Robotics 2

Linear parametrization and identification of robot dynamics

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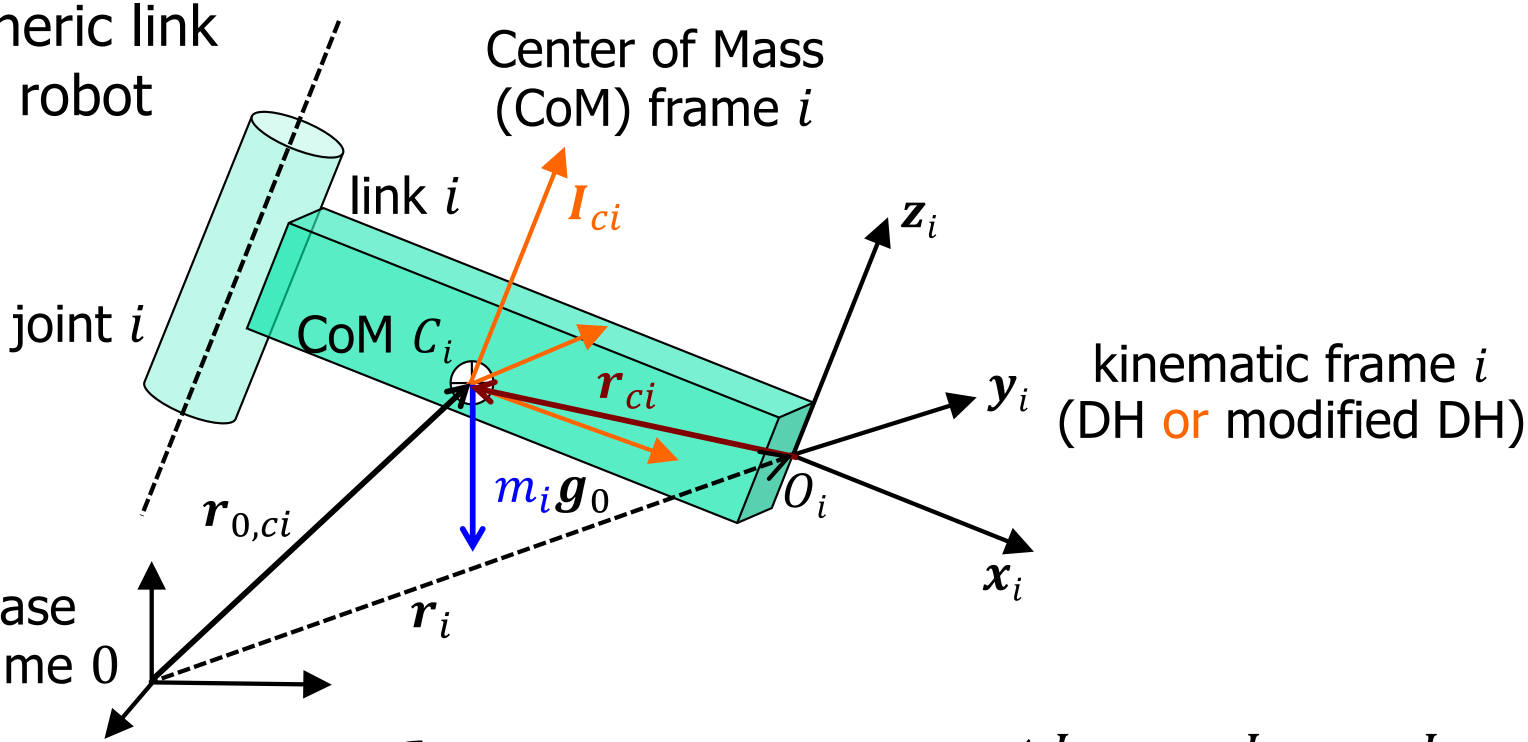
Dynamic parameters of robot links

- consider a generic link of a fully rigid robot



- each link is characterized by 10 dynamic parameters

$$\left\{ \; m_i \quad \boldsymbol{r}_{ci} = \begin{pmatrix} r_{xi} \\ r_{yi} \\ r_{zi} \end{pmatrix} \quad \boldsymbol{I}_{ci} = \begin{pmatrix} I_{ci,xx} & I_{ci,xy} & I_{ci,xz} \\ & I_{ci,yy} & I_{ci,yz} \\ \text{symm} & & I_{ci,zz} \end{pmatrix} \right.$$

- however, robot dynamics depends only on some of these parameters and possibly in a nonlinear way (e.g., via the combination $I_{ci,zz} + m_i r_{xi}^2$)

Dynamic parameters of robots

- both the kinetic energy and the gravity potential energy can be rewritten so that a new set of dynamic parameters appears only in a linear way
  - need to re-express link inertia and CoM position in (any) known kinematic frame attached to the link (same orientation as the barycentric frame)
- fundamental kinematic relation

$$v_{ci} = v_i + \omega_i \times r_{ci} = v_i + S(\omega_i)\, r_{ci} = v_i - S(r_{ci})\, \omega_i$$

- kinetic energy of link i

$$T_i = \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \omega_i^T I_{ci} \omega_i \quad \Leftrightarrow \text{ "reversing" König theorem now ...}$$

$$= \frac{1}{2} m_i (v_i - S(r_{ci})\omega_i)^T (v_i - S(r_{ci})\omega_i) + \frac{1}{2} \omega_i^T I_{ci} \omega_i$$

$$= \frac{1}{2} m_i v_i^T v_i + \frac{1}{2} \omega_i^T \underbrace{\left(I_{ci} + m_i S^T(r_{ci}) S(r_{ci})\right)}_{\text{Steiner theorem} \;\to\; I_i} \omega_i - v_i^T S(m_i r_{ci}) \omega_i$$

Steiner theorem

$$I_i = \begin{pmatrix} I_{i,xx} & I_{i,xy} & I_{i,xz} \\ & I_{i,yy} & I_{i,yz} \\ \text{symm} & & I_{i,zz} \end{pmatrix}$$

Standard dynamic parameters of robots

- gravitational potential energy of link i

$$U_i = -m_i g_0^T r_{0,ci} = -m_i g_0^T (r_i + r_{ci}) = -m_i g_0^T r_i - g_0^T (m_i r_{ci})$$

