Shear Centre, Flexural Centre and Flexural Axis

An Attempt to Clear up Current Confusion and Provide Definitions Differentiating Between the Three Terms

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Introduction

AERONAUTICAL engineers are afflicted by various 'centres' which will not stay put. Weights engineers spend much of their time chasing the elusive centre of gravity up and down the fuselage, the aerodynamicist worries about the centre of pressure, and the structural engineer, in addition to these, is cursed with the flexural centre and the shear centre. The main trouble connected with the flexural and shear centre seems to be historical in origin. No real attempt appears to have been made to keep pace with the development of wing structures from the old simple two-spar, fabric-covered, constant-section wing. It is still quite common to see the flexural centre of a wing defined as the point at which a load must be applied, so as to produce bending of the wing without twist. In general, something more precise is required, and, particularly in these days of tapered, swept-back, stressed-skin wings, it would seem desirable to review the old definitions and usages of terms such as flexural centre, and flexural axis in order to avoid confusion and possible misstatements or misapplications. The advent of stressed-skin construction has brought into fairly general use the term shear centre, which is often confused with the flexural centre, although the two points do not necessarily coincide. The use of sweep-back has caused further confusion concerning the definition of twist. It is hoped that this brief survey may indicate the nature of the confusion that is known to exist—and, perhaps, lead to an authoritative decision, which may standardize terms and definitions.

Simple Concept of Flexural Centre

In the early days of aircraft construction, the wing structure consisted in most cases of two or more spars and a number of ribs, with diagonal bracing in the chordal plane. The spars transmitted the entire shear, normal to the plane of the wing, in bending and took care of the torque by differential bending, there being no skin to form a torsion tube. Furthermore, quite often the wings were of rectangular plan-form and of constant section, each spar having a constant moment of inertia throughout its length. It was probably in connexion with this type of wing that flexural centre and flexural axis were first defined, and, unfortunately, the same definitions are often used today, although they have become meaningless, without further qualification, when applied to present-day structures.

Consider the simple type of structure shown in FIG. 1, with two spars, each of constant section, assumed cantilevered at the root and connected by a rib ABCD at the tip, the attachment of the rib to the spars being such that the rib may be considered as simply supported at the spars.

A load, P, applied as shown, would then be taken by the two spars in the inverse ratio of the distances a and b.

i.e. Load on front spar$=Pb/(a+b)$

Load on rear spar$=Pa/(a+b)$

The deflexions of the two spars, δ_F and δ_R would be calculated as

$$\delta_F=\frac{Pb}{(a+b)}\cdot\frac{L^3}{3EI_F} \text{ and } \delta_R=\frac{Pa}{(a+b)}\cdot\frac{L^3}{3EI_R}$$

If there is to be no twisting of section ABCD, the two spars must deflect equal amounts so that $\delta_F=\delta_R$, from which it follows that

$$a/b=I_R/I_F \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

i.e. for equal deflexions of the two spars, and consequently no rotation of the section ABCD, the point of application of the load must be the centroid of the moments of inertia of the spars.

This is the point originally defined as the flexural centre of the wing at the section ABCD.

If the spars are not of constant section, Equation (1) defines a point, 0, such that a load applied there will produce zero twist per unit length (or zero rate of change of twist) at the section ABCD, but it will not necessarily follow that there will be no twist of this section relative to the root. Similarly the definition of a point such that a load applied there will produce no twist of section ABCD relative to the root does not imply that there will be no twist at any intermediate section. Consequently, to refer to 'bending without twist' is meaningless for a non-uniform wing unless we specify that twist means either twist per unit length at the section considered or twist of the section considered relative to the root. These two concepts will, in general, define two distinct points of application of load.

When we add a stress-carrying skin, the problem becomes more complex, but it is still possible to define a point in any section such that when a load is applied there either (1) **there is no twist of that section relative to the root**, or (2) **there is no twist per unit length at that section.** It is suggested that it is the first of these two definitions which properly relates to what is today called the **flexural centre**. Confirmation of this is obtained from the standard experimental method of locating the flexural centre. It must be emphasized that the centroid of the moments of inertia of the spars (or effective flexural material) has no specific connexion with the flexural centre as now defined, except for the type of structure in FIG. 1. It is perhaps unfortunate that the term 'flexural centre' has been retained in its present sense, in view of its early identification with the point defined by Equation (1). The second definition above defines a point now known as the **shear centre**, which we proceed to discuss briefly in the next paragraph.

