Machine learning

encoding & decoding;
linear & non-linear models

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AARHUS UNIVERSITY

INTERACTING MINDS CENTRE

CFIN

Contents

Machine learning

Machine learning: what is it?

- multivariate pattern analysis,
  multivoxel pattern analysis,
  predictive modelling,
  statistical learning

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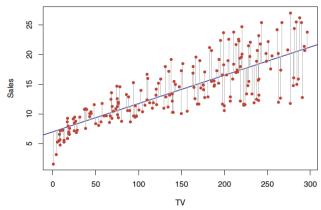

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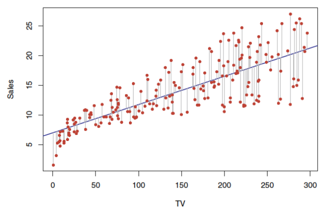

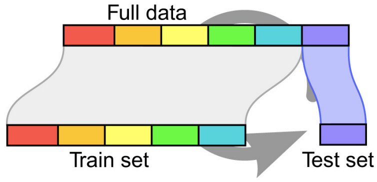

Top figure from James et al. (2013)

bottom figure from Varoquaux et al. (2017)

Machine learning: pipeline

- create features
- make cross-validation scheme
- fit model
- interpret model

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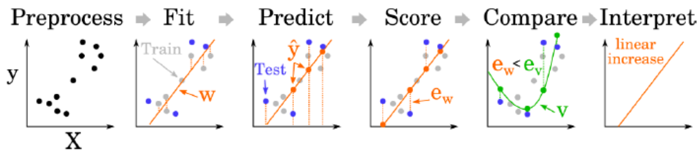

(Figure from King et al., 2018)

Features

What are the question to be investigated?

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- whole brian (all sensors) vs ROIs

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- raw signals vs transformed signals
- all times together vs time bins vs across time

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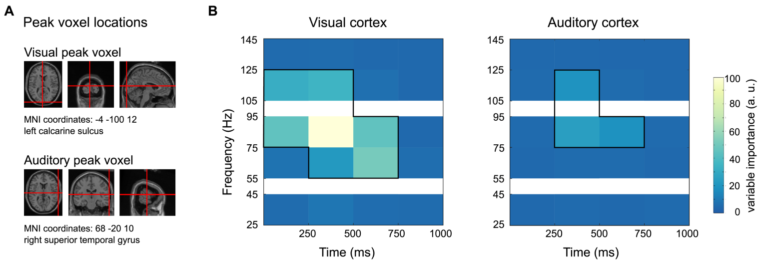

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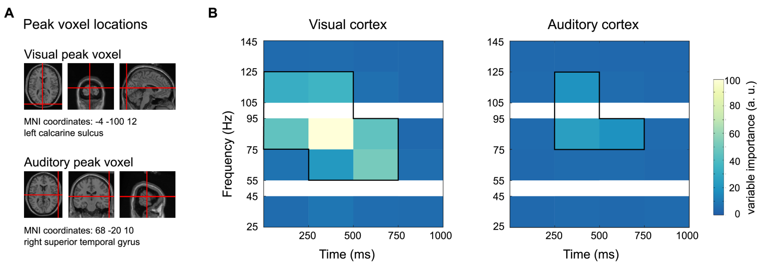

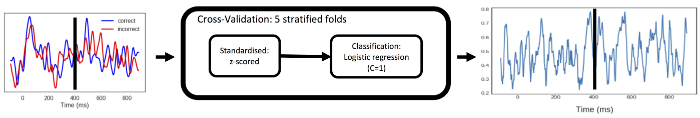

Top figure from Westner et al. (2018)

bottom figure mine.

Encoding

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- That is, given a specific stimulus, e.g. a sound, can we then predict the how the brain reacts.
- Often used algorithm is a linear regression
- Encoding is a world to brain relation.

