Robotics 2

Kinematic calibration

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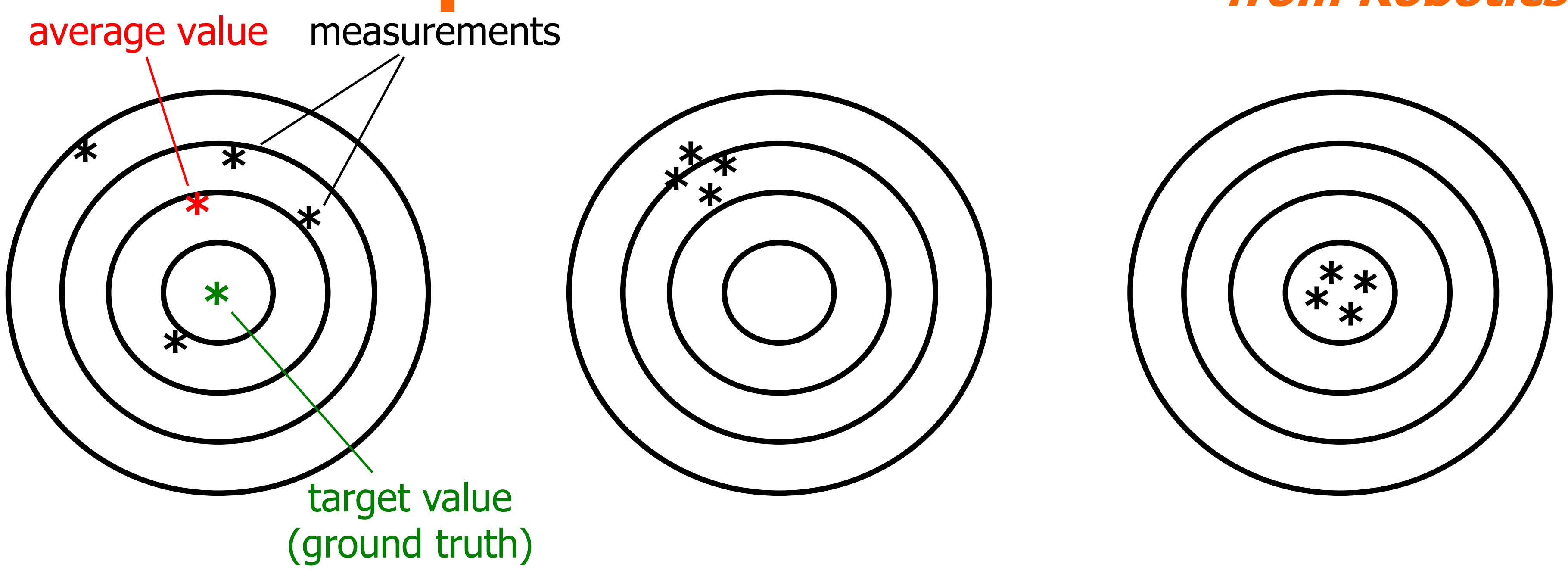
Accuracy and Repeatability

robot as a measuring device

from Robotics 1



low accuracy low repeatability	low accuracy high repeatability	high accuracy high repeatability

better components!

calibration!

Direct kinematics

- nominal set of Denavit-Hartenberg (D-H) parameters

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \quad \boldsymbol{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad \boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix}$$

for simplicity, suppose an all-revolute joints manipulator

- nominal direct kinematics

$$\boldsymbol{r}_{nom} = f(\boldsymbol{\alpha}, \boldsymbol{a}, \boldsymbol{d}, \boldsymbol{\theta})$$

