Journal of Financial Economics 120 (2016) 464–490

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/finec

Roughing up beta: Continuous versus discontinuous betas and the cross section of expected stock returns[☆]

Tim Bollerslev[a,b,c], Sophia Zhengzi Li[d,*], Viktor Todorov[e]

[a] Department of Economics, Duke University, Durham, NC 27708, USA
[b] National Bureau of Economic Research, USA
[c] CREATES, Denmark
[d] Department of Finance, Eli Broad College of Business, Michigan State University, East Lansing, MI 48824, USA
[e] Department of Finance, Kellogg School of Management, Northwestern University, Evanston, IL 60208, USA

ARTICLE INFO

Article history:
Received 30 March 2015
Revised 6 August 2015
Accepted 10 August 2015
Available online 24 February 2016

JEL Classification:
C13
C14
G11
G12

Keywords:
Market price risks
Jump betas
High-frequency data
Cross-sectional return variation

ABSTRACT

We investigate how individual equity prices respond to continuous and jumpy market price moves and how these different market price risks, or betas, are priced in the cross section of expected stock returns. Based on a novel high-frequency data set of almost 1,000 stocks over two decades, we find that the two rough betas associated with intraday discontinuous and overnight returns entail significant risk premiums, while the intraday continuous beta does not. These higher risk premiums for the discontinuous and overnight market betas remain significant after controlling for a long list of other firm characteristics and explanatory variables.

1. Introduction

The idea that only systematic market price risk should be priced represents one of the cornerstones of finance. Even though numerous studies over the past half-century have called into question the ability of the capital asset pricing model (CAPM) to fully explain the cross section of expected stock returns, the beta of an asset arguably remains the most commonly used systematic risk measure in financial practice. Early work by Fama, Fisher, Jensen, and Roll (1969) and Blume (1970) generally supports the CAPM. Subsequent prominent empirical studies that call into question the explanatory power of market betas for satisfactorily explaining the cross section of expected returns include Basu (1977, 1983), Roll (1977), Banz (1981), Stattman (1983), Rosenberg, Reid, and Lanstein (1985), Bhandari (1988), and Fama and French (1992). Meanwhile,

☆ An earlier version of the paper by Tim Bollerslev and Sophia Zhengzi Li was circulated under the title “Roughing up the CAPM: Jump betas and the cross section of expected stock returns.” We are grateful to Bill Schwert (the editor), and an anonymous referee for numerous helpful suggestions, which greatly improved the paper. We would also like to thank Turan Bali, Jia Li, Andrew Patton, Mark Schroder, and George Tauchen, along with seminar participants at several universities, financial institutions, and conferences for their helpful comments. The research was partly funded by a grant from the National Science Foundation (SES-0957330) to the National Bureau of Economic Research. Bollerslev and Li also gratefully acknowledge support from CREATES funded by the Danish National Research Foundation (DNRF78) and the 2012 Morgan Stanley Prize for Excellence in Financial Markets, respectively.

* Corresponding author. Tel: +1 517 353 3115; fax: +1 517 884 3718.
E-mail addresses: boller@duke.edu (T. Bollerslev), lizhengzi@broad.msu.edu (S.Z. Li), v-todorov@northwestern.edu (V. Todorov).

http://dx.doi.org/10.1016/j.jfineco.2016.02.001
S0304-405X(16)30001-0/© 2016 Published by Elsevier B.V.

more recent empirical evidence pertaining to the equity risk premium and the pricing of risk at the aggregate market level suggests that the expected return variation associated with discontinuous price moves, or jumps, is priced higher than the expected continuous price variation.[1]

Set against this background, we propose a general pricing framework involving three separate market betas: a continuous beta reflecting smooth intraday co-movements with the market and two rough betas associated with intraday price discontinuities, or jumps, during the active part of the trading day and the overnight close-to-open return, respectively. The seminal paper by Merton (1976) hypothesizes that jump risks for individual stocks are likely to be nonsystematic. Empirical evidence of increased cross-asset correlations for higher (in an absolute sense) returns shown in Ang and Chen (2002), among many others, indirectly suggests nonzero systematic jump risk, as does the downside risk asset pricing model recently explored by Lettau, Maggiori, and Weber (2014). Consistent with the idea that investors view intraday smooth and that easier to hedge price moves differently from intraday rough and day-to-day overnight price changes, we find that the risk premiums associated with the two jump betas are both statistically significant and indistinguishable, while the continuous beta does not appear to be priced in the cross-section.[2]

The theoretical framework motivating our empirical investigations and the separate cross-sectional pricing of continuous and discontinuous market price risks is very general and merely assumes the existence of a generic pricing kernel along the lines of Duffie, Pan, and Singleton (2000). Importantly, we make no explicit assumptions about the pricing of other nonmarket price risks. As such, our setup includes the popular long-run risk model of Bansal and Yaron (2004), the habit persistence model of Campbell and Cochrane (1999), and the rare disaster model of Gabaix (2012), as special cases obtained by further restricting the functional form of the pricing kernel, the set of other priced risk factors, and the connections with fundamentals.

The statistical theory underlying our estimation of the separate betas builds on recent advances in financial econometrics related to the use of high-frequency intraday data and so-called realized volatilities. Bollerslev and Zhang (2003), Barndorff-Nielsen and Shephard (2004a), and Andersen, Bollerslev, Diebold, and Wu (2005, 2006), in particular, have explored the use of high-frequency data and the asymptotic notion of increasingly finer sampled returns over fixed time intervals for more accurately estimating realized betas. In contrast to these earlier studies, which do not differentiate among different types of market price moves, we rely on the theory originally developed by Todorov and Bollerslev (2010) for explicitly estimating separate continuous and discontinuous betas for the open-to-close active part of the trading day, together with overnight betas for the close-to-open returns.[3]

Our actual empirical investigations are based on a novel high-frequency data set of all the 985 stocks included in the Standard & Poor's (S&P) 500 index over the 1993–2010 sample period. We begin by estimating the three separate betas as well as a standard CAPM regression-based beta for each of the individual stocks on a rolling one-year basis. Consistent with the basic tenets of the simple CAPM, we find that sorting the stocks in our sample on the basis of their betas results in a positive return differential between the high- and low-beta quintile portfolios for all of the four different beta estimates. However, even though all of the return differentials are large numerically, the difference in the monthly returns between the high- and low-beta portfolios constructed on the basis of the standard CAPM betas is not significantly different from zero at conventional levels. Similarly, sorting by our continuous beta estimates, the monthly long–short excess return for the high- minus low-beta quintile portfolios is not significantly different from zero. Sorting stocks on the basis of their discontinuous and overnight betas, as well as their relative betas defined by the difference between either of the two jump betas and the standard beta, results in significantly positive risk-adjusted returns on the high–low portfolios.[4] More important from a practical perspective, we show that these same significant contemporaneous return differentials carry over to a predictive setting, in which we compare the subsequent realized monthly returns of the quintile portfolios based on grouping the stocks according to their past rolling one-year beta estimates.

These predictive return differentials associated with the discontinuous and overnight betas remain statistically significant in double portfolio sorts designed to control for a number of other firm characteristics and risk factors previously associated with the cross section of expected returns, including firm size, book-to-market ratio, momentum, short-term reversal, idiosyncratic volatility, maximum daily return, illiquidity, and various measures of skewness and kurtosis. Standard predictive Fama–MacBeth regressions further corroborate the idea that only rough market risks are priced. While the estimated risk premiums associated with the intraday discontinuous and overnight betas are both significant after simultaneously controlling for a long list of firm characteristics and other risk factors, the estimated risk premium associated with the continuous beta is not.

Our main empirical findings rely on a relatively coarse 75-minute intraday sampling frequency for the one-year

[1] Empirical evidence based on aggregate equity index options in support of this hypothesis is presented by Pan (2002), Eraker, Johannes, and Polson (2003), Bollerslev and Todorov (2011), and Gabaix (2012), among others.

[2] Optimally managing market diffusive and jump price risks require the use of different hedging tools and derivative instruments; see, e.g., the theoretical analysis in Liu, Longstaff, and Pan (2003a, 2003b). The increased availability of short-maturity out-of-the-money options, which provide a particular convenient tool for managing jump tail risk, also directly speaks to the practical importance of separately accounting for these different types of risks.

[3] Branch and Ma (2012), Cliff, Cooper, and Gulen (2008), and Berkman, Koch, Tuttle, and Zhang (2012) also show distinctly different return patterns during trading and non-trading hours.

[4] As discussed further in Section 5.2, this contrasts with the recent results in Frazzini and Pedersen (2014), who report an almost flat security market line and highly significant positive CAPM alphas for portfolios betting against beta.

rolling continuous and jump beta estimation, as a way to guard against nonsynchronous trading effects and other market microstructure complications that arise at the highest intraday sampling frequency. However, our results remain robust to the use of other sampling frequencies and inference procedures for the estimation of the betas. Similarly, while our main cross-sectional regressions are based on a standard one-year estimation and subsequent one-month holding period, even stronger results hold true for other estimation windows and return holding periods. Also, while some of the jumps that occur at the aggregate market level are naturally associated with news about the economy, our results remain robust to the exclusion of several important macroeconomic news announcement days.[5]

The idea of allowing for time-varying market betas to help explain the cross section of expected stock returns is related to the large literature on testing conditional versions of the CAPM.[6] In contrast to this literature, our empirical investigations should not be interpreted as a test of the conditional CAPM per se. Instead, motivated by our general pricing framework, we simply show that market risks with different degrees of jumpiness, as determined by our high-frequency-based estimates of the time-varying continuous and jump betas, are priced differently and that these cross-sectional differences in the returns cannot be explained by other firm characteristics or commonly used risk factors. We are not arguing that market risk is the only source of priced risk in the cross section.

Our work is also related to, but fundamentally different from, several recent studies that have examined how jump risk can help explain the cross section of expected stock returns. Jiang and Yao (2013) argue that the size premium, the liquidity premium and, to a lesser extent, the value premium are all realized in the cross-sectional differences of jump returns. Cremers, Halling, and Weinbaum (2015) show that market expectations of aggregate jump risk implied from options prices are useful for explaining the cross-sectional variation in expected returns, and Yan (2011) shows that expected stock returns are negatively related to average jump sizes. Our work differs from these studies in at least two important dimensions. First, we focus explicitly on systematic jump risk, as measured by the exposure to nondiversifiable market-wide jumps and the two rough betas. Second, our use of high-frequency data to directly identify the intraday jumps and estimate the betas sets our study apart from other research inferring the jump risk from daily or lower-frequency data.

Our cross-sectional pricing results also complement recent time series estimates of the equity risk premium reported in Bollerslev and Todorov (2011) and Gabaix (2012), among others, which suggest that a large portion of the aggregate equity premium and the temporal variation therein could be attributable to jump tail risk. In line with these findings for the aggregate market, the two rough betas associated with intraday jumps and day-to-day overnight price changes directly reflect the individual stocks' systematic response to jump risk and, in turn, receive the largest compensation in the cross section. Intuitively, large stock price movements likely provide better signals about true changes in fundamentals and equity valuations than do smaller within-day price fluctuations, which could simply represent noise in the price formation process.

The remainder of the paper is organized as follows. Section 2 formally defines the different betas and the theory underlying their separate pricing within a conventional equilibrium-based asset pricing framework. The statistical procedures used for estimating the separate betas are discussed in Section 3. Section 4 describes the high-frequency data that we use to estimate the betas and the control variables employed in our empirical investigations. Section 5 presents our initial empirical evidence pertaining to various portfolio sorts. Section 6 discusses the results from the predictive firm-level cross-sectional pricing regressions and the estimates of the risk premiums for the different betas. Section 7 presents a series of robustness checks related to the intraday sampling frequency used in the estimation of the betas, possible nonsynchronous trading effects, errors-in-variables in the cross-sectional pricing regressions, the length of the beta estimation and return holding periods, and the influence of specific macroeconomic news announcements. Section 8 concludes. Appendix details the high-frequency data cleaning rules and the definitions of the explanatory variables used in the analysis.

2. Continuous and discontinuous market risk pricing

Our theoretical framework motivating the different betas and the separate pricing of continuous and discontinuous market price risks is very general and merely relies on no-arbitrage and the existence of a pricing kernel. By the same token, we do not provide explicit equilibrium-based expressions for the separate risk premiums. Doing so would require additional assumptions beyond the ones necessary for simply separating the continuous and discontinuous market risk premiums and the corresponding market betas.

To set out the notation, let the price of the aggregate market portfolio be denoted by $P_t^{(0)}$, with the corresponding logarithmic price denoted by lowercase $p_t^{(0)} \equiv \log P_t^{(0)}$. We assume the following general dynamic representation for the instantaneous return on the market:

$$dp_t^{(0)} = \alpha_t^{(0)} dt + \sigma_t dW_t + \int_{\mathbb{R}} x \widetilde{\mu}(dt, dx), \tag{1}$$

where W_t denotes a Brownian motion describing continuous Gaussian, or smooth, market price shocks with diffusive volatility σ_t and $\widetilde{\mu}$ is a (compensated) jump counting measure accounting for discontinuous, or rough,

[5] Initial studies showing large changes in high-frequency intraday returns in response to macroeconomic news announcements include Fleming and Remolona (1999) and Andersen, Bollerslev, Diebold, and Vega (2003, 2007b).

[6] Early contributions to this literature include Ferson, Kandel, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), and Harvey (1989), among others, along with more recent cross-sectionally oriented studies by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001). Bali, Engle, and Tang (2015) have also recently argued that generalized autoregressive conditional heteroskedasticity (GARCH)-based time-varying conditional betas help explain the cross-sectional variation in expected stock returns.

market price moves.[7] The drift term $\alpha_t^{(0)}$ is explicitly related to the pricing of these separate market risks.

We denote the cross section of individual stock prices by $P_t^{(i)}$, $i = 1, \ldots, n$. In parallel to the representation for the market portfolio, we assume that the instantaneous logarithmic price process, $p_t^{(i)} \equiv \log P_t^{(i)}$, for each of the n individual stocks could be expressed as

$$dp_t^{(i)} = \alpha_t^{(i)} dt + \beta_t^{(c,i)} \sigma_t^{(i)} dW_t + \int_{\mathbb{R}} \beta_t^{(d,i)} x \widetilde{\mu}(dt, dx) + \widetilde{\sigma}_t^{(i)} dW_t^{(i)} + \int_{\mathbb{R}} x \widetilde{\mu}^{(i)}(dt, dx), \quad (2)$$

where the $W_t^{(i)}$ Brownian motion is orthogonal to W_t, but possibly correlated with $W_t^{(j)}$ for $i \neq j$, and the $\mu^{(i)}$ jump measure is orthogonal to μ in the sense that $\mu(\{t\}, \mathbb{R})\mu^{(i)}(\{t\}, \mathbb{R}^p) = 0$ for every t, so that $\mu^{(i)}$ counts only firm-specific jumps occurring at times when the market does not jump. By explicitly allowing the individual loadings, or betas, associated with the market diffusive and jump risks to be time-varying, this decomposition of the continuous and discontinuous martingale parts of asset i's return into separate components directly related to their market counterparts and orthogonal components (in a martingale sense) is extremely general. For the diffusive part, this entails no assumptions and follows merely from the partition of a correlated bivariate Brownian motion into its orthogonal components (see, e.g., Theorem 2.1.2 in Jacod and Protter, 2012). For the discontinuous part, the decomposition implicitly assumes that the relation between the systematic jumps in the asset and the market index, while time-varying, does not depend on the size of the jumps.[8] This type of restriction is arguably unavoidable. By their very nature, systematic jumps are relatively rare, and as such it is not feasible to identify different jump betas for different jump sizes, let alone identify the small jumps in the first place. This assumption also maps directly into the way in which we empirically estimate jump betas for each of the individual stocks based solely on the large-size jumps.

To analyze the pricing of continuous and discontinuous market price risks, we follow standard practice in the asset pricing literature and assume the existence of an economy-wide pricing kernel of the form (see, e.g., Duffie, Pan, and Singleton, 2000)

$$M_t = e^{-\int_0^t r_s ds} \mathcal{E}\left(- \int_0^t \lambda_s dW_s + \int_0^t \int_{\mathbb{R}} (\kappa(s, x) - 1) \widetilde{\mu}(ds, dx) \right) M_t', \quad (3)$$

where r_t denotes the instantaneous risk-free interest rate and $\mathcal{E}(\cdot)$ refers to the stochastic exponential.[9] The càdlàg λ_t process and the predictable $\kappa(t, x)$ function account for the pricing of diffusive and jump market price risks, respectively. The last term, M_t', encapsulates the pricing of all other (orthogonalized to the market price risks) systematic risk factors. In parallel to the first part of the expression for M_t, we assume that this additional part of the pricing kernel takes the form,

$$M_t' = \mathcal{E}\left(- \int_0^t \lambda_s' dW_s' + \int_0^t \int_{\mathbb{R}} (\kappa'(s, \mathbf{x}) - 1) \widetilde{\mu}'(ds, d\mathbf{x}) \right), \quad (4)$$

where the W_t' Brownian motion is orthogonal to W_t and the two jump measures μ and μ' are orthogonal in the sense that $\mu(\{t\}, \mathbb{R})\mu'(\{t\}, \mathbb{R}^p) = 0$ for every t, so that the respective jumps never arrive at the exact same instant. The pricing kernel jointly defined by Eqs. (3) and (4) encompasses almost all parametric asset pricing models hitherto analyzed in the literature as special cases.

To help fix ideas, consider the case of a static pure-endowment economy, with independent and identically distributed consumption growth and a representative agent with Epstein–Zin preferences. In this basic consumption-based CAPM (CCAPM) setup, the dynamics of the pricing kernel are driven solely by consumption. Assuming that the market portfolio represents a claim on total consumption, it therefore follows that $M_t' \equiv 1$, resulting in a pricing kernel that solely depends on the diffusive Gaussian and discontinuous market price shocks. This same analysis continues to hold true for a representative agent with habit persistence as in Campbell and Cochrane (1999), the only difference being that in this situation the prices of the diffusive and jump market risks are time-varying due to the temporal variation in the degree of risk-aversion of the representative agent. In general, temporal variation in the investment opportunity set, as in the intertemporal CAPM (ICAPM) of Merton (1973), could induce additional sources of priced risks. Leading examples of other state variables that could affect the pricing kernel include the conditional mean and volatility of consumption growth as in Bansal and Yaron (2004) and the time-varying probability of a disaster as in Gabaix (2012) and Wachter (2013).[10] However, given our primary focus on the pricing of market price risk, we purposely do not take a stand on what these other risk factors could be, instead simply relegating their influence over and above what can be spanned by the market to the additional M_t' part of the pricing kernel.

The pricing kernel in Eq. (3) has also been widely used in the literature on derivatives pricing. For reasons of analytical tractability, in that literature the common assumptions are that λ_t is proportional to the market diffusive

[7] The compensated jump counting measure is formally related to the actual counting measure μ for the jumps in $P^{(0)}$ by the expression $\widetilde{\mu}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t(dx)$, where $\nu_t(dx)$ denotes the (possibly time-varying) intensity of the jumps, thus rendering the $\widetilde{\mu}$ measure a martingale.

[8] Formally, let s denote a time when the market jumps and $\Delta p_s^{(0)} \neq 0$. The representation in Eq. (2) then implies that $\Delta p_s^{(i)} / \Delta p_s^{(0)} = \beta_s^{(d,i)}$, allowing the jump beta to vary with the time s but not the actual size of the jump.

[9] Formally, for some arbitrary process Z, $\mathcal{E}(Z)$ is defined by the solution to the stochastic differential equation $\frac{dY}{Y_-} = dZ$, with initial condition $Y_0 = 1$.