- by expressing vectors and matrices in frame i, both T_i and U_i will be linear in the set of 10 (constant) standard parameters $\boldsymbol{\pi}_i \in \mathbb{R}^{10}$

$$T_i = \frac{1}{2} m_i \, {}^i v_i^T \, {}^i v_i + m_i \, {}^i r_{ci}^T S({}^i v_i) \, {}^i \omega_i + \frac{1}{2} \, {}^i \omega_i^T \, {}^i I_i \, {}^i \omega_i$$

$$U_i = -m_i g_0^T r_i - g_0^T \, {}^0 R_i \, (m_i \, {}^i r_{ci})$$

$$\boldsymbol{\pi}_i = \begin{pmatrix} m_i \\ m_i \, {}^i r_{ci} \\ vect\{{}^i I_i\} \end{pmatrix} = (\underbrace{m_i}_{\substack{\text{mass of link } i \\ \text{(0-th order moment)}}} \; \underbrace{m_i \, {}^i r_{ci,x} \;\; m_i \, {}^i r_{ci,y} \;\; m_i \, {}^i r_{ci,z}}_{\substack{\text{mass} \times \text{CoM position of link } i \\ \text{(1-st order moment)}}} \; \underbrace{{}^i I_{i,xx} \;\; {}^i I_{i,xy} \;\; {}^i I_{i,xz} \;\; {}^i I_{i,yy} \;\; {}^i I_{i,yz} \;\; {}^i I_{i,zz}}_{\substack{\text{inertia of link } i \\ \text{(2-nd order moment)}}})^T$$

- since the E-L equations involve only linear operations on T and U, also the robot dynamic model is linear in the standard parameters $\boldsymbol{\pi} \in \mathbb{R}^{10N}$

Linearity in the dynamic parameters

- using a $N \times 10N$ regression matrix Y_π that depends only on kinematic quantities, the robot dynamic equations can always be rewritten linearly in the standard dynamic parameters as

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = Y_\pi(q,\dot{q},\ddot{q})\,\pi = u$$

$$\pi^T = (\pi_1^T \quad \pi_2^T \quad \cdots \quad \pi_N^T)$$

- the open kinematic chain structure of the manipulator implies that the i-th dynamic equation can depend only on the dynamic parameters of links from i to N $\Rightarrow$ Y_π has a block upper triangular structure

$$Y_\pi(q,\dot{q},\ddot{q}) = \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ 0 & Y_{22} & \cdots & Y_{2N} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & Y_{NN} \end{pmatrix}$$

with row vectors Y_{ij} of size $1{\times}10$

Property: element m_{ij} of $M(q)$ depends at most on $(q_{k+1}, \cdots, q_N)$, with $k = \min\{i,j\}$, and at most on the dynamic parameters of links h to N, with $h = \max\{i,j\}$

dynamic parameters are pi. So for a 6R m_24 depends at most on pi_4, pi_5, pi_6

Linearity in the dynamic coefficients

- many standard parameters do not appear ("play no role") in the dynamic model of a given robot ⇒ the associated columns of Y_π are 0!
- some standard parameters may appear only in fixed combinations with others ⇒ the associated columns of Y_π are linearly dependent!
- one can isolate $p \ll 10N$ groups of parameters π (associated to p functionally independent columns Y_{indep} of Y_π) and partition matrix Y_π in two blocks, the second containing dependent (or zero) columns as $Y_{dep} = Y_{indep} T$, for a suitable $p \times (10N - p)$ constant matrix T

$$Y_\pi(q,\dot{q},\ddot{q})\,\pi = \begin{pmatrix} Y_{indep} & Y_{dep} \end{pmatrix} \begin{pmatrix} \pi_{indep} \\ \pi_{dep} \end{pmatrix} = \begin{pmatrix} Y_{indep} & Y_{indep}T \end{pmatrix} \begin{pmatrix} \pi_{indep} \\ \pi_{dep} \end{pmatrix}$$

$$= Y_{indep}\left(\pi_{indep} + T\pi_{dep}\right) = \boxed{Y(q,\dot{q},\ddot{q})\,a}$$

- these grouped parameters are called dynamic coefficients $a \in \mathbb{R}^p$, "the only that matter" in robot dynamics (= base parameters by W. Khalil)
- the minimal number p of dynamic coefficients that is needed can also be checked numerically (see → identification)

Linear parametrization of robot dynamics

it is always possible to rewrite the dynamic model in the form

regression matrix — a = vector of dynamic coefficients

$$\boxed{M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \underbrace{Y(q,\dot{q},\ddot{q})}_{N \times p}\ \underbrace{a}_{p \times 1} = u}$$

e.g., the heuristic grouping (found by inspection) on the planar 2R robot

$$\begin{pmatrix} \ddot{q}_1 & c_2(2\ddot{q}_1 + \ddot{q}_2) - s_2(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) & \ddot{q}_2 & c_1 & c_{12} \\ 0 & c_2\ddot{q}_1 + s_2\dot{q}_1^2 & \ddot{q}_1 + \ddot{q}_2 & 0 & c_{12} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{aligned} a_1 &= I_{c1,zz} + m_1 d_1^2 + I_{c2,zz} + m_2 d_2^2 + m_2 l_1^2 \\ a_2 &= m_2 l_1 d_2 \\ a_3 &= I_{c2,zz} + m_2 d_2^2 \\ a_4 &= g_0(m_1 d_1 + m_2 l_1) \\ a_5 &= g_0 m_2 d_2 \end{aligned}$$

Note: 4 more coefficients are added when including the coefficients $F_{V,i}$ and $F_{C,i}$ of viscous and Coulomb friction at the joints ("decoupled" terms appearing only in i –th equation, for $i = 1,2$)

Linear parametrization
of a 2R planar robot ($N = 2$)

- being the kinematics known (i.e., l_1 and g_0), the number of dynamic coefficients can be reduced since we can merge the two coefficients

$$a_2 = m_2 l_1 d_2 \;\&\; a_5 = g_0 m_2 d_2 \quad \Rightarrow \quad a_2 = m_2 d_2 \quad \text{(factoring out } l_1 \text{ and } g_0\text{)}$$

