THE INTERNET: Problem Solving Friend or Foe?

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A few years ago, students could turn to advanced mathematical software like Mathematica, Maple, or Derive to simplify complicated expressions, factor polynomials, extrapolate missing data from a given set, and plot a variety of different graphs simultaneously. Today, many of these tasks are also readily performed for free on a number of Web sites that anyone can access easily. When they are used to increase understanding through exploration and discovery, these Web sites hold a great deal of potential and promise in our mathematics classrooms. But these Web sites can also be used to find direct or indirect solutions to some problem-solving tasks. This disconnect between the problems and the available technology exposes a common misconception held by novice problem solvers—that their goal is simply to get an answer to the problem. Understanding the difference between "locating a solution" (obtaining an answer without actually solving the problem) and "finding a solution" (using mathematical problem-solving strategies to arrive at an answer) lies at the crux of the matter.

In my work with preservice teachers who are learning about problem solving, I have encountered a number of situations where the Internet has unexpectedly pushed my teaching in a different direction. In this article, I explore several issues that

Problem 1
How many different triangles with integer side lengths have a perimeter of 48?

arise from using the Internet with problem-solving tasks—drawing from ten sample problems that I have used with preservice teachers—and describe ways in which my students and I have learned to appreciate the role that the technology can play in mathematical exploration and solving problems.

PROBLEMS VS. EXERCISES

First, it is important to understand how a problem-solving task and a more traditional mathematics exercise differ. One commonly made distinction is that a problem has no immediate solution and/or obvious strategy, while an exercise is an application of a known algorithm, formula, or method (Zeitz 1999). It is also widely recognized that a question that is classified as a problem for one student may be an exercise for another (Schoenfeld 1985), which makes it difficult to determine whether a question will make for a good problem-solving exercise in a class of students. But a new level of complexity arises when a problem is unexpectedly turned into an exercise by the use of the Internet. This situation presented itself in my classroom when I posed a question that was, to me, a problem.

To all of my students, **problem 1** was initially a problem-solving task and not an exercise. Most students worked on problem 1 in ways that I had anticipated—looking at simpler problems, making tables of possibilities, and/or searching for patterns in their data. What I had not anticipated, though, was the direct application of a formula that can be used to find the total number of different triangles with integer side lengths for any given perimeter. One student found such a formula on the MathWorld Web site (www.mathworld.com), a comprehensive collection of mathematical topics and discoveries. Entering a few key words into the site's search engine, he quickly found a listing for "Integer Triangle," which includes the following formulas for the total number of unique integer triangles for a given perimeter, *n:*

$$\left[\frac{n^2}{12}\right]-\left\lfloor\frac{n}{4}\right\rfloor\left\lfloor\frac{n+2}{4}\right\rfloor=\begin{cases}\left[\dfrac{n^2}{48}\right] & \text{for } n \text{ even}\\[2ex] \left[\dfrac{(n+3)^2}{48}\right] & \text{for } n \text{ odd}\end{cases}$$

where $[x]$ is defined as the nearest integer function (rounding the value of x to the nearest integer) and $\lfloor x \rfloor$ is defined as the floor function (giving the largest integer less than or equal to x) (Weisstein 2005). Applying either of these formulae to the question quickly yields the correct solution, but with no mathematical problem solving being done.

If the student had collected data for a number of different situations with a variety of triangle perimeters and had inductively discovered either of these functions (which would have been quite impressive), this would have been an application of good mathematical problem-solving skills. But by locating a general formula through Internet savvy, he circumvented the nature of the assignment. My goal was for students not simply to locate a solution but to apply their problem-solving skills in the pursuit of such.

It should also be noted that having a problem solver use inductive reasoning (when this is one of the possible problem-solving strategies) can be subverted by the use of the Internet. For example, one Web site, the On-Line Encyclopedia of Integer Sequences (www.research.att.com/~njas/sequences/), provides an extensive listing of sequences that can be searched by entering known numbers in the sequence or other relevant information. If students were to explore problem 1 by investigating simpler problems, they might start by creating a table that relates the perimeter (n) with the number of different triangles that can be created (T) (**table 1**).