[10] In models involving nonfinancial wealth, so that the market portfolio and the total wealth portfolio are not perfect substitutes, additional sources of risks also naturally arise.

volatility σ_t, the jump intensity $\nu_t(dx)$ is affine in σ_t^2, and the price of jump risk $\kappa(t, x)$ is time-invariant. See, e.g., Duffie, Pan, and Singleton (2000), who show that these assumptions greatly facilitate the calculation of closed-form derivatives pricing formulas. These same assumptions also imply that the equity risk premium should be proportional to the variance of the aggregate market portfolio.[11]

In general, it follows readily by a standard change-of-measure (see, e.g., Jacod and Shiryaev, 2002) that without any additional restrictions on the pricing kernel defined by Eqs. (3) and (4), the instantaneous market risk premium must satisfy

$$\alpha_t^{(0)} - r_t - \delta_t^{(0)} - q_t^{(0)} = \gamma_t^c + \gamma_t^d, \tag{5}$$

where $\delta_t^{(0)}$ refers to the dividend yield on the market portfolio, and the compensation for continuous and discontinuous market price risks are determined by

$$\gamma_t^c \equiv \sigma_t \lambda_t, \quad \text{and} \quad \gamma_t^d \equiv \int_{\mathbb{R}} x\kappa(t,x)\nu_t(dx), \tag{6}$$

respectively, and $q_t^{(0)}$ represents a standard convexity adjustment term.[12] Because the compensation stemming from M_t' is orthogonal to the compensation for market price risk, this expression for $\alpha_t^{(0)}$ depends only on the first part of the pricing kernel.

For the individual assets, even though the $W_t^{(i)}$ and $\mu^{(i)}$ diffusive and jump risks are orthogonal to the corresponding market diffusive and jump risk components, they could nevertheless be priced in the cross section as they could be correlated with the W_t' and μ' risks that appear in the M_t' part of the pricing kernel. Denoting the part of the instantaneous risk premium for asset i arising from this separate pricing of $W_t^{(i)}$ and $\mu^{(i)}$ by $\widetilde{\alpha}_t^{(i)}$, it follows again by standard arguments that

$$\alpha_t^{(i)} - r_t - \delta_t^{(i)} - q_t^{(i)} = \beta_t^{(c,i)}\gamma_t^c + \beta_t^{(d,i)}\gamma_t^d + \widetilde{\alpha}_t^{(i)}, \tag{7}$$

where $\delta_t^{(i)}$ refers to the dividend yield of asset i and $q_t^{(i)}$ denotes a standard convexity adjustment term stemming from the pricing of market price risks.[13]

If $\widetilde{\alpha}_t^{(i)} \equiv 0$, as would be implied by $M_t' \equiv 1$, and if $\beta_t^{(c,i)}$ and $\beta_t^{(d,i)}$ were also the same, the expression in Eq. (7) trivially reduces to a simple continuous-time one-factor CAPM that linearly relates the instantaneous return on stock i to its single beta. The restriction that $\beta_t^{(c,i)} = \beta_t^{(d,i)}$ implies that the asset responds the same to market diffusive and jump price increments or, intuitively, that the asset and the market co-move the same during normal times and periods of extreme market moves. If $\beta_t^{(c,i)}$ and $\beta_t^{(d,i)}$ differ, empirical evidence for which is provided below, the cross-sectional variation in the continuous and jump betas could be used to identify their separate pricing. Importantly, this remains true in the presence of other priced risk factors, when $\widetilde{\alpha}_t^{(i)}$ is not necessarily equal to zero.

[11] This simple relation has been extensively investigated in the empirical asset pricing literature. See, e.g., Bollerslev, Sizova, and Tauchen (2012) and the many additional references therein.

[12] The $q_t^{(0)}$ term is formally given by $\frac{1}{2}\sigma_t^2 + \int_{\mathbb{R}}(e^x - 1 - x)\nu_t(dx)$.

[13] In parallel to the expression for $q_t^{(0)}$ above, $q_t^{(i)} = \frac{1}{2}\left(\beta_t^{(c,i)}\sigma_t\right)^2 + \int_{\mathbb{R}}\left(e^{\beta_t^{(d,i)}x} - 1 - \beta_t^{(d,i)}x\right)\nu_t(dx)$.

In practice, the returns on the assets have to be measured over some nontrivial time interval, say, $h > 0$. Let $r_{t,t+h}^{(i)} \equiv p_{t+h}^{(i)} - p_t^{(i)}$ denote the corresponding logarithmic return on asset i. For empirical tractability, assume that the betas remain constant over that same (short) time interval. The integrated conditional risk premium for asset i could then be expressed as

$$\begin{aligned}
&\mathbb{E}_t\left(r_{t,t+h}^{(i)} - \int_t^{t+h}(r_s + \delta_s^{(i)} + q_s^{(i)})ds\right) \\
&\quad = \beta_t^{(c,i)}\mathbb{E}_t\left(\int_t^{t+h}\gamma_s^c ds\right) + \beta_t^{(d,i)}\mathbb{E}_t\left(\int_t^{t+h}\gamma_s^d ds\right) \\
&\qquad + \mathbb{E}_t\left(\int_t^{t+h}\widetilde{\alpha}_s^{(i)} ds\right).
\end{aligned} \tag{8}$$

This expression for the discrete-time expected excess return maintains the same two-beta structure as the expression for the instantaneous risk premiums in Eq. (7).[14] It clearly highlights how the pricing of continuous and discontinuous market price risks could manifest differently in the cross section of expected stock returns and, in turn, how separately estimating $\beta_t^{(c,i)}$ and $\beta_t^{(d,i)}$ could allow for more accurate empirical predictions of the actual realized returns.

3. Continuous and discontinuous beta estimation

The decompositions of the prices for the market and each of the individual assets into separate diffusive and jump components that formally underly $\beta_t^{(c,i)}$ and $\beta_t^{(d,i)}$ in Eqs. (1) and (2) are not directly observable. Instead, the different continuous-time price components and, in turn, the betas have to be deduced from observed discrete-time prices and returns.

To this end, we assume that high-frequency intraday prices are available at time grids of length $1/n$ over the active intraday part of the trading day $[t, t+1)$. For notational simplicity, we denote the corresponding logarithmic discrete-time return on the market over the τth intraday time interval by $r_{t:\tau}^{(0)} \equiv p_{t+\tau/n}^{(0)} - p_{t+(\tau-1)/n}^{(0)}$, with the τth intraday return for asset i defined accordingly as $r_{t:\tau}^{(i)} \equiv p_{t+\tau/n}^{(i)} - p_{t+(\tau-1)/n}^{(i)}$. The theory underlying our estimation is formally based on the notion of fill-in asymptotics and $n \to \infty$, or ever finer sampled high-frequency returns.[15] To allow for reliable estimation, we further assume that the

[14] This contrast with the derivations in Longstaff (1989), who shows how temporally aggregating the simple continuous-time CAPM results in a multifactor model, and the more recent paper by Corradi, Distaso, and Fernandes (2013) that delivers conditional time-varying alphas and betas within a similar setting. Instead, our derivation is based on a general continuous-time jump-diffusion representation and arrives at a consistent two-factor discrete-time pricing relation under the assumption that the separate jump and diffusive betas remain constant over the (short) return horizons.

[15] Host of practical market microstructure complications invariably prevents us from sampling too finely. To assess the sensitive of our results to the specific choice of n, we experiment with the use of several different sampling schemes, including ones in which $n^{(i)}$ varies across stocks.

betas stay constant over multi-day time-intervals of length $l > 1$.[16]

To begin, consider the estimation of the continuous betas. To convey the intuition, suppose that neither the market nor stock i jumps, so that $\mu \equiv 0$ and $\mu^{(i)} \equiv 0$ almost surely. For simplicity, suppose also that the drift terms in Eqs. (5) and (7) are both equal to zero, so that

$$r_{s:\tau}^{(i)} = \beta_t^{(i,c)} r_{s:\tau}^{(0)} + \tilde{r}_{s:\tau}^{(i)}, \quad \text{where} \quad \tilde{r}_{s:\tau}^{(i)} \equiv \int_{s+(\tau-1)/n}^{s+\tau/n} \tilde{\sigma}_u^{(i)} dW_u^{(i)}, \tag{9}$$

for any $s \in [t-l, t]$. Thus, in this situation, the continuous beta could simply be estimated by an ordinary least squares (OLS) regression of the discrete-time high-frequency returns for stock i on the high-frequency returns for the market. Using a standard polarization of the covariance term, the resulting regression coefficient can be expressed as

$$\frac{\sum_{s=t-l}^{t-1} \sum_\tau r_{s:\tau}^{(i)} r_{s:\tau}^{(0)}}{\sum_{s=t-l}^{t-1} \sum_\tau (r_{s:\tau}^{(0)})^2} \equiv \frac{\sum_{s=t-l}^{t-1} \sum_\tau \left[(r_{s:\tau}^{(i)} + r_{s:\tau}^{(0)})^2 - (r_{s:\tau}^{(i)} - r_{s:\tau}^{(0)})^2 \right]}{4 \sum_{s=t-l}^{t-1} \sum_\tau (r_{s:\tau}^{(0)})^2}. \tag{10}$$

In general, the market and stock i could both jump over the $[t-l, t]$ time interval, and the drift terms are not identically equal to zero. Meanwhile, it follows readily by standard arguments that for $n \to \infty$, the impact of the drift terms are asymptotically negligible. However, to allow for the possible occurrence of jumps, the simple estimator defined above needs to be appropriately modified by removing the discontinuous components. The polarization of the covariance provides a particularly convenient way of doing so by expressing the estimator in terms of sample portfolio variances. In particular, as shown by Todorov and Bollerslev (2010), the truncation-based estimator defined by

$$\widehat{\beta}_t^{(c,i)} = \frac{\sum_{s=t-l}^{t-1} \sum_{\tau=1}^{n} \left[(r_{s:\tau}^{(i)} + r_{s:\tau}^{(0)})^2 \mathbf{1}_{\{|r_{s:\tau}^{(i)} + r_{s:\tau}^{(0)}| \le k_{s,\tau}^{(i+0)}\}} - (r_{s:\tau}^{(i)} - r_{s:\tau}^{(0)})^2 \mathbf{1}_{\{|r_{s:\tau}^{(i)} - r_{s:\tau}^{(0)}| \le k_{s,\tau}^{(i-0)}\}} \right]}{4 \sum_{s=t-l}^{t-1} \sum_{\tau=1}^{n} (r_{s:\tau}^{(0)})^2 \mathbf{1}_{\{|r_{s:\tau}^{(0)}| \le k_{s,\tau}^{(0)}\}}}, \tag{11}$$

consistently estimates the continuous beta for $n \to \infty$ under very general conditions.[17]

Next, consider the estimation of the discontinuous beta. Assuming that $\beta_t^{(d,i)}$ is positive, it follows that for any $s \in [t-l, t]$ such that $\Delta p_s^{(0)} \neq 0$, the discontinuous beta is uniquely identified by

$$\beta_t^{(d,i)} \equiv \sqrt{\frac{\left(\Delta p_s^{(i)} \Delta p_s^{(0)}\right)^2}{\left(\Delta p_s^{(0)}\right)^4}}. \tag{12}$$

Moreover, assuming that the beta is constant over the $[t-l, t]$ time interval, this same ratio holds true for all of the market jumps that occurred between time $t-l$ and t. The observed high-frequency returns contain both diffusive and jump risk components. However, by raising the high-frequency returns to powers of order greater than two (four in the expression above), the diffusive martingale components become negligible, so that the systematic jumps dominate asymptotically for $n \to \infty$.[18] This naturally suggests the following sample analogue to the expression for $\beta_t^{(d,i)}$ above as an estimator for the discontinuous beta[19]

$$\widehat{\beta}_t^{(d,i)} = \sqrt{\frac{\sum_{s=t-l}^{t-1} \sum_{\tau=1}^{n} \left(r_{s:\tau}^{(i)} r_{s:\tau}^{(0)}\right)^2}{\sum_{s=t-l}^{t-1} \sum_{\tau=1}^{n} \left(r_{s:\tau}^{(0)}\right)^4}}. \tag{13}$$

As formally shown in Todorov and Bollerslev (2010), this estimator is consistent for $\beta_t^{(d,i)}$ for $n \to \infty$.

The continuous-time processes in Eqs. (1) and (2) underlying the definitions of the separate betas portray the prices as continuously evolving over time. In practice, we have access to high-frequency prices only for the active part of the trading day when the stock exchanges are officially open. It is natural to think of the change in the price from the close on day t to the opening on day $t+1$ as a discontinuity, or a jump.[20] As such, the general continuous-time setup discussed in Section 2 needs to be augmented with a separate jump term and jump beta measure $\beta_t^{(n,i)}$ accounting for the overnight co-movements. The notion of an ever-increasing number of observations for identifying the intraday discontinuous price moves underlying the $\widehat{\beta}_t^{(d,i)}$ estimator in Eq. (13) does not apply with the overnight jump returns. However, $\beta_t^{(n,i)}$ could be similarly estimated by applying the same formula to all of the l overnight jump return pairs.

In addition to the high-frequency-based separate intraday and overnight betas, we calculate standard regression-based CAPM betas for each of the individual stocks, say, $\widehat{\beta}_t^{(s,i)}$. These are simply obtained by regressing the l daily returns for stock i on the corresponding daily returns for the market. In the following, we refer to each of these

[16] Due to the relatively rare nature of jumps, in our main empirical results, we base the estimation on a full year. However, we also experiment with the use of shorter estimation periods, if anything, resulting in even stronger results and more pronounced patterns.

[17] In our empirical analysis, we follow Bollerslev, Todorov, and Li (2013) in setting $k_{t,\tau}^{(\cdot)} = 3 \times n^{-0.49} (\mathrm{RV}_t^{(\cdot)} \wedge \mathrm{BV}_t^{(\cdot)} \times \mathrm{TOD}_\tau^{(\cdot)})^{1/2}$, where $\mathrm{RV}_t^{(\cdot)}$ and $\mathrm{BV}_t^{(\cdot)}$ denote the so-called realized variation and bipower variation on day t, respectively, and $\mathrm{TOD}_\tau^{(\cdot)}$ refers to an estimate of the intraday time-of-day volatility pattern.

[18] The basic idea of relying on higher order powers of returns to isolate the jump component of the price has previously been used in many other situations, both parametrically and nonparametrically. See, e.g., Barndorff-Nielsen and Shephard (2003) and Aït-Sahalia (2004).

[19] Because the sign of the jump betas gets lost by this transformation, our actual implementation also involves a sign correction, as detailed in Todorov and Bollerslev (2010). From a practical empirical perspective, this is immaterial, as all of the estimated jump betas in our sample are positive.

[20] This characterization of the overnight returns as discontinuous movements occurring at deterministic times mirrors the high-frequency modeling approach recently advocated by Andersen, Bollerslev, and Huang (2011).

four different beta estimates for stock i without the explicit time subscript and hat as β_i^c, β_i^d, β_i^n, and β_i^s for short.

4. Data and variables

We begin this section with a discussion of the high-frequency data that we use in our analysis, followed by an examination of the key properties of the resulting beta estimates. We also briefly consider the other explanatory variables and controls that we use in our double portfolio sorts and cross-sectional pricing regressions.

4.1. Data

The individual stocks included in our analysis are composed of the 985 constituents of the S&P 500 index over the January 1993 to December 2010 sample period.[21] All the high-frequency data for the individual stocks are obtained from the Trade and Quote (TAQ) database. The TAQ database provides all the necessary information to create our data set containing second-by-second observations of trading volume, number of trades, and transaction prices between 9:30 a.m. and 4:00 p.m. Eastern Standard Time for the 4,535 trading days in the sample.[22] We rely on high-frequency intraday S&P 500 futures prices from Tick Data Inc. as our proxy for the aggregate market portfolio.

Our cleaning rule for the TAQ data follows (Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2009). It consists of two main steps: removing and assigning. The removing step filters out recording errors in prices and trade sizes. This step also deletes data points that TAQ flags as "problematic." The assigning step ensures that every second of the trading day has a single price. Additional details are provided in Appendix A.1.

The sample consists of 738 stocks per month on average. Altogether, these stocks account for approximately three-quarters of the total market capitalization of the entire stock universe in the Center for Research in Security Prices (CRSP) database. Average daily trading volume for each stock increases from 302,026 in 1993 to 5,683,923 in 2010. Similarly, the daily number of trades for each stock rises from an average of 177 in 1993 to 20,197 in 2010. Conversely, the average trade size declines from 1,724 shares per trade in 1993 to just 202 in 2010.

We supplement the TAQ data with data from CRSP on total daily and monthly stock returns, number of shares outstanding, and daily and monthly trading volumes for each individual stock. To guard against survivorship biases associated with delistings, we take the delisting return from CRSP as the return on the last trading day following the delisting of a particular stock. We also use stock distribution information from CRSP to adjust overnight returns computed from the high-frequency prices.[23] We rely on Kenneth R. French's website[24] for daily and monthly returns on the Fama–French–Carhart four-factor (FFC4) portfolios. Lastly, we use the Compustat database for book values and other accounting information required for some of the control variables.

4.2. Beta estimation results

Our main empirical results are based on continuous, discontinuous, and overnight betas estimated from high-frequency data for each of the individual stocks in the sample. We rely on a one-year rolling overlapping monthly estimation scheme to balance the number of observations available for the estimation with the possible temporal variation in the systematic risks.[25] We also experiment with the use of shorter three- and six-month estimation windows. If anything, as further discussed in Section 7, these shorter estimation windows tend to result in even stronger return-beta patterns than the ones from the one-year moving windows.

We rely on a fixed intraday sampling frequency of 75 minutes in our estimation of the continuous and jump betas, with the returns spanning 9:45 a.m. to 4:00 p.m.[26] A 75-minute sampling frequency can seem coarse compared with the five-minute sampling frequency commonly advocated in the literature on realized volatility estimation. See, e.g., Andersen, Bollerslev, Diebold, and Labys (2001) and the survey by Hansen and Lunde (2006). Yet, estimation of multivariate realized variation measures, including betas, is invariably plagued by additional market microstructure complications relative to the estimation of univariate realized volatility measures. Coarser sampling frequencies are often used as a simple way to guard against any biases induced by these complications. See, e.g., the discussion in Sheppard (2006) and Bollerslev, Law, and Tauchen (2008), along with the survey by Barndorff-Nielsen and Shephard (2007). We also experiment with a number of other intraday sampling frequencies, ranging from five minutes to three hours, as well as a mixed frequency explicitly related to the trading activity of each of the individual stocks. As further detailed in Section 7, our key empirical findings remain robust across all of these different sampling schemes.

In parallel to our high-frequency-based estimates for β^c, β^d, and β^n, our estimates for the monthly standard CAPM β^ss are based on rolling overlapping regressions of the daily returns for each of the individual stocks over

[21] This more liquid S&P sample has the advantage of allowing for relatively reliable high-frequency estimation.

[22] The original data set on average consists of more than 17 million observations per day for each trading day.

[23] The TAQ database provides only the raw prices without considering price differences before and after distributions. We use the variable Cumulative Factor to Adjust Price (CFACPR) from CRSP to adjust the high-frequency overnight returns after a distribution.

[24] The website address is http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data-library.html.

[25] The use of a relatively long estimation period is especially important for the discontinuous betas, as there can be few or even no systematic jumps for a particular stock during a particular month. See also the discussion in Todorov and Bollerslev (2010). Annual horizon moving windows are also commonly used for the estimation of traditional CAPM betas based on coarser daily or monthly observations, as in, e.g., Ang, Chen, and Xing (2006a) and Fama and French (2006).

[26] Starting the trading day at 9:45 a.m. ensures that on most days most stocks will have traded at least once by that time. Patton and Verardo (2012) adopt a similar trading day convention in their high-frequency-based realized beta estimation.