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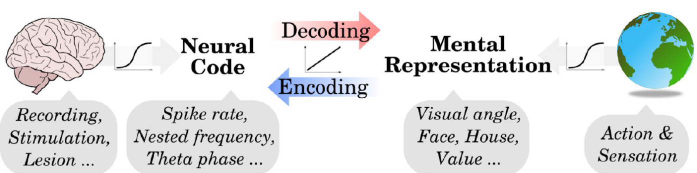

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Encoding

Linear regression problem

$$activity(t) = \sum_{i}^{N_{features}} features_i(t) * weight_i + error(t)$$

Where the neural activity at time t is modelled as a weighted sum of N stimulus features. (Holdgraf et al., 2017, p. 5)[1]

[1] Adapted from Hastie et al. (2009)

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Time lagged version

$$activity(t) = \sum_{j}^{N_{lags}} \sum_{i}^{N_{features}} features_i(t-j) * weight_{i,j} + error(t)$$

[1] Adapted from Hastie et al. (2009)

Encoding

Linear algebra terms:

$$activity = Sw + \epsilon$$

"In this case S is the stimulus matrix where each row corresponds to a timepoint of the response, and the columns are the feature values at that timepoint and time-lag (there are N_{lags} * $N_{features}$ columns). w is a vector of model weights (one for each feature * time lag), and ϵ is a vector of random noise at each timepoint (most often to be Gaussian for continuous signals or Poisson for discrete signals)." (Holdgraf et al., 2017, p. 5)

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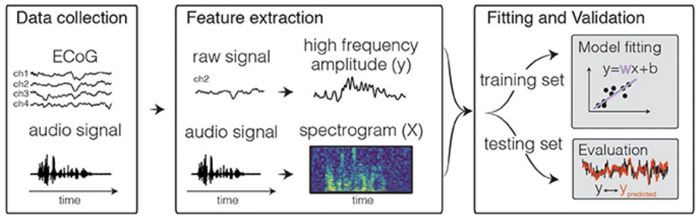

(Figure from Holdgraf et al., 2017)

Holdgraf et al 2016

ARTICLE

Received 27 May 2016 | Accepted 20 Oct 2016 | Published 20 Dec 2016 DOI: 10.1038/ncomms13654 OPEN

Rapid tuning shifts in human auditory cortex enhance speech intelligibility

Christopher R. Holdgraf[1], Wendy de Heer[2], Brian Pasley[1], Jochem Rieger[1], Nathan Crone[3], Jack J. Lin[4], Robert T. Knight[1,2,3] & Frédéric E. Theunissen[1,2]

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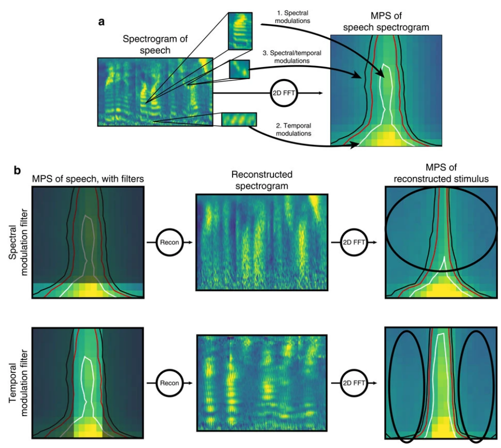

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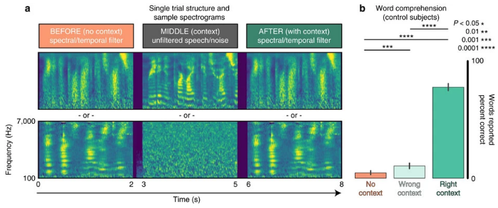

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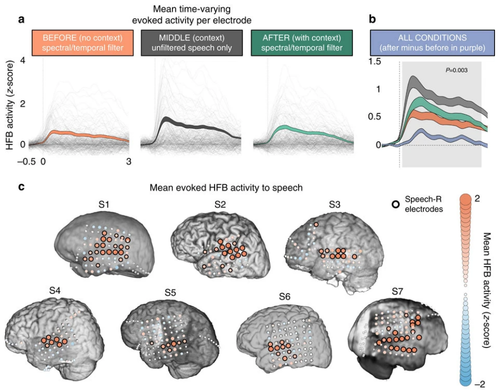

