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# **A Critical Analysis of Design Flaws in the Death Star**

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Report submitted in partial fulfilment of the requirements of the module  
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Electrical and Electronic Engineering at Stellenbosch University.

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# Abstract

## **English**

The English abstract.

## **Afrikaans**

Die Afrikaanse uittreksel.

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# Nomenclature

## Variables and functions

$p(x)$	Probability density function with respect to variable $x$ .
$P(A)$	Probability of event $A$ occurring.
$\varepsilon$	The Bayes error.
$\varepsilon_u$	The Bhattacharyya bound.
$B$	The Bhattacharyya distance.
$s$	An HMM state. A subscript is used to refer to a particular state, e.g. $s_i$ refers to the $i^{\text{th}}$ state of an HMM.
$\mathbf{S}$	A set of HMM states.
$\mathbf{F}$	A set of frames.
$\mathbf{o}_f$	Observation (feature) vector associated with frame $f$ .
$\gamma_s(\mathbf{o}_f)$	A posteriori probability of the observation vector $\mathbf{o}_f$ being generated by HMM state $s$ .
$\mu$	Statistical mean vector.
$\Sigma$	Statistical covariance matrix.
$L(\mathbf{S})$	Log likelihood of the set of HMM states $\mathbf{S}$ generating the training set observation vectors assigned to the states in that set.
$\mathcal{N}(\mathbf{x} \mu, \Sigma)$	Multivariate Gaussian PDF with mean $\mu$ and covariance matrix $\Sigma$ .
$a_{ij}$	The probability of a transition from HMM state $s_i$ to state $s_j$ .
$N$	Total number of frames or number of tokens, depending on the context.
$D$	Number of deletion errors.
$I$	Number of insertion errors.
$S$	Number of substitution errors.

**Acronyms and abbreviations**

AE	Afrikaans English
AID	accent identification
ASR	automatic speech recognition
AST	African Speech Technology
CE	Cape Flats English
DCD	dialect-context-dependent
DNN	deep neural network
G2P	grapheme-to-phoneme
GMM	Gaussian mixture model
HMM	hidden Markov model
HTK	Hidden Markov Model Toolkit
IE	Indian South African English
IPA	International Phonetic Alphabet
LM	language model
LMS	language model scaling factor
MFCC	Mel-frequency cepstral coefficient
MLLR	maximum likelihood linear regression
OOV	out-of-vocabulary
PD	pronunciation dictionary
PDF	probability density function
SAE	South African English
SAMPA	Speech Assessment Methods Phonetic Alphabet

# Chapter 1

## Introduction

The last few years have seen great advances in speech recognition. Much of this progress is due to the resurgence of neural networks; most speech systems now rely on deep neural networks (DNNs) with millions of parameters [?, 1]. However, as the complexity of these models has grown, so has their reliance on labelled training data. Currently, system development requires large corpora of transcribed speech audio data, texts for language modelling, and pronunciation dictionaries. Despite speech applications becoming available in more languages, it is hard to imagine that resource collection at the required scale would be possible for all 7000 languages spoken in the world today.

I really like apples.

