External Sorting

Why can't we simply do quicksort on a disk?

To get a[i] on

- **internal memory – O(1)**
- **hard disk**

1. **find the track;**
2. **find the sector;**
3. **find a[i] and transmit.**

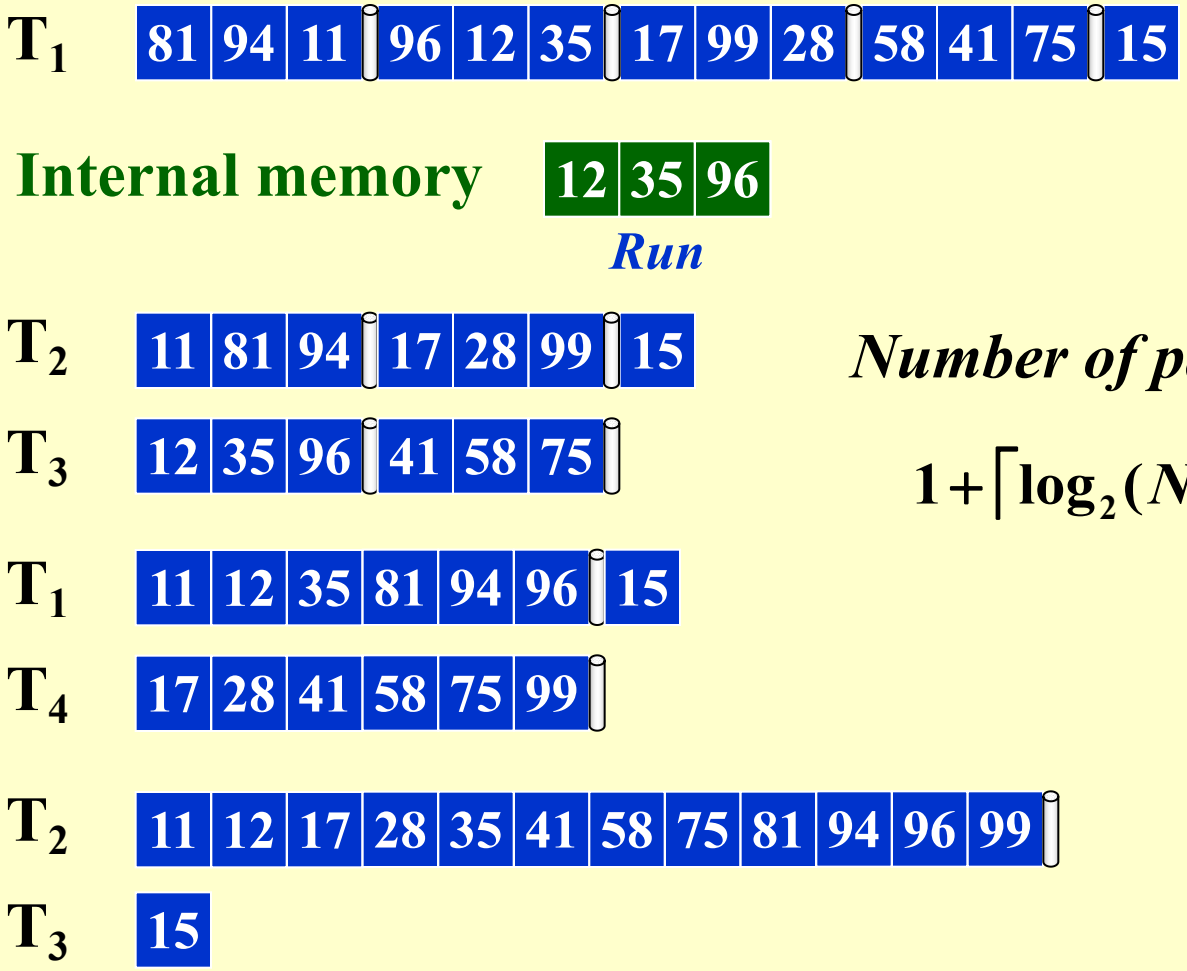
device-dependent

Tool : Mergesort

To simplify –

- **Store data on tapes (can only be accessed sequentially)**
- **Can use at least 3 tape drives**

〖Example〗 Suppose that the internal memory can handle $M = 3$ records at a time.



