Physics C: Center of Mass

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1 Introduction

The Center of Mass of an object is:

$$x_{com} = \frac{1}{M_{total}} \int x dm = \frac{1}{M} \int_a^b \rho dx$$

How? Recall the formula taught to you in algebra based physics:

$$\frac{1}{M} \sum_i x_i m_i$$

This is a sum for every point mass in the object, so we must sum through every mass in the object.
Let us have very tiny masses for a lot of x.

$$\sum_{i=x_0}^{x_f} x dm$$

This yields:

$$x_{com} = \frac{1}{M_{total}} \int x dm$$

Recall that $mass = density * volume = \rho v$, this implies the following line of logic:

$$m = \rho v \implies dm = \rho dv$$

In the case of a linear density function in one variable:

$$dm = \lambda dx$$

Yielding:

$$\frac{1}{M} \int_a^b \lambda dx$$

Where a and b represent bounds for which our density function is defined and our object exist.

Why? The center of mass is where you may put the torque of gravity on an object.
Recall the parallel axis theorem: $I = I_c + md^2$
In order to utilize it you need your moment of inertia about your center of mass!
By definition too, your center of mass is the point in which your torques from gravity are balanced too.

What isn't right? While the equations in this paper are correct, the way I formulated it was for understanding sake. Derive the proper Riemann sum and its bounds based on the center of mass formula. [1]

[1] Hint: You can exchange into intermediary forms seen here to check your progress.