### 1.1. Section heading

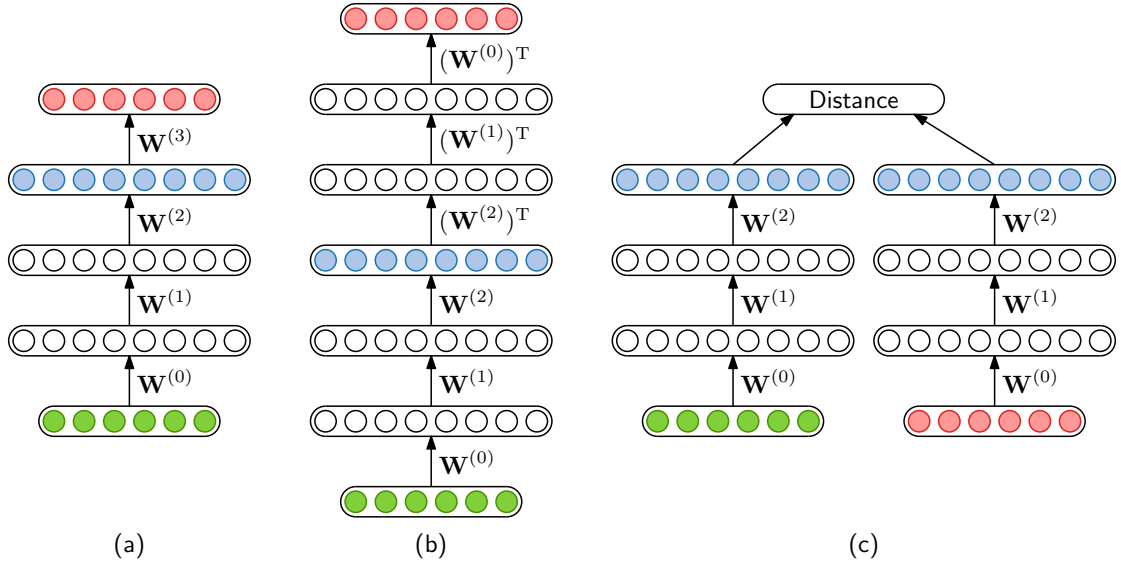
This is some section with two table in it: Table 1.1 and Table 1.2.

**Table 1.1:** Performance of the unconstrained segmental Bayesian model on TIDigits1 over iterations in which the reference set is refined.

Metric	1	2	3	4	5
WER (%)	35.4	23.5	21.5	21.2	22.9
Average cluster purity (%)	86.5	89.7	89.2	88.5	86.6
Word boundary $F$ -score (%)	70.6	72.2	71.8	70.9	69.4
Clusters covering 90% of data	20	13	13	13	13

**Table 1.2:** A table with an example of using multiple columns.

Model	Accuracy (%)		
	Intermediate	Output	Bitrate
Baseline	27.5	26.4	116
VQ-VAE	26.0	22.1	190
CatVAE	28.7	24.3	215



**Figure 1.1:** (a) The cAE as used in this chapter. The encoding layer (blue) is chosen based on performance on a development set. (b) The cAE with symmetrical tied weights. The encoding from the middle layer (blue) is always used. (c) The siamese DNN. The cosine distance between aligned frames (green and red) is either minimized or maximized depending on whether the frames belong to the same (discovered) word or not. A cAE can be seen as a type of DNN [?].

This is a new page, showing what the page headings looks like, and showing how to refer to a figure like Figure 1.1.

The following is an example of an equation:

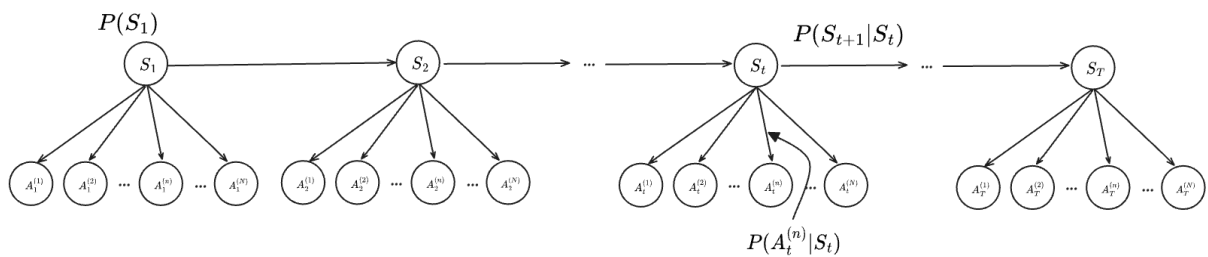
$$P(\mathbf{z}|\boldsymbol{\alpha}) = \int_{\boldsymbol{\pi}} P(\mathbf{z}|\boldsymbol{\pi}) p(\boldsymbol{\pi}|\boldsymbol{\alpha}) d\boldsymbol{\pi} = \int_{\boldsymbol{\pi}} \prod_{k=1}^K \pi_k^{N_k} \frac{1}{B(\boldsymbol{\alpha})} \prod_{k=1}^K \pi_k^{\alpha_k-1} d\boldsymbol{\pi} \quad (1.1)$$

which you can subsequently refer to as (1.1) or Equation 1.1. But make sure to consistently use the one or the other (and not mix the two ways of referring to equations).

