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The influence of self-efficacy and working memory capacity on problem-solving efficiency

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ABSTRACT

We investigated the influence of self-efficacy beliefs and working memory capacity on mathematical problem-solving performance, response time, and efficiency (i.e., the ratio of problems solved correctly to time). Students completed a letter-recoding task (Experiment 1) or an operation span task (Experiment 2), rated their self-efficacy for solving mental multiplication problems, and then solved similar problems of varying complexity. We tested the *motivational efficiency hypothesis,* which predicted that motivational beliefs, such as self-efficacy, increase problem-solving efficiency through focused effort and strategy use. Experiments 1 and 2 reported a significant effect for self-efficacy on problem-solving performance and efficiency, but limited effects for time. A self-efficacy by working memory interaction occurred in Experiment 1, suggesting self-efficacy is beneficial as demands on working memory increase. These findings suggested that self-efficacy increased problem-solving efficiency through strategic performance rather than faster solution times, and were consistent with the motivational efficiency hypothesis.

1. Introduction

Solving problems accurately and quickly continues to be the hallmark of efficient learners. Problem solving has been studied for decades and a great deal is known about general problem solving (Anderson, 1993; Dewey, 1910; Mayer, 1998), as well as task-specific problem solving in a variety of domains such as mathematics (Ashcraft, 1992; Campbell & Hackett, 1986; Kaye, deWinstanley, Chen, & Bonnefil, 1989; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Both task- and domain knowledge are essential components of problem-solving accuracy, however, a variety of other variables are related to problem-solving accuracy above and beyond domain knowledge, including self-efficacy beliefs (Lopez, Lent, Brown, & Gore, 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994), working memory capacity (Adams & Hitch, 1997; DeStefano & LeFevre, 2004; Klein & Bisanz, 2000; Seitz & Schumann-Hengsteler, 2000; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004), and problem complexity (Campbell & Xue, 2001; Hitch, 1978; Logie, Gilhooly, & Wynn, 1994; Mabbott & Bisanz, 2003).

Surprisingly, few studies have examined the role of motivational variables and variables such as working memory capacity, or problem complexity on problem-solving efficiency, which we define as the ratio of the number of problems solved correctly to the amount of time needed to solve them (Mory, 1992). Of special importance to the current research, there are no studies we know of that specifically have examined the influence of self-efficacy beliefs on problem-solving efficiency. The need for efficiency is important as some problem solving situations, such as post-secondary instruction, impose rigid time constraints.

The main goal of the present study was to examine whether self-efficacy beliefs enhance problem-solving efficiency of mental multiplication, while controlling for the variables of working memory capacity and problem complexity. This question is important for both theoretical and practical reasons. From a theoretical perspective, it is unknown whether self-efficacy increases problem-solving efficiency, even though self-efficacy is related to math problem-solving accuracy (Lopez et al., 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994). We tested the *motivational efficiency hypothesis* (Hoffman & Spatariu, 2008), which predicted that motivational beliefs, such as self-efficacy, increase problem-solving efficiency. It is possible that self-efficacy increases, decreases, or has no effect on response time. Although prior research has shown the positive influence of self-efficacy upon problem-solving accuracy, empirical support for problem-solving efficiency outcomes is lacking.

From a practical perspective, the possible positive effect of self-efficacy on problem-solving efficiency has important implications for understanding the optimal use of limited cognitive resources (Mayer & Moreno, 2003; Paas & van Merriënboer, 1993; Van Gerven, Paas, Van Merriënboer, & Schmidt, 2002). Given that many classrooms have constraints on the problem-solving process, and especially limited time (Marks, 2000), outcomes that seek to understand the relationship between motivational variables and efficiency warrant investigation.

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1.1. Self-efficacy and mathematical problem-solving

Self-efficacy, defined as the belief in one's ability to organize and execute courses of action to meet desired outcomes (Bandura, 1986), is a powerful intervening variable. The belief that one can perform a task successfully, in many cases, is a better predictor of eventual performance than previous attainments (Bandura, 1986; Pajares, 2002, 2003). Specifically, when controlling for performance accuracy, self-efficacy beliefs have been shown to operate independently of underlying skills, and mediate individual difference variables such as background knowledge (Pajares, 2003; Pajares & Miller, 1994), metacognitive awareness (McCombs & Marzano, 1990; Schunk & Ertmer, 2000), and overall ability (Bandura, 1986; Campbell & Hackett, 1986; Pajares & Kranzler; 1995; Pajares & Miller, 1994).

Self-efficacy beliefs play a powerful role concerning choice, persistence, effort, strategy use, and interest in mathematical problem solving (Lopez et al., 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Self-efficacy for math performance has been linked to selection of a college major (Hackett, 1985), and holds implications for mathematics career choice (Hackett & Betz, 1989). In addition, self-efficacy is positively related to mathematics achievement and problem solving over and above mathematics ability (Pajares & Kranzler, 1995; Pajares & Miller, 1994). Path analysis studies emphasize the predictive ability of self-efficacy by demonstrating self-efficacy either accounts for unique variance beyond factors such as gender, prior experience, perceived usefulness of math and self-concept (Pajares & Miller, 1994), or when controlling for background knowledge (Pajares & Kranzler, 1995). Although ability beliefs are a critical component of self-efficacy assessments, domain interest may be more of a reflection upon outcomes than capability (Lopez et al., 1997). Several meta-analyses have shown that effect sizes attributed to the mediating influence of self-efficacy on performance has ranged from .08 to .38 (Multon, Brown, & Lent, 1991; Stajkovic & Luthans, 1998; Valentine, DuBois, & Cooper, 2004).

Self-efficacy persists over time (Pajares & Graham, 1999), although judgments may change as the perception of task difficulty fluctuates. For example, Stajkovic and Luthans (1998) reported the role of self-efficacy diminished as task complexity increased. Campbell and Hackett (1986) found that self-efficacy assessments for easy tasks are higher due to the expectations of greater task success. However, it is not clear from these studies whether task difficulty and self-efficacy are related to problem-solving efficiency.

Self-efficacy is related to the tactics students will use when solving problems. Bandura (1986) described the connection between a learner's ability to control the learning environment and self-regulation. The sense of control, in turn, enhances the belief about their capabilities and potential to control their destiny (Pajares, 2002). The learner who believes s/he is capable of goal attainment uses more productive metacognitive strategies (Butler & Winne, 1995), works harder, expends more effort, and persists longer (Bandura, 1986; Bouffard-Bouchard, 2001; Lodewyk & Winne, 2005; Schunk & Zimmerman, 2006).

The self-efficacy research mentioned implies two main conclusions. First, self-efficacy is a powerful individual difference variable that is strongly related to academic achievement (Pajares, 1996; Zimmerman, Bandura, & Martinez-Pons, 1992). Strong self-efficacy beliefs can minimize other individual difference factors such as anxiety, physiological predisposition, and interest (Lent, Lopez, Brown, & Gore, 1996).

