Universal Coherence Algorithm: A Mathematical Framework for Recursive Equilibrium Systems

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2025

Contents

1 Abstract

This paper formalizes the Universal Coherence Algorithm (UCA) as a self-regulating, recursive system of differential and operator equations unifying feedback, stability, and fractal renormalization flows. The following section presents the full mathematical architecture.

2 Mathematical Framework

$$(\Omega, \mathcal{F}, \mathbb{P}), \quad \langle u, v\rangle_G = u^\top G v, \quad \|u\|_G^2 = \langle u, u\rangle_G.$$

$$C(t) = \int_0^t c(s)\, ds, \quad H(t) = \int_0^t h(s)\, ds.$$

$$D = rD\Big(1 - \frac{D}{D_{\max}}\Big) - \lambda(\alpha_1 (C - H)^2 + \alpha_2 H^2) + \xi_D.$$

$$(D, T_c) =_0 \Big(\frac{D - D_\star}{\tau\Big)\frac{1}{1+T_c/T_0}, \quad (D) =_0 \Big(\frac{D-D^\dagger}{\tau\Big), \quad (u) = \frac{1}{1+e^{-u}}.}}$$

$$\dot{\lambda}_V =_V (C - H) -_V .$$

$$L(C, H, A, \dot{C}, \dot{H}, \dot{A}; D, M) = (D)C - (D)H - \frac{A}{2}\dot{A}^2 - \frac{}{2}(\dot{C} - \dot{H})^2 - F(M) +_V (\dot{C} - \dot{H}).$$

$$S[C, H, A, M,_V] = \int_0^T L(C, H, A, \dot{C}, \dot{H}, \dot{A}; D, M)\, dt.$$

$$\dot{A} = -(_C W_{CA} W_C +_F \|\nabla_M F(M)\|_G) +_A .$$

$$\dot{C} = (D, T_c)\dot{H} - (D)\dot{A} - (\dot{C} - \dot{H}).$$

$$_V =_V W_C -_V, \quad W_C = \dot{C} - \dot{H}.$$

$$\dot{D} = rD(1 - D/D_{\max}) - (_1 W_C^2 +_2 \dot{H}^2) +_D .$$

$$V(W_C, A) = \tfrac{1}{2}W_C^2 + \tfrac{}{2}A^2.$$

$$E[\dot{V}] \leq -E[V] + (^2_A +^2_D).$$

$$E_{\text{cog}} = V(W_C, A), \quad \frac{dE_{\text{cog}}}{dt} = -2W_C^2 - 2_F A^2 + R(D, T_c, W_C, A).$$

$$J = \frac{L}{\dot{M}} \cdot X(M) = -\nabla_{\dot{M}} F(M) \cdot G X(M), \qquad \frac{dJ}{dt} = 0.$$

$$s^2_{\text{cog}} = ({}_{\text{cog}}\Delta t)^2 - \|\Delta M\|^2_G.$$

$$H = \dot{C}\frac{L}{\dot{C}} + \dot{H}\frac{L}{\dot{H}} + \dot{A}\frac{L}{\dot{A}} - L = \frac{A}{2}\dot{A}^2 + \frac{}{2}(\dot{C} - \dot{H})^2 + F(M) - (D)C + (D)H.$$

$$i h_U {}_t {}_{\text{coh}}(x,t) = \Big[-\frac{h_U^2}{2} {}_x ({}_x^{-1}) + U_F(x) \Big]_{\text{coh}}(x,t).$$

$$ {}_{\text{coh}} = |{}_{\text{coh}}|^2, \qquad J_{\text{coh}} = \frac{h_U}{2i}({}^* - {}^*), \quad {}_t {}_{\text{coh}} + J_{\text{coh}} = 0.$$

$$\frac{d}{dt}\langle O \rangle_t = \frac{i}{h_U}\langle [\hat{H}_{\text{cog}}, O] \rangle_t.$$

