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Calculating Acceleration Noise Parameters

Before we discuss the derivation of the process covariance matrix Q , you might be curious about how to choose values for σ_{ax}^2 and σ_{ay}^2 .

For the extended Kalman filter project, you will be given values for both. The unscented Kalman filter lessons will describe how to choose values for the acceleration noise parameters.

Process Covariance Matrix Q - Intuition

As a reminder, here are the state covariance matrix update equation and the equation for Q .

$$P' = F P F^T + Q$$

$$Q = \begin{pmatrix} \frac{\Delta t^4}{4} \sigma_{ax}^2 & 0 & \frac{\Delta t^3}{2} \sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^4}{4} \sigma_{ay}^2 & 0 & \frac{\Delta t^3}{2} \sigma_{ay}^2 \\ \frac{\Delta t^3}{2} \sigma_{ax}^2 & 0 & \Delta t^2 \sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^3}{2} \sigma_{ay}^2 & 0 & \Delta t^2 \sigma_{ay}^2 \end{pmatrix}$$

Because our state vector only tracks position and velocity, we are modeling acceleration as a random noise. The Q matrix includes time Δt to account for the fact that as more time passes, we become more uncertain about our position and velocity. So as Δt increases, we add more uncertainty to the state covariance matrix P .

Combining both 2D position and 2D velocity equations previously deducted formulas we have:

$$\begin{cases} p'_x = p_x + v_x \Delta t + \frac{a_x \Delta t^2}{2} \\ p'_y = p_y + v_y \Delta t + \frac{a_y \Delta t^2}{2} \\ v'_x = v_x + a_x \Delta t \\ v'_y = v_y + a_y \Delta t \end{cases}$$

vector v in this form:

$$v = \begin{pmatrix} v_{px} \\ v_{py} \\ v_{vx} \\ v_{vy} \end{pmatrix} = \begin{pmatrix} \frac{a_x \Delta t^2}{2} \\ \frac{a_y \Delta t^2}{2} \\ a_x \Delta t \\ a_y \Delta t \end{pmatrix}$$

which is described by a zero mean and a covariance matrix Q , so $v \sim N(0, Q)$.

The vector v can be decomposed into two components a 4 by 2 matrix G which does not contain random variables and a 2 by 1 matrix a which contains the random acceleration components:

$$v = \begin{pmatrix} \frac{a_x \Delta t^2}{2} \\ \frac{a_y \Delta t^2}{2} \\ a_x \Delta t \\ a_y \Delta t \end{pmatrix} = \begin{pmatrix} \frac{\Delta t^2}{2} & 0 \\ 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 \\ 0 & \Delta t \end{pmatrix} \begin{pmatrix} a_x \\ a_y \end{pmatrix} = Ga$$

Δt is computed at each Kalman Filter step and the acceleration is a random vector with zero mean and standard deviations σ_{ax} and σ_{ay} .

Based on our noise vector we can define now the new covariance matrix Q . The covariance matrix is defined as the expectation value of the noise vector v times the noise vector v^T . So let's write this down:

$$Q = E[vv^T] = E[Gaa^T G^T]$$

As G does not contain random variables, we can put it outside the expectation calculation.

$$Q = GE[aa^T]G^T = G \begin{pmatrix} \sigma_{ax}^2 & \sigma_{axy} \\ \sigma_{axy} & \sigma_{ay}^2 \end{pmatrix} G^T = GQ_v G^T$$

This leaves us with three statistical moments:

- the expectation of a_x times a_x , which is the variance of a_x : σ_{ax}^2 squared. the expectation of a_y times a_y , which is the variance of a_y : σ_{ay}^2 squared. and the expectation of a_x times a_y , which is the covariance of a_x and a_y : σ_{axy} .

a_x and a_y are assumed uncorrelated noise processes. This means that the covariance σ_{axy} in Q_v is zero:

$$Q_v = \begin{pmatrix} \sigma_{ax}^2 & \sigma_{axy} \\ \sigma_{axy} & \sigma_{ay}^2 \end{pmatrix} = \begin{pmatrix} \sigma_{ax}^2 & 0 \\ 0 & \sigma_{ay}^2 \end{pmatrix}$$

So after combining everything in one matrix we obtain our 4 by 4 Q matrix:

$$Q = GQ_v G^T = \begin{pmatrix} \frac{\Delta t^4}{4} \sigma_{ax}^2 & 0 & \frac{\Delta t^3}{2} \sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^4}{4} \sigma_{ay}^2 & 0 & \frac{\Delta t^3}{2} \sigma_{ay}^2 \\ \frac{\Delta t^3}{2} \sigma_{ax}^2 & 0 & \Delta t^2 \sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^3}{2} \sigma_{ay}^2 & 0 & \Delta t^2 \sigma_{ay}^2 \end{pmatrix}$$

Note on Notation

Some authors describe Q as the complete process noise covariance matrix. And some authors describe Q as the covariance matrix of the individual noise processes. In our case, the covariance matrix of the individual noise processes matrix is called Q_v , which is something to be aware of.