MIT 6.034 Artificial Intelligence

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Contents

1 Introduction

1.1 Definition

Algorithms enabled by constraints exposed by representations that support models targeted at thinking, perception, and action.

Artificial Intelligence is applied through problem solving procedures, methods, techniques and algorithms.

1.2 Basic Idea

1.2.1 Generate-And-Test Algorithm

Generate-and-test search algorithm is a very simple algorithm that guarantees to find a solution if done systematically and there exists a solution.

Algorithms:

1. Generate a possible solution.
2. Test to see if this is the expected solution.
3. If the solution has been found quit, else go to step 1.

This approach involved building generators with certain properties: not redundant (should not give the same solution twice), they should also be informable (able to select a category and disregard other)

1.2.2 Rumpelstiltsin Principle

Being able to name what you're talking about gives you power over it, to understand and solve problems. Naming things grants power over concepts.

1.2.3 Simple $\neq$ Trival

Trivial ideas implies that they are worthless, useless. In AI, the most simple ideas are often the most powerful.

2 Reasoning: Goal Trees and Problem Solving

2.1 Question

$$\int \frac{-5x^4}{(1-x^2)^{\frac{5}{2}}} dx$$

Is the program, which can solve this integral problem, in any sense of the word **INTELLIGENT**?

2.2 Model

2.2.1 Problem Rediction or AND-OR Tree or Goal Tree

An and–or tree is a graphical representation of the reduction of problems (or goals) to conjunctions and disjunctions of subproblems (or subgoals).

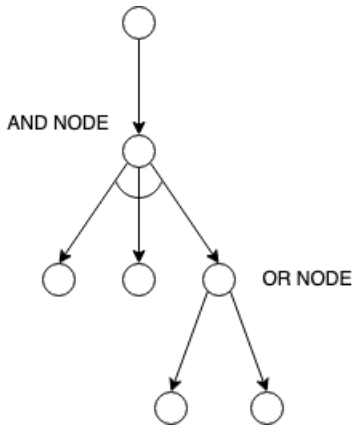


Figure 1: AND-OR Tree

2.2.2 Architecture

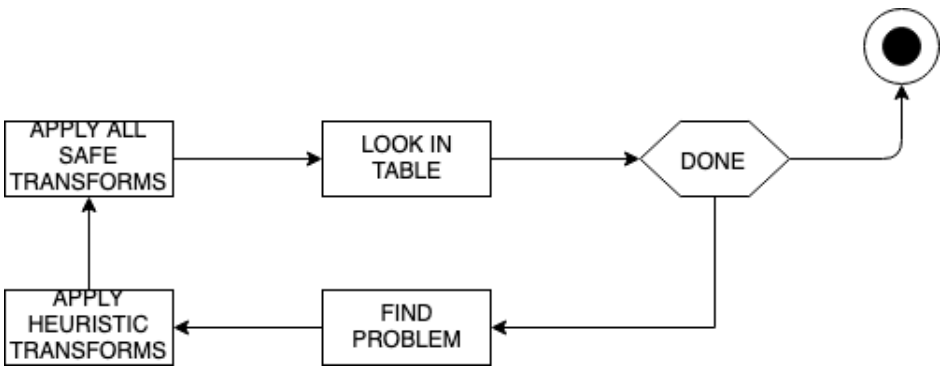


Figure 2: Problem Solving Architecture

2.3 Transformation

2.3.1 Safe Transformation

1. $\int -f(x)dx = -\int f(x)dx$
2. $\int cf(x)dx = c\int f(x)dx$
3. $\int \sum f_i(x)dx = \sum \int f_i(x)dx$
4. $\int \frac{P(x)}{Q(x)} \to DIVIDE$

2.3.2 Heuristic Transformation

A $f(\sin x, \cos x, \tan x, \cot x, \sec x, \csc x) \to g_1(\sin x, \cos x)\ or\ g_2(\tan x, \cot x)\ or\ g_3(\sec x, \csc x)$

B $\int f(\tan x)dx = \int \frac{f(y)}{1+y^2}dy$

C $\begin{cases} 1-x^2 \to x = \sin y \\ 1+x^2 \to x = \tan y \end{cases}$

2.3.3 Procedure

$$\begin{aligned}
\int \frac{-5x^4}{(1-x^2)^{\frac{5}{2}}}dx &\xrightarrow{1} \int \frac{5x^4}{(1-x^2)^{\frac{5}{2}}}dx \\
&\xrightarrow{2} \int \frac{x^4}{(1-x^2)^{\frac{5}{2}}}dx \\
&\xrightarrow{C} \int \frac{\sin^4 y}{\cos^4 y}dy \\
&\xrightarrow{A} \begin{cases} \int \frac{1}{\cot^4 s}dx \\ \int \tan^4 s dx \ \ (Choose) \end{cases} \\
&\xrightarrow{B} \int \frac{y^4}{1+y^2}dy \\
&\xrightarrow{4} \int y^2 - 1 + \frac{1}{1+y^2}dy \\
&\xrightarrow{3} \int y^2 dy + \int -1dy + \int \frac{1}{1+y^2}dy
\end{aligned}$$

$$\int -1dy \xrightarrow{1} \int dy \ \ and \ \ \int \frac{1}{1+y^2}dy \xrightarrow{C} \int dz$$

2.4 Reflection

KNOWLEDGE ABOUT KNOWLEDGE IS POWER.
When we understand how something works, intelligence seems to vanish

2.5 Questions about the nature of knowledge

1. WHAT kind of knowledge is involved?
2. HOW is the knowledge represented?
3. HOW is it used?
4. HOW much knowledge is required?
   Table of integrals: 26 elements.
   Safe transformations: 12.
   Heuristic transformations: 12.
   Then, the integral program is done by **JAMES SLAGLE**
5. WHAT exactly?

3 Reasoning: Goal Trees and Rule-Based Expert Systems

3.1 Goal Centered Programming

3.1.1 Goal

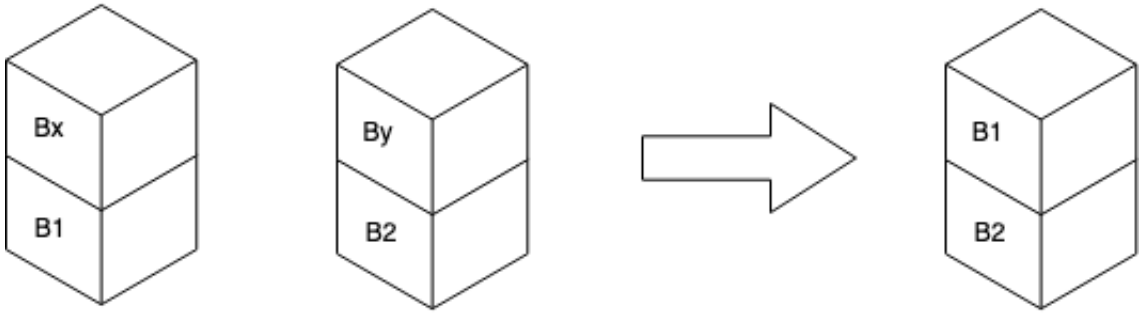


Figure 3: Put B_1 on B_2

3.1.2 Function

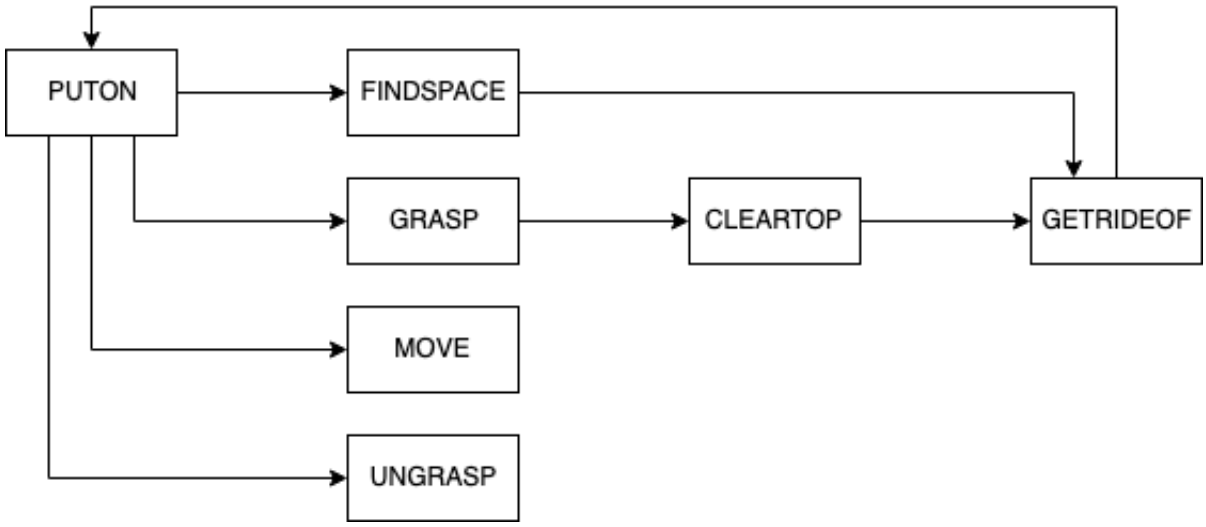


Figure 4: Related functions

3.1.3 Procedure

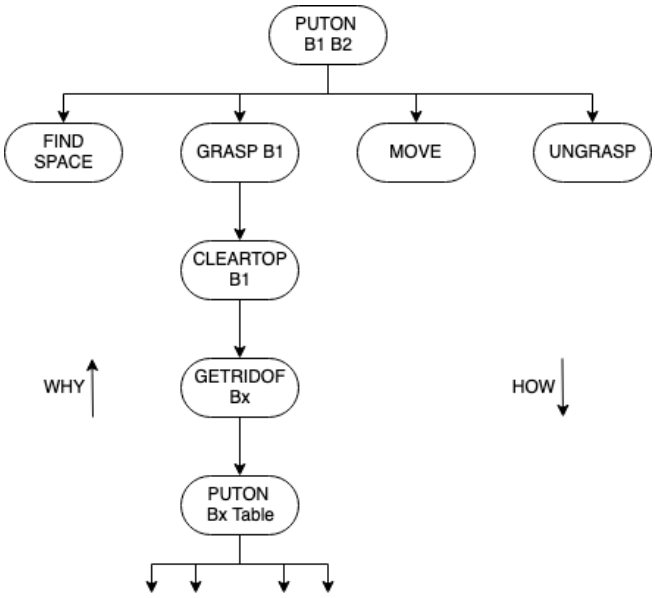


Figure 5: Related functions

For achieve their **GOALS**, the program leaves a **TRACE**, which is a **GOAL TREE**.