# Chapter 2

## Body

### 2.1. Model Definition



**Figure 2.1:** I am a caption below the figure of course

This is a reference to Figure 2.1

RVs:

- Hidden Drought State RVs  $\equiv S_t = \{1, 2, \dots, m\}$
- Attribute RVs  $\equiv A_t^{(n)} = \{1, 2, \dots, C_n\}$

Some further notation:

- $\mathbf{S}_{1:T} = \{S_1, S_2, \dots, S_T\}$
- $\mathbf{A}_{1:T} = \{\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_T\}$ 
  - Where  $\mathbf{A}_t = \{A_t^{(1)}, A_t^{(2)}, \dots, A_t^{(N)}\}$

#### 2.1.1. Joint Distribution

$$\begin{aligned}
 & p(S_1, S_2, \dots, S_T, A_1^{(1)}, A_1^{(2)}, \dots, A_1^{(N)}, A_2^{(1)}, \dots, A_T^{(N)}) \\
 &= p(S_1, S_2, \dots, S_T, \mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_T) \\
 &= p(\mathbf{S}_{1:T}, \mathbf{A}_{1:T}) \\
 &= p(S_1) \cdot \prod_{t=1}^{T-1} p(S_{t+1} | S_t) \cdot \prod_{n=1}^N \prod_{t=1}^T p(A_t^{(n)} | S_t)
 \end{aligned}$$

## 2.2. Factors

Priors

$S_1$	$p(S_1)$
1	$\pi_1$
2	$\pi_2$
$\vdots$	$\vdots$
$m$	$\pi_m$

Transition

$S_t$	$S_{t+1}$	$p(S_{t+1}   S_t)$	
1	1	$a_{1,1}$	$\equiv P^1 = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,m} \\ a_{2,1} & a_{2,2} & \dots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \dots & a_{m,m} \end{bmatrix}$
1	2	$a_{1,2}$	
$\vdots$	$\vdots$	$\vdots$	
1	$m$	$a_{1,m}$	
2	1	$a_{2,1}$	
2	2	$a_{2,2}$	
$\vdots$	$\vdots$	$\vdots$	
$m$	$m$	$a_{m,m}$	

Emission

$A_t^{(n)}$	$S_t$	$p(A_t^{(n)}   S_t)$
1	1	$b_1^{(n)}(1)$
1	2	$b_2^{(n)}(1)$
$\vdots$	$\vdots$	$\vdots$
1	$m$	$b_m^{(n)}(1)$
2	1	$b_1^{(n)}(2)$
2	2	$b_2^{(n)}(2)$
$\vdots$	$\vdots$	$\vdots$
$C_n$	$m$	$b_m^{(n)}(C_n)$

## 2.3. EM Theory

- $\mathcal{H} = (S_t)_{t=1}^T$
- $\mathcal{D} = (\mathbf{A}_t)_{t=1}^T$
- $\Theta = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3)$
- $\boldsymbol{\theta}_1 = \{\pi_1, \pi_2, \dots, \pi_m\} \equiv S_1 \text{ Priors}$

- $\theta_2 = \{a_{1,1}, a_{1,2}, \dots, a_{m,m}\} = P^1 \equiv \text{Transition Probabilities}$
- $\theta_3 = \{b_1^{(n)}(1), b_2^{(n)}(1), \dots, b_1^{(N)}(m)\} = P^1 \equiv \text{Emission Probabilities}$

### 2.3.1. E-Step

Hold  $\Theta$  fixed and choose  $q$  such that

$$\begin{aligned} q(\mathcal{H}) &= p(\mathcal{H} \mid \mathcal{D}, \Theta) \\ &= p(\mathbf{S}_{1:T} \mid A_{1:T}, \Theta) \end{aligned}$$