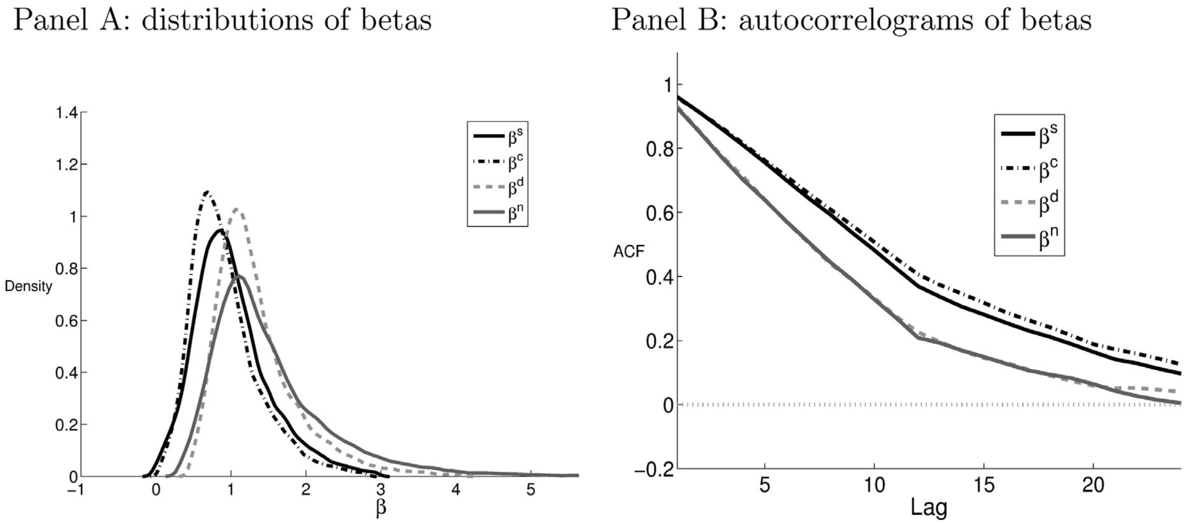

Fig. 1. Distributions and autocorrelograms of betas. Panel A displays kernel density estimates of the unconditional distributions of the four different betas averaged across firms and time. Panel B shows the monthly autocorrelograms for the four different betas averaged across firms.

the past year on the daily returns for the S&P 500 market portfolio.[27]

Turning to the actual estimation results, Panel A in Fig. 1 depicts kernel density estimates of the unconditional distributions of the four different betas averaged across time and stocks. The discontinuous and overnight betas both tend to be somewhat higher on average and more right-skewed than the continuous and standard betas.[28] At the same time, the figure suggests that the continuous betas are the least dispersed of the four betas across time and stocks. Part of the dispersion in the betas could be attributed to estimation errors. Based on the expressions derived in Todorov and Bollerslev (2010), the asymptotic standard errors for β^c and β^d averaged across all of the stocks and months in the sample equal 0.06 and 0.12, respectively, compared with 0.14 for the conventional OLS-based standard errors for the β^s estimates.[29]

Panel B of Fig. 1 shows the autocorrelograms for the four different betas averaged across stocks. The apparent kink in all four correlograms at the 11th lag is directly attributable to the use of overlapping annual windows in the monthly beta estimation. Still, the figure clearly suggests a higher degree of persistence in β^c and β^s than in β^d and β^n. This complements the existing high-frequency-based empirical evidence showing that continuous variation for most financial assets tends to be much more persistent and predictable than variation due to jumps. See, e.g., Barndorff-Nielsen and Shephard (2004b, 2006) and Andersen, Bollerslev, and Diebold (2007a).

To visualize the temporal and cross-sectional variation in the different betas, Fig. 2 shows the time series of equally weighted portfolio betas, based on monthly quintile sorts for each of the four different betas and all of the individual stocks in the sample. The variation in the β^s and β^c sorted portfolios in Panels A and B are evidently fairly close. The plots for the β^d and β^n quintile portfolios in Panels C and D, however, are distinctly different and more dispersed than the standard and continuous beta quintile portfolios.

Table 1
Cross-sectional relation of β^s, β^c, β^d, and β^n.
The table reports the estimated regression coefficients, robust t-statistics (in parentheses), and adjusted R^2s from Fama–MacBeth type regressions for explaining the cross-sectional variation in the standard β^s as a function of the continuous beta β^c, the discontinuous beta β^d, and the overnight beta β^n. All of the betas are computed from high-frequency data using a 12-month overlapping monthly estimation scheme.

Regression	β^c	β^d	β^n	Adjusted R^2
I	1.03			0.76
	(58.67)			
II		0.79		0.62
		(26.72)		
III			0.51	0.46
			(16.15)	
IV	0.78	0.17	0.10	0.81
	(29.64)	(6.87)	(7.10)	

[27] As an alternative to the standard CAPM betas, we have investigated high-frequency realized betas as in Andersen, Bollerslev, Diebold, and Wu (2005, 2006). The cross-sectional pricing results for these alternative standard beta estimates are very similar to the ones reported for the standard daily CAPM betas. Further details on these additional results are available upon request.

[28] The value-weighted averages of all the different betas should be equal to unity when averaged across the exact 500 stocks included in the S&P 500 index at a particular time. In practice, we are measuring the betas over nontrivial annual time intervals, and the S&P 500 constituents and their weights also change over time, so the averages will not be exactly equal to one. For example, the value-weighted averages for β^s, β^c, β^d, and β^n based on the exact 500 stocks included in the index at the very end of the sample equal 1.04, 0.98, 1.01, and 1.06, respectively.

[29] Intuitively, the continuous beta estimator could be interpreted as a regression based on truncated high-frequency intraday returns. As such, the standard errors should be reduced by a factor of approximately $1/\sqrt{n}$, relative to the standard errors for the standard betas based on daily returns, where n denotes the number of intradaily observations used in the estimation.

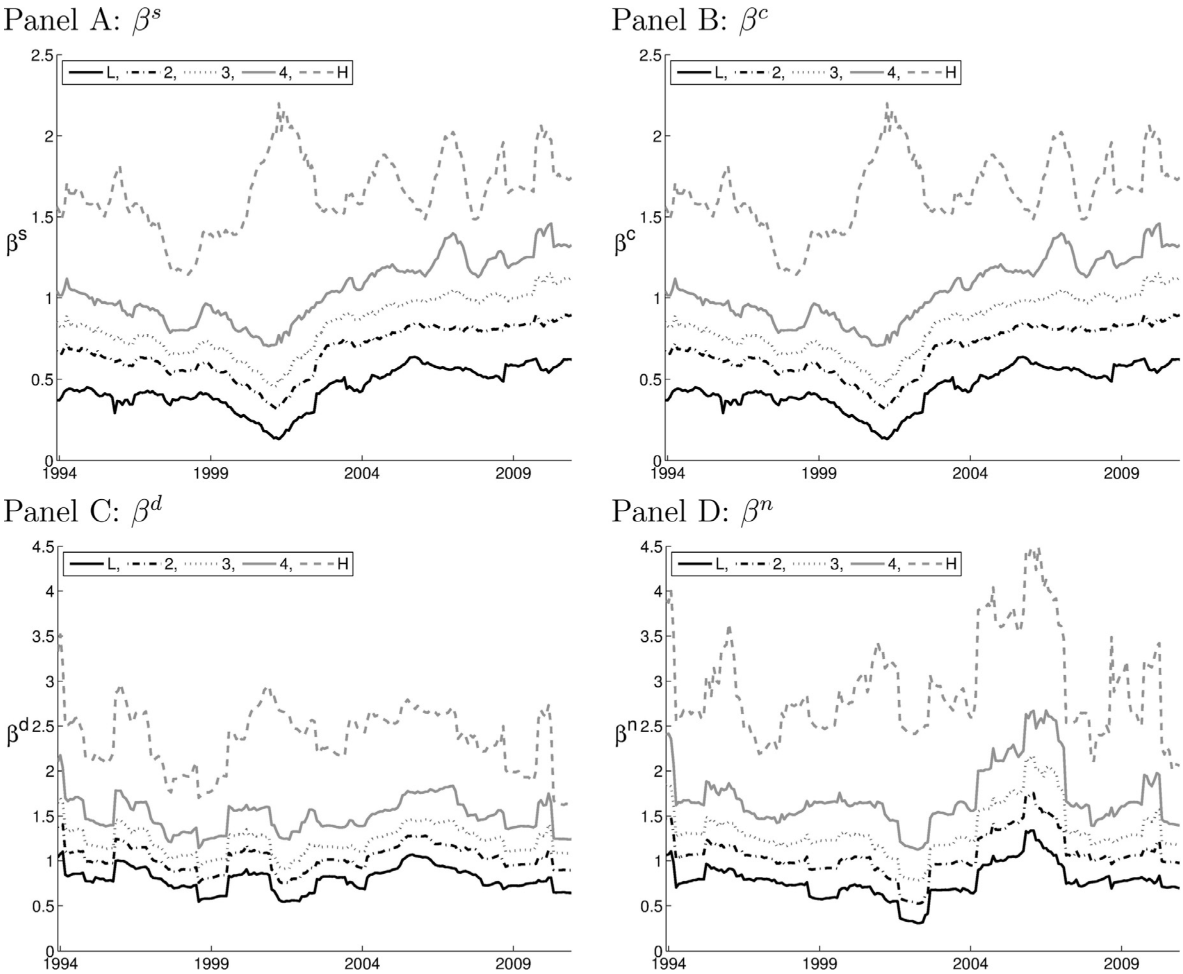

Fig. 2. Time series plots of betas. The figure displays the times series of the averages of the betas for each of the beta-sorted quintile portfolios. Panel A shows the results for the standard beta β^s-sorted portfolios; Panel B, the continuous beta β^c-sorted portfolios; Panel C, the discontinuous beta β^d-sorted portfolios; and Panel D, the overnight beta β^n-sorted portfolios.

To further illuminate these relations, Table 1 reports the results from Fama–MacBeth style regressions for explaining the cross-sectional variation in the standard betas as a function of the variation in the three other betas. Consistent with the results in Figs. 1 and 2, the continuous beta β^c exhibits the highest explanatory power for β^s, with an average adjusted R^2 of 0.76. The two jump betas β^d and β^n each explain 62% and 46% of the variation in β^s, respectively. Altogether, 81% of the cross-sectional variation in β^s can be accounted for by the high-frequency betas, with β^c having by far the largest and most significant effect.

The differences in information content of the betas also manifest in different relations with the underlying continuous and discontinuous price variation. Relying on the truncation rules discussed in Section 3, the intraday discontinuous variation and the overnight variation account for approximately 9% and 30% of the total variation at the aggregate market level. Applying the same truncation rule to the individual stocks, the discontinuous and overnight variation account for an average of 10% and 32%, respectively, at the individual firm level. Meanwhile, when sorting the stocks according to the four different betas, the sorts reveal a clear monotonic relation between β^d and the jump contribution and between β^n and the overnight contribution, but an inverse relation between β^c and the proportion of the total variation accounted for by jumps.

4.3. Other explanatory variables and controls

A long list of prior empirical studies have sought to relate the cross-sectional variation in stock returns to other explanatory variables and firm characteristics. To guard against some of the most prominent previously shown effects and anomalies vis-à-vis the standard CAPM in the double portfolio sorts and cross-sectional regressions reported below, we explicitly control for firm size (ME), book-to-market ratio (BM), momentum (MOM), reversal (REV), idiosyncratic volatility (IVOL), coskewness (CSK), cokurtosis (CKT), realized skewness (RSK), realized kurtosis (RKT), maximum daily return (MAX), and illiquidity (ILLIQ). Our construction of these different control variables follows standard procedures in the literature, as discussed in more detail in Appendix A.2.

Table 2 displays time series averages of monthly firm-level cross-sectional correlations between the four different betas and the various explanatory variables listed above. All of the four betas are negatively related to book-to-market and positively correlated with momentum. The betas are also generally positively correlated with idiosyncratic volatility, and the two jump betas more strongly so. While β^s and β^c are negatively correlated with illiquidity, β^d and β^n appear to be positively related to illiquidity.

To help further gauge these relations, Table 3 reports the results from a series of simple single-sorts. At the end of each month, we sort stocks by each of their betas. We then form five equal-size portfolios and compute the time series averages of the various firm characteristics for the stocks within each of these quintile portfolios. Consistent with the results discussed above, the portfolio sorts reveal a strong positive relation between all of the four different betas. Meanwhile, it also follows from Panels C and D that stocks with higher β^d and β^n tend to be smaller firms, with lower book-to-market ratios, higher momentum, and higher idiosyncratic volatility.[30] Higher discontinuous and overnight betas also tend to be associated with higher illiquidity, and the differences in illiquidity between high- and low-quintile portfolios for the continuous and standard beta sorts in Panels A and B are both negative.[31]

5. Portfolio sorts

We begin our empirical investigations pertaining to the pricing of different market price risks with an examination of the return differentials among portfolios sorted according to the different betas. We consider single-sorted contemporaneous and predictive portfolios, double-sorted predictive portfolios designed to control for other risk factors and firm characteristics, and reverse double-sorted predictive portfolios by first sorting on betas and then on explanatory variables.

5.1. Contemporaneous single-sorted portfolios

We estimate the four different betas at the beginning of each month based on the next 12-month returns. We then sort the stocks into quintile portfolios based on their betas and record the returns over the same 12-month period. Rebalancing monthly, we record the excess returns on each portfolio, starting with the first portfolio formation period spanning the first full year of the sample, ending with the last full year of the sample. This approach directly mirrors the single portfolio sorts commonly employed in the literature (see, e.g., Ang, Chen, and Xing, 2006a, among numerous other studies).

Panel A of Table 4 reports the average monthly returns for portfolios sorted by the standard beta. Consistent with the standard CAPM, the average excess returns increase

Table 2
Sample correlations.
The table displays time series averages of monthly cross-sectional correlations. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P 500) index over 1993–2010. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of the firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT are the measures of coskewness and cokurtosis, respectively. RSK and RKT denote the realized skewness and the realized kurtosis, respectively, computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t. * and ** indicate significance at the 5% and 1% level, respectively.

	β^s	β^c	β^d	β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
β^s	1	0.88**	0.76**	0.63**	−0.12**	−0.15**	0.10**	0.01	0.46**	0.04*	0.38**	−0.03**	−0.07**	0.47**	−0.04*
β^c		1	0.77**	0.60**	−0.02	−0.17**	0.09**	0.01	0.43**	0.07**	0.26**	−0.03**	−0.15**	0.43**	−0.08**
β^d			1	0.74**	−0.27**	−0.11**	0.08**	0.01	0.58**	0.01	0.06**	−0.05**	−0.02**	0.53**	0.15**
β^n				1	−0.22**	−0.11**	0.03**	0.01	0.53**	0.01	−0.01	−0.04**	−0.02	0.48**	0.13**
ME					1	−0.15**	−0.04**	−0.03**	−0.34**	0.04**	0.30**	0.00	−0.40**	−0.28**	−0.91**
BM						1	−0.04	0.00	−0.08**	−0.06**	−0.07**	0.01	0.05**	−0.07**	0.14**
MOM							1	0.02*	0.00	−0.07**	0.06*	−0.02	0.04**	0.00	0.05**
REV								1	0.03*	−0.01	−0.02**	0.37**	0.02**	0.30**	−0.01*
IVOL									1	0.00	−0.22**	−0.04**	0.11**	0.81**	0.26**
CSK										1	−0.03	0.02**	−0.05**	0.03*	−0.05**
CKT											1	0.00	−0.16**	−0.11**	−0.27**
RSK												1	0.04**	0.04**	0.00
RKT													1	0.07**	0.41**
MAX														1	0.21**
ILLIQ															1

[30] In a recent study, Alexeev, Dungey, and Yao (2015) find that smaller stocks tend to have higher discontinuous betas than larger stocks and that, during periods of financial distress, high leverage stocks are more exposed to continuous risks.

[31] Bali, Engle, and Tang (2015) and Fu (2009) also report a negative relation between standard betas and illiquidity.

Table 3
Portfolio characteristics sorted by betas.
The table displays time series averages of equal-weighted characteristics of stocks sorted by the four different betas. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P 500) index over 1993–2010. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT are the measures of coskewness and cokurtosis, respectively. RSK and RKT denote the realized skewness and the realized kurtosis computed from high-frequency data. MAX represents the maximum daily raw return over month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t. Panel A displays the results sorted by β^s; Panel B, by β^c; Panel C, by β^d; and Panel D, by β^n.

Quintile	β^s	β^c	β^d	β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
Panel A: Sorted by β^s															
1 (Low)	0.45	0.52	1.02	1.11	8.48	0.56	8.37	0.76	1.37	−0.10	1.51	0.03	5.31	3.54	−2.78
2	0.73	0.70	1.12	1.22	8.56	0.54	9.59	0.86	1.48	−0.10	2.04	0.04	5.42	4.03	−3.08
3	0.93	0.83	1.23	1.37	8.58	0.47	10.59	0.95	1.60	−0.10	2.31	0.03	5.39	4.52	−3.18
4	1.18	1.03	1.45	1.67	8.62	0.50	12.80	1.26	1.83	−0.09	2.48	0.02	5.26	5.33	−3.28
5 (High)	1.83	1.58	2.08	2.43	8.43	0.43	13.84	1.48	2.57	−0.09	2.52	0.02	5.10	7.78	−3.40
Panel B: Sorted by β^c															
1 (Low)	0.57	0.44	0.99	1.14	8.54	0.54	7.80	0.81	1.39	−0.11	1.73	0.03	5.54	3.68	−2.32
2	0.79	0.67	1.09	1.22	8.73	0.50	11.28	0.83	1.47	−0.11	2.12	0.03	5.40	4.07	−2.86
3	0.95	0.83	1.23	1.38	8.77	0.45	10.55	0.95	1.59	−0.10	2.31	0.03	5.31	4.53	−3.29
4	1.18	1.05	1.46	1.67	8.86	0.49	11.40	1.13	1.83	−0.10	2.45	0.03	5.13	5.29	−3.52
5 (High)	1.80	1.65	2.15	2.45	8.63	0.41	12.60	1.44	2.54	−0.08	2.44	0.02	4.94	7.64	−3.57
Panel C: Sorted by β^d															
1 (Low)	0.60	0.53	0.80	0.93	8.94	0.54	9.16	0.78	1.15	−0.10	1.96	0.04	5.46	3.14	−3.35
2	0.81	0.71	1.03	1.17	8.91	0.49	9.68	0.83	1.39	−0.10	2.22	0.04	5.32	3.90	−3.23
3	0.96	0.85	1.22	1.40	8.86	0.49	10.62	0.90	1.60	−0.10	2.28	0.03	5.22	4.54	−3.26
4	1.18	1.03	1.50	1.72	8.68	0.49	11.24	1.09	1.92	−0.09	2.32	0.03	5.16	5.51	−3.19
5 (High)	1.73	1.51	2.37	2.63	8.14	0.37	13.33	1.56	2.77	−0.09	2.27	0.02	5.16	8.16	−2.82
Panel D: Sorted by β^n															
1 (Low)	0.64	0.59	0.90	0.78	8.95	0.55	9.48	0.71	1.14	−0.10	2.06	0.04	5.38	3.11	−3.56
2	0.83	0.74	1.09	1.08	8.91	0.50	10.11	0.81	1.41	−0.10	2.24	0.04	5.30	3.93	−3.27
3	0.98	0.86	1.26	1.34	8.81	0.49	10.83	0.93	1.61	−0.10	2.27	0.03	5.26	4.58	−3.23
4	1.20	1.04	1.50	1.72	8.66	0.48	11.83	1.12	1.93	−0.10	2.32	0.02	5.18	5.56	−3.20
5 (High)	1.65	1.42	2.17	2.95	8.20	0.35	14.51	1.60	2.75	−0.09	2.17	0.02	5.19	8.09	−2.97

across the β^s segments.[32] The spread between the high- and low-β^s quintile portfolios is only weakly statistically significant, however. The results for the continuous beta portfolio sorts reported in Panel B are comparable, with the return spread and t-statistic for the high–low β^c-sorted portfolios equal to 1.61% and 1.81, respectively. By comparison, the results for the two rough beta sorts reported in Panels C and D, respectively, both show a stronger and more reliable relation between the betas and the contemporaneous portfolio returns. For the β^d sorts the return spread for the high–low portfolio equals 1.71% with a t-statistic of 2.63, and for the β^n sorts the spread and the corresponding t-statistic equal 1.64% and 2.59, respectively.

