The Optimization of Alcoholism under a Hypothetical Bartering System

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1 Introduction to the hypothetical bartering system

You have \$10, and a beer is 2\$. Very quickly you can see that if you spend all 10\$, you will get 5 beers. Once you've drunken the 5 beers, you are left with 5 beer bottles and 5 caps. The store owner strikes you a deal. If you give him two bottles, he'll give you a new fresh bottle of beer. You can give him four bottle caps and he'll also give you a new fresh bottle of beer. He is also kind enough to let your drink before you pay. You are interested in how many drinks you can get.

2 Modeling amount of caps and bottles

2.1 Vector representation of caps and bottles

$$\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$$

Will be the vector that represents the bottles and caps such that a is the amount of bottles, and b is the amount of caps. 1 is the homogenization of the vector.

2.2 Matrix representation of the bartering system

If we can spend two empty bottles and receive a full drink, that is equivalent to spending two bottles and getting one bottle and one cap. We will represent this operation as the following translational matrix.

$$B = \begin{bmatrix} 1 & 0 & -2+1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

And since we can drink before we pay, having merely one empty bottle is enough to drink.

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

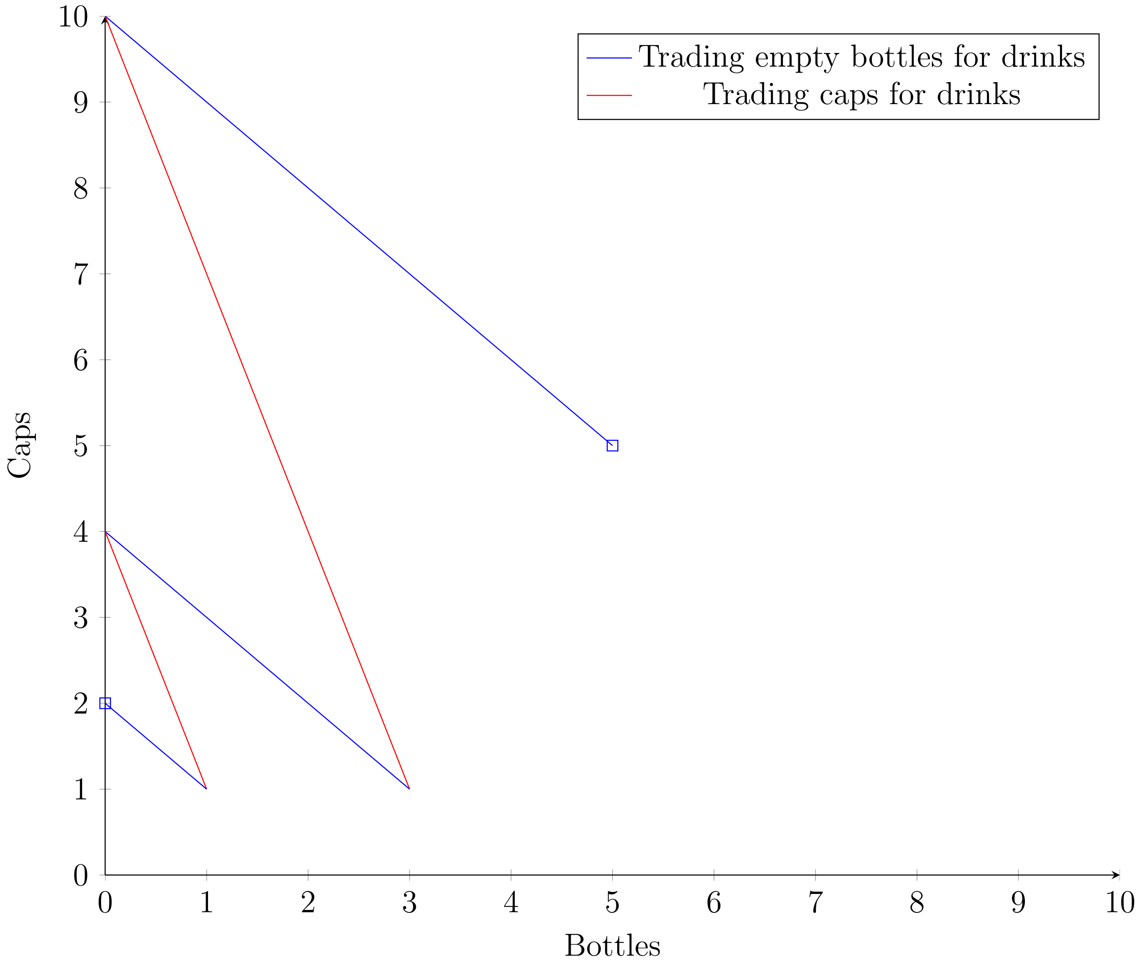
The purchasing of a full drink using 4 caps can be similarly represented as a translational matrix.

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -4+1 \\ 0 & 0 & 1 \end{bmatrix}$$

Which can simply be evaluated to.

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

These two operations can be represented geometrically as a translation of a point on a 2 dimensional Cartesian plane. For example, if we start with 5 empty bottles and 5 empty caps, we can trace the motion of the point as following.



3 Proof of the final state of the vector

FIGURE THIS OUT

4 Using the final state of the vector to deduce the amount of drinks one had

Since two empty bottles can get you a drink and a drink is worth \$2, then that means one bottle is worth \$1. Similarly since four bottle caps can get you a drink and a drink is worth \$2, then that means bottle cap is worth \$0.5.

Since a full drink is consisted of one cap, one bottle, and some drink, we can use simple algebra to deduce that:

$$\$2 = d + \$1 + \$0.5 \tag{1}$$

$$d = \$0.5 \tag{2}$$

the worth of the drink is \$0.5. If we started with \$10 dollars, and we are left with a bottles and b caps, then:

$$\$10 = \$0.5x + \$a + \$0.5b \tag{3}$$

$$x = \frac{\$10 - \$a - \$0.5b}{\$0.5} \tag{4}$$