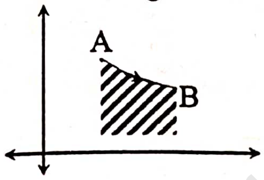
4. Thermodynamics

Important Formulae and Shortcut Methods

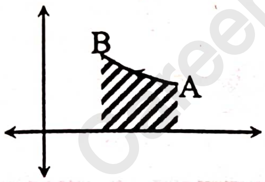
1. In P–V diagram, for a closed path, work done is always area of that closed path.
2. For the following cases :

(1)



Work done is +ve.

(2)



Work done is −ve.

When a gas expands, work done is positive.
When a gas contracts, work done is negative.

3. **For a closed curve**
   i) Work done in clockwise direction is +ve.
   ii) Work done in anticlockwise direction is −ve.
4. Internal energy is a state function. So ΔU for a closed path is zero.
5. The adiabatic exponent of a gaseous mixture is given by

$$\frac{n_1+n_2}{\gamma-1}=\frac{n_2}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}$$

6. $dU = nC_v dT$ (always)
   $dQ = nC_p dT$ (constant pressure process)
7. For an isothermal process, T is constant and hence PV = constant
   For an isochoric process, V is constant and hence $\frac{P}{T}$ = constant
   For an isobaric process, P is constant and hence $\frac{V}{T}$ = constant
   For an adiabatic process :
   (i) PV^{γ} = constant
   (ii) $TV^{\gamma-1}$ = constant
   (iii) $P^{1-\gamma}T^{\gamma}$ = constant
8. In questions, identify the system properly and identify correctly the type of processes taking place.
   (1) e.g. If gas is in thermally insulated vessel undergoing volume change then processes are adiabatic.
   (2) If gas is in diathermic conducting ($\Delta Q = 0$) undergoing reversible changes then it is isothermic.
   (3) Number of moles for closed system is constant in absence of any chemical reaction.

- The work done by a gas in irreversible cycle cannot be calculated from p–v diagram.
- For specific heat of a gas, we may use the following formulae

$$C_v = \frac{R}{\gamma - 1} \quad \text{and} \quad C_p = \frac{\gamma R}{\gamma - 1}$$

- The function on heat energy used to increase internal energy of a gas is

$$\frac{dU}{dQ} = \frac{C_v}{C_p} = \frac{1}{\gamma}$$

9. Work done on an ideal gas :

isothermal process, $W = nRT\,\ell n\left(\frac{V_f}{V_i}\right)$

isobaric process, $W = P(V_f - V_i)$

isochoric process, $W = 0$

adiabatic process, $W = \frac{P_iV_i - P_fV_f}{\gamma - 1}$

Multiple Choice Questions

MHT-CET 2004

1. The state of a thermodynamic system is represented by
(A) pressure only (B) volume only
(C) pressure, volume and temperature (D) number of moles

2. A gas expands adiabatically at constant pressure, such that its temperature $T \propto \frac{1}{\sqrt{V}}$. The value of C_P/C_v of the gas is
(A) 1.30 (B) 1.50 (C) 1.67 (D) 2.00

MHT-CET 2005

3. With same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways the work done by the gas is W_1, if the process is isothermal, W_2 if isobaric and W_3 if adiabatic, then
(A) $W_2 > W_1 > W_3$ (B) $W_2 > W_3 > W_1$ (C) $W_1 > W_2 > W_3$ (D) $W_1 > W_3 > W_2$

MHT-CET 2007

4. We consider a thermodynamic system. If ΔU represents the increase in its internal energy and W the work done by the system, which of the following statements are true?
(A) $\Delta U = -W$ is an adiabatic process (B) $\Delta U = W$ in an isothermal process
(C) $\Delta U = -W$ in an isothermal process (D) $\Delta U = W$ in an adiabatic process

MHT-CET 2019

*5. If α is the coefficient of performance of a refrigerator and 'Q_1' is heat released to the hot reservoir, then the heat extracted from the cold reservoir 'Q_2' is
(A) $\frac{\alpha Q_1}{\alpha - 1}$ (B) $\frac{\alpha Q_1}{1 + \alpha}$ (C) $\frac{1+\alpha}{\alpha} Q_1$ (D) $\frac{\alpha - 1}{\alpha} Q_1$

6. The equation of state for 2g of oxygen at a pressure 'P' and temperature 'T', when occupying a volume 'V' will be
(A) $PV = \frac{1}{16} RT$ (B) $PV = RT$ (C) $PV = 2RT$ (D) $PV = 16\,RT$

MHT-CET 2020

*7. For a heat engine operating between temperatures t_1 °C and t_2 °C, its efficiency will be

(A) $\frac{t_1 - t_2}{t_2}$ (B) $\frac{t_1 - t_2}{t_1 + 273}$ (C) $\frac{t_1}{t_2}$ (D) $1 - \frac{t_2}{t_1}$

8. The initial pressure and volume of a gas is 'P' and 'V' respectively. First by isothermal process gas is expanded to volume '9V' and then by adiabatic process its volume is compressed to 'V' then its final pressure is (Ratio of specific heat at constant pressure to constant volume = $\frac{3}{2}$)

