Linear Differential Equations

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Abstract

Strategy works with first order, linear differential equations

1 Declarations

$variable$; variable description; $variable\quad domain\quad and\quad range,\quad if\quad applicable$

2 Rule

$$\frac{dy}{dx} + p(x)y = f(x)$$
$$\implies \frac{d}{dx}[\mu(x)y] = \mu(x)f(x)$$
$$\implies y = \frac{\int \mu(x)f(x)}{\mu(x)}$$
$$\text{Where } \mu(x) = e^{\int p(x)dx}$$

3 Pre-Derivation

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

Must first get equation into standard form, with the derivative of y alone...

$$\frac{dy}{dx} + \frac{a_0(x)}{a_1(x)}y = \frac{g(x)}{a_1(x)}$$

$$\text{Let } \frac{a_0(x)}{a_1(x)} = p(x), \frac{g(x)}{a_1(x)} = f(x)$$

$$\therefore \frac{dy}{dx} + p(x)y = f(x)$$

4 Derivation

$$
\begin{aligned}
\frac{dy}{dx} + p(x)y &= f(x) \\
\mu(x)\frac{dy}{dx} + \mu(x)p(x)y &= \mu(x)f(x)
\end{aligned}
$$

Finding the integration coeffecient, $\mu(x)$...

$$
\begin{aligned}
\exists \mu(x) | \frac{d}{dx}[\mu(x)y] &= \mu(x)\frac{dy}{dx} + \mu(x)p(x)y \\
\frac{d}{dx}[\mu(x)y] &= \mu(x)\frac{dy}{dx} + \frac{d}{dx}[\mu(x)]y \\
\implies \frac{d}{dx}[\mu(x)]y &= \mu(x)p(x)y \\
\frac{d}{dx}\mu(x) &= \mu(x)p(x) \\
\frac{1}{\mu(x)}du &= p(x)dx \\
\int \frac{1}{\mu(x)}du &= \int p(x)dx \\
\implies \ln \mu(x) &= \int p(x)dx \\
\mu(x) &= e^{\int p(x)dx}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dx}[\mu(x)y] &= \mu(x)f(x) \\
\int \frac{d}{dx}[\mu(x)y] &= \int \mu(x)f(x) \\
\mu(x)y &= \int \mu(x)f(x) \\
y &= \frac{\int \mu(x)f(x)}{\mu(x)}
\end{aligned}
$$

5 Exempli Gratia

Examples of important instances