Second, self-efficacy influences performance beyond existing skills and ability. Even when controlling for general intelligence (Pajares & Kranzler, 1995) or prior math experience (Pajares & Miller, 1994), judgments of self-efficacy predicted achievement outcomes. The beliefs individuals possess concerning anticipated success also determine what challenges individuals attempt (Pajares & Kranzler, 1995). In sum, these studies demonstrate a significant influence of self-efficacy on performance outcomes.

1.2. Working memory, problem complexity, and mathematical problem-solving

Mental arithmetic requires the problem solver to encode the presented information, perform a mental calculation, and provide a response (DeStefano & LeFevre, 2004; Logie et al., 1994). Solving of mental arithmetic involves cognitive processes beyond mere fact retrieval (Seitz & Schumann-Hengsteler, 2000), and is assumed to include both the storage and processing of information (Hitch, 1978; Mabbott & Bisanz, 2003; Swanson & Beebe-Frankenberger, 2004).

Previous research indicates that individuals with higher levels of working memory capacity (WMC) perform better on learning tasks because they have more cognitive resources (Daneman & Carpenter, 1980; Mayer, 2001; Mousavi, Low, & Sweller, 1995). WMC also is positively correlated with general fluid intelligence, (g), (Engle, Kane, & Tuholski, 1999) and speed of processing (Bjorklund, 2005). It is likely that WMC affects cognitive efficiency due to the processing and storage requirements necessary to solve mental problems.

The effect of WMC on math problem solving has been documented in many studies (Adams & Hitch, 1997; DeStefano & LeFevre, 2004; Klein & Bisanz, 2000; Seitz & Schumann-Hengsteler, 2000; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). The current research used an operations span task to measure WMC and a recoding task to measure short-term memory. Span tasks require participants to solve problems while concurrently remembering either the cumulative sums of a series of problems or a list of words or numbers that follow a sequence of problems. These tasks presumably measure both a basic retrieval mechanism as well as central executive processing (Engle, Tuholski, & Laughlin, 1999; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). In contrast, a letter-recoding task (Benton, Kraft, Glover, & Plake, 1984) provides a measure of basic storage, such as temporary storage of verbal information in Baddeley's (1998) articulatory loop.

The processing efficiency of working memory is contingent upon both problem complexity (number of digits) and task demand (carry and load demands) (DeStefano & LeFevre, 2004). Kaye et al. (1989) used a dual-task paradigm requiring problem solving and concurrent detection of auditory probes. Dual task conditions are designed to divert memory resources from the primary task of remembering problem solutions. Kaye et al. concluded when participants were required to maintain sums in memory, or attend to dual tasks, response time increased. In a related study, Logie et al. (1994) used adding-span techniques, which involve addition of individual problems while concurrently maintaining a cumulative running total, to study mental arithmetic. Volunteer participants were required to solve either “single carry” or “multiple carry” (p. 399) mental arithmetic problems in both single and dual-task conditions. These studies demonstrated that problem-solving ability was related to the activation of working memory resources, including the central executive, which generally is regarded as the active processing component of working memory (DeStefano & LeFevre, 2004). They also supported the conclusion that as task complexity increases, processing become less efficient by imposing greater demands upon working memory that impede performance (Pollock, Chandler, & Sweller, 2002; Sweller & Chandler, 1994).

A critical variable influencing problem-solving performance is problem complexity. Solving basic multiplication problems, such as 3×4, often involves use of an automated calculation algorithm (Logie et al., 1994) that requires few working memory resources. Solving problems of multiple digits involves greater complexity (Hitch, 1978; Logie et al., 1994; Mabbott & Bisanz, 2003), takes longer (Hitch, 1978; Royer et al., 1999; Siegler, 1988), and requires time consuming mental computations (Campbell & Xue, 2001) resulting in far greater demand on limited cognitive resources.

The research on WMC leads to two main conclusions. First, math problem-solving ability is mediated by WMC (DeStefano & LeFevre, 2004; Logie et al., 1994; Passolunghi & Siegel, 2001; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). WMC influences how material is encoded, strategies used, problem-solving time, and overall performance accuracy.

Second, problem complexity and problem length determine the efficiency of problem-solving performance (Adams & Hitch, 1997; Campbell & Xue, 2001; Siegler, 1988). Problems requiring multiple stages of calculation take longer to solve, result in reduced accuracy, and presumably use more working memory resources. Thus, more complex problems may be solved less efficiently (Ashcraft, 1992; Kaye et al., 1989; Logie et al., 1994). Determining the processing limitations imposed by time, ability, and instructional constraints constitutes an important step in defining overall problem-solving efficiency.

1.3. The present study

The present study investigated whether self-efficacy beliefs are related to problem-solving performance, problem-solving time, and problem-solving efficiency under unrestricted time conditions. We controlled for the effect of working memory capacity and problem complexity to better examine the unique variation shared by self-efficacy, performance accuracy, and efficiency. Before solving a series of multiplication problems, participants assessed their self-efficacy for mental multiplication accuracy, and completed either an alphabetic recoding task in Experiment 1 or an operation span task in Experiment 2. Problem complexity was manipulated using four difficulty levels of math problems in Experiment 1 and two levels in Experiment 2.

We proposed the motivational efficiency hypothesis (Hoffman & Spatariu, 2008), which states that positive motivational beliefs such as self-efficacy (Bandura, 1986, 1997; Butler & Winne, 1995; Pintrich, 2000; Pintrich & De Groot, 1990), personal goal orientations (Ames & Archer, 1988; Stone, 2000), intrinsic motivation (Zimmerman, 1989), engagement, or attributions to metacognitive strategy use (Fredericks, Blumenfeld, & Paris, 2004; Linnenbrink & Pintrich, 2003) are related to more efficient problem solving. Our primary goal was to investigate whether self-efficacy is related to efficiency in addition to performance. Previous research has failed to investigate this important question. We believe that self-efficacy is related to better problem-solving efficiency for three inter-related reasons. One reason is that higher self-efficacy individuals use the most appropriate problem-solving strategy (Siegler; 1988; Walczyk, 1994). A second reason is individuals with greater self-efficacy also engage in attentional focusing, in which they assess the complexity of the problem and monitor their solution in the most efficient manner (Cowan, 2005; Mayer & Moreno, 2003). A third reason is that higher self-efficacy individuals experience fewer distractions due to anxiety and handicapping (Ashcraft & Kirk, 2001; Eysenck & Calvo, 1992; Hopko, Hunt, & Armento, 2005).

