceleration is 'β' ($0 < \alpha < \beta$). The distance between the two positions is

(A) $\frac{u^2-v^2}{\alpha+\beta}$ (B) $\frac{u^2+v^2}{\alpha+\beta}$ (C) $\frac{u^2-v^2}{\alpha-\beta}$ (D) $\frac{u^2+v^2}{\alpha-\beta}$

MHT-CET 2018

37. The path length of oscillation of simple pendulum of length 1 metre is 16 cm. Its maximum velocity is ($g = \pi^2$ m/s^2)

(A) 2π cm/s (B) 4π cm/s (C) 8π cm/s (D) 16π cm/s

38. For a particle performing linear S.H.M., its average speed over one oscillation is (a = amplitude of S.H.M., n = frequency of oscillation)

(A) 2 an (B) 4 an (C) 6 an (D) 8 an

39. A mass is suspended from a vertical spring which is executing S.H.M. of frequency 5Hz. The spring is upstretched at the highest point of oscillation. Maximum speed of the mass is acceleration due to gravity $g = 10\ m/s^2$

(A) 2π m/s (B) π m/s (C) $\frac{1}{2\pi}$ m/s (D) $\frac{1}{\pi}$ m/s

MHT-CET 2019

40. A clock pendulum having coefficient of linear expansion $\alpha = 9 \times 10^{-7}$ /°C has a period of 0.5 s at 20°C. If the clock is used in a climate where the temperature is 30°C, how much time does the clock lose in each oscillation? (g = constant)

(A) 2.25×10^{-6} s (B) 5×10^{-7} s (C) 2.5×10^{-7} s (D) 1.125×10^{-6} s

41. The quantity which does not vary periodically for a particle performing S.H.M. is

(A) velocity (B) total energy (C) acceleration (D) displacement

42. A particle executes the simple harmonic motion with an amplitude 'A'. The distance travelled by it in one periodic time is

(A) 2A (B) 4A (C) A (D) $\frac{A}{2}$

43. Two pendulums begin to swing simultaneously. The first pendulum makes nine full oscillations when the other makes seven. The ratio of the lengths of the two pendulums is.

(A) $\frac{64}{81}$ (B) $\frac{7}{9}$ (C) $\frac{49}{81}$ (D) $\frac{8}{9}$

44. A particle is performing a linear simple harmonic motion of amplitude 'A'. When it is midway between its mean and extreme position, the magnitudes of its velocity and acceleration are equal. What is the periodic time of the motion ?

(A) $2\pi\sqrt{3}$ s (B) $\frac{\sqrt{3}}{2\pi}$ s (C) $\frac{2\pi}{\sqrt{3}}$ s (D) $\frac{1}{2\pi\sqrt{3}}$ s

45. If 'x', 'V' and 'a' denote the displacement, velocity and acceleration of a particle respectively executing S.H.M. of periodic time 'T', then which one of the following does not change with time ?

(A) $at + 2\pi V$ (B) $\frac{aT}{V}$ (C) $\frac{aT}{x}$ (D) $aT + 4\pi^2V^2$

46. The total energy of a simple harmonic oscillator is proportional to

(A) square of the amplitude (B) frequency
(C) square root of displacement (D) amplitude

47. A person measures a time period of a simple pendulum inside a stationary lift and finds it to be 'T'. If the lift start accelerating upwards with an acceleration $\left(\frac{g}{3}\right)$, the time period of the pendulum will be

(A) $\frac{T}{3}$ (B) $\frac{T}{\sqrt{3}}$ (C) $\sqrt{3}T$ (D) $\sqrt{3}\frac{T}{2}$

48. In damped S.H.M., the SI unit of damping constant is

(A) $\frac{N}{s}$ (B) $\frac{kg}{s}$ (C) $\frac{kg}{m}$ (D) $\frac{N}{m}$

49. In phase difference between two S.H.M.s of same amplitude 'a' is '$\frac{\pi}{2}$' radian, then the resultant amplitude will be sin 0° = cos 90° = 0

(A) 0.866 a (B) 0.7071 a (C) 1.414 a (D) $\frac{a}{1.732}$

50. The ratio of kinetic energy to the potential energy of a particle executing S.H.M. at a distance equal to (1/3)rd of its amplitude is

(A) 9 : 1 (B) 2 : 1 (C) 8 :1 (D) 3 :1

51. If the maximum velocity of a particle performing S.H.M. is 'v', then the average velocity during its motion from one extreme position to the other will be

(A) $\frac{2}{\pi}$V (B) $\frac{\pi}{4}$V (C) $\frac{4}{\pi}$V (D) $\frac{\pi}{2}$V

52. A pendulum is hanging in a uniformly rising lift. If lift's motion becomes uniformly accelerated, how will the period of oscillation of pendulum change?

(A) The period will remain unaffected (B) The period will be shorter
(C) The period will be longer (D) The period will become zero suddenly

53. The time period of a simple pendulum inside a stationary lift is 'T'. When the lift moves upwards with an acceleration 'g/3', the periodic time of the simple pendulum will be

(A) $\frac{T}{2}$ (B) $\frac{\sqrt{3}}{2}$T (C) $\frac{T}{4}$ (D) $\frac{\sqrt{3}}{4}$T

MHT-CET 2020

54. The bob of a simple pendulum is released at time t = 0 from a position of small angular displacement. Its linear displacement is (ℓ = length of simple pendulum and g = acceleration due to gravity, A = amplitude of S.H.M.)

(A) A sin $\left(\sqrt{\frac{\ell}{g}}\right)t$ (B) A cos $\left(\sqrt{\frac{g}{\ell}}\right)t$ (C) A sin $\left(\sqrt{\frac{g}{\ell}}\right)t$ (D) A cos $\left(\sqrt{\frac{\ell}{g}}\right)t$

55. A particle starting from mean position oscillates simple harmonically with period 4 s. After what time will its kinetic energy be 75% of the total energy ?

$\left(\cos 30° = \frac{\sqrt{3}}{2}\right)$

(A) $\frac{1}{4}$ s (B) $\frac{1}{3}$ s (C) $\frac{1}{2}$ s (D) $\frac{1}{5}$ s

56. If the frequency of oscillation of a simple pendulum in simple harmonic motion is 'n', then frequency of oscillation of simple pendulum when length is 4 times is

(A) $\frac{n}{2}$ (B) 4 n (C) n (D) 2 n

57. The length of the seconds pendulum is 1 m on the earth. If the mass and diameter of the planet is half that of the earth then the length of the seconds pendulum on the planet will be

(A) 1.5 m (B) 2 m (C) 0.5 m (D) 1 m

58. In a damped oscillatory motion, if 'T' is its time period and 'A' is its amplitude, then damping

(A) increases both 'A' and 'T' (B) decreases 'A' and increases 'T'.
(C) increases 'A' and decreases 'T'. (D) decreases both 'A' and 'T'.

59. The total energy of a simple harmonic oscillator is directly proportional to
(A) Square of amplitude
(B) frequency
(C) amplitude
(D) velocity

60. A block of mass 16 kg moving with velocity 4 m/s on a frictionless surface compresses an ideal spring and comes to rest. If force constant of the spring is 100 N/m then how much will be the spring compressed?
(A) 0.4 m
(B) 1.6 m
(C) 1.2 m
(D) 0.8 m

61. A body performing simple harmonic motion has potential energy 'P_1' at displacement 'x_1'. Its potential energy is 'P_2' at displacement 'x_2'. The potential energy 'P' at displacement $(x_1 + x_2)$ is
(A) $P_1 + P_2$
(B) $P_1 + P_2 + 2\sqrt{P_1P_2}$
(C) $\sqrt{P_1P_2}$
(D) $\sqrt{P_1^2 + P_2^2}$

62. A spring produces extension 'x' by applying a force 'F' N. A body of mass 'm' suspended from spring oscillates vertically with a period 'T', The mass of the suspended body is (neglect mass of spring)
(A) $\frac{T^2F}{4\pi^2x}$
(B) $\frac{2T^2F}{\pi^2x}$
(C) $\frac{T^2F}{2\pi^2x}$
(D) $\frac{T^2F}{\pi^2x}$

63. The kinetic energy of a particle performing S.H.M. is $\frac{1}{n}$ times its potential energy. If the amplitude of S.H.M. is 'A', then the displacement of the particle will be
(A) nA
(B) $\sqrt{\frac{(n+1)A^2}{n}}$
(C) $\frac{A}{n}$
(D) $\sqrt{\frac{nA^2}{n+1}}$

64. A simple pendulum oscillates with an angular amplitude 'θ'. If the maximum tension in the string is twice the minimum tension then 'θ' is
(A) $\cos^{-1}(0.50)$
(B) $\cos^{-1}(0.75)$
(C) $\cos^{-1}(0.10)$
(D) $\cos^{-1}(0.25)$

65. Two springs of spring constants 'K' and '2K' are stretched by same force. If 'E_1' and 'E_2' are the potential energies stored in them respectively, then
(A) $E_1 = \frac{1}{4}E_2$
(B) $E_1 = 2E_2$
(C) $E_2 = 2E_1$
(D) $E_1 = E_2$

66. A body oscillates simple harmonically with a period of 2 second, starting from the origin. Its kinetic energy will be 75% of the total energy after time
($\sin 30° = \cos 60° = \frac{1}{2}$)
(A) $\frac{1}{6}$s
(B) $\frac{1}{3}$s
(C) $\frac{1}{12}$s
(D) $\frac{1}{4}$s

67. Two bodies 'A' and 'B of equal mass are suspended from two separate massless springs of force constant 'k_1' and 'k_2' respectively. The bodies oscillate vertically such that their maximum velocities are equal. The ratio of the amplitudes of body A to that of body B is
(A) $\sqrt{\frac{k_2}{k_1}}$
(B) $\frac{k_2}{k_1}$
(C) $\sqrt{\frac{k_1}{k_2}}$
(D) $\frac{k_1}{k_2}$

68. A bob of a simple pendulum has mass 'm' and is oscillating with an amplitude 'a'. If the length of the pendulum is 'L' then the maximum tension in the string is
$\cos 0° = 1$, g = acceleratin due to gravity

(A) $mg\left[1-\left(\frac{L}{a}\right)^2\right]$ (B) $mg\left[1+\left(\frac{a}{L}\right)^2\right]$ (C) $mg\left[1-\left(\frac{a}{L}\right)^2\right]$ (D) $mg\left[1+\left(\frac{L}{a}\right)^2\right]$

69. A body of mass 64 g is made to oscillate turn by turn on two different springs A and B. Spring A and B has force constant $4\frac{N}{m}$ and $16\frac{N}{m}$ respectively. If T_1 and T_2 are period of oscillations of springs A and B respectively then $\frac{T_1+T_2}{T_1-T_2}$ will be

(A) 3 : 1 (B) 1 : 3 (C) 2 : 1 (D) 1 : 2

70. A block of mass 'M' is suspended from one end of a spring of force constant 'k'. The other end is rigidly attached to a horizontal platform. The mass oscillates vertically with a time period 'T'. If the mass gets detached from the spring, then the length of the spring will be shortened by
(g = acceleration due to gravity)

(A) $\frac{gT^2}{2\pi^2}$ (B) $\frac{gT^2}{2\pi}$ (C) $\frac{gT^2}{4\pi}$ (D) $\frac{gT^2}{4\pi^2}$

71. The maximum kinetic energy of a simple pendulum is 'K'. Its displacement in terms of amplitude 'a', when kinetic energy becomes $\left(\frac{K}{2}\right)$ is

(A) $\frac{a}{\sqrt{2}}$ (B) $\frac{a}{\sqrt{3}}$ (C) $\frac{a}{3}$ (D) $\frac{a}{2}$

72. A particle starting from mean position performs linear S.H.M. Its amplitude is 'A' and total energy is 'E'. At what displacement its kinetic energy is 3E/4?