To more directly explore the idea that most of the premiums for market price risks stem from the compensation for jump risk, Panels E and F report the results based on portfolios sorted by the relative betas $\beta^d - \beta^s$ and $\beta^n - \beta^s$, respectively. As evident from the almost flat β^s loadings coupled with the increasing β^d or β^n loadings over the different quintiles, the relative betas effectively eliminate the part of the cross-sectional variation in each of the two jump betas that could be explained by the variation in the standard beta.[33] Even though the spreads in the returns are smaller when sorting on these relative jump betas compared with the sorts based on the individual betas, the t-statistics equal to 3.34 and 3.18 are both higher than the t-statistics associated with any of the individual beta sorts. Similarly, sorting the stocks into portfolios according to the two jump betas in excess of the continuous beta, $\beta^d - \beta^c$ and $\beta^n - \beta^c$, generate return spreads of 1.21% and 0.77% (Panels G and H), with statistically significant t-statistics of 3.34 and 2.48, respectively. As such, these relative beta sorts clearly highlight the differences in the risks measured by the jump betas and the standard and continuous betas, and the pricing thereof.

5.2. Predictive single-sorted portfolios

The contemporaneous portfolio sorts pertain to returns and betas estimated over the same holding period. While this represents the essence of the risk-return relation implied by the theoretical framework in Section 2, these results are not of much practical value if the betas cannot be used to predict the future returns. In this subsection, we extend the previous sorts to a predictive setting.

[32] Even though the relation is monotonic, most of the spread in the returns between the high and low portfolios comes from the spread between the fourth and highest quintile. This is true for many of the other portfolio sorts discussed below as well.

[33] Analogous relative beta measures have also been used by Ang, Chen, and Xing (2006a) in their study of downside beta risk and by Bali, Engle, and Tang (2015) in their study of dynamic conditional betas.

Table 4
Contemporaneous single-sorted portfolios.
The table reports the average returns and betas for single-sorted portfolios. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P 500) index over 1993–2010. At the beginning of each month, stocks are sorted into quintiles according to betas computed from the next 12-month returns. Each equal-weighted portfolio is held for 12 months. The column labeled "Return" reports the average monthly excess return in the 12-month holding period for each portfolio. The row labeled "High–Low" reports the difference in returns between Portfolios 5 and 1, with Newey–West robust t-statistics in parentheses. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. Panel A displays the results sorted by β^s; Panel B, by β^c; Panel C, by β^d; Panel D, by β^n; Panel E, by $\beta^d-\beta^s$; Panel F, by $\beta^n-\beta^s$; Panel G, by $\beta^d-\beta^c$; and Panel H, by $\beta^n-\beta^c$.

Quintile	β^s	β^c	β^d	β^n	Return	Quintile	β^s	β^c	β^d	β^n	Return
Panel A: Sorted by β^s						*Panel E: Sorted by $\beta^d-\beta^s$*					
1 (Low)	0.45	0.52	1.02	1.11	0.77	1 (Low)	1.17	0.96	1.22	1.43	0.97
					(3.66)						(2.68)
2	0.73	0.70	1.12	1.22	0.81	2	0.96	0.87	1.13	1.30	0.94
					(3.14)						(3.44)
3	0.93	0.83	1.23	1.37	0.97	3	0.95	0.84	1.20	1.37	1.08
					(3.21)						(3.88)
4	1.18	1.03	1.45	1.67	1.17	4	1.02	0.87	1.36	1.55	1.25
					(3.18)						(4.13)
5 (High)	1.83	1.58	2.08	2.43	2.39	5 (High)	1.19	1.10	2.02	2.22	1.91
					(2.90)						(3.56)
High–Low	1.38	1.06	1.06	1.32	1.62	High–Low	0.02	0.14	0.80	0.79	0.94
					(1.85)						(3.34)
Panel B: Sorted by β^c						*Panel F: Sorted by $\beta^n-\beta^s$*					
1 (Low)	0.57	0.44	0.99	1.14	0.80	1 (Low)	1.16	1.03	1.29	1.06	0.98
					(3.55)						(3.22)
2	0.79	0.67	1.09	1.22	0.82	2	0.93	0.84	1.17	1.13	1.17
					(3.14)						(3.87)
3	0.95	0.83	1.23	1.38	0.91	3	0.97	0.86	1.24	1.33	1.17
					(3.15)						(4.16)
4	1.18	1.05	1.46	1.67	1.20	4	1.07	0.91	1.38	1.62	1.29
					(3.26)						(3.80)
5 (High)	1.80	1.65	2.15	2.45	2.41	5 (High)	1.15	1.00	1.85	2.73	1.68
					(2.92)						(3.01)
High–Low	1.23	1.21	1.16	1.31	1.61	High–Low	−0.01	−0.03	0.56	1.67	0.69
					(1.81)						(3.18)
Panel C: Sorted by β^d						*Panel G: Sorted by $\beta^d-\beta^c$*					
1 (Low)	0.60	0.53	0.80	0.93	0.78	1 (Low)	1.08	1.01	1.17	1.32	0.88
					(3.80)						(2.55)
2	0.81	0.71	1.03	1.17	0.83	2	0.96	0.88	1.12	1.28	0.96
					(3.21)						(3.51)
3	0.96	0.85	1.22	1.40	0.85	3	0.96	0.88	1.21	1.39	1.09
					(2.99)						(3.95)
4	1.18	1.03	1.50	1.72	1.21	4	1.05	0.94	1.40	1.60	1.29
					(3.31)						(4.33)
5 (High)	1.73	1.51	2.37	2.63	2.48	5 (High)	1.21	1.05	2.02	2.26	2.08
					(2.97)						(3.64)
High–Low	1.13	0.98	1.57	1.70	1.71	High–Low	0.13	0.04	0.84	0.93	1.21
					(2.63)						(3.34)
Panel D: Sorted by β^n						*Panel H: Sorted by $\beta^n-\beta^c$*					
1 (Low)	0.64	0.59	0.90	0.78	0.75	1 (Low)	1.02	1.09	1.25	1.01	0.93
					(3.84)						(3.01)
2	0.83	0.74	1.09	1.08	0.83	2	0.93	0.86	1.17	1.13	1.06
					(3.37)						(3.91)
3	0.98	0.86	1.26	1.34	0.96	3	0.97	0.86	1.24	1.33	1.23
					(3.29)						(4.29)
4	1.20	1.04	1.50	1.72	1.22	4	1.07	0.90	1.38	1.63	1.38
					(3.29)						(3.95)
5 (High)	1.65	1.42	2.17	2.95	2.39	5 (High)	1.26	1.04	1.87	2.77	1.70
					(2.91)						(3.22)
High–Low	1.01	0.83	1.27	2.17	1.64	High–Low	0.24	−0.05	0.62	1.75	0.77
					(2.59)						(2.48)

In parallel to the previous contemporaneous sorts, we estimate the different betas based on the past 12-month returns. We then sort the stocks according to each of the different betas and record the returns for the following month. In addition to the resulting pre-formation beta estimates and the predictive ex post excess returns for each of these equally weighted quintile portfolios, we record the ex post betas for the different portfolios based on the beta estimates for the 12 months proceeding the pre-formation period, together with the risk-adjusted excess returns, as measured by the intercept from a time series regression of the monthly portfolio returns on the four Fama–French–Carhart factors.

Table 5 summarizes the results. Comparing the ex post betas with the pre-formation beta estimates, the high–low quintile spreads are naturally dampened somewhat relative to the ex ante measures. However, consistent with the slowly decaying autocorrelations for the betas shown in Fig. 2, the high–low spreads remain quite sizable for all of the four individual beta sorts in Panels A–D. The spreads in the ex post relative betas for the corresponding sorts reported in Panels E–H, are, not surprisingly, reduced by more than the spreads in the ex post betas for the individual beta sorts, but the spreads remain nontrivial.

This persistence in the betas translate into similar predictive return-beta relations to the ones showed for the contemporaneous return in Section 5.1. We continue to see a monotone relation between the future portfolio returns and the past betas. Directly in line with the previous contemporaneous portfolio sorts, the relations are stronger and more statistically significant for the rough betas than for the standard and continuous betas. In particular, focusing on the risk-adjusted FFC4 alphas, the *t*-statistics for the high–low quintile portfolios based on the β^s and β^c sorts equal 1.76 and 1.44, respectively, compared with 2.04 and 2.74 for the β^d and β^n predictive sorts, and the *t*-statistics for the four relative $\beta^d - \beta^s$, $\beta^n - \beta^s$, $\beta^d - \beta^c$, and $\beta^n - \beta^c$ sorts equal 2.29, 3.05, 1.91, and 2.90, respectively. Thus, the results suggest that the discontinuous and overnight betas are better able to predict the cross-sectional variation in the future returns than the continuous and standard betas. Furthermore, the relations between the rough betas and the future returns cannot be explained by the size, book-to-market ratio, and momentum effects captured by the Fama–French–Carhart factors.

The positive relations between betas and future returns contrast with the recent results in Frazzini and Pedersen (2014), who report an almost flat security market line and highly significant positive risk-adjusted alphas for portfolios betting against beta (BAB). Compared with our investigations, which are limited by the availability of reliable high-frequency intraday data and as such involves only 985 relatively large company stocks over the past two decades, the results in Frazzini and Pedersen (2014) are based on a much larger sample of more than 20 thousand stocks spanning almost a full century. Both our more recent sample and our sample of stocks help explain the differences. Restricting the sample period to be the same as the one used here, the FFC4 alpha of the monthly BAB factor equals 0.41%, with a *t*-statistic of 1.40.[34] This indicates a weaker BAB effect compared with the alpha of 0.55%, with a *t*-statistic of 5.59, for the much longer sample period analyzed in Frazzini and Pedersen (2014). However, the more important explanation for the difference in the results arguably stems from the differences in the samples of stocks. Constructing a BAB factor from the same 985 S&P 500 constituent stocks used here based on the standard betas and the same approach as in Frazzini and Pedersen (2014) results in an FFC4 alpha for the high–low portfolio of -0.38%, with a *t*-statistic of -1.42, consistent with the results for the single-sorts reported in Panel A of Table 5.[35] Irrespective of these results, it is important to recognize that a priced BAB factor is not at odds with the idea that continuous and discontinuous market price moves could be priced differently by investors.

5.3. Predictive double-sorted portfolios

Our single portfolio sorts reveal that stocks with high discontinuous and overnight betas tend to have high returns and that the return differences are greater than the differences for stocks sorted by their continuous and standard betas. This predictive power of the discontinuous and overnight betas cannot be explained by the standard or continuous betas, as little variation exists in those betas across the corresponding relative beta quintiles. However, the relative beta sorts do not explicitly separate the effects of β^d or β^n from those of β^s or β^c. In an effort to more directly pin down the source of the risk premiums for the different betas, we therefore augment the single-sorts with a series of double-sorts that first control for each of the different betas. Moreover, as discussed in Section 4.3, because the cross-sectional variation in the betas could be related to other firm characteristics and explanatory variables that have previously been shown to help predict the cross-sectional variation in stock returns, we also report the results from double-sorts designed to control for some of these other explanatory variables.

To implement the double-sorts, we first sort all of the stocks into five quintiles according to each of the different explanatory variables for each of the months in the sample. Within each quintile, we then sort stocks into five additional quintiles according to one of the four different beta measures. Finally, we average the returns on the five beta portfolios across the five different control variable portfolios to produce beta portfolios with large cross-portfolio variations in their betas, but little variation in the control variable.

The first two columns in Panels A–D in Table 6 display the results from these double-sorts for the different beta controls. In Panel A, for example, the two columns labeled "β^d" and "β^n" report the average returns for the β^s quintile portfolios after controlling for β^d and β^n, respectively. The resulting FFC4 alphas of the high–low spreads equal 0.34% and 0.23%, respectively, with *t*-statistics of 1.56 and 1.00. Both of these alphas and *t*-statistics are smaller than the alpha of 0.58% and the *t*-statistic of 1.76 in Panel A of Table 5, suggesting that β^d and β^n absorb much of the predictability inherent in β^s. Similarly, looking at Panel B shows that first controlling for β^d or β^n absorbs almost all of the spread in the β^c- sorted portfolios. By contrast, first controlling for β^s or β^c, the predicability of β^d or

[34] The monthly BAB factor is available from the AQR data library via https://www.aqr.com/library/data-sets/betting-against-beta-equity-factors-monthly.

[35] It would be interesting, but beyond the scope and main focus of the present paper, to further explore whether these differences in the significance of the BAB factor over different time periods and different samples of stocks can be explained by differences in liquidity or the influence of financially constrained investors, or both.

Table 5

Predictive single-sorted portfolios.

The table reports the average returns and betas for predictive single-sorted portfolios. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P) 500 index over 1993–2010. At the end of each month, stocks are sorted into quintiles according to betas computed from previous 12-month returns. Each portfolio is held for one month. The column labeled "Ex Post" reports the ex-post betas compute from the subsequent 12-month returns. The column labeled "Return" reports the average one-month-ahead excess returns of each portfolio. The column labeled "FFC4 alpha" reports the corresponding Fama–French–Carhart four-factor alpha for each portfolio. The row labeled "High–Low" reports the difference in returns between Portfolios 10 and 1, with Newey–West robust t-statistics in parentheses. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. Panel A displays the results sorted by β^s; Panel B, by β^c; Panel C, by β^d; Panel D, by β^n; Panel E, by $\beta^d-\beta^s$; Panel F, by $\beta^n-\beta^s$; Panel G, by $\beta^d-\beta^c$; and Panel H, by $\beta^n-\beta^c$.

Panel A: Sorted by β^s

Quintile	β^s	β^c	β^d	β^n	Ex Post β^s	Return	FFC4 alpha
1 (Low)	0.45	0.52	1.02	1.11	0.60	0.76	0.32
						(3.09)	(2.07)
2	0.73	0.70	1.12	1.22	0.80	0.85	0.31
						(2.74)	(2.08)
3	0.93	0.83	1.23	1.37	0.95	1.00	0.38
						(2.76)	(2.95)
4	1.18	1.03	1.45	1.67	1.12	1.23	0.57
						(2.86)	(4.12)
5 (High)	1.83	1.58	2.08	2.43	1.63	1.59	0.90
						(2.29)	(3.87)
High–Low	1.38	1.06	1.06	1.32	1.04	0.83	0.58
						(1.40)	(1.76)

Panel B: Sorted by β^c

Quintile	β^s	β^c	β^d	β^n	Ex Post β^c	Return	FFC4 alpha
1 (Low)	0.57	0.44	0.99	1.14	0.57	0.79	0.33
						(3.05)	(2.20)
2	0.79	0.67	1.09	1.22	0.74	0.79	0.29
						(2.55)	(1.97)
3	0.95	0.83	1.23	1.38	0.84	0.97	0.35
						(2.65)	(2.59)
4	1.18	1.05	1.46	1.67	1.02	1.19	0.54
						(2.80)	(4.01)
5 (High)	1.80	1.65	2.15	2.45	1.51	1.50	0.82
						(2.18)	(3.45)
High–Low	1.23	1.21	1.16	1.31	0.93	0.71	0.49
						(1.22)	(1.44)

Panel C: Sorted by β^d

Quintile	β^s	β^c	β^d	β^n	Ex Post β^d	Return	FFC4 alpha
1 (Low)	0.60	0.53	0.80	0.93	0.93	0.72	0.30
						(2.94)	(2.20)
2	0.81	0.71	1.03	1.17	1.12	0.76	0.23
						(2.37)	(1.73)
3	0.96	0.85	1.22	1.40	1.24	0.95	0.36
						(2.62)	(2.77)
4	1.18	1.03	1.50	1.72	1.46	1.17	0.52
						(2.72)	(3.71)
5 (High)	1.73	1.51	2.37	2.63	2.07	1.65	0.94
						(2.39)	(4.08)
High–Low	1.13	0.98	1.57	1.70	1.13	0.93	0.64
						(1.55)	(2.04)

Panel D: Sorted by β^n

Quintile	β^s	β^c	β^d	β^n	Ex Post β^n	Return	FFC4 alpha
1 (Low)	0.64	0.59	0.90	0.78	1.02	0.64	0.21
						(2.53)	(1.52)
2	0.83	0.74	1.09	1.08	1.26	0.76	0.24
						(2.45)	(2.06)
3	0.98	0.86	1.26	1.34	1.45	0.96	0.35
						(2.67)	(2.69)
4	1.20	1.04	1.50	1.72	1.72	1.14	0.47
						(2.61)	(3.58)
5 (High)	1.65	1.42	2.17	2.95	2.34	1.75	1.06
						(2.60)	(4.69)
High–Low	1.01	0.83	1.27	2.17	1.32	1.11	0.85
						(1.93)	(2.74)

Panel E: Sorted by $\beta^d-\beta^s$

Quintile	β^s	β^c	β^d	β^n	Ex Post $\beta^d-\beta^s$	Return	FFC4 alpha
1 (Low)	1.17	0.96	1.22	1.43	0.20	0.99	0.47
						(2.35)	(3.18)
2	0.96	0.87	1.13	1.30	0.24	0.81	0.27
						(2.41)	(2.51)
3	0.95	0.84	1.20	1.37	0.27	0.93	0.38
						(2.77)	(3.61)
4	1.02	0.87	1.36	1.55	0.30	1.02	0.44
						(2.87)	(3.40)
5 (High)	1.19	1.10	2.02	2.22	0.54	1.49	0.78
						(2.91)	(4.54)
High–Low	0.02	0.14	0.80	0.79	0.34	0.51	0.31
						(1.85)	(2.29)

Panel F: Sorted by $\beta^n-\beta^s$

Quintile	β^s	β^c	β^d	β^n	Ex Post $\beta^n-\beta^s$	Return	FFC4 alpha
1 (Low)	1.16	0.98	1.29	1.06	0.31	0.68	0.15
						(1.80)	(0.82)
2	0.93	0.84	1.17	1.13	0.38	0.81	0.23
						(2.40)	(1.91)
3	0.97	0.86	1.24	1.33	0.47	1.01	0.43
						(2.99)	(3.74)
4	1.07	0.91	1.38	1.62	0.57	1.03	0.45
						(2.67)	(3.44)
5 (High)	1.15	1.05	1.85	2.73	0.88	1.71	1.09
						(3.18)	(5.81)
High–Low	−0.01	0.07	0.56	1.67	0.57	1.03	0.94
						(3.09)	(3.05)

Panel G: Sorted by $\beta^d-\beta^c$

Quintile	β^s	β^c	β^d	β^n	Ex Post $\beta^d-\beta^c$	Return	FFC4 alpha
1 (Low)	1.08	1.01	1.17	1.32	0.27	0.94	0.42
						(2.32)	(2.91)
2	0.96	0.88	1.12	1.28	0.33	0.80	0.24
						(2.30)	(2.27)
3	0.96	0.88	1.21	1.39	0.37	0.97	0.40
						(2.90)	(3.70)
4	1.05	0.94	1.40	1.60	0.44	0.96	0.36
						(2.60)	(2.69)
5 (High)	1.21	1.05	2.02	2.26	0.74	1.63	0.90
						(3.08)	(5.08)
High–Low	0.13	0.04	0.84	0.93	0.47	0.69	0.47
						(2.24)	(1.91)

Panel H: Sorted by $\beta^n-\beta^c$

Quintile	β^s	β^c	β^d	β^n	Ex Post $\beta^n-\beta^c$	Return	FFC4 alpha
1 (Low)	1.02	1.09	1.25	1.01	0.41	0.68	0.14
						(1.78)	(0.72)
2	0.93	0.86	1.17	1.13	0.49	0.79	0.20
						(2.35)	(1.74)
3	0.97	0.86	1.24	1.33	0.58	1.04	0.44
						(3.01)	(3.78)
4	1.07	0.90	1.38	1.63	0.68	1.06	0.45
						(2.75)	(3.40)
5 (High)	1.26	1.04	1.87	2.77	0.98	1.72	1.08
						(3.16)	(5.58)
High–Low	0.24	−0.05	0.62	1.75	0.58	1.03	0.95
						(2.95)	(2.90)

Table 6
Predictive double-sorted portfolios.
The table reports the average returns for predictive double-sorted portfolios. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P) 500 index over 1993–2010. For each month, all stocks in the sample are first sorted into five quintiles on the basis of one control variable. Within each quintile, the stocks are then sorted into five quintiles according to their betas. These five beta portfolios are then averaged across the five control variable portfolios to produce beta portfolios with large cross-portfolio variation in their betas but little variation in the control variable. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT are the measures of coskewness and cokurtosis, respectively. RSK and RKT denote the realized skewness and realized kurtosis computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t. The first five rows in each panel report time series averages of monthly excess returns for the beta quintile portfolios. The row labeled "High–Low" reports the difference in the returns between Portfolios 5 and 1. The row labeled "FFC4 alpha" reports the average Fama–French–Carhart four-factor alphas. The corresponding Newey–West robust t-statistics are reported in parentheses. Panels A, B, C, and D display the results for the portfolios first sorted by the control variables listed in the columns and then by β^s, β^c, β^d, and β^n, respectively.

