Question 1:

 $\Pi_{P.pid}(P \bowtie_{K1.pid1=P.pid} K1 \bowtie_{K2.pid1=P.pid \land K1.pid2 <> K2.pid2} K2 \bowtie_{(W.pid=K1.pid2 \lor W.pid=K2.pid2) \land (W.cname='Apple' \lor W.cname='Netflix')} W)$

 $\Pi_{P.pid}(P\bowtie_{K1.pid1=P.pid}K1\bowtie_{K2.pid1=P.pid\wedge K1.pid2<>K2.pid2}K2\bowtie_{(W.pid=K1.pid2\vee W.pid=K2.pid2)}(\sigma_{W.cname='Apple'\vee W.cname='Netflix'}W))$

-> Push selection in

 $\Pi_{P,pid}(P\bowtie_{K1,pid1=P,pid}K1\bowtie_{K2,pid1=P,pid}\Lambda_{K1,pid2<>K2,pid2}K2\bowtie_{(W,pid=K1,pid2\vee W,pid=K2,pid2)}\Pi_{W,pid}(\sigma_{W,cname='Apple'V})$ $_{W,cname='Netflix'}W))$

-> Attribute elimination

 $\Pi_{\mathsf{P.pid}}(\mathsf{P}^{\bowtie}(\mathsf{K1}\bowtie_{\mathsf{K2.pid1}=\mathsf{P.pid}\wedge\mathsf{K1.pid2}<>\mathsf{K2.pid2}}\mathsf{K2}\bowtie_{(\mathsf{W.pid}=\mathsf{K1.pid2}\vee\mathsf{W.pid}=\mathsf{K2.pid2})}\Pi_{\mathsf{W.pid}}(\sigma_{\mathsf{W.cname}=\mathsf{Apple}^{:}\mathsf{V}\;\mathsf{W.cname}=\mathsf{Netflix}^{:}\mathsf{W})))$

-> Semi Join

Question 2:

$$\begin{aligned} Q_1 &= \Pi_{P,pid,\ C.cname}(P\bowtie W\bowtie C\bowtie_{M.eid=P,pid} M\bowtie_{pS1.pid=M.mid} pS1\bowtie_{pS2.pid=M.mid \land pS1.skill <>pS2.skill} pS2) \\ Q_1 &= \Pi_{W,pid,\ W.cname}(W\bowtie_{M.eid=W.pid} M\bowtie_{pS1.pid=M.mid} pS1\bowtie_{pS2.pid=M.mid \land pS1.skill <>pS2.skill} pS2) \\ &-> FK\ elimination \end{aligned}$$

$$\begin{aligned} Q_2 &= \Pi_{\text{P.pid, C.cname}}(P \bowtie W \bowtie C) \\ Q_2 &= \Pi_{\text{W.pid, W.cname}}(W) -> \text{FK elimination} \end{aligned}$$

$$\begin{split} Q_3 &= \Pi_{P.pid, \ C.cname}(P\bowtie W\bowtie C\bowtie_{K.pid1=P.pid} K\bowtie_{P1.pid=K.pid2\land P1.city='Seattle'}P1) \\ Q_3 &= \Pi_{W.pid, \ W.cname}(W\bowtie_{K.pid1=W.pid} K\bowtie_{P1.pid=K.pid2\land P1.city='Seattle'}P1) \ -> \ FK \ elimination \\ Q_3 &= \Pi_{W.pid, \ W.cname}(W\bowtie_{K.pid1=W.pid} K\bowtie_{P1.pid=K.pid2}(\sigma_{P1.city='Seattle'}P1)) \ -> \ Push \ Selection \ In \end{split}$$

$$Q_1 \cap (Q_2 - Q_3)$$

Question 3:

 $\Pi_{\text{S.Skill}}(\text{SMpS1Mp}_{\text{pS1.Skill-pS2.Skill}}) = \text{pS2.Skill} \\ \text{pS2.Skill} \\ \text{pS2.Skill} \\ \text{pS2.pid} \\ \text{pS3.pid} \\ \text{pS3.pi$

 $\Pi_{\text{S.Skill}}(\text{SMpS1M}_{\text{pS1.Skill}=\text{pS2.Skill}}) \times \text{PS2.pid} \times$

-> Commutativity Rule

 $\Pi_{\text{S.Skill}}(\text{SMpS1} \bowtie_{\text{pS1.Skill}=\text{pS2.Skill}} \text{ppS1.pid} \sim_{\text{pS2.pid}} \text{pS2} \bowtie_{\text{P.pid}=\text{pS1.pid}} \text{pS2.pid}) (\sigma_{\text{P.city='Bloomington'}} P))$