(A) $\frac{A}{4}$ (B) $\frac{A}{3}$ (C) $\frac{A}{2}$ (D) A

73. A spring has length 'L' and force constant 'k'. It is cut into two springs of length 'L_1' and 'L_2' such that $L_1 = NL_2$ (N is an integer). The force constant of spring of length 'L_1' is

(A) $\frac{K(N-1)}{N}$ (B) $\frac{K(N+1)}{2N}$ (C) $\frac{K(N+1)}{N}$ (D) $\frac{K(N-1)}{2N}$

74. A simple pendulum of length 'L' has mass 'M' and it oscillates freely with amplitude 'A'. At extreme position its potential energy is (g = acceleration due to gravity)

(A) $\frac{MgA^2}{L}$ (B) $\frac{MgA}{2L}$ (C) $\frac{2MgA}{L}$ (D) $\frac{MgA^2}{2L}$

75. If 'α' and 'β' are the maximum velocity and maximum acceleration respectively, of a particle performing linear simple harmonic motion, then the path length of the particle is

(A) $\frac{2\alpha^2}{\beta}$ (B) $\frac{2\beta^2}{\alpha}$ (C) $\frac{2\alpha}{\beta}$ (D) $\frac{\alpha}{\beta}$

76. A vertical spring oscillates with period of 6 s with mass 'm' suspended from it. When the mass is at rest, the spring is stretched through a distance of (Take $g = \pi^2$)

(A) 5 cm (B) 7 cm (C) 9 cm (D) 3 cm

77. The displacement of a particle is 'y'= $2\sin\left[\frac{\pi t}{2}+\phi\right]$, where 'y' is cm and 't' in second. What is the maximum acceleration of the particle executing simple harmonic motion ?
(ϕ = phase difference)

(A) $\frac{\pi^2}{4}$ cm/s^2 (B) $\frac{\pi}{2}$ cm/s^2 (C) $\frac{\pi^2}{2}$ cm/s^2 (D) $\frac{\pi}{4}$ cm/s^2

78. A simple pendulum oscillates about its mean position with amplitude 'a' and periodic time 'T'. The linear speed of pendulum when its displacement is half the amplitude is

(A) $\frac{\pi a}{T}$ (B) $\frac{\pi a\sqrt{3}}{T}$ (C) $\frac{3\pi^2 a}{T}$ (D) $\frac{\pi a\sqrt{3}}{2T}$

79. A rectangular block of mass 'm' and cross-sectional area 'A' floats on a liquid of density ρ. If it is given a small vertical displacement from equilibrium, it starts oscillating with frequency 'n' equal to (g = acceleration due to gravity)

(A) $2\pi\sqrt{\frac{m}{A\rho g}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$ (C) $2\pi\sqrt{\frac{A\rho g}{m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{m}{A\rho g}}$

80. A body of mass m performs linear S.H.M. given by equation,
$x = P\sin\omega t + Q\sin\left(\omega t+\frac{\pi}{2}\right)$. The total energy of the particle at any instant is

(A) $\frac{1}{2}m\omega^2(P^2+Q^2)$ (B) $\frac{1}{2}m\omega^2/PQ$

(C) $\frac{1}{2}m\omega^2 PQ$ (D) $\frac{1}{2}m\omega^2 P^2Q^2$

81. Time period of a simple pendulum will be doubled if we

(A) increase the length two times (B) decrease the length four times

(C) decrease the length two times (D) increase the length four times

82. A particle of mass 'm' is executing simple harmonic motion about its mean position. If 'A' is the amplitude and 'T' is the period of S.H.M., then the total energy of the particle is

(A) $\frac{8\pi^2 mA^2}{T^2}$ (B) $\frac{\pi^2 mA^2}{T^2}$ (C) $\frac{2\pi^2 mA^2}{T^2}$ (D) $\frac{4\pi^2 mA^2}{T^2}$

83. The length of seconds pendulum is 1m on the earth. If the mass and diameter of the planet is double than that of the earth, then the length of the seconds pendulum on the planet will be

(A) 0.3 m (B) 0.4 m (C) 0.5 m (D) 0.2 m

84. When a mass 'm' is suspended from a spring of length 'ℓ', the length of the spring becomes 'L'. The mass is pulled down by a distance 'd' and released. If the equation of motion of the mass is $\frac{d^2x}{dt^2}+P^2x=0$, then P is equal to (g = acceleration due to gravity)

(A) $\sqrt{\frac{L-\ell}{g}}$ (B) $\frac{g}{L-\ell}$ (C) $\sqrt{\frac{g}{L-\ell}}$ (D) $\frac{L-\ell}{g}$

85. A particle performs S.H.M. Its potential energies are 'U_1' and 'U_2' at displacements 'x_1' and 'x_2' respectively. At displacement $(x_1 + x_2)$, its potential energy 'U' is

(A) $\sqrt{U} = \left(\sqrt{U_1} + \sqrt{U_2}\right)^2$
(B) $\sqrt{U} = \left(\sqrt{U_1} - \sqrt{U_2}\right)^2$
(C) $\sqrt{U} = \sqrt{U_1} + \sqrt{U_2}$
(D) $\sqrt{U} = \sqrt{U_1} - \sqrt{U_2}$

86. A horizontal spring executes S.H.M. with amplitude 'A_1', when mass 'm_1' is attached to it. When it passes through mean position another mass 'm_2' is placed on it. Both masses move together with amplitude 'A_2'. Therefore $A_2 : A_1$ is

(A) $\left[\frac{m_1+m_2}{m_2}\right]^{\frac{1}{2}}$
(B) $\left[\frac{m_1+m_2}{m_1}\right]^{\frac{1}{2}}$
(C) $\left[\frac{m_2}{m_1+m_2}\right]^{\frac{1}{2}}$
(D) $\left[\frac{m_1}{m_1+m_2}\right]^{\frac{1}{2}}$

87. The linear displacement 'x' of the bob of simple pendulum from its mean position varies as $x = a\sin\left(\frac{\pi}{\sqrt{2}}t\right)$ where 'a' is its amplitude expressed in metre and 't' is in second. The length of simple pendulum is (Take 'g' = π^2 m/s^2)

(A) 2.5 m
(B) 1.5 m
(C) 2.0 m
(D) 3.0 m

***88.** The damping force of an oscillator is directly proportional to the velocity. The unit of constant of proportionality is

(A) kg s
(B) kg m s^{-1}
(C) kg m s^{-2}
(D) kg s^{-1}

89. A particle performs simple harmonic motion with period of 3 second. The time taken by it to cover a distance equal to half the amplitude from mean position is sin 30° = 0.5

(A) $\frac{3}{4}$ s
(B) $\frac{1}{2}$ s
(C) $\frac{3}{2}$ s
(D) $\frac{1}{4}$ s

90. A simple pendulum of length 'ℓ' has a bob of mass 'm'. It executes S.H.M. of small amplitude 'A'. The maximum tension in the string is (g = acceleration due to gravity)

(A) $mg\left(\frac{A}{\ell}+1\right)$
(B) 2 mg
(C) mg
(D) $mg\left(\frac{A^2}{\ell^2}+1\right)$

91. A block of mass 'm' attached to one end of the vertical spring produces extension 'x'. It the block is pulled and released, the periodic time of oscillation is

(A) $2\pi\sqrt{\frac{x}{g}}$
(B) $2\pi\sqrt{\frac{2x}{g}}$
(C) $2\pi\sqrt{\frac{x}{2g}}$
(D) $2\pi\sqrt{\frac{x}{4g}}$

92. For a particle performing S.H.M., when displacement is 'x', the potential energy and restoring force acting on it is denoted by 'E' and 'F' respectively. The relation between x, E and F is

(A) $\frac{E}{F} - x = 0$
(B) $\frac{2E}{F} + x = 0$
(C) $\frac{2E}{F} - x = 0$
(D) $\frac{E}{F} + x = 0$

93. A simple pendulum of length 'L' has mass 'm' and it oscillates freely with amplitude 'A'. At extreme position, its potential energy is (g = acceleration due to gravity)

(A) $\frac{mgA}{L}$
(B) $\frac{mgA^2}{L}$
(C) $\frac{mgA^2}{2L}$
(D) $\frac{mgA}{2L}$

94. The displacement of the particle executing linear S.H.M. is x = 0.25 sin (11t + 0.5)m. The period of S.H.M. is $\left(\pi = \frac{22}{7}\right)$

(A) $\frac{2}{7}$s (B) $\frac{3}{7}$s (C) $\frac{1}{7}$s (D) $\frac{4}{7}$s

95. A particle starts from mean position and performs S.H.M. with period 6 second. At what time its kinetic energy is 50% of total energy? $\left(\cos 45° = \frac{1}{\sqrt{2}}\right)$

(A) 1 second (B) 0.50 second (C) 0.25 second (D) 0.75 second

96. A simple pendulum has length 2m and a bob of mass 100 gram. It is whirled in a horizontal plane. If the string breaks under a tension of 10 N, the angle made by the string with vertical is ($g = 10\ m/s^2$)

(A) $\cos^{-1}(0.05)$ (B) $\cos^{-1}(0.2)$ (C) $\cos^{-1}(0.4)$ (D) $\cos^{-1}(0.1)$

97. A particle executes simple harmonic motion with amplitude 'A' and period 'T'. If it is half way between mean position and extreme position, then its speed at that point is

(A) $\frac{\sqrt{3}\,\pi A}{2T}$ (B) $\frac{\pi A}{T}$ (C) $\frac{\sqrt{3}\pi A}{T}$ (D) $\frac{3\pi A}{T}$

98. A body performs linear S.H.M. with amplitude 'a'. When it is at a distance $\frac{'a'}{3}$ from extreme position, the magnitude of velocity is $\frac{1}{3}$ times the magnitude of acceleration. The period of S.H.M. is

(A) $\frac{4\pi}{3\sqrt{5}}$s (B) $\frac{\pi}{3\sqrt{5}}$s (C) $\frac{5\pi}{3\sqrt{5}}$s (D) $\frac{3\pi}{2\sqrt{5}}$s

99. A pendulum performs S.H.M. with period $\sqrt{3}$ second in a stationary lift. If lift moves up with acceleration $\frac{g}{3}$, the period of pendulum is g = acceleration due to gravity

(A) 2.5 second (B) 2.00 second (C) 1.5 second (D) 1.75 second

100. A coin is placed on the horizontal plate. Plate performs S.H.M. vertically with angular frequency 'ω'. The amplitude (A) of oscillations is gradually increased. The coin will lose contact with plate for the first time when amplitude is (g = acceleration due to gravity)

(A) zero (B) $\frac{A}{2}$ (C) $\frac{\omega^2}{g}$ (D) $\frac{g}{\omega^2}$

101. A man of mass 'M' is standing on the platform. The platform is executing S.H.M. of frequency 'f' in vertical direction. The span of oscillation is 'L'. Then the acceleration of the platform at the top of the oscillation is

(A) $\frac{4\pi^2 f^2 L}{M}$ (B) $\frac{2\pi^2 f^2 L}{M}$ (C) $2\pi^2 f^2 L$ (D) $4\pi^2 f^2 L$

102. If length of oscillating simple pendulum is made $\frac{1}{3}$ times at a place keeping amplitude same, then its total energy (E) will be

(A) 2E (B) 6E (C) 4E (D) 3E

103. A simple pendulum of mass 'm' having length 'L' is oscillating with amplitude 'A'. The maximum tension in the string is

(A) $\frac{mgA}{L}$ (B) $mg\left[1-\left(\frac{A}{L}\right)^2\right]$ (C) $mg\left[1+\left(\frac{A}{L}\right)^2\right]$ (D) $\frac{mgA^2}{L^2}$

104. The displacement of the particle performing S.H.M. is given by $x = 4 \sin \pi t$, where x is in cm and t is in second. The time taken by the particle in second to move from the equilibrium position to the position of half the maximum displacement, is

$$\left[\begin{array}{l}\sin 30° = \cos 60° = 0.5\\ \cos 30° = \sin 60° = \sqrt{3}/2\end{array}\right]$$

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$

105. A pendulum clock is running fast. To correct its time, we should

(A) reduce the mass of the bob.
(B) reduce the amplitude of oscillation
(C) reduce the length of pendulum
(D) increase the length of the pendulum.

