Separable Equations

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Abstract

Abstract goes here...

1 Declarations

$\frac{dy}{dx} \triangleq y'$; Differential and prime notation equivalence;
$p(y) = \frac{1}{h(y)}$;P is the reciprocal of H, for neatness' sake;

2 Rule

$$\frac{dy}{dx} = g(x)\,h(y)$$

$$\implies \int p(y)\,dy = \int g(x)\,dx$$

3 Pre-Derivation

This rule only holds true if a differential equation is of first order and holds the following form, that being the product of two functions, one of x and one of y. See below...

$$\frac{dy}{dx} = g(x)\,h(y)$$

4 Derivation

$$
\begin{aligned}
\frac{dy}{dx} &= g(x)\,h(y) \\
\frac{1}{h(y)}\frac{dy}{dx} &= g(x) \\
\text{Let } p(y) &= \frac{1}{h(y)},\ h(y) \neq 0 \\
\therefore \frac{dy}{dx} p(y) &= g(x)
\end{aligned}
$$

$$
\begin{aligned}
\text{Assume } y &= \phi(x),\ \text{the solution.} \\
\frac{dy}{dx} &= \phi'(x)
\end{aligned}
$$

$$
\begin{aligned}
p(\phi(x))\phi'(x) &= g(x) \\
\int p(\phi(x))\phi'(x)dx &= \int g(x)dx
\end{aligned}
$$

$$
\begin{aligned}
\text{But since } y &= \phi(x) \\
\implies dy &= \phi'(x)dx
\end{aligned}
$$

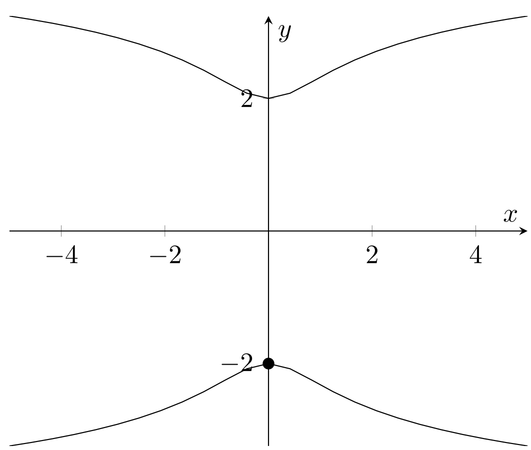
$$
\therefore \int p(y)\,dy = \int g(x)\,dx
$$

5 Exempli Gratia

5.1 Solution branches with initial value

$$\frac{dy}{dx} = \frac{2x}{y + x^2 y},\ y(0) = -2$$
$$y\,dy = \frac{2x}{x^2}dx$$
$$\int y\,dy = \int \frac{2x}{1+x^2}dx$$
$$\frac{y^2}{2} = ln|1 + x^2| + C$$
$$\implies \frac{y^2}{2} = ln(1 + x^2) + C$$
$$y = \pm\sqrt{2\,ln(1 + x^2) + C}$$

$$y(0) = 2 \implies x = 0,\ y = -2$$
$$\text{using } y^2 = 2\,ln(1 + x^2) + C$$
$$4 = 2\,ln(1 + 0) + C$$
$$\implies C = 4$$



The bottom branch is the solution to the given differential equation because it contains the point. No two discontinuous branches can be solutions at the same time.