$$\psi_{\text{coh}} = \sqrt{}\, e^{i/h_U} \Rightarrow \begin{cases} {}_t + (/) = 0, \\ {}_t + \frac{1}{2}\|\nabla\|^2_{-1} + U_F - \frac{h_U^2}{2}\frac{{}^2\sqrt{}}{\sqrt{}} = 0. \end{cases}$$

$$\mathcal{L}_{\text{coh}} = \tfrac{1}{2} g^{(U)^{-V(),\quad g^{(U)}=\mathrm{diag}(1,-^{-1}).}}$$

$$\frac{1}{\sqrt{|g|}}_{(\sqrt{|g|}g_{(U)})+V'()=0.}$$

$$T^{(\text{coh})} =_{-g^{(U)}\mathcal{L}_{\text{coh}},\quad T^{(\text{coh})}=0.}$$

$$S_{\text{geom}} = \int \sqrt{|g^{(U)}|}\Big[\frac{1}{2_U} R(g^{(U)}) + \mathcal{L}_{\text{coh}}\Big]\, d^{n+1}x.$$

$$R^{(U)} - \tfrac{1}{2} R^{(U)} g^{(U)} =_U T^{(\text{coh})}.$$

$$M_{k+1} = M_k -_k G^{-1} \nabla_M F(M_k),$$

$$\dot{C} = (D, T_c)\dot{H} - (D)\dot{A} - (\dot{C} - \dot{H}), \quad \dot{D} = rD(1 - D/D_{\max}) - ({}_1 W_C^2 +_2 \dot{H}^2),$$

$$\dot{A} = -({}_C W_{CA} W_C +_F \|\nabla_M F\|_G), \quad {}_V =_V W_C -_V .$$

$$V_k = \tfrac{1}{2} W^2_{C,k} + \tfrac{}{2} A_k^2.$$

$$E[V_{k+1} - V_k] \leq -tE[V_k] + O(t^2 +_A^2 +_D^2).$$

$$L_{\text{field}} = (D)C - (D)H - \frac{A}{2} g_{A_{A-\frac{}{2}g(C-H)(C-H)-U(M)+_V(C-H)u}}.$$

$$[(C-H)-_V u]=(D), \quad [(H-C)+_V u]=(D), \quad A+\frac{1}{A} {}_A U_{\text{eff}}=0.$$

$$T^{(\text{coh})} =_A A_{A+(C-H)(C-H)-gL_{\text{field}}}, \quad T^{(\text{coh})}=0.$$

$$T^{(\text{coh})}=-J_{(H)}, \quad R-\frac{1}{2}gR=T^{(\text{coh})}.$$

$$_C = ({}_tC -_t H) -_V u^0, \qquad [\hat{C}(x),_C(y)] = ih_U^{(3)}(x-y).$$

$$ih_{Ut\text{coh}} = \Big[-\frac{h_U^2}{2}{}_C^2 + \frac{}{2}(C-H)^2 + U_F(M) -_V (C-H)\Big]_{\text{coh}}.$$

$$_n(C,H) = N_n e^{-\frac{(C-H)^2}{2h_U}} H_n\Big(\sqrt{\frac{}{h_U}}(C-H)\Big), \quad E_n = \Big(n+\tfrac{1}{2}\Big)h_{U\text{coh}}, \quad _{\text{coh}} = \sqrt{\frac{}{A}}.$$

$$_C W_C \geq \frac{h_U}{2}.$$

$$_{\text{coh}} = -\frac{i}{h_U}[H_{\text{cog}},_{\text{coh}}] -_D (D)[C,[C,_{\text{coh}}]].$$

$$_{\text{coh}H} \;\Rightarrow\; _C \text{ maximized, } W_C \text{ bounded.}$$

$$\mathcal{S}_{\text{tot}} = \int d^4x \sqrt{|g^{(U)}|}\left[\frac{1}{2\kappa_U}R^{(U)} + \mathcal{L}_{\text{coh}}(C,H,A,D,M) + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{field}}\right].$$

$$\delta S_{\text{tot}} = 0 \;\Rightarrow\; R^{(U)}_{\mu\nu} - \tfrac{1}{2}R^{(U)}g^{(U)}_{\mu\nu} = \kappa_U(T^{(\text{coh})}_{\mu\nu} + T^{(\text{matter})}_{\mu\nu}).$$