Number of passes = 1+3

$$1+\lceil \log_2(N/M) \rceil$$

What are the concerns?

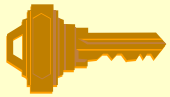
- **Seek time — *O(number of passes)***
- **Time to read or write one block of records**
- **Time to internally sort *M* records**
- **Time to merge *N* records from input buffers to the output buffer**

Computer can carry out I\O and CPU processing in parallel

Targets:

- ***Reduction of the number of passes***
- ***Run merging***
- ***Buffer handling for parallel operation***
- ***Run generation***

How to reduce the number of passes?



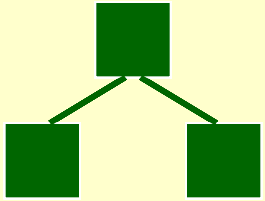
Use a *k-way* merge!

T_1 | 81 | 94 | 11 | 96 | 12 | 35 | 17 | 99 | 28 | 58 | 41 | 75 | 15 |

T_2 | 11 | 81 | 94 | 41 | 58 | 75 |

T_3 | 12 | 35 | 96 | 15 |

T_4 | 17 | 28 | 99 |



T_1 | 11 | 12 | 17 | 28 | 35 | 81 | 94 | 96 | 99 |

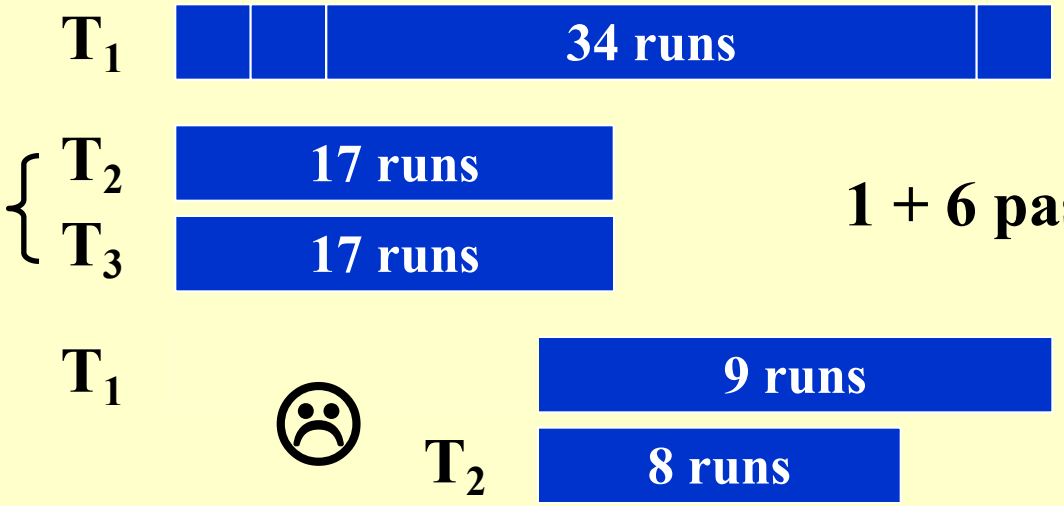
T_5 | 15 | 41 | 58 | 75 |

T_6

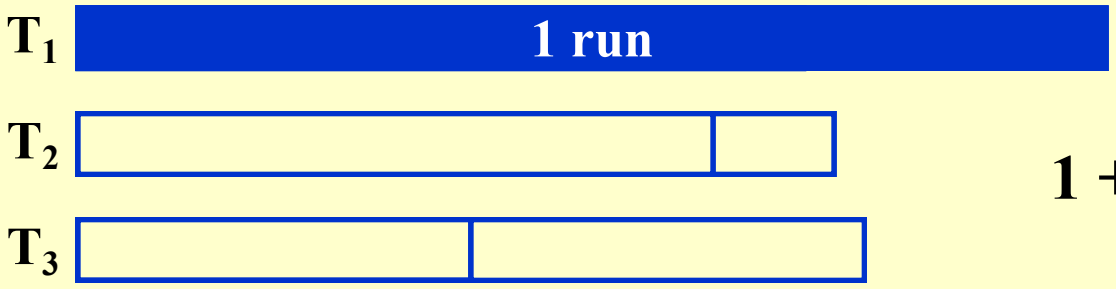
$$Number\ of\ passes = 1 + \lceil \log_k (N / M) \rceil$$

Require $2k$ tapes!

Can we use 3 tapes for a 2-way merge?



☝ A smarter way – split *unevenly*



Discussion 20:

What will happen if 22 runs are placed on T_2, with 12 on T_3 ?

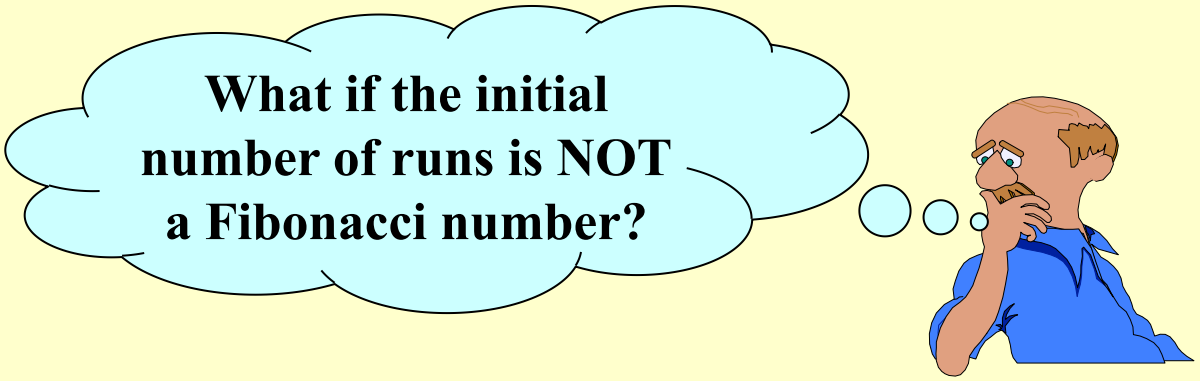
Claim: **If the number of runs is a Fibonacci number F_N, then the best way to distribute them is to split them into F_{N-1} and F_{N-2} .**

Claim: **For a k-way merge, $F_N^{(k)} = F_{N-1}^{(k)} + \cdots + F_{N-k}^{(k)}$**

