

# WIP: The Optimization of Alcoholism under a Hypothetical Bartering System

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## Contents

|          |   |          |
|----------|---|----------|
| <b>1</b> | <b>Introduction to the hypothetical bartering system</b>                        | <b>1</b> |
| <b>2</b> | <b>Modeling amount of caps and bottles</b>                                      | <b>1</b> |
| 2.1      | Vector representation of caps and bottles . . . . .                             | 1        |
| 2.2      | Matrix representation of the bartering system . . . . .                         | 2        |
| 2.3      | Algebraic Solution . . . . .  | 3        |
| <b>3</b> | <b>Using the final state of the vector to deduce the amount of drinks drunk</b> | <b>4</b> |

## 1 Introduction to the hypothetical bartering system

You have \$10, and a beer is 2\$. Very quickly you can see that if you spend all 10\$, you will get 5 beers. Once you've drunken the 5 beers, you are left with 5 beer bottles and 5 caps. The store owner strikes you a deal. If you give him two empty bottles or four bottle caps, he'll give you a new bottle of beer. He is also kind enough to let you drink before you pay. How many drinks you can get?

## 2 Modeling amount of caps and bottles

### 2.1 Vector representation of caps and bottles

$$\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$$

Will be the vector that represents the bottles and caps such that  $a$  is the amount of bottles, and  $b$  is the amount of caps. The 1 is a homogenization of the vector.

## 2.2 Matrix representation of the bartering system

If we can spend two empty bottles and receive a full drink, that is equivalent to spending two bottles and getting one bottle and one cap. We will represent this operation as the following translational matrix.

$$B = \begin{bmatrix} 1 & 0 & -2 + 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

And since we can drink before we pay, having only one empty bottle is enough to drink.

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

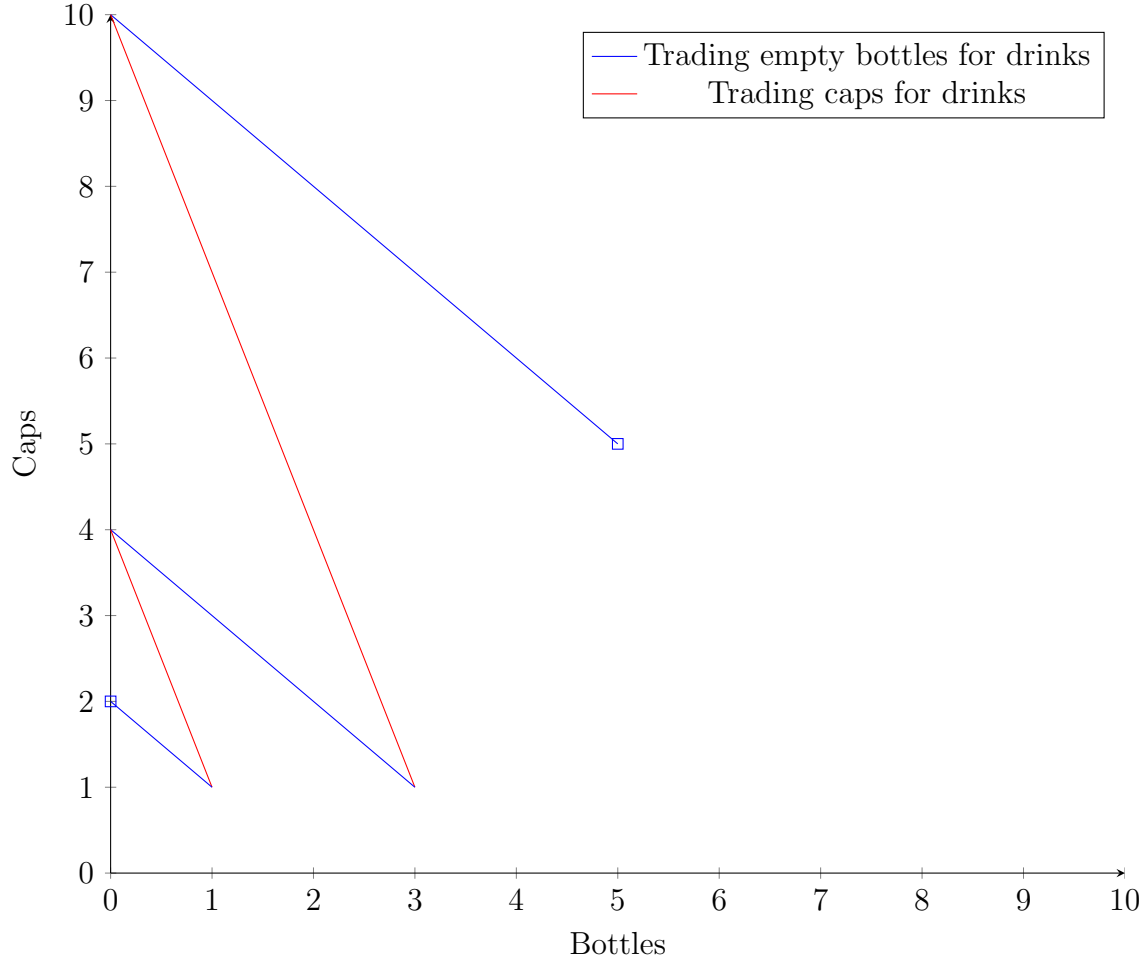
The purchasing of a full drink using 4 caps can be similarly represented as a translational matrix.