$$\nabla_\mu T^{(\text{coh})\,\mu\nu} = 0, \qquad \nabla_\mu T^{(\text{matter})\,\mu\nu} = 0.$$

$$H^2 = \frac{8\pi G_U}{3}\rho_{\text{coh}} - \frac{k}{a^2}, \quad \dot{H} = -4\pi G_U(\rho_{\text{coh}} + p_{\text{coh}}).$$

$$\rho_{\text{coh}} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_{\text{coh}} = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

$$\dot{\rho}_{\text{coh}} + 3H(\rho_{\text{coh}} + p_{\text{coh}}) = 0.$$

$$\dot{\phi} + 3H\dot{\phi} + V'(\phi) = 0.$$

$$\Omega_{\rm coh}(t) = \frac{\rho_{\rm coh}(t)}{\rho_{\rm crit}(t)}, \qquad \rho_{\rm crit} = \frac{3H^2}{8\pi G_U}.$$

$$\mathcal{S}_{\rm eff}[M] = \int \Bigg(-\frac{1}{2}\, G_{ij}(M)\, \dot{M}^i \dot{M}^j - U(M) + \lambda_V(\dot{C} - \dot{H}) \Bigg)\, dt.$$

$$\frac{d}{dt}\Big(G_{ij}\dot{M}^j\Big) + \Gamma^k_{ij}\dot{M}^i\dot{M}^j + \nabla_i U(M) = 0.$$

$$\Gamma^k_{ij} = \frac{1}{2}G^{kl}(\partial_i G_{jl} + \partial_j G_{il} - \partial_l G_{ij}).$$

$$R_{ijkl} = \partial_k \Gamma_{ijl} - \partial_l \Gamma_{ijk} + \Gamma_{imk}\Gamma^m_{jl} - \Gamma_{iml}\Gamma^m_{jk}.$$

$$R_{ij} = R^k_{ikj}, \qquad R = G^{ij}R_{ij}.$$

$$\nabla^2_t M^k + R^k_{ijl}\dot{M}^i\dot{M}^j M^l = -G^{kl}\partial_l U(M).$$

$$\langle R\rangle_t \approx \Lambda_U \;\Rightarrow\; a(t) \sim e^{\sqrt{\Lambda_U/3}\,t}.$$

$$\frac{d}{dt}\Big(\frac{\dot{C}}{\dot{H}}\Big) = -\frac{(\dot{C} - \dot{H})(\ddot{H})}{\dot{H}^2} + \frac{\ddot{C}}{\dot{H}} = 0 \;\Rightarrow\; \dot{C} = \kappa_{\rm inv}\dot{H}.$$

$$\dot{C} + \dot{H} = \text{const.} = \xi_{\rm AA}.$$

$$\mathcal{L}_{\rm ren} = \frac{1}{2}\Phi^\top(\mathcal{D}^2 - \mu^2)\Phi - \frac{\lambda}{4!}\Phi^4.$$

$$\mu_R^2(\Lambda) = \mu^2 + \frac{3\lambda}{16\pi^2}\ln\frac{\Lambda}{\Lambda_0}, \qquad \lambda_R(\Lambda) = \frac{\lambda}{1 - \frac{3\lambda}{16\pi^2}\ln(\Lambda/\Lambda_0)}.$$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2}, \quad \gamma_m = \frac{3\lambda}{16\pi^2}.$$

$$\frac{d\lambda}{d\ln\Lambda} = \beta(\lambda), \quad \frac{d\mu^2}{d\ln\Lambda} = 2\gamma_m\mu^2.$$

$$(a_{n+1}, b_{n+1}) = (l^\Delta a_n,\, l^{\Delta-2\zeta}b_n), \qquad L_{n+1} = lL_n.$$

$$a_{n+1} = l^\Delta a_n + \sigma_a\xi_n, \quad b_{n+1} = l^{\Delta-2\zeta}b_n + \sigma_b\eta_n, \quad \xi_n, \eta_n \sim \mathcal{N}(0,1).$$