Then they could enter the sequence {0, 0, 1, 0, 1, 1, 2, 1, 3, 2} into the On-Line Encyclopedia of Integer Sequences search engine, which yields six different sequences that contain this set of ten numbers. The first one listed is named Alcuin's Sequence and is described as (among other things), the "number of triangles with integer sides and perimeter n" (Sloane 2005). Here, the sequence begins with $n = 0$ (the trivial case) and goes through $n = 73$ (120 different triangles) and provides an easy answer to problem 1. Students could generate some

Table 1
Integer Triangle Data

n	T	
1	0	
2	0	
3	1	(1-1-1)
4	0	
5	1	(2-2-1)
6	1	(2-2-2)
7	2	(3-2-2) & (3-3-1)
8	1	(3-3-2)
9	3	(4-4-1), (4-3-2) & (3-3-3)
10	2	(4-4-2) & (4-3-3)

Problem 2
The number $2^{48} - 1$ has two factors between 60 and 70. What are they?

Problem 3
If $\frac{1}{4}$ of 2^{42} is 4^x, what is the value of x?

Problem 4
What is the sum of the coefficients in the expansion of $(a + b)^{10}$?

Problem 5
Solve for all real values of k:

$$\sqrt{(k+3)(k+2)(k+1)(k)+1} = 71$$

data through investigating simpler versions of the problem, but the inductive reasoning for finding the solution to the original problem is left to an Internet search engine to perform, not by induction, but by sheer exhaustion. Thus, this mathematical problem is reduced to an exercise in a different way.

TECHNOLOGY THAT PRODUCES SOLUTIONS

There is another set of problem-solving tasks for which Web sites exist that can aid students in finding solutions, although the solution may not already be given online. One of the more powerful sites is Factoris (wims.unice.fr/wims/en_tool~algebra~factor.en.phtml), an online calculator found on the WWW Interactive Multipurpose Server that can be used primarily to factor numbers and algebraic expressions. For example, in **problem 2**, while the teacher's intent may be to have students solve the problem by applying the laws of exponents and the factorization of $a^2 - b^2$, students could use Factoris to break $2^{48} - 1$ down into its prime factorization: $3^2 \times 5 \times 7 \times 13 \times 17 \times 97 \times 241 \times 257 \times 673$, from which the factors 63 and 65 can be obtained.

But students have also found that Factoris is useful in accomplishing other mathematical tasks as well. In **problem 3**, a teacher may want students only to apply rules of exponents in finding a solution (since 2^{42} is too large to fit on most calculator displays). Instead, students could use Factoris to calculate part of the answer by entering $2^{42}/4$. Factoris returns the answer "$1099511627776 = 2^{40}$" (since it is Factoris, it takes the opportunity to factor this 13-digit number as 2^{40}), from which students could reason that forty factors of 2 could be grouped as twenty factors of (2×2) or twenty factors of 4, so $2^{40} = 4^{20}$, giving a solution of $x = 20$. Factoris will also expand numerical and algebraic expressions, making solutions to questions like **problem 4** easy to find without using Pascal's triangle or any other method for finding patterns.

There are many other Web sites that can be extremely useful in exploring mathematical concepts but that could also be used to undermine problem-solving efforts. There are Web sites for graphing functions that can easily identify roots of equations or intersections of two graphs (e.g., mathdartmouth.edu/~klbooksite/appfolder/tools/Grapher.html), converting between different number bases (e.g., www.projects.ex.ac.uk/trol/scol/calnumba.htm), plotting and calculating the area between two curves (e.g., cs.jsu.edu/mcis/faculty/leathrum/Mathlests/twocurve.html), and almost everything that you can think of. And since there are already sites that solve simple one- or two-step algebraic equations, it would not be too surprising within five to ten years to find Web sites that would solve more complicated equations like the one in **problem 5**, thus removing all aspects of problem solving and discovery that can be gleaned from such classic problems.

SOLUTIONS FOR TODAY'S CLASSROOMS

Obviously, the Internet does not signal the death of problem solving. On the contrary, the advent of mathematical Web sites and the technology used to create them challenge us as teachers to redefine the problems that we use in our classes and to meet the demands of the emerging technology by educating students who are prepared to use the tools available to them in meaningful ways. As teachers, we need to be aware of the different types of resources that are available to our students on the Internet—recognizing that problems and their solutions often appear online and finding ways to work with the technology instead of against it.

Problems and Solutions Posted Online

Along with the many Web sites that offer teacher support through lesson plans and curriculum ideas, there has been a proliferation of sites that post interesting problems for students to solve. For example, the NCTM Web site (www.nctm.org) currently posts "Weekly Problems" that are very useful for developing students' problem-solving skills. However, teachers need to be aware that in addition to the problems, the site posts "Past Problems and Solutions," which details the answers to the problems as well as at least one strategy that students might use in reaching a solution. If students were to type one of these problems word for word into a search engine such as Google (www.google.com)—especially if they enclosed the text in

Problem 6

A pencil, eraser, and notebook together cost one dollar. A notebook costs more than two pencils, and three pencils cost more than four erasers. If three erasers cost more than a notebook, how much does each cost?

Problem 7

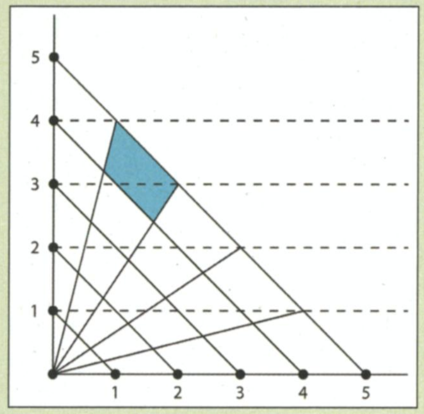
The product of the ages of three children is 1872. The middle child is as much older than the youngest child as she is younger than the oldest child. What are the ages of the three children?

Problem 8

Find the surface area of the solid defined by $|x| + |y| + |z| \leq 1$.

Problem 9

What is the area of the shaded region in the figure below?



quotation marks, forcing the search engine to look for the complete block of text online—they could locate the original problem and the solution.

Teachers need to be aware that problems found in print sources can show up online as well. In fact, when this article appears online, I will be contributing to this situation by including these ten problems as well as their answers, although not all of their complete solutions (**fig. 1**). The "Weekly Problems" on the NCTM Web site are actually reprints of some of the "Calendar" problems that have been a mainstay of the *Mathematics Teacher* for more than twenty years. In addition, some of the Web sites on which problems are posed and solutions are posted freely "borrow" good problems from books and journals; students who have learned to perform extensive searches of the Internet are thereby given access to solutions without doing the requisite mathematics that their teacher may intend.

When I find a problem in print that I would like to pose to my students, I first search for that problem online myself. This way, I know whether students would have easy access to locating a solution. If I do find the problem and solution posted online, or if my initial source for the problem is an existing Web site, I have learned that some simple editing of the problem can usually take care of the situation.

For example, I found **problem 6**, which is a good application of solving a system of inequalities, in an issue of *Mathematics Teacher* from the 1970s. I was able to find parts of the text of this problem on two Web sites—neither of which was the NCTM site. A simple change of "one dollar" to "$1.00" eliminated these hits. Further changes to the products mentioned in the problem could also make it more difficult for students to find online references to the problem. In addition, rewording the problem but keeping all of the basic elements the same makes the problem harder to find online. Sometimes, a simple change of the names used in a problem (e.g., changing "Sue" to "Penelope") can thwart search-engine junkies.

Another solution is to alter the numbers that are used in a problem. However, this approach can sometimes backfire when the underlying mathematics of the problem is dependent on the numbers that are used. For example, in **problem 7**, not every number can be used in place of the product 1872. In fact, interesting extensions of this problem examine whether a specific product is possible (e.g., 950 or 960) or look at the different possible products within a given range.

Another way to avoid the situation of students finding problems and their solutions online is to use problems that are currently not easily traceable using Internet search engines. In particular, problems that involve expressions with fractions, exponents, and mathematical symbols that cannot be entered from a keyboard are good candidates (e.g., **problem 8**). Also, problems that depend primarily on accompanying diagrams are next to impossible to locate online, as long as the accompanying text does not give the problem away (e.g., **problem 9**).

Of course, the underlying issue in students' direct copying of solutions they find online is one of plagiarism. Students need to understand that simply copying down solutions from any source (the Internet, a textbook, another student) is not acceptable and is a serious form of academic dishonesty. But by making some of these minor modifications, teachers can help eliminate the possibility of academic dishonesty before it has an opportunity to transpire. These changes can often deter students from finding online postings of problems that teachers may want to use in class, but there

Problem 10
The product of two numbers is 4 and their sum is 12. Find the sum of the reciprocals of the numbers.

Answers to Problems (complete solutions are left to the reader to supply):
1. 48
2. 63 and 65
3. $x = 20$
4. 1024
5. $k = -10$ or $k = 7$
6. Pencil: \$0.26, eraser: \$0.19, notebook: \$0.55
7. 8, 13, and 18 years old
8. $4\sqrt{3}$
9. 9/10
10. 3

Fig. 1 Answers to problems

are issues with broader implications that also need to be addressed. As professionals, teachers must consider three important steps that can help keep students from thinking that "using the Internet" is the first and only problem-solving heuristic they need to know.

