Taylor Series

Logan Grosz

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Abstract

Abstract goes here...

1 Declarations

a; domain value of which the series is about;
n; term in series; $x \in \mathbb{Z}^+$
f^n; nth derivative of a function;$n \in \mathbb{Z}^+, f^0(a) = f(a)$

2 Rule

$$\sum_{n=0}^{\infty} f^n(a) \frac{x-a^n}{n!}$$

$$= f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + f^{\mathrm{iv}}(a)\frac{(x-a)^4}{4!}$$

$$\vdots$$

$$+f^{\text{n-1}}(a)\frac{(x-a)^{n-1}}{(n-1)!} + f^{\mathrm{n}}(a)\frac{(x-a)^n}{n!}$$

3 Pre-Derivation

Anything that the derivation relies on goes here

4 Derivation

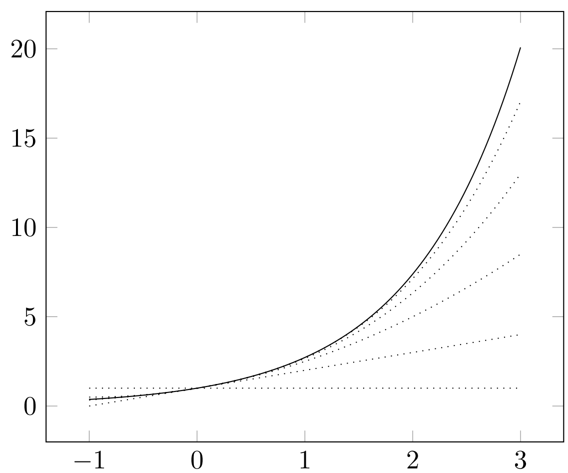
Derivation goes here

5 Exempli Gratia

5.1 Taylor Series Expansion of e^x about 0

$$
\begin{aligned}
f(x) &= e^x & f(0) &= 1 \\
f'(x) &= e^x & f'(0) &= 1 \\
f''(x) &= e^x & f''(0) &= 1 \\
f'''(x) &= e^x & f'''(0) &= 1 \\
f^{\mathrm{iv}}(x) &= e^x & f^{\mathrm{iv}}(0) &= 1 \\
&\vdots & &\vdots
\end{aligned}
$$

$$
\begin{aligned}
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\
&= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{!4} + \dots
\end{aligned}
$$



5.2 Taylor Series Expansion of cos about 0

$$
\begin{aligned}
f(x) &= \cos x & f(0) &= 1 \\
f'(x) &= -\sin x & f'(0) &= 0 \\
f''(x) &= -\cos x & f''(0) &= -1 \\
f'''(x) &= \sin x & f'''(0) &= 0 \\
f^{\text{iv}}(x) &= \cos x & f^{\text{iv}}(0) &= 1 \\
&\vdots & &\vdots
\end{aligned}
$$

$$
\begin{aligned}
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \\
&= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots
\end{aligned}
$$

5.3 Taylor Series Expansion of sin about 0

$$
\begin{aligned}
f(x) &= \sin x & f(0) &= 0 \\
f'(x) &= \cos x & f'(0) &= 1 \\
f''(x) &= -\sin x & f''(0) &= 0 \\
f'''(x) &= -\cos x & f'''(0) &= -1 \\
f^{\text{iv}}(x) &= \sin x & f^{\text{iv}}(0) &= 0 \\
&\vdots & &\vdots
\end{aligned}
$$

$$
\begin{aligned}
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
&= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \dots
\end{aligned}
$$