Separable Equations

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Abstract

Abstract goes here...

1 Declarations

$\frac{dy}{dx} \triangleq y'$; Differential and prime notation equivalence;
$p(y) = \frac{1}{h(y)}$;P is the reciprocal of H, for neatness' sake;

2 Rule

$$\frac{dy}{dx} = g(x)\,h(y)$$

$$\implies \int p(y)\,dy = \int g(x)\,dx$$

3 Pre-Derivation

This rule only holds true if a differential equation is of first order and holds the following form, that being the product of two functions, one of x and one of y. See below...

$$\frac{dy}{dx} = g(x)\,h(y)$$

4 Derivation

$$\frac{dy}{dx} = g(x)\,h(y)$$
$$\frac{1}{h(y)}\frac{dx}{dy} = g(x)$$
$$\text{Let } p(y) = \frac{1}{h(y)},\ h(y) \neq 0$$
$$\therefore \frac{dy}{dx}p(y) = g(x)$$

$$\text{Assume } y = \phi(x),\ \text{ the solution.}$$
$$\frac{dy}{dx} = \phi'(x)$$

$$p(\phi(x))\phi'(x) = g(x)$$
$$\int p(\phi(x))\phi'(x)dx = \int g(x)dx$$

$$\text{But since } y = \phi(x)$$
$$\implies dy = \phi'(x)dx$$

$$\therefore \int p(y)\,dy = \int g(x)\,dx$$

5 Exempli Gratia

Examples of important instances