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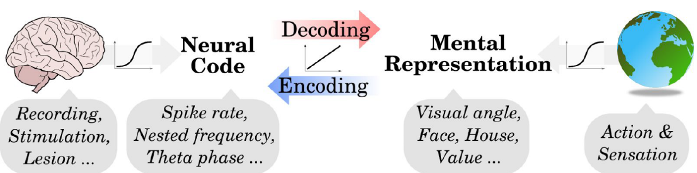


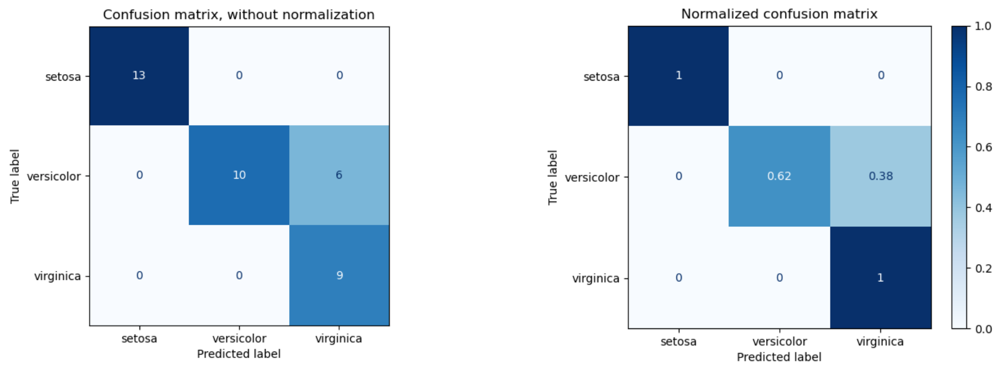
Decoding

$$feature(t) = \sum_{j}^{N_{lags}} \sum_{i}^{N_{channels}} activity_i(t+j) * weight_{i,j} + error(t)$$

$$s = Xw + \epsilon$$

"where s is a vector of stimulus feature values recorded over time, and X is the channel activity matrix where each row is a timepoint and each column is a neural feature (with time-lags being treated as a separate column each). w is a vector of model weights (one for each neural feature * time lag), and ϵ is a vector of random noise at each timepoint (often assumed to be Gaussian noise)." (Holdgraf et al., 2017, p. 5)

Confusion matrix



(Figure from https://scikit-learn.org/stable/auto_examples/model_selection/plot_confusion_matrix.html)

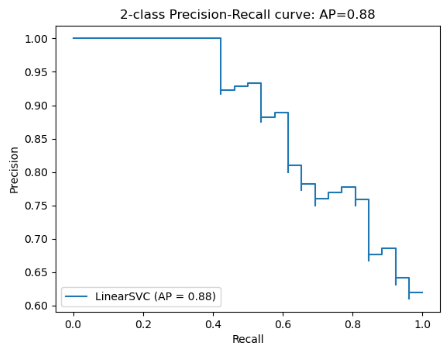
Precision-Recall

Precision (P) is defined as the number of true positives (T_p) over the number of true positives plus the number of false positives (F_p).

$$P = \frac{T_p}{T_p + F_p}$$

Recall (R) is defined as the number of true positives (T_p) over the number of true positives plus the number of false negatives (F_n).

$$R = \frac{T_p}{T_p + F_n}$$



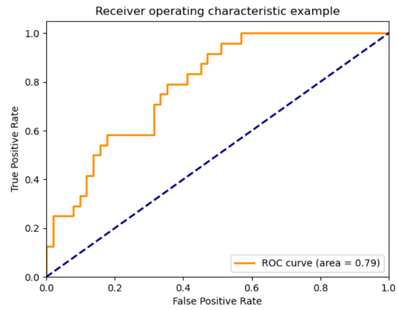
(Figure from https://scikit-learn.org/stable/auto_examples/model_selection/plot_precision_recall.html)

Receiver Operating Characteristic (ROC)