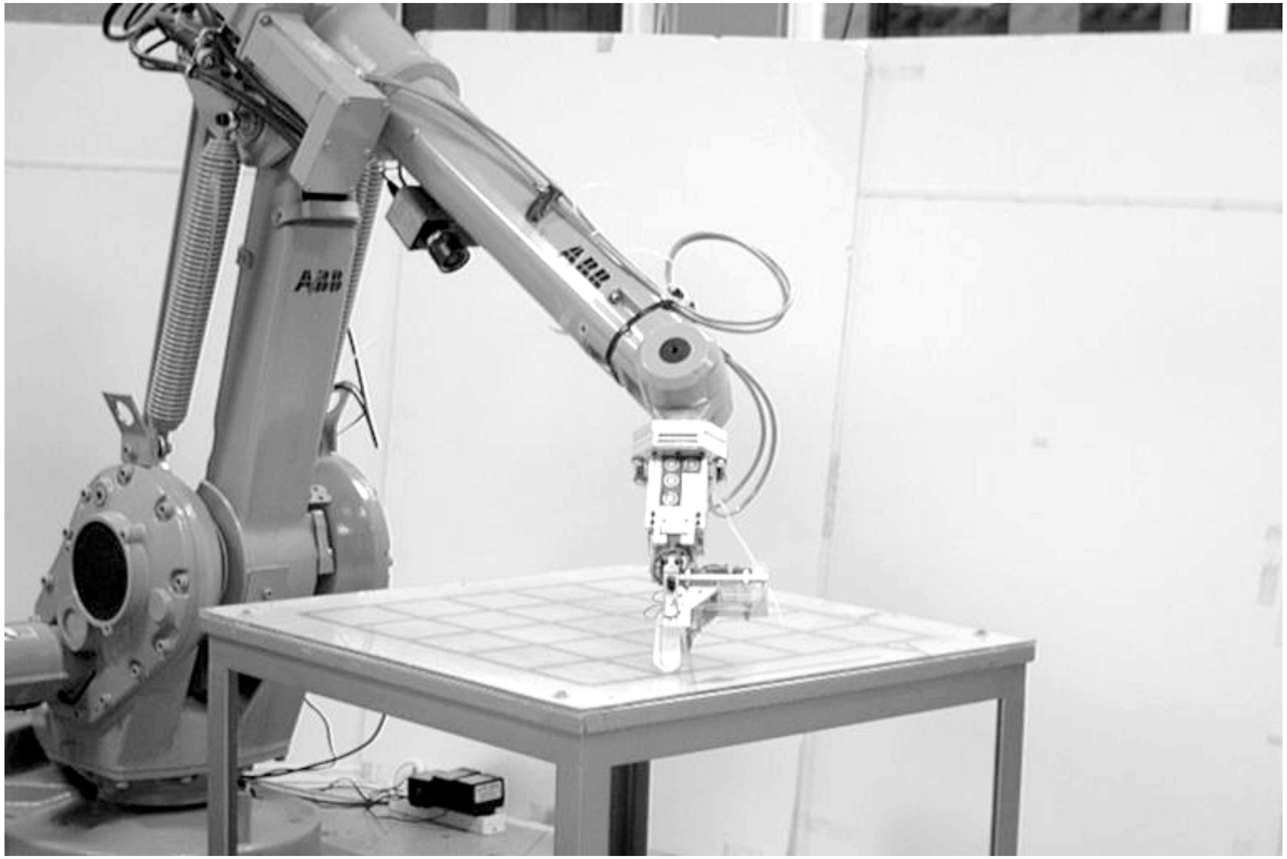
θ are typically measured by encoders

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}$$

Need for calibration

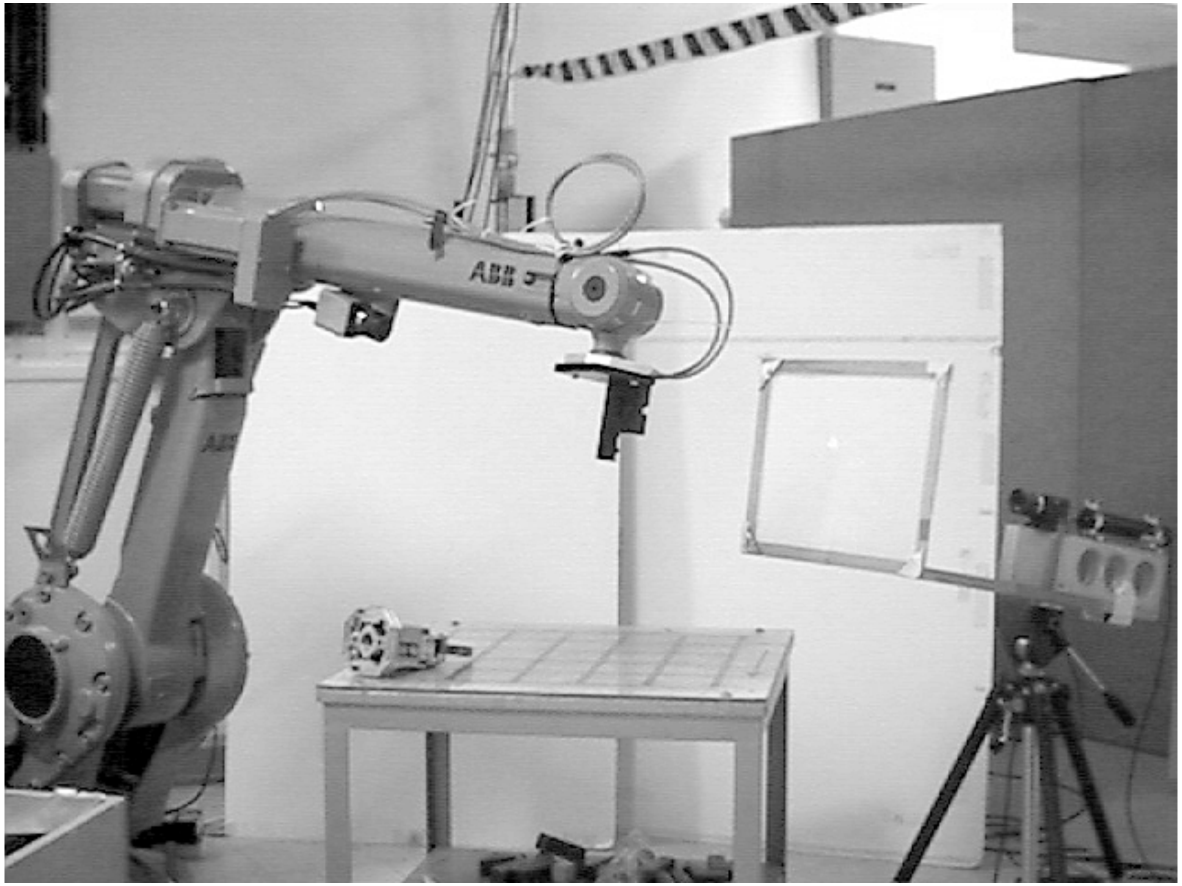
- tolerances in mechanical construction and in assembly of links/joints imply small errors in actual end-effector pose (real $\neq$ nominal parameters)
- encoder mounting on motor axes may not be consistent with the "zero reference" of the robot direct kinematics (joint angle measures are constantly biased)
- errors distributed "along" the arm are amplified, due to the open chain kinematic structure of most robots
- calibration goal: recover as much as possible E-E pose errors by correcting the nominal set of D-H parameters, based on independent external (accurate!) measurements
- experiments to be done once for each robot, before starting operation... (and maybe repeated from time to time)

