Dot Product

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Abstract

Dot product, or scalar product, is a form of multiplication of vectors such that $\vec{a} \bullet \vec{b}$ is a scalar. A dot product of vectors can also be a measure of how parallel the two vectors are.

1 Declarations

$\bullet$; dot product; Simple multiplication is used as follows 1 N $\equiv$ 1 kg$\cdot$m/s^2 v_a;the a component of vector v; often denoted with subscripts$_x$, $_y$, and $_z$

2 Rule

$$\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\vec{a} \bullet \vec{b} = a_x\, b_x + a_y\, b_y + a_z\, b_z$$

3 Pre-Derivation

Orthogonal vectors Distributive property of dot product Associative property of dot product

4 Derivation

4.1 Geometric Dot Product

4.2 Unit Vector Dot Product

$$\vec{a} =< a_x, a_y, a_z >= a_x \cdot \hat{i} + a_y \cdot \hat{j} + a_z \cdot \hat{k}$$
$$\vec{b} =< b_x, b_y, b_z >= b_x \cdot \hat{i} + b_y \cdot \hat{j} + b_z \cdot \hat{k}$$

$$\begin{aligned}\vec{a} \bullet \vec{b} &= a_x\, b_x\, (\hat{\imath} \bullet \hat{\imath}) + a_x\, b_y\, (\hat{\imath} \bullet \hat{\jmath}) + a_x\, b_z\, (\hat{\imath} \bullet \hat{k}) \\ &= a_y\, b_x\, (\hat{\jmath} \bullet \hat{\imath}) + a_y\, b_y\, (\hat{\jmath} \bullet \hat{\jmath}) + a_y\, b_z\, (\hat{\jmath} \bullet \hat{k}) \\ &= a_z\, b_x\, (\hat{k} \bullet \hat{\imath}) + a_z\, b_y\, (\hat{k} \bullet \hat{\jmath}) + a_z\, b_z\, (\hat{\imath} \bullet \hat{k})\end{aligned}$$

Recall different unit vectors are orthogonal and orthogonal vectors have a dot product of 0.

$$\vec{a} \bullet \vec{b} = a_x\, b_x\, (\hat{\imath} \bullet \hat{\imath}) + a_y\, b_y (\hat{\jmath} \bullet \hat{\jmath}) + a_z\, b_z\, (\hat{k} \bullet \hat{k})$$

Recall parallel vectors have a dot product of 1.

$$\therefore \vec{a} \bullet \vec{b} = a_x\, b_x + a_y\, b_y + a_z\, b_z$$

5 Exempli Gratia

Examples of important instances