Robotics 2

Iterative Learning
for Gravity Compensation

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Control goal

- **regulation of arbitrary equilibrium configurations in the presence of gravity**

  - **without** explicit knowledge of robot dynamic coefficients (nor of the structure of the gravity term)

  - **without** the need of "high" position gain

  - **without** complex conditions on the control gains

- **based on an iterative control scheme that uses**

  1. PD control on joint position error + constant feedforward term

  2. iterative update of the feedforward term at successive steady-state conditions

- **derive sufficient conditions for the global convergence of the iterative scheme with zero final error**

Preliminaries

- robot dynamic model

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = u$$

- available bound on the gradient of the gravity term

$$\left\| \frac{\partial g(q)}{\partial q} \right\| \leq \alpha$$

- regulation attempted with a joint-based PD law (without gravity cancellation nor compensation)

$$u = K_P(q_d - q) - K_D\dot{q} \qquad K_P > 0, K_D > 0$$

- at steady state, there is a non-zero error left

$$q = \bar{q}, \dot{q} = 0 \qquad g(\bar{q}) = K_P(q_d - \bar{q}) \qquad \bar{e} = q_d - \bar{q} \neq 0$$

Iterative control scheme

- **control law** at the i-th iteration (for $i = 1, 2, ...$)

$$u = \gamma K_P(q_d - q) - K_D \dot{q} + u_{i-1} \qquad \gamma > 0$$

with a constant compensation term u_{i-1} (feedforward)
  - $K_P > 0, K_D > 0$ are chosen diagonal for simplicity
  - q_0 is the initial robot configuration
  - $u_0 = 0$ is the 'easiest' initialization of the feedforward term
  - at the steady state of the i-th iteration ($q = q_i, \dot{q} = 0$), one has

$$g(q_i) = \gamma K_P(q_d - q_i) + u_{i-1}$$

- **update law** of the compensation term (for next iteration)

$$u_i = \gamma K_P(q_d - q_i) + u_{i-1} \qquad [= g(q_i)]$$

$\leftarrow$ for implementation $\rightarrow$ [for analysis]

Convergence analysis

Theorem

(a) $\lambda_{\min}(K_P) > \alpha$

(b) $\gamma \geq 2$

guarantee that the sequence $\{q_0, q_1, q_2, \dots\}$ converges to q_d (and $\dot{q} = 0$) from any initial value q_0 (and $\dot{q}_0$), i.e., globally

- condition (a) is sufficient for the global asymptotic stability of the desired equilibrium state when using

$$u = K_P(q_d - q) - K_D\dot{q} + g(q_d)$$

  with a known gravity term and diagonal gain matrices

- the additional sufficient condition (b) guarantees the convergence of the iterative scheme, yielding

$$\lim_{i \to \infty} u_i = g(q_d)$$

Proof

- let $e_i = q_d - q_i$ be the error at the end of the i-th iteration; based on the update law, it is $u_i = g(q_i)$ and thus

$$\|u_i - u_{i-1}\| = \|g(q_i) - g(q_{i-1})\| \leq \alpha \|q_i - q_{i-1}\|$$

$$\leq \alpha(\|e_i\| + \|e_{i-1}\|)$$

adding and subtracting q_d

- on the other hand, from the update law it is

$$\|u_i - u_{i-1}\| = \gamma \|K_P e_i\|$$

- combining the two above relations under (a), we have

$$\gamma\alpha\|e_i\| < \gamma\lambda_{\min}(K_P)\|e_i\| \leq \gamma\|K_P e_i\| \leq \alpha(\|e_i\| + \|e_{i-1}\|)$$

$$\text{or } \|e_i\| < \frac{1}{\gamma}(\|e_i\| + \|e_{i-1}\|)$$

■ condition (b) guarantees that the error sequence $\{e_0, e_1, e_2, \dots\}$

$$\|e_i\| < \frac{\frac{1}{\gamma}}{1 - \frac{1}{\gamma}} \|e_{i-1}\| = \frac{1}{\gamma - 1} \|e_{i-1}\|$$

is a contraction mapping, so that

$$\lim_{i \to \infty} \|e_i\| = 0$$

with asymptotic convergence from any initial state ◄

⇒ the robot progressively approaches the desired configuration through successive steady-state conditions

- ■ K_P and K_D affect each transient phase

- ■ coefficient γ drives the convergence rate of intermediate steady states to the final one

Remarks

- combining (a) and (b), the sufficient condition only requires the doubling of the proportional gain w.r.t. the known gravity case

$$\widehat{K}_P = \gamma K_P \quad \Longrightarrow \quad \boxed{\lambda_{\min}(\widehat{K}_P) > 2\alpha}$$

- for a diagonal $\widehat{K}_P$, this condition implies a (positive) lower bound on the single diagonal elements of the matrix

- again, it is only a sufficient condition

  - the scheme may converge even if this condition is violated ...