Then the program can answer questions about its own behavior as long as it builds an and/or tree.

Simon's ant: The complexity of the behavior is the max of the complexity of program and the complexity of environment.

3.2 Rule-Based Expert System

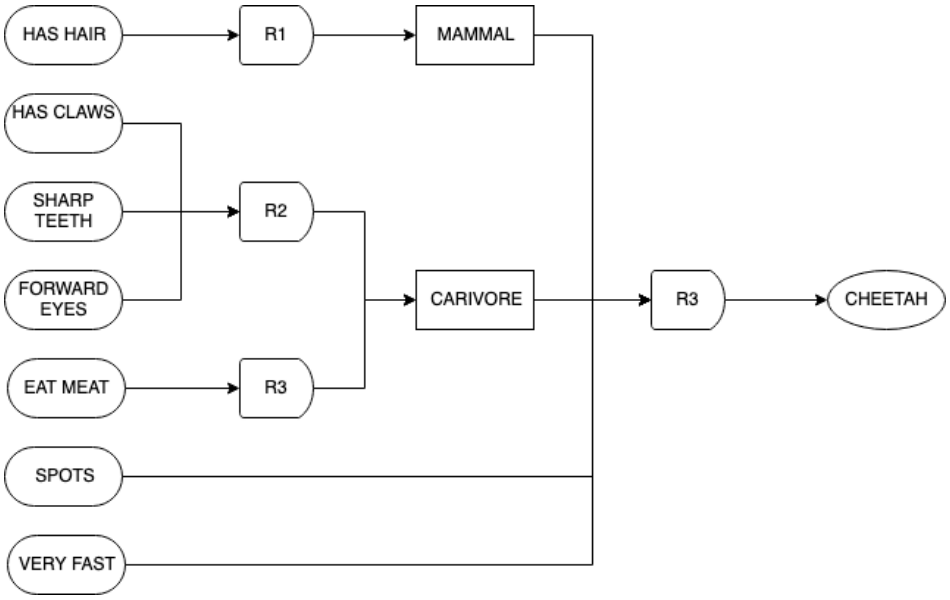


Figure 6: Forward Chaining Rule-Based Expert System

4 Search: Depth-First, Hill Climbing, Beam

4.1 Question

Find a path from S to G.

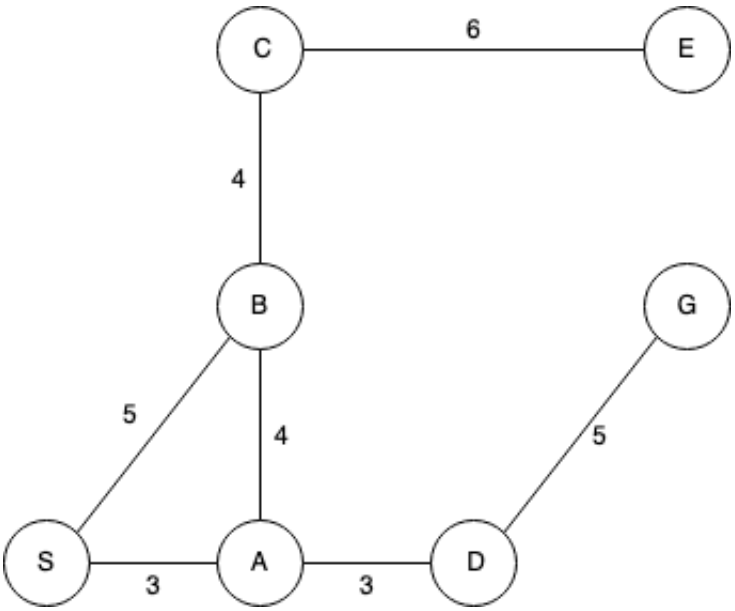


Figure 7: Map

4.2 British Museum Approach–find every possible path

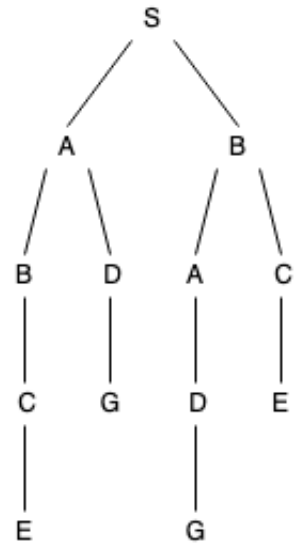


Figure 8: British Museum Algorithm

4.3 Depth First Search/Breadth First Search

e,g: Depth First Search

(S, A) (S, B)
(S, A, B) (S, A, D) (S, B)
(S, A, B, C) (S, A, D) (S, B)
(S, A, B, C, E) (S, A, D) (S, B)
(S, A, D, G) (S, B)

4.4 Hill Climbing

just like depth first search instead of using lexical order to break ties, we're going to break ties according to which node is closer to the goal.

PROBLEM

1. Local
2. Telephone Pole Problem
3. Ridge

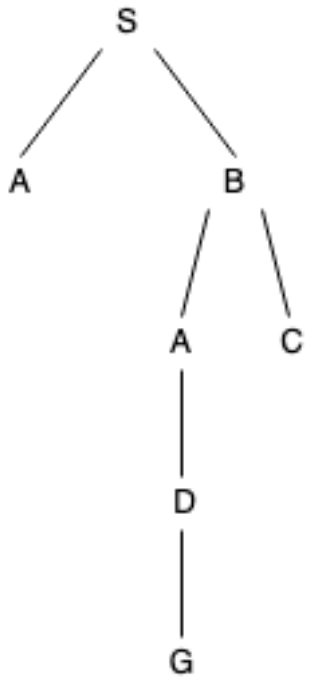


Figure 9: Hill Climbing

4.5 Beam Search

complement, addition of and informing heuristic to breadth first search. Limit the number of paths to be considered at any level to some small fixed number.

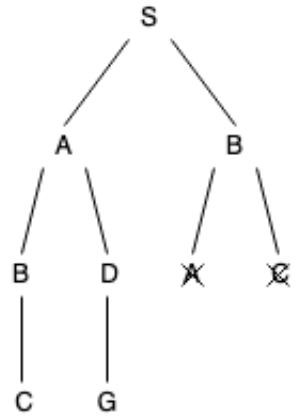


Figure 10: Beam Search (w=2)

4.6 Procedure

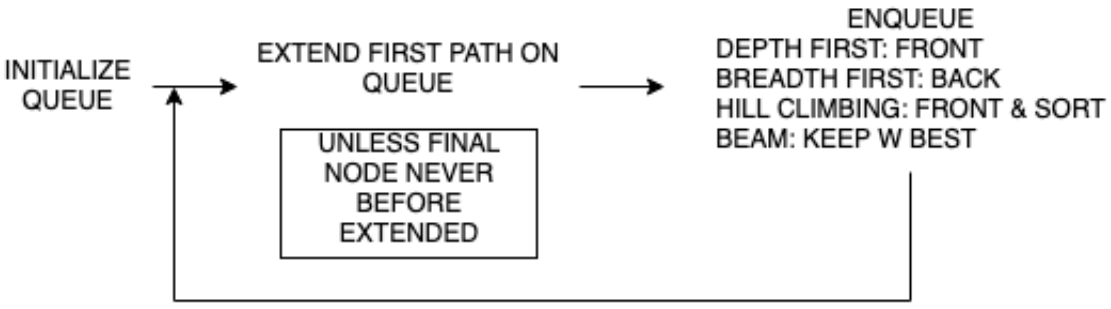


Figure 11: Procedure

4.7 Property

Method	BACKTRACKING	USE ENQEUE LIST	INFORMED
BRITISH MUSEUM	X	X	X
DEPTH FIRST	V	V	X
BREADTH FIRST	X	V	X
HILL CLIMBING	V	V	V
BEAM	V	V	V