### 2.3.2. M-Step

Hold  $q$  fixed and optimise  $\mathcal{L}(q, \Theta)$  w.r.t  $\Theta$

After some math, this means finding  $\Theta$  such that:

$$\begin{aligned} \Theta &= \underset{\Theta}{\operatorname{argmax}} Q(\Theta) \\ &= \underset{\Theta}{\operatorname{argmax}} \sum_{\mathcal{H}} q(\mathcal{H}) \cdot \log p(\mathcal{D}, \mathcal{H} \mid \Theta) \end{aligned}$$

## 2.4. Update Equations

Priors:

$$\pi_i^{\text{new}} = q(S_1 = i) \quad (2.1)$$

Transition Probabilities:

$$a_{ij}^{\text{new}} = \frac{\sum_{t=1}^{T-1} q(S_t = i, S_{t+1} = j)}{\sum_{t=1}^{T-1} q(S_t = i)} \quad (2.2)$$

Emission Probabilities:

$$b_i^{(n)}(j)^{\text{new}} = \frac{\sum_{t=1}^T q(S_t = i) \cdot I(A_t^{(n)} = j)}{\sum_{t=1}^T q(S_t = i)} \quad (2.3)$$

We can now reference these equations by their label: Equation 2.1, Equation 2.2 or Equation 2.3. This is wicked, lemme tell you



## 2.5. FIGURES TIME

things before figure

## 2.6. Determining $m$

We are using AIC, BIC and maximum log likelihood to do this. Here are the formulas:

$$AIC = -2 \cdot \log L(\Theta) + 2p$$

$$BIC = -2 \cdot \log L(\Theta) + p \cdot \log k$$

Where:

- $L(\Theta) \equiv$  the maximized value of the likelihood function for the estimated model
- $p \equiv$  the number of free parameters,
- $k \equiv$  the number of data points.

The idea is that we are going to sweep  $m$ , this means creating many models with different values of  $m$ , and choose the model that minimizes both AIC & BIC, whilst maximising  $L(\Theta)$ . Here is a more comprehensive criteria for choosing a particular  $m$ :

1. Primary: lowest BIC (preferred if you want parsimony, BIC penalizes complexity strongly).
2. Secondary: lowest AIC.
3. Also look at the log-likelihood curve: if  $\log L(\Theta)$  improves only marginally as  $m$  increases, choose the simpler model (elbow rule).

But okay, lets see what the particular values are for our BIC & AIC calculations

### 2.6.1. What is $k$

We will first look at  $k \equiv$  the number of data points: Not sure, can either see each observation as a vector, therefore we have  $T$ , or we can see each  $a_t^{(n)}$  as an observation, and in that case we would have  $T \times N$  observations.

### 2.6.2. What is $p$

- Next we look at  $p \equiv$  the number of free parameters. Look more into this. See Occam's Razor (A principled method to model selection and how BIC is an approximation of this), in the, use  $m - 1$ , so we will have:

$$\begin{aligned}
p &= (m-1) + m(m-1) + \sum_{n=1}^N m(C_n - 1) \\
&= m^2 - 1 + m \sum_{n=1}^N (C_n - 1)
\end{aligned}$$

### 2.6.3. How To Get $\ell(\Theta)$

This is the real kicker...

Of course the likelihood is the probability our data is observed, this means finding:

$$\begin{aligned}
\ell(\Theta) &= p(A_1^{(1)} = a_1^{(1)}, A_1^{(2)} = a_1^{(2)}, \dots, A_T^{(N)} = a_T^{(N)} \mid \Theta) \\
&= p(A_{1:T}^{\text{obs}} \mid \Theta)
\end{aligned}$$

Note: Observed Data =  $(a_1^{(1)}, a_1^{(2)}, \dots, a_T^{(N)})$

To do this, we will use the Forward Algorithm (also note that have this (obs) superset makes things very verbose and will be omitted from here on out):

