2nd Order, Linear, Homogeneous Differential Equations

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Abstract

Abstract goes here...

1 Declarations

variable; variable description; *variable domain and range, if applicable*

2 Rule

Math in text mode... $f(x) = \sum_{i=0}^{n} \frac{a_i}{1+x}$

Display mode:

$$f(x) = \sum_{i=0}^{n} \frac{a_i}{1+x}$$

3 Pre-Derivation

Anything that the derivation relies on goes here

4 Derivation

$$a\,y'' + b\,y' + c\,y = 0$$
Where $a, b, y \in k$

$$\text{Let } y = e^{m\,x}$$
$$\implies y' = m\,e^{m\,x}$$
$$\implies y'' = m^2\,e^{m\,x}$$

$$a\,m^2\,e^{m\,x} + b\,m\,e^{m\,x} + c\,e^{m\,x} = 0 \qquad \text{Substitution}$$
$$e^{m\,x}(a\,m^2 + b\,m + c) = 0 \qquad \text{Distribution}$$

$$\forall x(e^{m\,x}) \neq 0$$
$$\implies a\,m^2 + b\,m + c = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a} \qquad \text{Quadratic Formula}$$
$$\text{Let } \Delta = b^2 - 4\,a\,c \qquad \text{Discriminant}$$

$$\text{Case 1: } \Delta > 0 \implies m_1, m_2 \in \mathbb{R}$$
$$\text{Case 2: } \Delta = 0 \implies m_1 = m_2 = \frac{-b}{2\,a}$$
$$\text{Case 3: } \Delta < 0 \implies m_1, m_2 \in \mathbb{C}$$

$$y = c_1\,e^{m_1\,x} + c_2\,e^{m_2\,x}$$

$$\text{Note: Case 2} \implies y_2 = y_1 \int \frac{e^{\int P(x)dx}}{{y_1}^2} dx$$

5 Exempli Gratia

Examples of important instances