106. The displacement of the particle at a distance 'x' from the origin is given by $Y = A \sin \omega\left(\frac{x}{v} - k\right)$, where 'ω' is the angular velocity and 'v' is the linear velocity. The dimensions of 'k' are

(A) $L^0M^0T^{-1}$ (B) $L^0M^0T^2$ (C) $L^0M^0T^1$ (D) $L^1M^0T^1$

107. A particle executes linear S.H.M. with amplitude 4 cm. The magnitude of velocity and acceleration is equal when it is at 3 cm from mean position. The time period is

(A) $\frac{3\pi}{7}$ (B) $\frac{3\pi}{\sqrt{7}}$ (C) $\frac{6\pi}{\sqrt{7}}$ (D) $\frac{6\pi}{7}$

108. In the differential equation of linear simple harmonic motion, $\frac{d^2x}{dt^2} + \omega^2 x = 0$, the term ω^2 represents

(A) restoring force per unit mass.
(B) restoring force per unit mass per unit displacement.
(C) restoring force per unit displacement.
(D) acceleration per unit mass per unit displacement.

109. A simple pendulum has length 'L_1'. When its amplitude is 'a' cm, its energy is 'E_1'. When length is $2L_1$, its energy is 'E_2' Amplitude is same for both the lengths The relation between E_1 and E_2 is

(A) $E_2 = 4E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = \frac{E_1}{2}$ (D) $E_2 = E_1$

110. The displacement of a particle in S.H.M. is $x = A\cos\left(\omega t + \frac{\pi}{6}\right)$. Its speed will be maximum at time

(A) $\frac{\pi}{6\omega}$ second (B) $\frac{\pi}{4\omega}$ (C) $\frac{\pi}{3\omega}$ (D) $\frac{\pi}{2\omega}$

111. A body is executing S.H.M. Its potential energy is E_1 and E_2 at displacements x and y respectively. The potential energy at displacement (x + y) is

(A) $E_1 - E_2 = E$ (B) $\sqrt{E_1} - \sqrt{E_2} = \sqrt{E}$ (C) $E_1 + E_2 = E$ (D) $\sqrt{E_1} + \sqrt{E_2} = \sqrt{E}$

112. A particle executing S.H.M. has velocities 'v_1' and 'v_2' at distance 'x_1' and 'x_2' respectively from mean position. The angular velocity (ω) of the particle is given by

(A) $\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$ (B) $\sqrt{\frac{v_2^2 - v_1^2}{x_2^2 - x_1^2}}$ (C) $\sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}$ (D) $\sqrt{\frac{v_1^2 - v_2^2}{x_1^2 - x_2^2}}$

113. Total energy of a particle performing S.H.M. is 'NOT' proportional to

(A) its mass
(B) square of amplitude of oscillation
(C) displacement from mean position.
(D) square of frequency of oscillation.

114. The period of oscillation of a mass 'M' suspended from a spring of negligible mass is 'T'. If along with it another mass M is also suspended, the period of oscillation now will be

(A) $\frac{T}{\sqrt{2}}$ (B) $\sqrt{2}T$ (C) T (D) 2T

115. A body performs S.H.M. due to force 'F_1', with time period 0.8 s. If force is changed to 'F_2', it executes S.H.M. with time period 0.6 s. Now both the forces act simultaneously in the same direction on the same body. New periodic time is

(A) 0.12 s (B) 0.24 s (C) 0.48 s (D) 0.36 s

116. A particle performs S.H.M. of period 24 s. Three second after passing through the mean position it acquires a velocity of 2 π m/s. Its path length is

$\sin 45° = \cos 45° = 1/\sqrt{2}$

(A) $24\sqrt{2}$ m (B) $48\sqrt{2}$ m (C) $36\sqrt{2}$ m (D) $12\sqrt{2}$ m

117. The maximum speed of a particle in S.H.M. is 'V'. The average speed is

(A) $\frac{2V}{\pi^2}$ (B) $\frac{2\pi}{V}$ (C) $2V^2/\pi$ (D) $2V/\pi$

118. A body attached to a spring oscillates in horizontal plane with frequency 'n'. Its total energy is 'E'. If the velocity in the mean position is V, then the spring constant is

(A) $\frac{E\pi^2 n^2}{v^2}$ (B) $\frac{2E\pi^2 n^2}{v^2}$ (C) $\frac{8E\pi^2 n^2}{v^2}$ (D) $\frac{4E\pi^2 n^2}{v^2}$

119. The frequency of a particle performing linear S.H.M. is $\frac{7}{2\pi}$ Hz. The differential equation of S.H.M. is

(A) $\frac{d^2x}{dt^2} + 64x = 0$ (B) $\frac{d^2x}{dt^2} + 49x = 0$ (C) $\frac{d^2x}{dt^2} + 14x = 0$ (D) $\frac{d^2x}{dt^2} + 25x = 0$

120. A cube of mass 'M' and side 'L' is fixed on the horizontal surface. Modulus of rigidity of the material of cube is 'η'. A force is applied perpendicular to one of the side faces. When the force is removed, cube executes small oscillations. The time period

(A) $2\pi\sqrt{\frac{M}{\eta L}}$ (B) $2\pi\sqrt{\frac{\eta}{ML}}$ (C) $\frac{M}{2\pi\eta L}$ (D) $\frac{2\pi\eta L}{M}$

121. A simple pendulum of length 'L' is suspended from a roof of a trolley. A trolley moves in horizontal direction with an acceleration 'a'. What would be the period of oscillation of a simple pendulum? g is acceleration due to gravity

(A) $2\pi\sqrt{\frac{L}{g+a}}$ (B) $2\pi\sqrt{L}(a^2+g^2)^{-\frac{1}{4}}$ (C) $2\pi\sqrt{\frac{L}{g-a}}$ (D) $2\pi\sqrt{L}(a^2+g^2)^{-\frac{1}{2}}$

122. The displacements of two particles executing simple harmonic motion are represented as $y_1 = 2\sin(10t+\theta)$ and $y_2 = 3\cos 10t$. The phase difference between the velocities of these waves is

(A) θ (B) $-\theta$ (C) $\left(\theta-\frac{\pi}{2}\right)$ (D) $\left(\theta+\frac{\pi}{2}\right)$

123. The period of seconds pendulum on a planet, whose mass and radius are three times that of earth, is

(A) $2\sqrt{2}$ second (B) $3\sqrt{2}$ second (C) $2\sqrt{3}$ second (D) $\sqrt{3}$ second

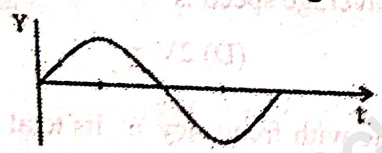
124. The weight suspended from a spring oscillates up and down. The acceleration of weight will be zero at

(A) highest position (B) mean position (C) lowest position (D) half of the amplitude.

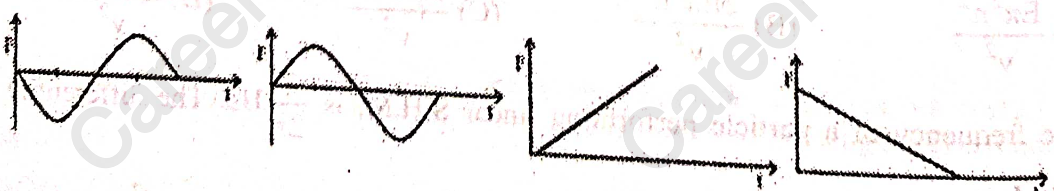
125. A mass 'M' is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of period T. If the mass is increased by 'm', the time period becomes $\frac{5T}{3}$. What is the ratio $\left(\frac{M}{m}\right)$?

(A) $\frac{25}{9}$ (B) $\frac{9}{16}$ (C) $\frac{16}{9}$ (D) $\frac{9}{25}$

126. For a particle performing S.H.M., the displacement-time graph is as shown.



For that particle the force-time graph is correctly shown in graph



(1) (2) (3) (4)

(A) (2) (B) (1) (C) (4) (D) (3)

127.A body perform linear simple harmonic motion of amplitude 'A'. At what displacement from the mean position, the potential energy of the body is one fourth of its total energy?

(A) $\frac{A}{2}$ (B) $\frac{A}{4}$ (C) $\frac{3A}{4}$ (D) $\frac{A}{3}$

128.A mass M attached to a horizontal spring executes S.H.M. of amplitude A_1. When the mass M passes through its mean position, then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

(A) $\frac{M}{M+m}$ (B) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$ (C) $\frac{M+m}{M}$ (D) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$

129.Two springs of spring constants 'K' and '2K' are stretched by same force. If 'w_1' and 'w_2' are the energies stored in them respectively then

(A) $w_1 = \frac{w_2}{4}$ (B) $w_1 = 2w_2$ (C) $w_1 = w_2$ (D) $w_2 = 2w_1$

130.A particle performs S.H.M. with amplitude 'A'. Its speed is tripled at the instant when it is at a distance of $\frac{2A}{3}$ from the mean position. The new amplitude of the motion is

(A) $\frac{2A}{3}$ (B) $\frac{A}{3}$ (C) $\frac{5A}{3}$ (D) $\frac{7A}{3}$

131.A particle performs S.H.M. from the mean position. Its amplitude is 'A' and total energy is 'E'. At a particular instant its kinetic energy is $\frac{3E}{4}$. The displacement of the particle at that instant is

(A) $\frac{A}{8}$ (B) $\frac{A}{2}$ (C) A (D) $\frac{A}{4}$

132.The length of the seconds pendulum is decreased by 0.3 cm when it is shifted from place A to place B. If the acceleration due to gravity at place A is 981 cm/s^2, the acceleration due to gravity at place B is (Take $\pi^2 = 10$)