(A) 6 P (B) 27 P (C) 3 P (D) 9 P

9. If a gas is compressed isothermally then the r.m.s. velocity of the molecules

(A) increases (B) decreases
(C) first decreases and then increases (D) remains the same

10. If 'ΔQ' is the amount of heat supplied to 'n' moles of a diatomic gas at constant pressure, 'ΔU' is the change in internal energy and 'ΔW' is the work done, then $\Delta W : \Delta U : \Delta Q$ is

(A) 1 : 2 : 3 (B) 2 : 5 : 7 (C) 2 : 3 : 4 (D) 5 : 7 : 9

11. Ideal gas for which 'γ' = 1.5 is suddenly compressed to $\frac{1}{4}$th of its initial volume. The ratio of the final pressure to the initial pressure is $\left(\gamma = \frac{C_P}{C_V}\right)$

(A) 4:1 (B) 8:1 (C) 1:16 (D) 1:8

12. An ideal gas at 27°C is compressed adiabatically to (8/27) of its original volume. If ratio of specific heats, $\gamma = 5/3$ then the rise in temperature of the gas is

(A) 500 K (B) 125 K (C) 250 K (D) 375 K

SOLUTIONS

1. (C)
The state of thermodynamics system is represented by pressure, volume and temperature.

2. (B)
For adiabatic expansion, we have the formula

$$pV^{\gamma} = \text{constant} \quad \ldots\text{(i)}$$

Gas equation is,

$$pV = RT$$

$$\Rightarrow \quad p = \frac{RT}{V} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we obtain

$$\left(\frac{RT}{V}\right)V^{\gamma} = \text{constant}$$

$$\Rightarrow \quad TV^{\gamma-1} = \text{constant} \quad \ldots\text{(iii)}$$

But $\quad T \propto \frac{1}{\sqrt{V}}$ (given)

as $\quad TV^{1/2} = \text{constant} \quad \ldots\text{(iv)}$

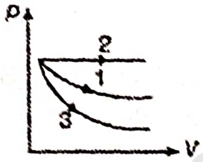
Thus, using Eqs. (iii) and (iv) together, we get

$$\gamma - 1 = \frac{1}{2} \quad \text{or} \quad \gamma = \frac{3}{2} = 1.5 \quad \Rightarrow \quad \frac{C_P}{C_V} = 1.5$$

3. (A)

Here, for isobaric process, work done is largest, next for isothermal and then for adiabatic process,

$$W_2 > W_1 > W_3$$



4. (A)

An isothermal process is a constant temperature process. In this process

$$T = \text{constant or} \quad \Delta T = 0$$

$$\therefore \quad \Delta U = nC_v \Delta T = 0$$

An adiabatic process is defined as one with no heat transfer into or out of a system

Therefore, Q = 0. From the first law of thermodynamics.

$$W = -\Delta U$$

or $\quad \Delta U = -W$

5. (B)

$$\alpha = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\frac{1}{\alpha} = \frac{Q_1 - Q_2}{Q_2} = \frac{Q_1}{Q_2} - 1$$

$$\frac{Q_1}{Q_2} = \frac{1}{\alpha} + 1 = \frac{1+\alpha}{\alpha}$$

$$Q_2 = \frac{\alpha Q_1}{1+\alpha}$$

6. (A)

$$PV = nRT$$

$$n = \frac{m}{M} = \frac{2}{32} = \frac{1}{16}$$

7. (B)

$$\text{Efficiency } \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$= \frac{t_1 - t_2}{t_1 + 273}$$

8. (C)

$$\frac{C_p}{C_v} = \frac{3}{2}$$

Case I : Isothermal process

$$P_1V_1 = P_2V_2$$

$$PV = P_2 \times 9\,V$$

$$\therefore \quad P_2 = \frac{P}{9}$$

Case II : Adiabatic process

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$\frac{P}{9}(9V)^{\gamma} = P_3(V)^{\gamma}$$

$$P_3 = \frac{P}{9}9^{\gamma}\frac{V^{\gamma}}{V^{\gamma}} = \frac{P}{9}9^{3/2} = \frac{P}{9}\times 27 = 3P$$

9. (D)
In an isothermal process there is no change of internal energy. Hence RMS velocity remains unchanged.

10. (B)
We know,

$$\Delta Q = nC_p\Delta T$$

$$\Delta U = nC_v\Delta T$$

$$W = P\,\Delta V = nR\,\Delta T$$

$$\Delta W : \Delta U : \Delta Q = R : C_v : C_p$$

For a diatomic gas f = 5

$$\therefore\ C_v = \frac{f}{2}R = \frac{5}{2}R$$

$$C_p = C_v + R = \frac{7}{2}R$$

$$\therefore\ R : C_v : C_p = 1 : \frac{5}{2} : \frac{7}{2} = 2 : 5 : 7$$

11. (B)

For adiabatic process

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\therefore\ \frac{P_2}{P_2} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (4)^{\frac{3}{2}} = 8$$

12. (D)

For an adiabatic process $TV^{\gamma-1}$ = constant

$$\therefore\ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{27}{8}\right)^{\frac{5}{3}-1} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{9}{4}$$

$$\therefore\ T_2 = \frac{4}{9}\cdot T_1 = \frac{4}{9}\times 300 = 675\ \text{K}$$

$$\therefore\ T_2 - T_1 = 675 - 300 = 375\ \text{K}$$