We made several predictions regarding the relationship between self-efficacy and WMC. First, we expected individuals with higher self-efficacy and greater WMC to be more efficient, solving problems both more accurately and quickly. In addition, individuals should be more efficient solving easier problems (e.g., one digit multiplied by two digits) than more difficult ones (e.g., two digits multiplied by two digits). A second prediction was that self-efficacy would interact with WMC in a compensatory manner. We expected self-efficacy to influence performance and efficiency more when WMC was low.

2. Experiment 1

2.1. Method

2.1.1. Participants

Study participants were students enrolled in an introductory educational psychology course from a large southwestern university in the United States ($N=58$, 16 = males, 42 = females), who volunteered as partial fulfillment of a class requirement. The study employed a regression design to assess the relationships among self-efficacy and WMC on three outcome measures. Each participant solved 32 problems of mental multiplication at four different levels of problem complexity. No subject attrition was encountered.

2.1.2. Materials and procedures

Standardized instructions were provided to all participants. Each participant completed three distinct tasks. The first task was a measure of WMC, which has been identified as a significant contributor to the variance associated with problem-solving and general intellectual ability (Conway, Kane, Bunting, Hambrick, Wilhelm, & Engle, 2005; Engle, Kane et al., 1999; Engle, Tuholski et al., 1999), and was used to account for participants' problem-solving processing ability.

The task consisted of letter recoding, in which individuals heard 15 trials of either a four, five, or six letter sequence read from an audiotape. After hearing the string of letters, participants had up to ten seconds to recode the letters in ascending alphabetical order (e.g., C, T, X, B → B, C, T, X) and record their results. The task was a replication of the ordered letters task used by Benton et al. (1984). Each letter correctly recoded in the exact serial order was allocated one point for scoring purposes. There were no time limits during any portion of the procedure, with the exception of time allotted to recoding.

We assessed self-efficacy for solving mental multiplication problems by asking participants to rate their confidence for solving eight different mental multiplication problems accurately. Ratings were made prior to the actual problem-solving task; thus, participants did not know that they would be asked to solve problems as efficiently as possible, or that the problem-solving task would be timed. Participants were told to indicate their self-efficacy assessments for accurately solving the problems in a reasonable amount of time, under the expectation that if actually solving the problems, they would *not* have an unlimited amount of time.

The problems in the self-efficacy assessment were identical in length and difficulty to items solved in the actual study. We controlled for digit size and frequency and the problems that were part of the self-efficacy assessment had the same number of digits in the solution as the actual task. This method of measuring self-efficacy was equivalent to Lopez et al. (1997). Students rated problems in 10-point increments from no confidence at all (0) in solving, to total confidence in solving (100). Participants were required to rate their level of confidence for solving each problem. Ratings for the eight items were averaged to create a single composite score that ranged from 0 to 100.

In the third part of the study, individuals solved 34 mental multiplication problems at four different levels of complexity, presented individually, on a computer screen. The problems were solved without the aid of computers, paper and pencil, or any other aid. The problems differed in complexity based upon number of digits in the equation and the solution. We used two- and three digit mental multiplication problems in both experiments for several reasons. First, adults have extensive experience with mental multiplication. Second, it is possible to manipulate and control the complexity of multiplication problems in order to control problem complexity as documented by the problem size effect (Campbell & Xue, 2001). Third, most individuals should be able to make accurate self-efficacy judgments of their ability to solve math problems (Chen, 2002; Stone, 2000).

The first two items of the 34-item instrument were designated as practice problems designed to familiarize each student with the process and content of solving mental arithmetic, and were not included in the statistical analysis. The scored trials consisted of (8) 2 digit × 1 digit problems with three digit solutions ($49 \times 9 = 441$, level a); (8) 2 digit × 2 digit problems with three digit solutions ($45 \times 12 = 540$, level b); (8) 2 digit × 2 digit problems with four digit solutions ($98 \times 52 = 5096$, level c); and (8) 3 digit × 2 digit problems with four digit solutions ($291 \times 17 = 4947$, level d).

Each problem was presented individually, one appearing on the computer screen at a time. Order of problem presentation was determined randomly. The randomized order was presented in the same sequence to each student. Presentation of problems was determined

Table 1
Descriptive statistics—Experiment 1

Variable	Mean	SD	Observed range	Possible range
Performance (number correct)				
Level a	6.982	1.277	1.00–8.00	.00–8.00
Level b	3.534	2.295	.00–7.00	.00–8.00
Level c	3.086	2.079	.00–8.00	.00–8.00
Level d	2.862	2.243	.00–8.00	.00–8.00
Time (milliseconds × 1000)				
Level a	147.853	48.691	66.88–298.47	.001–∞
Level b	374.976	107.487	144.16–621.40	.001–∞
Level c	397.080	121.020	131.67–723.21	.001–∞
Level d	474.031	114.019	158.48–791–27	.001–∞
Efficiency (performance/time × 1000)				
Level a	52.734	20.109	4.55–119.62	.001–∞
Level b	10.246	7.795	.00–36.35	.001–∞
Level c	8.260	6.026	.00–27.56	.001–∞
Level d	6.165	5.452	.00–21.44	.001–∞
Self-efficacy	69.353	22.116	20.00–100.00	.00–100.00
Working memory capacity	9.380	2.961	2.00–15.00	.00–15.00

randomly and each participant solved the problems in the same order to insure consistency in difficulty levels across sets (Seitz & Schumann-Hengsteler, 2000). Instructions were presented on the computer screen and also read aloud to the students as a group.

Students used the computer keyboard to input answers to individual problems in a data entry field immediately below each problem. After designating an answer to each problem, students clicked “continue”. Upon clicking “continue”, the next problem was presented and the computer recorded the completion time for providing an answer and submitting the response. Students were informed that they could not view problems or change answers on previous screens once they advanced to the next screen. Before beginning, the researcher indicated that participants should read at their normal rate and click “continue” when ready to read the next problem. Students were instructed to solve problems as accurately and as quickly as possible without sacrificing accuracy of response. Additionally, students were instructed not to use the computer to input and temporarily store partial problem solutions before indicating the complete answer to the problem. Students were aware that both accuracy and problem-solving time were recorded. There were no completion time limits during any portion of the procedure; however, students were told to try as best as possible to arrive at the correct solution to the problem even if they thought the problem was not readily solvable. Inputting a result was required to advance to the subsequent problem. Students were closely monitored by the lead author for task compliance to ensure participants completed the task according to instructions. Although answers could be modified before submission, changing answers was permitted only in the event of a typographical error. The results of any student suspected of manipulating answers or not completing the task according to instructions were removed from the analysis.

Three dependent measures were recorded: number of fully correct responses to each of 34 multiplication problems; aggregate time in milliseconds (converted to seconds) to complete and submit answers to the multiplication problems; and problem-solving efficiency, the aggregate number of correct responses divided by response time (multiplied by 1000 for ease of representation). Results were aggregated for each level of item complexity to determine any differences between less complex problems, versus problems that were more complex.

