Kinematics and Projectile Motion

AP Physics 1

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1 What *is* Kinematics

Kinematics is etymologically: "Motion" (Kine) and "Charaterized By" (matics). In physics we describe it as **the study of motion**.

Recall: From an analysis of Newton's three laws, we arrived upon an equation.

$$\vec{F} = m\vec{a}$$

Where we may remove the vector notation for one dimensional analysis.

2 Derivation of Constant Acceleration Equations

This is a tricky bit unfortunately, the *true* derivation requires some introductory calculus. However calculus gave us confirmation on a couple facts which aren't horribly unusual so we may use them. First let us define our quantities:

$$x \equiv Position$$

$$v = \frac{\Delta x}{\Delta t}$$

$$a = \frac{\Delta v}{\Delta t}$$

A friendly reminder is that these fractions are really just the slope of the single letter above. Now a convenient method in mathematics to handle problems is to break a complex problem into a simple case to begin. [1] A simple case would be that of a constant force, hence a constant acceleration. Let us proceed to outline a process to derive kinematic equations which will help us calculate the trajectories of objects under constant forces.

[1] This process will be helpful later.

Suppose a is a constant real number.

Proposition: A quantity is equal to the area under the graph of its rate of change.
Explanation: This sounds far fetched however when you consider how we write rate of change it comes out nicely for constant rates.

$$a = \frac{\Delta v}{\Delta t} \implies \Delta v = a\Delta t$$

If a is a constant then we get that our Δv or just v if $v_0 = 0$. Again, if our original t, t_0, is equal to 0 then: $v = at$. To arrive to the conclusion we want for our constant, a is our height, and t (or Δt) is the width; v is really just the area under the graph of a which happens to be a rectangle.

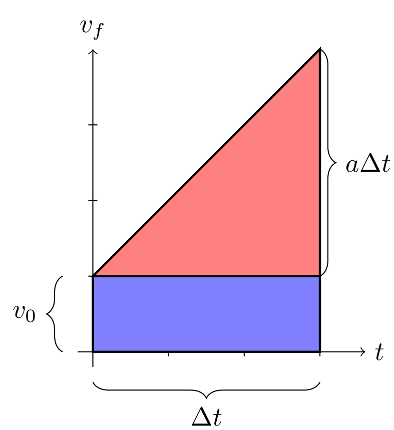
Using our proposition:

$$\Delta v = a\Delta t$$

We don't necessarily need $v_0 = 0$, so:

$$v_f = a\Delta t + v_0$$

Now our graph looks like this:



Recall: The area of a triangle is $\frac{1}{2}bh$ or in this case: $\frac{1}{2}(\Delta t)(a\Delta t)$.
Adding up the areas under our graphs we get:

$$\Delta x = \frac{1}{2}a(\Delta t)^2 + v_0\Delta t$$

Yielding:

$$x_f = \frac{1}{2}a(\Delta t)^2 + v_0\Delta t + x_0$$

Backtracking and cleaning up gives us the set of equations below, often called the kinematic equations for constant acceleration.

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$
$$v = at + v_0$$
$$a = a$$

We shall note the following properties and fact. First, to solve a system of n many equations, you need n known independent values to solve for a constant value of every variable. However if we leave time constant then we need $n - 1$ variables.

1. We have 4 variables across our two non-obvious equations.
2. To find $v(t)$ we need two variables, a and v_0, if we know x_0 we may use the area under $v(t)$'s curve to find $x(t)$

3 Process For Solving Problems:

1. What does the problem *actually* want?
2. Write what is given.
3. Write how many variables you know for each equation.
4. Work on the equation with the least amount of variables missing.
5. Can you plug one equation's solution for a single variable into the other equation?

Example:

4 Example Problem

4.1 Problem:

A car starts from rest ($v_0 = 0$) and accelerates at $a = 2$ m/s^2 for $\Delta t = 5$ s.
Find the following values:
1. Final velocity v_f
2. Displacement Δx

4.2 Solution:

Step 1: Final velocity From the constant acceleration equation:

$$v_f = v_0 + a\Delta t$$

Substitute the values:

$$v_f = 0 + 2 \cdot 5 = 10 \text{ m/s}$$

Step 2: Displacement Displacement is the area under the velocity-time graph: rectangle plus triangle.

$$\Delta x = v_0 \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$\Delta x = 0 \cdot 5 + \frac{1}{2} \cdot 2 \cdot 5^2$$

$$\Delta x = 25 \text{ m}$$

4.3 Answer:

$$v_f = 10 \text{ m/s}, \quad \Delta x = 25 \text{ m}$$

5 Problems

Problem 1: You are biking and decide to start accelerating at 0.5 m/s^2. After 20 seconds how fast will you be? Where are you position wise at 10 and 20 seconds, assuming your initial velocity and position are both equal to 0 (this is called starting at rest).

