Physics C–M–2

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1 The Grand Ole Moment of Inertia

Here lie your first great integration practice. Let us remember the formula for moment of inertia.

$$\sum_i mr^2$$

This isn't continuous though! What if we have a tiny piece of mass, matter of a fact let us call it density. For a uniform line the density is $\lambda = \frac{mass}{length}$ which really means that each small piece of mass is actually $dm = \lambda dx$ where x is our tiny piece of length.

$$\sum_i r^2 dm = \sum_i \lambda r^2 dx$$

Then let there be infinite [1] dm along our finite axis:

$$\int_0^b \lambda r^2 dx$$

This is reasonable and rather an easy integral — Well atleast when lambda is a constant (challenge problem upcoming...). "b" is the length from the origin or the point you integrate out from.

2 Rationalization:

- If this process was more generalized, how would it look?
- Where do you expect to see this again?
- How will the process differ for a density which is a function of radius?

[1] The discussion of infinity is a slippery slope, in this case it means an infinity with the cardinality or "size" of the real numbers. A countable amount is where if you had infinite fingers you could count them — which would still have the size of zero...

3 Investigate:

- Suppose you have the second most general case, from a to b on the rod, and $[\lambda \in \mathbb{R}]$[2].
- Pick a function $[\lambda : \mathbb{R} \to \mathbb{R}]$ [3] such as a polynomial or real valued exponential and see what formula you get. This function is the density function.
- What would happen with a piecewise density (like the heaviside step function), something you may see in welded metal or any coupled objects of differing densities.
- Remember these problems aren't to test any wrought ability, they are special cases and general cases that I want you to investigate and discover properties of. There is little point of specifically wrought activities so I allow you to either practice on your own or get the experience wrought activity gives through just doing multiple investigations.
- Think of a problem you don't know off the top of your head, now exchange it with your peer for one they thought of. Discuss how you solved each other's problems. What did you learn?
- Could vectors work here? How would you do it? [4]

4 Parallel Axis Theorem

The center of mass of an object is special, it is the point which *minimizes* the moment of inertia. Yet an object can rotate around any axis drawn through it — consequently physicists calculated an expression for the moment of inertia of an object (on an axis which lie on the normal plane of the already calculated center of mass axis.).

$$I_{parallel} = I_c + md^2$$

- Consider one of the previous questions. Calculate the moment of inertia about the endpoint assuming the original calculation was the center of mass.
- How does your intuition interpret what the parallel axis theorem states?
- Make a challenge question for a peer and exchange questions. Discuss how you solved each other's problems.

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[2] constant real number

[3] This is notation for a function which takes a real number, and gives a real number as an output.

[4] This is a very nuanced discussion because the vectors associated with angular processes is very unintuitive and a consequence of how mathematicians define what you should consider a "perpendicular product". There is a branch of mathematics which can simplify this into objects called bi-vectors or Blades (See Clifford Algebra and Geometric Algebra).