We begin by defining the factor  $p(\mathbf{A}_t \mid S_t = i)$

- Recall,  $\mathbf{A}_t = \{A_t^{(1)}, A_t^{(2)}, \dots, A_t^{(N)}\}$
- Because of our model and the independent properties and things, we know that:

$$\begin{aligned}
p(\mathbf{A}_t \mid S_t = i) &= p(A_t^{(1)}, A_t^{(2)}, \dots, A_t^{(N)} \mid S_t = i) \\
&= p(A_t^{(1)} \mid S_t = i) \cdot p(A_t^{(2)} \mid S_t = i) \dots p(A_t^{(N)} \mid S_t = i) \\
&= \prod_{n=1}^N p(A_t^{(n)} \mid S_t = i) \\
&= \prod_{n=1}^N e_i^{(n)}(a_t^{(n)})
\end{aligned}$$

Lets now look at the Forward Algorithm

- We first define

$$\begin{aligned}
\alpha_t(i) &= p(S_t = i, A_{1:t}^{\text{obs}} \mid \Theta) \\
&= p(S_t = i, \mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_t \mid \Theta)
\end{aligned}$$

- We begin at  $t = 1$ :

$$\begin{aligned}
\alpha_1(i) &= p(S_1 = i, A_{1:1}^{\text{obs}} \mid \Theta) \\
&= p(S_1 = i, \mathbf{A}_1 \mid \Theta) \\
&= p(S_1) \cdot p(\mathbf{A}_1 \mid S_1 = i, \Theta)
\end{aligned}$$

- Then to move to the next time step:

$$\alpha_{t+1}(j) = \left( \sum_{i=1}^m \alpha_t(i) \cdot p_{i,j} \right) \cdot p(\mathbf{A}_{t+1} \mid S_{t+1} = j, \Theta)$$

- Thus, by the end we will have  $\alpha_T(i) = p(A_{1:T}, S_T = i \mid \Theta)$
- Then finally we can obtain our likelihood by marginalising out  $S_T$

$$\begin{aligned} \sum_{i=1}^m \alpha_T(i) &= \sum_{S_t} p(A_{1:t}, S_t \mid \Theta) \\ &= p(A_{1:t} \mid \Theta) \\ &= \ell(\Theta) \end{aligned}$$

Okay, lets regroup. Just so we fully get it. This is done after our model has been fitted and we already have our final  $\Theta$ . There is also no factors going on here. Our parameters ( $\Theta$ ) are the probabilities in themselves. So when we observed our data  $p(A_t^{(n)} = a_t^{(n)} \mid S_t = i)$  we are in essence choosing the value  $b_i^{(n)}(a_t^{(n)})$  (Note, these  $b$ 's will have to change to  $e$  or something...). And thus we have actual values we are playing with, thus pure math, not factors.

Reminder: These are factors we have available:

- $q(\mathcal{H}) = p(\mathbf{S}_{1:T} \mid A_{1:T}^{\text{obs}}, \Theta)$
- $p(S_1)$
- $p(S_{t+1} \mid S_t)$
- $p(A_t^{(n)} \mid S_t)$

We want  $\log \ell(\Theta) = \log p(A_{1:T}^{\text{obs}} \mid \Theta)$

Math to get there:

1. Start With Each  $p(A_t^{(n)} \mid S_t)$
2. For Each  $A_t^{(n)}$ 
  - (a) Observe data point  $a_t^{(n)}$  to get:  $p(A_t^{(n)} = a_t^{(n)} \mid S_t)$
  - (b) Get  $p(A_t^{(n)} = a_t^{(n)})$  understanding that:

$$p(A_t^{(n)} = a_t^{(n)}) = \sum_{S_t} p(A_t^{(n)} = a_t^{(n)} \mid S_t) \cdot p(S_t)$$

This  $p(S_t)$  is our  $q(\mathcal{H})$

3. Then of course, we multiply these to get

$$p(A_{1:T}^{\text{obs}}) = \prod_{n=1}^N \prod_{t=1}^T p(A_t^{(n)} = a_t^{(n)})$$

4. These will likely underflow so we insert log now:

$$\log \ell(\Theta) = \log p(A_{1:T}^{\text{obs}}) = \sum_{n=1}^N \sum_{t=1}^T \log p(A_t^{(n)} = a_t^{(n)})$$

Thoughts?