(A) 975 cm/s^2 (B) 984 cm/s^2 (C) 978 cm/s^2 (D) 981 cm/s^2

133.A spring executes S.H.M. with mass 10 kg attached to it. The force constant of the spring is 10 N/m. If at any instant its velocity is 40 cm/s, the displacement at that instant is (Amplitude of S.H.M. = 0.5 m)

(A) 0.3 m (B) 0.4 m (C) 0.2 m (D) 0.45 m

134.The ratio of frequencies of oscillations of two simple pendulums is 3 : 4, then their lengths are in the ratio

(A) 16 : 9 (B) $\sqrt{4} : \sqrt{3}$ (C) 9 : 16 (D) $\sqrt{3} : \sqrt{4}$

135.A small mass 'm' is suspended at the end of a wire having (negligible mass) length 'L' and cross-sectional area 'A'. The frequency of oscillation for the S.H.M. along the vertical lines is (Y = Young's modulus of the wire)

(A) $2\pi\left(\frac{YA}{mL}\right)^{\frac{1}{2}}$ (B) $\frac{YA}{2\pi mL}$ (C) $\frac{2\pi YA}{mL}$ (D) $\frac{1}{2\pi}\left(\frac{YA}{mL}\right)^{\frac{1}{2}}$

SOLUTIONS

1. (D)

The relation for velocity of a particle executing SHM is given by, $v = \omega\sqrt{A^2 - y^2}$. Hence, at mean position $y = 0$

The velocity is maximum

$$V_{maximum} = A\omega \quad \text{(where, A is amplitude)}$$

2. (D)

Acceleration of particle executing SHM is given by acceleration $= -\omega^2 y$

At mean position, i.e. $y = 0$ acceleration is minimum $= 0$

3. (B)

Using the relation for velocity, $v = \omega\sqrt{(a^2 - y^2)}$

and $v_{max} = a\omega$ and $v' = \frac{v_{max}}{2} = \frac{a\omega}{2}$

So, $\omega\sqrt{a^2 - y^2} = \frac{\omega a}{2} \Rightarrow a^2 - y^2 = \frac{a^2}{4}$

So, $y^2 = \frac{3}{4}a^2 \Rightarrow y = \frac{a\sqrt{3}}{2}$

4. (A)

The angular frequency of spring is given by

$$\omega = \sqrt{\frac{k}{m}} \propto \sqrt{k}$$

For equal maximum velocities, we have

$$A_1\omega_1 = A_2\omega_2$$

or $\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{k_1}} = \left(\frac{k_2}{k_1}\right)^{\frac{1}{2}}$ $(\because m = m_1 = m_2)$

5. (B)

When the bob is displaced to position P, through a small angle θ from the vertical, the various forces acting on the bob at P are

(i) the weight mg of the bob acting vertically downwards

(ii) the tension T in the string acting along PS. If the string neither slackens nor breaks but remains taut, then

$$T = mg\cos\theta$$

The force $mg\sin\theta$ tends to bring the bob back to its mean position O. Therefore, restoring force acting on the bob is $F = -mg\sin\theta$

6. (A)

A harmonic oscillation of constant amplitude and of single frequency is called simple harmonic oscillation.

Here, $x = 8\sin\omega t + 6\cos\omega t$

$a_1 = 8$ cm and $a_2 = 6$ cm

$\therefore$ Amplitude of motion, $A = \sqrt{a_1^2 + a_2^2} - \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$ cm

7. (A)
Frequency depends on spring factor and inertia factor.

In this case, stress $=\frac{mg}{A}$ (where, ℓ is extension)

Now, Young's modulus Y is given by

$$Y=\frac{\text{stress}}{\text{strain}}=\frac{mg/A}{\ell/L} \quad \text{or} \quad mg=\frac{YA\ell}{L}$$

So, $k\ell=\frac{YA\ell}{L}$ ($\because$ mg = kl) (k is force constant)

$\therefore$ $k=\frac{YA}{L}$

Now, frequency is given by, $n=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\left(\frac{YA}{mL}\right)}$

8. (B)
Phase difference between any two particles in a wave determines lack of harmony in the vibrating state of two particles, i.e. how for one particle leads the other or lags behind the other.

Here, $y_1=\frac{1}{2}\sin\omega t+\frac{\sqrt{3}}{2}\cos\omega t$

$$=\cos\frac{\pi}{3}\sin\omega t+\sin\frac{\pi}{3}\cos\omega t$$

$\therefore$ $y_1=\sin\left(\omega t+\frac{\pi}{3}\right)$ and $y_2=\sqrt{2}\sin\left(\omega t+\frac{\pi}{4}\right)$

$\therefore$ Phase difference, $\Delta\phi=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$

9. (A)
Here, $x=5\sqrt{2}$ (sin $2\pi t$ + cos $2\pi t$)

$\Rightarrow x=5\sqrt{2}\sin 2\pi t+5\sqrt{2}\cos 2\pi t$ ...(i)

The standard equation of simple harmonic motion is given by

$x=A_1\sin\omega t+A_2\cos\omega t$ ...(ii)

Now, comparing Eqs. (i) and (ii), we obtain

$A_1=5\sqrt{2}$ and $A_2=5\sqrt{2}$

So, the resultant amplitude of the motion

$$A=\sqrt{A_1^2+A_2^2}=\sqrt{(5\sqrt{2})^2+(5\sqrt{2})^2}=10\,\text{cm}$$

10. (D)

The formula for time period is, $T=2\pi\sqrt{\frac{\ell}{g}}\propto\sqrt{\ell}$

If the length is increased by four times, then time period will be doubled.

11. (D)
Equation of simple harmonic wave, y = A sin ($\omega t+\phi$)

Here, $y=\frac{A}{2}$

$\therefore$ A sin $(\omega t+\phi)=\frac{A}{2}$ $\Rightarrow \delta=\omega t+\phi=\frac{\pi}{6}$ or $\frac{5\pi}{6}$

So, the phase difference of the two particles when they are crossing each other at $y = \frac{A}{2}$ in opposite directions are

$$\delta = \delta_1 - \delta_2 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

12. (C)

Displacement equation, $y = A \sin \omega t - B \cos \omega t$

Let $A = a \cos \theta$ and $B = a \sin \theta$

So, $A^2 + B^2 = a^2 \Rightarrow a = \sqrt{A^2 + B^2}$

Then, $y = a \cos \theta \sin \omega t - a \sin \theta \cos \omega t$

$= a \sin (\omega t - \theta)$

which is the equation of simple harmonic oscillator.

The amplitude of the oscillator $= a = \sqrt{A^2 + B^2}$

13. (A)

Potential energy of a simple harmonic oscillator,

$$U = \frac{1}{2} m\omega^2 y^2$$

Kinetic energy of a simple harmonic oscillator.

$$K = \frac{1}{2} m\omega^2 (A^2 - y^2)$$

Here, y = displacement from mean position

A = maximum displacement of amplitude from mean position

$$E = U + K = \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} m\omega^2 (A^2 - y^2) = \frac{1}{2} m\omega^2 A^2$$

When the particle is half way to its end point, i.e. at half of its amplitude, then,

$$y = \frac{A}{2}$$

Hence, potential energy,

$$U = \frac{1}{2} m\omega^2 \left(\frac{A}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2} m\omega^2 A^2\right) = \frac{E}{4}$$

14. (B)

Acceleration of simple harmonic motion is

$$a_{max} = -\omega^2 A$$

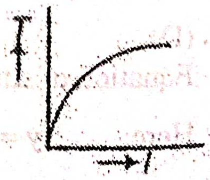
or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{\omega_1^2}{\omega_2^2}$ (as A remains same)

or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1 : 10^2$

15. (D)

The time period of a simple pendulum is,

$T = 2\pi\sqrt{\frac{\ell}{g}}$ Here, I is the length of the pendulum.



On squaring both sides $T^2 = \frac{4\pi^2 I}{g} \Rightarrow T^2 \propto I$

So, the graph between time period T and length I of the pendulum is a parabola..

16. (D)

$$y = A\sin\left(\frac{2\pi}{T}\right)t \Rightarrow \frac{A}{2} = A\sin\left(\frac{2\pi}{4}\right)t \text{ or } \frac{\pi t}{2} = \frac{\pi}{6} \Rightarrow t = \frac{1}{3}s$$

17. (B)

$$v_{max} = a\omega = a \times \frac{2\pi}{T} = (50 \times 10^{-3}) \times \frac{2\pi}{2} = 0.15\, ms^{-1}$$

18. (C)

The average acceleration of a particle performing SHM over one complete oscillation is zero.

19. (B)

The potential energy, $U = \frac{1}{2}kx^2$

$$2U = kx^2 = -Fx \qquad \because F = -kx$$

or $\frac{2U}{F} = -x$ or $\frac{2U}{F} + x = 0$

20. (D)

$$y = a\sin\omega t = \frac{a\sin 2\pi}{T}t$$

At half amplitude, $y = \frac{a}{2} \Rightarrow \frac{a}{2} = a\frac{\sin 2\pi t}{T}$ and $t = \frac{T}{12}$

21. (C)

In SHM, the total energy = potential energy + kinetic energy

or $E = U + K = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$

$$= \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$$

where, k = force constant = $m\omega^2$

Thus, total energy depends on k and A.

22. (A)

Total energy of a particle executing SHM is

$$E = \frac{1}{2}m\omega a^2 \quad \text{or} \quad E \propto a^2$$

It means total energy is directly proportional to square of amplitude.

23. (B)

Potential energy of a particle executing SHM at a displacement y from the mean position is

$$PE = \frac{1}{2}ky^2 \qquad \ldots(i)$$

Final kinetic energy of the particle is

$$KE = \frac{1}{2}k(A^2 - y^2) \qquad \ldots(ii)$$

As PE and KE are equal, so equating them, we get

$$\frac{1}{2}ky^2 = \frac{1}{2}k(A^2 - y^2)$$

or $A^2 - y^2 = y^2 \Rightarrow 2y^2 = A^2$

or $y = \frac{A}{\sqrt{2}} = 0.707A = 0.71A$

24. (D)

At extreme position, displacement of particle executing SHM is maximum.

Maximum velocity of particle executing SHM is given by

$\therefore \quad v_{max} = \omega A$

Maximum acceleration of particle executing SHM is given by

by, $\quad a = -\omega^2 A \quad$ or $\quad a_{max} = |A| = \omega^2 A$

As $\quad a_{max} = (A\omega)\omega = v_{max}\,\omega$

$\therefore \quad \omega = \dfrac{a_{max}}{v_{max}} = \dfrac{4}{2} = 2 \text{ rad s}^{-1}$

25. (A)

When restoring force will become equal to the frictional force, block will start to slip.

$\therefore$ Restoring force = Friction force

$\Rightarrow \quad kA = \mu\, mg \qquad \ldots\text{(i)}$

Frequency, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ and from Eq. (i) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\mu g}{A}}$

26. (B)

$x = A \sin \omega t$

$\therefore \quad 12.5 = 25 \sin \omega t$

$\therefore \quad \sin \omega t = 0.5 \qquad \therefore \quad \omega t = \dfrac{\pi}{6}$

$\therefore \quad \dfrac{2\pi}{T}\cdot t = \dfrac{\pi}{6} \qquad \therefore \quad t = \dfrac{T}{12} = \dfrac{3}{12} = 0.25 \text{ s}$

This is the time required between mean position and x = 12.5 cm.

The time required to move between two point at 12.5 cm on either side of the mean position will be double of this i.e. 0.5 s.