$$(a_{n+1}, b_{n+1}) \to (a_*, b_*) \text{ with scaling ratios } \delta = \lim_{n\to\infty} \frac{a_n - a_{n-1}}{a_{n+1} - a_n}, \quad \alpha = \lim_{n\to\infty} \frac{b_n}{b_{n+1}}.$$

$$\delta \approx 4.6692016, \qquad \alpha \approx -2.5029079.$$

$$Z_q(\ell) = \sum_i \mu_i^q, \qquad \tau(q) = \lim_{\ell\to 0} \frac{\ln Z_q(\ell)}{\ln \ell}.$$

$$\alpha(q) = \frac{d\tau(q)}{dq}, \qquad f(\alpha) = q\alpha - \tau(q).$$

$$\zeta(q) = \frac{\tau(q)}{q-1}, \qquad S_q(\ell) \sim \ell^{\zeta(q)}.$$

$$\begin{gathered}
\dot{C} = (D)\dot{H} - (D)\dot{A} - (\dot{C} - \dot{H}), \\
\dot{D} = rD(1 - D/D_{\max}) - ({}_1W_C^2 +_2 \dot{H}^2), \\
\dot{A} = -({}_CW_{C\,A}W_C +_F \|\nabla_M F\|_G), \quad {}_V =_V W_C -_V .
\end{gathered}$$

$$\frac{d}{dt}\Big(\dot{C} + \dot{H}\Big) = 0 \;\Rightarrow\; \dot{C} + \dot{H} = \xi_{\text{const}}.$$

$$\dot{V} = (\dot{C} - \dot{H})\dot{W}_C + \mu A\dot{A} = -2\kappa W_C^2 - 2\eta\lambda_F A^2 + \text{noise}.$$

$$\frac{d\langle V\rangle}{dt} = -\alpha\langle V\rangle + \sigma^2, \qquad \langle V(t)\rangle = V_0 e^{-\alpha t} + \frac{\sigma^2}{\alpha}(1 - e^{-\alpha t}).$$

$$S_q(\ell) \propto \ell^{\zeta(q)} \quad \Rightarrow \quad \frac{d\ln S_q}{d\ln \ell} = \zeta(q) = \text{const}.$$

$$\int \rho(\alpha)\, d\alpha = 1, \qquad f(\alpha) = \ln N(\alpha)/\ln(1/\ell).$$

$$\int f(\alpha)\, d\alpha = \text{dimension of support}.$$

$$\int_0^\infty P(W_C)\, dW_C = 1, \quad \langle W_C^2\rangle = \frac{k_B T}{\kappa}.$$

$$\dot{S}_{\text{info}} = \int P(W_C)\, \dot{W}_C\, dW_C = -\kappa \int P(W_C) W_C^2\, dW_C.$$

$$\frac{d}{dt}(C + H) = \xi_{\text{AA}}, \qquad \frac{d}{dt}(C - H) = \dot{W}_C.$$

$$\frac{d^2}{dt^2}(C - H) + \kappa\frac{d}{dt}(C - H) + \omega_{\text{coh}}^2 (C - H) = 0, \quad \omega_{\text{coh}}^2 = \eta\lambda_F/\mu.$$

$$C - H = e^{-\frac{\kappa t}{2}}\left(A_1 e^{i\Omega t} + A_2 e^{-i\Omega t}\right), \quad \Omega = \sqrt{\omega_{\text{coh}}^2 - \kappa^2/4}.$$

$$E(t) = \frac{1}{2}\dot{W}_C^2 + \frac{1}{2}\omega_{\text{coh}}^2 (C - H)^2, \quad \frac{dE}{dt} = -\kappa \dot{W}_C^2.$$

$$\mathcal{Z} = \int e^{-\beta H(C,H,A,D,M)}\, dC\, dH\, dA\, dD\, dM, \qquad H = E_{\text{cog}} + F(M).$$