If I can still use the forward equations, let me know. Otherwise I need to calculate the full joint distr and marginalise out?

A little bit embarrassing but how exactly do we get the log likelihood, the naive way. My understanding is that we:

1. Calculate the full joint distribution:

$$p(\mathbf{S}_{1:T}, A_{1:T}) = p(S_1) \cdot \prod_{t=1}^{T-1} p(S_{t+1} | S_t) \cdot \prod_{n=1}^N \prod_{t=1}^T p(A_t^{(n)} | S_t)$$

2. Marginalise out all  $S_t$ :

$$p(A_{1:T} | \Theta) = \sum_{S_{1:T}} p(\mathbf{S}_{1:T}, A_{1:T} | \Theta)$$

3. Then Observe Actual Data and sum the probs?

$$\text{Likelihood} = \sum p(A_{1:T} = \mathcal{D})??$$

I don't know what you mean by me being stuck. I get parameters which are the probabilities to the factors I am looking for. The model output is the max value  $S_t$  which i get from my  $q$  function. Let me know if I am overlooking something.

With regards to the AIC & BIC calcs. you have here  $\ell(\Theta) = \sum_{S_{1:T}} p(\mathbf{S}_{1:T}, A_{1:T})$  but this leaves us with a factor not a single value? This is required for AIC and BIC which are single values? This is why I thought you must sum over observations?

Okay Ill stop faffing. Forward-Backward is new, I didnt want to waste a time sink learning it. Especially because this LBU + EM is much more flexible of a route which is good for me since I plan to expand this model. One idea I have is to introduce second-order markov property to the thing. Is Forward-Backward still feasible for a DNBC with the second order markov property? If not I am sticking to LBU. And thus maybe need an alternative to AIC and BIC. But let me know your thoughts.

## 2.7. Questions

1. Forward-Backward Equations. I have to right? From what I can see it applies to second order as well when we vectorise our states?
  - Its only really a problem to try and get  $\log \ell(\Theta)$  for AIC and BIC. Besides this it works fine?
  - is extracting  $S_t$  from  $q(\mathcal{H})$  fine and correct? Since we want  $p(S_t)$  not  $p(S_t \mid A_{1:T})\dots$
2. Based on this as about LBU things:
  - (a) emdw has this `#include "lbu2_cg.hpp"`. What is this??
  - (b) Ask about LTRIP vs other methods  $\rightarrow$  Other Methods: BETHE, JTREE
3. Breaking symmetry for the priors  $p(S_1)$ . Is it necessary?
4. With regards to BIC & AIC, we need  $k \equiv$  Number of Free Parameters (See calcs on `model-dev-clean` pg 11)
5. Discrete vs Cts Attribute RVs. See `model-dev-clean` pg 10
6. `main.cpp` line 951

## 2.8. Forward-Backward

- Initial:  $\pi_i = P(S_1 = i)$ .
- Transition:  $a_{ij} = P(S_{t+1} = j \mid S_t = i)$ .
- Emission Probabilities:  $b_i^{(n)}(j) = p(A_t^{(n)} = j \mid S_t = i)$ .

### 2.8.1. Define Forward Values

$$\alpha_t(i) = p(S_t = i, A_{1:t}^{\text{obs}} \mid \Theta)$$

### 2.8.2. Define Backward Values

$$\beta_t(i) = p(A_{t+1:T}^{\text{obs}} \mid S_t = i, \Theta)$$

### 2.8.3. Updates

First Define:

$$\gamma_t(i) = p(S_t = i \mid A_{1:T}^{\text{obs}}, \Theta) = \frac{p(S_t = i, A_{1:T}^{\text{obs}} \mid \Theta)}{p(A_{1:T}^{\text{obs}} \mid \theta)} = \frac{\alpha_t(i)\beta_t(i)}{\sum_{k=1}^m \alpha_t(k)\beta_t(k)}$$

