Ast 713

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Chpt 5 #5,6

5 a) First, we have Eqn' 5.58 for the thermal energy of the gas.

$$\epsilon = \frac{1}{\gamma-1}\frac{kT}{\bar{m}}$$

We need the temperature, given by Eqn' 5.65, of the shocked gas.

$$T = \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{\bar{m}}{k}V_0^2$$

Combining these, we have: $\epsilon = \frac{1}{\cancel{\gamma-1}}\frac{\cancel{k}}{\cancel{\bar{m}}}\cdot\frac{2\cancel{(\gamma-1)}}{(\gamma+1)^2}\frac{\cancel{\bar{m}}}{\cancel{k}}V_0^2 \Rightarrow \epsilon = \frac{2}{(\gamma+1)^2}V_0^2$

Now, $\frac{\epsilon}{E_{K,0}} = \frac{2\cancel{V_0^2}}{(\gamma+1)^2}\cdot\frac{2}{\cancel{V_0^2}} \Rightarrow \boxed{\frac{\epsilon}{E_{K,0}} = \frac{4}{(\gamma+1)^2} = \frac{9}{16}}$

b) The kinetic energy of the shocked gas is, $E_{K,1} = \frac{V_1^2}{2}$

From Eqn' 5.54, $\rho_0 V_0 = \rho_1 V_1 \Rightarrow \frac{\rho_1}{\rho_0} = \frac{V_0}{V_1}$

And, in a strong shock, we have: $\frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1} \Rightarrow V_1 = V_0\frac{\gamma-1}{\gamma+1}$

So, $\frac{E_{K,1}}{E_{K,0}} = \frac{\cancel{\frac{1}{2}}V_1^2}{\cancel{\frac{1}{2}}V_0^2} = \frac{\cancel{V_0^2}\left(\frac{\gamma-1}{\gamma+1}\right)^2}{\cancel{V_0^2}} \Rightarrow \boxed{\frac{E_{K,1}}{E_{K,0}} = \frac{(\gamma-1)^2}{(\gamma+1)^2} = \frac{1}{16}}$

c) In the observers frame, $E_{K,1}' = \frac{V^2}{2}$ where $V = V_0 - V_1$

From part (b), $V_1 = V_0\frac{\gamma-1}{\gamma+1}$ So, $V = V_0 - V_0\frac{\gamma-1}{\gamma+1}$

Now, $V = V_0\left(1 - \frac{\gamma-1}{\gamma+1}\right) = V_0\left(\frac{\cancel{\gamma}+1-\cancel{\gamma}+1}{\gamma+1}\right) \Rightarrow V = V_0\frac{2}{\gamma+1}$

Finally, $\frac{E_{K,1}'}{E_{K,0}} = \frac{\cancel{2}}{\cancel{2}}\frac{V_0^2\frac{4}{(\gamma+1)^2}}{V_0^2} \Rightarrow \boxed{\frac{E_{K,1}'}{E_{K,0}} = \frac{4}{(\gamma+1)^2} = \frac{9}{16}}$