Quintile	β^d	β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
Panel A: Final sort by β^s													
1 (Low)	0.96	0.91	0.74	0.72	0.92	0.73	0.89	0.85	0.72	0.69	0.59	0.87	0.77
2	0.94	1.08	0.93	0.88	0.96	0.97	1.02	0.82	0.81	0.81	0.86	0.93	0.86
3	0.98	0.98	0.95	1.00	1.11	0.97	1.04	0.96	1.00	0.98	0.95	0.93	0.98
4	0.99	0.99	1.20	1.18	1.13	1.27	1.19	1.20	1.14	1.32	1.23	1.22	1.17
5 (High)	1.38	1.28	1.61	1.58	1.24	1.52	1.30	1.61	1.75	1.50	1.67	1.48	1.63
High–Low	0.41	0.37	0.87	0.86	0.32	0.78	0.41	0.76	1.03	0.81	1.08	0.61	0.86
	(1.46)	(1.12)	(1.54)	(1.64)	(1.47)	(1.55)	(1.01)	(1.43)	(1.91)	(1.36)	(1.84)	(1.58)	(1.57)
FFC4 alpha	0.34	0.23	0.63	0.57	0.08	0.52	0.22	0.46	0.70	0.52	0.77	0.42	0.60
	(1.56)	(1.00)	(2.01)	(1.87)	(1.33)	(1.33)	(0.84)	(1.50)	(2.34)	(1.56)	(2.30)	(1.72)	(2.19)
Panel B: Final sort by β^c													
1 (Low)	1.11	1.11	0.76	0.74	0.93	0.81	0.88	0.81	0.80	0.77	0.75	0.90	0.80
2	1.02	1.07	0.90	0.86	0.96	0.93	0.96	0.88	0.81	0.87	0.87	0.87	0.78
3	0.94	0.96	0.90	0.98	0.97	0.91	1.02	0.93	0.95	0.94	0.88	1.00	0.97
4	0.98	0.96	1.10	1.14	1.19	1.10	1.11	1.06	1.06	1.19	1.19	1.04	1.06
5 (High)	1.19	1.17	1.57	1.45	1.23	1.56	1.30	1.59	1.65	1.49	1.60	1.46	1.64
High–Low	0.08	0.06	0.81	0.71	0.30	0.74	0.42	0.78	0.85	0.72	0.85	0.55	0.84
	(0.32)	(0.21)	(1.46)	(1.42)	(1.61)	(1.52)	(1.08)	(1.50)	(1.67)	(1.27)	(1.51)	(1.47)	(1.53)
FFC4 alpha	0.07	−0.01	0.60	0.42	0.13	0.51	0.25	0.51	0.55	0.49	0.57	0.37	0.60
	(0.32)	(0.06)	(1.75)	(1.35)	(1.44)	(1.66)	(0.90)	(1.55)	(1.72)	(1.45)	(1.66)	(1.50)	(2.11)
Panel C: Final sort by β^d													
1 (Low)	0.93	0.79	0.74	0.70	0.82	0.72	0.82	0.73	0.71	0.74	0.70	0.74	0.73
2	1.00	0.94	0.88	0.76	0.90	0.81	0.79	0.81	0.85	0.78	0.78	0.74	0.87
3	0.89	0.89	0.96	0.92	0.97	0.90	1.12	0.96	0.88	0.90	0.92	1.06	0.93
4	0.96	1.00	1.19	1.17	1.15	1.12	1.17	1.07	1.11	1.18	1.12	1.26	1.13
5 (High)	1.44	1.63	1.47	1.62	1.30	1.73	1.33	1.68	1.70	1.66	1.75	1.46	1.56
High–Low	0.51	0.85	0.73	0.92	0.48	1.02	0.50	0.95	0.99	0.92	1.05	0.72	0.83
	(1.79)	(2.74)	(1.32)	(1.70)	(1.95)	(2.02)	(1.36)	(1.79)	(1.87)	(1.58)	(1.84)	(1.90)	(1.78)
FFC4 alpha	0.35	0.69	0.51	0.60	0.26	0.73	0.28	0.65	0.66	0.61	0.73	0.48	0.56
	(1.84)	(3.76)	(1.70)	(2.10)	(1.92)	(2.59)	(1.21)	(2.29)	(2.33)	(1.95)	(2.43)	(2.12)	(2.06)
Panel D: Final sort by β^n													
1 (Low)	0.83	0.78	0.69	0.64	0.79	0.66	0.72	0.62	0.63	0.63	0.64	0.66	0.68
2	0.94	0.83	0.78	0.74	0.86	0.75	0.93	0.83	0.82	0.81	0.79	0.86	0.82
3	0.93	0.98	0.95	0.90	1.05	0.95	0.99	0.91	0.95	0.93	0.87	0.99	0.94
4	1.07	1.08	1.18	1.13	1.12	1.13	1.20	1.18	1.15	1.12	1.12	1.20	1.05
5 (High)	1.43	1.59	1.63	1.75	1.44	1.78	1.40	1.70	1.70	1.75	1.85	1.53	1.75
High–Low	0.60	0.81	0.94	1.10	0.65	1.12	0.68	1.07	1.07	1.12	1.21	0.86	1.07
	(2.33)	(2.89)	(1.83)	(2.20)	(2.11)	(2.35)	(1.99)	(2.09)	(2.13)	(2.00)	(2.24)	(2.46)	(2.04)
FFC4 alpha	0.48	0.68	0.76	0.83	0.43	0.86	0.50	0.80	0.78	0.85	0.91	0.66	0.85
	(2.53)	(3.62)	(2.57)	(2.96)	(1.98)	(3.01)	(2.24)	(2.68)	(2.74)	(2.81)	(3.04)	(3.01)	(3.17)

β^n remains intact. The FFC4 alphas for the high–low β^d- and β^n- sorted quintile portfolios that first control for β^c equal 0.69% and 0.68%, respectively, with highly significant t-statistics of 3.76 and 3.62. First controlling for the standard beta also produces a spread in both of the β^d and β^n sorts, although the magnitude of the spreads and the significance of the t-statistics are somewhat reduced. This is to be expected because the standard betas already contain some of the information in the discontinuous betas, as the discontinuous returns are included in their calculation.

Turning next to the controls for the other firm characteristics and explanatory variables, the results in Panels C and D again show that higher discontinuous and overnight betas are always associated with higher portfolio returns. For β^d in particular, the spread in the returns between the high- and low-quintile portfolios ranges from 0.48% (MOM) to 1.05% (RKT), and the spreads in the FFC4 alphas range from 0.26% (MOM) to 0.73% (REV and RKT). Similarly, for the β^n portfolio sorts, the spreads range from 0.65% (MOM) to 1.21% (RKT), and the spreads in the FFC4

Table 7

Predictive reverse double-sorted portfolios.

The table reports the average returns for predicative reverse double-sorted portfolios. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P) 500 index over 1993–2010. For each month, all stocks in the sample are first sorted into five quintiles on the basis of one beta. Within each quintile, the stocks are then sorted into five quintiles according to one control variable. These five control variable portfolios are then averaged across the five beta portfolios to produce control variable portfolios with large cross-portfolio variation in their control variables but little variation in beta. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT are the measures of coskewness and cokurtosis, respectively. RSK and RKT denote the realized skewness and realized kurtosis computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t. The first five rows in each panel report time series averages of monthly excess returns for the control variable quintile portfolios. The row labeled "High–Low" reports the difference in the returns between Portfolios 5 and 1. The row labeled "FFC4 alpha" reports the average Fama–French–Carhart four-factor alphas. The corresponding Newey–West robust t-statistics are reported in parentheses. Panels A, B, C, and D display the results for the portfolios first sorted by β^s, β^c, β^d, and β^n, respectively, and then by the control variables listed in the columns.

Quintile	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
Panel A: First sort by β^s											
1 (Low)	1.99	1.06	1.17	1.36	0.85	1.22	1.64	1.21	0.85	1.10	0.57
2	1.28	1.08	1.04	1.34	0.91	1.17	1.22	1.18	0.94	0.95	0.76
3	0.98	1.03	0.85	0.92	0.98	1.07	1.01	1.04	0.96	1.00	1.01
4	0.63	1.08	0.96	0.89	1.16	1.06	0.77	0.95	1.18	0.95	1.28
5 (High)	0.56	1.13	1.41	0.92	1.54	0.90	0.80	0.92	1.37	1.43	1.81
High–Low	−1.43	0.07	0.23	−0.44	0.69	−0.32	−0.84	−0.28	0.52	0.33	1.24
	(−5.25)	(0.34)	(0.64)	(−1.73)	(2.31)	(−1.41)	(−3.11)	(−1.76)	(2.45)	(1.28)	(4.41)
FFC4 alpha	−1.09	−0.20	−0.18	−0.44	0.52	−0.10	−0.56	−0.25	0.25	0.22	0.99
	(−7.63)	(−1.38)	(−0.90)	(−1.72)	(2.73)	(−0.44)	(−2.93)	(−1.56)	(1.24)	(1.19)	(7.03)
Panel B: First sort by β^c											
1 (Low)	1.90	1.03	1.16	1.34	0.77	1.19	1.37	1.20	0.88	1.02	0.54
2	1.20	0.97	0.96	1.20	0.91	1.08	1.17	1.16	0.87	0.88	0.79
3	0.93	1.04	0.90	0.98	0.93	1.09	1.06	1.04	0.99	0.95	0.93
4	0.73	1.05	0.92	0.86	1.07	1.02	0.76	0.93	1.18	1.02	1.32
5 (High)	0.49	1.07	1.31	0.89	1.58	0.84	0.89	0.92	1.37	1.39	1.66
High–Low	−1.41	0.04	0.15	−0.45	0.81	−0.34	−0.48	−0.29	0.49	0.37	1.12
	(−4.90)	(0.21)	(0.38)	(−1.69)	(2.50)	(−1.50)	(−1.85)	(−1.82)	(2.30)	(1.30)	(3.74)
FFC4 alpha	−1.09	−0.20	−0.29	−0.43	0.66	−0.10	−0.27	−0.26	0.24	0.26	0.91
	(−6.82)	(−1.41)	(−1.38)	(−1.55)	(3.32)	(−0.43)	(−1.19)	(−1.65)	(1.17)	(1.37)	(6.32)
Panel C: First sort by β^d											
1 (Low)	1.65	0.96	1.13	1.28	0.90	1.07	1.28	1.25	0.86	0.97	0.61
2	1.34	1.06	0.97	1.26	0.92	1.18	1.16	1.10	0.98	1.07	0.83
3	0.93	1.07	0.91	1.07	0.99	1.07	0.99	1.03	0.99	0.97	1.02
4	0.79	1.06	0.96	0.79	1.13	1.05	0.97	0.90	1.10	1.06	1.29
5 (High)	0.55	1.03	1.26	0.86	1.31	0.85	0.86	0.97	1.35	1.18	1.48
High–Low	−1.09	0.06	0.13	−0.42	0.41	−0.21	−0.42	−0.28	0.49	0.22	0.87
	(−4.46)	(0.31)	(0.34)	(−1.63)	(1.55)	(−0.95)	(−1.71)	(−1.81)	(2.52)	(0.85)	(3.15)
FFC4 alpha	−0.81	−0.21	−0.30	−0.40	0.32	0.03	−0.24	−0.24	0.26	0.14	0.69
	(−5.15)	(−1.50)	(−1.49)	(−1.51)	(1.66)	(0.15)	(−1.10)	(−1.61)	(1.34)	(0.70)	(4.49)
Panel D: First sort by β^n											
1 (Low)	1.59	1.00	0.99	1.28	0.87	1.15	1.22	1.22	0.89	1.04	0.66
2	1.34	1.09	1.04	1.24	0.99	1.17	1.20	1.13	0.94	0.98	0.85
3	1.03	1.04	0.85	1.05	1.02	1.08	1.02	1.02	0.96	1.04	0.98
4	0.73	0.99	1.04	0.94	1.21	1.02	0.86	0.98	1.14	1.12	1.25
5 (High)	0.57	1.05	1.30	0.76	1.17	0.80	0.98	0.90	1.35	1.05	1.50
High–Low	−1.02	0.05	0.30	−0.52	0.30	−0.35	−0.25	−0.32	0.46	0.01	0.85
	(−4.01)	(0.22)	(0.77)	(−1.93)	(1.09)	(−1.44)	(−0.99)	(−2.04)	(2.36)	(0.04)	(2.99)
FFC4 alpha	−0.73	−0.20	−0.13	−0.50	0.17	−0.06	−0.09	−0.26	0.22	−0.11	0.65
	(−4.91)	(−1.33)	(−0.65)	(−1.81)	(0.99)	(−0.27)	(−0.45)	(−1.74)	(1.18)	(−0.53)	(4.49)

alphas range from 0.43% (MOM) to 0.91% (RKT). Most of these alphas are not only statistically significant at the usual 5% level, but they also translate into economically meaningful differences, ranging from $0.26\% \times 12 = 3.1\%$ to $1.21\% \times 12 = 14.5\%$ per year. Comparison of the results across different betas in the four different panels for a given control variable also reveals that the high–low portfolio return differences are generally the greatest for the β^d- and β^n- based double-sorts, further corroborating the idea that systematic jump risk is priced higher than continuous market price risk.

The double-sorts presented in Table 6 indicate that the firm characteristics and explanatory variables used as controls cannot fully account for the predictive power of the jump betas. Conversely, to investigate whether the jump betas could help explain some of the abnormal returns associated with the different firm characteristics and other explanatory variables, Table 7 presents the results from reversing the order of the sorts, by first sorting on the betas and then on the explanatory variables.[36] Comparing the spreads in the same column across different panels shows that the predictive powers of the other explanatory variables are generally reduced when controlling for the jump betas, with almost all of the high–low portfolio returns being lower (in an absolute sense) in Panels C and D than in Panels A and B. Looking specifically at the results in Panel A that control for the variation in the standard β^s, the ME, IVOL, CKT, and ILLIQ sorts all yield t-statistics for the FFC4 alphas greater than the usual critical value of two. Meanwhile, in the double-sorts that control for the jump betas reported in Panels C and D, only the t-statistics for ME and ILLIQ are significant. Hence, the separate pricing of systematic jump risk could at least in part help explain the abnormal returns associated with idiosyncratic volatility and cokurtosis observed over the present sample.[37]

6. Cross-sectional pricing regressions

The portfolio sorts discussed in Section 5 impose no model assumptions. However, they ignore potentially important cross-sectional firm-level information by aggregating the stocks into quintile portfolios. Also, even though the double-sorted portfolios do control for other explanatory variables, they control only for one variable at a time. Hence, we turn next to a standard (Fama and MacBeth, 1973)- type cross-sectional approach based on firm-level data for estimating the risk premiums associated with the different betas, while simultaneously controlling for multiple explanatory variables.

For ease of notation, let the unit time interval be a month. The cross-sectional pricing regression for each of the months $t = 1, 2, \ldots, T$, and all of the stocks $i = 1, 2, \ldots, N_t$ available for a particular month t in the sample, could then be expressed as

$$r_{t,t+1}^{(i)} = \gamma_{0,t} + \gamma_{\beta,t}^{c}\beta_t^{(c,i)} + \gamma_{\beta,t}^{d}\beta_t^{(d,i)} + \gamma_{\beta,t}^{n}\beta_t^{(n,i)} + \sum_{j=1}^{p}\gamma_{j,t}Z_{j,t}^{(i)} + \epsilon_{t,t+1}^{(i)}, \quad (14)$$

where $r_{t,t+1}^{(i)}$ denotes the excess return for stock i from month t to month $t+1$ and the explanatory variables $Z_{j,t}^{(i)}$ and the betas $\beta_t^{(c,i)}$, $\beta_t^{(d,i)}$, and $\beta_t^{(n,i)}$ are measured at the end of month t.[38] For comparison, we also estimate similar regressions by replacing the three betas by the standard CAPM beta $\beta_t^{(s,i)}$. Based on these cross-sectional regression results, we then estimate the risk premiums associated with the different betas and explanatory variables as the time series means of the $T = 204$ individual monthly gamma estimates. For $k = s, c, d, n$ and $j = 1, \ldots, p$,

$$\hat{\gamma}_\beta^k = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{\beta,t}^k, \quad \text{and} \quad \hat{\gamma}_j = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{j,t}. \quad (15)$$

The average risk premium estimates, with robust t-statistics in parentheses, are reported in Table 8 for a range of different combinations of explanatory variables. Panel A gives the results from simple univariate regressions involving a single beta measure or a single explanatory variable. Consistent with the standard CAPM, all the beta risk premiums are estimated to be positive. The premium associated with β^c is the highest of the four, and that of β^n is the lowest, although the t-statistic associated with β^n is the highest and the t-statistic with β^c the lowest. Many of the previously showed CAPM-related anomalies appear fairly weak in the present sample of relatively large liquid stocks. Still, the significant positive premiums for ILLIQ and RKT do corroborate the empirical findings in Amihud (2002) and Amaya, Christoffersen, Jacobs, and Vasquez (2015), respectively. The negative estimates for ME, CSK, and RSK are also in line with the empirical evidence reported in Fama and French (1992), Harvey and Siddique (2000), and Amaya, Christoffersen, Jacobs, and Vasquez (2015), among many others, while as previously noted the positive albeit statistically weak premium for IVOL is counter to Ang, Hodrick, Xing, and Zhang (2006b).

Turning to the multiple regression results in Panel B, Regression I shows that the standard beta β^s becomes insignificant when controlling for all the other explanatory variables. Similarly, the risk premium for β^c in Regression II is also insignificant, suggesting that the explanatory power of the continuous beta is effectively subsumed by the other explanatory variables. The t-statistic for the discontinuous beta β^d in Regression III, however, is largely unchanged from the results in the simple regression in Panel A. For the overnight beta β^n, the t-statistic for Regression IV is even higher than in the simple regression in Panel A.

Regressions V–XIII show the results from simultaneously including the continuous, discontinuous, and overnight betas, controlling for ME, BM, MOM, and each of the other explanatory variables in turn. The high correlations across the different beta estimates, discussed in Table 2, invariably render lower slope coefficients and t-statistics than in Regressions II–IV. Nonetheless, the estimated risk premiums associated with β^d and β^n remain close to significant across all the specifications when judged by their one-sided t-statistics at the usual 5% level. The t-statistics for β^c are practically zero for all specifications, suggesting that the premium for systematic

[36] An interesting recent study, Lou, Polk, and Skouras (2015), finds that most of the abnormal returns on momentum strategies tend to occur overnight, while the abnormal returns on other strategies primarily occur intraday.

[37] The positive premium for IVOL observed here is counter to the idiosyncratic volatility puzzle first highlighted by Ang, Hodrick, Xing, and Zhang (2006b). However, as previously showed in the literature, the idiosyncratic volatility puzzle is primarily driven by small firms (Fu, 2009), firms that are dominated by retail investors (Han and Kumar, 2013), and lottery-like firms (Bali, Cakici, and Whitelaw, 2011). By contrast, our sample of S&P 500 constituents consists entirely of relatively large firms. For a recent discussion of the idiosyncratic volatility puzzle, see Stambaugh, Yu, and Yuan (2015).

[38] Following common practice in the literature (e.g., Ang, Chen, and Xing, 2006a, among many others), in an effort to reduce the effect of extreme observations or outliers, we winsorize the independent variables at their 0.5% and 99.5% levels. The results from the non-winsorized regressions, available upon request, are very similar to the results reported here.