A simultaneous multiple regression was performed for each dependent variable at each level of complexity (labeled a, b, c and d) with self-efficacy and working memory capacity as the independent variables. The analysis was performed using SPSS 14.0 for Windows. Diagnostics were conducted for each set of regressions.

In order to calculate the joint effects of both self-efficacy and WMC, an interaction term was created between the independent variables of self-efficacy and WMC, and mean centering was used to control for multicollinearity. The interaction was created by multiplying the crossproducts of the standardized variables for each respective variable. The initial regression model included the interaction term and the independent variables. A separate model analyzing only the unique contributions of the independent variables was created excluding the interaction term, if the interaction term was not significant (Aiken & West, 1991).

Reliability coefficients using Cronbach's alpha were calculated to determine the reliability of the eight-item self-efficacy measure. Results indicated the measure was highly reliable, $\alpha = .954$.

3. Results

Preliminary *t*-tests examined if results related to problem-solving performance and problem-solving response time were influenced by gender. Results indicated gender did not influence problem-solving performance or problem-solving time, $t_{(56)} = -1.767$, $p = .083$ ($M_{\text{female}} = 17.39$, $SD_{\text{female}} = 6.06$ and $M_{\text{male}} = 14.06$, $SD_{\text{male}} = 7.21$) and $t_{(56)} = -1.269$, $p = .403$, ($M_{\text{female}} = 1412.15$, $SD_{\text{female}} = 281.58$ and $M_{\text{male}} = 1346.14$, $SD_{\text{male}} = 373.47$) respectively.

3.1. Descriptive statistics and correlations

Means and standard deviations for each dependent measure and for the self-efficacy and WMC variables are presented in Table 1. Zero-order correlations were conducted and are presented in Table 2. Highlights of the correlation table revealed a significant positive relationship between the self-efficacy variable and WMC. Additionally, a positive relationship between self-efficacy and both problem-solving performance and

Table 2
Correlations—Experiment 1

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. GPA	–	.087	−.019	−.030	−.019	.003	−.073	−.108	−.185	−.082	−.303*	.091	.069	.004	−.005
2. Self-efficacy		–	.316*	.079	.285*	.409**	.398**	−.082	−.252	−.095	−.164	.124	.356**	.440**	.453**
3. Memory			–	.201	.179	.220	.349**	−.142	−.310*	−.109	−.159	.243	.231	.229	.382**
4. Performance level a				–	.249	.390**	.311*	−.201	−.041	−.007	.069	.538**	.223	.345**	.294*
5. Performance level b					–	.659**	.726**	−.301*	−.076	.111	.145	.261*	.905**	.586**	.696**
6. Performance level c						–	.755**	−.220	−.093	.050	−.033	.336**	.684**	.909**	.754**
7. Performance level d							–	−.251	−.115	.145	.118	.318*	.724**	.660**	.953**
8. Time level a								–	.224	.122	.268*	−.846**	−.356**	−.243	−.309*
9. Time level b									–	.638**	.627**	−.183	−.375**	−.303*	−.217
10. Time level c										–	.668**	−.044	−.075	−.271*	.002
11. Time level d											–	−.209	−.057	−.257	−.099
12. Efficiency level a												–	.320*	.328*	.378**
13. Efficiency level b													–	.695**	.747**
14. Efficiency level c														–	.745**
15. Efficiency level d															–

* Correlation is significant at the .05 level (2-tailed).
** Correlation is significant at the .01 level (2-tailed).

Table 3
Results of multiple regression analysis for problem-solving performance, Experiment 1

Predictor variable	β	SE	T	p	F	p	R^2
Performance-level a							
Model with interaction					.768	.517	.041
Self-efficacy × WMC*	−.013	.003	−.094	.926			
Model w/o interaction					1.169	.318	.041
Self-efficacy	.017	.008	.121	. 964			
WMC	.196	.060	1.408	.165			
Performance-level b							
Model with interaction					2.34	.084	.115
Self-efficacy × WMC	.168	.005	1.242	.220			
Model w/o interaction					2.712	.075	.090
Self-efficacy	.253	.014	1.869	.067			
WMC	.099	.105	.727	.470			
Performance-level c							
Model with interaction					10.324	<.001	.364
Self-efficacy × WMC*	.458	.004	3.999	<.001			
Model w/o interaction					5.884	.005	.176
Self-efficacy	.377	.012	2.923	.005			
WMC	.101	.091	.779	.439			
Performance-level d							
Model with interaction					9.364	<.001	.342
Self-efficacy × WMC*	.378	.004	3.247	.002			
Model w/o interaction					7.475	.001	.214
Self-efficacy	.320	.013	2.539	.014			
WMC	.248	.095	1.965	.054			

* β reflects use of centered means.

Table 4
Results of multiple regression analysis for problem-solving time, Experiment 1

Predictor variable	β	SE	T	p	F	p	R^2
Time level a							
Model with interaction					1.165	.332	.061
Self-efficacy × WMC*	−.209	.111	−1.50	.139			
Model w/o interaction					.609	.548	.022
Self-efficacy	−.041	.309	−.290	.773			
WMC	−.129	2.312	−918	.363			
Time level b							
Model with interaction					3.486	.022	.162
Self-efficacy × WMC*	.211	.232	1.606	.114			
Model w/o interaction					3.829	.028	.122
Self-efficacy	−.171	.647	−1.282	.205			
WMC	−.256	4.834	−1.922	.060			
Time level c							
Model with interaction					1.041	.382	.055
Self-efficacy × WMC*	.208	.278	1.488	.143			
Model w/o interaction					.445	.643	.016
Self-efficacy	−.067	.771	−.477	.635			
WMC	−.088	5.763	−.621	.537			
Time level d							
Model with interaction					.771	.515	.041
Self-efficacy × WMC*	.039	.264	.279	.781			
Model w/o interaction					1.137	.328	.040
Self-efficacy	−.127	.718	−.910	.367			
WMC	−.119	5.363	−.853	.397			

* β reflects use of centered means.

problem-solving efficiency was found. Strong relationships, with changes across complexity levels, were observed among the problem-solving performance and problem-solving efficiency outcome variables.

3.2. Problem-solving performance

At the third level of problem difficulty, the overall model was significant, $F_{(3,\ 54)}=10.324$, MSR = 2.902, $p<.001$, $R^2=.364$. As can be seen in Table 3, a significant self-efficacy × WMC interaction was observed, $\beta=.458$, SE = .004, $t=3.999$, $p<.001$. Similar results were observed at level (d) of difficulty, $F_{(3,\ 54)}=9.364$, MSR = 3.495, $p<.001$, $R^2=.342$ indicating overall model significance. The interaction term was statistically significant, $\beta=.378$, SE = .004, $t=3.247$, $p<.005$, indicating the combination of high self-efficacy and high WMC led to significantly better problem-solving performance.