Cartesian measurement systems - 1



calibration table

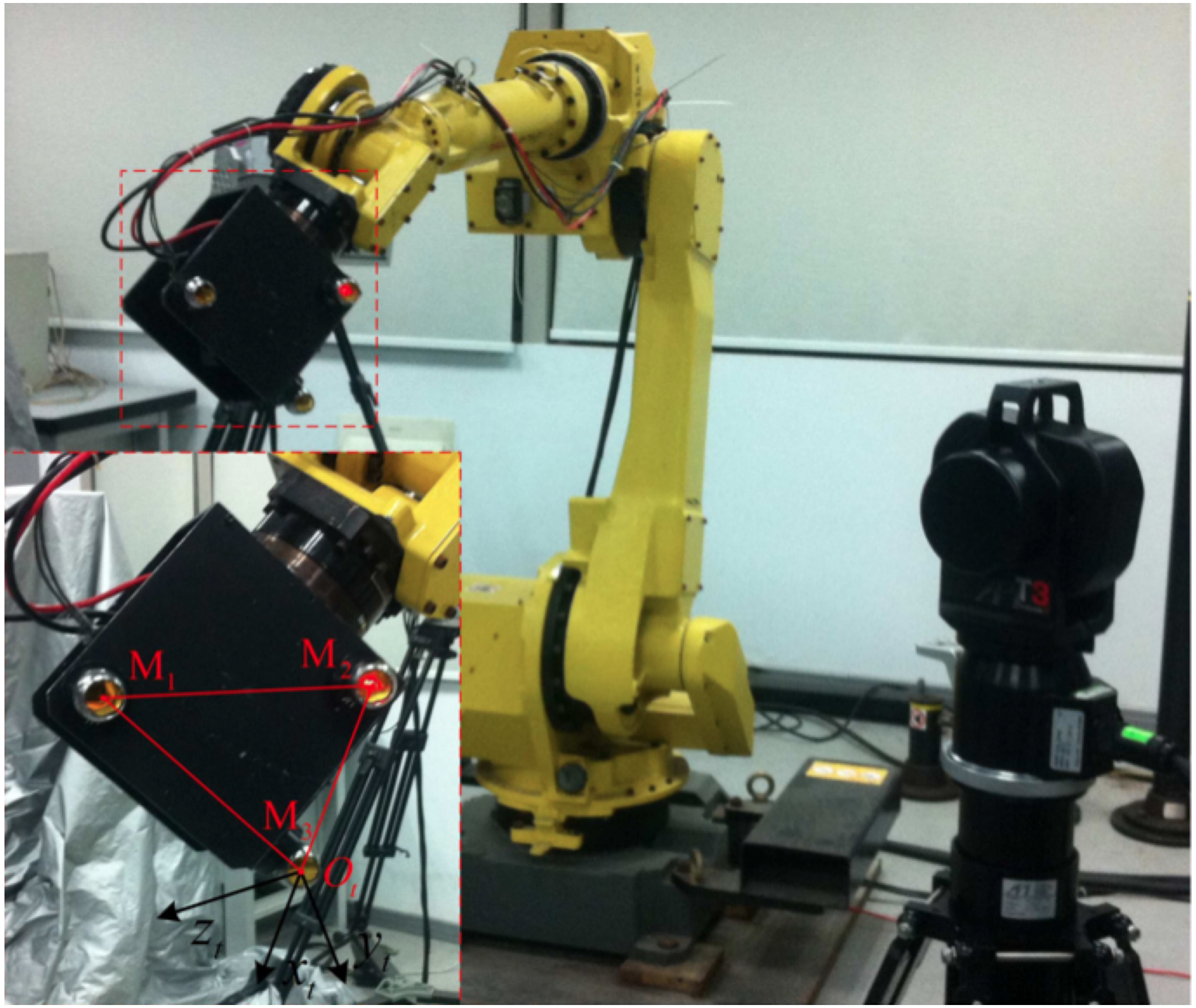
Cartesian measurement systems - 2



laser/camera system + triangulation

Cartesian measurement systems - 3

FANUC 6R robot
M-710iC/50



3 SMRs
(Spherically-