Shear Centre

This is a term which has come into common use as a consequence of the adoption of thin-walled torsion members, and it is often confused with flexural centre. The shear centre of any section is the point about which the resultant moment of the shear flows in the walls of the section due to bending alone (i.e. without torque) is zero. Consequently, a load applied at the shear centre implies that no torque is applied at the section. For a constant section tube, therefore, a load applied at the shear centre will produce no twist relative to the root, and the shear centre and flexural centre coincide. For a tapered tube, the shear centre at any section defines a point

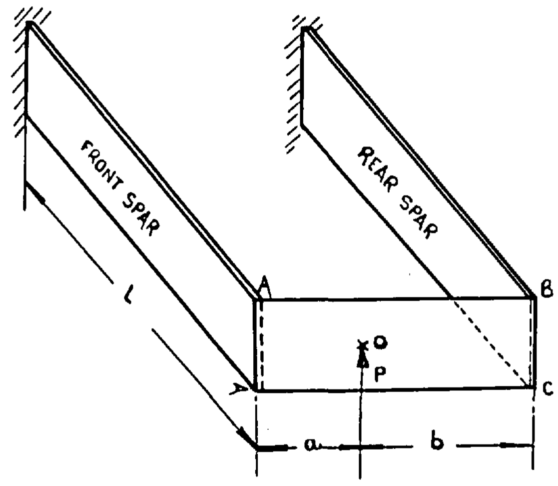

Fig. 1

such that a load applied there produces zero twist per unit length at that section. This point is appropriately defined as it is, indeed, a 'centre of shear'. It may be noted that for a two-spar wing without stressed-skin covering the shear centre coincides with the point defined by Equation (1).

The shear centre is a characteristic of the section alone, not of the section considered as part of the complete wing, and so influenced by the remainder of the wing.

Flexural Axis

In a structure such as FIG. 1 the flexural centres at all sections would lie on a straight line and it could be said that a load applied at all points on that line would produce 'bending without twist'. Further, this line would be perpendicular to the plane of symmetry and it could be said also that the application of a torque would produce twisting about this axis. With present structures no such straight line will necessarily exist, but attempts have been made to define a 'flexural axis' in similar terms. The old definition gave the flexural axis as the locus of the flexural centres along the spar. Retention of this definition leads in general to a locus which is curved. It is true to say that a load applied at any point on this locus will produce zero twist of the section on which it is applied relative to the root. It is, however, no longer true to say that a torque will produce twist of the section about this axis. Another definition is to say that the flexural axis at any section is the straight line through the flexural centre perpendicular to the plane of symmetry. This will give the *apparent* or *effective* axis of twist of the wing at the section when a torque is applied there. It will not be the true axis of twist for the section alone—only the axis about which the section appears to twist when the cumulative effect of twisting at all sections along the span is taken into account. Since this definition (which is the one generally accepted nowadays) gives a different flexural axis for each section, it is obviously meaningless to speak of the flexural axis of the wing, without specifying the particular station along the wing span under consideration. One more definition sometimes used is the line through the shear centre perpendicular to the plane of symmetry, which would give the axis of twist for the section considered alone (assuming the section parallel to the plane of symmetry). It appears therefore that the concept of the flexural axis as the locus of the flexural centres has no significance with present types of wing structure and that there is a definite need for tidying up the definitions here too.

Current Usage (for Unswept Wings)

In the light of the above paragraphs it seems possible to adopt the following definitions in

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current practice; it should be noted that the problems arising from the use of sweep-back have not yet been mentioned, so that these definitions relate to unswept wings.

(1) FLEXURAL CENTRE
The point on a wing section at which a load must be applied so as to produce **zero twist of that section relative to the root.**

(2) SHEAR CENTRE
The point on a section at which a load must be applied so as to produce **zero rate of change of twist (or zero twist per unit length) at the section.**

(3) FLEXURAL AXIS
A straight line through the flexural centre perpendicular to the plane of symmetry (or root plane).

It will be observed that the flexural centre and flexural axis are **characteristics of the wing at a particular section,** while the shear centre is a **characteristic of a particular section alone.**

FIG. 2 shows hypothetical curves of twist per unit length for loads applied at (*a*) the flexural centre and (*b*) the shear centre at station A, and FIG. 3 shows a plot of total twist along the span, obtained as integrals of the curves in FIG. 2.

In structural analysis, the shear centre is the point which is of importance. The location of the flexural centre, as defined above, may be required for the determination of the wing stiffness and, as such, is primarily of experimental importance.