5 Search: Optimal, Branch and Bound, A*

5.1 Question: find the best path

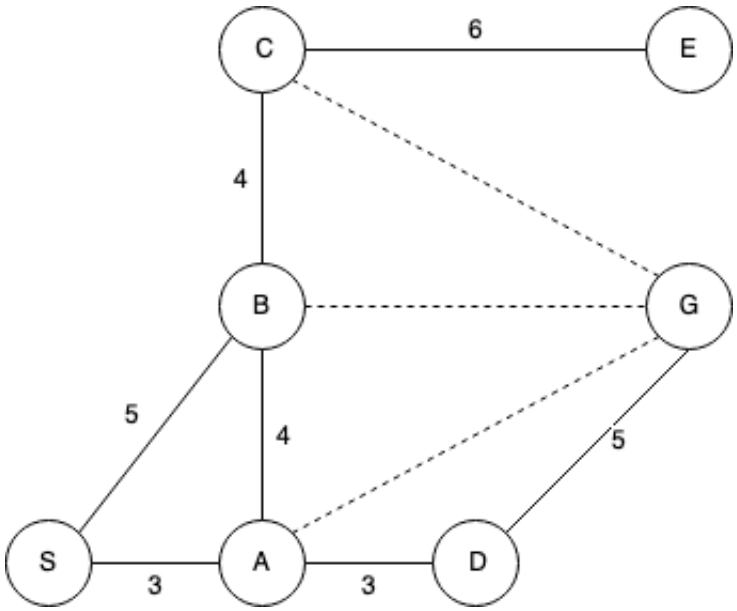


Figure 12: Map with actual distance & airline distance

5.2 Branch & Bound

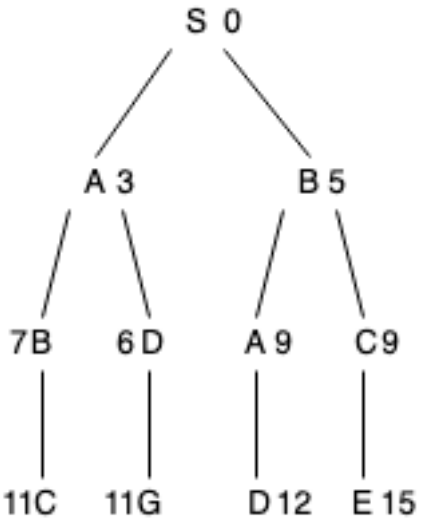


Figure 13: Branch & Bound

5.3 with Extended List

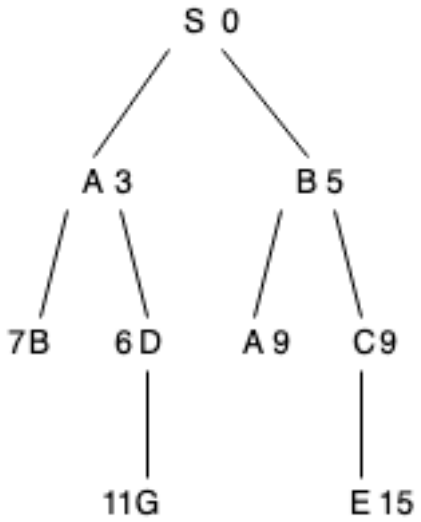


Figure 14: Branch & Bound + Admissible Heuristic

5.4 with Admissible Heuristic

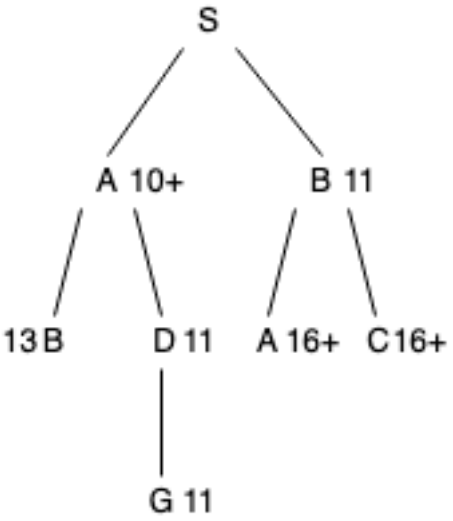


Figure 15: Branch & Bound + Admissible Heuristic

5.5 A*

Figure 16: Flow Chart

ADMISSIBLE: $H(X, G) \leq D(X, G)$
CONSISTENCE: $|H(X, G) - H(Y, G)) \leq D(X, Y)$
'Extended' in this part is the pure pass through. Not same as **Dijkstra's Algorithm**, so **A*** may fail at **non-Euclidean** arrangements

6 Search: Games, Minimax, and Alpha-Beta

6.1 Ways to Play

1. **Analysis** + **Strategy** + **Tactics** → **Move**
2. **If** · **Then** Rules
3. Look Ahead + Evaluate

$$\begin{aligned} S =& g(f_1, f_2, \cdots, f_n) \\ =& c_1 f_1 + c_2 f_2 + \cdots + c_n f_n \end{aligned}$$

4. **British Museum Algorithm**
5. *Look Ahead as far as possible

6.2 Minimax Algorithm

Minimax is a kind of backtracking algorithm that is used in decision making and game theory to find the optimal move for a player.

In Minimax the two players are called maximizer and minimizer. The maximizer tries to get the highest score possible while the minimizer tries to do the opposite and get the lowest score possible.

Every board state has a value associated with it. In a given state if the maximizer has upper hand then, the score of the board will tend to be some positive value. If the minimizer has the upper hand in that board state then it will tend to be some negative value. The values of the board are calculated by some heuristics which are unique for every type of game.

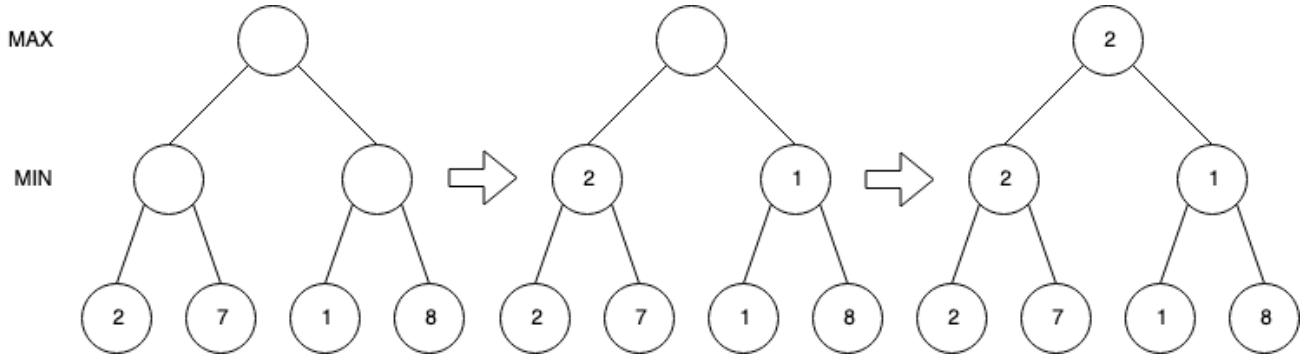


Figure 17: Minimax Algorithm

6.3 Alpha-Beta Pruning

Alpha-Beta pruning is not actually a new algorithm, rather an optimization technique for minimax algorithm. It cuts off branches in the game tree which need not be searched because there already exists a better move available.

Alpha is the best value that the maximizer currently can guarantee at that level or above.

Beta is the best value that the minimizer currently can guarantee at that level or above.

Example 1

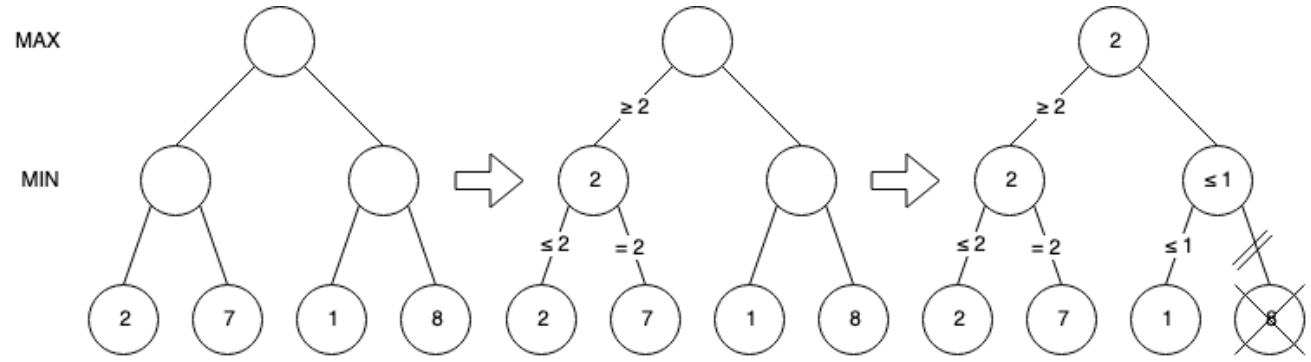


Figure 18: Alpha-Beta Pruning Algorithm

Example 2

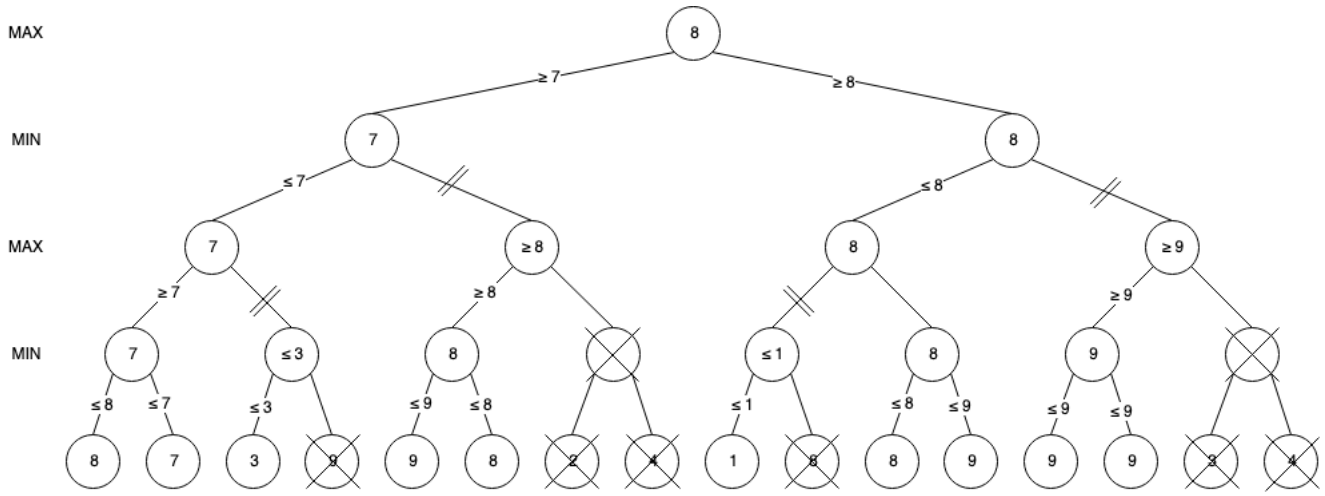


Figure 19: Alpha-Beta Pruning Algorithm

6.4 Progressive Deepening–Anytime Algorithm

Start from the very first level,give an insurance policy for every level we try to calculate

$$C = 1 + b + \cdots + b^{d-1} = \frac{1 - b^d}{1 - b} = \frac{b^d - 1}{b - 1} \approx b^{d-1}$$

6.5 Deep Blue

DEEP BLUE = Minimax + $\alpha - \beta$ + Progressive Deepening + Parallel Computing + Opening Book + Special-purpose Stuff for the End Game + **Uneven Tree Development**

7 Constraints: Interpreting Line Drawings

Object: Determine how many objects are in a line drawing?

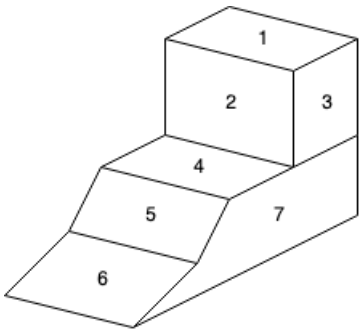


Figure 20: Line Drawing

7.1 Guzman's Solution

Two perspectives

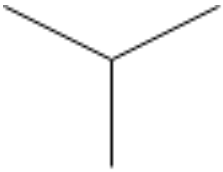


Figure 21: Fork type junction

Figure 22: Arrow type junction

1. Fork type junctions: three pairs of faces seem to belong to the same object.
2. Arrow type junctions: faces on either side of she shaft belong to the same object.