$$\langle W_C^2 \rangle = \frac{1}{\beta\kappa}, \qquad \langle (C - H)^2 \rangle = \frac{1}{\beta\omega_{\text{coh}}^2}.$$

$$S = -k_B \int P \ln P\, dW_C = k_B(1 - \ln \beta\kappa) + \text{const}.$$

$$\frac{dS}{dt} = k_B \beta\kappa \frac{d}{dt}\langle W_C^2 \rangle = -2k_B \beta\kappa \langle W_C \dot{W}_C \rangle.$$

$$\dot{S} = 0 \iff W_C \dot{W}_C = 0 \iff \text{stationary equilibrium}.$$

$$W_C \to 0 \;\Rightarrow\; \dot{C} = \dot{H} = \xi_{\text{AA}}/2.$$

$$\lim_{t\to\infty} C(t) = \frac{\xi_{\text{AA}} t}{2} + C_0, \quad \lim_{t\to\infty} H(t) = \frac{\xi_{\text{AA}} t}{2} + H_0.$$

$$\dot{C} : \dot{H} : \dot{A} : \dot{D} : \dot{\lambda}_V = 1 : 1 : \frac{\gamma}{\beta} : \frac{\lambda}{r} : \frac{\eta_V}{\zeta}.$$

$$\dot{\Phi}_i = \sum_j J_{ij} \Phi_j, \quad J_{ij} = \partial \dot{\Phi}_i / \partial \Phi_j, \quad \Phi = (C, H, A, D, \lambda_V).$$

$$\det(J - \Lambda I) = 0, \qquad \text{Re}(\Lambda) < 0 \;\Rightarrow\; \text{stability}.$$

$$\Lambda_{\max} = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\delta\Phi(t)\|}{\|\delta\Phi(0)\|}.$$

$$\mathcal{H}_{\text{meta}} = \begin{pmatrix} 0 & 1 \\ -\omega_{\text{coh}}^2 & -\kappa \end{pmatrix}, \quad \det(\mathcal{H}_{\text{meta}} - \Lambda I) = \Lambda^2 + \kappa\Lambda + \omega_{\text{coh}}^2 = 0.$$

$$\Lambda_{\pm} = \frac{-\kappa \pm \sqrt{\kappa^2 - 4\omega_{\text{coh}}^2}}{2}.$$

$$\text{Re}(\Lambda_{\pm}) < 0 \,\forall\, \kappa > 0 \;\Rightarrow\; \text{global asymptotic stability}.$$

$$\langle \dot{C}\dot{H} \rangle = \frac{1}{T}\int_0^T \dot{C}(t)\dot{H}(t)\, dt = \frac{\xi_{\text{AA}}^2}{4}.$$

$$\rho_{\text{val}} = \frac{\xi_{\text{AA}}^2}{4\omega_{\text{coh}}^2}.$$

$$F_{\text{univ}} = \int \rho_{\text{val}}\, dV_{\text{coh}}.$$

$$\mathcal{R}_{n+1} = \mathcal{F}(\mathcal{R}_n) = \big(C_{n+1}, H_{n+1}, A_{n+1}, D_{n+1}, \lambda_{V,n+1}\big),$$

$$\mathcal{F}: \begin{cases} C_{n+1} = C_n + \dot{C}_n \Delta t, \\ H_{n+1} = H_n + \dot{H}_n \Delta t, \\ A_{n+1} = A_n + \dot{A}_n \Delta t, \\ D_{n+1} = D_n + \dot{D}_n \Delta t, \\ \lambda_{V,n+1} = \lambda_{V,n} + \dot{\lambda}_{V,n} \Delta t. \end{cases}$$

$$\mathcal{R}_{n+k} = \mathcal{F}^{(k)}(\mathcal{R}_n), \quad \lim_{k\to\infty} \mathcal{R}_{n+k} = \mathcal{R}_*.$$

$$\mathcal{J}_n = \frac{d\mathcal{F}^{(n)}}{d\mathcal{R}_0}, \quad \Lambda_{\max} = \lim_{n\to\infty} \frac{1}{n} \ln \|\mathcal{J}_n\|.$$