- therefore, after regrouping, $\boldsymbol{p = 4}$ dynamic coefficients are sufficient

$$\begin{pmatrix} \ddot{q}_1 & l_1 c_2(2\ddot{q}_1 + \ddot{q}_2) - l_1 s_2(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) + g_0 c_{12} & \ddot{q}_2 & g_0 c_1 \\ 0 & l_1(c_2\ddot{q}_1 + s_2\dot{q}_1^2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = Y\,a = u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$a_1 = I_{c1,zz} + m_1 d_1^2 + I_{c2,zz} + m_2 d_2^2 + m_2 l_1^2 \qquad a_3 = I_{c2,zz} + m_2 d_2^2$$

$$a_2 = m_2 d_2 \qquad a_4 = m_1 d_1 + m_2 l_1$$

- this (minimal) linear parametrization of robot dynamics is not unique, both in terms of the chosen set of dynamic coefficients a and for the associated regression matrix Y
- a systematic procedure for its derivation would be preferable

Linear parametrization
of a 2R planar robot ($N = 2$)

- as alternative to the previous heuristic method, apply the general procedure
  - $10N = 20$ standard parameters $\boldsymbol{\pi}$ are defined for the two links
  - from the assumptions made on CoM locations, only 5 such parameters actually appear, namely (with ${}^{i}r_{ci,x} = -l_i + d_i$)

link 1: $\underset{\pi_1}{m_1 d_1}$ $\quad \underset{\pi_2}{I_{1,zz}} = I_{c1,zz} + m_1 d_1^2$ $\qquad$ link 2: $\underset{\pi_3}{m_2}$ $\quad \underset{\pi_4}{m_2 d_2}$ $\quad \underset{\pi_5}{I_{2,zz}} = I_{c2,zz} + m_2 d_2^2$

  - in the 2×5 matrix Y_π, the 3rd column (associated to $\boxed{m_2}$) is $Y_{\pi 3} = Y_{\pi 1} l_1 + Y_{\pi 2} l_1^2$
  - after regrouping/reordering, $\boldsymbol{p} = \mathbf{4}$ dynamic coefficients are again sufficient

$$\begin{pmatrix} g_0 c_1 & \ddot{q}_1 & l_1 c_2(2\ddot{q}_1 + \ddot{q}_2) - l_1 s_2(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 \\ 0 & 0 & l_1(c_2\ddot{q}_1 + s_2\dot{q}_1^2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = Y\,a = u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$a_1 = m_1 d_1 + \boxed{m_2} l_1 \qquad a_2 = I_{1,zz} + \boxed{m_2} l_1^2 = \left(I_{c1,zz} + m_1 d_1^2\right) + m_1 l_1^2 \qquad \begin{aligned} a_3 &= m_2 d_2 \\ a_4 &= I_{2,zz} = I_{c2,zz} + m_2 d_2^2 \end{aligned}$$

- determining a minimal parameterization (i.e., minimizing p) is important for
  - experimental identification of dynamic coefficients
  - adaptive/robust control design in the presence of uncertain parameters

See blackboard (xournalapp) to understand more

Identification of dynamic coefficients

- in order to "use" the model, one needs to know the numeric values of the robot dynamic coefficients
  - robot manufacturers provide at most only a few principal dynamic parameters (e.g., link masses)
- estimates can be found with CAD tools (e.g., assuming uniform mass)
- friction coefficients are (slowly) varying over time
  - lubrication of joints/transmissions
- for an added payload (attached to the E-E)
  - a change in the 10 dynamic parameters of last link …
  - … implies a variation of (almost) all robot dynamic coefficients!
- preliminary identification experiments are needed
  - robot in motion (dynamic issues, not just static or geometric ones!)
  - only the robot dynamic coefficients can be identified (and not all the link standard parameters!)

Identification experiments

1. choose a motion trajectory $q_d(t)$ that is sufficiently "exciting", i.e.,
   - explores the robot workspace and involves all components in the robot dynamic model
   - is periodic, with multiple frequency components
2. execute this motion (approximately) by means of a control law
   - taking advantage of any available information on the robot model
   - often $u = K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$ (PD, no model information used)
3. measure q (encoders) in n_c time instants (and, if available, also $\dot{q}$)
   - joint velocity $\dot{q}$ and acceleration $\ddot{q}$ can be estimated later off line by numerical differentiation (use of non-causal filters is feasible)
4. with such measures/estimates, evaluate the regression matrix Y (on the left) and use the applied commands u (on the right) in the expression

$$Y\big(q(t_k), \dot{q}(t_k), \ddot{q}(t_k)\big)\, a = u(t_k) \quad k = 1, \cdots, n_c$$

Least Squares (LS) identification

- set up the system of linear equations

$$n_c \times N \left(\begin{array}{c} Y\big(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)\big) \\ \vdots \\ Y\left(q(t_{n_c}), \dot{q}(t_{n_c}), \ddot{q}(t_{n_c})\right) \end{array} \right) a = \begin{pmatrix} u(t_1) \\ \vdots \\ u(t_{n_c}) \end{pmatrix} \Leftrightarrow \boxed{\bar{Y} a = \bar{u}}$$

- sufficiently "exciting" trajectories, large enough number of samples ($n_c \times N \gg p$), and their suitable selection/position guarantee that rank($\bar{Y}$) = p (full column rank)
- solution by pseudoinversion of matrix $\bar{Y}$

$$a = \bar{Y}^{\#} \bar{u} = (\bar{Y}^T \bar{Y})^{-1} \bar{Y}^T \bar{u} \quad (\in \mathbb{R}^p)$$