27. (D)

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28. (B)

$T = 2\pi\sqrt{\dfrac{m}{K}}$

$\therefore \quad T^2 = 4\pi^2 \cdot \dfrac{m}{K} \qquad \therefore \quad m = \dfrac{KT^2}{4\pi^2}$

Weight $= mg = \dfrac{KT^2 g}{4\pi^2}$

29. (C)

$$K.E. = \frac{1}{2}m\omega^2(A^2 - x^2);\ P.E. = \frac{1}{2}m\omega^2x^2$$

$$\therefore \frac{K.E.}{P.E.} = \frac{A^2 - x^2}{x^2}$$

30. (A)

If the velocity of mass m_1 while passing through the mean position is υ and υ_1 is the velocity of $(m_1 + m_2)$ just after m_2 is placed on m_1, then by law of conservation of momentum we have,

$m_1 \upsilon = (m_1 + m_2)\upsilon_1$

$$\therefore \frac{\upsilon_1}{\upsilon} = \frac{m_1}{m_1 + m_2} \quad \text{...(1)}$$

Total energy initially is

$$\frac{1}{2}kA^2 = \frac{1}{2}m_1\upsilon^2 \quad \text{...(2)}$$

Total energy after m_2 is placed is

$$\frac{1}{2}kA_1^2 = \frac{1}{2}(m_1 + m_2)\upsilon_1^2 \quad \text{...(3)}$$

By Eqs. (2) and (3):

$$\frac{A_1}{A} = \left(\frac{\upsilon_1}{\upsilon}\right)\left(\frac{m_1 + m_2}{m_1}\right)^{\frac{1}{2}}$$

Putting value of $\frac{\upsilon_1}{\upsilon}$ from Eq. (1),

$$\frac{A_1}{A} = \left(\frac{m_1}{m_1 + m_2}\right)^{\frac{1}{2}}$$

31. (A)

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

In air $g_{eff} = g$, where as in water

$$g_{eff} = g\left(\frac{\sigma - \rho}{\sigma}\right) = g\left(\frac{\frac{9}{8}\times 10^3 - 10^3}{\frac{9}{8}\times 10^3}\right) = g\left(\frac{\frac{9}{8} - 1}{\frac{9}{8}}\right) = g \times \left(\frac{1}{8}\times\frac{8}{9}\right) = \frac{g}{9}$$

$$\therefore T_1 = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{9\ell}{g}} \quad \Rightarrow T_1 = \sqrt{9}\,T = 3T$$

32. (C)

33. (D)

Displacement of the particle $= x = A \sin\omega t$

Velocity of the particle $= \nu = \frac{dx}{dt} = A\omega\cos\omega t$

$$\nu = \pi \text{ m/s},\ T = 16 \text{ s},\ \omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8} \text{ rad/s}$$

$\therefore \quad \pi = A \times \frac{\pi}{8} \times \cos\frac{\pi}{8} \times 2$

$\therefore \quad 1 = \frac{A}{8}\cos\frac{\pi}{4} = \frac{A}{8}\cdot\frac{1}{\sqrt{2}}$

$\therefore \quad A = 8\sqrt{2}\ \text{m}$

34. (D)

35. (A)

Potential energy $= \frac{1}{2}M\omega^2A^2 = \frac{1}{2}M\cdot\frac{g}{L}\cdot A^2 \quad \left(\because \omega = \sqrt{\frac{g}{\ell}}\right)$

36. (A)

Let distance be x_1 when velocity is u and acceleration α.
Let distance by x_2 when velocity is v and acceleration β.
If ω is the angular frequency then

$\alpha = \omega^2 x_1$

and $\beta = \omega^2 x_2$

$\therefore \quad \alpha + \beta = \omega^2 (x_1 + x_2)$ .... (1)

Also $\quad u^2 = \omega^2A^2 - \omega^2x_1^2$

and $\quad v^2 = \omega^2A^2 - \omega^2x_2^2$

$v^2 - u^2 = \omega^2\left(x_1^2 - x_2^2\right)$

$v^2 - u^2 = \omega^2 (x_1 - x_2)(x_1 + x_2)$ .... (2)

By Eq. (1) we get $v^2 - u^2 = (x_1 - x_2)(\alpha + \beta)$

$\therefore \quad x_1 - x_2 = \frac{v^2 - u^2}{\alpha + \beta}$

or $\quad x_2 - x_1 = \frac{u^2 - v^2}{\alpha + \beta}$

37. (C)

$T = 2\pi\sqrt{\frac{\ell}{g}}, \qquad \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{\pi^2}{1}} = \pi$

$A = \frac{16}{2} = 8\ \text{cm}$

Maximum velocity $V_{max} = A\omega = 8\pi$ cm/s

38. (B)

Total distance = 4a

Total time $= \frac{1}{n}$

Average speed $= \frac{4a}{\left(\frac{1}{n}\right)} = 4an$

39. (D)

$f = 5\text{ Hz}, T = \frac{1}{5}\text{s}$

$T = 2\pi\sqrt{\frac{m}{K}}$

$Kx = mg$ x is the amplitude

$\therefore \frac{m}{K} = \frac{x}{g}$

$\therefore T = 2\pi\sqrt{\frac{x}{g}}$ $\frac{1}{5} = 2\pi\sqrt{\frac{x}{g}}$

$\frac{1}{25} = 4\pi^2 \cdot \frac{x}{g}$

$\therefore x = \frac{g}{100\pi^2} = \frac{10}{100\pi^2} = \frac{1}{10\pi^2}$

$\therefore v_{max} = x\omega = x \times 2\pi f = \frac{1}{10\pi^2} \times 2\pi \times 5 = \frac{1}{\pi}$

40. (A)

$T = 2\pi\sqrt{\frac{\ell_0(1+\alpha\Delta\theta)}{g}} \approx 2\pi\sqrt{\frac{\ell_0}{g}}\left(1+\frac{\alpha}{2}\Delta\theta\right)$ & $T_0 = 2\pi\sqrt{\frac{\ell_0}{g}}$

$T = T_0\left(1+\frac{\alpha}{2}\Delta\theta\right)$

$\therefore T = 0.5\left(1+\frac{9\times10^{-7}}{2}\times10\right)$

$T - T_0 = 2.25 \times 10^{-6}$ sec

41. (B)

Total energy remains constant.

42. (B)

43. (C)

$T \propto \sqrt{\ell}$

$T_1 \propto \frac{1}{9}$ $T_2 \propto \frac{1}{7}$

$\frac{T_1}{T_2} = \frac{7}{9} = \sqrt{\frac{\ell_1}{\ell_2}}$ $\therefore \frac{\ell_1}{\ell_2} = \frac{49}{81}$

44. (C)

Velocity $= \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - \frac{A^2}{4}} = \omega\sqrt{\frac{3A^2}{4}}$

Acceleration $= -\omega^2 x = \left|\omega^2 \frac{A}{2}\right|$

$\omega^2\frac{A}{2} = \frac{\omega}{2}A\sqrt{3}$

$\frac{2\pi}{T} = \sqrt{3} \Rightarrow T = \frac{2\pi}{\sqrt{3}}$

45. (B)

$$aT = \frac{x}{T^2}.T = \frac{x}{T}$$

$$V = \frac{x}{T}.$$

$$\therefore \frac{aT}{V} = \text{No dimension; so will not change with time}$$

46. (A)

$$E = \frac{1}{2}m\omega^2 a^2$$

47. (D)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g_2 = g + \frac{g}{3} = \frac{4g}{3}$$

$$T_2 = 2\pi\sqrt{\frac{3\ell}{4g}} = \frac{\sqrt{3}}{2}T$$

48. (B)

$$F = -bv$$

$$\therefore \quad b = \frac{F}{v} = \frac{kg\,m/s^2}{m/s} = \frac{kg}{s}$$

49. (C)

$$R = \sqrt{a^2 + a^2 + 2a^2\cos\frac{\pi}{2}} = \sqrt{2}a = 0.7071\,a$$

50. (C)

$$KE = \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{9}\right) = \frac{1}{2}m\omega^2\frac{8a^2}{9}$$

$$PE = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2\frac{a^2}{9} \quad \therefore \quad \frac{KE}{PE} = 8$$

51. (A)

$$V_{max} = \omega A = \frac{2\pi}{T}A$$

$$v_{Av} = \frac{4A}{T} = \frac{4V_{max}}{2\pi} = \frac{2V}{\pi}$$

52. (B)

$$T = 2\pi\sqrt{\frac{\ell}{g'}} = 2\pi\sqrt{\frac{\ell}{g+a}} \quad \text{as a increases, T decreases.}$$

53. (B)

$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{4g_1/3}{g_1}} = \sqrt{\frac{4}{3}}$$

$$T_2 = \frac{T_1}{2}\sqrt{3}$$

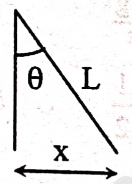
54. (B)

Equation of SHM

$x = A\cos\omega t$ where $\omega =$ $\left[\because\ T = 2\pi\sqrt{\frac{\ell}{g}}\right.$

$x = A\cos\sqrt{\frac{g}{\ell}}\,t$

$\left.\therefore\ \omega = \frac{2\pi}{2\pi\sqrt{\frac{\ell}{g}}} = \sqrt{\frac{g}{\ell}}\right]$



55. (B)

$T = 4$ s

$TE = \frac{1}{2}m\omega^2A^2 \qquad KE = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

$\frac{3}{4}\times\frac{1}{2}m\omega^2A^2 = \frac{1}{2}m\omega^2\left(A^2 - \omega^2\right)$

$\frac{3}{4}A^2 = A^2 - x^2$

$\therefore\ x^2 = A^2 - \frac{3}{4}A^2 = \frac{1}{4}A^2$

$x = \frac{A}{2}$

$x = A\sin\omega t$

$\frac{A}{2} = A\sin\frac{\pi}{2}t$

$\sin\frac{\pi}{6} = \sin\frac{\pi t}{2} \qquad \therefore\ \frac{\pi}{6} = \frac{\pi t}{2} \qquad \therefore\ t = \frac{1}{3}\,s$

56. (A)

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \qquad \therefore\ n \propto \frac{1}{\sqrt{\ell}}$

$\frac{n_1}{n_2} = \sqrt{\frac{\ell_2}{\ell_1}} = \sqrt{\frac{4\ell}{\ell}} = 2$

$\therefore\ n_2 = \frac{n}{2}$

57. (B)

$\ell = 1\,m$

$g = \frac{GM}{R^2}$

$g' = \frac{GM/2}{\frac{R^2}{4}} = \frac{GM}{2}\times\frac{4}{R^2} = 2g$

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\ell = \frac{T^2g}{4\pi^2}$

$$\ell_1 = \frac{T^2 2g}{4\pi^2}$$

$$\therefore \frac{\ell}{\ell_1} = \frac{1}{2} \qquad \therefore \ell_1 = 2\ell = 2m$$

58. (B)

59. (A)

$$E = \frac{1}{2}m\omega^2 A^2$$

60. (B)