Table 8
Fama–MacBeth regressions.
The table reports the estimated regression coefficients and robust t-statistics (in parentheses) from Fama–MacBeth cross-sectional regressions for monthly stock returns. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P) 500 index over 1993–2010. β^s, β^c, β^d, and β^n are the standard, continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT denote the measures of coskewness and cokurtosis, respectively. RSK and RKT are the realized skewness and realized kurtosis, respectively, computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t. Panel A reports the results of simple regressions with a single explanatory variable. Panel B reports the results of multiple regressions with more than one explanatory variable.

Panel A: Simple regressions

β^s	β^c	β^d	β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
0.86	0.95	0.82	0.58	−0.37	−0.20	0.01	−0.01	0.23	−1.56	−0.25	−0.63	0.15	0.06	0.31
(1.94)	(1.91)	(1.99)	(2.14)	(−4.08)	(−0.79)	(0.82)	(−0.74)	(1.36)	(−1.83)	(−1.43)	(−1.91)	(1.94)	(1.28)	(3.33)

Panel B: Multiple regressions

Regression	β^s	β^c	β^d	β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
I	0.71				−0.45	−0.28	0.01	−0.02	−0.19	0.77	−0.41	−0.18	−0.05	0.02	−0.05
	(1.41)				(−3.03)	(−1.51)	(1.31)	(−2.06)	(−1.73)	(1.35)	(−2.25)	(−0.67)	(−0.92)	(0.69)	(−0.43)
II		0.58			−0.47	−0.31	0.01	−0.02	−0.15	0.78	−0.18	−0.21	−0.05	0.03	−0.03
		(1.46)			(−3.03)	(−1.66)	(1.19)	(−2.04)	(−1.36)	(1.39)	(−1.27)	(−0.78)	(−1.00)	(0.93)	(−0.22)
III			0.55		−0.49	−0.33	0.01	−0.02	−0.17	0.97	−0.15	−0.18	−0.04	0.03	−0.10
			(1.97)		(−3.10)	(−1.71)	(1.27)	(−2.07)	(−1.62)	(1.67)	(−1.06)	(−0.66)	(−0.77)	(0.80)	(−0.73)
IV				0.43	−0.51	−0.32	0.01	−0.03	−0.17	0.86	−0.13	−0.12	−0.03	0.02	−0.11
				(2.25)	(−3.09)	(−1.63)	(1.45)	(−2.16)	(−1.56)	(1.48)	(−0.90)	(−0.47)	(−0.62)	(0.50)	(−0.81)
V		−0.06	0.29	0.30	−0.37	−0.25	0.01								
		(−0.13)	(1.68)	(2.16)	(−4.45)	(−0.97)	(1.15)								
V		−0.02	0.28	0.31	−0.38	−0.23	0.00	−0.02							
		(−0.04)	(1.74)	(2.22)	(−4.59)	(−1.19)	(0.97)	(−2.17)							
VII		−0.05	0.36	0.32	−0.38	−0.26	0.01		−0.10						
		(−0.12)	(1.61)	(2.11)	(−4.96)	(−1.03)	(1.16)		(−0.99)						
VIII		−0.07	0.33	0.29	−0.37	−0.28	0.01			−0.14					
		(−0.15)	(1.72)	(2.01)	(−4.50)	(−1.12)	(1.15)			(−0.26)					
IX		−0.01	0.30	0.28	−0.36	−0.28	0.01				−0.05				
		(−0.03)	(1.76)	(2.02)	(−4.70)	(−1.14)	(1.12)				(−0.36)				
X		−0.02	0.28	0.30	−0.37	−0.27	0.01					−0.57			
		(−0.04)	(1.71)	(2.05)	(−4.49)	(−1.04)	(1.11)					(−1.94)			
XI		−0.07	0.27	0.29	−0.40	−0.31	0.01						−0.05		
		(−0.15)	(1.73)	(2.00)	(−4.69)	(−1.28)	(1.27)						(−0.85)		
XII		−0.03	0.39	0.35	−0.38	−0.26	0.01							−0.05	
		(−0.07)	(1.74)	(2.47)	(−4.86)	(−1.02)	(1.14)							(−1.68)	
XIII		−0.06	0.29	0.28	−0.42	−0.24	0.01								−0.06
		(−0.13)	(1.76)	(1.96)	(−2.65)	(−0.92)	(1.34)								(−0.48)
XIV		0.02	0.31	0.31	−0.47	−0.28	0.01	−0.02	−0.20	0.80	−0.13	−0.18	−0.04	0.03	−0.08
		(0.05)	(2.33)		(−3.17)	(−1.48)	(1.25)	(−2.09)	(−1.93)	(1.44)	(−0.91)	(−0.69)	(−0.80)	(0.77)	(−0.61)
XV		0.25	0.25	0.25	−0.45	−0.28	0.01	−0.02	−0.21	0.81	−0.17	−0.18	−0.02	0.02	−0.06
			(1.96)		(−2.95)	(−1.46)	(1.33)	(−2.09)	(−2.00)	(1.41)	(−1.26)	(−0.66)	(−0.39)	(0.49)	(−0.48)

continuous market risk is fully absorbed by the premiums for the two rough betas and the other explanatory variables.

The estimated premiums for β^d and β^n risks are also remarkably robust across the different specifications, with typical values of around 0.3% for each of the rough betas. The t-statistic for testing that the two premiums are the same after controlling for all the other explanatory variables equals just 0.26. Hence, in Regression XIV, we report the results including the three betas and all control variables, explicitly restricting the premiums for β^d and β^n risks to be the same. The estimated common rough beta risk premium equals 0.31% with a t-statistic of 2.33.[39] Given that the cross-sectional standard deviations of β^d and β^n are equal to 1.14 and 1.20, respectively, a two standard deviation change in each of the two rough betas also translates into large and economically meaningful expected return differences of about $2 \times 1.14 \times 0.33\% \times 12 = 9.03\%$ and $2 \times 1.20 \times 0.33\% \times 12 = 9.50\%$ per year, respectively.

Regression XV further constrains all three β^c, β^d, and β^n risks to have the same premium. This results in a marginally significant t-statistic of 1.96 for the beta risk premium. However, a robust F-test easily rejects the null

[39] This value is very much in line with the options-based estimate of the aggregate equity risk premium attributable to jump tail risk of close to 5% per year reported in Bollerslev and Todorov (2011). That study also suggests that the premium for jump tail risk could change over time with changes in investors' attitude to risk or fear. To investigate the sensitivity of our results to the financial crisis, we have also redone the estimation excluding January 2007 to December 2008 from the sample, resulting in a jump beta risk premium of 0.43% and an even more significant t-statistic of 3.07.

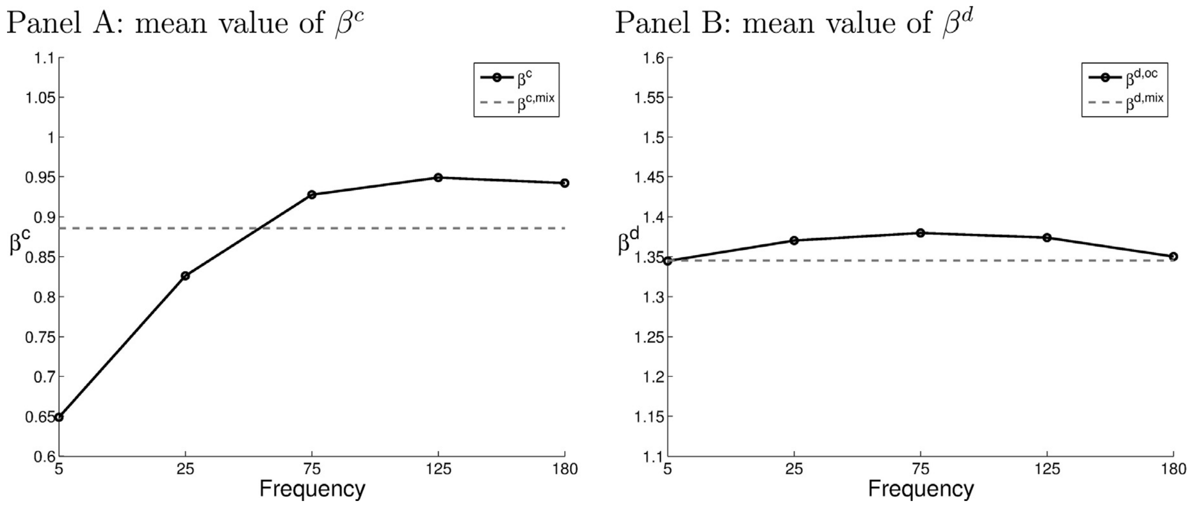

Fig. 3. Signature plots for betas. Panel A shows the mean value of β^c (solid line) averaged across stocks and time for different sampling frequencies (labeled in minutes on the *X*-axis). The dashed line gives the mean value of the mixed-frequency β^c. Panel B plots the same averaged estimates for β^d.

hypothesis that the three risk premiums are the same. By contrast, the assumption that the risk premiums for β^d and β^n are the same and different from the premium for β^c, as in Regression XIV, cannot be rejected.

7. Robustness checks

To further help corroborate the robustness of our findings, we carry out a series of additional tests and empirical investigations. To begin, we investigate the sensitivity of our main empirical findings to the choice of intraday sampling frequency used in the estimation of the betas, possible biases in the estimation of the betas induced by nonsynchronous trading effects, and errors-in-variables in the cross-sectional pricing regressions stemming from estimation errors in the betas. Next, we analyze how the cross-sectional regression results and the estimated risk premiums for the different betas are affected by the length of the sample period used in the estimation of the betas and the holding period of the future returns. Finally, we compare our main results with those obtained by excluding specific macroeconomic news announcement days in the estimation of the betas.

7.1. Sampling frequency and beta estimation

The continuous-time framework of the empirical investigations and the consistency of the β^c and β^d estimates hinge on increasingly finer sampled intraday returns. In practice, nonsynchronous trading and other market microstructure effects invariably limit the frequency of the data available for estimation. To assess the sensitivity of the beta estimates to the choice of sampling frequency, we compute betas for five different fixed sampling frequencies: 5-, 25-, 75-, 125-, and 180-minute. These five sampling schemes, ranging from a total of 75 observations per day (five-minute) to only two observations per day (180-minute), span most of the frequencies used in the literature for computing multivariate realized variation measures. The extent of market microstructure frictions varies across different stocks. Less frequently traded stocks are likely more prone to estimation biases in their betas from too frequent sampling than more liquid stocks. Thus, we also adopt a mixed-frequency strategy in which we apply different sampling frequencies to different stocks. We sort all stocks into quintiles according to their ILLIQ measure at the end of each month *t*. We then use the *i*th highest of the five fixed sampling frequencies for stocks in the *i*th illiquidity quintile; i.e., five-minute frequency for stocks in the lowest ILLIQ quintile (the most liquid) and 180-minute frequency for stocks in the highest ILLIQ quintile (the least liquid).

Fig. 3 plots the sample means averaged across time and stocks for the resulting β^c and β^d estimates as a function of the five different fixed sampling frequencies. The sample means of the mixed-frequency beta estimates are shown as a flat dashed line in both panels. The average β^c estimates, reported in Panel A, increase substantially from the 5- to the 25-minute sampling frequency but appear to flatten at around 0.93 at the 75-minute sampling frequency used in our empirical results reported so far. The average β^d estimates reported in Panel B, however, are remarkably stable across different sampling frequencies and close to the average mixed-frequency value of 1.35. The specific choice of sampling frequency within the range of values considered here appears largely irrelevant to the two discontinuous beta estimates.

To further investigate the role of sampling frequency in our key empirical findings, Table 9 reports results of cross-sectional pricing regressions based on the different beta estimates. Panel A gives the results obtained by varying the sampling frequency used in the estimation of β^c, keeping the sampling frequency for the β^d estimation fixed at 75 minutes. Panel B reports the results for the different β^d estimates, using the same 75-minute β^c estimates. To conserve space, we report only results corresponding to

Table 9
Fama–MacBeth regressions with different beta estimation frequencies.
The table reports the estimated regression coefficients and robust *t*-statistics (in parentheses) from monthly Fama–MacBeth cross-sectional regressions simultaneously controlling for all explanatory variables, restricting the coefficients for β^d and β^n to be the same. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P) 500 index over 1993–2010. β^c, β^d, and β^n refer to the continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT denote the measures of coskewness and cokurtosis, respectively. RSK and RKT refer to the realized skewness and realized kurtosis, respectively, computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t. Panel A reports the results for different β^c estimates computed using the sampling frequencies listed in the first column labeled "Frequency." Panel B reports the results for different β^d estimates based on the sampling frequencies in the "Frequency" column.

Frequency	β^c	β^d, β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
Panel A: Different β^c estimates													
5 minutes	−0.36	0.34	−0.52	−0.30	0.01	−2.45	−0.21	0.86	−0.06	−0.15	−0.04	0.02	−0.16
	(−0.87)	(2.57)	(−3.34)	(−1.60)	(1.37)	(−2.13)	(−1.96)	(1.54)	(−0.42)	(−0.55)	(−0.73)	(0.69)	(−1.21)
25 minutes	−0.02	0.31	−0.51	−0.31	0.01	−2.39	−0.21	0.82	−0.08	−0.21	−0.03	0.03	−0.10
	(−0.04)	(2.35)	(−3.37)	(−1.66)	(1.36)	(−2.10)	(−2.01)	(1.50)	(−0.61)	(−0.79)	(−0.69)	(0.73)	(−0.82)
75 minutes	0.02	0.31	−0.47	−0.28	0.01	−2.38	−0.20	0.80	−0.13	−0.18	−0.04	0.03	−0.08
	(0.05)	(2.33)	(−3.17)	(−1.48)	(1.25)	(−2.09)	(−1.94)	(1.44)	(−0.91)	(−0.69)	(−0.80)	(0.77)	(−0.61)
125 minutes	0.12	0.27	−0.47	−0.30	0.01	−2.43	−0.21	0.82	−0.15	−0.17	−0.03	0.02	−0.08
	(0.35)	(2.10)	(−3.10)	(−1.58)	(1.34)	(−2.11)	(−1.99)	(1.46)	(−1.05)	(−0.65)	(−0.59)	(0.62)	(−0.60)
180 minutes	0.05	0.29	−0.48	−0.31	0.01	−2.52	−0.20	0.88	−0.15	−0.17	−0.03	0.02	−0.10
	(0.14)	(2.22)	(−3.17)	(−1.62)	(1.40)	(−2.20)	(−1.94)	(1.59)	(−1.11)	(−0.62)	(−0.64)	(0.68)	(−0.73)
Mix	0.06	0.30	−0.42	−0.31	0.01	−2.49	−0.21	0.86	−0.09	−0.18	−0.03	0.02	−0.08
	(0.17)	(2.24)	(−2.87)	(−1.63)	(1.42)	(−2.18)	(−1.96)	(1.53)	(−0.66)	(−0.67)	(−0.63)	(0.61)	(−0.58)
Panel B: Different β^d estimates													
5 minutes	0.12	0.28	−0.48	−0.30	0.01	−2.39	−0.19	0.78	−0.11	−0.17	−0.04	0.03	−0.08
	(0.32)	(2.10)	(−3.19)	(−1.57)	(1.28)	(−2.10)	(−1.80)	(1.40)	(−0.81)	(−0.66)	(−0.70)	(0.76)	(−0.61)
25 minutes	0.05	0.29	−0.48	−0.28	0.01	−2.45	−0.20	0.80	−0.12	−0.16	−0.04	0.03	−0.08
	(0.15)	(2.32)	(−3.19)	(−1.47)	(1.30)	(−2.14)	(−1.88)	(1.44)	(−0.87)	(−0.59)	(−0.76)	(0.80)	(−0.61)
75 minutes	0.02	0.31	−0.47	−0.28	0.01	−2.38	−0.20	0.80	−0.13	−0.18	−0.04	0.03	−0.08
	(0.05)	(2.33)	(−3.17)	(−1.48)	(1.25)	(−2.09)	(−1.94)	(1.44)	(−0.91)	(−0.69)	(−0.80)	(0.77)	(−0.61)
125 minutes	0.04	0.30	−0.47	−0.28	0.01	−2.44	−0.21	0.79	−0.14	−0.19	−0.04	0.03	−0.07
	(0.10)	(2.28)	(−3.14)	(−1.51)	(1.24)	(−2.15)	(−2.02)	(1.41)	(−0.99)	(−0.71)	(−0.77)	(0.85)	(−0.58)
180 minutes	0.04	0.29	−0.47	−0.28	0.01	−2.47	−0.21	0.75	−0.13	−0.20	−0.04	0.03	−0.07
	(0.12)	(2.22)	(−3.15)	(−1.49)	(1.25)	(−2.16)	(−2.01)	(1.35)	(−0.94)	(−0.75)	(−0.80)	(0.88)	(−0.59)
Mix	0.08	0.28	−0.45	−0.28	0.01	−2.44	−0.20	0.77	−0.13	−0.18	−0.04	0.03	−0.07
	(0.22)	(2.20)	(−3.07)	(−1.50)	(1.26)	(−2.13)	(−1.95)	(1.37)	(−0.93)	(−0.70)	(−0.72)	(0.85)	(−0.56)

the full Regression XIV reported in Panel B of Table 8 that restricts the premiums for the two rough betas to be the same.

None of the t-statistics for the continuous systematic risk premiums in Panel A is close to significant. All of the t-statistics for the rough beta risk premiums are higher than two. The estimated risk premiums are also very similar across the different regressions and close to the value of 0.31% for the benchmark Regression XIV in Table 8. The regressions in Panel B for the different β^d estimates tell a very similar story. The risk premiums for the rough betas are always significant, and those for the continuous betas are not. Overall, our key cross-sectional pricing results appear robust to choice of intraday sampling frequency used in the estimation of the β^c and β^d risk measures.

7.2. Nonsynchronous trading and beta estimation

The results in Section 7.1 indicate that the estimated jump betas are very stable across different sampling frequencies, and the continuous betas appear to be downward-biased for the highest sampling frequencies. This downward bias could in part be attributed to nonsynchronous trading effects. To more directly investigate this, following the original ideas of Scholes and Williams (1977) and Dimson (1979), we calculate high-frequency based lead and lag continuous betas as

$$\widehat{\beta}^{(c,i)}_{t,-} = \frac{n}{n-1}\frac{\sum_{s=t-l}^{t-1}\sum_{\tau=2}^{n}\left[(r^{(i)}_{s:\tau}+r^{(0)}_{s:\tau-1})^2\mathbf{1}_{\{|r^{(i)}_{s:\tau}+r^{(0)}_{s:\tau-1}|\le k^{(i+0)}_{s,\tau}\}}-(r^{(i)}_{s:\tau}-r^{(0)}_{s:\tau-1})^2\mathbf{1}_{\{|r^{(i)}_{s:\tau}-r^{(0)}_{s:\tau-1}|\le k^{(i-0)}_{s,\tau}\}}\right]}{4\sum_{s=t-l}^{t-1}\sum_{\tau=1}^{n}(r^{(0)}_{s:\tau})^2\mathbf{1}_{\{|r^{(0)}_{s:\tau}|\le k^{(0)}_{s,\tau}\}}} \tag{16}$$

and

$$\widehat{\beta}^{(c,i)}_{t,+} = \frac{n}{n-1}\frac{\sum_{s=t-l}^{t-1}\sum_{\tau=2}^{n}\left[(r^{(i)}_{s:\tau-1}+r^{(0)}_{s:\tau})^2\mathbf{1}_{\{|r^{(i)}_{s:\tau-1}+r^{(0)}_{s:\tau}|\le k^{(i+0)}_{s,\tau}\}}-(r^{(i)}_{s:\tau-1}-r^{(0)}_{s:\tau})^2\mathbf{1}_{\{|r^{(i)}_{s:\tau-1}-r^{(0)}_{s:\tau}|\le k^{(i-0)}_{s,\tau}\}}\right]}{4\sum_{s=t-l}^{t-1}\sum_{\tau=1}^{n}(r^{(0)}_{s:\tau})^2\mathbf{1}_{\{|r^{(0)}_{s:\tau}|\le k^{(0)}_{s,\tau}\}}}, \tag{17}$$

where n denotes the number of high-frequency observations within a day used in the estimation; i.e., $n = 5$ for the 75-minute sampling underlying our main empirical results. The theory behind the high-frequency betas implies that the lead and lag betas should be asymptotically negligible and thus have no significant impact on the cross-sectional pricing.[40]

To test for this, we repeat the single-sorts in Table 4 by instead sorting the stocks according to their $\widehat{\beta}_{t,-}^{(c,i)}$ and $\widehat{\beta}_{t,+}^{(c,i)}$ estimates. The monthly return differences between the resulting high and low quintile portfolios equal −0.37% with a t-statistic of −0.77 for the lagged continuous beta sorts and 0.03% with a t-statistic of 0.17 for the lead continuous beta sorts, thus corroborating the idea that neither the lead nor the lagged continuous betas are priced in the cross section. Further along these lines, we also calculate an adjusted continuous beta by adding all three continuous beta estimates, $\widehat{\beta}_{t,\mathrm{adj}}^{(c,i)} \equiv \widehat{\beta}_{t}^{(c,i)} + \widehat{\beta}_{t,-}^{(c,i)} + \widehat{\beta}_{t,+}^{(c,i)}$. Sorting by these adjusted continuous betas produces a spread in the returns between the high and low quintile portfolios of a 1.51%, very close to the value of 1.61% reported in Table 4.