ROC are *true positive rate* (TPR; recall) plotted against the *false positive rate* (FPR)

$$TPR = \frac{T_p}{T_p + F_n}$$

$$FPR = \frac{F_p}{F_p + T_n}$$



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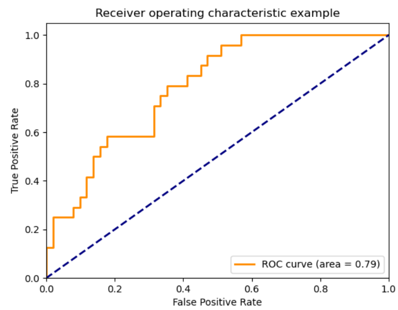

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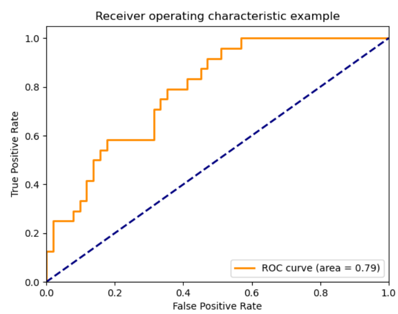
If the classifier is performing at chance, then the AUC will be 0.5, and if it correctly labels all datapoints for all decision thresholds, then the AUC will be 1 " (Holdgraf et al., 2017, p. 15)



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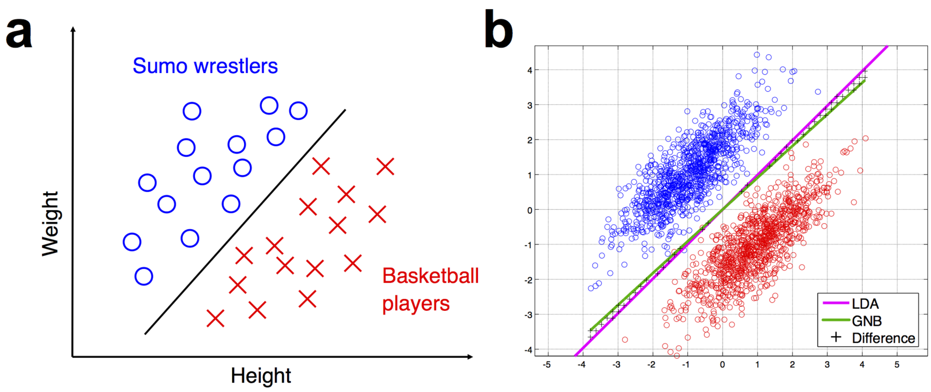
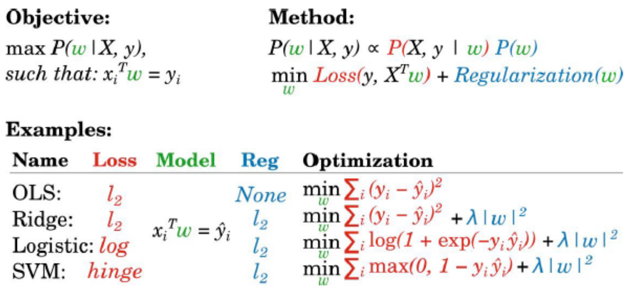
Decision boundaries



(Figure from Raizada & Lee, 2013)

Regularisation



Objective:
max $P(w \mid X, y)$,
such that: $x_i^T w = y_i$

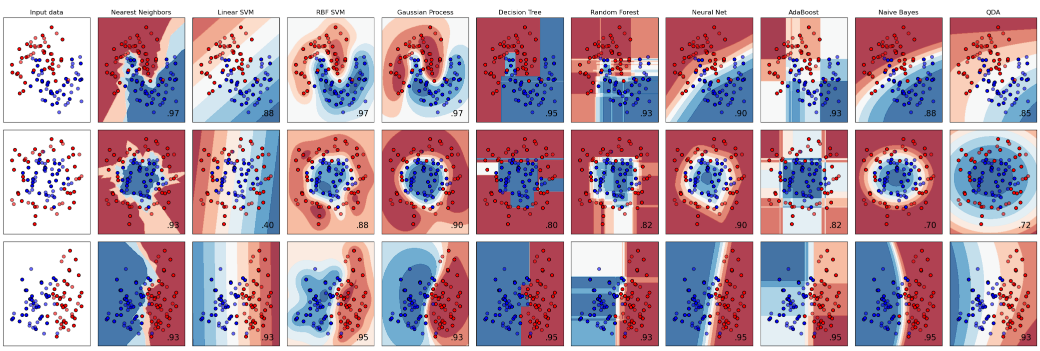
Method:
$P(w \mid X, y) \propto P(X, y \mid w)\, P(w)$
$\min_w Loss(y, X^T w) + Regularization(w)$

