Chapter 4

Discounted Cash Flow Valuation
折现现金流估值

Key Concepts and Skills

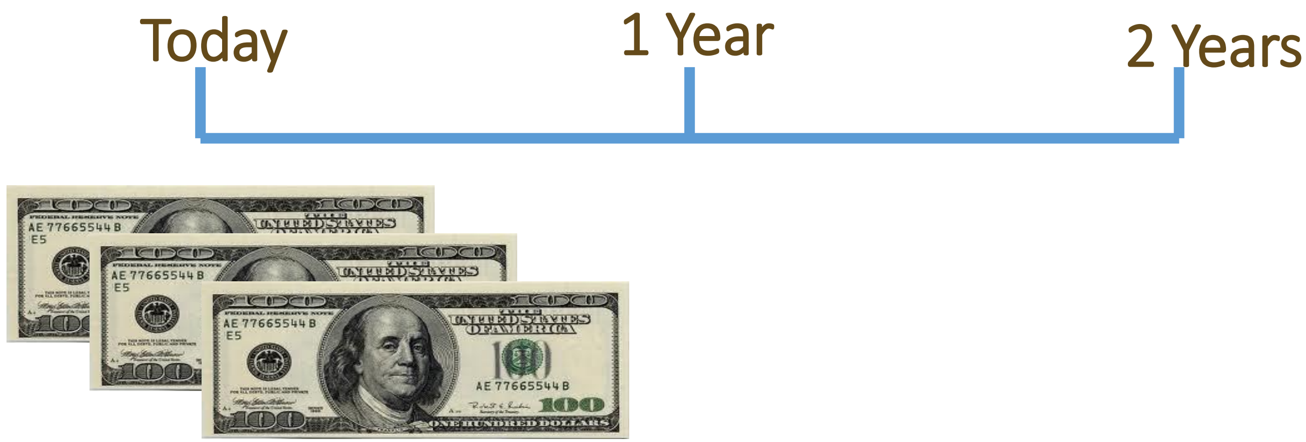
- Be able to compute the future value and/or present value of a single cash flow or series of cash flows
- Be able to compute the return on an investment
- Understand perpetuities and annuities

Time and Money

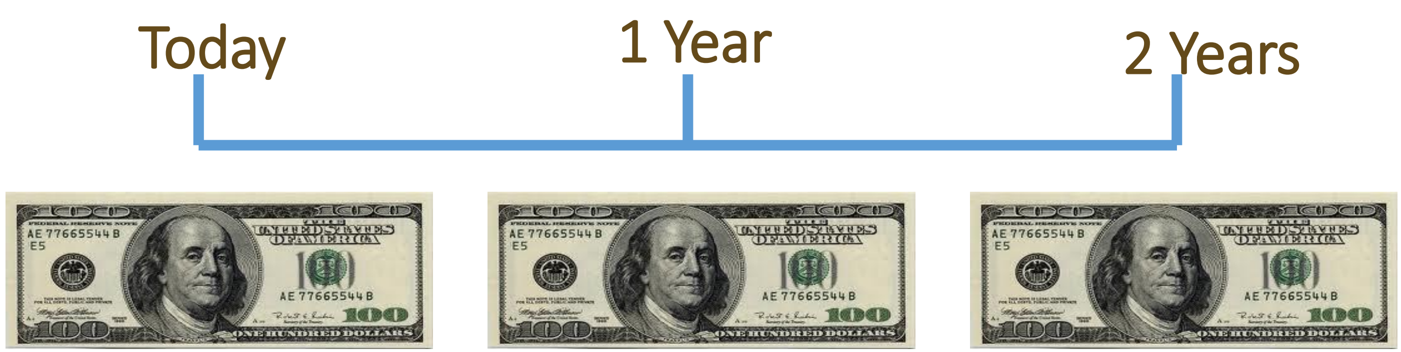
Which would you rather receive: A or B?

A is better because you get all of the $300 today instead of having to wait two years.

A



B



The Intuitive Basis for Time Value of Money

- Why a cash flow in the future is worth less than a similar cash flow today?

1. **Individuals prefer present consumption to future consumption**. People would have to be offered more in the future to give up present consumption. If the preference for current consumption is strong, individuals will have to be offered much more in terms of future consumption to give up current consumption, a trade-off that is captured by a high "real" rate of return or discount rate. Conversely, when the preference for current consumption is weaker, individuals will settle for much less in terms of future consumption and, by extension, a low real rate of return or discount rate.

2. **When there is monetary inflation, the value of currency decreases over time**. The greater the inflation, the greater the difference in value between a nominal cash flow today and the same cash flow in the future.

3. **A promised cash flow might not be delivered** for a number of reasons: The promisor might default on the payment, the promisee might not be around to receive payment, or some other contingency might intervene to prevent the promised payment or to reduce it. Any uncertainty (risk) associated with the cash flow in the future reduces the value of the cash flow.

The One-Period Case

- If you were to invest \$10,000 at 5-percent interest for one year, your investment would grow to \$10,500.