where $F_N^{(k)} = 0 \ \ (0 \le N \le k-2), \ F_{k-1}^{(k)} = 1$

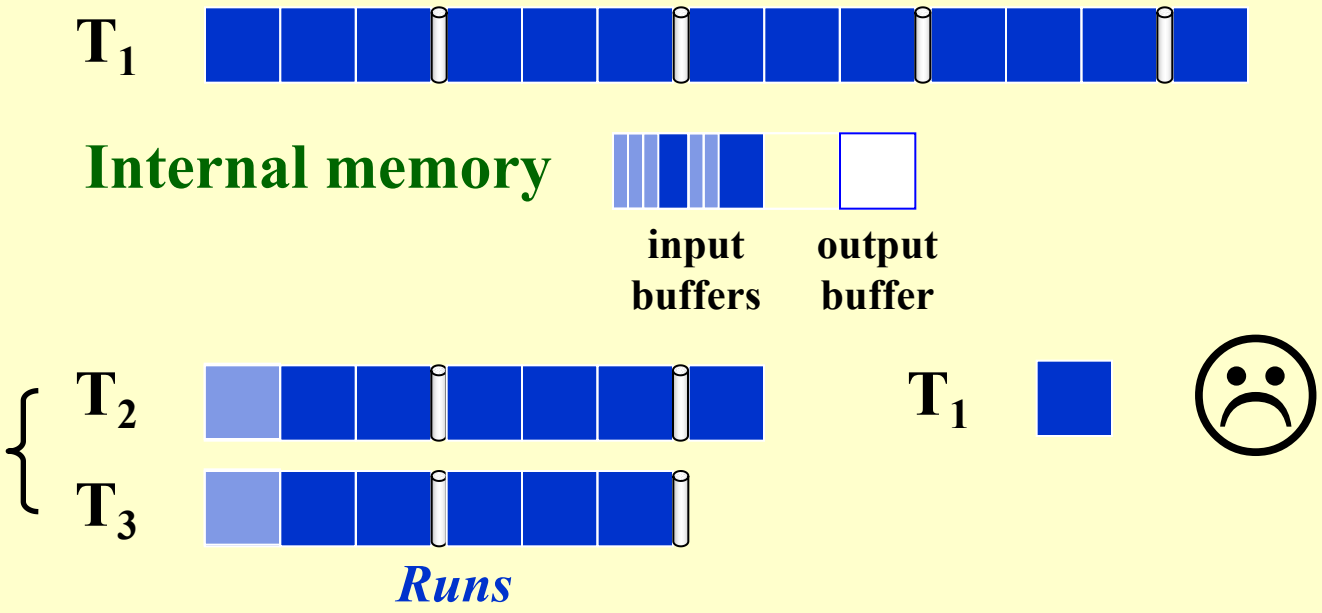
Polyphase Merge

$k+1$ tapes only



How to handle the buffers for parallel operation?

〖Example〗 Sort a file containing 3250 records, using a computer with an internal memory capable of sorting at most 750 records. The input file has a block length of 250 records.



2-way merge

input buffers output buffer

In general, for a *k*-way merge we need 2*k* input buffers and 2 output buffers for parallel operations.

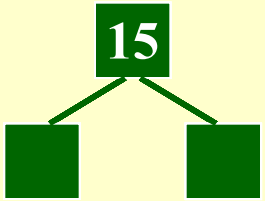
k → # of input buffers → buffer size

seek time ← block size on disk

Beyond a certain *k* value, the I\O time would actually increase despite the decrease in the number of passes being made. The optimal value for *k* clearly depends on disk parameters and the amount of internal memory available for buffers.

Can we generate a longer run?

81	94	11	96	12	35	17	99	28	58	41	75	15



Replacement selection

11	81	94	96

12	17	28	35	41	58	75	99

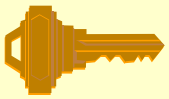
15

$L_{avg} = 2M$

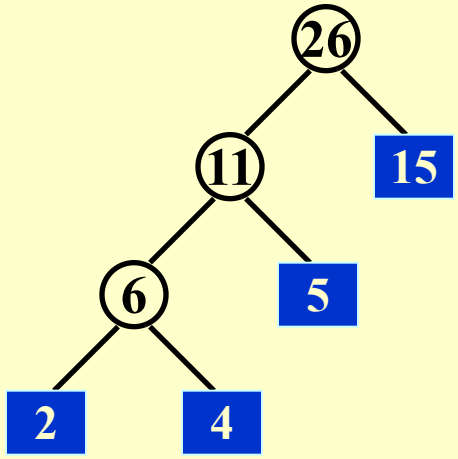
Powerful when input is often *nearly sorted* for external sorting.

How to minimize the merge time?

〖Example〗 Suppose we have 4 runs of length 2, 4, 5, and 15, respectively. How can we arrange the merging to obtain minimum merge times?



Huffman Tree!



$$2\times3+4\times3+5\times2+15\times1=43$$

Total merge time = O (*the weighted external path length*)

Reference:

Data Structure and Algorithm Analysis in C (2nd Edition)：Ch.6，p.222-227；*M.A.Weiss著、陈越改编，人民邮件出版社，2005*

The Fibonacci Numbers and Polyphase Sorting; *W. C. LYNCH, Case Institute of Technology, Cleveland, Ohio*