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H versus h(x)

The H matrix from the lidar lesson and $h(x)$ equations from the radar lesson are actually accomplishing the same thing; they are both needed to solve $y = z - Hx'$ in the update step.

But for radar, there is no H matrix that will map the state vector x into polar coordinates; instead, you need to calculate the mapping manually to convert from cartesian coordinates to polar coordinates.

Here is the h function that specifies how the predicted position and speed get mapped to the polar coordinates of range, bearing and range rate.

$$h(x') = \begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} \sqrt{p_x'^2 + p_y'^2} \\ \arctan(p_y'/p_x') \\ \frac{p_x' v_x' + p_y' v_y'}{\sqrt{p_x'^2 + p_y'^2}} \end{pmatrix}$$

Hence for radar $y = z - Hx'$ becomes $y = z - h(x')$.

Definition of Radar Variables

- The range, (ρ), is the distance to the pedestrian. The range is basically the magnitude of the position vector ρ which can be defined as $\rho = sqrt(p_x^2 + p_y^2).\varphi = atan(p_y/p_x)$. Note that φ is referenced counter-clockwise from the x-axis, so φ from the video clip above in that situation would actually be negative.The range rate, $\dot{\rho}$, is the projection of the velocity, v, onto the line, L.

Deriving the Radar Measurement Function

The measurement function is composed of three components that show how the predicted state, $x' = (p_x', p_y', v_x', v_y')^T$, is mapped into the measurement space, $z = (\rho, \varphi, \dot{\rho})^T$:

1. The range, ρ, is the distance to the pedestrian which can be defined as:
2. $\rho = \sqrt{p_x^2 + p_y^2}$
3. φ is the angle between ρ and the x direction and can be defined as:
4. $\varphi = \arctan(p_y/p_x)$
5. There are two ways to do the range rate $\dot{\rho}(t)$ derivation:
    1. Generally we can explicitly describe the range, ρ, as a function of time:

time derivative of ρ:

4. $\dot{\rho} = \frac{\partial \rho(t)}{\partial t} = \frac{\partial}{\partial t}\sqrt{p_x(t)^2 + p_y(t)^2} = \frac{1}{2\sqrt{p_x(t)^2+p_y(t)^2}}(\frac{\partial}{\partial t}p_x(t)^2 + \frac{\partial}{\partial t}p_y(t)^2)$

5. $= \frac{1}{2\sqrt{p_x(t)^2+p_y(t)^2}}(2p_x(t)\frac{\partial}{\partial t}p_x(t) + 2p_y(t)\frac{\partial}{\partial t}p_y(t))$

6. $\frac{\partial}{\partial t}p_x(t)$ is nothing else than $v_x(t)$, similarly $\frac{\partial}{\partial t}p_y(t)$ is $v_y(t)$. So we have:

7. $\dot{\rho} = \frac{\partial \rho(t)}{\partial t} = \frac{1}{2\sqrt{p_x(t)^2+p_y(t)^2}}(2p_x(t)v_x(t) + 2p_y(t)v_y(t)) = \frac{2(p_x(t)v_x(t)+p_y(t)v_y(t))}{2\sqrt{p_x(t)^2+p_y(t)^2}}$

8. $= \frac{p_x(t)v_x(t)+p_y(t)v_y(t)}{\sqrt{p_x(t)^2+p_y(t)^2}}$

9. For simplicity we just use the following notation:

10. $\dot{\rho} = \frac{p_x v_x + p_y v_y}{\sqrt{p_x^2+p_y^2}}$

11. The range rate, $\dot{\rho}$, can be seen as a scalar projection of the velocity vector, $\vec{v}$, onto $\vec{\rho}$. Both $\vec{\rho}$ and $\vec{v}$ are 2D vectors defined as:

12. $\vec{\rho} = \begin{pmatrix} p_x \\ p_y \end{pmatrix},\ \vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

13. The scalar projection of the velocity vector $\vec{v}$ onto $\vec{\rho}$ is defined as:

14. $\dot{\rho} = \frac{\vec{v}\vec{\rho}}{|\vec{\rho}|} = \frac{\begin{pmatrix} v_x & v_y \end{pmatrix}\begin{pmatrix} p_x \\ p_y \end{pmatrix}}{\sqrt{p_x^2+p_y^2}} = \frac{p_x v_x + p_y v_y}{\sqrt{p_x^2+p_y^2}}$

15. where $|\vec{\rho}|$ is the length of $\vec{\rho}$. In our case it is actually the range, so $\rho = |\vec{\rho}|$.

The Next Quiz

$$\begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} \leftarrow h(x) \begin{pmatrix} p_x' \\ p_y' \\ v_x' \\ v_y' \end{pmatrix}$$

h is a nonlinear function. In the next quiz I would like to check your intuition about what that means.