- the scheme can be interpreted as using an integral term

  - updated only in correspondence of a discrete sequence of time instants

  - with a guaranteed global convergence (and implicit stability)

Numerical results

- 3R robot with uniform links, moving in the vertical plane

$$l_1 = l_2 = l_3 = 0.5 \ [\text{m}]$$

$$m_1 = 30, m_2 = 20, m_3 = 10 \ [\text{kg}] \quad \Longrightarrow \quad \boxed{\alpha \cong 400}$$

- with saturations of the actuating torques

$$U_{1,\max} = 800, U_{2,\max} = 400, U_{3,\max} = 200 \ [\text{Nm}]$$

- three cases, from the downward position $q_0 = (0,0,0)$

I: $\quad q_d = (\pi/2, 0, 0)$

II: $\quad q_d = (3\pi/4, 0, 0)$

$$\widehat{K}_P = \text{diag}\{1000, 600, 280\}$$
$$K_D = \text{diag}\{200, 100, 20\}$$

III: $q_d = (3\pi/4, 0, 0)$

$$\widehat{K}_P = \text{diag}\{500, 500, 500\}$$
$$K_D = \text{as before}$$

Case I: $q_d = (\pi/2, 0, 0)$
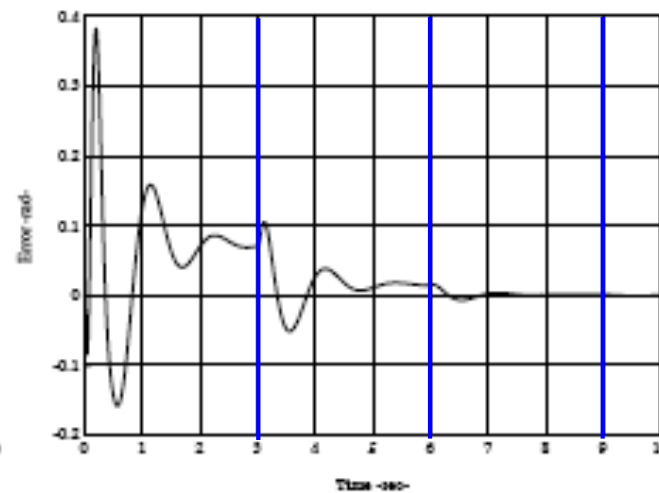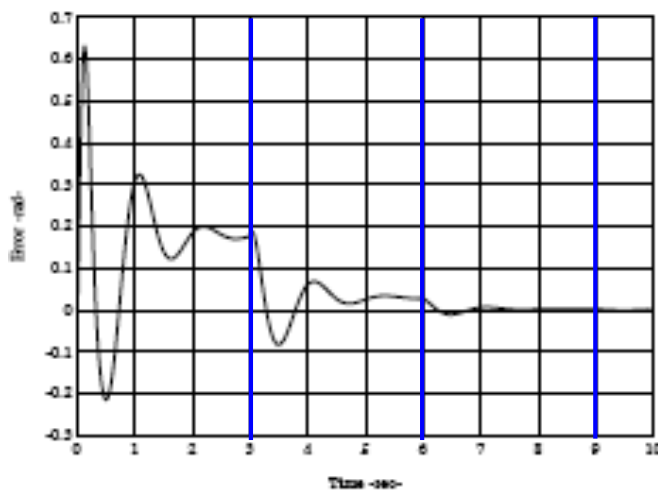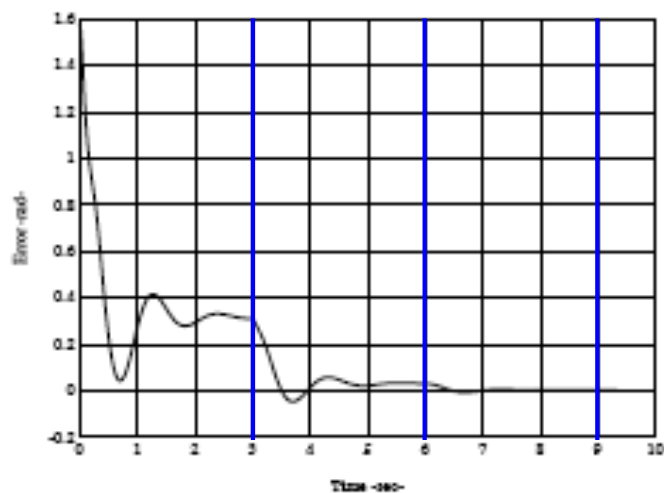
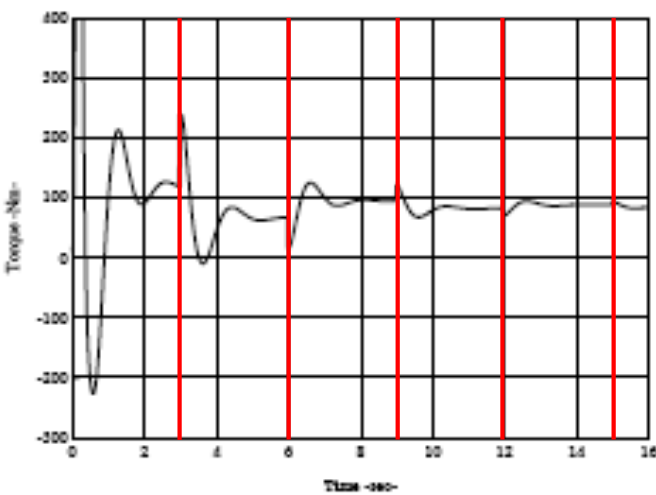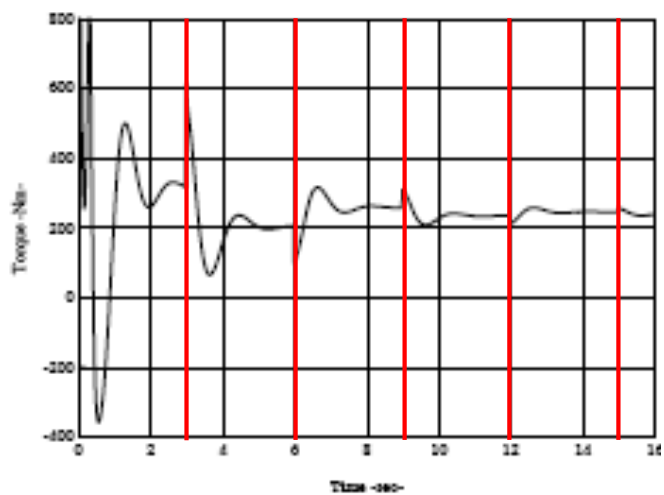joint position errors (zero after 3 updates)
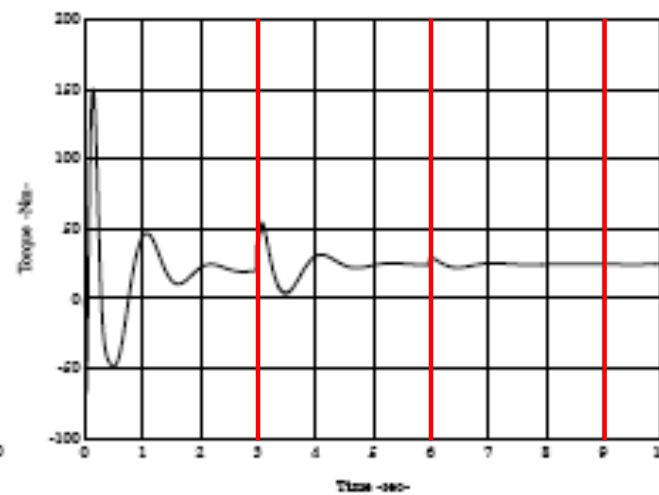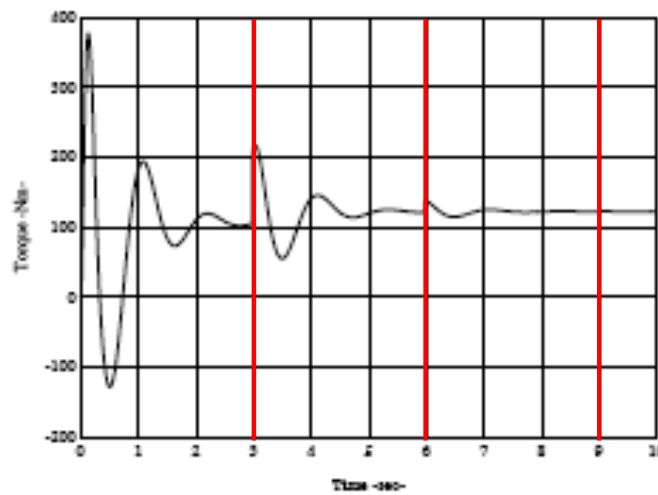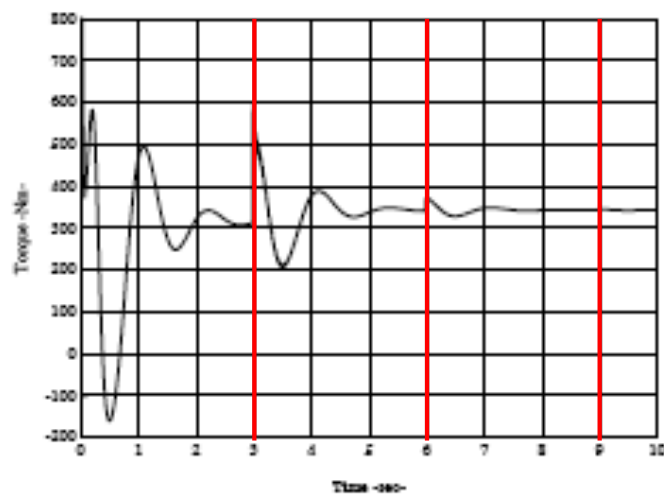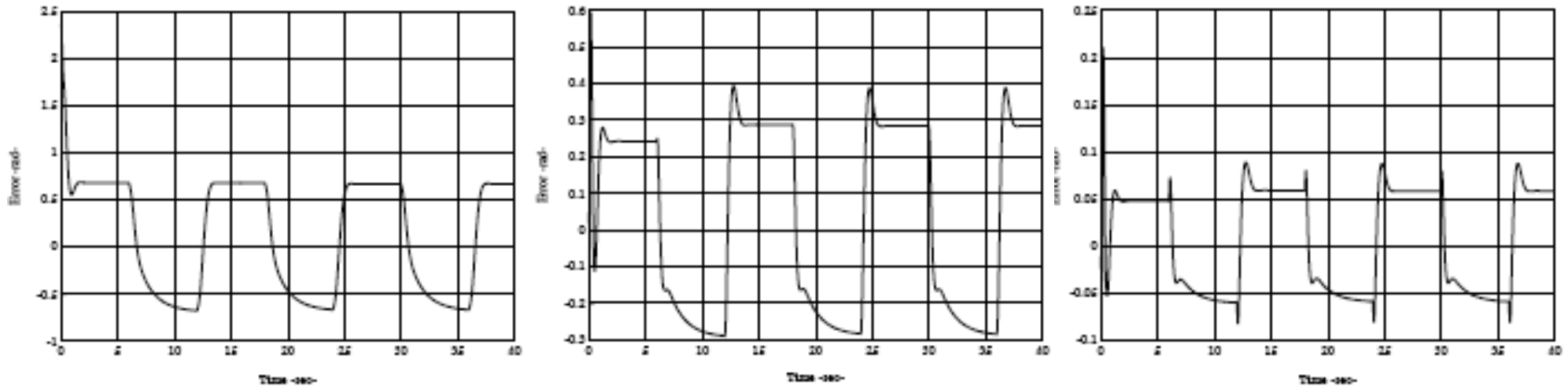
control torques