Taken as a whole, the results discussed in Section 7.1 together with the results for the lead–lag beta adjustments discussed above indicate that nonsynchronous trading effects and biases in the high-frequency betas are not of great concern.[41]

7.3. Errors-in-variables in the cross-sectional pricing regressions

Another potential concern when testing linear factor pricing models relates to the errors-in-variables problem arising from the first-stage estimation of the betas. As formally shown by Shanken (1992), the first-stage estimation error generally results in an increase in the asymptotic variance of the risk premium estimates from the second-stage cross-sectional regressions. In our setting, however, the betas are estimated from high-frequency data, resulting in lower measurement errors and, in turn, less of an errors-in-variables problem than in traditional Fama–MacBeth type regressions that rely on betas estimated with lower frequency data. At the same time, this also means that the standard adjustment procedures, as in, e.g., Shanken (1992), are not applicable in the present context.[42]

Instead, we conduct a small-scale Monte Carlo experiment by appropriately perturbing the high-frequency beta estimates. For $\widehat{\beta}_{t}^{(c,i)}$ and $\widehat{\beta}_{t}^{(d,i)}$, we rely on the results in Todorov and Bollerslev (2010) to generate replicates $\{\widehat{\beta}_{t}^{(c,i,\mathrm{rep})}\}$ and $\{\widehat{\beta}_{t}^{(d,i,\mathrm{rep})}\}$ from two independent normal distributions with means equal to the estimated $\widehat{\beta}_{t}^{(c,i)}$ and $\widehat{\beta}_{t}^{(d,i)}$, respectively, and standard deviations equal to the corresponding theoretical asymptotic standard errors. For $\widehat{\beta}_{t}^{(n,i)}$, we rely on a bootstrap procedure to generate random samples of $\widehat{\beta}_{t}^{(n,i,\mathrm{rep})}$ from the actual sampling distribution. Given a random sample of the three betas ($\widehat{\beta}_{t}^{(c,i,\mathrm{rep})}$, $\widehat{\beta}_{t}^{(d,i,\mathrm{rep})}$, and $\widehat{\beta}_{t}^{(n,i,\mathrm{rep})}$), we then estimate the key Fama–MacBeth Regression XIV in Table 8 based on the perturbed beta estimates keeping all of the other controls the same. We repeat the simulations a total of 100 times.

The resulting simulation-based estimates for the risk premiums are in the range of -0.12 to 0.27 for β^c with t-statistic between -0.24 and 0.87 and in the range of 0.20 to 0.38 with t-statistic between 1.62 and 3.16 for β^d and β^n. The magnitudes of these simulated risk premiums and their t-statistics are all fairly close to the values for the actual regression reported in Table 8, thus confirming that the errors-in-variables problem is not of major concern in the present context and that it does not materially affect the statistical or economic significance of the rough betas.

7.4. Beta estimation and return holding periods

All the cross-sectional pricing regressions in Tables 8 and 9 are based on betas estimated from returns over the past year and a future one-month return holding period. These are typical estimation and holding periods used to test for the significant pricing ability of explanatory variables and risk factors. To assess the robustness of our results to different lagged beta estimation periods (L) and longer future return horizons (H), Table 10 reports results based on shorter three- and six-month beta estimates and longer 3-, 6- and 12-month prediction horizons.[43]

Regressions I–V pertain to the standard beta. Although the regression coefficients associated with the standard beta seem to increase with the forecast horizon, their t-statistics are at most weakly significant. Regressions VI–X pertain to the continuous and rough betas. The regressions show that the t-statistics associated with the two rough betas are always significant and that the continuous systematic risk is not priced in the cross section. In fact, if anything, the results for the shorter estimation periods and longer return horizons are even more significant than the results for the baseline Regression XIV in Panel B of Table 8 and the typical choice of $L = 12$ and $H = 1$.

[40] It is not possible to similarly adjust the jump betas by including leads and lags in their calculation. The lead–lag adjustment for the continuous betas relies on the notion that the true high-frequency returns are approximately serially uncorrelated. However, the construction of the jump betas is based on higher order powers of the high-frequency returns, and the squared returns, in particular, are clearly not serially uncorrelated. Meanwhile, the signature plots for the jump betas in Panel B of Fig. 3 show that the estimates of the jump betas are very robust to the choice of sampling frequency and, as such, much less prone to any systematic biases arising from nonsynchronous trading effects.

[41] We also experimented with the use of alternative discontinuous beta estimates based on simple OLS regressions for the high-frequency returns that exceed a jump threshold. The results from the corresponding portfolio sorts and Fama–MacBeth regressions were generally close to the results based on the discontinuous beta estimator in Eq. (13) formally developed in Todorov and Bollerslev (2010) that we rely on throughout the paper.

[42] Formally accounting for the estimation errors in the high-frequency betas would require a new asymptotic framework in which both the time span of the data used for the cross-sectional regression-based estimates of the risk premiums and the sampling frequency used for the estimation of the betas go to infinity. We leave this for future work.

[43] All of the cross-sectional regressions are estimated monthly. The robust t-statistics for the longer H = 3-, 6- and 12-month return horizons explicitly adjust for the resulting overlap in the return observations.

Table 10
Fama–MacBeth regressions with different beta estimation periods and return holding horizons.
The table reports the estimated regression coefficients and robust t-statistics (in parentheses) from Fama–MacBeth cross-sectional regressions for predicting the next H-month cumulative returns. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P) 500 index over 1993–2010. The regressions simultaneously control for all explanatory variables, restricting the coefficients for β^d and β^n to be the same. The betas are computed from the previous L-month high-frequency returns. β^s, β^c, β^d, and β^n refer to the standard, continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT denote the measures of coskewness and cokurtosis, respectively. RSK and RKT refer to the realized skewness and realized kurtosis, respectively, computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t.

Regression	L	H	β^s	β^c	β^d, β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
I	3	1	0.45			−0.41	−0.34	0.00	−0.03	−0.09	0.88	−0.45	−0.18	−0.14	0.05	0.09
			(1.63)			(−2.32)	(−1.66)	(0.99)	(−2.75)	(−0.69)	(1.40)	(−3.04)	(−0.63)	(−2.24)	(1.44)	(0.59)
II	6	1	0.67			−0.38	−0.34	0.00	−0.03	−0.14	0.87	−0.53	−0.18	−0.12	0.06	0.10
			(1.64)			(−2.28)	(−1.71)	(0.96)	(−2.81)	(−1.17)	(1.37)	(−3.27)	(−0.63)	(−2.01)	(1.55)	(0.70)
III	12	3	2.05			−0.70	−0.73	0.02	−0.05	−0.16	2.04	−1.57	−0.37	−0.36	0.13	0.66
			(1.63)			(−1.45)	(−1.64)	(1.37)	(−1.86)	(−0.70)	(1.46)	(−3.11)	(−0.67)	(−2.77)	(1.82)	(1.38)
IV	12	6	4.25			−1.59	−1.53	0.03	0.00	0.36	3.28	−2.42	−0.96	−0.49	0.03	1.21
			(1.48)			(−1.39)	(−1.74)	(1.09)	(−0.06)	(0.87)	(1.32)	(−2.43)	(−0.89)	(−1.45)	(0.29)	(1.05)
V	12	12	9.57			−5.55	−3.76	0.05	0.09	0.72	3.73	−3.34	−1.35	−0.59	0.07	0.24
			(1.42)			(−1.95)	(−1.73)	(1.02)	(0.96)	(1.23)	(0.79)	(−1.77)	(−0.73)	(−0.67)	(0.36)	(0.10)
VI	3	1		−0.30	0.43	−0.41	−0.34	0.00	−0.03	−0.19	0.76	−0.23	−0.14	−0.14	0.05	0.07
				(−1.10)	(3.48)	(−2.58)	(−1.69)	(0.94)	(−2.78)	(−1.68)	(1.24)	(−1.66)	(−0.48)	(−2.53)	(1.49)	(0.53)
VII	6	1		−0.45	0.55	−0.42	−0.34	0.00	−0.03	−0.21	0.85	−0.22	−0.16	−0.12	0.06	0.05
				(−1.32)	(4.15)	(−2.63)	(−1.70)	(0.90)	(−2.87)	(−1.91)	(1.37)	(−1.61)	(−0.56)	(−2.26)	(1.70)	(0.39)
VIII	12	3		−1.66	1.65	−0.84	−0.72	0.01	−0.05	−0.32	2.50	−0.65	−0.39	−0.34	0.14	0.46
				(−1.39)	(3.94)	(−1.86)	(−1.60)	(1.27)	(−1.97)	(−1.52)	(1.86)	(−2.12)	(−0.74)	(−2.85)	(1.92)	(1.11)
IX	12	6		−1.35	2.73	−1.73	−1.40	0.03	−0.01	0.02	4.01	−1.05	−1.02	−0.39	0.05	0.96
				(−0.50)	(3.45)	(−1.67)	(−1.59)	(1.06)	(−0.23)	(0.06)	(1.64)	(−1.65)	(−1.00)	(−1.40)	(0.50)	(0.99)
X	12	12		3.82	4.07	−5.42	−3.15	0.05	0.08	−0.08	4.85	−1.65	−1.33	−0.18	0.08	0.35
				(0.65)	(3.28)	(−2.04)	(−1.59)	(1.00)	(0.89)	(−0.11)	(1.01)	(−1.09)	(−0.72)	(−0.23)	(0.39)	(0.17)

The significance of the results for the longer 3-, 6-, and 12-month return horizons also highlights nontrivial persistence in the cross-sectional predictability. Converting the resulting estimates for the different return horizons to an annual level implies rough beta risk premiums of $1.65\% \times 4 = 6.60\%$, $2.73\% \times 2 = 5.46\%$, and $4.07 \times 1 = 4.07\%$ per year, respectively, compared with $0.31 \times 12 = 3.72\%$ per year for the one-month future return horizon implied by Regression XIV in Table 8.

While ME, REV, and RKT are each significant in one or more of the regressions reported in Table 10, they are not systematically so. Short-term reversal and realized kurtosis, in particular, both lose their significance for the longer 6- and 12-month holding periods. The only variable that remains highly significant across all different estimation periods and return predictability horizons is the rough beta risk premium.

7.5. Betas and macroeconomic news announcements

An extensive literature has been devoted to studying the effects of macroeconomic news announcements on asset prices. Andersen, Bollerslev, and Diebold (2007a), Lee (2012), and Lahaye, Laurent, and Neely (2011) have all sought to relate jumps in high-frequency asset prices, with the significant jumps identified through similar techniques to the ones used here, to regularly scheduled macroeconomic news releases. Related to this, Savor and Wilson (2014) have also recently argued that cross-sectional return patterns are different on news announcement days.[44]

To investigate whether macroeconomic announcement days confound our beta estimates and the significant cross-sectional relation between the two rough betas and expected stock returns, we exclude three specific announcement days in our estimation of β^c, β^d, and β^n, including days when the employment report (Employment) and the US Producer Price Index (PPI) are announced by the Bureau of Labor Statistics and days when scheduled interest rates are announced by the Federal Open Market Committee (FOMC). Employment and PPI are both announced monthly at 8:30 a.m. before the stock market officially opens, and FOMC is announced at 2:15 p.m. every six weeks.[45]

Relying on the same test for significant intraday jumps used above, Fig. 4 compares the average jump intensity for the S&P 500 market portfolio for the three announcement days and all other days in the sample (Non-Ann), as a function of the time of day. Not surprisingly, the FOMC announcements at 2:15 p.m. have the greatest intraday effect, increasing the jump intensity from an average of about 1% per day on non-announcement days to 9% on FOMC days.[46]

[44] Patton and Verardo (2012) also show that standard realized betas calculated from high-frequency intraday data tend to be higher on individual firms' earnings announcement days.

[45] Andersen, Bollerslev, Diebold, and Vega (2003) provide a comprehensive list of US macroeconomic news announcements and their release times.

[46] Lucca and Moench (2015) have also recently shown large average pre-FOMC one-day equity returns in anticipation of monetary policy decisions.

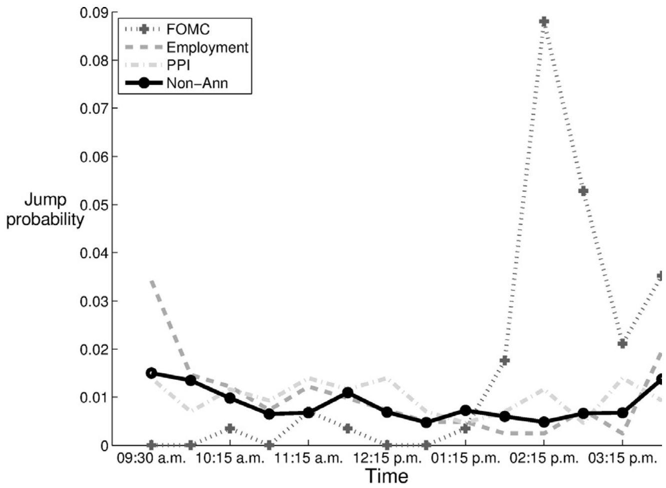

Fig. 4. Jump intensity and macro announcement. The figure plots the average estimated jump intensity (probability) for the Standard & Poor's (S&P) 500 market portfolio across regular trading hours on Federal Open Market Committee (FOMC) announcement days, employment report (Employment) announcement days, US Producer Price Index (PPI) announcement days, and all other days (Non-Ann).

Table 11
Fama–MacBeth regressions excluding macroeconomic news announcement days.
The table reports the estimated regression coefficients and robust t-statistics (in parentheses) from monthly Fama–MacBeth cross-sectional regressions simultaneously controlling for all explanatory variables, restricting the coefficients for β^d and β^n to be the same. The sample consists of the 985 individual stocks included in the Standard & Poor's (S&P500) index over 1993–2010. The betas are calculated excluding Federal Open Market Committee (FOMC), employment report (Employment), and US Producer Price Index (PPI) announcement days in the estimation. β^c, β^d, and β^n refer to the continuous, discontinuous, and overnight betas, respectively. ME denotes the logarithm of the market capitalization of firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from month $t-11$ through month $t-1$. REV is the month t return. IVOL is a measure of idiosyncratic volatility. CSK and CKT denote the measures of coskewness and cokurtosis, respectively. RSK and RKT refer to the realized skewness and realized kurtosis, respectively, computed from high-frequency data. MAX represents the maximum daily raw return for month t. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t.

Regression	β^c	β^d, β^n	ME	BM	MOM	REV	IVOL	CSK	CKT	RSK	RKT	MAX	ILLIQ
I	−0.11	0.30	−0.37	−0.25	0.01								
	(−0.25)	(2.32)	(−4.40)	(−0.93)	(1.03)								
II	−0.09	0.30	−0.38	−0.23	0.00	−0.02							
	(−0.21)	(2.28)	(−4.51)	(−1.13)	(0.84)	(−2.12)							
III	−0.07	0.33	−0.39	−0.26	0.00		−0.09						
	(−0.15)	(2.31)	(−5.04)	(−1.00)	(1.04)		(−0.92)						
IV	−0.09	0.30	−0.38	−0.28	0.00			−0.23					
	(−0.21)	(2.26)	(−4.51)	(−1.09)	(1.02)			(−0.43)					
V	−0.04	0.29	−0.37	−0.29	0.00				−0.04				
	(−0.10)	(2.30)	(−4.72)	(−1.12)	(0.99)				(−0.31)				
VI	−0.08	0.29	−0.37	−0.26	0.00					−0.59			
	(−0.18)	(2.30)	(−4.43)	(−0.98)	(0.99)					(−2.01)			
VII	−0.11	0.28	−0.41	−0.31	0.01						−0.06		
	(−0.25)	(2.27)	(−4.63)	(−1.24)	(1.16)						(−0.89)		
VIII	−0.05	0.36	−0.39	−0.26	0.00							−0.05	
	(−0.12)	(2.56)	(−4.87)	(−1.00)	(1.02)							(−1.61)	
IX	−0.11	0.28	−0.44	−0.23	0.01								−0.08
	(−0.26)	(2.29)	(−2.74)	(−0.87)	(1.21)								(−0.60)
X	−0.01	0.31	−0.50	−0.29	0.00	−0.02	−0.22	0.81	−0.12	−0.17	−0.05	0.03	−0.10
	(−0.04)	(2.33)	(−3.33)	(−1.54)	(1.14)	(−2.16)	(−2.05)	(1.46)	(−0.84)	(−0.63)	(−0.89)	(0.95)	(−0.77)
XI	0.24		−0.47	−0.28	0.01	−0.02	−0.22	0.81	−0.17	−0.17	−0.02	0.02	−0.08
	(1.95)		(−3.05)	(−1.49)	(1.27)	(−2.13)	(−2.08)	(1.41)	(−1.23)	(−0.61)	(−0.45)	(0.66)	(−0.62)

The employment report also makes a market jump more likely in the first few minutes of trading, although not dramatically so. Employment and PPI are both announced before the market officially opens and, thus, can be expected to affect estimation of the overnight betas the most.

Table 11 reports results of the full firm-level cross-sectional regressions excluding the three different types of announcement days. Both the size and the statistical significance of the risk premium estimates are very similar to those in Table 8, Panel B. In fact, the estimated risk premium for the discontinuous and overnight betas in Regression X in Table 11 is identical within two decimal points to the estimate from Regression XIV in Table 8. The predictive power and significant cross-sectional pricing ability of the two rough betas do not appear to be solely driven by important macroeconomic news announcements.

8. Conclusions

Building on a general continuous-time representation for the return on the aggregate market portfolio coupled with an economy-wide pricing kernel that separately prices market diffusive and jump risks, we show how standard asset pricing theory naturally results in separate risk premiums for continuous, or smooth, market betas and discontinuous, or rough, market betas. Importantly, our theoretical framework explicitly allows for other systematic risk factors to enter the pricing kernel and possibly affect the cross-sectional pricing. Only if nonmarket risks are not priced, and the premiums for continuous and jump market risks are the same, does the standard conditional CAPM hold.

Motivated by these theoretical results, we empirically investigate whether market diffusive and jump risks are priced differently in the cross section of expected stock returns. Our empirical investigations rely on a novel high-frequency data set for a large cross section of individual stocks together with new econometric techniques for separately estimating continuous, discontinuous, and overnight betas. We find that the discontinuous and overnight betas are different from, and more cross-sectionally dispersed than, the continuous and standard CAPM betas. When we sort individual stocks by the different betas, we find that stocks with high discontinuous and overnight betas earn significantly higher returns than stocks with low discontinuous and overnight betas, while at best only a weak relation exists between a stock's return and its continuous beta. We also find that the estimated risk premiums for the discontinuous and overnight betas to be both statistically significant and indistinguishable from one another and that this rough beta risk cannot be explained by a long list of other firm characteristics and explanatory variables commonly employed in the literature. In contrast, the estimated continuous beta risk premium is insignificant.