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -4 + 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Which can simply be evaluated to.

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

These two operations can be represented geometrically as a translation of a point on a 2 dimensional Cartesian plane. For example, if we start with 5 empty bottles and 5 empty caps, we can trace the motion of the point as following.



## 2.3 Algebraic Solution

Let  $\vec{i}$  be the initial state,  $\vec{f}$  be the final state,  $c$  be the cap-based transactions, and  $b$  be the bottle-based transaction.

$$\begin{aligned}\vec{i} + c \begin{bmatrix} -3 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \end{bmatrix} &= \vec{f} \\ c \begin{bmatrix} -3 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \end{bmatrix} &= \vec{f} - \vec{i} \\ \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} c \\ b \end{bmatrix} &= \vec{f} - \vec{i} \\ \begin{bmatrix} c \\ b \end{bmatrix} &= \frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & -3 \end{bmatrix} (\vec{f} - \vec{i})\end{aligned}$$

The goal is to get as drunk as possible, which is reaching as many transactions as possible, thus we want to maximize  $b+c$  with respect to the elements of  $f$ . The above matrix equation

can be represented as the following set of equations.

$$\begin{aligned}c &= \frac{1}{2}(-(f_c - i_c) - (f_b - i_b)) \\b &= \frac{1}{2}(-(f_c - i_c) - 3(f_b - i_b)) \\c + b &= \frac{1}{2}(-2(f_c - i_c) - 4(f_b - i_b))\end{aligned}$$

Which can be simplified to:

$$\begin{aligned}c &= \frac{1}{2}(-f_c - f_b + i_c + i_b) \\b &= \frac{1}{2}(-f_c - 3f_b + i_c + i_b) \\c + b &= (-f_c - 4f_b + i_c + 2i_b)\end{aligned}$$

Using the typical optimization with derivatives isn't going to help us here, because the function is linear with respect to both  $f_c$  and  $f_b$ . Instead we can use the constraint that  $c$ ,  $b$ , and  $c + b$  must be natural numbers (in my definition, the natural numbers include 0).

$$\begin{aligned}c &\in \mathbb{N} \\-f_c - f_b + i_b + i_c &\in 2\mathbb{N}\end{aligned}$$

Since  $i_b = i_c$ ,  $i_b + i_c \in 2\mathbb{N}$

$$\begin{aligned}-f_c - f_b &\in 2\mathbb{Z} \\f_c + f_b &\in 2\mathbb{N}\end{aligned}$$

### 3 Using the final state of the vector to deduce the amount of drinks drunk

Since two empty bottles can get you a drink and a drink is worth \$2, then that means one bottle is worth \$1. Similarly since four bottle caps can get you a drink and a drink is worth \$2, then that means bottle cap is worth \$0.5.

Since a full drink is consisted of one cap, one bottle, and some drink, we can use simple algebra to deduce that:

$$\text{\$2} = d + \text{\$1} + \text{\$0.5} \tag{1}$$

$$d = \text{\$0.5} \tag{2}$$

the worth of the drink is \$0.5. If we started with \$10 dollars, and we are left with  $a$  bottles and  $b$  caps, then:

$$\text{\$10} = \text{\$0.5}x + \text{\$}a + \text{\$0.5}b \tag{3}$$

$$x = \frac{\text{\$10} - \text{\$}a - \text{\$0.5}b}{\text{\$0.5}} \tag{4}$$