Case II: $q_d = (3\pi/4, 0, 0)$
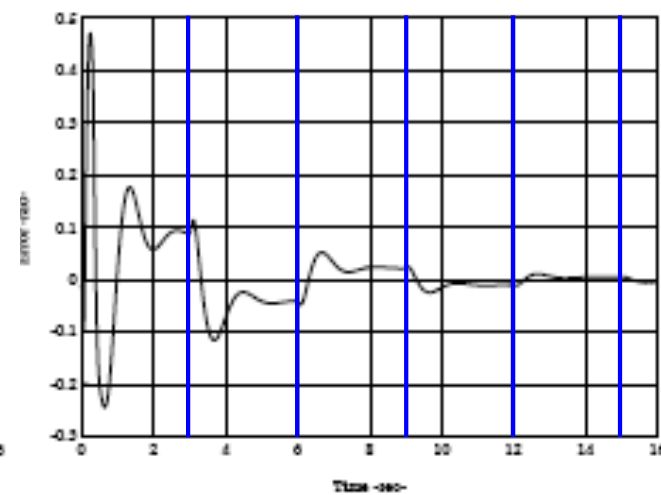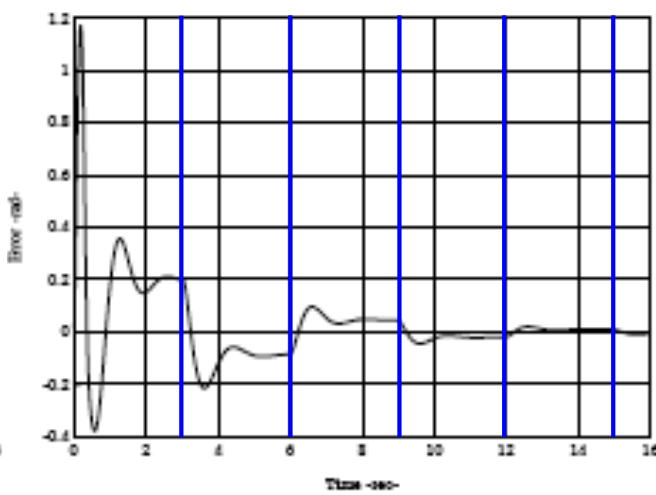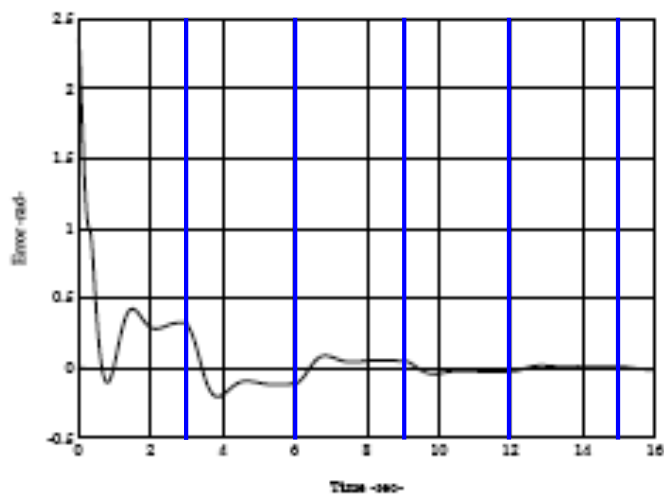
joint position errors (zero after 5 updates)
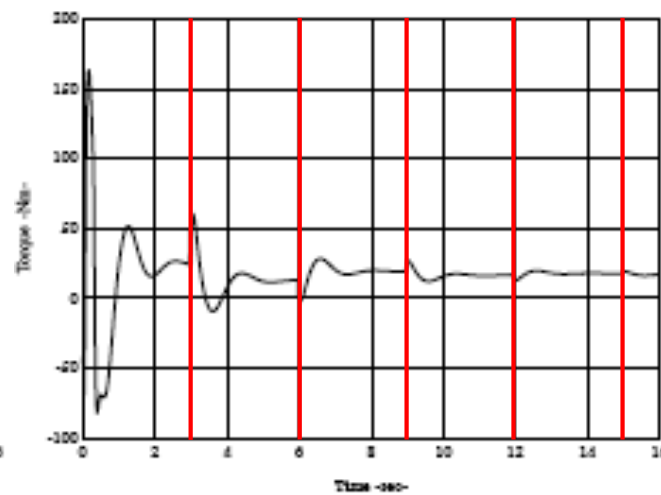
control torques