Transform the line drawing to a GRAPH

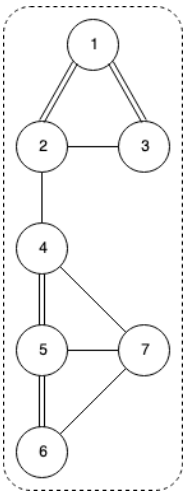


Figure 23: 1 LINK

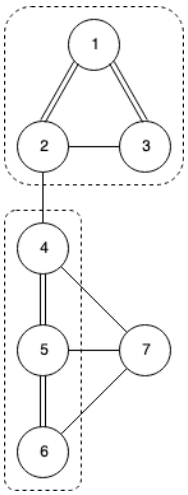


Figure 24: 2 LINK

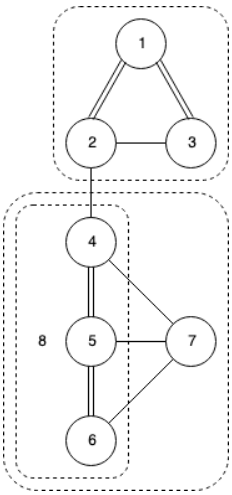


Figure 25: 2 LINK*

7.2 Dave Huffman's Solution

Construct a mathematical model (Three Assumptions)

1. General Position

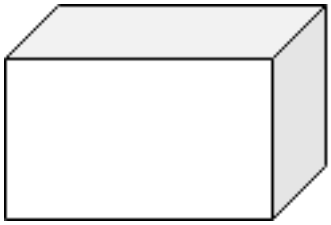


Figure 26: Nice case

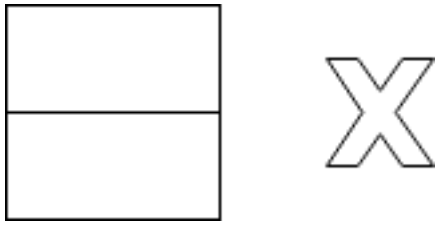


Figure 27: Weird case

2. Trihedral: all vertexes out there are going to be formed from three planes.
3. Four kinds of symbols

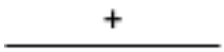


Figure 28: Convex

Figure 29: Concave

Figure 30: Boundary

Figure 31: Boundary

***Boundary**: which side you would see the object if you walk along the direction of this arrow

All possible ways that junctions can have line labels arranged around them

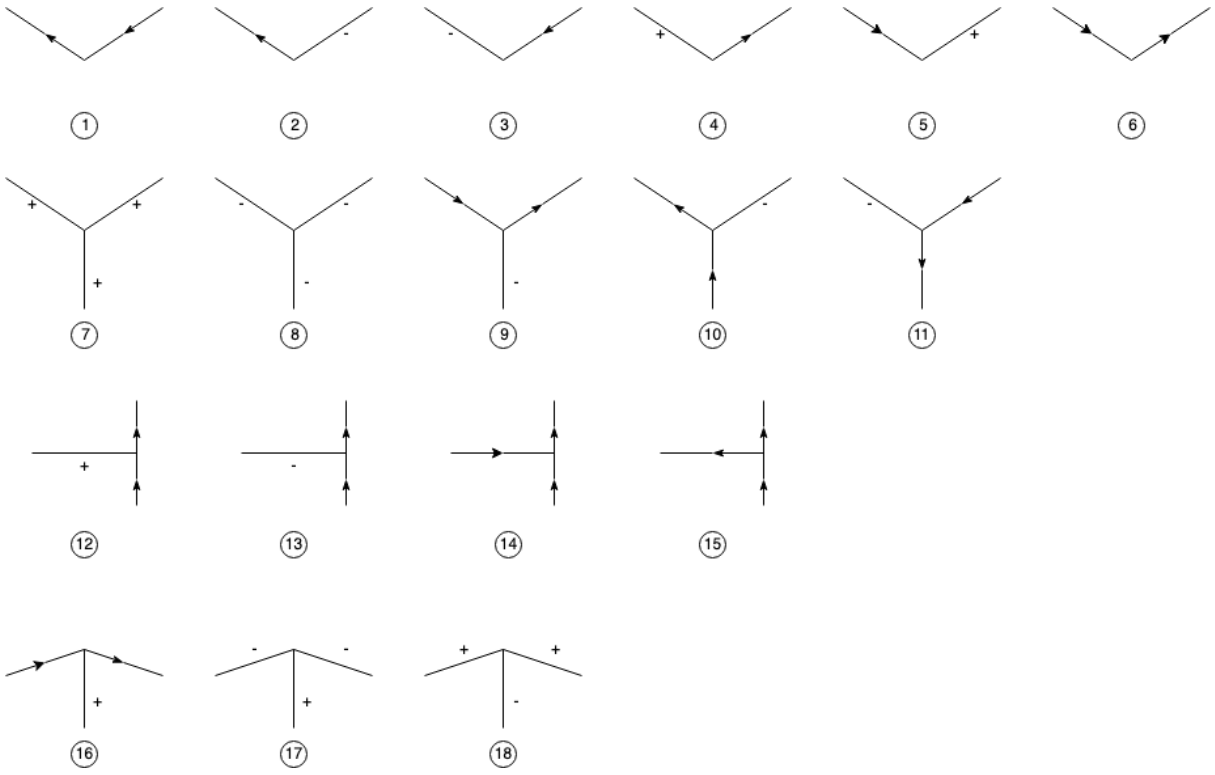


Figure 32: All kinds of junctions

7.3 David Waltz's Solution

+ CRACKS
+ SHADOWS
+ NON-TRIHEDRAL VERTEXES
+ LIGHT
Then, 4 labels $\Longrightarrow$ 50^{+} labels, 18 junctions $\Longrightarrow$ $1000's$ junctions.

Waltz's Algorithms (use Huffman's set to demonstrate Waltz's algorithm)

8 Constraints: Search, Domain Reduction

Object: Graph Coloring

8.1 Term

1. **Variable** v: something that can have assignment
2. **Value** x: something can be an assignment
3. **Domain** D: bag of value
4. **Constrain** C: limit on variable values

8.2 Procedure–Domain Reduction Algorithm

for each DFS assignment **do**
  for each variable v_i considered **do**
    for each x_i in D_i **do**
      for each constraint $C(x_i, x_j)$ where $x_j \in D_j$ **do**
        if $\nexists\ x_j \ni C(x_i, x_j)$ satisfied **then**
          remove x_i from D_i
        end if
        if D_i is empty **then**
          backup
        end if
      end for
    end for
  end for
end for

8.3 Consider

1. Nothing
2. Assignment
3. Check Neighbors
4. **Propagate** Checking v with D Reduced to 1 Value
5. **Propagate** Checking through v with Reduced D
6. Everything

9 Constraints: Visual Object Recognition

9.1 David Marr's idea–JUST A IDEA

IDEA: Based on the idea that you start off by looking at edges and you end up in several steps of transformation, producing something that you can look up in a library of descriptions.

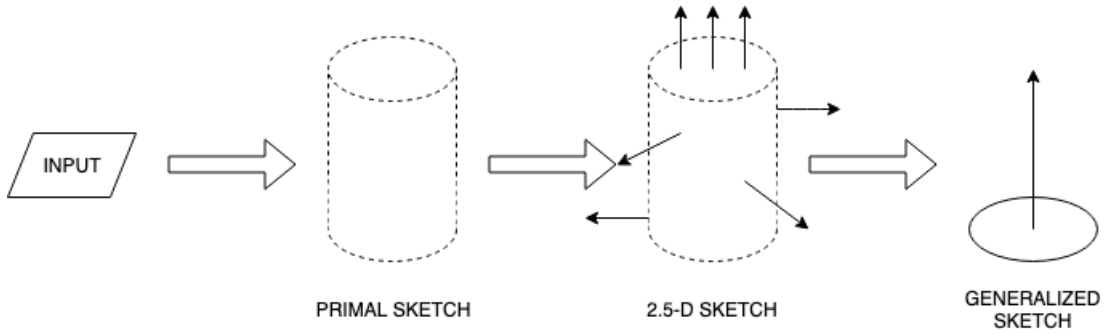


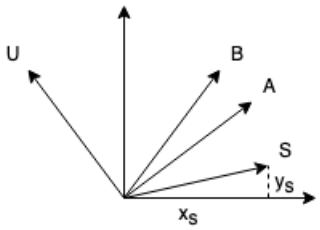
Figure 33: Marr's idea

1. Input from camera
2. Form this edge based description of what's out there in the world
3. Decorate the primal sketch with some vectors, some surface normals, showing where the faces on the object are oriented
4. Transform this 2.5 dimensional sketch to generalized sketch (function+direction)
5. Match against the library of such descriptions, and results in recognition

9.2 Shimon Ullman–Alignment Theory

IDEA: Assume we have a transparent object where all the vertexes are visible, and neglect the effect of perspectives, then we can reconstruct any view of that object.

Θ_A, Θ_B and Θ_U are corresponding angles between $\vec{S}$ and $\vec{A}$, $\vec{B}$, $\vec{U}$. If we only consider 2D rotation, we need two picture as follows.



$$
\begin{aligned}
x_A &= x_S \cos\Theta_A - y_S \sin\Theta_A \\
x_B &= x_S \cos\Theta_B - y_S \sin\Theta_B \\
x_U &= x_S \cos\Theta_U - y_S \sin\Theta_U \\
\Longrightarrow x_U &= \alpha x_A + \beta x_B
\end{aligned}
$$

Figure 34: Example

If include translation element, we need add a constant item τ.

9.3 Correlation Theory

$$\max_{X,Y} \int f(x,y)g(x-X,y-Y)$$

To some extends, it's like convolution. However, this course was opened in 2010, but CNN was presented in 2012, so there is not much explanation here.