The individual effects of self-efficacy and WMC were calculated with the interaction term removed from the regression equation in order to obtain an estimate of the unique variation explained by each variable. Self-efficacy was a significant predictor of problem-solving performance at level (c) of difficulty, $\beta=.377$, SE = .012, $p<.005$ and at difficulty level (d), $\beta=.320$, SE = .013, $p<.025$. Examination of squared semipartial correlations between self-efficacy and problem-solving performance at level (c) revealed that 12.8% of the variation in problem-solving performance was accounted for by self-efficacy, while at level (d) 9.2% of the variability in problem-solving performance was exclusively a function of the self-efficacy variable.

WMC was not a significant predictor of problem-solving performance at any difficulty level. No other significant findings for problem-solving performance were observed.

3.3. Problem-solving time

Problem-solving time was determined by the latency of response for each of the 32 problems of mental multiplication (Table 4). Response time was recorded in milliseconds, and converted to seconds for ease of analysis. Results were aggregated to provide a total problem-solving time for each level of problem complexity. Results were screened for extreme scores prior to analysis. There was no evidence of exceptionally fast or slow responses during the problem-solving phase of the experiment. No effects for time were observed in any of the data analyses.

3.4. Problem-solving efficiency

Problem-solving efficiency was determined by computing the ratio between the number of problems solved correctly and problem-solving time for each of the 32 problems of mental multiplication (Table 5). Results were summed to provide a total problem-solving efficiency score for each level of problem complexity. The ratio of

Table 5
Results of multiple regression analysis for problem-solving efficiency, Experiment 1

Predictor variable	β	SE	T	p	F	p	R^2
Efficiency level a							
Model with interaction					1.586	.204	.081
Self-efficacy × WMC*	.146	.046	1.065	.292			
Model w/o interaction					1.807	.174	.062
Self-efficacy	.052	.125	.381	.705			
WMC	.227	.935	1.647	.105			
Efficiency level b							
Model with interaction					3.540	.021	.164
Self-efficacy × WMC*	.157	.017	1.197	.236			
Model w/o interaction					4.557	.015	.142
Self-efficacy	.314	.046	2.386	.021			
WMC	.132	.347	1.001	.321			
Efficiency level c							
Model with interaction					8.982	<.001	.333
Self-efficacy × WMC*	.381	.012	3.247	.002			
Model w/o interaction					6.988	.002	.203
Self-efficacy	.409	.035	3.220	.002			
WMC	.100	.258	.786	.436			
Efficiency level d							
Model with interaction					11.445	<.001	.389
Self-efficacy × WMC*	.366	.010	3.259	.002			
Model w/o interaction					10.094	<.001	.268
Self-efficacy	.370	.030	3.040	.004			
WMC	.265	.224	2.179	.034			

* β reflects use of centered means.

problem-solving performance to problem-solving time was multiplied by 1000 for ease of comparison.

A significant self-efficacy × WMC interaction at was observed at difficulty levels (c & d) of as both models were statistically significant, $F_{(3, 54)}$ = 8.982, MSR = 25.575, $p < .001$, $R^2 = .333$ and $F_{(3, 54)}$ = 11.445, MSR = 19.181, $p < .001$, $R^2 = .389$, respectively. The interaction was a significant predictor of problem-solving efficiency at level (c), $\beta = .381$, SE = .012, $t = 3.247$, $p < .005$ and at level (d), $\beta = .366$, SE = .010, $t = 3.259$, $p < .001$. Consistent with the correlation results, both self-efficacy and WMC were related to more efficient problem-solving.

We examined the influence of the self-efficacy variable after removing the interaction term from the regression model. For difficulty levels (b, c, and d) self-efficacy was a significant predictor of problem-solving efficiency, at level (b), $\beta = .314$, SE = .046, $t = 2.386$, $p < .05$, at level (c), $\beta = .409$, SE = .035, $t = 3.220$, $p < .005$ and at level (d), $\beta = .370$, SE = .030, $t = 3.040$, $p < .005$. These results indicated self-efficacy for mental multiplication is positively related to problem-solving efficiency.

In addition, for problems of the greatest complexity WMC was a significant predictor of problem-solving efficiency, $\beta = .265$, SE = .224, $t = 2.179$, $p < .05$, revealing that 6.3% of unique variance can be associated with WMC. No other significant findings were observed.

4. Summary of results—Experiment 1

Our findings revealed a two-way interaction between self-efficacy and WMC on performance. This result suggested the combination of high WMC and high self-efficacy was related to improved problem solving. Also as problem complexity increased, self-efficacy became a better predictor of problem-solving accuracy. Self-efficacy accounted for a significant proportion of variance in performance accuracy and efficiency at high levels of problem complexity over and above the effect of other variables. This result supported the motivational efficiency hypothesis' claim that self-efficacy is related to problem-solving efficiency.

In contrast, the assumption that higher WMC consistently improves problem-solving performance and efficiency was not supported. We believe this result occurred because the cognitive effort imposed by more complex problems of mental arithmetic exceeded the processing capabilities of the even the most capable individuals. A comparison of performance means between 2 × 1 digit problems ($M_{\text{level a}} = 6.98$) with three digit solutions and problems of greater complexity ($M_{\text{level b}} = 3.53$, $M_{\text{level c}} = 3.09$ and $M_{\text{level d}} = 2.86$ respectively) indicated significant differences. Once the threshold of the easiest problems was exceeded, performance and efficiency decreased dramatically.

5. Experiment 2

Experiment 2 was conducted for two reasons. First, Experiment 2 utilized an operation span working memory task, which is thought to provide a more comprehensive method of measuring working memory capacity (Engle, Kane et al., 1999; Engle, Tuholski et al., 1999; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). It is possible that a measure of memory span may be sensitive to changes in efficiency that the measure of letter recoding did not detect. Second, we used two levels of problem difficulty with more items at each level to create a more sensitive test. This change was motivated by the fact that three of the four levels in Experiment 1 were answered with the same relative degree of accuracy. Approximately twice as many problems were used at each of the two levels to better assess the effects of the independent variables.

6. Methods

6.1. Participants

Participants consisted of 81 (21 males and 60 females) college undergraduates from the pool described in Experiment 1. Participants volunteered as partial fulfillment of a class requirement. No subject attrition was encountered.