joint position errors (limit cycles, no convergence!)
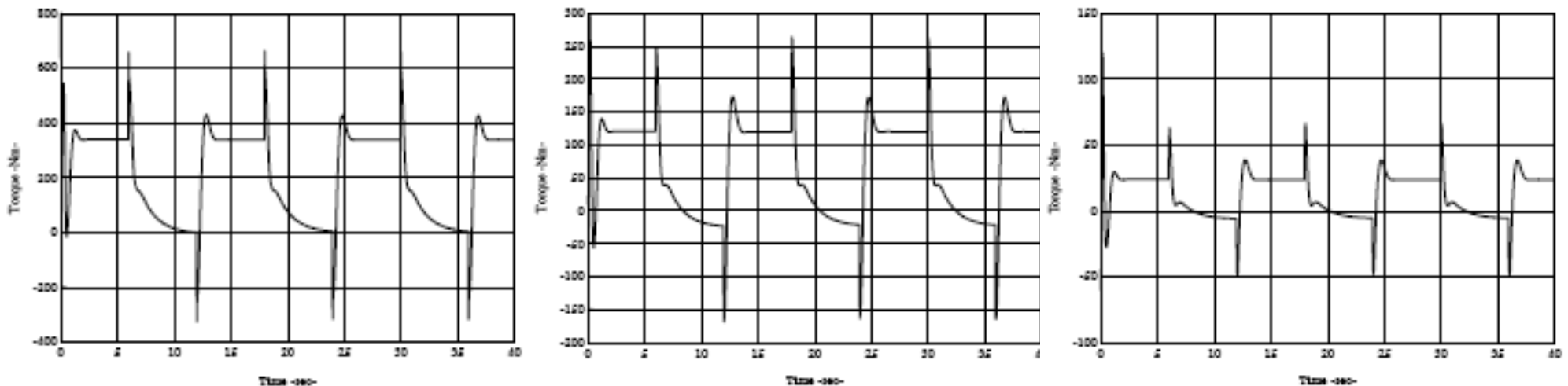
control torques

Final comments

- only few iterations are needed for obtaining convergence, learning the correct gravity compensation at the desired q_d

- sufficiency of the condition on the P gain

  - even if violated, convergence can still be obtained (first two cases); otherwise, a limit motion cycle takes place between two equilibrium configurations that are both incorrect (as in the third case)

  - this shows 'how far' is sufficiency from necessity

- analysis can be refined to get lower bounds on the K_{P_i} (diagonal case) that are smaller, but still sufficient for convergence

  - intuitively, lower values for K_{P_i} should still work for distal joints

- in practice, update of the feedforward term occurs when the robot is close enough to a steady state (joint velocities and position variations are below suitable thresholds)