Mounted
Reflectors)

API
laser tracker III
www.apisensor.com

laser tracker + targets on end-effector

Acquiring data for calibration

FARO ION
laser tracker

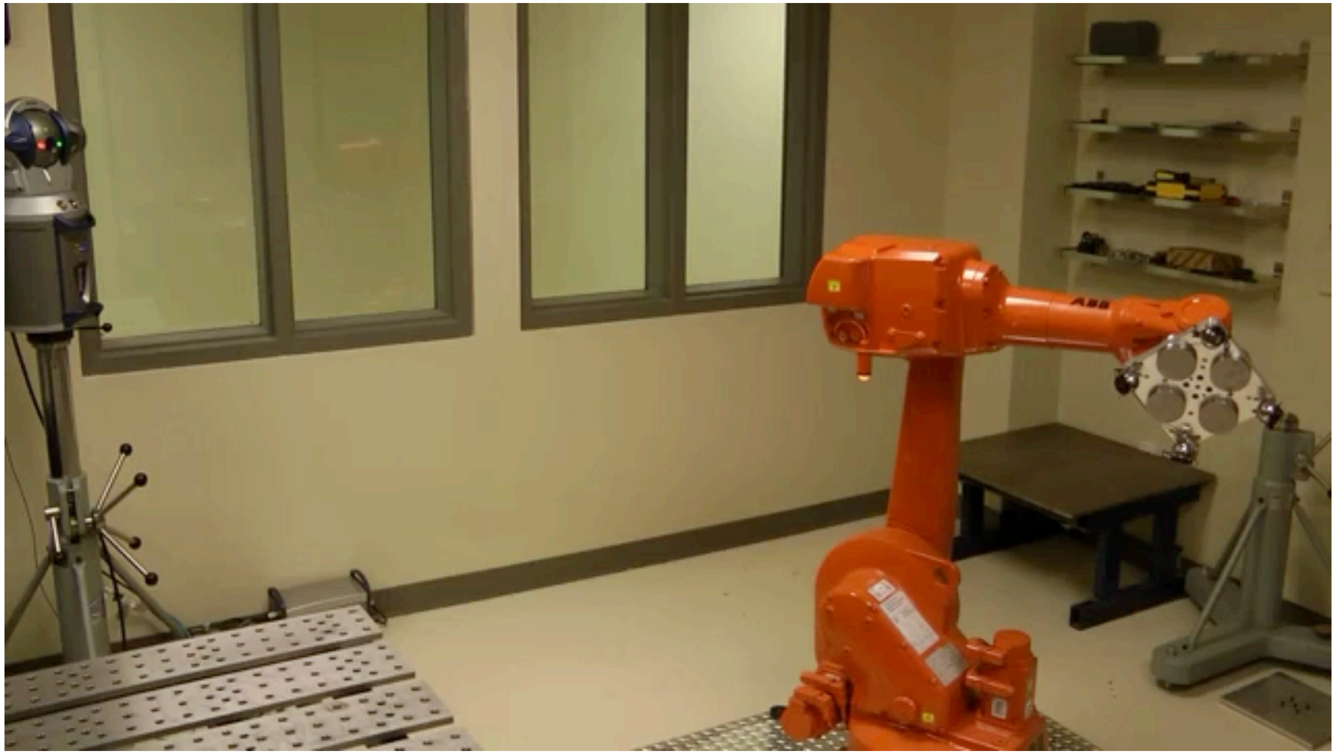
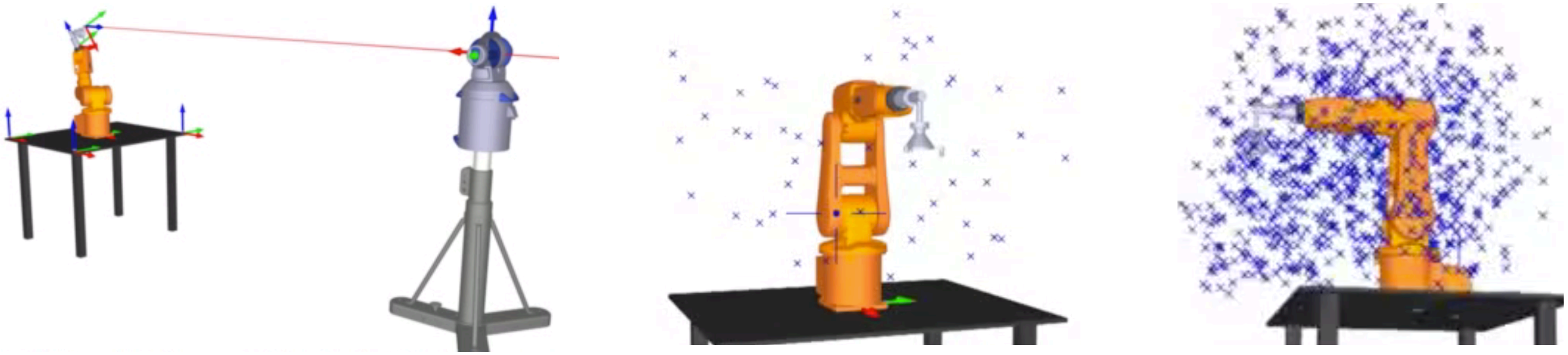