-> Push selection In

 $\Pi_{\text{S.Skill}}(\text{SMpS1M}_{\text{pS1.Skill-pS2.Skill}},\text{pS1.pid} \sim \text{pS2.pid}) \\ PS2M_{\text{(P.pid-pS1.pidVP.pid-pS2.pid)}}\Pi_{\text{P.pid}}(\sigma_{\text{P.city='Bloomington'}}P))$

-> Attribute Elimination

 $\Pi_{pS1.Skill}(pS1\bowtie_{pS1.Skill=pS2.Skill\wedge pS1.pid <> pS2.pid}pS2\bowtie_{(P.pid=pS1.pidVP.pid=pS2.pid)}\Pi_{P.pid}(\sigma_{P.city='Bloomington'}P))$

-> FK Elimination

Question 4:

$$Q_1 = \Pi_{P.pid}(P \bowtie_{W.pid=P.pid} W \bowtie_{W.cname=C.cname \land C.headquarter='Mountain View'} C)$$

 $Q_1 = \prod_{P.pid} (P \bowtie_{W.pid=P.pid} W \bowtie_{W.cname=C.cname} (\sigma_{C.headquarter='MountainView'} C))$

-> Push Selection In

 $Q_1 = \Pi_{W.pid}(W \bowtie_{W.cname=C.cname} (\sigma_{C.headquarter='MountainView'}C))$

-> FK Elimination

 $Q_1 = \Pi_{W.pid}(W \ltimes (\sigma_{C.headquarter='MountainView'}C))$

-> Semi Join

$$Q_2 = \prod_{Q1.Pid, W.salary} (Q_1 \bowtie_{K.pid2=Q1.pid} K \bowtie_{W.pid=K.pid1} W)$$

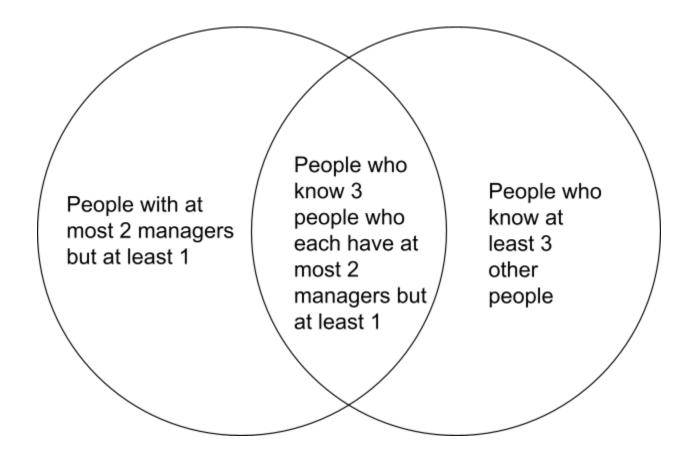
$$\Pi_{pid,salary}(Q_2 - \Pi_{Q2.*}(Q_2 \bowtie_{Q2.salary>Q2} ^1.salary \land_{Q2.pid=Q2} ^1.pid Q_2^1))$$

Question 5:

 $\begin{aligned} Q_1 &= \Pi_{C.cname}(C\bowtie W\bowtie_{(pS,pid=W,pid\land pS.skill='OperatingSystems)}pS) \\ Q_1 &= \Pi_{C.cname}(C\bowtie W\bowtie_{pS,pid=W,pid}(\sigma_{pS.skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_1 &= \Pi_{W.cname}(W\bowtie_{pS,pid=W,pid}(\sigma_{pS.skill='OperatingSystems}pS)) \text{ -> FK Elimination} \\ Q_1 &= \Pi_{W.cname}(W\bowtie_{pS,pid=W,pid}\Pi_{pS,pid}(\sigma_{pS.skill='OperatingSystems}pS)) \text{ -> Attribute Elimination} \\ Q_1 &= \Pi_{W.cname}(W\bowtie_{pS,pid}(\sigma_{pS.skill='OperatingSystems}pS)) \text{ -> Semi Join} \end{aligned}$ $Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid}(\sigma_{pS.skill='OperatingSystems}pS)) \text{ -> Semi Join} \\ Q_2 &= \Pi_{W.cname}(C\bowtie_{yS,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_1 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_2 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_3 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_4 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_4 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_4 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_4 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_4 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_4 &= \Pi_{W.cname}(V\bowtie_{pS,pid=W,pid}(\sigma_{pS,skill='OperatingSystems}pS)) \text{ -> Push Selection In} \\ Q_5 &= \Pi_{W$

 $Q_1 \cap Q_2$

Question 9:



Question 10:

