

Introduction to Graphs

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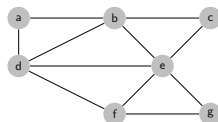
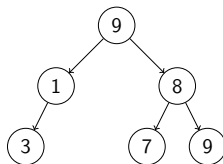
Objectives

- Become familiar with graph terminology.
- How graphs are structured.
- How graphs are represented with adjacency matrices.
- Practice implementing some basic graph operations using adj matrices.

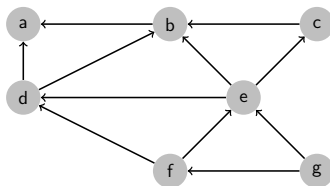
Objectives

- **Week 10:** Introduction to graphs via Adj Matrices.
- **Week 11:** Introduction into Adj. Lists and Graphs Search Algorithms. Implementation 5 released.
- **Week 12:** Election day, checkpoint 1 due.
- **Week 13:** Shortest Path and Minimum Spanning Tree Algorithms. Implementation 5 due and 6 released.
- **Week 14:** Thanksgiving break.
- **Week 15:** Conclusion and Application 3 released.

LinkedLists → Trees → Graphs

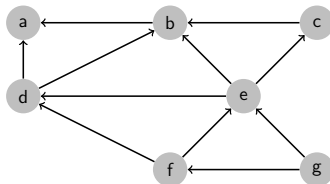


Terms



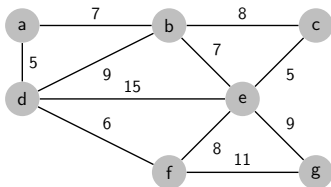
- **Vertex:** A “Node” in the graph.
- **Edge:** A connection between two vertices.
- **Graph:** A set of edges (E) and a set of vertices (V) and is often denoted as $G = (V, E)$.

Terms

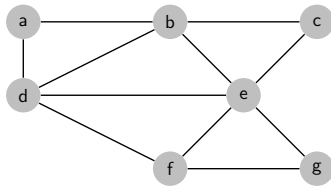


- **Directed v Undirected:** Can we move between vertices in either direction or only in one? Examples of directed graphs include:
 - **Linked Lists:** Once you move forward you can't move back.
 - **Trees:** Once you proceed left you can *directly* move back up to the parent.
- **Weighted v Unweighted:** Does it cost us anything to move between vertices?

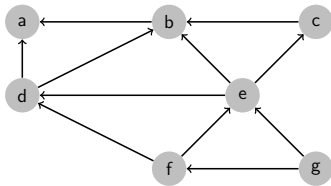
Directed vs Undirected and Weighted vs Unweighted



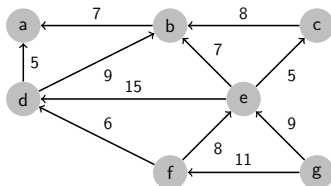
Undirected, Weighted



Undirected, Unweighted

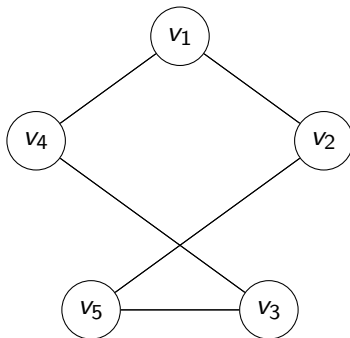


Directed, Unweighted



Directed, Weighted

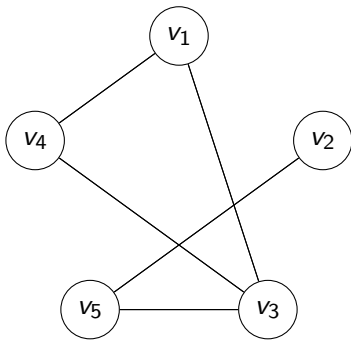
Adjacency Matrix vs Adjacency List



	1	2	3	4	5
1	0	1	0	1	0
2	1	0	0	0	1
3	0	0	0	1	1
4	1	0	1	0	0
5	0	1	1	0	0

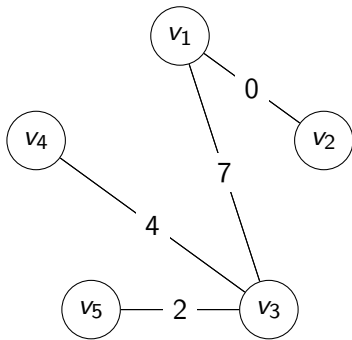
- 1 source vertex is the row.
- 2 target vertex is the column.
- 3 1 for edge exist and 0 for it doesn't.

Constructing Adj. Matrix - Unweighted, Undirected



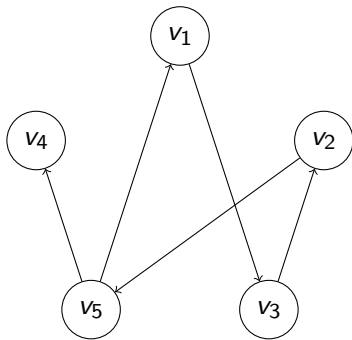
	1	2	3	4	5
1					
2					
3					
4					
5					

Constructing Adj. Matrix - Weighted, Undirected



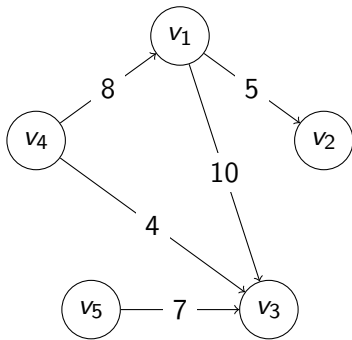
	1	2	3	4	5
1					
2					
3					
4					
5					

Constructing Adj. Matrix - Unweighted, Directed



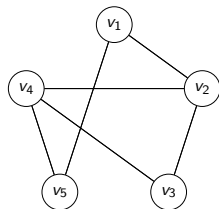
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1					
2					
3					
4					
5					

Constructing Adj. Matrix - Weighted, Undirected

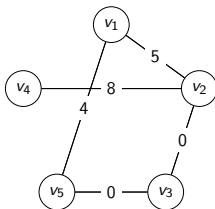


	1	2	3	4	5
1					
2					
3					
4					
5					

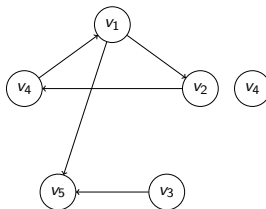
Adj Matrix - Practice



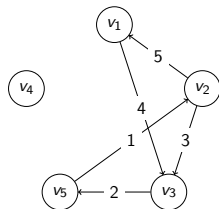
	1	2	3	4	5
1					
2					
3					
4					
5					



	1	2	3	4	5
1					
2					
3					
4					
5					

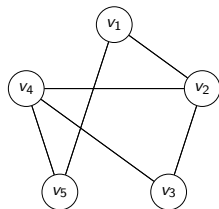


	1	2	3	4	5
1					
2					
3					
4					
5					

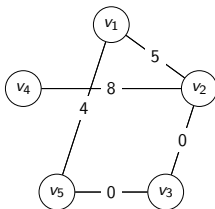


	1	2	3	4	5
1					
2					
3					
4					
5					

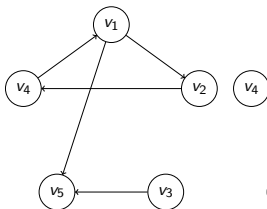
Adj Matrix - Practice



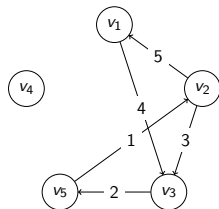
	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	0
3	0	1	0	1	0
4	0	1	1	0	1
5	1	0	0	1	0



	1	2	3	4	5
1		5			4
2	5		0	8	
3		0			0
4		8			
5	4		0		



	1	2	3	4	5
1	0	1	0	0	1
2	0	0	0	1	0
3	0	0	0	0	1
4	1	0	0	0	0
5	0	0	0	0	0



	1	2	3	4	5
1			4		
2	5		3		
3					2
4					
5		1			

Adj Matrix - Vertex Count and Edge List to Adj Matrix

```
public void printAdjMatrix(List<Integer[]> edges, int vertexCount){
    //Step 1: Create a NxN matrix
    int [][] matrix = ??

    //Step 2: Populate the matrix using vertex list
    for(Integer[] pair: edges){

    }
    //Step 3: Print the matrix
}
```

- 1 **Input:** Vertex count and list of edges
- 2 **Steps:**
 - Step 1: Initialize the 2d Array to be an $N \times N$ matrix where N is the vertex count.
 - Step 2: Iterate over the list of edges (source, target) and set
 - Recall that target is the row and source is the column
 - Step 3: Iterate over the matrix in order to display its contents.

Adj Matrix - Class

```
class MatrixGraph {  
    private Integer[][] graph;  
    private boolean isDirected;  
  
    // Step 1: Initialize an NxN graph with all 0s  
    MatrixGraph(int numVertices, boolean isDirected){ ... }  
  
    // Step 2: Construct an AddEdge Method  
    public void addEdge(int v1, int v2, int edgeWeight){ ... }  
}
```

- ❶ **Step 1 - Constructor:** The constructor should take a vertexCount and an isDirected. It should initialize the graph to be an NxN array and populate it
- ❷ **Step 2 - Add Edge:** This method should add populate the cell are the row/column as specified by the source/target. If isDirected is false, it should also add the edge going in the other direction.