Intuitively, market jumps more likely reflect true information surprises than do continuous price moves, which could simply be attributed to random noise in the price formation process. Moreover, important news is often announced during overnight non-trading hours.[47] As such, the two rough betas could more accurately reflect the systematic market price risks that are priced than do the continuous betas and the standard conditional CAPM betas that do not differentiate between smooth and rough market price moves.

The theoretical setup in Section 2 that motivates our empirical investigations is deliberately very general. However, a more formal investigation into the reasons behind the differences in the pricing of the smooth and rough betas and whether the differences could be explained by different loadings on diffusive and jump fundamental shocks or differences of opinion and learning, possibly influenced by behavioral effects, would be very interesting. We leave this for future research.

[47] Conversely, Berkman, Koch, Tuttle, and Zhang (2012) and Lou, Polk, and Skouras (2015) find that institutional investors tend to trade relatively more during the day and individual investors trade relatively more overnight, thus indirectly suggesting that the overnight betas could be more susceptible to the influence of noise trading.

Appendix A

A.1. High-frequency data cleaning

We begin by removing entries that satisfy at least one of the following criteria: a time stamp outside the exchange open window between 9:30 a.m. and 4:00 p.m.; a price less than or equal to zero; a trade size less than or equal to zero; corrected trades, i.e., trades with Correction Indicator, CORR, other than 0, 1, or 2; and an abnormal sale condition, i.e., trades for which the Sale Condition, COND, has a letter code other than @, *, E, F, @E, @F, *E and *F. We then assign a single value to each variable for each second within the 9:30 a.m.–4:00 p.m. time interval. If one or multiple transactions have occurred in that second, we calculate the sum of volumes, the sum of trades, and the volume-weighted average price within that second. If no transaction has occurred in that second, we enter zero for volume and trades. For the volume-weighted average price, we use the entry from the nearest previous second, i.e., forward-filtering. If no transaction has occurred before that second, we use the entry from the nearest subsequent second, i.e., backward-filtering. Motivated by our analysis of the trading volume distribution across different exchanges over time, we purposely incorporate information from all exchanges covered by the TAQ database.[48]

A.2. Additional explanatory variables

Our empirical investigations rely on the following explanatory variables and firm characteristics.

- Size (ME): Following Fama and French (1993), a firm's size is measured at the end of June by its market value of equity—the product of the closing price and the number of shares outstanding (in millions of dollars). Market equity is updated annually and is used to explain returns over the subsequent 12 months. Following common practice, we also transform the size variable by its natural logarithm to reduce skewness.
- Book-to-market ratio (BM): Following Fama and French (1992), the book-to-market ratio in June of year t is computed as the ratio of the book value of common equity in fiscal year $t-1$ to the market value of equity (size) in December of year $t-1$.[49] BM for fiscal year t is used to explain returns from July of year $t+1$ through June of year $t+2$. The time gap between BM and returns ensures that information on BM is publicly available prior to the returns.
- Momentum (MOM): Following Jegadeesh and Titman (1993), the momentum variable at the end of month t

[48] Further details on the exchange analysis are available upon request.

[49] Book common equity is defined as book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus book value of preferred stock for fiscal year $t-1$.

is defined as the compound gross return from month $t-11$ through month $t-1$; i.e., skipping the short-term reversal month t.[50]

- Reversal (REV): Following Jegadeesh (1990) and Lehmann (1990), the short-term reversal variable at the end of month t is defined as the return over that same month t.
- Idiosyncratic volatility (IVOL): Following Ang, Hodrick, Xing, and Zhang (2006b), a firm's idiosyncratic volatility at the end of month t is computed as the standard deviation of the residuals from the regression based on the daily return regression:

$$r_{i,d} - r_{f,d} = \alpha_i + \beta_i(r_{0,d} - r_{f,d}) + \gamma_i \mathrm{SMB}_d + \phi_i \mathrm{HML}_d + \epsilon_{i,d}, \tag{18}$$

where $r_{i,\,d}$ and $r_{0,\,d}$ are the daily returns of stock i and the market portfolio on day d, respectively, and SMB_d and HML_d denote the daily Fama and French (1993) size and book-to-market factors.

- Coskewness (CSK): Following Harvey and Siddique (2000) and Ang, Chen, and Xing (2006a), the coskewness of stock i at the end of month t is estimated using daily returns for month t as

$$\widehat{\mathrm{CSK}}_{i,t} = \frac{\frac{1}{N}\sum_d (r_{i,d} - \bar{r}_i)(r_{0,d} - \bar{r}_0)^2}{\sqrt{\frac{1}{N}\sum_d (r_{i,d} - \bar{r}_i)^2}\left(\frac{1}{N}\sum_d (r_{0,d} - \bar{r}_0)^2\right)}, \tag{19}$$

where N denotes the number of trading days in month t, $r_{i,\,d}$ and $r_{0,\,d}$ are the daily returns of stock i and the market portfolio on day d, respectively, and $\bar{r}_i$ and $\bar{r}_0$ denote the corresponding average daily returns.

- Cokurtosis (CKT): Following Ang, Chen, and Xing (2006a), the cokurtosis of stock i at the end of month t is estimated using the daily returns for month t as

$$\widehat{\mathrm{CKT}}_{i,t} = \frac{\frac{1}{N}\sum_d (r_{i,d} - \bar{r}_i)(r_{0,d} - \bar{r}_0)^3}{\sqrt{\frac{1}{N}\sum_d (r_{i,d} - \bar{r}_i)^2}\left(\frac{1}{N}\sum_d (r_{0,d} - \bar{r}_0)^2\right)^{3/2}}, \tag{20}$$

where variables are the same as for CSK.

- Realized skewness (RSK): Following Amaya, Christoffersen, Jacobs, and Vasquez (2015), the realized skewness for stock i on day d is constructed from high-frequency data as

$$\mathrm{RSK}_{i,d} = \frac{\sqrt{L}\sum_{l=1}^{L} r_{i,d,l}^3}{\left(\sum_{l=1}^{L} r_{i,d,l}^2\right)^{3/2}}, \tag{21}$$

where $r_{i,\,d,\,l}$ refers to the lth intraday return on day d for stock i and L denotes the number of intraday returns available on day d. Consistent with Amaya, Christoffersen, Jacobs, and Vasquez (2015), we use five-minute returns from 9:45 a.m. to 4:00 p.m. so that for the full intraday time period $L = 75$. The RSK for stock i at the end of month t is computed as the average of the daily $\mathrm{RSK}_{i,d}$ for that month.

- Realized kurtosis (RKT): Following Amaya, Christoffersen, Jacobs, and Vasquez (2015), the realized kurtosis for stock i on day d is computed as

$$\mathrm{RKT}_{i,d} = \frac{L\sum_{l=1}^{L} r_{i,d,l}^4}{\left(\sum_{l=1}^{L} r_{i,d,l}^2\right)^2}, \tag{22}$$

where variables and estimation details are the same as for RSK.

- Maximum daily return (MAX): Following Bali, Cakici, and Whitelaw (2011), the MAX variable for stock i and month t is defined as the largest total daily return observed over that month.
- Illiquidity (ILLIQ): Following Amihud (2002), the illiquidity for stock i at the end of month t is measured as the average daily ratio of the absolute stock return to the dollar trading volume from month $t-11$ through month t:

$$\mathrm{ILLIQ}_{i,t} = \frac{1}{N}\sum_d \left(\frac{|r_{i,d}|}{\mathrm{volume}_{i,d} \times \mathrm{price}_{i,d}}\right), \tag{23}$$

where $\mathrm{volume}_{i,\,d}$ is the daily trading volume, $\mathrm{price}_{i,\,d}$ is the daily price, and other variables are as defined before. We further transform the illiquidity measure by its natural logarithm to reduce skewness.

[50] Jegadeesh (1990) shows that monthly returns on many individual stocks are significantly and negatively serially correlated.

References

Aït-Sahalia, Y., 2004. Disentangling volatility from jumps. Journal of Financial Economics 74, 487–528.

Alexeev, V., Dungey, M., Yao, W., 2015. Time-varying continuous and jump betas: the role of firm characteristics and periods of stress. Unpublished working paper. University of Tasmania and Cambridge University, Tasmania, Australia, and Cambridge, UK.

Amaya, D., Christoffersen, P., Jacobs, K., Vasquez, A., 2015. Does realized skewness predict the cross section of equity returns? Journal of Financial Economics 118, 135–167.

Amihud, Y., 2002. Illiquidity and stock returns: cross section and time series effects. Journal of Financial Markets 5, 31–56.

Andersen, T.G., Bollerslev, T., Diebold, F.X., 2007a. Roughing it up: disentangling continuous and jump components in measuring, modeling, and forecasting asset return volatility. Review of Economics and Statistics 89, 701–720.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2001. The distribution of realized exchange rate volatility. Journal of the American Statistical Association 42, 42–55.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Vega, C., 2003. Micro effects of macro annoucements: real-time price discovery in foreign exchange. American Economic Review 93, 38–62.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Vega, C., 2007b. Real-time price discovery in stock, bond, and foreign exchange markets. Journal of International Economics 73, 251–277.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Wu, G., 2005. A framework for exploring the macroeconomic determinants of systematic risk. American Economic Review 95, 398–404.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Wu, G., 2006. Realized beta: persistence and predictability. In: Fomby, T., Terrell, D. (Eds.), In: Advances in Econometrics: Econometric Analysis of Economic and Financial Time Series, Vol. 20, Part 2. Emerald Group Publishing Limited, Bingley, UK, pp. 1–39.

Andersen, T.G., Bollerslev, T., Huang, X., 2011. A reduced form framework for modeling volatility of speculative prices based on realized variation measures. Journal of Econometrics 160, 176–189.

Ang, A., Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Economics 63, 443–494.

Ang, A., Chen, J., Xing, Y., 2006a. Downside risk. Review of Financial Studies 19, 1191–1239.

Ang, A., Hodrick, R., Xing, Y., Zhang, X., 2006b. The cross section of volatility and expected returns. Journal of Finance 61, 259–299.

Bali, T.G., Cakici, N., Whitelaw, R., 2011. Maxing out: stocks as lotteries and the cross section of expected returns. Journal of Financial Economics 99, 427–446.

Bali, T. G., Engle, R. F., Tang, Y., 2015. Dynamic conditional beta is alive and well in the cross section of daily stock returns. Unpublished working paper. Georgetown University, New York University, and Fordham University, Washington, DC, and New York, NY.

Bansal, R., Yaron, A., 2004. Risks for the long run: a potential resolution of asset pricing puzzles. Journal of Finance 59, 1481–1509.

Banz, R.W., 1981. The relationship between return and market value of common stock. Journal of Financial Economics 9, 3–18.

Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A., Shephard, N., 2009. Realized kernels in practice: trades and quotes. Econometrics Journal 12, C1–C32.

Barndorff-Nielsen, O.E., Shephard, N., 2003. Realized power variation and stochastic volatility models. Bernoulli 9, 243–265.

Barndorff-Nielsen, O.E., Shephard, N., 2004a. Econometric analysis of realized covariation: high-frequency-based covariance, regression, and correlation in financial economics. Econometrica 72, 885–925.

Barndorff-Nielsen, O.E., Shephard, N., 2004b. Power and bipower variation with stochastic volatility and jumps. Journal of Financial Econometrics 2, 1–37.

Barndorff-Nielsen, O.E., Shephard, N., 2006. Econometrics of testing for jumps in financial economics using bipower variation. Journal of Financial Econometrics 4, 1–30.

Barndorff-Nielsen, O.E., Shephard, N., 2007. Variation, jumps, market frictions and high frequency data in financial econometrics. In: Blundell, R., Torsten, P., Newey, W.K (Eds.), Advances in Economics and Econometrics: Theory and Applications. Econometric Society monographs. Cambridge University Press, Cambridge, UK, pp. 328–372.

Basu, S., 1977. Investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis. Journal of Finance 32, 663–682.

Basu, S., 1983. The relationship between earnings yield, market value, and return for NYSE common stocks: further evidence. Journal of Financial Economics 12, 129–156.

Berkman, H., Koch, P., Tuttle, L., Zhang, Y., 2012. Paying attention: overnight returns and the hidden cost of buying at the open. Journal of Financial and Quantitative Analysis 47, 715–741.

Bhandari, L.C., 1988. Debt/equity ratio and expected common stock returns: empirical evidence. Journal of Finance 43, 507–528.

Blume, M., 1970. Portfolio theory: a step toward its practical application. Journal of Business 43, 152–173.

Bollerslev, T., Engle, R., Wooldridge, J., 1988. A capital asset pricing model with time-varying covariances. Journal of Political Economy 96, 116–131.

Bollerslev, T., Law, T.H., Tauchen, G., 2008. Risk, jumps, and diversification. Journal of Econometrics 144, 234–256.

Bollerslev, T., Sizova, N., Tauchen, G., 2012. Volatility in equilibrium: asymmetries and dynamic dependencies. Review of Finance 16, 31–80.

Bollerslev, T., Todorov, V., 2011. Tails, fears, and risk premia. Journal of Finance 66, 2165–2221.

Bollerslev, T., Todorov, V., Li, S.Z., 2013. Jump tails, extreme dependencies, and the distribution of stock returns. Journal of Econometrics 172, 307–324.

Bollerslev, T., Zhang, B.Y., 2003. Measuring and modeling systematic risk in factor pricing models using high-frequency data. Journal of Empirical Finance 10, 533–558.

Branch, B., Ma, A., 2012. The overnight return: the invisible hand behind the intraday return. Journal of Applied Finance 22 (2), 90–100.

Campbell, J., Cochrane, J., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251.

Cliff, M., Cooper, M., Gulen, H., 2008. Return differences between trading and non-trading hours: like night and day. Unpublished working paper. University of Utah, Analysis Group, and Purdue University, Salt Lake City, UT, Boston, MA, and West Lafayette, IN.

Corradi, V., Distaso, W., Fernandes, M., 2013. Conditional alphas and realized betas. Unpublished working paper. University of Warwick, Imperial College London, and Queen Mary University of London, Coventry, UK, and London, UK.

Cremers, M., Halling, M., Weinbaum, D., 2015. Aggregate jump and volatility risk in the cross section of stock returns. Journal of Finance 70, 577–614.

Dimson, E., 1979. Risk measurement when shares are subject to infrequent trading. Journal of Financial Economics 7, 197–226.

Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. Econometrica 68, 1343–1376.

Eraker, B., Johannes, M., Polson, N., 2003. The impact of jumps in volatility and returns. Journal of Finance 58, 1269–1300.

Fama, E.F., Fisher, L., Jensen, M., Roll, R., 1969. The adjustment of stock prices to new information. International Economic Review 10, 1–21.

Fama, E.F., French, K.R., 1992. The cross section of expected stock returns. Journal of Finance 47, 427–465.

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.

Fama, E.F., French, K.R., 2006. The value premium and the CAPM. Journal of Finance 61, 2163–2185.

Fama, E.F., MacBeth, J.D., 1973. Risk, return, and equilibrium: empirical tests. Journal of Political Economy 81 (3), 607–636.

Ferson, W., Kandel, S., Stambaugh, R.F., 1987. Tests of asset pricing with time-varying expected risk premiums and market betas. Journal of Finance 42, 201–220.

Fleming, M.J., Remolona, E.M., 1999. Price formation and liquidity in the US Treasury market: the response to public information. Journal of Finance 54, 1901–1915.

Frazzini, A., Pedersen, L., 2014. Betting against beta. Journal of Financial Economics 111, 1–25.

Fu, F., 2009. Idiosyncratic risk and the cross section of expected stock returns. Journal of Financial Economics 91, 24–37.

Gabaix, X., 2012. Variable rare disasters: an exactly solved framework for ten puzzles in macrofinance. Quarterly Journal of Economics 127, 645–700.

Han, B., Kumar, A., 2013. Speculative retail trading and asset prices. Journal of Financial and Quantitative Analysis 48, 377–404.

Hansen, P., Lunde, A., 2006. Realized variance and market microstructure noise. Journal of Business and Economic Statistics 24, 127–161.

Harvey, C., 1989. Time-varying conditional covariances in tests of asset pricing models. Journal of Financial Economics 24, 289–317.

Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263–1295.

Jacod, J., Protter, P., 2012. Discretization of Processes. Springer-Verlag, Berlin, Germany.

Jacod, J., Shiryaev, A., 2002. Limit Theorems for Stochastic Processes. Springer-Verlag, Berlin, Germany.

Jagannathan, R., Wang, Z., 1996. The conditional CAPM and the cross section of expected stock returns. Journal of Finance 51, 3–53.

Jegadeesh, N., 1990. Evidence of predictable behavior of security returns. Journal of Finance 45, 881–898.

Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. Journal of Finance 48, 65–92.

Jiang, G., Yao, T., 2013. Stock price jumps and cross-sectional return predictability. Journal of Financial and Quantitative Analysis 48 (5), 1519–1544.

Lahaye, J., Laurent, S., Neely, C., 2011. Jumps, co-jumps, and macro announcements. Journal of Applied Econometrics 26, 893–921.

Lee, S., 2012. Jumps and information flow in financial markets. Review of Financial Studies 25 (2), 439–479.

Lehmann, B.N., 1990. Fads, martingales, and market efficiency. Quarterly Journal of Economics 105, 1–28.

Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying. Journal of Political Economy 109, 1238–1287.

Lettau, M., Maggiori, M., Weber, M., 2014. Conditional risk premia in currency markets and other asset classes. Journal of Financial Economics 114, 197–225.

Liu, J., Longstaff, F., Pan, J., 2003a. Dynamic asset allocation with event risk. Journal of Finance 58, 231–259.

Liu, J., Longstaff, F., Pan, J., 2003b. Dynamic derivative strategies. Journal of Financial Economics 69, 401–430.

Longstaff, F., 1989. Temporal aggregation and the continuous-time capital asset pricing model. Journal of Finance 44, 871–887.

Lou, D., Polk, C., Skouras, S., 2015. A tug of war: overnight versus intraday expected returns. Unpublished working paper. London School of Economics and Athens University, London, UK, and Athens, Greece.

Lucca, D., Moench, E., 2015. The pre-FOMC announcement drift. Journal of Finance, 70, 329–371.

Merton, R.C., 1973. An intertemporal capital asset pricing model. Econometrica 41, 867–887.

Merton, R.C., 1976. Option pricing when underlying asset returns are discontinuous. Journal of Financial Economics 3, 125–144.

Pan, J., 2002. The jump-risk premium implicit in options: evidence from an integrated time series study. Journal of Financial Economics 53, 3–50.

Patton, A., Verardo, M., 2012. Does beta move with news? Firm-specific information flows and learning about profitability. Review of Financial Studies 25, 2789–2839.

Roll, R., 1977. A critique of the asset pricing theory's tests, Part I: on past and potential testability of the theory. Journal of Financial Economics 4, 129–176.

Rosenberg, B., Reid, K., Lanstein, R., 1985. Persuasive evidence of market inefficiency. Journal of Portfolio Management 11, 9–16.

Savor, P., Wilson, M., 2014. Asset pricing: a tale of two days. Journal of Financial Economics 113, 171–201.

Scholes, M., Williams, J., 1977. Estimating betas from nonsynchronous data. Journal of Financial Economics 5, 308–328.

Shanken, J., 1992. On the estimation of beta-pricing models. Review of Financial Studies 5, 1–55.

Sheppard, K., 2006. Realized covariance and scrambling. Unpublished working paper. University of Oxford, Oxford, UK.

Stambaugh, R.F., Yu, J., Yuan, Y., 2015. Arbitrage asymmetry and the idiosyncratic volatility puzzle. Journal of Finance 70 (5), 1903–1948.

Stattman, D., 1983. Book values and stock returns. Chicago MBA: A Journal of Selected Papers 4, 25–45.

Todorov, V., Bollerslev, T., 2010. Jumps and betas: a new framework for disentangling and estimating systematic risks. Journal of Econometrics 157, 220–235.

Wachter, J.A., 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility? Journal of Finance 68, 987–1035.

Yan, S., 2011. Jump risk, stock returns, and slope of implied volatility smile. Journal of Financial Economics 99, 216–233.