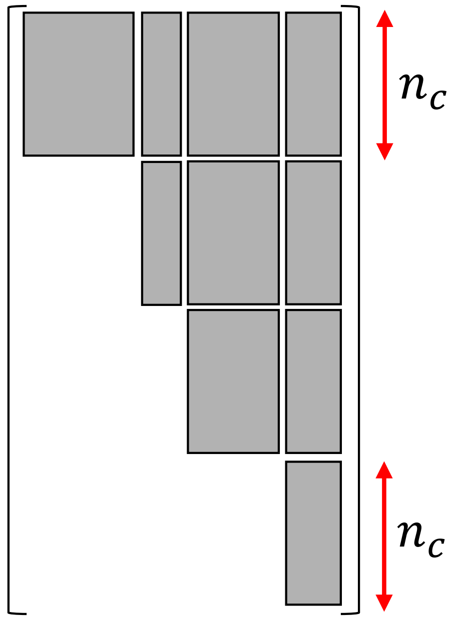
- one can also use a weighted pseudoinverse, to take into account different levels of noise in the collected measures

Additional remarks on LS identification

- it is convenient to preserve the block (upper) triangular structure of the regression matrix, by “stacking” all time evaluations in row by row sequence of the original Y matrix

$$
N\times\left\{\begin{array}{l} n_c\left\{\begin{pmatrix} Y_1\left(q(t_1),\dot q(t_1),\ddot q(t_1)\right)\\ \vdots \\ Y_1\left(q(t_{n_c}),\dot q(t_{n_c}),\ddot q(t_{n_c})\right)\\ Y_2\left(q(t_1),\dot q(t_1),\ddot q(t_1)\right)\\ \vdots \\ Y_2\left(q(t_{n_c}),\dot q(t_{n_c}),\ddot q(t_{n_c})\right)\\ \vdots \\ Y_N\left(q(t_1),\dot q(t_1),\ddot q(t_1)\right)\\ \vdots \\ Y_N\left(q(t_{n_c}),\dot q(t_{n_c}),\ddot q(t_{n_c})\right)\end{pmatrix}\right. \end{array}\right. a = \begin{pmatrix} u_1(t_1)\\ \vdots \\ u_1(t_{n_c})\\ u_2(t_1)\\ \vdots \\ u_2(t_{n_c})\\ \vdots \\ u_N(t_1)\\ \vdots \\ u_N(t_{n_c})\end{pmatrix} \Leftrightarrow \boxed{\bar Y a = \bar u}
$$

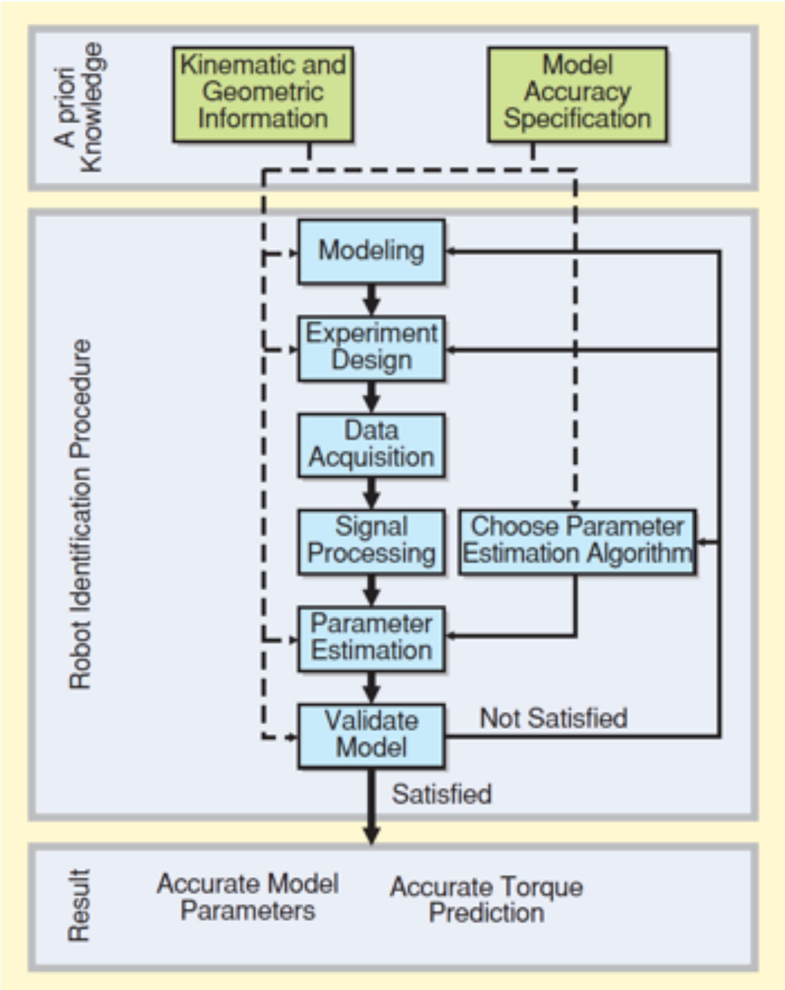
$\bar Y =$



- numerical check of full column rank is more robust $\Leftrightarrow$ rank = p (# of col’s)