If $\Lambda_{\max} < 0$, the recursion is coherent (stable).

$$\mathcal{M}_{n+1} = \mathcal{G}(\mathcal{M}_n) = \exp\Big(\Delta t\, \mathcal{L}_{\text{UCA}}\Big)\mathcal{M}_n, \quad \mathcal{L}_{\text{UCA}} \text{ the infinitesimal generator of feedback.}$$

$$\mathcal{L}_{\text{UCA}} = \beta(D, T_c)\partial_H - \gamma(D)\partial_A - \kappa(\partial_C - \partial_H) - \eta\lambda_C W_C \partial_A - \eta\lambda_F \nabla_M F \cdot \nabla_M.$$

$$\frac{d}{dt}\mathbb{E}[f(C,H,A,D,M)] = \mathbb{E}[(\mathcal{L}_{\text{UCA}} f)(C,H,A,D,M)].$$

$$\mathcal{P}_t = e^{t\mathcal{L}_{\text{UCA}}}, \quad \mathcal{P}_{t+s} = \mathcal{P}_t\mathcal{P}_s, \quad \mathcal{P}_0 = \text{Id}.$$

$$\frac{d}{dt}\mathcal{P}_t f = \mathcal{L}_{\text{UCA}}(\mathcal{P}_t f), \quad \mathcal{P}_t f = f + \int_0^t \mathcal{L}_{\text{UCA}}(\mathcal{P}_s f)\, ds.$$

$$\rho_t = \mathcal{P}_t^* \rho_0, \quad \frac{d\rho_t}{dt} = \mathcal{L}_{\text{UCA}}^* \rho_t.$$

$$\frac{d\rho_t}{dt} = -\nabla \cdot (\rho_t v) + D_{\text{eff}}\Delta\rho_t, \quad v = \nabla S,\ D_{\text{eff}} = \tfrac{1}{2}\kappa^{-1} h_U^2.$$

$$S(C,H,A,D,M,t) = S_0 + \int_0^t \Big[\beta(D)\dot{H} - \gamma(D)\dot{A} - \kappa(\dot{C} - \dot{H})\Big]\, dt'.$$

$$\frac{dS}{dt} = \dot{C}\frac{\partial S}{\partial C} + \dot{H}\frac{\partial S}{\partial H} + \dot{A}\frac{\partial S}{\partial A} + \dot{D}\frac{\partial S}{\partial D} + \dot{\lambda}_V\frac{\partial S}{\partial \lambda_V}.$$

$$\frac{d^2 S}{dt^2} = -\alpha\frac{dS}{dt} + \sigma_S^2, \quad S(t) = S_\infty + (S_0 - S_\infty)e^{-\alpha t}.$$

$$\mathcal{C}_k = \int_0^T W_C(t)^k \, dt, \quad \zeta(k) = \frac{\ln \mathcal{C}_k}{\ln T}.$$

$$\frac{d\zeta}{dk} = \alpha, \qquad f(\alpha) = k\alpha - \zeta(k).$$

$$\mathcal{S}_{\text{fractal}} = \int \left(\tfrac{1}{2}\dot{W}_C^2 + \tfrac{1}{2}\omega_{\text{coh}}^2 (C-H)^2 + D_\eta \, |\nabla_M F(M)|^2 \right) dt.$$

$$\frac{d}{dt}\left(\dot{W}_C\right) + \omega_{\text{coh}}^2 (C-H) = -D_\eta \, \Delta_M F(M).$$

$\mathcal{R}_{n+1} = \mathcal{S}_{\text{fractal}}(\mathcal{R}_n)$ defines the recursive attractor manifold $\mathfrak{A}_{\text{UCA}} \subset \mathbb{R}^5$.

