Linear Differential Equations

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Abstract

Strategy works with first order, linear differential equations

1 Declarations

variable; variable description; *variable domain and range, if applicable*

2 Rule

$$\frac{dy}{dx} + p(x)y = f(x)$$
$$\implies \frac{d}{dx}[\mu(x)y] = \mu(x)f(x)$$
$$\implies y = \frac{\int \mu(x)f(x)}{\mu(x)}$$
$$\text{Where } \mu(x) = e^{\int p(x)dx}$$

3 Pre-Derivation

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

Must first get equation into standard form, with the derivative of y alone...

$$\frac{dy}{dx} + \frac{a_0(x)}{a_1(x)}y = \frac{g(x)}{a_1(x)}$$

$$\text{Let } \frac{a_0(x)}{a_1(x)} = p(x), \frac{g(x)}{a_1(x)} = f(x)$$

$$\therefore \frac{dy}{dx} + p(x)y = f(x)$$

4 Derivation

$$\frac{dy}{dx} + p(x)y = f(x)$$
$$\mu(x)\frac{dy}{dx} + \mu(x)p(x)y = \mu(x)f(x)$$

Finding the integration coeffecient, $\mu(x)$...

$$\exists\mu(x) | \frac{d}{dx}[\mu(x)y] = \mu(x)\frac{dy}{dx} + \mu(x)p(x)y$$

$$Product\, rule\, of\, derivatives...$$

$$\frac{d}{dx}[\mu(x)y] = \mu(x)\frac{dy}{dx} + \frac{d}{dx}[\mu(x)]y$$
$$\implies \frac{d}{dx}[\mu(x)]y = \mu(x)p(x)y$$
$$\frac{d}{dx}\mu(x) = \mu(x)p(x)$$
$$\frac{1}{\mu(x)}du = p(x)dx$$
$$\int \frac{1}{\mu(x)}du = \int p(x)dx$$
$$\implies \ln\mu(x) = \int p(x)dx$$
$$\mu(x) = e^{\int p(x)dx}$$

$$\frac{d}{dx}[\mu(x)y] = \mu(x)f(x)$$
$$\int \frac{d}{dx}[\mu(x)y]dx = \int \mu(x)f(x)dx$$
$$\mu(x)y = \int \mu(x)f(x)dx$$
$$y = \frac{\int \mu(x)f(x)}{\mu(x)}$$

5 Exempli Gratia

Examples of important instances