10 Introduction to Learning, Nearest Neighbors

10.1 Types of Learning

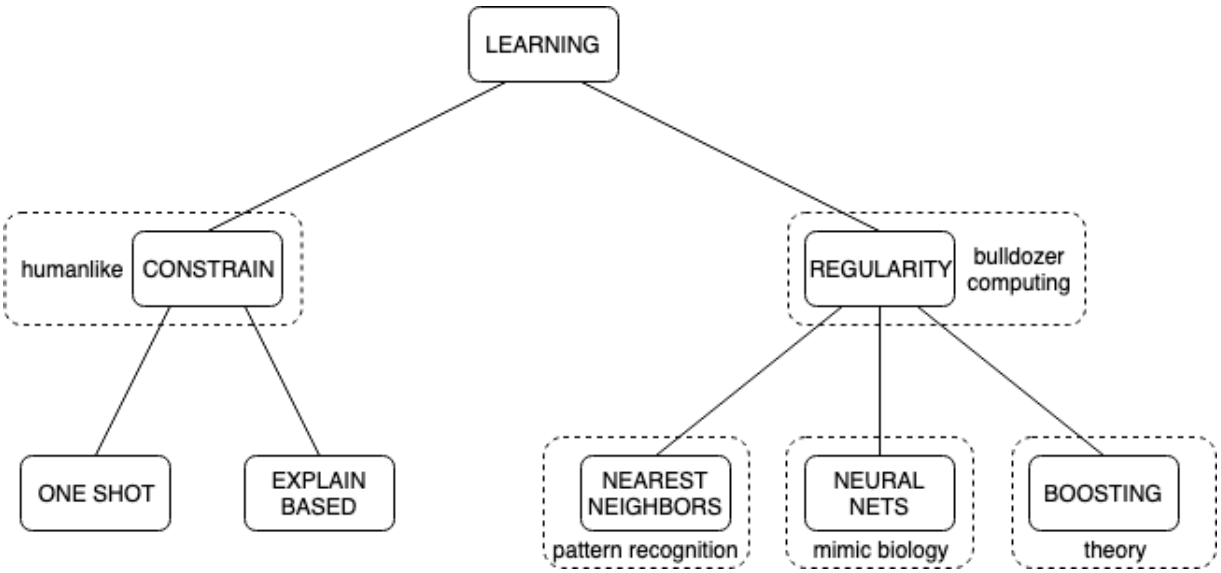


Figure 35: Learning types

10.2 Mechanism of Pattern Recognition

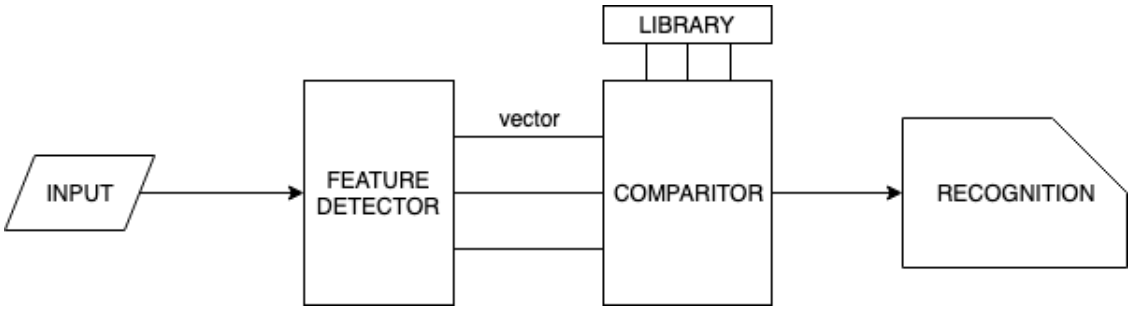


Figure 36: Mechanism of Pattern Recognition

Use Different Distance Metric to Generate Decision Boundary

10.3 Learning

Why LEARNING?

11 Learning: Identification Trees, Disorder

11.1 Target

Use data to build a recognition mechanism

Vampire	Shadow	Garlic	Complexion	Accent
No	?	Yes	PALE	None
No	Yes	Yes	Ruddy	None
Yes	?	No	Ruddy	None
Yes	No	No	Average	Heavy
Yes	?	No	Average	Odd
No	Yes	No	Pale	Heavy
No	Yes	No	Average	Heavy
No	?	Yes	Ruddy	Odd

11.2 Difference between the Dataset of Identification Tree and Nearest Neighbor

1. Some features are non-numeric
2. Some features don't matter
3. Some features only some of time
4. Some tests Cost
5. *Identification tree is talking in terms of **tests**, not a **vector of real values**

11.3 Procedure——Occam's Razor

11.3.1 Naive Strategy

Based on the number of sample individuals are put into a homogeneous set.

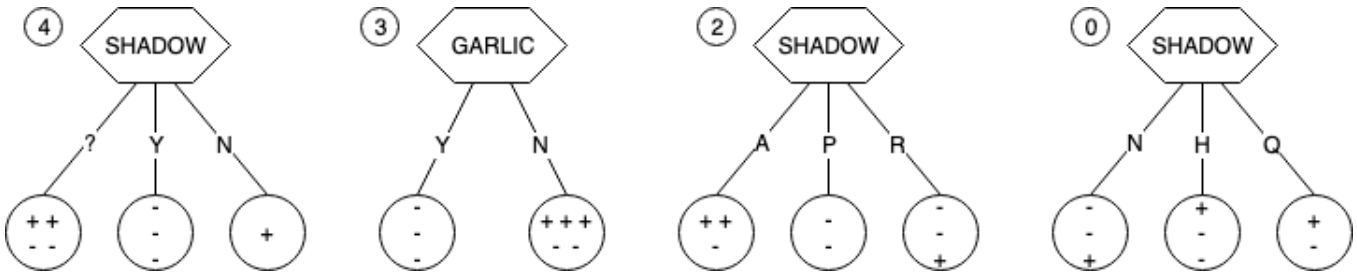


Figure 37: First test

Vampire	Shadow	Garlic	Complexion	Accent
No	?	Yes	PALE	None
Yes	?	No	Ruddy	None
Yes	?	No	Average	Odd
No	?	Yes	Ruddy	Odd

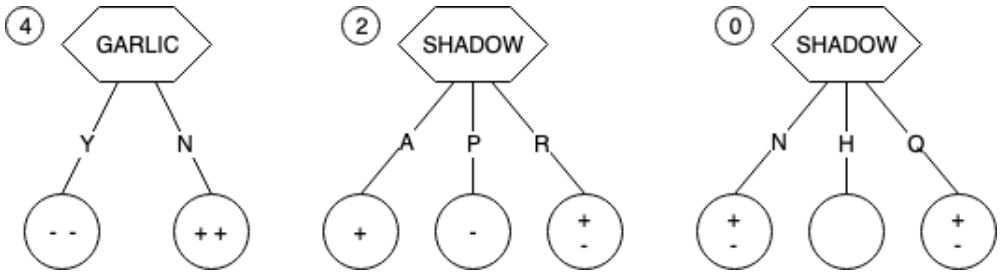


Figure 38: Second test

Then, we can construct a identification tree

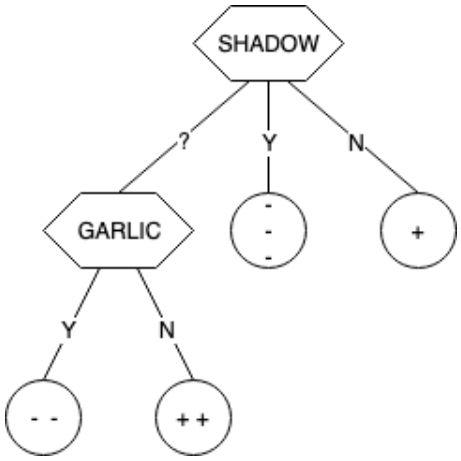


Figure 39: Identification tree

11.3.2 Large Dataset

Evaluate the disorder in sets.

$$D(set) = -\frac{P}{T}\log_2\frac{P}{T} - \frac{N}{T}\log_2\frac{N}{T}$$

$$Q(test) = \sum_{setsproduced} D(set)\frac{\#of\ samples\ in\ set}{\#of\ samples\ handled\ in\ test}$$

Then we will get the same answer as shown in Figure 39

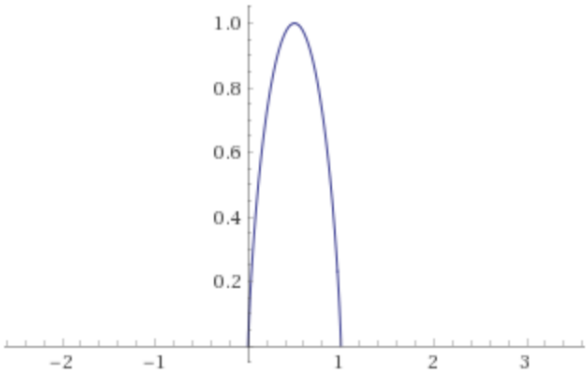


Figure 40: The relationship between $D(set)$ and $\frac{P}{T}$

11.4 Decision Boundary

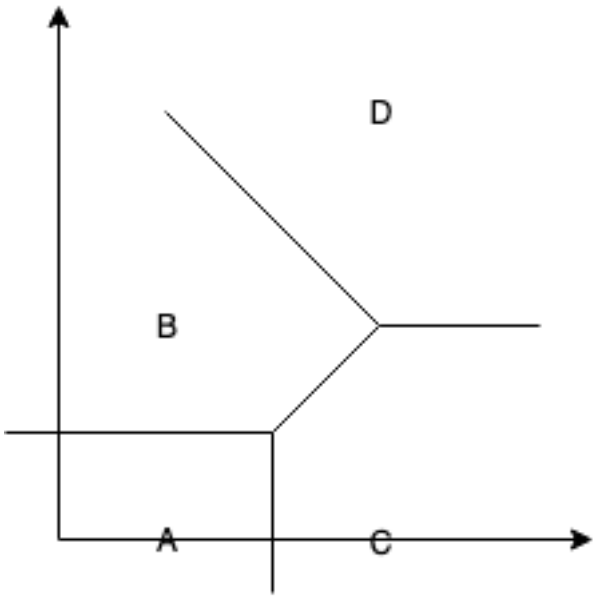


Figure 41: Nearest neighbors

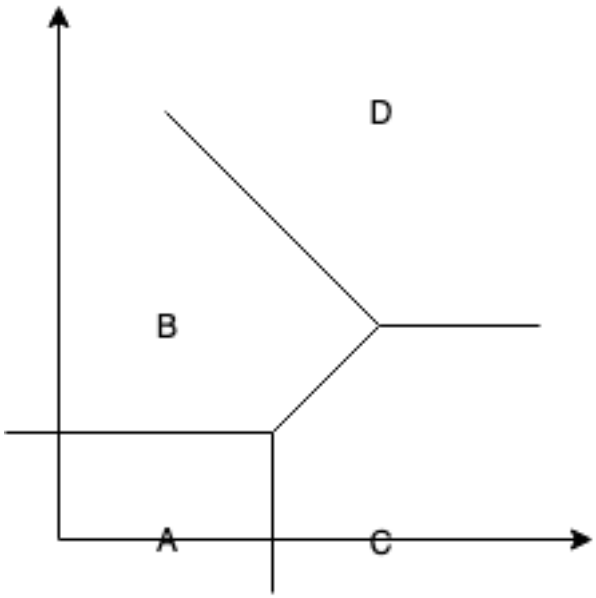


Figure 42: Identification tree

11.5 Tree to Rule

Go down each branch to the leaf. Sometimes, we can simplify the description.