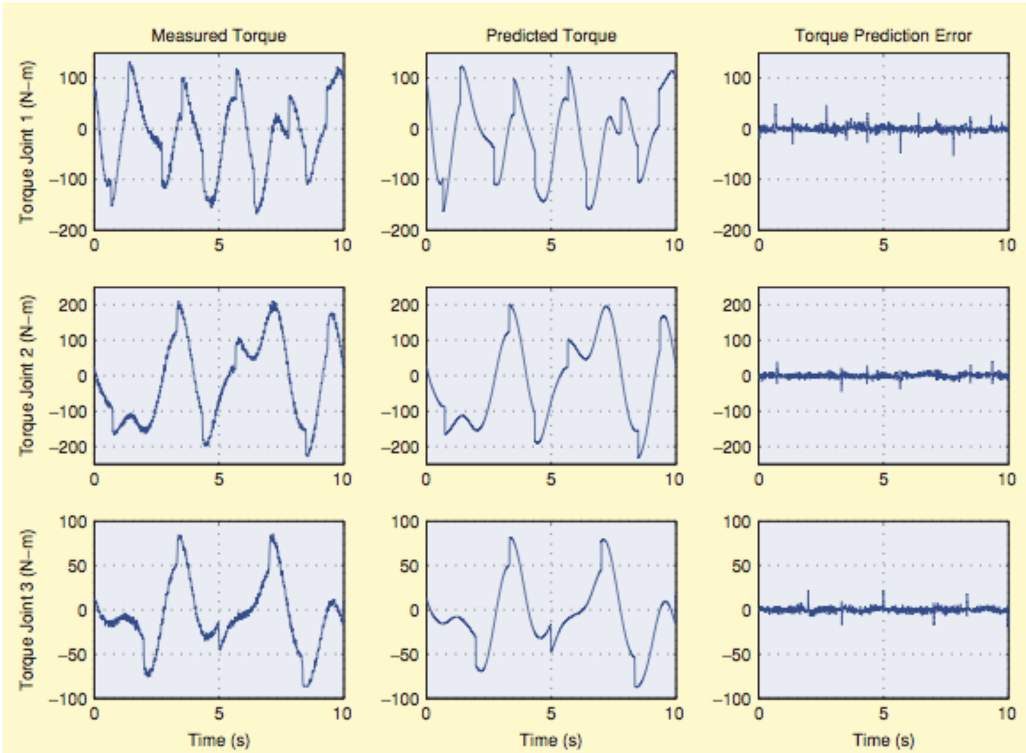
- further practical hints
  - outlier data can be eliminated in advance (when building Y)
  - if sufficiently rich friction models are not included in Ya, discard the data collected at joint velocities close to zero

Summary on dynamic identification



J. Swevers, W. Verdonck, and J. De Schutter:
"Dynamic model identification for industrial robots"
IEEE Control Systems Mag., Oct 2007

KUKA IR 361 robot and optimal excitation trajectory



results after identification (first three joints only)

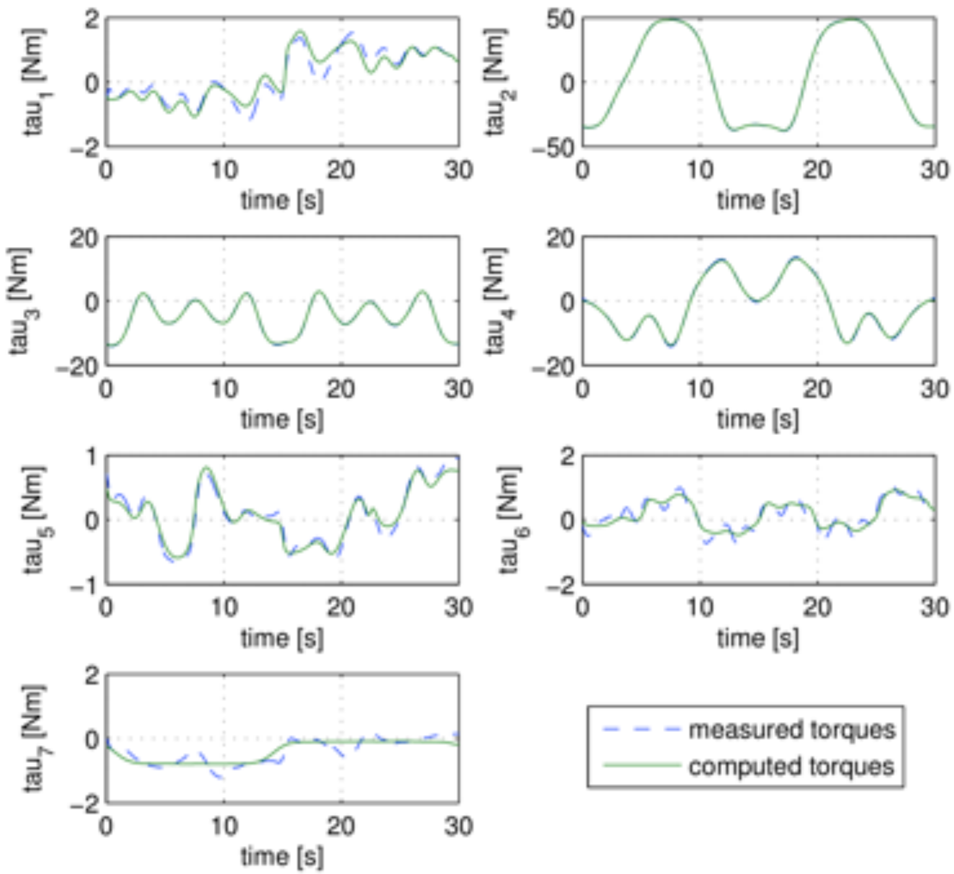
Dynamic identification of KUKA LWR4

video

data acquisition for identification

dynamic coefficients: 30 inertial, 12 for gravity

C. Gaz, F. Flacco, A. De Luca:
"Identifying the dynamic model used by the KUKA LWR:
A reverse engineering approach", IEEE ICRA 2014



validation after identification (for all 7 joints):
on new desired trajectories, compare
torques computed with the identified model
and torques measured by joint torque sensors

Identification of LWR4 gravity terms

using the linear parametrization, gravity terms can also be identified separately

$$\boxed{\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}_g(\boldsymbol{q})\boldsymbol{\pi}_g}$$

$$\boldsymbol{\pi}_g = \begin{pmatrix} c_{7y}m_7 \\ c_{7x}m_7 \\ c_{6x}m_6 \\ c_{6z}m_6 + c_{7z}m_7 \\ c_{5z}m_5 - c_{6y}m_6 \\ c_{5x}m_5 \\ c_{5y}m_5 + c_{4z}m_4 + d_2(m_5+m_6+m_7) \\ c_{4x}m_4 \\ c_{4y}m_4 + c_{3z}m_3 \\ c_{2x}m_2 \\ c_{3x}m_3 \\ c_{2z}m_2 - c_{3y}m_3 + d_1(m_3+m_4+m_5+m_6+m_7) \end{pmatrix}$$