Examples:

Name	Loss	Model	Reg	Optimization
OLS:	l_2		*None*	$\min_w \sum_i (y_i - \hat{y}_i)^2$
Ridge:	l_2	$x_i^T w = \hat{y}_i$	l_2	$\min_w \sum_i (y_i - \hat{y}_i)^2 + \lambda \lvert w \rvert^2$
Logistic:	*log*		l_2	$\min_w \sum_i \log(1 + \exp(-y_i \hat{y}_i)) + \lambda \lvert w \rvert^2$
SVM:	*hinge*		l_2	$\min_w \sum_i \max(0, 1 - y_i \hat{y}_i) + \lambda \lvert w \rvert^2$

(Figure from King et al., 2018)

Linear vs non-linear models



(Figure from https://scikit-learn.org/stable/auto_examples/classification/plot_classifier_comparison.html)

Linear vs non-linear models

Examples of non-linear classifiers:

- Random forest (Breiman, 2001)
- XGBoost (Chen & Guestrin, 2016)

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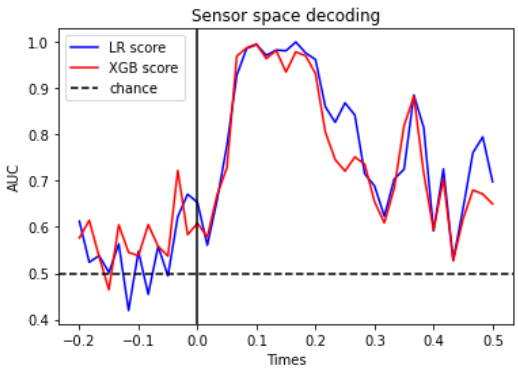
- can estimate non-linear decision boundaries

Cons non-linear classifiers

- require more data that linear classifiers
- require tuning of hyper-paremeters
- often does not perform better
- hard to interpret



Time generalisation

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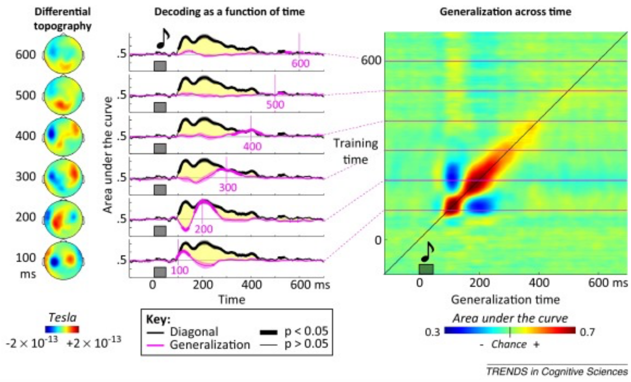

(Figure from King & Dehaene, 2014)

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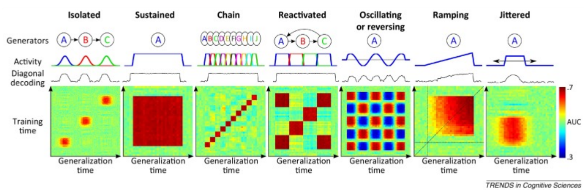

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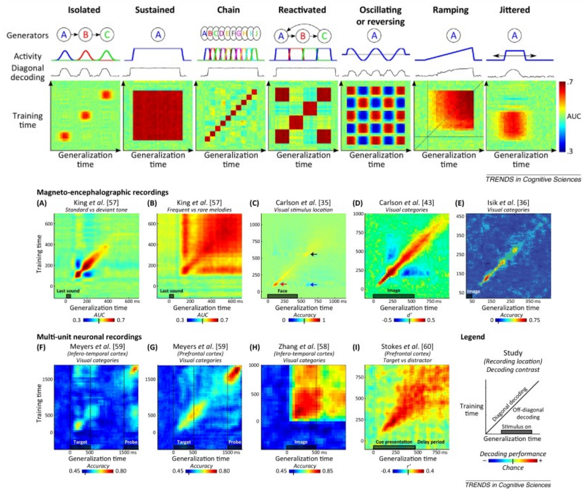