ABB
IRB 1600
robot

4 SMRs

video
@CoRo Lab
ETS Montréal



Linearization of direct kinematics

partial Jacobians evaluated in nominal conditions

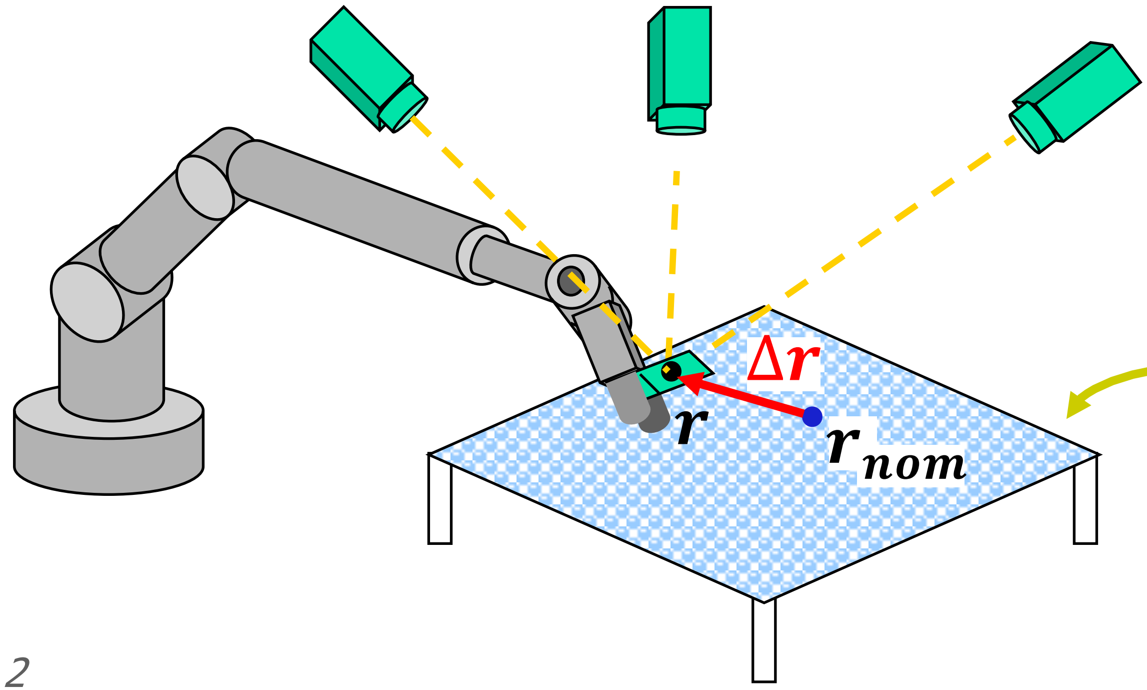
Taylor Series

$$\Delta r = r - r_{nom} = \frac{\partial f}{\partial \alpha} \cdot \Delta\alpha + \frac{\partial f}{\partial a} \cdot \Delta a + \frac{\partial f}{\partial d} \cdot \Delta d + \frac{\partial f}{\partial \theta} \cdot \Delta\theta$$

"small" errors

obtained by external measurement system

first-order variations



cameras + triangulation

calibration table

Calibration equation

$4n \times 1$ alpha, a, d and theta are n-dimensional since we have n joints

$$\Delta\varphi = \begin{pmatrix} \Delta\alpha \\ \Delta a \\ \Delta d \\ \Delta\theta \end{pmatrix} \qquad \Phi = \begin{pmatrix} \frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial a} & \frac{\partial f}{\partial d} & \frac{\partial f}{\partial \theta} \end{pmatrix}$$

Note: delta f w.r.t. to theta is just the analytic jacobian, we are extending it

$\boxed{6 \times 4n}$ 6 because we are assuming the task space has 6 dimensions i.e. f has 6 components

$$\boxed{\Delta r = \Phi \cdot \Delta\varphi}$$

ℓ experiments ($\ell \gg$ n)

Doing L experiments, so L points

$6\ell \times 1$ $\qquad$ $6\ell \times 4n$

$$\Delta\bar{r} = \begin{pmatrix} \Delta r_1 \\ \Delta r_2 \\ \vdots \\ \Delta r_\ell \end{pmatrix} \qquad \bar{\Phi} = \boxed{\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_\ell \end{pmatrix}}$$

$$\Delta\bar{r} = \bar{\Phi} \cdot \Delta\varphi$$

measures $\qquad$ unknowns

regressor matrix evaluated at nominal parameters

full column rank (for sufficiently large ℓ)

Phi_bar matrix has many rows and few columns.
If you take a sufficient number of experiments and they are independent (not aligned in one direction) we have full column rank

Note that delta phi doesn't not grow with experiments, they are the same for all the experiments -> overconstrained system! We choose a vector delta phi that reduces the error on all equations.