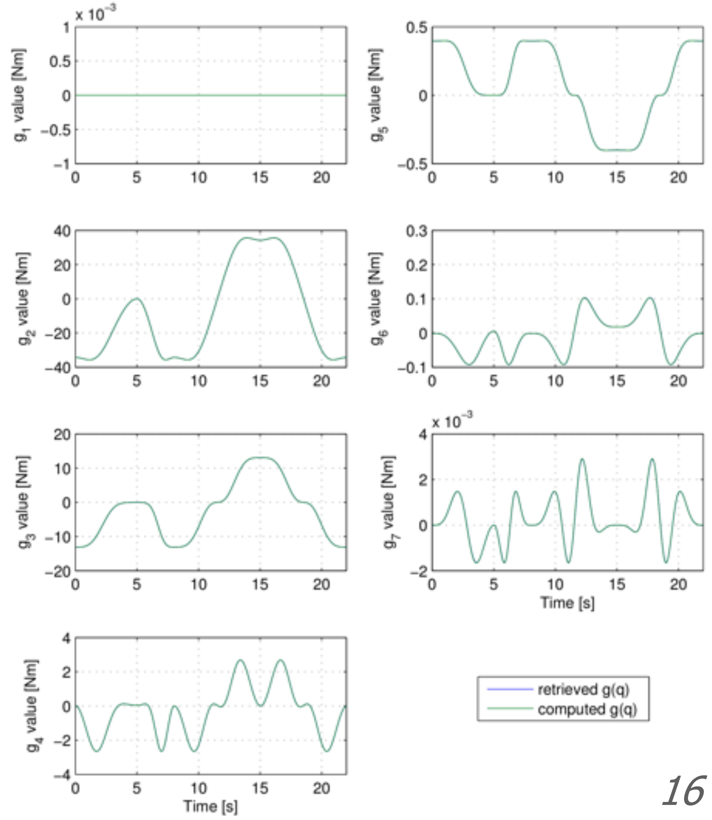
symbolic expressions of gravity-related dynamic coefficients

some small values may also be discarded …

$$\hat{\boldsymbol{\pi}}_g = \begin{pmatrix} 9.5457\times10^{-4} \\ -2.9826\times10^{-4} \\ 8.3524\times10^{-4} \\ 0.0286 \\ -0.0407 \\ -6.5637\times10^{-4} \\ 1.334 \\ -0.0035 \\ -4.7258\times10^{-4} \\ 0.0014 \\ 9.4532\times10^{-4} \\ 3.4568 \end{pmatrix}$$

numerical values identified through experiments

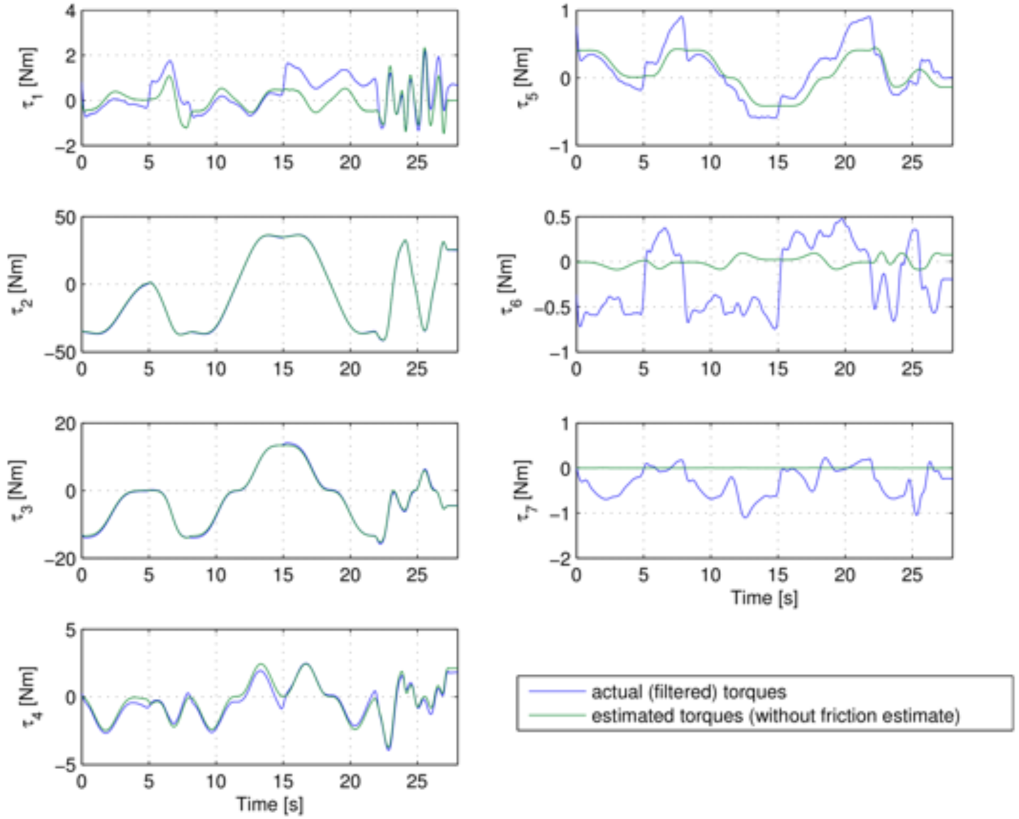
gravity joint torques prediction/evaluation on new validation trajectory