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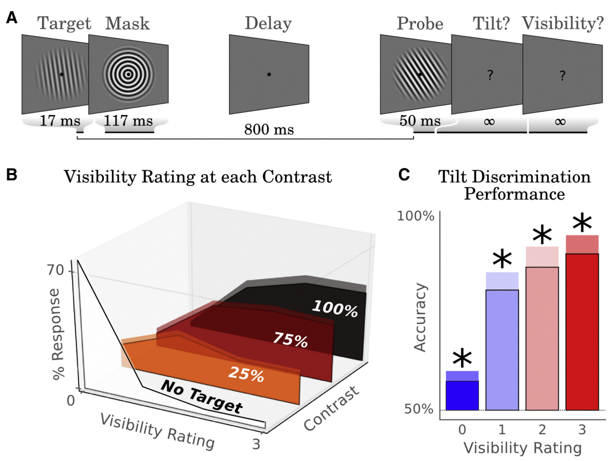
King et al. 2016

Brain Mechanisms Underlying the Brief Maintenance of Seen and Unseen Sensory Information

Jean-Rémi King, Niccolo Pescetelli, and Stanislas Dehaene
[1] Department of Psychology, New York University, New York, NY 10003, USA
[2] Neuroscience Department, Frankfurt Institute for Advanced Studies, 60438 Frankfurt, Germany
[3] Department of Experimental Psychology, University of Oxford, OX1 3UD Oxford, UK
[4] Cognitive Neuroimaging Unit, CEA DSV/I2BM, INSERM, Université Paris-Sud, Université Paris-Saclay, NeuroSpin Center, 91191 Gif/Yvette, France

King et al. 2016



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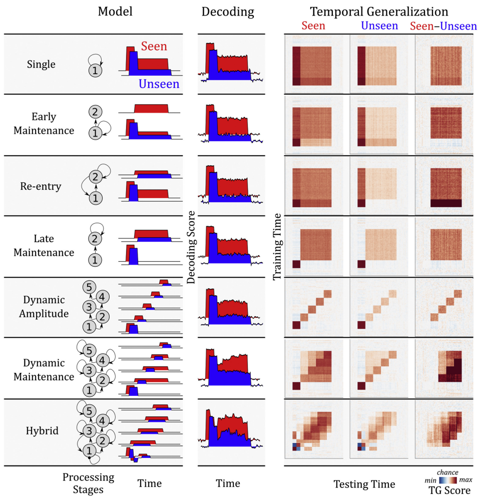

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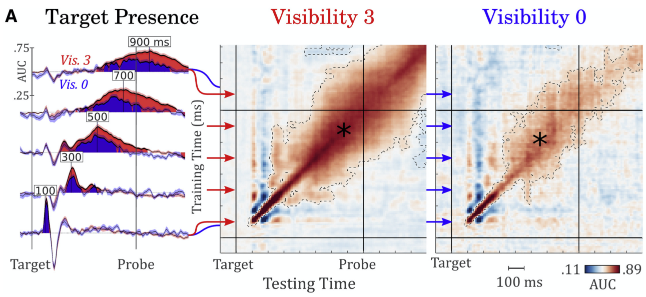

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General paper feedback

General small points

- No reason for an abstract or keywords
- Citations are good!
  direct quotes need page number(s)

Abstract

Abstract

An abstract is a short summary of your (published or unpublished) research paper, usually about a paragraph (c. 6-7 sentences, 150-250 words) long. A well-written abstract serves multiple purposes:

- an abstract lets readers get the gist or essence of your paper or article quickly, in order to decide whether to read the full paper;
- an abstract prepares readers to follow the detailed information, analyses, and arguments in your full paper;
- and, later, an abstract helps readers remember key points from your paper.