Control experiments with flexible robots
without gravity

even for just a single but flexible link in the absence of gravity, a rest-to-rest maneuver without residual oscillations is difficult to be performed by a pure PD joint control action
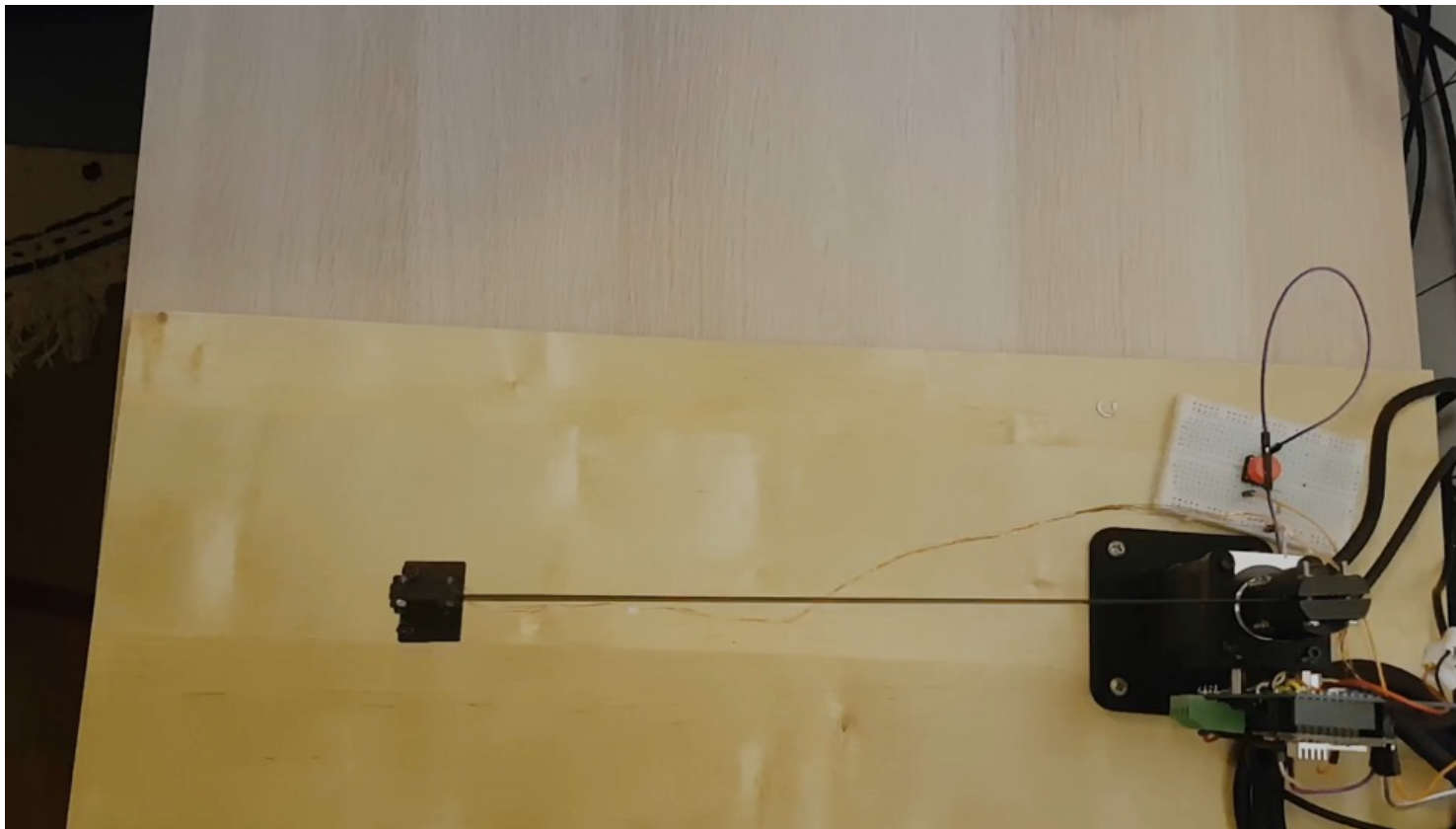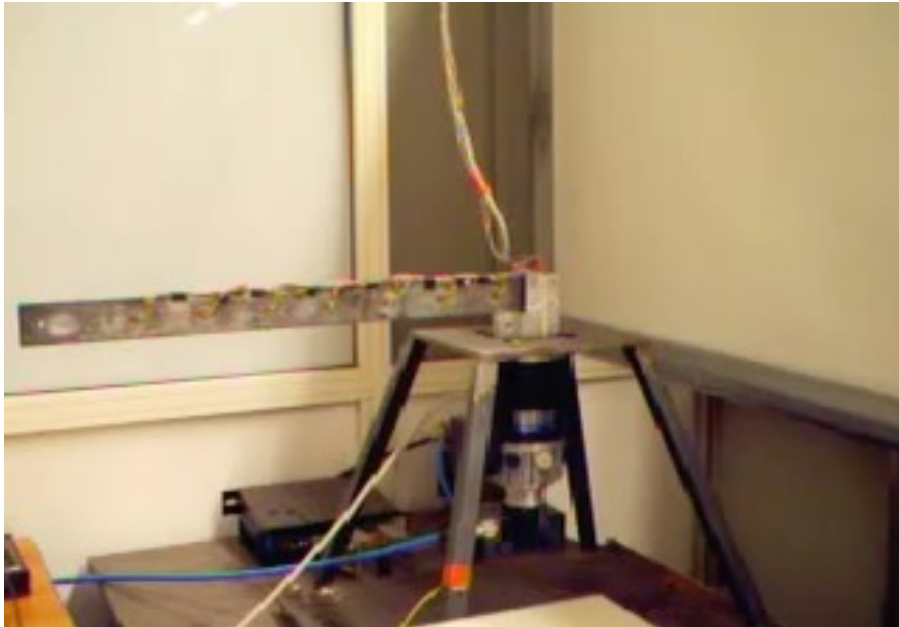

video

ICRA 2023

S. Drost. P. Pustina. F. Angelini, A. De Luca, G. Smit, C. Della Santina,
"Experimental validation of functional iterative learning control on a one-link flexible arm"

Control experiments with flexible robots
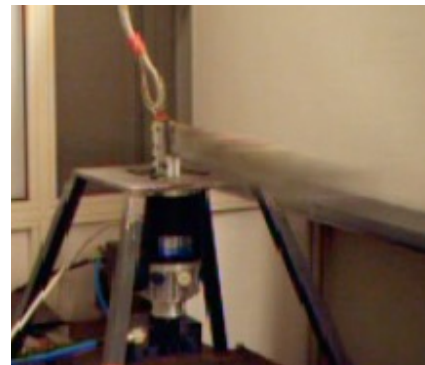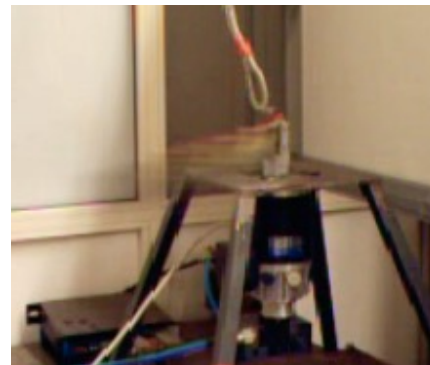## without gravity