Under- and over-determined systems of linear equations

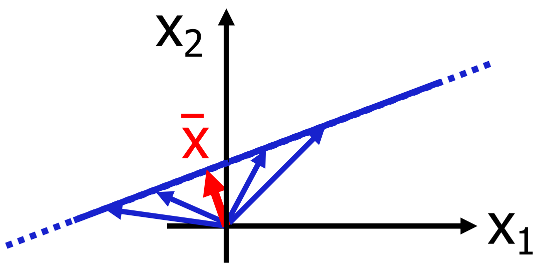
$$A\,x = b$$

full row rank full rank \

A

Here all the points on the line are solution
so we have an infinite numbers of solutions

1 equation
2 unknowns



minimum norm solution

$$\min \frac{1}{2}\|x\|^2, \text{ among } x: \; A\,x = b$$

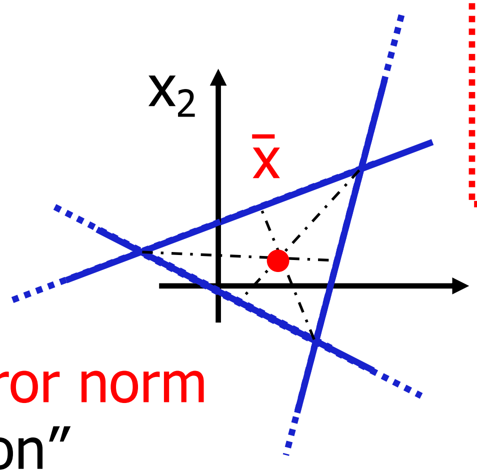
$$\boxed{A^{\#} = A^T\left(A\,A^T\right)^{-1}}$$

A single solution is the intersection between the three. We don't have it in this case. So we find an approximate solution minimizing the distances between these lines.

full rank full column rank

A

3 equations
2 unknowns



minimum error norm "solution"

$$\min \frac{1}{2}\|A\,x - b\|^2$$

$$\boxed{A^{\#} = \left(A^T A\right)^{-1} A^T}$$

$$\bar{x} = A^{\#} b$$

pseudoinverse !!

if multiple solutions x exist (having the same minimum **error** norm), the pseudoinverse provides the one having **minimum norm**

Calibration algorithm

$$\boxed{\Delta\bar{r} = \bar{\Phi} \cdot \Delta\varphi}$$

$$\boxed{\Delta\varphi = \bar{\Phi}^{\#}\Delta\bar{r} = (\bar{\Phi}^T\bar{\Phi})^{-1}\bar{\Phi}^T \cdot \Delta\bar{r}}$$

minimizes $\frac{1}{2}\|\bar{\Phi}\,\Delta\varphi - \Delta\bar{r}\|^2$

$$\boxed{\varphi_{nom} + \Delta\varphi = \varphi'}$$

new set of DH parameters
+ "bias" on measures of θ

We have linearized locally the non-linear system of equation with Taylor. So this is an updated version but I need to iterate in order to reduce more the error

$$\boxed{\Delta\bar{r}' = \bar{\Phi}' \cdot \Delta\varphi}$$

evaluated with new values φ'

... ITERATE!

Improvement by kinematic calibration

- ABB IRB 120 6R industrial robot
- 1000 random configurations (collision-free by simulation)
- 50 arbitrary configurations used for measurement in calibration
- 950 configurations used for validation
- Cartesian position errors

	before calibration	after calibration
Average	1.746 mm	0.193 mm
Median	1.567 mm	0.180 mm
Standard Deviation	1.043 mm	0.085 mm
Min	0.050 mm	0.010 mm
Max	4.423 mm	0.516 mm

- Improvement by **a factor 8÷10**

Final comments

- an iterative least squares method
  - original problem is nonlinear in the unknowns, then linearized using first-order Taylor expansion
- it is useful to calibrate first and separately those quantities that are less accurate (typically, the encoder bias)
  - keeping the remaining ones at their nominal values

