Geometric Series

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Abstract

A geometric series is defined as a sum of terms where each successive term is its previous multiplied by some common ratio. If this ratio is a fraction with a magnitude less than one, then the converging point of the sum can be determined. If this ratio is greater than one, each successive addend will grow without bound, resulting in an a diverging series.

1 Declarations

a; the first term of the geometric sequence;
r; the multiplier for each term in the sequence; $a_n = a_{n-1}\, r$
A_n; the sequence of terms;

2 Rule

$$\sum_{n=0}^{\infty} a\, r^n = \begin{cases} \dfrac{a}{1-r} & |r| < 1 \\ \text{divergence} & |r| \geq 1 \end{cases} \tag{1}$$

3 Pre-Derivation

1) Prove that when $|r| \geq 1$, the sum diverges

Finding a and r

Given sigma notation:

$$\sum_{n=0}^{\infty} a\, r^n$$

Given a sequence of terms:

Assuming a geometric series, there's a common ratio between a term and its successor, such that:

$$r = \frac{A_{n+1}}{A_n}, \forall\, n$$

$$a = A_1$$

4 Derivations

$$\begin{aligned}
Let |r| &< 1 \\
S &= \sum_{n=0}^{\infty} a\, r^n \\
&= a\, r^0 + a\, r^1 + a\, r^2 + a\, r^3 + \cdots a\, r^{n-2} + a\, r^{n-1} + a\, r^n \\
&= a + a\, r^1 + a\, r^2 + a\, r^3 + \cdots a\, r^{n-2} + a\, r^{n-1} + a\, r^n \\
\\
r\, S &= a\, r^1 + a\, r^2 + a\, r^3 + \cdots a\, r^{n-2} + a\, r^{n-1} + a\, r^n \\
\\
a + r\, S &= a + a\, r^1 + a\, r^2 + a\, r^3 + \cdots a\, r^{n-2} + a\, r^{n-1} + a\, r^n \\
a + r\, S &= S \\
\\
S &= \frac{a}{1 - r}
\end{aligned}$$

5 Exempli Gratia

5.1 Finding range such that a sum converges

Find range of x such that the following geometric sum converges:

$$\begin{aligned}
S &= \sum_{n=0}^{\infty} (-4)^n (x)^n \\
&= \sum_{n=0}^{\infty} (-4x)^n
\end{aligned}$$

$$\text{Let } a = 1, r = -4x$$
$$\text{Let } |r| < 1$$

$$\begin{aligned}
|-4x| &< 1 \\
-1 < 4x &< 1 \\
\frac{-1}{4} < x &< \frac{1}{4}
\end{aligned}$$