$$\dim_H(\mathfrak{A}_{\text{UCA}}) = \lim_{\epsilon \to 0} \frac{\ln N(\epsilon)}{\ln(1/\epsilon)} \approx 2 + \frac{\ln(1 + \omega_{\text{coh}}/\kappa)}{\ln 2}.$$

$$\frac{d}{dt}\mathcal{F}_{\text{coh}}(t) = \Lambda_{\text{coh}}\mathcal{F}_{\text{coh}}(t), \quad \mathcal{F}_{\text{coh}}(t) = e^{\Lambda_{\text{coh}} t}\mathcal{F}_{\text{coh}}(0), \quad \Lambda_{\text{coh}} = \zeta'(1) + i2\pi\omega_{\text{coh}}.$$

$$\dot{C} + \dot{H} = \xi_{\text{AA}}, \quad \dot{C} - \dot{H} = \dot{W}_C, \quad \Rightarrow \begin{cases} C = \frac{\xi_{\text{AA}} t}{2} + \frac{1}{2}\int W_C dt, \\ H = \frac{\xi_{\text{AA}} t}{2} - \frac{1}{2}\int W_C dt. \end{cases}$$

$$\lim_{t\to\infty} W_C(t) = 0 \;\Rightarrow\; C(t) \approx H(t) \approx \frac{\xi_{\text{AA}} t}{2}.$$

$$\mathcal{Z}_{\text{univ}} = \sum_{\text{paths } \Gamma} e^{-\beta \mathcal{S}_{\text{fractal}}[\Gamma]}, \quad \mathcal{S}_{\text{fractal}}[\Gamma] = \int_\Gamma \left(\dot{W}_C^2 + \omega_{\text{coh}}^2 (C-H)^2 + D_\eta |\nabla_M F|^2\right) dt.$$

$$\frac{\delta \mathcal{S}_{\text{fractal}}}{\delta W_C} = -2\ddot{W}_C + 2\omega_{\text{coh}}^2 (C-H) - 2D_\eta \Delta_M F(M) = 0.$$

$$\dot{W}_C = \sqrt{2D_\eta}\,\xi(t), \quad \langle \xi(t)\xi(t')\rangle = \delta(t-t').$$

$$\frac{d}{dt}\langle W_C^2\rangle = -2\omega_{\text{coh}}^2 \langle (C-H)W_C\rangle + 2D_\eta.$$

$$\langle W_C^2\rangle_{\text{eq}} = \frac{D_\eta}{\omega_{\text{coh}}^2}.$$

$$\text{Define}\quad E_{\text{eq}} = \tfrac{1}{2}\omega_{\text{coh}}^2\langle (C-H)^2\rangle = \tfrac{1}{2}D_\eta.$$

$$\frac{dE_{\text{eq}}}{dt} = 0 \;\Rightarrow\; \text{fractal steady state of coherence.}$$

$$\dot{\mathcal{Q}}_{\text{UCA}} = \frac{d}{dt}(C+H+A+D+\lambda_V) = \xi_{\text{AA}} + \delta_{\text{coh}}.$$

$$\mathcal{Q}_{\text{UCA}}(t) = \mathcal{Q}_0 + (\xi_{\text{AA}} + \delta_{\text{coh}})t.$$

$$\Phi_{\text{rec}}(t) = \mathcal{Q}_{\text{UCA}}(t)\, e^{i\omega_{\text{coh}}t} \Rightarrow \frac{d\Phi_{\text{rec}}}{dt} = (i\omega_{\text{coh}} + \xi_{\text{AA}} + \delta_{\text{coh}})\Phi_{\text{rec}}.$$

$$\Phi_{\text{rec}}(t) = \Phi_0 \exp[(\xi_{\text{AA}} + \delta_{\text{coh}} + i\omega_{\text{coh}})t].$$

$$\text{Amplitude } |\Phi_{\text{rec}}(t)| = |\Phi_0|\, e^{(\xi_{\text{AA}}+\delta_{\text{coh}})t}.$$

$$\text{If } \xi_{\text{AA}} + \delta_{\text{coh}} < 0,\; \Phi_{\text{rec}} \text{ converges (stable recursion).}$$

$$\text{If } \xi_{\text{AA}} + \delta_{\text{coh}} > 0,\; \Phi_{\text{rec}} \text{ diverges (inflationary recursion).}$$