12 Neural Nets

12.1 Model Real Neuron

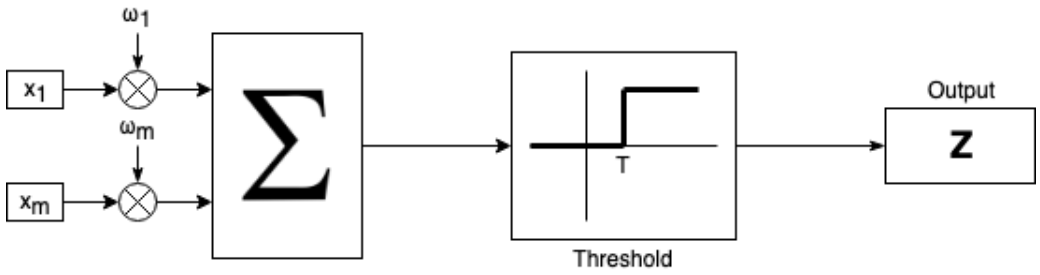


Figure 43: Neural Model

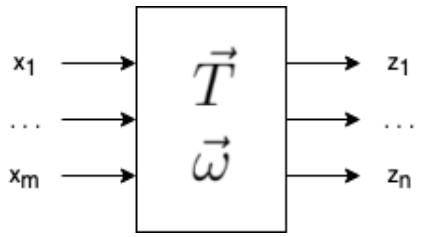
INCLUDE

1. All or none
2. Cumulative influence
3. Synaptic weight

EXCLUDE

1. Refractory period
2. Axonal bifurcation
3. Time patterns

12.2 Training



$$\vec{Z} = f(\vec{x},\ \vec{\omega},\ \vec{T})$$

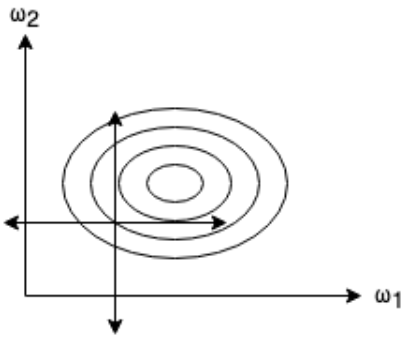
Figure 44: Neural Model

Training a neural net is adjusting these weights and thresholds.

$$Neural\ Net \implies Function\ Approximator$$

desired: $\vec{d} = g(\vec{X})$
performance: $P = -\frac{1}{2}||\vec{d} - \vec{a}||^2$ (define for mathematical convenient)

12.3 Gradient Descent



$$\Delta\vec{\omega} = r(\frac{\partial P}{\partial \omega_1}i + \frac{\partial P}{\partial \omega_2}j)$$

Figure 45: Gradient Descent

But we cannot use gradient descent/ascent–Discontinuous for our function.

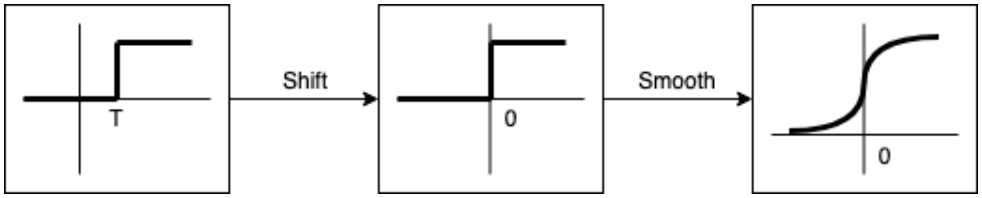


Figure 46: Change threshold function

12.4

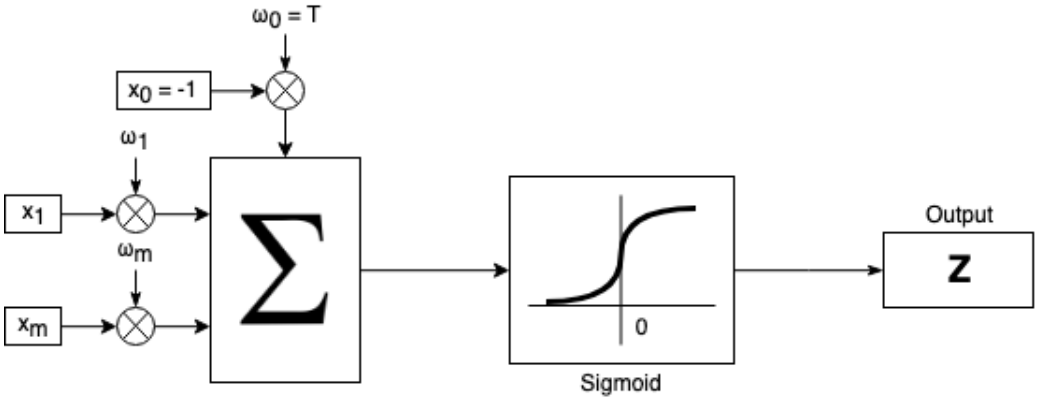


Figure 47: New Neural Model

For sigmoid function $\beta = \frac{1}{1+e^{-\alpha}}$:

$$\begin{aligned}\frac{d\beta}{d\alpha} =& \frac{d}{d\alpha}(1+e^{-\alpha})^{-1} \\ =& e^{-\alpha}(1+e^{-\alpha})^{-2} \\ =& \frac{e^{-\alpha}}{1+e^{-\alpha}} \cdot \frac{1}{1+e^{-\alpha}} \\ =& \beta(1-\beta)\end{aligned}$$

12.5 Backpropagation Algorithm

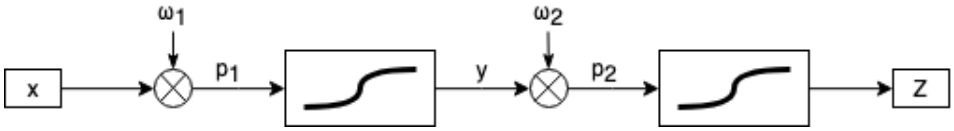


Figure 48: Sample Neural Net

$$\frac{\partial P}{\partial \omega_2} = \frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial \omega_2}$$

$$\frac{\partial P}{\partial \omega_1} = \frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial y} \cdot \frac{\partial y}{\partial p_1} \cdot \frac{\partial p_1}{\partial \omega_1}$$

$$\frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial \omega_2} = y \cdot z(1-z) \cdot (d-z)$$

$$\frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial y} \cdot \frac{\partial y}{\partial p_1} \cdot \frac{\partial p_1}{\partial \omega_1} = x \cdot y(1-y) \cdot \omega_2 \cdot z(1-z) \cdot (d-z)$$

13 Learning: Genetic Algorithms

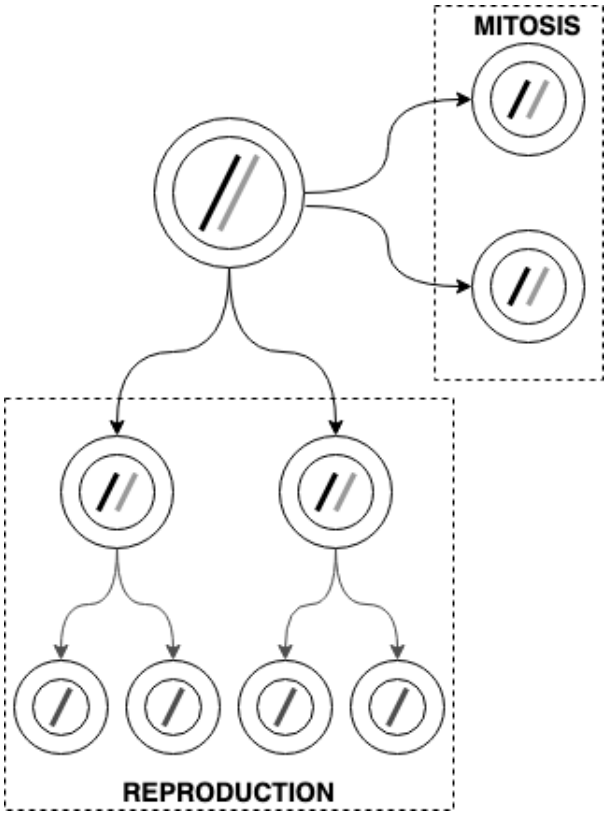


Figure 49: Genetic Biology

13.1 Imitation (ATCG $\longrightarrow$ 01)

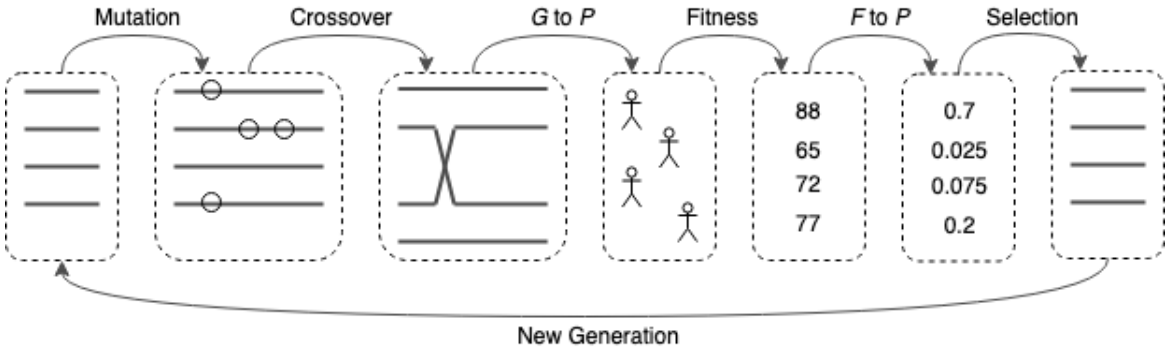


Figure 50: Genetic Algorithm

G: Genotype
P: Phenotype
F: Fitness
P: Probability

13.2 Mechanism

13.2.1 Fitness

$$P_i = \frac{f_i}{\sum_j f_j}$$

$$f(x, y) = (\sin \omega x)^2 (\sin \omega y)^2 e^{-\frac{x+y}{\sigma}}$$

13.2.2 Rank Space

$$\begin{aligned} P_1 =& P_c \\ P_2 =& (1 - P_c) P_c \\ & \cdots \\ P_{n-1} =& (1 - P_c)^{n-2} P_c \\ P_n =& (1 - P_c)^{n-1} \end{aligned}$$