6.2. Materials and procedures

The methodology for Experiment 2 was the same as Experiment 1 with two exceptions, the working memory measure and levels of complexity. First, the working memory measure employed was an operation span working memory task, employing the methodology and scoring procedure suggested by Conway et al. (2005). The working memory task is consistent with Daneman and Carpenter's (1980) seminal measures of working memory capacity. The operation span task requires concurrent processing and storage of information (Swanson, 2004). Participants completed the task individually with the first author. Each of 42 computer-based trials consisted of providing participants with a simple math equation such as (6/3) + 2 = 5, followed by a single syllable word. Participants were required to verbally articulate the equation, verbally verify to the researcher if the equation was correct or incorrect by saying "yes" if correct or "no" if incorrect, and then attempted to remember the word following the equation. When instructed, participants recalled the series of words that followed the equations. Accuracy on the working memory task was measured by the number of words recalled correctly in serial order for each set of equation-word pairs. Equations were counter-balanced with equal addition and subtraction tasks and equality in the correctness or incorrectness of the equation. Twelve trials consisting of between two to six equation-word pairs were presented to each participant. Presenting equations in the same order to each participant provides a mechanism to control for task difficulty across sets (Seitz & Schumann-Hengsteler, 2000) and allows for consistent fatigue and practice effects across participants.

The 12 equation-word pairs trials were presented in the same order to each participant, and consisted of word-equation pairs in the following combinations; three 2-item pairs, three 3-item pairs, three 4-item pairs, and three 5-item pairs. Each trial was untimed; however, the researcher monitored participants to ensure the verbal verification of the equation and solution was followed by the verbalization of the to-be-remembered words. After each trial, participants recalled and recorded the to-be-remembered words in exact serial order on a worksheet. After completion of each trial and recall, the participants proceeded to the next trial, at their own pace, until all trials were completed.

Scoring of the operation span task involved assessment of the equation-processing component and the serial recall task. Partial credit unit scoring (Conway et al., 2005) was used for the recall task. Partial credit scoring calculates the mean proportion of words within a trial that are recalled in correct serial order. Words recalled inaccurately or in the wrong serial position were counted as incorrect. Conway et al. (2005) recommend removal of individuals who score less than 85% accuracy on the equation-processing component of the operation span task, based upon inattention to the processing component. In the current study, all participants achieved greater than 85% accuracy on the equation-processing component of the task. The aggregate number of word items recalled correctly was used to determine performance on the operation span.

Experiment 2 used two levels of problem complexity. The problems differed in complexity based upon number of digits in the equation and the solution. The first two items of the 42-item instrument were designated as practice problems to familiarize each student with the process and content of solving mental arithmetic, and were not included in the statistical analysis. The scored trials consisted of (20) 2 digit × 1 digit problems with three digit solutions (49 × 9 = 441), and (20) 2 digit × 2 digit problems with three digit solutions (45 × 12 = 540).

After completion of the first 20 problems, participants received a message on their computer screens indicating a two-minute break

Table 6
Descriptive statistics—Experiment 2

Variable	Mean	SD	Observed range	Possible range
Performance (number correct)				
Level a	17.177	2.257	11.00–20.00	.00–20.00
Level b	10.924	5.741	.00–19.00	.00–20.00
Time (milliseconds × 1000)				
Level a	338.662	116.694	66.88–298.47	.001–∞
Level b	867.699	298.144	144.16–621.40	.001–∞
Efficiency (performance/time × 1000)				
Level a	57.157	22.606	4.55–119.62	.001–∞
Level b	14.405	10.921	.00–36.35	.001–∞
Self-efficacy	67.104	24.531	13.75–100.00	.00–100.00
Working memory capacity	31.27	4.687	19.00–41.00	.00–45.00

would elapse before students could complete the remaining 20 problems. A break was instituted to avoid fatigue effects between the first 20 and second 20 problems. As in Experiment 1, the amount of time participants available to solve problems was not restricted. In all other respects Experiment 2 was identical to Experiment 1. Reliability coefficients using Cronbach's alpha were calculated to determine the reliability of the eight-item self-efficacy measure. The measure was reliable, α = .944.

7. Results

Problem-solving performance, time and efficiency were calculated and analyzed as in Experiment 1. Results were screened for violation of assumptions and outliers, resulting in two cases being removed from the statistical analysis due to excessive problem-solving time. A preliminary comparison of males and females was performed using problem-solving performance and problem-solving response time. Both analyses indicated that gender did not affect results on the dependent measures, $t_{(77)} = -1.986$, $p = .051$ ($M_{\text{female}} = 27.07$, SD $_{\text{female}} = 7.09$ and $M_{\text{male}} = 30.95$, SD $_{\text{male}} = 7.03$) and $t_{(77)} = -.137$, $p = .891$ ($M_{\text{female}} = 1211.04$, SD $_{\text{female}} = 324.83$ and $M_{\text{male}} = 1193.43$, SD $_{\text{female}} = 512.53$), respectively.

7.1. Descriptive statistics and correlations

Means and standard deviations for performance, time, and efficiency measured at the two complexity levels (labeled as a and b) and for self-efficacy and WMC are presented in Table 6. Zero-order correlations are presented in Table 7. Highlights of the correlation table revealed self-efficacy was significantly correlated with both levels of problem-solving performance, problem-solving time at the first complexity level, and both levels of problem-solving efficiency. Unlike in experiment 1, a correlation between self-efficacy and WMC was not found, likely due to the difference in the working memory measure used in Experiment 2.

7.2. Problem-solving performance

As can be seen in Table 8, no significant interactions between the self-efficacy variable and the working memory variable were observed in Experiment 2.

After removing the interaction term from the regression model at level (b) of complexity, the regression model was statistically significant, $F_{(2,76)} = 14.153$, MSR = 24.654, $p < .001$, $R^2 = .271$. Self-efficacy was a statistically significant predictor of problem-solving performance, $\beta = .446$, SE = .023, $t = 4.462$, $p < .001$. Examination of squared semipartial correlations between self-efficacy and problem-solving

Table 7
Correlations—Experiment 2

Measure	1	2	3	4	5	6	7	8	9
1. GPA	–	.183	.243*	.221	.214	.016	−.032	.128	.277*
2. Self-efficacy		–	.205	.257*	.486**	−.278*	.069	.322**	.409**
3. Memory			–	.088	.284*	−.005	.113	.061	.188
4. Performance level a				–	.556**	−.274*	−.052	.560**	.464**
5. Performance level b					–	−.238*	.037	.414**	.763**
6. Time level a						–	.604**	−.837**	−.520**
7. Time level b							–	−.577**	−.490**
8. Efficiency level a								–	.739**
9. Efficiency level b									–

* Correlation is significant at the .05 level (2-tailed).
** Correlation is significant at the .01 level (2-tailed).

