

# Journal Template

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## Abstract

Abstract goes here...

## 1 Declarations

*variable*; variable description; *variable domain and range, if applicable*

## 2 Rule

$$\frac{dy}{dx} + p(x)y = f(x)y^n, n \neq 0 \vee 1$$

$$u = y^{1-n}$$

## 3 Pre-Derivation

Function must be of first order and in standard form...

$$\frac{dy}{dx} + p(x)y = f(x)y^n, n \neq 0 \vee 1$$

## 4 Derivation

Derivation goes here

## 5 Exempli Gratia

### 5.1 A basic problem...

Get in standard form and find n

$$\begin{aligned}\frac{dy}{dx} - \frac{3y}{2x} &= \frac{2x}{y} \\ \implies n &= -1\end{aligned}$$

Use rule to determine the substitutive variable and its derivative

We know  $u = y^{1-n}$ , so  $u = y^2$

$$\begin{aligned}\implies y &= u^{\frac{1}{2}} \\ \implies \frac{dy}{dx} &= \frac{1}{2u^{\frac{1}{2}}} \frac{du}{dx}\end{aligned}$$

Substitute

$$\frac{1}{2u^{\frac{1}{2}}} \frac{du}{dx} - \frac{3u^{\frac{1}{2}}}{2x} = \frac{2x}{u^{\frac{1}{2}}}$$

Get to standard form of a 1st order linear equation.

$$\frac{du}{dx} - \frac{3u}{x} = 4x$$

Solve differential equation with respect to  $u$

We know,  $\mu(x) = e^{\int p(x)dx}$

$$\text{Let } p(x) = \frac{-3}{x}$$

$$\implies \mu(x) = e^{\int \frac{-3}{x} dx} = e^{-3 \ln(x)} = x^{-3}$$

$$\text{Multiple by } \mu(x) \frac{du}{dx} x^{-3} - 3ux^{-4} = 4x^{-2}$$

Apply product rule

$$\frac{d}{dx}[x^{-3}u] = 4x^{-2}$$

Integrate

$$\begin{aligned}\int \frac{d}{dx}[x^{-3}u] dx &= \int 4x^{-2} dx \\ x^{-3}u &= \frac{-4}{x} + C \\ u &= -4x^2 + Cx^3\end{aligned}$$

Substitute in for  $u$

$$y^2 = -4x^2 + Cx^3$$