Taylor Series

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Abstract

Abstract goes here...

1 Declarations

a; domain value of which the series is about;
n; term in series; $x \in \mathbb{Z}^+$
f^n; nth derivative of a function;$n \in \mathbb{Z}^+, f^0(a) = f(a)$

2 Rule

$$\sum_{n=0}^{\infty} f^n(a)\frac{x-a^n}{n!}$$
$$= f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + f^{\mathrm{iv}}(a)\frac{(x-a)^4}{4!}$$
$$\vdots$$
$$+f^{\mathrm{n\text{-}1}}(a)\frac{(x-a)^{n-1}}{(n-1)!} + f^{\mathrm{n}}(a)\frac{(x-a)^n}{n!}$$

3 Pre-Derivation

Anything that the derivation relies on goes here

4 Derivation

Derivation goes here

5 Exempli Gratia

5.1 Taylor Series Expansion of e^x about 0

$$\begin{aligned} f(x) &= e^x & f(0) &= 1 \\ f'(x) &= e^x & f'(0) &= 1 \\ f''(x) &= e^x & f''(0) &= 1 \\ f'''(x) &= e^x & f'''(0) &= 1 \\ f^{\mathrm{iv}}(x) &= e^x & f^{\mathrm{iv}}(0) &= 1 \\ &\vdots & &\vdots \end{aligned}$$

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\ &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{!4} + \dots \end{aligned}$$

5.2 Taylor Series Expansion of cos about 0

$$\begin{aligned} f(x) &= \cos x & f(0) &= 1 \\ f'(x) &= -\sin x & f'(0) &= 0 \\ f''(x) &= -\cos x & f''(0) &= -1 \\ f'''(x) &= \sin x & f'''(0) &= 0 \\ f^{\mathrm{iv}}(x) &= \cos x & f^{\mathrm{iv}}(0) &= 1 \\ &\vdots & &\vdots \end{aligned}$$

$$\begin{aligned} \cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots \end{aligned}$$

5.3 Taylor Series Expansion of sin about 0

$$
\begin{array}{rr}
f(x) = \sin x & f(0) = 0 \\
f'(x) = \cos x & f'(0) = 1 \\
f''(x) = -\sin x & f''(0) = 0 \\
f'''(x) = -\cos x & f'''(0) = -1 \\
f^{\mathrm{iv}}(x) = \sin x & f^{\mathrm{iv}}(0) = 0 \\
\vdots & \vdots
\end{array}
$$

$$
\begin{aligned}
\sin x &= \sum n = 0^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
&=
\end{aligned}
$$