The Optimization of Alcoholism under a Hypothetical Bartering System

Kevin Palani and Kevin Zheng

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1 Introduction to the hypothetical bartering system

You have \$10, and a beer is 2\$. Very quickly you can see that if you spend all 10\$, you will get 5 beers. Once you've drunken the 5 beers, you are left with 5 beer bottles and 5 caps. The store owner strikes you a deal. If you give him two bottles, he'll give you a new fresh bottle of beer. You can give him four bottle caps and he'll also give you a new fresh bottle of beer. He is also kind enough to let your drink before you pay. You are interested in how many drinks you can get.

2 Modeling amount of caps and bottles

2.1 Vector representation of caps and bottles

$$\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$$

Will be the vector that represents the bottles and caps such that a is the amount of bottles, and b is the amount of caps. 1 is the homogenization of the vector.

2.2 Matrix representation of the bartering system

If we can spend two empty bottles and receive a full drink, that is equivalent to spending two bottles and getting one bottle and one cap. We will represent this operation as the following translational matrix.

$$B = \begin{bmatrix} 1 & 0 & -2+1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

And since we can drink before we pay, having merely one empty bottle is enough to drink.

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

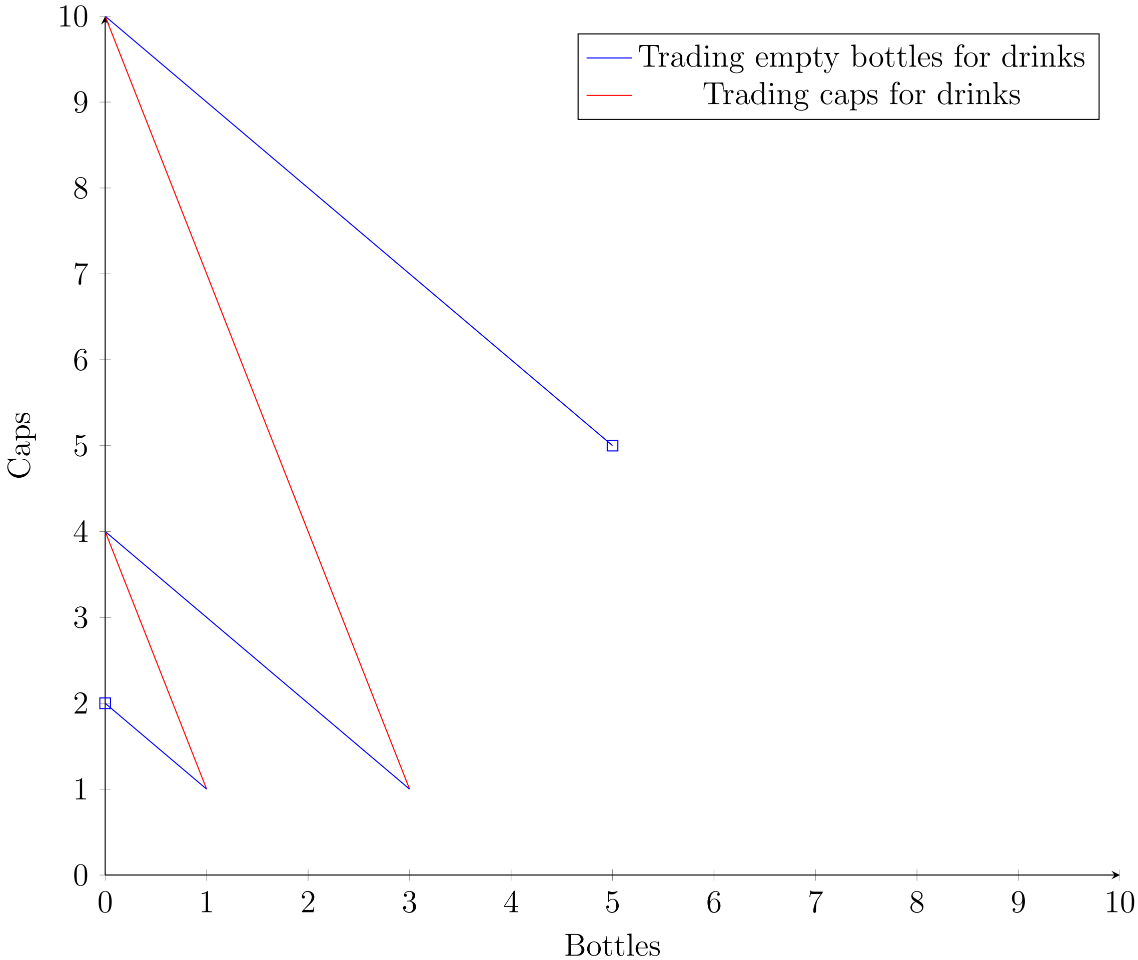
The purchasing of a full drink using 4 caps can be similarly represented as a translational matrix.

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -4+1 \\ 0 & 0 & 1 \end{bmatrix}$$

Which can simply be evaluated to.

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

These two operations can be represented geometrically as a translation of a point on a 2 dimensional Cartesian plane. For example, if we start with 5 empty bottles and 5 empty caps, we can trace the motion of the point as following.



3 Proof of the final state of the vector

3.1 Algorithm

```
cap = 4 - 1
bot = 2 - 1

capcount = 0
botcount = 0

def final(a, b):
    global capcount
    global botcount
    if a / cap > 0:
        capcount = capcount + 1
        return final(a % cap , b)
    elif b / bot > 0:
        botcount = botcount + 1
```

```
        return final(a, b % bot)
    return (a, b)

print final(7, 7)
print "capcount: " + str(capcount)
print "botcount: " + str(botcount)
```

3.2 Linear algebra

The final state can be represented as the following:

$$\begin{bmatrix}5\\5\end{bmatrix} + c\begin{bmatrix}-3\\1\end{bmatrix} + b\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$$

Which seems to be some kind of reformulation of modular arithmetic for vectors. This can also be thought of as:

$$c\begin{bmatrix}-3\\1\end{bmatrix} + b\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}5\\5\end{bmatrix}$$

Which simplifies to:

$$\begin{bmatrix}-3 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}c\\b\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}5\\5\end{bmatrix}$$

$$\begin{bmatrix}c\\b\end{bmatrix} = \begin{bmatrix}-\frac{1}{2} & -\frac{1}{2}\\-\frac{1}{2} & -\frac{3}{2}\end{bmatrix}(\begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}5\\5\end{bmatrix})$$

Such that b and c are integer solutions. Which is the largest vector that can be represented as an integer linear combination of the basis vectors that represent the trades.

3.3 WIP: Ideas

- GCD? The algorithm is quite similar to Euclid's algorithm for greatest common denominator

4 Using the final state of the vector to deduce the amount of drinks one had

Since two empty bottles can get you a drink and a drink is worth \$2, then that means one bottle is worth \$1. Similarly since four bottle caps can get you a drink and a drink is worth \$2, then that means bottle cap is worth \$0.5.

Since a full drink is consisted of one cap, one bottle, and some drink, we can use simple algebra to deduce that:

$$\$2 = d + \$1 + \$0.5 \tag{1}$$

$$d = \$0.5 \tag{2}$$

the worth of the drink is \$0.5. If we started with \$10 dollars, and we are left with a bottles and b caps, then:

$$\$10 = \$0.5x + \$a + \$0.5b \tag{3}$$

$$x = \frac{\$10 - \$a - \$0.5b}{\$0.5} \tag{4}$$