Table 8
Results of multiple regression analysis by predictor and complexity level, Experiment 2

Predictor variable	β	SE	T	p	F	df	p	R^2
Accuracy-level a								
Model with interaction					1.82	3.75	.152	.068
Self-efficacy × WMC	.01	.002	.116	.908				
Model w/o interaction					2.75	2.76	.07	.068
Self-efficacy	.25	.010	2.20	.03				
WMC	.03	.055	.32	.748				
Accuracy-level b								
Model with interaction					10.09	3.75	.001	.288
Self-efficacy × WMC	−.13	.005	−1.30	.197				
Model w/o interaction					14.15	2.76	.001	.271
Self-efficacy	.45	.023	4.46	.001				
WMC	.19	.123	1.92	.058				
Time level a								
Model with interaction					2.23	3.75	.091	.082
Self-efficacy × WMC	.045	.113	.390	.697				
Model w/o interaction					3.31	2.76	.042	.08
Self-efficacy	−.289	.535	−2.57	.012				
WMC	.054	2.80	.48	.632				
Time level b								
Model with interaction					.864	3.75	.463	.033
Self-efficacy × WMC	−.142	.297	−1.19	.236				
Model w/o interaction					.580	2.76	.563	.015
Self-efficacy	.048	1.41	.68	.680				
WMC	.103	7.40	.888	.377				
Efficiency level a								
Model with interaction					2.90	3.75	.04	.104
Self-efficacy × WMC	.010	.022	.09	.931				
Model w/o interaction					4.41	2.76	.015	.104
Self-efficacy	.323	.102	2.92	.005				
WMC	−.005	.535	−.040	.961				
Efficiency level b					5.44	3.75	.002	.179
Self-efficacy × WMC	−.024	.010	−.22	.830				
Model w/o interaction					8.247	2.76	.001	.178
Self-efficacy	.386	.047	3.64	.001				
WMC	.109	.248	1.03	.308				

* β reflect use of centered means.

performance revealed that 19.1% of the variation in problem-solving performance was accounted for by self-efficacy.

7.3. Problem-solving time

No significant interactions were observed for problem-solving time. After removing the interaction term from the regression model at level (a), the model was statistically significant, $F_{(2,76)}=3.312$, MSR=12,855.412, $p<.05$, $R^2=.080$. Self-efficacy was a statistically significant predictor of problem-solving time, $\beta=-.289$, SE=.535, $t=-2.573$, $p<.025$. Squared semipartial correlations between self-efficacy and problem-solving time revealed that 8.0% of the variation in problem-solving time was accounted for by self-efficacy.

7.4. Problem-solving efficiency

No significant interactions were observed for problem-solving efficiency. After removing the interaction term from the regression model at complexity level (a), the model was statistically significant, $F_{(2,76)}=4.408$, MSR=469.988, $p<.025$, $R^2=.104$. Self-efficacy was a statistically significant predictor of problem-solving efficiency, $\beta=.323$, SE=.102, $t=2.916$, $p<.01$. Squared semipartial correlations between self-efficacy and problem-solving efficiency revealed 10.0% of the variation in problem-solving efficiency was accounted for by self-efficacy.

Similar results were observed at complexity level (b) after removing the interaction term, statistical significance was observed, $F_{(2,76)}=8.247$, MSR=100.579, $p<.005$, $R^2=.178$. The consistent finding of self-efficacy as a predictor of problem-solving efficiency was again supported, $\beta=.386$, SE=.047, $t=3.636$, $p<.005$. Examination of squared semipartial correlations between self-efficacy and problem-solving efficiency revealed 14.2% of the variation in problem-solving efficiency was accounted for by self-efficacy. As the complexity of problems increased the role of self-efficacy as a predictor of problem-solving efficiency increased.

8. Summary of results—Experiment 2

As in Experiment 1, individuals with higher levels of self-efficacy solved more problems correctly and were more efficient in the problem-solving process. As problem complexity increased, the influence of self-efficacy as a predictor of performance accuracy and efficiency increased.

WMC was unrelated to any of the three outcome measures. These findings indicate that the working memory operation span task, which measures complex processing capacity, did not predict performance on any of the outcome measures. No other significant findings were observed.

9. General discussion

This study investigated the effect of self-efficacy and WMC at multiple levels of problem complexity on problem-solving performance, response time, and problem-solving efficiency. We tested the motivational efficiency hypothesis, which states that motivational beliefs such as self-efficacy, are related to problem-solving efficiency. Although previous research has established a clear relationship between self-efficacy and performance, few studies have investigated the relationship between self-efficacy and efficiency. We hypothesized self-efficacy would predict performance and efficiency when WMC was lower, and self-efficacy would interact with working memory capacity when problems were increasingly complex.

Results in both experiments supported the motivational efficiency hypothesis. Self-efficacy was a significant predictor of performance and efficiency, even when controlling for others variables. Self-efficacy interacted with WMC in Experiment 1 at the third and fourth levels of problem complexity (levels c and d). This interaction occurred at higher levels of problem complexity but not at lower levels of problem complexity. The interaction suggested self-efficacy may boost performance and compensate for processing constraints created when demands strain limited cognitive resources, or when problems are overly complex (i.e. 2 digit × 3 digit multiplication).

Greater demands on WMC exist at higher levels of problem complexity (DeStefano & LeFevre, 2004) and individuals higher in WMC are typically more accurate problem solvers (Pollock et al., 2002; Sweller & Chandler, 1994). Although our data does not explicitly support the contention that self-efficacy is a predictor of problem-solving performance and efficiency when WMC is low, our data indicated affects for self-efficacy may boost performance when WMC is unable to account for problem-solving accuracy and efficiency alone as in Experiment 1. In the absence of affects for WMC, self-efficacy was a significant predictor of performance (levels c and d) and efficiency (levels b and c).

WMC did not have strong or consistent effects on problem-solving accuracy. This may be due to the type of problems that were being solved or the possibility that effective mental arithmetic strategies are more important than capacity (DeStefano & LeFevre, 2004; Hecht, 2002). Indeed, previous research has shown that strategy training positively influences tasks such as span performance, even for low capacity subjects (McNamara & Scott, 2001). In addition, high WMC does not guarantee the use of effective problem-solving strategies. (Hambrick, Kane, & Engle, 2005; Hecht, 2002). Another plausible explanation for WMC not consistently predicting accuracy or efficiency in this study may be due to use of compensatory strategies as a means to overcome subcomponent inefficiency in unrestricted conditions (Walczyk & Griffith-Ross, 2006).

The finding that self-efficacy explained a significant proportion of unique variation in both experiments suggested that self-efficacy is an important predictor of performance and efficiency. Participants with higher levels of self-efficacy may be more resourceful in the allocation and adaptation of alternative strategies compared to those individuals with lower levels of self-efficacy, and thus solve problems with greater accuracy and efficiency. Walczyk, Wei, Griffith-Ross, Goubert, Cooper, and Zha (2007) described compensatory processes in reading and indicated engaged readers “circumvent automatic processes that fail” (p. 882). We believe motivational variables such as self-efficacy may also help readers utilize compensatory processes that lead to greater problem-solving efficiency.