The Problems Arising from the Use of Sweepback

In the case of swept-back wings, difficulty arises in defining wing twist. Professor G. T. R. Hill pointed out this problem very clearly in the *Journal of the Royal Aeronautical Society*, Volume LII, page 186, 1948, when he expounded the difference between the aerodynamicist's idea of wing twist and that of the structural engineer. Even then, Professor Hill confined his discussion to the simplest type of structure, with parallel, uniform spars and no taper. With the unswept or lightly swept wing, the section, where twist is considered, is a section parallel to the plane of symmetry (or root), but in a heavily sweptback wing this will not necessarily be the case. Professor Hill points out that in the case of his simplified wing the aerodynamicist is interested in the twist of a section such as AB (FIG. 4), whilst the structural engineer will be more concerned with the twist of a section such as AC. Quite obviously, zero twist of one section will not give zero twist of the other. Consequently, in defining the flexural centre of the swept wing a stipulation must be made regarding the actual section to be considered. It is not sufficient to say that the section is at a certain percentage of the span, since this does not automatically define the section.

It may be noted that the A.R.B. Requirements for Civil Aeroplanes specify that the reference section for measurement of wing stiffness is a chordwise section *parallel* to the plane of symmetry—a section such as AB in FIG. 4. It seems reasonable to suggest, therefore, that the flexural centre should be defined with reference to such a section.

In a swept-back wing, with ribs which are not normal to the spars, induced torsion and bending arise which necessitate deeper consideration being given to the definition of shear centre and flexural centre. The addition of taper introduces more complexity and there appears to be a definite need for a comprehensive study of the problems associated with the definitions of the relevant points. In the meantime an earnest appeal is made to authors of technical reports and articles to give always precise indication of the sense in which these terms are used.

Conclusion

In a sphere of engineering in which so much readiness to adopt new ideas has been shown, it is rather surprising to find such reluctance to abandon the old terms and definitions, in spite of the confusion and misapprehension which must ensue. It is suggested that a new approach with a fresh terminology, taking into account the special problems arising from the use of sweep-back, would make things easier, not only for the budding aeronautical engineer, who finds the old definitions in most of the reference books, but also for the old hand, who cannot forget the old definitions he learned so well. A new term for flexural centre which emphasizes the idea of *twist* **relative to the root** might help to make the position clearer, particularly if the term flexural axis, which no longer has any real significance in its original sense as a locus of flexural centres and an axis of twist, were abandoned. Cumbersome, but perhaps more appropriate terms might be 'centre of no relative twist' and 'effective axis of rotation'. No doubt there are many engineers who have no difficulty whatever in distinguishing between the various points considered, but it has been the author's experience that many of these people use the same term in different senses, and that whilst the appropriate meaning is clear to them, it may not be so to others. Until definite standardization is introduced these terms should always be defined clearly whenever they are used. It must be emphasized that two distinct points are to be defined—one, the shear centre, relating to the properties of the section alone, and the other, at present called the flexural centre, relating to the properties of the whole wing, considered at a particular section. The author suggests that the definitions given under the heading 'Current Usage' above, are reasonably indicative of present-day practice—with the addition of the specification of the reference section for the flexural centre of swept-back wings. At the same time, it is felt that a new terminology would be of greater value.