video

rest-to-rest maneuver in given motion time for a single flexible link (PD + feedforward)
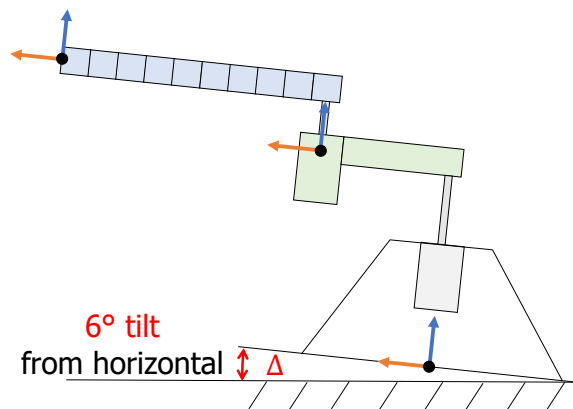
video

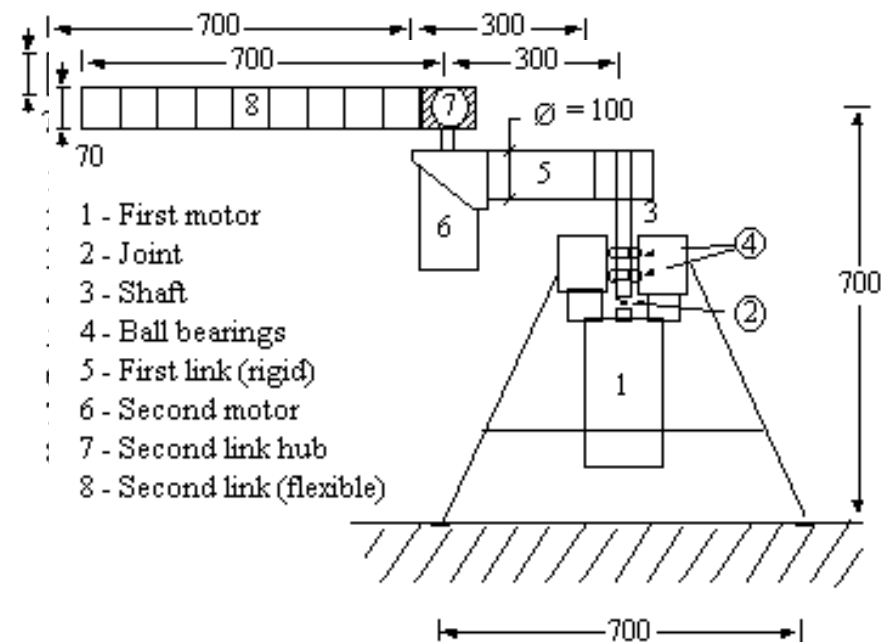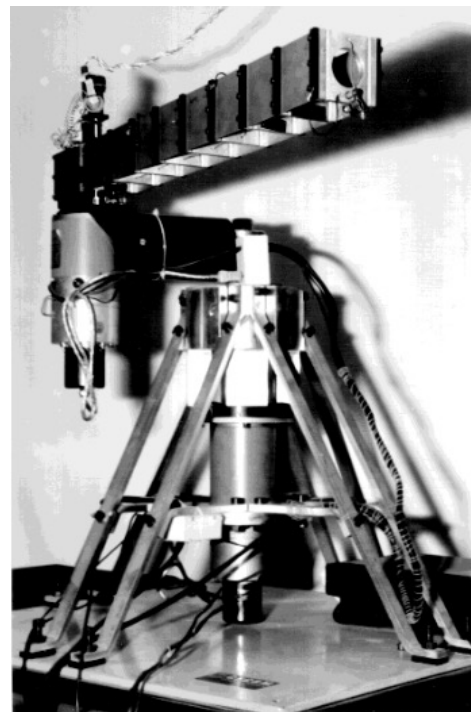end-effector trajectory tracking for FlexArm —a planar 2R robot with flexible forearm

Extension to flexible robots

- the same iterative learning control approach has been extended to position regulation in robots with flexible joints and/or links under gravity
    - at the motor/joint level
    - at the Cartesian level (end-effector tip position, beyond flexibility), using a double iterative scheme

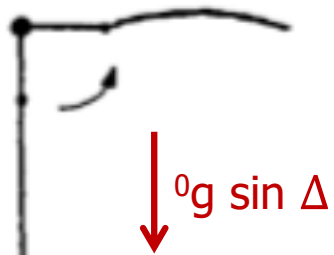- experimentally validated on the two-link FlexArm @ DIS (now DIAG!)