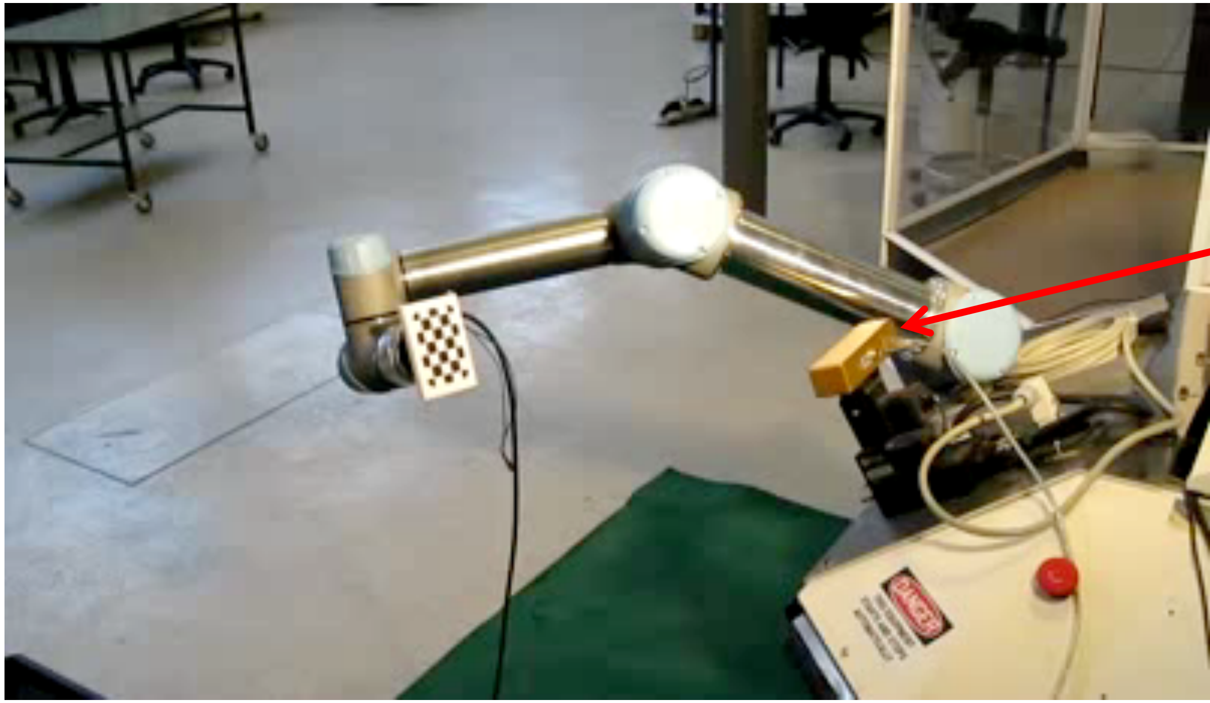
It's like normalizing values: if you have to minimize a parameters that go from 0 to 1 and another in the order of thousands first you normalize, optimize and then go back to the unnormalized space. Here is the same: theta has larger error, throwing all together, since minimizing the norm you are not doing any difference between components, we are spreading the variation on all parameters on an equal way. So we first capture the variation of the parameters which have larger effect on the positioning error. Now you just put the delta theta, keeping the other fixed, on their nominal values. You do the algorithm, getting the variation as usual, but only for delta theta. Once you have done this, the error is reduced, removed the component with larger effect, at this point you consider the full problem, allowing also the variation of alpha, a and d.,

- alternative kinematic descriptions can be used
  - more complex than D-H parameters, but leading to a better numerical conditioning of the regressor matrix in calibration algorithm
  - one such description uses the POE (Product Of Exponential) formula
- more in general, 6 base parameters should also be included
  - to locate 0-th robot frame w.r.t. world coordinate frame (of external sensor)
- accurate calibration/estimation of real parameters is a general problem in robotics (and beyond...)
  - for sensors (e.g., camera calibration)
  - for models (identification of dynamic parameters of a manipulator)

Calibration experiment

in a research environment

video



Videre Design
stereovision
camera

- automatic data acquisition for simultaneous calibration of
  - robot-camera transformation
  - DH parameters of the manipulator

Calibration experiment

in an industrial setting

video

FANUC
3D Laser
calibration
(with iR Vision)