Node marginals:

$$\gamma_t(i) = P(S_t = i \mid A_{1:T}, \Theta) = \frac{\alpha_t(i)\beta_t(i)}{\sum_{k=1}^m \alpha_t(k)\beta_t(k)}.$$

Pairwise marginals:

$$\xi_t(i, j) = P(S_t = i, S_{t+1} = j \mid A_{1:T}, \Theta) = \frac{\alpha_t(i)a_{ij}P(A_{t+1} \mid S_{t+1} = j)\beta_{t+1}(j)}{\sum_{p=1}^m \sum_{q=1}^m \alpha_t(p)a_{pq}P(A_{t+1} \mid S_{t+1} = q)\beta_{t+1}(q)}.$$

1. Forward  $\alpha_t(i) = P(A_{1:t}, S_t = i \mid \theta)$  and backward  $\beta_t(i) = P(A_{t+1:T} \mid S_t = i, \theta)$ . Use scaling or log-space. 2. Responsibilities:

$$\gamma_t(i) \equiv P(S_t = i \mid A_{1:T}, \theta) = \frac{\alpha_t(i) \beta_t(i)}{\sum_{\ell} \alpha_t(\ell) \beta_t(\ell)}.$$

3. Pairwise responsibilities:

$$\xi_t(i, j) \equiv P(S_t = i, S_{t+1} = j \mid A_{1:T}, \theta) = \frac{\alpha_t(i) a_{ij} P(A_{t+1} \mid S_{t+1} = j) \beta_{t+1}(j)}{\sum_{p,q} \alpha_t(p) a_{pq} P(A_{t+1} \mid S_{t+1} = q) \beta_{t+1}(q)}.$$

For multiple independent sequences  $n = 1, \dots, N$ , compute  $\gamma_t^{(n)}$  and  $\xi_t^{(n)}$  per sequence and sum where needed.

“Baum’s auxiliary function” = EM  $Q$ -function

$$Q(\theta \mid \theta^{\text{old}}) = \mathbb{E}_{S_{1:T} \mid A_{1:T}, \theta^{\text{old}}} [\log P(S_{1:T}, A_{1:T} \mid \theta)].$$

Expanding the complete-data log-likelihood under the DNBC factorization gives three decoupled blocks (initial, transition, emission). Maximizing  $Q$  with simplex constraints  $\sum_i \pi_i = 1$ ,  $\sum_j a_{ij} = 1$ ,  $\sum_k b_{i,m}(k) = 1$  via Lagrange multipliers yields normalized expected counts. That’s “Baum–Welch”. Same thing you should get if you derive EM straight.

M-step (discrete case, no priors)

Single sequence (replace sums with  $\sum_n$  for multiple sequences):

**\*\*Initial:\*\***

$$\pi_i^{\text{new}} = \gamma_1(i).$$

**\*\*Transition:\*\***

$$a_{ij}^{\text{new}} = \frac{\sum_{t=1}^{T-1} \xi_t(i, j)}{\sum_{t=1}^{T-1} \gamma_t(i)}.$$

**\*\*Emission for each attribute  $m$  and value  $k$ :**

$$b_{i,m}^{\text{new}}(k) = \frac{\sum_{t=1}^T \gamma_t(i) \mathbf{1}\{A_t^{(m)} = k\}}{\sum_{t=1}^T \gamma_t(i)}.$$

## **Chapter 3**

### **Summary and Conclusion**



# Bibliography

- [1] G. Hinton, L. Deng, D. Yu, G. E. Dahl, A.-R. Mohamed, N. Jaitly, A. Senior, V. Vanhoucke, P. Nguyen, T. N. Sainath, and B. Kingsbury, “Deep neural networks for acoustic modeling in speech recognition: The shared views of four research groups,” *IEEE Signal Process. Mag.*, vol. 29, no. 6, pp. 82–97, 2012.

# **Appendix A**

## **Project Planning Schedule**

This is an appendix.

# **Appendix B**

## **Outcomes Compliance**

This is another appendix.