13.2.3 Diversity

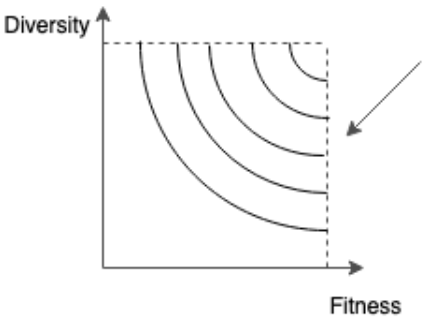


Figure 51: Genetic Biology

14 Learning: Sparse Spaces, Phonology

14.1 Point

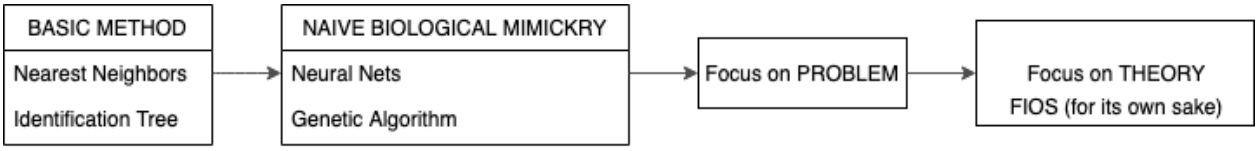


Figure 52: Learning Process

14.2 Phonological Rules

14.2.1 Distinctive features Theory

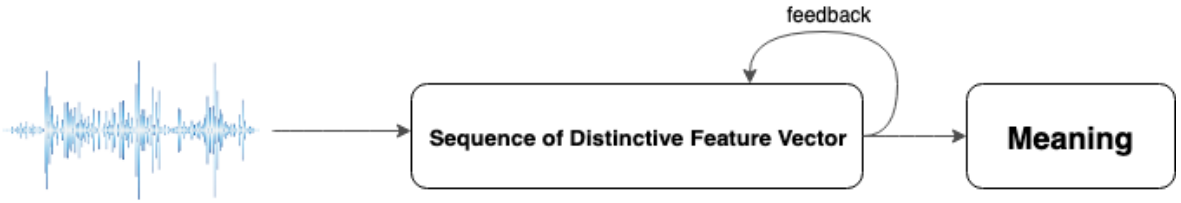


Figure 53: Distinctive features

14.2.2 YIP-SUSSMAN Machine/Learner

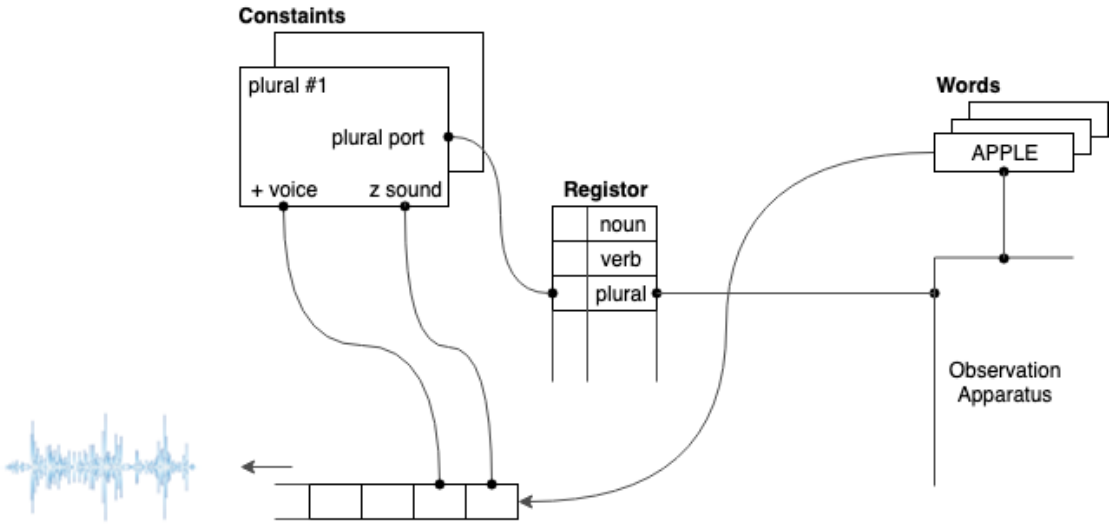


Figure 54: YIP-SUSSMAN Machine

/æ/	/p/	/l/	/z/	
+	−	−	−	syllabic
...	...	...	...	...
+	−	+	+	voiced
+	−	+	−	continuous
...	...	...	...	...
−	−	−	+	strided

Figure 55: Four distinctive features of "APPLES"

If we present this machine with a pair of apples.

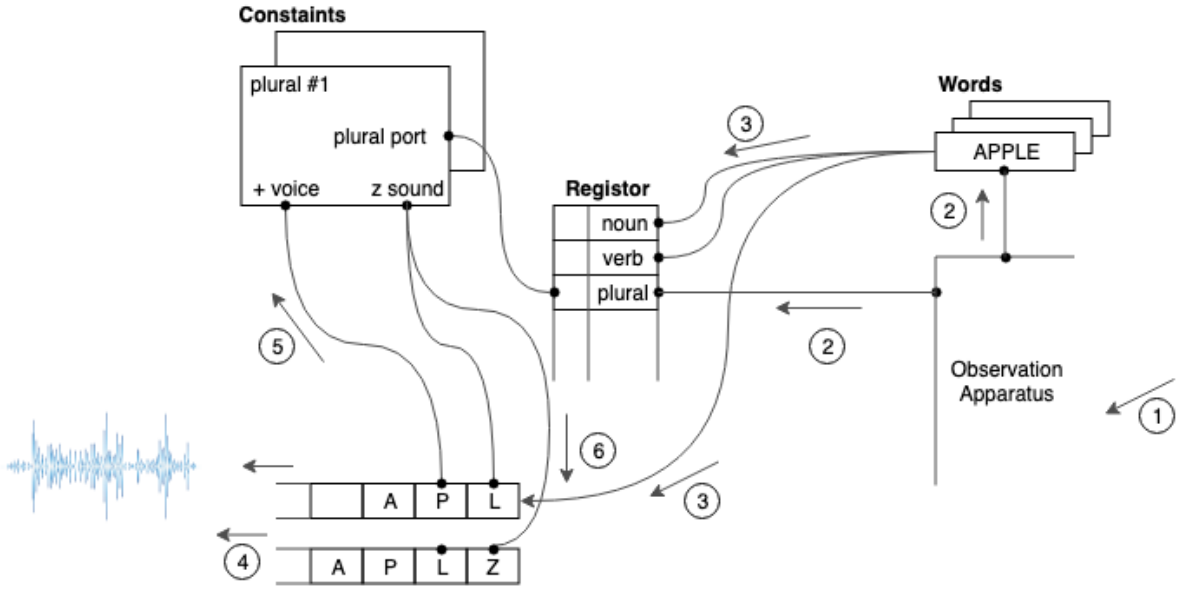


Figure 56: Work flow

1. The vision apparatus comes in and produces the notation, the concept of 2 apples.
2. Information flows from that meaning register to the "APPLE" word; information also flows to the register and mark it as plus plural
3. Information flows from word to register so as to indicate that it's a noun but not a verb; also writes "a", "p", and "l" into the buffer
4. Left flow of the word

14.3 Sparse Spaces(!More information need)

1. COLLECT positive(+) and negative(-) examples
2. PICK positive(+) seed
3. GENERALIZE + $\longrightarrow *$("*" means "Don't care")
4. UNTIL we admit or match a negative example

14.4 Marr's Catechism

1. Specify the problem
2. Devise a representation
3. Determine an approach/thought/method
4. Pick a mechanism/Devise a algorithm
5. Experiment

