11_02

EKF Algorithm Generalization

Extended Kalman Filter Equations

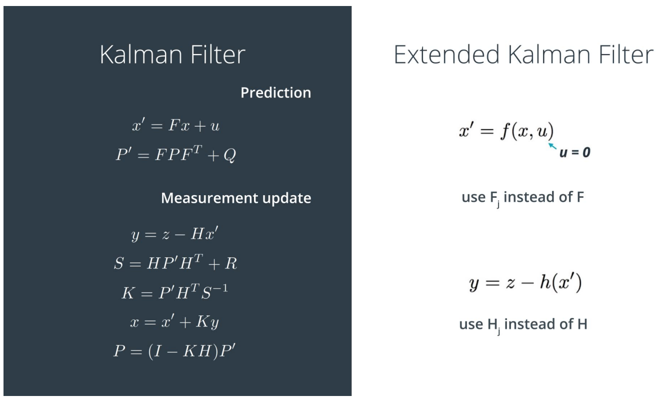
Although the mathematical proof is somewhat complex, it turns out that the Kalman filter equations and extended Kalman filter equations are very similar. The main differences are:

- the FFF matrix will be replaced by FjF_jFj when calculating P′P'P′.
- the HHH matrix in the Kalman filter will be replaced by the Jacobian matrix HjH_jHj when calculating SSS, KKK, and PP.
- to calculate x′x'x′, the prediction update function, fff, is used instead of the FFF matrix.
- to calculate yyy, the hhh function is used instead of the HHH matrix.

For this project, however, we do not need to use the fff function or FjF_jFj. If we had been using a non-linear model in the prediction step, we would need to replace the FFF matrix with its Jacobian, FjF_jFj. However, we are using a linear model for the prediction step. So, for the prediction step, we can still use the regular Kalman filter equations and the FFF matrix rather than the extended Kalman filter equations.

The measurement update for lidar will also use the regular Kalman filter equations, since lidar uses linear equations. Only the measurement update for the radar sensor will use the extended Kalman filter equations.

One important point to reiterate is that the equation y=z−Hx′y = z - Hx'y=z−Hx′ for the Kalman filter does not become y=z−Hjxy = z - H_jxy=z−Hjx for the extended Kalman filter. Instead, for extended Kalman filters, we'll use the h function directly to map predicted locations x′x'x′ from Cartesian to polar coordinates.



The comparison for reference.

Clarification of u = 0

In the above image, the prediction equation is written as x′=Fx+ux' = Fx + ux′=Fx+u and x′=f(x,u)x' = f(x,u)x′=f(x,u). Previously the equation was written x′=Fx+v.x' = Fx + \nu. x′=Fx+v.

It is just a question of notation where v\nuv is the greek letter "nu" and "u" is used in the code examples. Remember that v\nuv is represented by a gaussian distribution with mean zero. The equation x′=Fx+ux' = Fx + ux′=Fx+u or the equivalent equation x′=Fx+vx' = Fx + \nux′=Fx+v calculates the mean value of the state variable xxx; hence we set u = 0. The uncertainty in the gaussian distribution shows up in the QQQ matrix.

More Details About Calculations with Radar Versus Lidar

In the radar update step, the Jacobian matrix HjH_jHj is used to calculate SSS, KKK and PP. To calculate yyy, we use the equations that map the predicted location x′x'x′ from Cartesian coordinates to polar coordinates:

$$h(x')= \begin{pmatrix} \sqrt{ p'^2_{x} + p'^2_{y} } \\ \arctan(p_y' / p_x') \\ \frac{p_x' v_x' + p_y' v_y'}{\sqrt{p'^2_{x} + p'^2_{y}}} \end{pmatrix}$$

The predicted measurement vector x′x'x′ is a vector containing values in the form [px,py,vx,vy] \begin{bmatrix} p_x, p_y, v_x, v_y \end{bmatrix} [px,py,vx,vy]. The radar sensor will output values in polar coordinates:

$$\begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix}$$

In order to calculate yyy for the radar sensor, we need to convert x′x'x′ to polar coordinates. In other words, the function h(x)h(x)h(x) maps values from Cartesian coordinates to polar coordinates. So the equation for radar becomes y=zradar−h(x′)y = z_{radar} - h(x')y=zradar−h(x′).

One other important point when calculating yyy with radar sensor data: the second value in the polar coordinate vector is the angle ϕ\phiϕ. You'll need to make sure to normalize ϕ\phiϕ in the yyy vector so that its angle is between −pi-pi−pi and pipipi; in other words, add or subtract 2pi2pi2pi from ϕ\phiϕ until it is between −pi-pi−pi and pipipi.

To summarize:

- for measurement updates with lidar, we can use the HHH matrix for calculating yyy, SSS, KKK and PP.
- for radar, HjH_jHj is used to calculate SSS, KKK and PP.

Quiz Question

Compared to Kalman Filters, how would the Extended Kalman Filter result differ when the prediction function and measurement function are both linear?

- The Extended Kalman Filter's result would be the same as the standard Kalman Filter's result.
- The Extended Kalman Filter's result would vary unpredictably compared to the Kalman Filter's result.
- The Extended Kalman Filter's result would be less accurate than the Kalman Filter's result.
- The Extended Kalman Filter's result would be more accurate than the Kalman Filter's result.