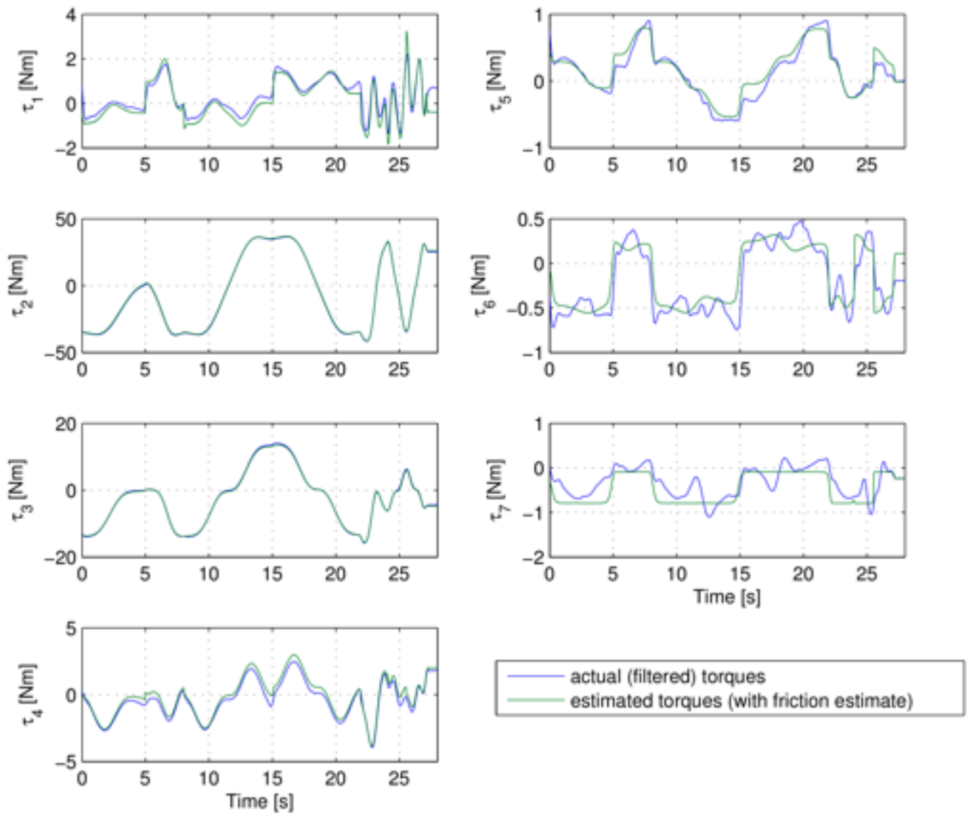


Role of friction in identification

KUKA LWR4 dynamic model estimation vs. joint torque sensor measurement

$$\boxed{\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}} \boxed{- \boldsymbol{\tau}_{friction}} \qquad \boxed{\boldsymbol{\tau}_{meas}}$$





without the use of a joint friction model

including an identified joint friction model

$$\tau_{f,j}(\dot{q}_j) = \frac{\varphi_{1,j}}{1 + e^{-\varphi_{2,j}(\dot{q}_j + \varphi_{3,j})}} - \frac{\varphi_{1,j}}{1 + e^{-\varphi_{2,j}\varphi_{3,j}}}$$

Dynamic identification of KUKA LWR4

video

using more dynamic robot motions for model identification

J. Hollerbach, W. Khalil, M. Gautier: "Ch. 6: Model Identification", Springer Handbook of Robotics (2nd Ed), 2016
free access to multimedia extension: http://handbookofrobotics.org

Adding a payload to the robot

- in several industrial applications, changes in the robot payload are often needed
  - using different tools for various technological operations such as polishing, welding, grinding, ...
  - pick-and-place tasks of objects having unknown mass
- what is the rule of change for dynamic parameters when there is an additional payload?
  - do we obtain again a linearly parameterized problem?
  - does this property rely on some specific choice of reference frames (e.g., conventional or modified D-H)?

Rule of change in dynamic parameters

- only the dynamic parameters of the link where a load is added will change (typically, added to the last one –link n– as payload)
  - last link dynamic parameters: m_n (mass), $\boldsymbol{c}_n = (c_{nx}\ c_{ny}\ c_{nz})^T$ (center of mass), $\boldsymbol{I}_n$ (inertia tensor expressed w.r.t. frame n)
  - payload dynamic parameters: m_L (mass), $\boldsymbol{c}_L = (c_{Lx}\ c_{Ly}\ c_{Lz})^T$ (center of mass), $\boldsymbol{I}_L$ (inertia tensor expressed w.r.t. frame n)

- mass $\quad m_n \to m_n + m_L$

- center of mass $\quad c_{ni} m_n \to \dfrac{c_{ni} m_n + c_{Li} m_L}{m_n + m_L}(m_n + m_L) = c_{ni} m_n + c_{Li} m_L$

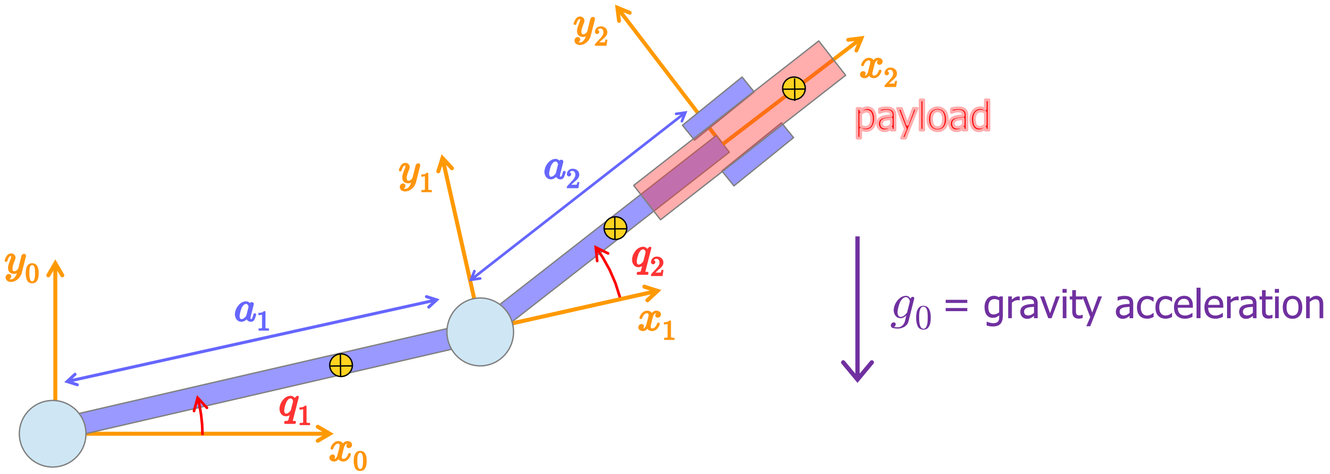
  (weighted average) where $i = x,\ y,\ z$

- inertia tensor $\quad \boldsymbol{I}_n \to \boldsymbol{I}_n + \boldsymbol{I}_L$

  valid only if tensors are expressed w.r.t. the same reference frame (i.e., frame n)!