15 Learning: Near Misses, Felicity Conditions

15.1 Learning Process

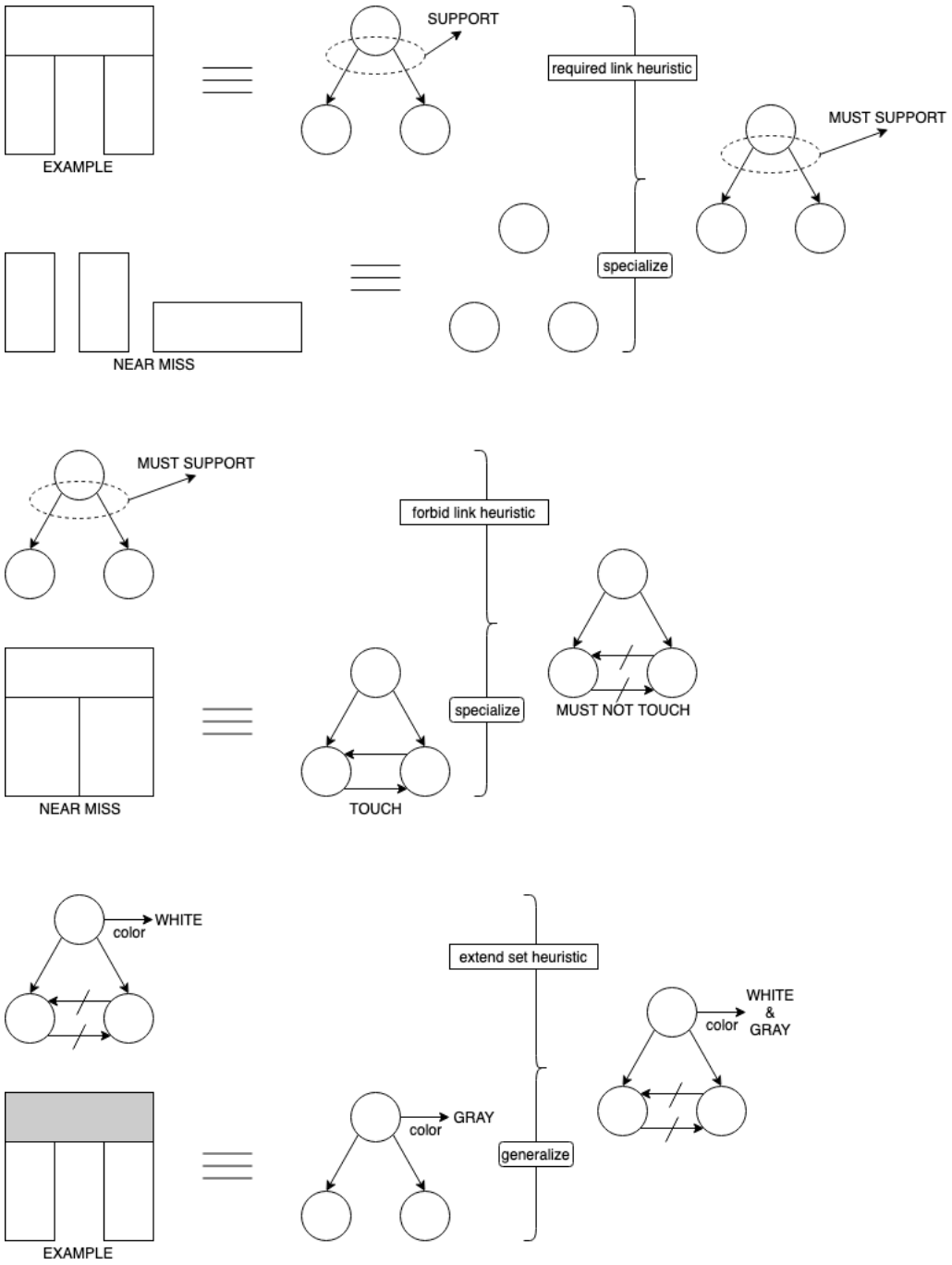


Figure 57: Specialization

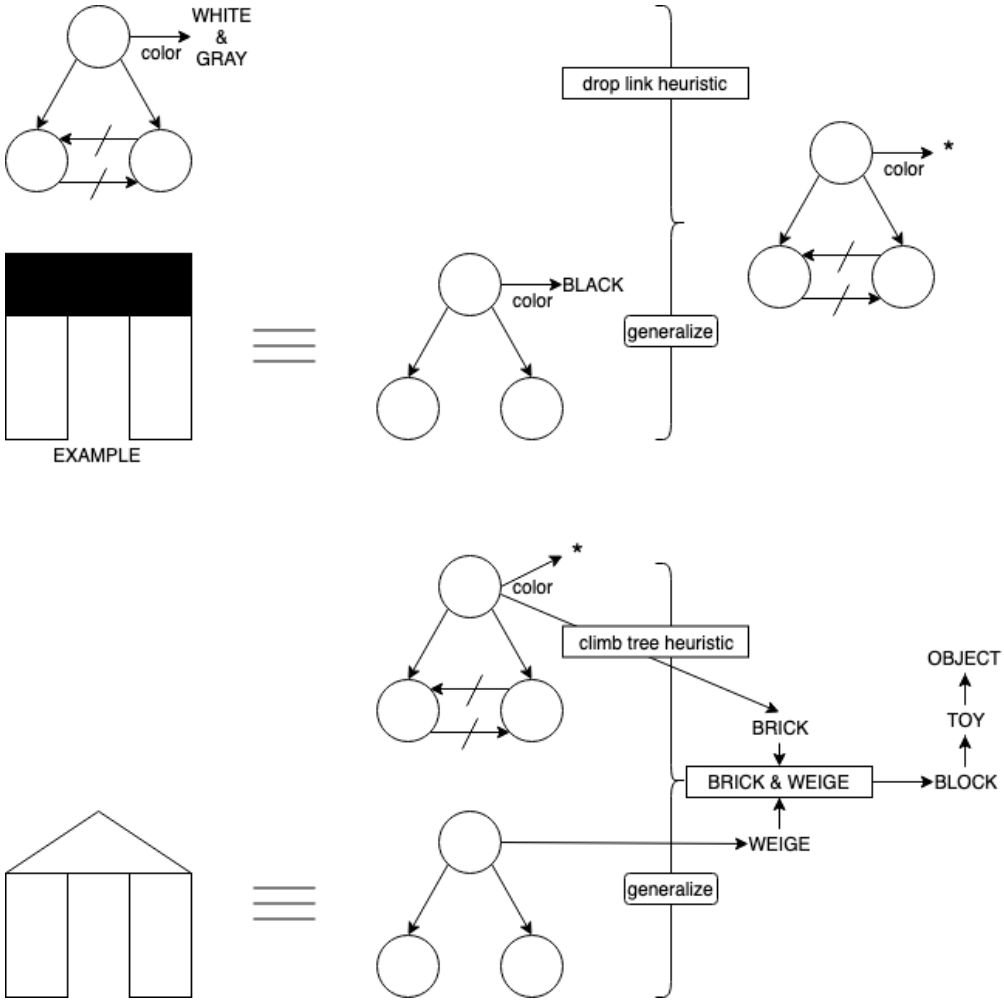


Figure 58: Generalization

15.2 Five Qualities

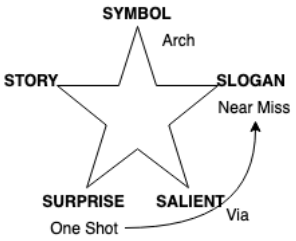


Figure 59: Five "S"

16 Learning: Support Vector Machines

16.1 Decision Boundary

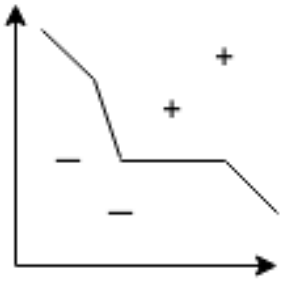


Figure 60: Nearest Neighbors

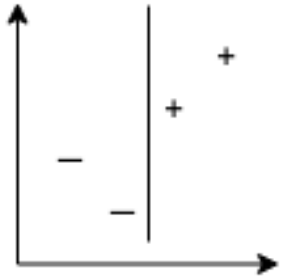


Figure 61: Identification Tree

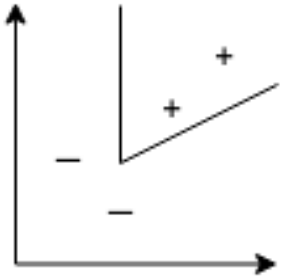


Figure 62: Neural Nets

16.2 Vladimir Vapnik's Idea–Support Vector Machine

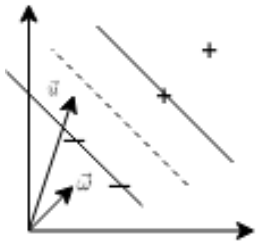


Figure 63: Sample Space

$$\boxed{\vec{w} \cdot \vec{u} \geq c}$$

Decision Rule: if $\vec{w} \cdot \vec{u} + b \geq 0$, then u is a positive point ($c = -b$)
Then, we are going to lay on some additional constraints

$$\begin{aligned} \vec{w} \cdot \vec{x_+} + b \geq & 1 \\ \vec{w} \cdot \vec{x_-} + b \leq & 1 \end{aligned}$$

Define y_i such that $y_i = +1$ for positive samples and $y_i = -1$ for negative samples

$$\begin{aligned} y_i(\vec{w} \cdot \vec{x_i} + b) &\geq 1 \\ y_i(\vec{w} \cdot \vec{x_i} + b) - 1 &\geq 0 \end{aligned}$$

For x_i located in the gutter

$$\boxed{y_i(\vec{w} \cdot \vec{x_i} + b) - 1 = 0}$$

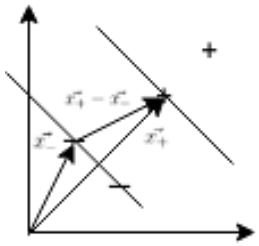


Figure 64: Sample Space

Let $\vec{w}$ be a normal vector

$$\begin{aligned} WIDTH =& (\vec{x_+} - \vec{x_-}) \cdot \frac{\vec{w}}{||\vec{w}||} \\ =& \frac{(\vec{x_+} \cdot \vec{w} - \vec{x_-} \cdot \vec{w})}{||\vec{w}||} \\ =& \frac{(1+b) - (b-1)}{||\vec{w}||} \\ =& \frac{2}{||\vec{w}||} \end{aligned}$$

MAX $WIDTH \Longrightarrow$ **MAX** $\frac{2}{||\vec{w}||} \Longrightarrow$ **MAX** $\frac{1}{||\vec{w}||} \Longrightarrow$ **MIN** $||\vec{w}|| \Longrightarrow$ **MIN** $\frac{1}{2}||\vec{w}||^2$
For finding the extremum of a function with constraints, we have to use ***Lagrange Multiplier***

$$\begin{aligned} L =& \frac{1}{2}||\vec{w}||^2 - \sum_i \alpha_i [y_i(\vec{w} \cdot \vec{x_i} + b) - 1] \\ \frac{\partial L}{\partial \vec{w}} =& \vec{w} - \sum_i \alpha_i y_i \vec{x_i} = 0 \Longrightarrow \boxed{\vec{w} = \sum_i \alpha_i y_i \vec{x_i}} \\ \frac{\partial L}{\partial b} =& -\sum_i \alpha_i y_i = 0 \Longrightarrow \boxed{\sum_i a_i y_i = 0} \end{aligned}$$

Plug the expression for $\vec{w}$ back in the expression for L

$$\begin{aligned} L =& \frac{1}{2}(\sum_i \alpha_i y_i \vec{x_i})(\sum_j \alpha_j y_j \vec{x_j}) - (\sum_i \alpha_i y_i \vec{x_i})(\sum_j \alpha_j y_j \vec{x_j}) - \sum_i \alpha_i y_i b + \sum_i \alpha_i \\ =& \sum_i \alpha_i - \frac{1}{2}(\sum_i \alpha_i y_i \vec{x_i})(\sum_j \alpha_j y_j \vec{x_j}) - b \sum_i \alpha_i y_i \\ =& \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j \boxed{\vec{x_i} \cdot \vec{x_j}} \end{aligned}$$

This maximization depends only on the dot product of pairs of samples
Plug the expression for $\vec{w}$ back in the decision rule

$$\sum_i \alpha_i y_i \vec{x_i} \cdot \vec{u} + b \geq 0$$

the decision rule also depends only on the dot product of those samples vectors and the unknown point.

If the samples are not linearly separable

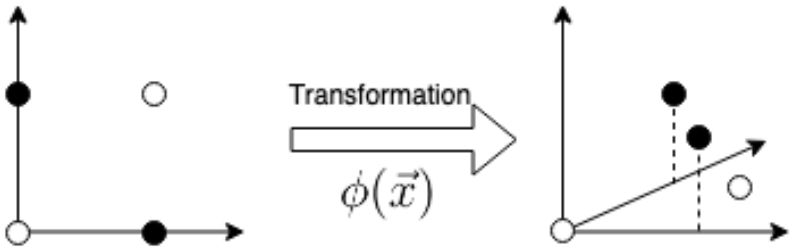


Figure 65: Space transformation

As the optimization only depends on the dot product, we just need to maximize

$$\phi(\vec{x_i}) \cdot \phi(\vec{x_j})$$

As the recognition also only depends on the dot product, we just need to maximize

$$\phi(\vec{x_i}) \cdot \phi(\vec{u})$$

What the matter is the dot product, even though we do not know the specific transformation. All we need is a ***Kernal Function***

$$K(\vec{x_i}, \vec{x_j}) = \phi(\vec{x_i}) \cdot \phi(\vec{x_j})$$

16.3 Kernal Function

1. Linear kernal
$$(\vec{x_i} \cdot \vec{x_j} + 1)^n$$
2. Radial basis kernal
$$e^{-\frac{||\vec{x_i} - \vec{x_j}||}{\sigma}}$$

17 Learning: Boosting

18 Representations: Classes, Trajectories, Transitions

19 Architectures: GPS, SOAR, Subsumption, Society of Mind

20 Probabilistic Inference I

21 Probabilistic Inference II

22 Model Merging, Cross-Modal Coupling, Course Summary