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}\times 100\times x^2 = \frac{1}{2}\times 16\times 4^2$$

$$50x^2 = 128 \qquad \therefore \qquad x = 1.6\ m$$

61.(B)

$$P = \frac{1}{2}k(x_1 + x_2)^2$$

$$= \frac{1}{2}k\left(x_1^2 + x_2^2 + 2x_1x_2\right)$$

$$= \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + 2\left(\sqrt{\frac{k}{2}}x\right)\left(\sqrt{\frac{k}{2}}\,x_2\right)$$

$$= P_1 + P_2 + 2P_1 P_2$$

62.(A)

$$F = -kx$$

$$|k| = \frac{F}{x}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{mx}{F}}$$

$$\therefore \ m = \frac{T^2 F}{4\pi^2 x}$$

63. (D)

$$K.E. = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

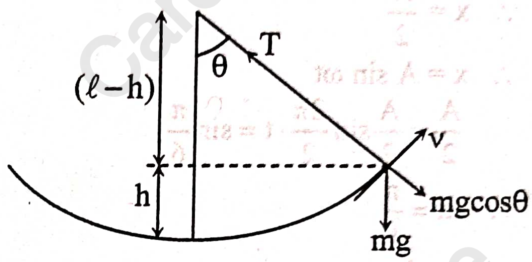
$$P.E.\ \frac{1}{2}m\omega^2 x^2$$

$$n\ KE = PE$$

$$n\left(A^2 - x^2\right) = x^2$$

$$nA^2 - nx^2 = x^2$$
$$x^2(1+n) = nA^2$$
$$x^2 = \frac{n}{1+n}A^2$$
$$x = \sqrt{\frac{n}{1+n}}A = \sqrt{\frac{nA^2}{n+1}}$$

64. (B)



$$T - mg\cos = \frac{mv^2}{\ell}$$
$$T = mg\cos\theta + \frac{mv^2}{\ell}$$
$$T_{max} = mg + \frac{mv^2}{\ell} \quad \text{(when } \cos\theta = 1)$$
$$T_{min} = mg\cos\theta \quad \text{(when } v = 0)$$
$$T_{max} = 2\,T_{min}$$
$$\therefore\ mg + \frac{mv^2}{\ell} = 2mg\cos\theta$$
$$g + \frac{v^2}{\ell} = 2g\cos\theta \quad \ldots(1)$$

Again $\frac{1}{2}mv^2 = mgh$

$$v = \sqrt{2gh}$$
$$h = \ell(1-\cos\theta)$$
$$\therefore\ v = \sqrt{2g\ell(1-\cos\theta)}$$
$$\therefore\ g + \frac{2g\ell(1-\cos\theta)}{\ell} = 2g\cos\theta \Rightarrow g + 2g - 2g\cos\theta = 2g\cos\theta$$
$$3 = 4\cos\theta$$
$$\cos\theta = \frac{3}{4}$$
$$\theta = \cos^{-1}(0.75)$$

65. (B)

$$F = kx_1 = (2k)x_2$$
$$\therefore\ kx_1 = 2kx_2$$
$$x_1 = 2x_2$$
$$E_1 = \frac{1}{2}kx_1^2$$
$$E_2 = \frac{1}{2}(2k)x_2^2 = \frac{1}{2}(2k)\frac{x_1^2}{4}$$
$$\therefore\ \frac{E_1}{E_2} = \frac{\frac{1}{2}kx_1^2}{\frac{1}{4}kx_1^2} = 2$$
$$E_1 = 2E_2$$

66. (A)

$$T = 2 \text{ sec}$$

$$\frac{3}{4}TE = PE$$

$$\frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right) = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

$$\frac{3}{4}A^2 = A^2 - x^2$$

$$\therefore \; x^2 = A^2 - \frac{3}{4}A^2 = \frac{A^2}{4}$$

$$\therefore \; x = \frac{A}{2}$$

$$\therefore \; x = A \sin \omega t$$

$$\frac{A}{2} = \frac{A}{2}\sin\frac{2\pi}{2}\cdot t = \sin\frac{\pi}{6}$$

$$\therefore \; \pi t = \frac{\pi}{6}$$

$$\therefore \; t = \frac{1}{6} \text{ sec}$$

67.(A)

If their maximum velocities are equal then their total energy is same.

If A_1, A_2 are their amplitudes, then

$$\frac{1}{2}K_1A_1^2 = \frac{1}{2}K_2A_2^2$$

$$\therefore \; \frac{A_1}{A_2} = \sqrt{\frac{K_2}{K_1}}$$

68.(B)

tension in the string is maximum when the bob passes through the mean position.

$$T_{max} = mg + \frac{mV^2}{L} \qquad \ldots(1)$$

In S.H.M. velocity at the mean position is given by $V = a\omega$

For simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

$$\therefore \; \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$\therefore \; V = a\sqrt{\frac{g}{L}} \quad \text{or} \quad V^2 = a^2\frac{g}{L}$$

Putting this value of V^2 in Eq. (1) we get

$$T_{max} = mg\left[1 + \left(\frac{a}{L}\right)^2\right]$$

69.(A)

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \therefore \frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{16}{4}} = \frac{2}{1}$$

$$\therefore \frac{T_1 + T_2}{T_1 - T_2} = \frac{2+1}{2-1} = \frac{3}{1}$$

70. (D)

Time period $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T^2 = 4\pi^2 \frac{m}{k}$

$$\Rightarrow \quad k = \frac{4\pi^2 m}{T^2}$$

$mg = kx$

$$\therefore \quad x = \frac{mg}{k} = mg\frac{T^2}{4\pi^2 m} = \frac{gT^2}{4\pi^2}$$

71. (A)

$$k = \frac{1}{2}m\omega^2 a^2$$

$$k \propto a^2 \quad \therefore \frac{k}{k/2} = \frac{a^2}{x^2} = 2$$

$$\therefore \quad x = \frac{a}{\sqrt{2}}$$

72.(C)

$$K.E. = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

$$T.E. = \frac{1}{2}m\omega^2 A^2$$

$$\frac{3}{4}\times\frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

$$x^2 = \left(1 - \frac{3}{4}\right)A^2 = \frac{A^2}{4}$$

$$x = \frac{A}{2}$$

73. (C)

$L_1 + L_2 = L$

$L_1 = NL_2$

$\therefore \; k_2 = Nk_1$

Before cutting, they are connected in series

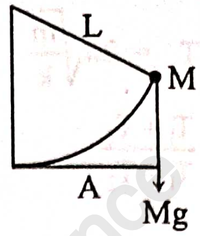
$$\therefore \; \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_1} + \frac{1}{Nk_1} = \frac{N+1}{Nk_1}$$

$$k = \frac{Nk_1}{N+1}$$

$$\therefore \; k_1 = \frac{k(N+1)}{N}$$

74. (D)

At extreme position the PE is total energy $= \frac{1}{2}M\omega^2A^2$



But $\omega = \sqrt{\frac{g}{L}}$ $\therefore$ PE $= \frac{1}{2}\frac{MgA^2}{L}$ $\left[\because\ T = 2\pi\sqrt{\frac{L}{g}} = \frac{2\pi}{\omega}\right]$

75. (A)

$\alpha = A\omega$

$\beta = A\omega^2$ Path length = 2A

$\alpha^2 = A^2\omega^2$

$\beta = A\omega^2$

$A = \frac{\alpha^2}{\beta}$ $\therefore$ $2A = \frac{2\alpha^2}{\beta}$

76. (C)

$$T = 6\text{ s} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\therefore\ \omega = \sqrt{\frac{g}{\ell}}$$

$$\therefore\ \omega = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\omega^2 = \frac{g}{\ell} = \frac{\pi^2}{9}$$

$$\ell = \frac{9g}{\pi^2} = \frac{9\pi^2}{\pi^2} = 9\text{ cm}$$

77. (C)

$$y = 2\sin\left(\frac{\pi t}{2} + \phi\right)$$

$$\frac{dy}{dt} = \frac{2\pi}{2}\cos\left(\frac{\pi t}{2} + \phi\right)$$

$$\frac{d^2y}{dt^2} = (1)\pi \times \frac{\pi}{2}\sin\left(\frac{\pi t}{2} + \phi\right)$$

$$\therefore\ \left.\frac{d^2y}{dt^2}\right|_{max} = (-)\frac{\pi^2}{2}\ \text{cm/s}^2$$

78. (B)

$$v = a\omega = a\frac{2\pi}{T}$$

$$v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\frac{\sqrt{3}}{2}a = \frac{2\pi}{T}\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}\pi a}{T}$$

79. (B)

$T = 2\pi\sqrt{\frac{\ell}{g}}$ ℓ = length of block dipped in liquid

Now, $mg = A\ell\rho g$

$$\ell = \frac{m}{A\rho}$$

$$T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

$$\therefore\ n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$$

80. (A)

$$x = P\sin\omega t + Q\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= P\sin\omega t + Q\cos\omega t$$

Let $P = A\cos\theta$ $\quad Q = A\sin\theta$

$\therefore\ P^2 + Q^2 = A^2$

$\therefore\ x = A\sin(\omega t + \theta)$

$$E_{total} = \frac{1}{2}m\omega^2A^2 \quad = \frac{1}{2}m\omega^2\left(P^2 + Q^2\right)$$

81. (D)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

82. (C)

$Y = A\sin(\omega t)$

$$E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}m\frac{4\pi^2}{T^2}A^2 = \frac{2\pi^2mA^2}{T^2}$$

83. (C)

$$\frac{T_e}{T_p} = \sqrt{\frac{\ell_e}{g_e}} \times \sqrt{\frac{g_p}{\ell_p}} \qquad \text{But } T_e = T_p$$

$$\therefore\ 1 = \sqrt{\frac{\ell_e \times g_p}{g_e \times \ell_p}}$$

But $g_p = \dfrac{G \times 2M}{(2R)^2} = \dfrac{GM \times 2}{4R^2} = \dfrac{g}{2}$

$g_e\ell_p = \ell_e g_p$

$$\ell_p = \ell_e\frac{g_p}{g_e} = 1 \times \frac{1}{2} = 0.5\text{ m}$$

84. (C)

$L - \ell = x,\ mg = kx \qquad \therefore\ k = \dfrac{mg}{x}$

$\dfrac{d^2x}{dt^2} = -p^2x \qquad \therefore\ p^2 = \dfrac{k}{m} = \dfrac{g}{x}$

$$\therefore\ p = \sqrt{\frac{g}{x}} = \sqrt{\frac{g}{L-\ell}}$$

85. (C)

$$U_1 = \frac{1}{2}kx_1^2 \quad \therefore \ x_1 = \sqrt{\frac{2U_1}{k}}$$

$$U_2 = \frac{1}{2}kx_2^2 \quad \therefore \ x_2 = \sqrt{\frac{2U_2}{k}}$$

$$\therefore \ U = \frac{1}{2}k(x_1 + x_2)^2$$

$$\therefore \ x_1 + x_2 = \sqrt{\frac{2U}{k}}$$

$$\sqrt{\frac{2U_1}{k}} + \sqrt{\frac{2U_2}{k}} = \sqrt{\frac{2U}{k}}$$

$$\therefore \ \sqrt{U} = \sqrt{U_1} + \sqrt{U_2}$$

86. (D)