6° tilt
from horizontal Δ

with supporting base
tilted by Δ ≈ 6°
(inclusion of gravity)

700   300
700   300
70   8   7   Ø = 100
5
6   4
3
700
2
1 - First motor
2 - Joint
3 - Shaft
4 - Ball bearings
5 - First link (rigid)
6 - Second motor
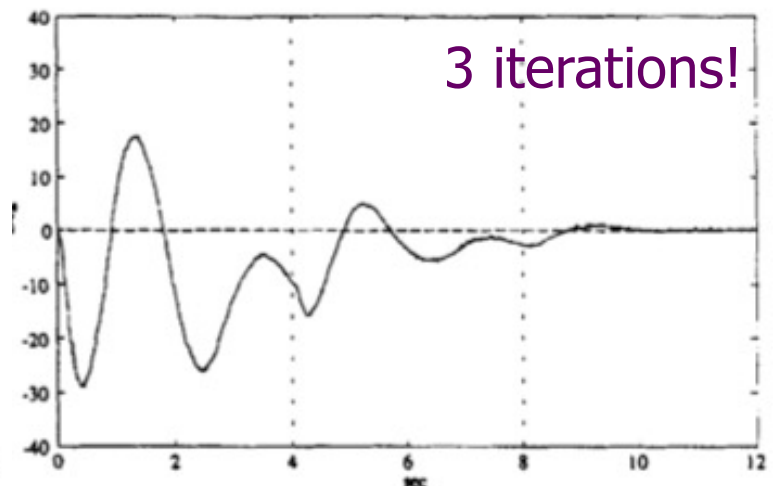7 - Second link hub
8 - Second link (flexible)
700

Experimental results for tip regulation

^{0}g sin Δ

motion task:
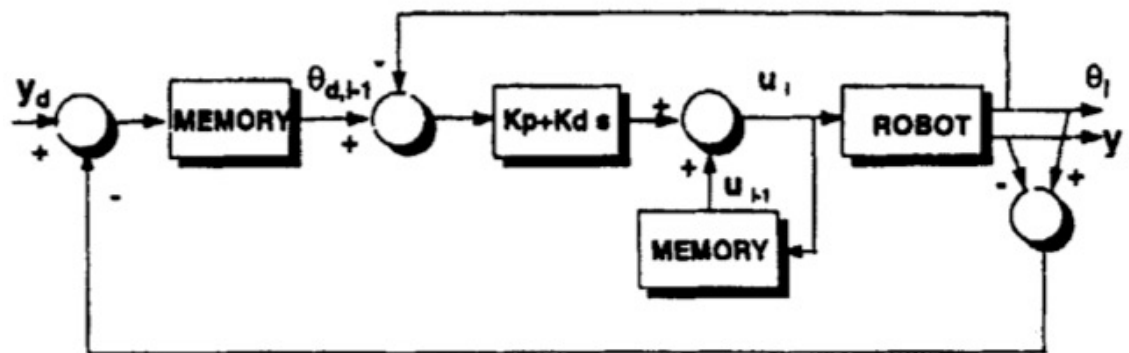$(0^\circ, 0^\circ) \Rightarrow (90^\circ, 0^\circ)$
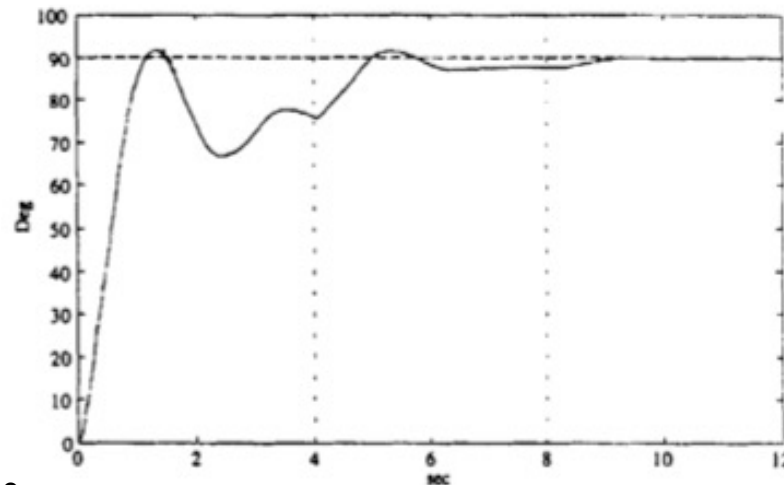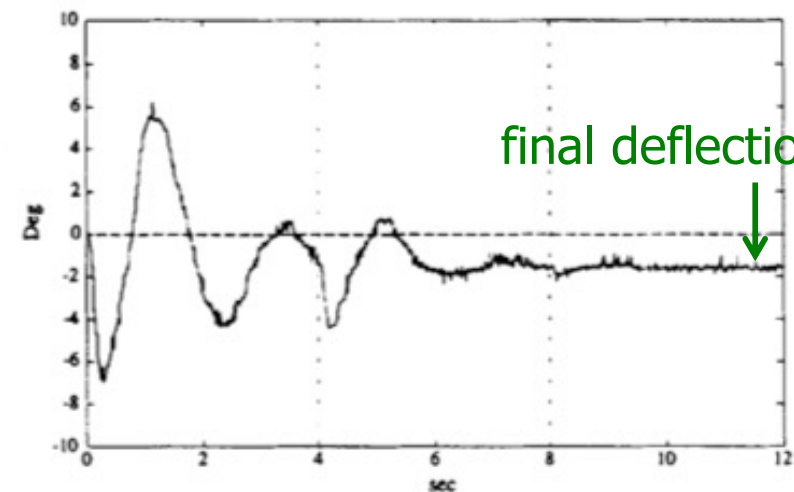
first link position

3 iterations!

tip angle w.r.t. first link

double iterative scheme

De Luca, Panzieri: Int J Adapt Cont & Sign Proc, 1996
(factor $\gamma \rightarrow 1/\beta$ in the original paper)

final deflection

second link deflection