In previous research, self-efficacy explained variance above and beyond math background knowledge (Pajares, 2003; Pajares & Miller, 1994). It is possible unique variance explained by self-efficacy may also be attributed to the use of different problem-solving strategies such as automaticity or better monitoring. This finding is consistent with Compensatory Efficiency Theory (C-ET), which contends under unrestricted conditions, as in the current study, individuals invoke compensatory processes to overcome processing obstacles such as problem complexity or subcomponent inefficiencies (Walczyk & Griffith-Ross, 2006). Another explanation is that high self-efficacy problem solvers experience less distraction due to anxiety. The potential compensatory effect of high self-efficacy should be investigated in future studies.

The results of Experiments 1 and 2 supported a main component of the motivational efficiency hypothesis in two ways. First, the present study extended findings from research on math problem-solving accuracy (Bandura, 1997; Pajares & Graham, 1999; Pajares & Kranzler, 1995; Pajares & Miller, 1994) to problem-solving efficiency. Second, the results involving self-efficacy in both experiments suggest that self-efficacy is most important when problem complexity is higher. High levels of self-efficacy appear to compensate for low working memory capacity and high problem complexity, presumably by motivating the problem solver in positive ways. We encourage researchers to test additional aspects of the motivational efficiency hypothesis, which predicts other motivational variables such as goal orientations, engagement, and strategy use enhance problem-solving efficiency.

There are several possible explanations for the effect of self-efficacy on efficiency, including strategy choice, cognitive savings, and attentional diversion. These explanations are not mutually exclusive; each may independently or collectively influence efficiency. High self-efficacy problem solvers likely possess a large, more flexible repertoire of problem-solving strategies (Horn, Bruning, Schraw, & Curry, 1993; Pajares, 2002; Schunk, 1990). As a result, these individuals should be able to solve problems more efficiently. High self-efficacy problem solvers also may save resources due to careful planning and monitoring of their problem solving. Previous research indicates that self-regulated learners are effective information processors because they are motivated, monitor, and direct their own learning (Walczyk, 1994; Wolters, 2004). These individuals may require less time to monitor problem accuracy, which provides additional time for subsequent problems (Walczyk & Griffith-Ross, 2006). It also is likely that high self-efficacy problem solvers experience less anxiety and distraction when solving problems. This explanation is consistent with the Processing Efficiency Theory proposed by Eysenck and Calvo (1992), which argues that anxiety invoking situations, such as mental arithmetic, draw from limited working memory resources. We assume that high self-efficacy problem solvers are more efficient in part because they do not divert attentional resources to manage stress and anxiety.

These findings are new to the literature and warrant two additional strands of research. One strand should investigate the effects of self-efficacy on processing time and efficiency in a variety of settings using different types of problems, but especially more complex problem-solving that places a greater demand on the working memory system. A second strand should examine the compensatory nature of self-efficacy in different settings, such as when students compose complex expository text in writing projects. This research has important implications for understanding the relationship between motivational beliefs and less malleable aspects of information processing.

Our results are consistent with previous research in that self-efficacy and problem complexity were related to problem-solving accuracy. In contrast, in both experiments we found WMC was not related to accuracy or related to efficiency as in several previous studies (DeStefano & LeFevre, 2004; Eysenck & Calvo, 1992). This may be due to the fact the memory measurement in Experiment 1 measured only the storage component of working memory, combined with a possible ceiling effect induced by the complexity of problems in both experiments. We advise other researchers to use only two levels of problem complexity as was employed in Experiment 2 to counteract potential ceiling effects based upon problem difficulty.

A second unexpected finding was that self-efficacy was not a consistent predictor of problem-solving time except for the first level of complexity in Experiment 2. In contrast, previous research found high self-efficacy problem solvers reported expending less effort, and presumably less time, than low self-efficacy counterparts (Bandura, 1997; Chen, 2002; Eysenck & Calvo, 1992). Thus, it is unclear whether high self-efficacy participants misjudged their effort expenditure, or if the present study utilized problems that did not require high- and low self-efficacy participants to differ with respect to expended effort. Participants were explicitly instructed to rate their self-efficacy to solve the problems accurately within a reasonable time frame. It is possible that participant's self-efficacy ratings for solving problems efficiently may differ from self-efficacy ratings to solve problems accurately. Additional research should investigate the degree to which self-efficacy for efficiency and accuracy are inter-correlated, as well as how each of these ratings are related to problem-solving time.

The current results support the conclusion that efficiency improves due to better performance. Efficiency does not appear to be related to problem-solving time under unrestricted conditions; however, differences may appear as problems become more difficult to solve or when time limits are imposed (Walczyk & Griffith-Ross, 2006). In addition, the fixed order of problem presentation may have influenced the observed results. Future research should examine these potential explanations using qualitative methods, or by attempting to experimentally measure or manipulate strategies, monitoring, and anxiety.

9.1. Limitations

Two limitations of the present research should be noted. One is that the experimental instructions (i.e., solve problems as quickly as you can without making mistakes) might have affected problem-solving strategy. Given that all participants received the same instructions, and that we did not include a comparison group that received no instructions or instructions to focus on accuracy only, it is unclear whether the instructions provided additional motivation to perform efficiently. This question should be addressed in future research by manipulating instructions that do and do not collectively emphasize speed and accuracy. The present study clearly suggests that self-efficacy for performance is related to performance accuracy and efficiency when participants are asked to perform quickly, yet accurately. It is not clear whether this relationship would replicate if participants were given different instructions.

A second limitation is that the present study used a familiar task, mental multiplication, which may be solved through the use of partially automated strategies. Complex tasks such as spatial (e.g., mental rotation problems) or verbal reasoning (e.g., verbal analogies) may yield a different pattern of results due to the inability to automatically retrieve solutions.

Our findings have broader relevance to the self-efficacy literature in that this is one of the first studies that we know of to investigate the effect of self-efficacy on problem-solving efficiency. The finding that self-efficacy consistently had a robust effect on efficiency has important implications for problem solving and instruction. Increasing self-efficacy may improve performance without increasing problem-solving time. We encourage others to replicate and extend our findings to other performance measures.

We believe further research is needed to replicate and extend the findings reported here, and to explore the effect of other beliefs such as goal orientations, attributions, and epistemological beliefs on problem-solving time and efficiency. Although self-efficacy was related to efficiency in this study, we believe that other motivational variables may contribute in a similar way. More research is needed to examine the effect of motivational variables on efficiency and to determine whether they provide a compensatory effect for WMC or other person-related variables such as ability and prior knowledge.

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