At mean position $f_{net} = 0$

$\therefore$ Applying conservation of momentum

$$m_1v_1 = (m_1 + m_2)v_2$$

$$m_1\omega_1A_1 = (m_1 + m_2)\omega_2A_2$$

But $\omega_1 = \sqrt{\frac{k}{m_1}}$ $\quad \omega_2 = \sqrt{\frac{k}{m_1 + m_2}}A_2$

$$\therefore \quad m_1\sqrt{\frac{k}{m_1}}A_1 = (m_1 + m_2)\sqrt{\frac{k}{m_1 + m_2}}A_2$$

$$\frac{A_1}{A_2} = \sqrt{\frac{m_1 + m_2}{m_1}}$$

87. (C)

$$x = a\sin\left(\frac{\pi}{\sqrt{2}}t\right)$$

$$\therefore \ \omega = \frac{\pi}{\sqrt{2}} = \frac{2\pi}{T} \quad \therefore \ T = 2\sqrt{2}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$2\sqrt{2} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$2 = \pi^2 \cdot \frac{\ell}{g}$$

$$\therefore \ \ell = \frac{2g}{\pi^2} = 2 \text{ m}$$

88. (D)

$$F = bv$$

$$b = \frac{F}{v} = \frac{N}{m/s} = \frac{MLT^{-2}}{L}T = ML^{-1}$$

$\Rightarrow$ kg sec^{-1}

89. (D)

$T = 3$ sec

$y = A \sin \omega t$

$$\frac{A}{2} = A \sin \frac{2\pi}{T} t$$

$$\sin \frac{\pi}{6} = \sin \frac{2\pi}{3} t$$

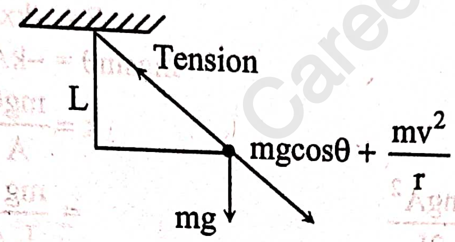
$$\frac{\pi}{6} = \frac{2\pi}{3} t \qquad \therefore\ t = \frac{\pi}{6} \times \frac{3}{2\pi} = \frac{1}{4}\ s$$

90. (D)

$y = A \sin \omega t$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} = \omega$$



$$\therefore\ \text{Tension} = mg\cos\theta + \frac{mv^2}{L}$$

$$T_{max} = mg + \frac{mv^2}{L} \qquad \cos\theta = 1$$

$$= mg\left(1 + \frac{v^2}{gL}\right)$$

Now, $y = A \sin \omega t$

$$\frac{dy}{dt} = A\omega \cos \omega t$$

$$\left.\frac{dy}{dt}\right|_{max} = A\omega = A\sqrt{\frac{g}{\ell}} = V_{max}$$

$$\therefore\ V_{max}^2 = A^2 \frac{g}{L}$$

$$\therefore\ T_{max} = mg\left(1 + \frac{A^2 g}{L^2 g}\right) = mg\left(1 + \frac{A^2}{L^2}\right)$$

91. (A)

$$mg = -kx \qquad \therefore\ k = \frac{mg}{x}$$

$$\omega^2 = \frac{k}{m}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$$

$$\frac{4\pi^2}{T^2} = \frac{k}{m}$$

$$\therefore\ T^2 = \frac{4\pi^2 m}{k} = \frac{4\pi^2 mx}{mg} \Rightarrow T = 2\pi\sqrt{\frac{x}{g}}$$

92. (B)

Displacement = x, P.E. = E, Force = F

$F = -kx$

$$E = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}kx^2$$

$$2E = (-)\frac{F}{x}\times x^2 = (-)\,Fx$$

$$\frac{2E}{F} + x = 0$$

93. (C)

$$PE = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2 = \frac{1}{2}\frac{mg}{L}A^2 = \frac{mgA^2}{2L}$$

$$F = -kx$$

$$mg\sin\theta = -kA$$

$$k = \frac{mg\theta}{A} = \frac{mg\,A}{L\,A}$$

94.(D)

Comparing with standard equation of S.H.M.

$x = A\sin(\omega t + \phi)$

we get $\omega = 11$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\times 22}{11\times 7} = \frac{4}{7}\,s$$

95. (D)

T = 6 sec

$$\frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}\left(\frac{1}{2}m\omega^2A^2\right)$$

$$A^2 - x^2 = \frac{A^2}{2}$$

$$\therefore x^2 = \frac{A^2}{2} \Rightarrow x = \frac{A}{\sqrt{2}}$$

$$\frac{A}{\sqrt{2}} = A\sin\omega t = A\sin\frac{2\pi}{T}t$$

$$\frac{1}{\sqrt{2}} = \sin\frac{2\pi}{6}t = \sin\frac{\pi}{3}t$$

$$\sin\frac{\pi}{4}t = \sin\frac{\pi}{3}t$$

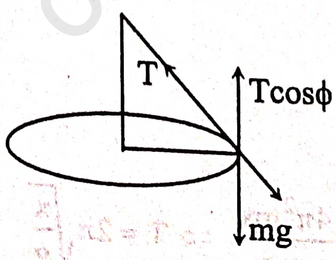
$$\therefore t = \frac{3}{4} = 0.75\text{ sec}$$

96. (D)

$L = 2\text{ m}$ $\quad m = \frac{100}{1000} = 0.1\text{ kg}$

$T\cos\phi = mg$

$$T\sin\phi = \frac{mv^2}{R}$$



$$\cos\phi = \frac{mg}{T} = \frac{0.1\times10}{10} = 0.1$$

$$\therefore \quad \phi = \cos^{-1}(0.1)$$

97. (C)

$$v = \omega\sqrt{(A^2 - x^2)} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}A}{2}\times\frac{2\pi}{T}$$

$$= \frac{\sqrt{3}\pi A}{T}$$

98. (A)

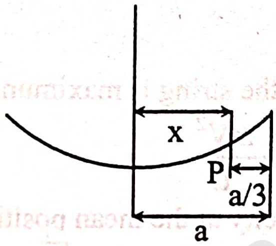
a = amplitude

$$x = a - \frac{a}{3} = \frac{2a}{3}$$

$$a_p = \omega^2 x = \omega^2\frac{2a}{3}$$

$$v_p = \omega\sqrt{a^2 - x^2} = \omega\sqrt{a^2 - \frac{4a^2}{9}} = \omega\sqrt{\frac{5a^2}{9}}$$

$$= \omega\sqrt{5}\frac{a}{3}$$



$$3v_p = a_p$$

$$\frac{v_p}{a_p} = \frac{1}{3} = \frac{\omega\sqrt{5}\,a/3}{\omega^2\frac{2a}{3}} = \frac{\sqrt{5}}{2\omega}$$

$$3 = \frac{2\omega}{\sqrt{5}} = \frac{2}{\sqrt{5}}\times\frac{2\pi}{T}$$

$$\therefore \quad T = \frac{4\pi}{3\sqrt{5}}$$

99. (C)

$$T = \sqrt{3} \qquad a = \frac{g}{3}$$

$$T' = 2\pi\sqrt{\frac{\ell}{g + \frac{g}{3}}}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{4g/3}} = \frac{\sqrt{3}}{2} \qquad T' = \sqrt{3}\times\frac{\sqrt{3}}{2} = \frac{3}{2} = 1.5\text{ sec}$$

100.(D)

The coin will lose contact with the plate when its acceleration in downward direction just exceeds acceleration due to gravity.

$$\therefore \quad \omega^2 x = g$$

$$\therefore \quad x = \frac{g}{\omega^2}$$

101.(D)

$$a = -\omega^2 L = -4\pi^2 f^2 L$$

102.(D)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\omega = \sqrt{\frac{g}{\ell}} \quad \therefore \quad \omega \propto \frac{1}{\sqrt{\ell}}$$

$$\therefore \quad \frac{\omega_2}{\omega_1} = \sqrt{\frac{\ell_1}{\ell_2}} = \sqrt{\frac{\ell_1}{\ell_1/3}} = \sqrt{3}$$

$$\therefore \quad \omega_2 = \sqrt{3}\,\omega_1$$

Now $\quad E \propto \omega^2$

$$\therefore \quad \frac{E_2}{E_1} = \frac{\omega_2^2}{\omega_1^2} = 3$$

$$\therefore \quad E_2 = 3E_1$$

103.(C)

The tension in the string is maximum when the bob passes through the mean position.

$$T_{max} = mg + \frac{mV^2}{L} \qquad \text{...(1)}$$

In S.H.M. velocity at the mean position is given by $V = A\omega$

For simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$\therefore \quad V = A\sqrt{\frac{g}{L}} \quad \text{or} \quad V^2 = \frac{A^2 g}{L}$$

Putting this value of V^2 in eq. (1) :

$$T_{max} = mg\left[1 + \frac{A^2}{L^2}\right]$$

104.(A)

$x = 4 \sin \pi t$

At $t = 0, x = 0$

when $x = 2$ cm we have

$2 = 4 \sin \pi t$

$$\therefore \quad \sin \pi t = \frac{1}{2}$$

$$\therefore \quad \pi t = \frac{\pi}{6} \quad \therefore \quad t = \frac{1}{6}\text{s}$$

105.(D)

The period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell}{9}} \quad \therefore \quad T \propto \sqrt{\ell}$$

The period of oscillation will increase if length is increased.

106.(C)

$$Y = A\sin\omega\left(\frac{x}{v} - k\right) = A\sin\left(\frac{wx}{v} - wk\right)$$

The argument of a trigonometric function should be dimensionless (since it is an angle).

$\therefore \omega k = M^0L^0T^0$

$$\therefore k = \frac{[M^0L^0T^0]}{[\omega]} = \frac{M^0L^0T^0}{[T^{-1}]} = L^0M^0T^1$$

107.(C)

$A = 4$ cm, $x = 3$ cm

$v = \omega\sqrt{A^2 - x^2}; a = \omega^2 x$

If $v = \omega$ then $\omega\sqrt{A^2 - x^2} = \omega^2 x$

$$\therefore \omega = \frac{\sqrt{A^2 - x^2}}{x} = \sqrt{\frac{16-9}{3}} = \frac{\sqrt{7}}{3}$$

$$\therefore \frac{2\pi}{T} = \frac{\sqrt{7}}{3}$$

$$\therefore T = \frac{6\pi}{\sqrt{7}}$$

108(B)

$$\omega^2 = \frac{k}{m} = \frac{\text{Restoring force per unit displacement}}{\text{mass}}$$

109. (C)

For simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{1}{2}} \qquad \because L_2 = 2L_1$$

Energy $E = \frac{1}{2}m\omega^2 A^2$

$$\therefore \quad E \propto \omega^2 \propto \frac{1}{T^2}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{T_1^2}{T_2^2} = \frac{1}{2}$$

$$\therefore \quad E_2 = \frac{E_1}{2}$$

110.(C)

$$x = A\cos\left(\omega t + \frac{\pi}{6}\right)$$

$$v = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \frac{\pi}{6}\right)$$

v is maximum when $\sin\left(\omega t+\frac{\pi}{6}\right)=1$

or $\omega t+\frac{\pi}{6}=\frac{\pi}{2}$

$\therefore \omega t=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$

$\therefore t=\frac{\pi}{3\omega}$

111.(D)

$\sqrt{E_1}+\sqrt{E_2}=\sqrt{E}$

112.(C)

$v_1=\omega\sqrt{(A^2-x_1^2)}$; $v_2=\omega\sqrt{(A^2-x_2^2)}$

$\therefore v_1^2=\omega^2(A^2-x_1^2)$; $v_2^2=\omega^2(A^2-x_2^2)$

$\therefore v_2^2-v_1^2=\omega^2\left(x_1^2-x_2^2\right)$

$\therefore \omega=\sqrt{\frac{v_2^2-v_1^2}{x_1^2-x_2^2}}$

113.(C)

Total energy $E=\frac{1}{2}m\omega^2A^2$

It is not proportional to displacement from mean position.