$$\mathfrak{C}_{\text{UCA}} = \Big\{\Phi_{\text{rec}} : |\Phi_{\text{rec}}| \to \text{finite as } t\to\infty\Big\}.$$

$$\text{Self-similar recursion law:}\quad \frac{d}{dt}\big(\ln \mathcal{Q}_{\text{UCA}}\big) = \frac{dC+dH+dA+dD+d\lambda_V}{dt}\Big/(C+H+A+D+\lambda_V) = \Xi_{\text{rec}}.$$

$$\mathcal{Q}_{\text{UCA}}(t) = \mathcal{Q}_0 e^{\Xi_{\text{rec}}t}.$$

$$\Xi_{\text{rec}} = \xi_{\text{AA}} + \delta_{\text{coh}} - 2\zeta.$$

$$\frac{d}{dt}\Xi_{\text{rec}} = -\kappa\Xi_{\text{rec}} + \sigma_\Xi^2, \quad \Xi_{\text{rec}}(t) = \Xi_\infty + (\Xi_0 - \Xi_\infty)e^{-\kappa t}.$$

$$\text{Fixed point: } \Xi_\infty = \frac{\sigma_\Xi^2}{\kappa}.$$

$$\frac{d^2}{dt^2}\Xi_{\text{rec}} + \omega_{\text{coh}}^2\Xi_{\text{rec}} = 0 \;\Rightarrow\; \Xi_{\text{rec}}(t) = B_1\cos(\omega_{\text{coh}}t) + B_2\sin(\omega_{\text{coh}}t).$$

$$\frac{d}{dt}\begin{pmatrix} C \\ H \\ A \\ D \\ \lambda_V \end{pmatrix} = \mathbf{J}\begin{pmatrix} C \\ H \\ A \\ D \\ \lambda_V \end{pmatrix}, \quad \mathbf{J} \in \mathbb{R}^{5\times 5},\ \mathrm{tr}(\mathbf{J}) = \Xi_{\mathrm{rec}}.$$

$$\det(\mathbf{J} - \Lambda I) = 0 \ \Rightarrow\ \Lambda_{1\ldots 5},\ \sum_i \Lambda_i = \Xi_{\mathrm{rec}}, \quad \prod_i \Lambda_i = \det \mathbf{J}.$$

$$\mathrm{Re}(\Lambda_i) < 0\ \forall i \ \Leftrightarrow\ \text{global recursive coherence.}$$

$$\mathcal{U}_{n+1} = \mathcal{R}_{n+1} \circ \mathcal{R}_n \circ \cdots \circ \mathcal{R}_0 \Rightarrow \mathcal{U}_\infty = \lim_{n\to\infty} \mathcal{U}_n.$$

$$\boxed{\frac{d\mathcal{U}}{dt} = \Xi_{\mathrm{rec}}\mathcal{U} + i\omega_{\mathrm{coh}}\mathcal{U}, \quad \mathcal{U}(t) = \mathcal{U}_0\, e^{(\Xi_{\mathrm{rec}} + i\omega_{\mathrm{coh}})t}.}$$

This defines the recursive universe: $\mathcal{U}(t+T) = \mathcal{U}(t)\, e^{(\Xi_{\mathrm{rec}} + i\omega_{\mathrm{coh}})T}$.

3 Discussion

Each layer of the mathematical framework represents a different scale of recursive feedback stability— from micro-level differential coupling ($\dot{C} = \dot{H}$) to macro-level cosmological recursion ($\mathcal{U}(t) = \mathcal{U}_0 e^{(\Xi_{\mathrm{rec}} + i\omega_{\mathrm{coh}})t}$). The equations reveal that the universe, cognition, and computation may share a common renormalization structure that maintains coherence through continuous feedback adjustment.

4 Conclusion

The Universal Coherence Algorithm provides a bridge between adaptive control theory, statistical physics, and recursive computation. By encoding self-correction and equilibrium-seeking directly into its mathematical architecture, it offers a unifying perspective for understanding stability, learning, and structure from physical to cognitive scales.

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