Problem 2: Repeat the example problem with an initial velocity of 4 m/s them solve for final position and final velocity at $t = 20$

Problem 3: A car starts from rest and accelerates at 4 m/s^2. It traveled a distance of 50 meters. How long did it travel for? [2]

Challenge Problem: The above problem but with an initial velocity of 6 m/s and initial position of 2 meters. Solve for how long the car traveled for.

5.1 Reflection:

Reflect on what you have figured out from the above problems? What are some effective methods for taking care of problems? Where are you lacking?

6 Multi-Dimensional Kinematics: Projectile Motion

The equations we derived for constant acceleration apply independently in each direction. This allows us to analyze complex motion by breaking it into simpler one-dimensional components. For projectile motion, the horizontal and vertical motions can be treated separately and then combined to describe the complete trajectory.

[2] Hint: You can't travel for a negative amount of time.

Inquiry Observation: Consider an object launched with an initial velocity at some angle above the horizontal. The horizontal component moves at constant velocity, while the vertical component accelerates downward due to gravity. Using the same kinematic equations from one-dimensional motion independently in each direction gives full control over predicting the motion.

Conceptual Reflection: Students should observe:

- Horizontal displacement depends only on horizontal velocity and time.
- Vertical displacement depends on gravitational acceleration, initial vertical velocity, and time.
- Combining independent components reconstructs the projectile's path.

Worked Example

A projectile is launched from ground level with an initial speed of $v_0 = 20$ m/s at a launch angle $\theta = 35°$. Determine:

1. Time of flight
2. Maximum height reached
3. Horizontal range

Step 1: Decompose the initial velocity

$$v_{0x} = v_0 \cos\theta = 20\cos(35°) \approx 16.38 \text{ m/s}$$
$$v_{0y} = v_0 \sin\theta = 20\sin(35°) \approx 11.47 \text{ m/s}$$

Step 2: Time to reach maximum height

At maximum height, $v_y = 0$.

$$0 = v_{0y} - g t_{\text{up}}$$

$$t_{\text{up}} = \frac{v_{0y}}{g} = \frac{11.47}{9.8} \approx 1.17 \text{ s}$$

Total flight time:

$$t_{\text{total}} = 2t_{\text{up}} \approx 2.34 \text{ s}$$

Step 3: Maximum height

$$h_{\max} = v_{0y} t_{\text{up}} - \frac{1}{2} g t_{\text{up}}^2$$

$$h_{\max} \approx 11.47(1.17) - \frac{1}{2}(9.8)(1.17^2) \approx 6.72 \text{ m}$$

Step 4: Horizontal range

$$R = v_{0x} \cdot t_{\text{total}} = 16.38(2.34) \approx 38.3 \text{ m}$$

$$\boxed{t_{\text{total}} \approx 2.34 \text{ s}}, \quad \boxed{h_{\max} \approx 6.72 \text{ m}}, \quad \boxed{R \approx 38.3 \text{ m}}$$

Concept Questions for Inquiry

1. A ball is thrown with an initial velocity of 10 m/s at 30° above the horizontal. Identify what must be calculated first to determine:
   (a) Time in the air
   (b) Maximum height
   (c) Horizontal distance
2. Two objects are thrown with the same initial speed: one vertically upward and one at 45°. Predict which returns to the ground first and justify your reasoning.
3. Explain how treating horizontal and vertical motion independently simplifies projectile motion. Include a brief sketch showing separate components.
4. (Challenge) A projectile is launched from a cliff of height $h = 20$ m with speed $v_0 = 15$ m/s at $\theta = 40°$.
   (a) Solve for the time it hits the ground.
   (b) Compute the horizontal distance from the base of the cliff.
   (c) Describe how velocity-time graph areas relate to this motion in each direction.