From
https://writing.wisc.edu/handbook/assignments/writing-an-abstract-for-your-research-paper/

Abstract

Here are the typical kinds of information found in most abstracts:

1. the context or background information for your research; the general topic under study; the specific topic of your research
2. the central questions or statement of the problem your research addresses
3. what's already known about this question, what previous research has done or shown
4. the main reason(s), the exigency, the rationale, the goals for your research—Why is it important to address these questions? Are you, for example, examining a new topic? Why is that topic worth examining? Are you filling a gap in previous research? Applying new methods to take a fresh look at existing ideas or data? Resolving a dispute within the literature in your field? ...
5. your research and/or analytical methods
6. your main findings, results, or arguments
7. the significance or implications of your findings or arguments.

https://writing.wisc.edu/handbook/assignments/writing-an-abstract-for-your-research-paper/

Parts of a paper

- introduction
- method
- results
- discussion
- general discussion

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- general discussion
  - related the results and discussion to the overall topic
  - inverse of the introduction

Building models of statements

If the program returns the correct result, the code and compiler works.
The compiler did *not* work
Therefore the program did not return the correct result.

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$A \rightarrow (B \wedge C)$
$\neg C$
$\therefore \neg A$

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When is an argument valid?

An argument is valid if and only if it takes a form that make it impossible for the premises to be true and the conclusion nevertheles to be false.

if then statements: material conditional

What we say:	What we write:
if p then q	$p \to q$

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p	p

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if p then q	$p \rightarrow q$
p	p
therefore q	$\therefore q$

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Truth table

p	q	p → q
T	T	T
T	F	F
F	T	T
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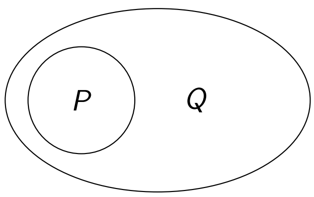

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$p \rightarrow q$
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Examples:

- *if* it rains *then* the street is wet
- *if* it is a Labrador *then* it is a dog

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if then statements: material conditional

$p \to q$
p
$\therefore q$

$p \to q$
$\neg p$

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Examples:

- *if* it rains *then* the street is wet
- *if* it is a Labrador *then* it is a dog

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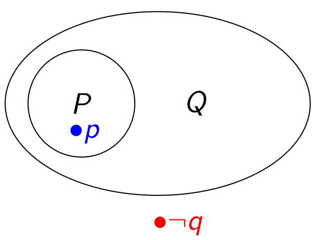

Truth tables

$\wedge$ (and)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth tables

$\wedge$ (and)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$\vee$ (or)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth tables

$\wedge$ (and)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
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p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$\oplus$ (Exclusive-or)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Truth tables

$\wedge$ (and)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$\vee$ (or)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$\oplus$ (Exclusive-or)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$$p \oplus q = (p \vee q) \wedge \neg(p \wedge q)$$

Example

If the program returns the correct result, the code and compiler works.
The compiler did *not* work
Therefore the program did not return the correct result.

Example

If the program returns the correct result, *then* the code *and* compiler works.
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$A \rightarrow (B \wedge C)$
$\neg C$
$\therefore \neg A$

Example

If the program returns the correct result, *then* the code *and* compiler works.
The compiler did not work
Therefore the program did not return the correct result.

$A \rightarrow (B \wedge C)$
$\neg C$
$\therefore \neg A$

$\wedge$ (and)

B	C	$(B \wedge C)$
T	T	T
T	F	F
F	T	F
F	F	F

A	$(B \wedge C)$	$A \rightarrow (B \wedge C)$
T	T	T
T	F	F
F	T	T
F	F	T

Questions?

1. Machine learning
2. Encoding
   - example: rapid tuning shifts in human auditory cortex ...
3. Decoding
   - evaluating algorithms
   - linear vs non-linear models
4. Time generalisation
   - example: brain mechanisms underlying the brief maintenance ...
5. General paper feedback
   - abstract
   - parts of a paper
   - arguments
   - material condition

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