Journal Template

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Abstract

Abstract goes here...

1   Declarations

variable; variable description; *variable   domain   and   range,   if   applicable*

2   Rule

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1{}^2}$$

3   Pre-Derivation

Anything that the derivation relies on goes here

4   Derivation

Assume y_1 is know. By preliminary theory...

$$\frac{y}{y_1} \notin k$$

$$\implies \frac{y_1}{y} = u(x) \implies y_2 = u(x)\, y_1(x)$$

5   Exempli Gratia

Find y_2 given:

$$x^2\, y'' - 3\, x\, y' + 4\, y = 0,\ y_1 = x^2$$

$$y = u(x)\,x^2 = x^2\,u$$
$$y' = 2\,x\,u + x^2\,u'$$
$$y'' = 2\,u + 2\,x\,u' + 2\,x\,u' + x^2\,u''$$
$$= 2\,u + 4\,x\,u' + x^2\,u''$$

$$0 = x^2(2\,u + 2\,x\,u' + x^2\,u) - 3\,x(2\,x\,u + x^2\,u') + 4(x^2\,u) \qquad \text{Substitution}$$
$$= 2\,x^2\,u + 4\,x^3\,u' + x^4\,u'' - 6\,x^2\,u - 3\,x^3\,u' + 4\,x^2\,u \qquad \text{Distribution}$$
$$= x^4\,u'' + x^3\,u'$$

$$\text{Let } w = u',\ w' = u''$$
$$\implies 0 = x^4\,w' + x^3\,w$$
$$\implies 0 = w' + \frac{1}{x}\,w \qquad\qquad\qquad \text{Divide by } x^3$$