First, we need to create an awareness of this situation—not only with the teachers, but with the students as well. In this article, I have raised issues to challenge teachers who may not be aware that students could be using Web sites under the pretext of problem solving. But none of these Web sites exists without a purpose. In fact, each site should be viewed as a powerful tool that could be used to explore mathematics and to make new discoveries about the nature and structure of the field.

Second—and closely connected to the first step—is the need to help students learn how to differentiate between the act of problem solving and the use of technology to locate solutions. When my student found a formula that made problem 1 an exercise, I used that opportunity to launch a class discussion about the nature of problem solving. As a class, we developed a statement to be included in future problem-solving assignments: "Work that can be attributed *only* to technology (graphing calculators, computer programs, Web sites, etc.) is not considered problem solving," and my scoring rubric was reworked to reflect this. This approach still allowed students to use technology for investigating the mathematics (a legitimate problem-solving strategy), but it discouraged them from using technology just to locate a solution.

Third, we must recognize the power of the

The product of two numbers is 4 and their sum is 12. Find the sum of the reciprocals of the numbers. Explore at least two other pairs of numbers with different products and sums. Find a general rule that could be used to find the sum of the reciprocals for any two numbers, given their product and their sum.

Fig. 2 Revised problem 10

emerging technology and use that understanding to create new problems that can draw upon this technology without undermining the development of many of the other problem-solving strategies. For example, **problem 10** is a classic problem-solving task that could (when used by a teacher who wants to use the problem for this purpose) lead to the generalization that

$$\frac{1}{a}+\frac{1}{b}=\frac{b}{ab}+\frac{a}{ab}=\frac{b+a}{ab},$$

verifying that the sum of the reciprocals of the two numbers can be found by taking the ratio of their sum to their product—without needing to know what the two numbers are. But with a function plotting tool that can identify points of intersections (as found on various Web sites), students could plot the functions $f(x) = 4/x$ and $g(x) = 12 - x$ to find that the two numbers are approximately 11.6568542495 and 0.343145750508. Then they could use a calculator (online or otherwise) to find the sum of the reciprocals of these numbers is approximately 3. This strategy completely avoids an interesting aspect of the problem that might only be exposed when other non-calculation-based approaches are used.

As a mathematics teacher, I am more interested in my students' discovery of the underlying algebraic principle that the sum of the reciprocals of two numbers is the same as the ratio of their sum to their product than I am in their finding a solution to the problem. A rewording of the problem that asks students to explore different situations could help them discover this principle (see revised problem 10, **fig. 2**). Students can still use the technology that is available to them for the exploration of the problem, but other problem-solving strategies must also be employed to answer the generalized question.

CONCLUSION

My students and I have greatly benefited from our increased awareness of problem solving and the evolving role that the Internet plays in solving problems. By focusing our understanding on the problem-solving process, we have learned to recognize and utilize more of the power behind these emerging technologies. We have also gained a more healthy respect for the goals of problem solving—that the process is infinitely more important than the product. As author Ursula K. LeGuin says, "It is good to have an end to journey toward; but it is the journey that matters, in the end" (quoted in Safransky 1990, p. 146).

REFERENCES

Safransky, S. *Sunbeams: A Book of Quotations.* Berkeley, CA: Sun Publishing Company, 1990.

Schoenfeld, A. *Mathematical Problem Solving.* San Diego, CA: Academic Press, 1985.

Sloane, N. J. A. The On-Line Encyclopedia of Integer Sequences. www.research.att.com/cgi-bin/access.cgi/as/njas/sequences/eisA.cgi?Anum=A005044. Retrieved September 29, 2005.

Weisstein, Eric W. "Integer Triangle." From Math World—Wolfram Web Resource. mathworld.wolfram.com/IntegerTriangle.html. Retrieved September 29, 2005.

Zeitz, P. *The Art and Craft of Problem Solving.* New York: John Wiley & Sons, 1999.

Web sites

Area between Two Curves:
cs.jsu.edu/mcis/faculty/leathrum/Mathlets/twocurves.html

Factoris:
wims.unice.fr/wims/en_tool~algebra~factor.en.phtml

Function Grapher/Plotter:
math.dartmouth.edu/~klbooksite/appfolder/tools/Grapher.html

Google: www.google.com

MathWorld: www.mathworld.com

NCTM: www.nctm.org

The Number Base Calculator:
www.projects.ex.ac.uk/trol/scol/calnumba.htm

The On-Line Encyclopedia of Integer Sequences:
www.research.att.com/~njas/sequences/. ∞

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