114.(B)

$T_1=2\pi\sqrt{\frac{M}{k}}$, $T_2=2\pi\sqrt{\frac{2M}{k}}$

$\therefore \frac{T_2}{T_1}=\sqrt{2}$

$\therefore T_2=\sqrt{2}T_1=\sqrt{2}T$

115.(C)

$F_1=k_1x$, $F_2=k_2x$

$F=F_1+F_2=k_1x+k_2x=(k_1+k_2)x$

$\therefore k=k_1+k_2$

$T_1=0.8=2\pi\sqrt{\frac{m}{k_1}}$

$\therefore 0.64=4\pi^2\frac{m}{k}$ or $k_1=\frac{4\pi^2 m}{0.64}$

$T_2=0.6=2\pi\sqrt{\frac{m}{k_2}}$ $\therefore k_2=\frac{4\pi^2 m}{0.36}$

$k=k_1+k_2=4\pi^2 m\left(\frac{1}{0.64}+\frac{1}{0.36}\right)=4\pi^2 m\left(\frac{1}{0.64\times0.36}\right)$

$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m\times0.64\times0.36}{4\pi^2 m}}=0.8\times0.6=0.48\text{ s}$

116.(B)

$T = 24\text{ s} \qquad \therefore\ \omega = \frac{2\pi}{T} = \frac{2\pi}{24} = \frac{\pi}{12}$ rad/s

$v = 2\pi$ m/s at $t = 3$

v is given by $v = A\omega \cos \omega t$

$$\therefore\ 2\pi = A \times \frac{\pi}{12} \times \cos\frac{\pi}{12}\cdot 3$$

$$\therefore\ 24 = A\cos\frac{\pi}{4}$$

$$\therefore\ 24 = A\cdot\frac{1}{\sqrt{2}}$$

$$\therefore\ A = 24\sqrt{2}\ \text{m}$$

Path length $= 2A = 48\sqrt{2}$ m

117.(D)

Maximum speed $V = A\omega$ where A is the amplitude and ω angular frequency

Average speed $V_A = \frac{\text{Total distance}}{\text{Total time}} = \frac{4A}{T}$

where $T = \frac{2\pi}{\omega}$ = Period of S.H.M.

$$\therefore\ V_A = \frac{4A\omega}{2\pi} = \frac{2A\omega}{\pi} = \frac{2V}{\pi}$$

118.(C)

$V = A\omega = A \times 2\pi f$

$$\therefore\ A = \frac{V}{2\pi f}$$

Total energy $E = \frac{1}{2}kA^2 = \frac{1}{2}k \times \frac{V^2}{4\pi^2 f^2}$

$$\therefore\ k = \frac{8E\pi^2 n^2}{V^2}$$

119.(B)

Frequency $f = \frac{7}{2\pi}$

$$\therefore\ \omega = 2\pi f = 2\pi \times \frac{7}{2\pi} = 7\ \text{rad/s}$$

The standard differential equation of S.H.M. is

$$\frac{dx^2}{dt^2} + \omega^2 x = 0$$

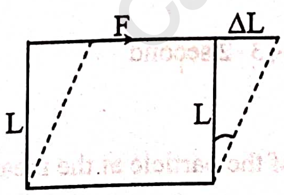
Putting the value of ω we get

$$\frac{dx^2}{dt^2} + 49x = 0$$

120.(A)

$$\eta = \frac{F}{L^2}\cdot\frac{L}{\Delta L}$$

$$= \frac{F}{L\Delta L}$$



$\therefore \quad F = \eta L \Delta L$

$F = k\Delta L$ where $k = \eta L$

$\therefore \quad T = 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{M}{\eta L}}$

121.(B)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is the effective value of acceleration due to gravity. When the trolley has acceleration 'a' in horizontal direction, the effective value of acceleration is the resultant of a and g which are at right angles to each other. Hence effective acceleration is $\sqrt{a^2+g^2}$ or $\left(a^2+g^2\right)^{\frac{1}{2}}$

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{(a^2+g^2)^{\frac{1}{2}}}} = \frac{2\pi\sqrt{L}}{\sqrt{\left(a^2+g^2\right)^{\frac{1}{2}}}}$$

$$= 2\pi\sqrt{L}\left(a^2+g^2\right)^{-\frac{1}{4}}$$

122.(C)

$y_1 = 2\sin(10t+\theta) \qquad \therefore \quad V_1 = \dfrac{dy_1}{dt} = 20\cos(10t+\theta)$

$y_2 = 3\cos 10t \qquad \therefore \quad V_2 = -30\sin 10t$

$$= -30\cos\left(\frac{\pi}{2}-10t\right)$$

$$= -30\cos\left(10t-\frac{\pi}{2}\right)$$

$$= 30\cos\left(10t-\frac{\pi}{2}+\pi\right)$$

$$= 30\cos\left(10t+\frac{\pi}{2}\right)$$

$\therefore$ phase difference between V_1 and V_2

$$= (10t+\theta) - \left(10t+\frac{\pi}{2}\right) = \left(\theta-\frac{\pi}{2}\right)$$

123.(C)

$$\frac{g'}{g} = \frac{M'}{M}\cdot\frac{R^2}{R'^2}$$

$$= 3\cdot\frac{1}{3^2} = \frac{1}{3}$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{3}$$

$\therefore \quad T' = \sqrt{3}\,T = \sqrt{3}\cdot 2$ second

124.(B)

The acceleration of the particle at the mean position is zero in simple harmonic motion.

125.(B)

$$T = 2\pi\sqrt{\frac{M}{k}}; \quad \frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\therefore \frac{5}{3}\times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\therefore \frac{25}{9}\cdot\frac{M}{k} = \frac{M+m}{k}$$

$$\therefore \frac{25}{9}M = M+m$$

Dividing by M, $\frac{25}{9} = 1+\frac{m}{M}$ $\quad \therefore \frac{m}{M} = \frac{25}{9}-1 = \frac{16}{9}$

$$\frac{M}{m} = \frac{9}{16}$$

126.(B)

For a particle performing S.H.M.

$F = -kx = -kA \sin \omega t$

The force time graph will be inverted distance time graph.

127.(A)

Total energy $= \frac{1}{2}KA^2$

Potential energy $= \frac{1}{2}kx^2$

If P.E. $= \frac{1}{4}$(K.E.) then

$$\frac{1}{2}kx^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right)$$

$$\therefore x^2 = \frac{A^2}{4} \text{ or } x = \frac{A}{2}$$

128.(D)

$$MV = (M+m)V'$$

$$\frac{1}{2}(M+m)V'^2 = \frac{1}{2}kA_2^2$$

$$\frac{1}{2}MV^2 = \frac{1}{2}kA_1^2$$

$$\frac{M+m}{M}\cdot\frac{V'^2}{V^2} = \frac{A_2^2}{A_1^2}$$

$$\frac{M+m}{M}\cdot\left(\frac{M}{M+m}\right)^2 = \frac{A_2^2}{A_1^2}$$

$$\frac{A_1^2}{A_2^2} = \frac{M+m}{M}$$

$$\therefore \frac{A_1}{A_2} = \left(\frac{M+m}{M}\right)^{\frac{1}{2}}$$

129.(B)

$w_1 = \frac{1}{2}kx_1^2$, $w_2 = \frac{1}{2}(2k)x_2^2 = kx_2^2$

$\therefore \frac{w_1}{w_2} = \frac{1}{2}\cdot\frac{x_1^2}{x_2^2}$

$F = kx_1 = 2kx_2$

$\therefore \frac{x_1}{x_2} = 2$

$\therefore \frac{w_1}{w_2} = \frac{1}{2}\times(2)^2 = \frac{1}{2}\times 4 = 2$

$\therefore w_1 = 2w_2$

130.(D)

Kinetic energy of a particle performing S.H.M. is given by

$$k = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

When $x = \frac{2}{3}A$,

$$k = \frac{1}{2}m\omega^2\left(A^2 - \frac{4}{9}A^2\right) = \frac{1}{2}m\omega^2A^2\times\frac{5}{9}.$$

If the velocity is tripled, its kinetic energy will become 9 times. The new kinetic energy will be

$$k' = \frac{1}{2}m\omega^2A^2\times 5$$

The potential energy $p = \frac{1}{2}m\omega^2\left(\frac{2}{3}A\right)^2 = \frac{1}{2}m\omega^2A^2\cdot\frac{4}{9}$

If A′ is the new amplitude then the total energy e is given by

$$E = \frac{1}{2}m\omega^2A'^2$$

Also, $E = P + k'$

$$\therefore \frac{1}{2}m\omega^2A'^2 = \frac{1}{2}m\omega^2A^2\cdot\frac{4}{9}+\frac{1}{2}m\omega^2A^2\cdot 5$$

$$\therefore A'^2 = \left(\frac{4}{9}+5\right)A^2$$

$$\therefore A' = \frac{7}{3}A$$

131.(B)

Total energy $= E = \frac{1}{2}m\omega^2A^2$

When kinetic energy is $\frac{3E}{4}$, its potential energy is

$$\left(E - \frac{3E}{4}\right) = \frac{E}{4}$$

Dividing Eq.(1) by Eq.(2), $4 = \frac{A^2}{x^2}$ $\therefore x = \frac{A}{2}$

132.(C)

For seconds pendulum $T = 2s$

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad \therefore \quad 2 = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\text{or } 1 = \pi\sqrt{\frac{\ell}{g}}$$

$$\text{squaring : } 1 = \pi^2\frac{\ell}{g}$$

$$\therefore \ell = \frac{9}{\pi^2} = \frac{981}{10} = 98.1 \text{ cm}$$

At place B, $\ell' = 98.1 - 0.3 = 97.8$ cm

$$\text{Again, } 2 = 2\pi\sqrt{\frac{\ell'}{g'}}$$

$$\therefore 1 = \pi^2\frac{\ell'}{g'}$$

$$\therefore g' = \pi^2\ell' = 10 \times 97.8 = 978 \text{ cm/s}^2$$

133.(A)

$$V = \omega\sqrt{A^2 - x^2} = \sqrt{\frac{k}{m}}\cdot\sqrt{A^2 - x^2}$$

$k = 10$ N/m, $m = 10$ kg, $A = 0.5$ m

$V = 40$ cm/s $= 0.4$ m/s

Substituting the values and solving we get $x = 0.3$ m

134.(A)

Frequency of simple pendulum is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \qquad \therefore \frac{f_1}{f_2} = \sqrt{\frac{\ell_2}{\ell_1}}$$

$$\therefore \frac{3}{4} = \sqrt{\frac{\ell_2}{\ell_1}} \quad \text{or} \quad \frac{9}{16} = \frac{\ell_2}{\ell_1}$$

$$\therefore \frac{\ell_1}{\ell_2} = \frac{16}{9}$$

135.(D)

If x is the extension of the wire then

$$F = \left(\frac{YA}{L}\right)x \qquad F = kx \quad \text{where } k = \frac{YA}{L}$$

frequency of oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$