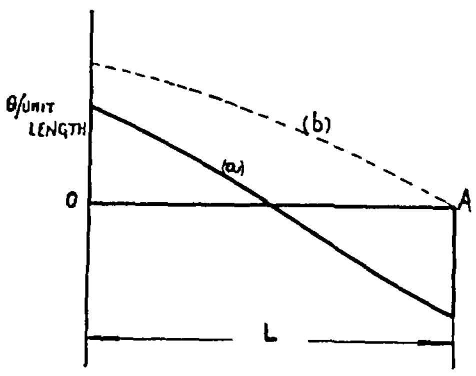

Fig. 2

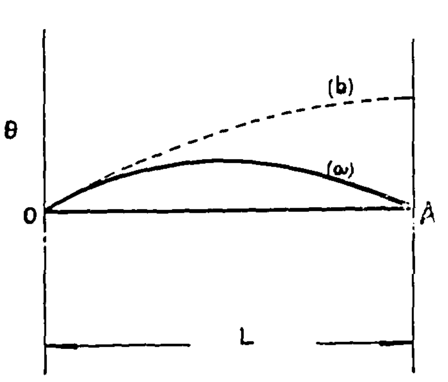

Fig. 3

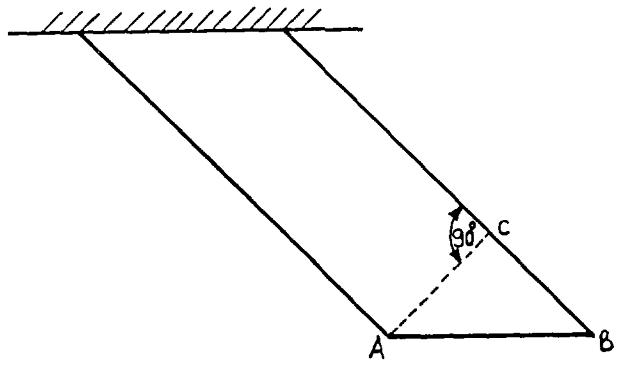

Fig. 4

Professional Publications

Under this heading are given each month the principal articles of aeronautical interest appearing in the current issues of the Journals of the leading Professional Societies and Institutions

Associazione Italiana di Anotecnica (Italy)

L'AEROTECNICA (*Alternate Months*)

Vol. XXXI, No. 2, April 15, 1951

Caratteristiche aerodinaneiche di ali a freccia con bordo d'attaco subsonico e bordo d'uscita supersonico. Part III. A. Eula.

Institute of the Aeronautical Sciences (U.S.A.)

JOURNAL OF THE AERONAUTICAL SCIENCES (*Monthly*)

Vol. 18, No. 5, May 1951

Some Roll Characteristics of Cruciform Delta Wings at Supersonic Speeds. Z. O. Bleviss.
Non-stationary Motion of Purely Supersonic Wings. J. E. Froehlich
Effects of Pressure Gradient on Stability and Skin Friction in Laminar Boundary Layer in Compressivle Fluids. H. Weil
The Performance of Axial-Flow Compressors as Affected by Single-stage Charactertistics. S. M. Boydonoff
Coupled Free Vibrations of a Swept Wing. A. E. Engelbrecht
An Analysis of the Elastic and Plastic Stability of Sandwich Plates by the Method of Split Rigidities—I. P. P. Bijlaard.

AERONAUTICAL ENGINEERING REVIEW (*Monthly*)

Vol. 10, No. 5, May 1951

Electronics for Aircraft. W. A. Shradee.
Electronics in Aviation. H. A. Kressly and J. S. Erickson.
Problems of Standardizing Electronic Equipment. C. R. Banks.
Packaging of Air-borne Electronic Equipment. O. M. Dunning
Air-borne Antenna Problems. H. Schuta
Frequency Allocation for Aviation Electronics. E. C. White
All-Weather Flying. R. C. Robson
Distance-Measuring Equipment for the Terminal Area. J. Lyman and G. B. Litchford
Air Traffic Control. Hon D. W. Rentzel

Society of Automotive Engineers (U.S.A.)

SAE JOURNAL (*Monthly*)

Vol. 59, No. 5, May 1951

Summaries:
Working for Air Defence. W. Voigt
German Aircraft Manufacturing Methods. A. Bringervald
Some Developments for Super Aircraft. A. L. Morse
Air Transport Developments 1949-1950 and a View to the Future. R. D. Spear

British Interplanatory Society

JOURNAL (*Monthly*)

Vol. 10, No. 3, May 1951

Orbital Rockets. K. W. Gattaud, A. E. Dixon and A. M. Kanisch
Combustion in the Rocket Rotor. A. D. Baxter

The Royal Aeronautical Society

THE AERONAUTICAL QUARTERLY

Vol. III, Part I, May 1951

The Symmetric Vibrations of Aircraft. R. W. Traile-Nash
Airscrews at Supersonic Forward Speeds. J. C. Burns
Pohlhausen's Method for Three-Dimensional Laminar Boundary Layers. J. C. Cooke
Supersonic Flow Past Bodies of Revolution with Thin Wings of Small Aspect Ratio. P. M. Stocker

JOURNAL (*Monthly*)

Vol. 55, No. 486, June 1951

Some Aspects of Flight Research. H. Davies
The Principles Underlying the Dynamic Stressing of Aeroplanes. D. Williams
Indication, Measurement and Control of Ice Accretion. A/C D. F. Lucking

The Society of Licensed Aircraft Engineers

THE TECHNICAL INSTRUCTOR (*Monthly*)

Vol. VI, No. 5, May 1951

Air Rescue and Survival. R. Bicknell