- linear parametrization is preserved with any kinematic convention (the parameters of the last link will always appear in the form shown above)

Example: 2R planar robot with payload



unloaded robot dynamics $\boldsymbol{Y\pi} = \boldsymbol{\tau}$ loaded robot dynamics $\boldsymbol{Y\pi}^L = \boldsymbol{\tau}^L$

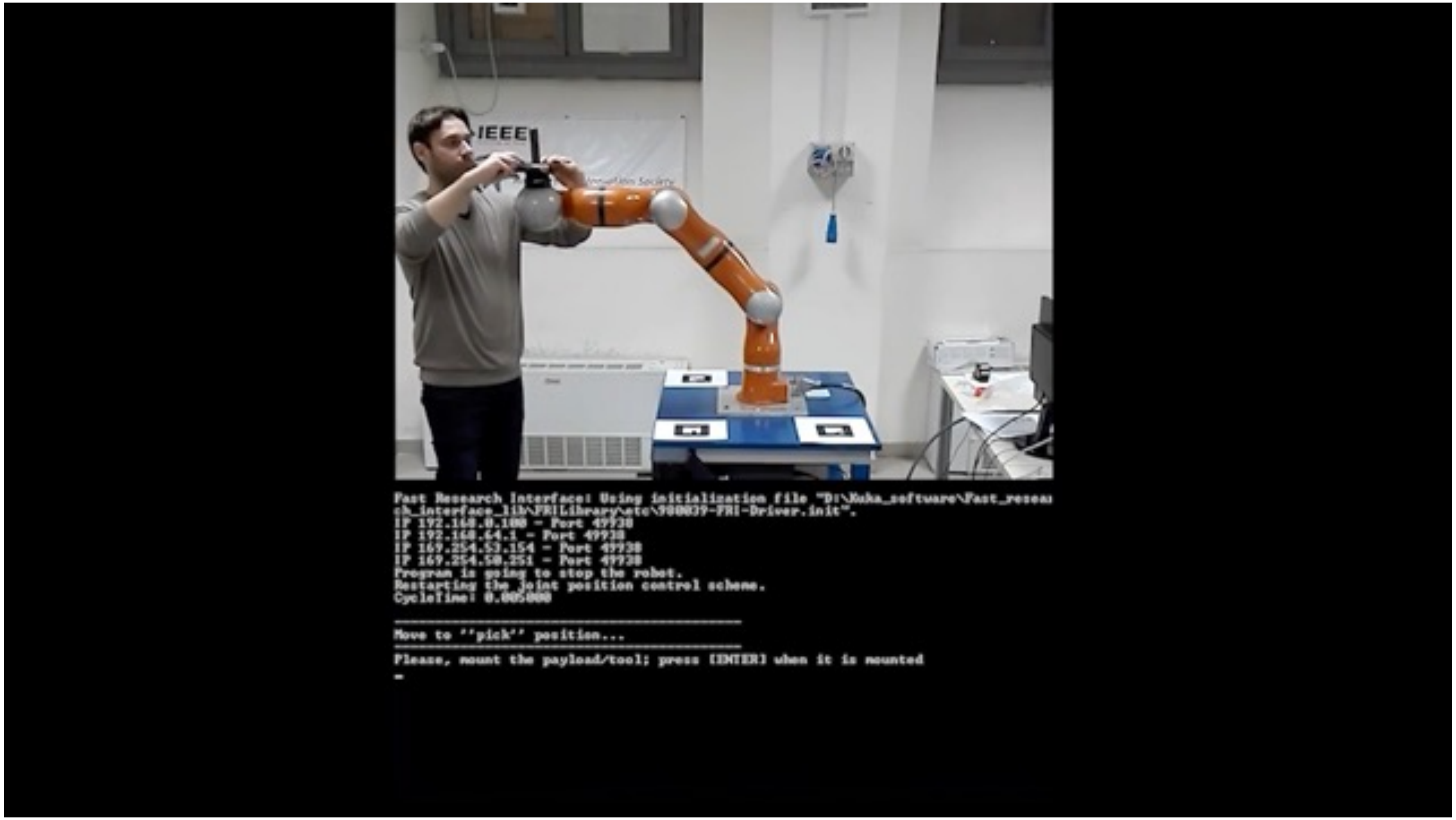
$$\boldsymbol{\pi} = \begin{pmatrix} \frac{1}{2}\left(m_2 a_2^2 + I_{2zz}\right) + a_2 c_{2x} m_2 \\ c_{2x} m_2 + a_2 m_2 \\ c_{2y} m_2 \\ \frac{1}{2}\left(I_{1zz} + a_1^2 m_1 + a_1^2 m_2\right) + a_1 c_{1x} m_1 \\ c_{1x} m_1 + a_1 m_1 + a_1 m_2 \\ c_{1y} m_1 \end{pmatrix} \Rightarrow \boldsymbol{\pi}^L = \begin{pmatrix} \frac{1}{2}\left(a_2^2\left(m_2 + m_L\right) + I_{2zz} + I_{Lzz}\right) + a_2\left(c_{2x} m_2 + c_{Lx} m_L\right) \\ c_{2x} m_2 + c_{Lx} m_L + a_2\left(m_2 + m_L\right) \\ c_{2y} m_2 + c_{Ly} m_L \\ \frac{1}{2}\left(I_{1zz} + a_1^2 m_1 + a_1^2\left(m_2 + m_L\right)\right) + a_1 c_{1x} m_1 \\ c_{1x} m_1 + a_1 m_1 + a_1\left(m_2 + m_L\right) \\ c_{1y} m_1 \end{pmatrix}$$

Note 1: position of the center of mass of the two links and of the payload may also be asymmetric

Note 2: link inertia & center of mass are expressed in the DH kinematic frame attached to the link (e.g., I_{2zz} is the inertia of the second link around the axis z_2)

Validation on the KUKA LWR4 robot

video



C. Gaz, A. De Luca: "Payload estimation based on identified coefficients of robot dynamics – with an application to collision detection" IEEE IROS 2017, Vancouver, September 2017

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KUKA LWR4 (7R)

Universal Robot UR10 (6R)

Franka Emika Panda (7R)