\$500 would be interest (\$10,000 × .05)

\$10,000 is the principal repayment (\$10,000 × 1)

\$10,500 is the total due. It can be calculated as:

$$\$10,500 = \$10,000 \times (1.05)$$

- The total amount due at the end of the investment is call the *Future Value* (*FV*).

Future Value 终值

- In the one-period case, the formula for *FV* can be written as:

$$FV = C_0 \times (1 + r)$$

Where C_0 is cash flow today (time zero), and
r is the appropriate interest rate.

Present Value 现值

- If you were to be promised \$10,000 due in one year when interest rates are 5-percent, your investment would be worth \$9,523.81 in today's dollars.

$$\$9,523.81 = \frac{\$10,000}{1.05}$$

The amount that a borrower would need to set aside today to be able to meet the promised payment of \$10,000 in one year is called the *Present Value* (*PV*).

Note that \$10,000 = \$9,523.81×(1.05).

Present Value

- In the one-period case, the formula for *PV* can be written as:

$$PV = \frac{C_1}{1 + r}$$

Where C_1 is cash flow at date 1, and

r is the appropriate interest rate.

Net Present Value 净现值

- The Net Present Value (*NPV*) of an investment is the present value of the expected cash flows, less the cost of the investment.
- Suppose an investment that promises to pay $10,000 in one year is offered for sale for $9,500. Your interest rate is 5%. Should you buy?

Net Present Value

$$NPV = -\$9{,}500 + \frac{\$10{,}000}{1.05}$$

$$NPV = -\$9{,}500 + \$9{,}523.81$$

$$NPV = \$23.81$$

The present value of the cash inflow is greater than the cost. In other words, the Net Present Value is positive, so the investment should be purchased.

Net Present Value

In the one-period case, the formula for *NPV* can be written as:

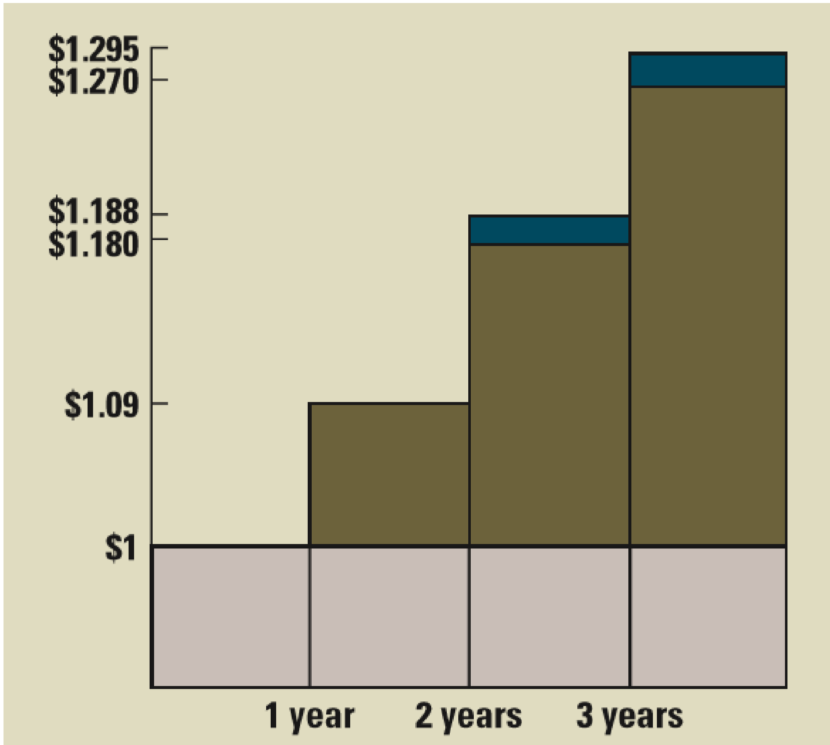
$$NPV = –Cost + PV$$

If we had *not* undertaken the positive *NPV* project considered on the last slide, and instead invested our \$9,500 elsewhere at 5 percent, our *FV* would be less than the \$10,000 the investment promised, and we would be worse off in *FV* terms :

$$\$9,500 \times (1.05) = \$9,975 < \$10,000$$

The Multi-period Case

- When cash is invested at **compound interest （复利）, each interest payment is reinvested**. With **simple interest （单利）**, the interest is not reinvested.
- “Money makes money and the money that money makes makes more money” Benjamin Franklin
- Simple interest:
  - *FV=$1+1*9%*2=$1.18*
- Compound interest:
  - *FV=$1*×(1 + *9%*)^2*=$1.1881*
- Interest on Interest
  - *$1.1881-$1.18=($1*9%)*9%*
    =$0.0081



Compound interest 复利

- The general formula for the future value of an investment over many periods can be written as:

$$FV = C_0 \times (1 + r)^T$$

Where

C_0 is cash flow at date 0,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Future Value

- Suppose a stock currently pays a dividend of $1.10, which is expected to grow at 40% per year for the next five years.
- What will the dividend be in five years?

$$FV = C_0 \times (1 + r)^T$$

$$\$5.92 = \$1.10 \times (1.40)^5$$

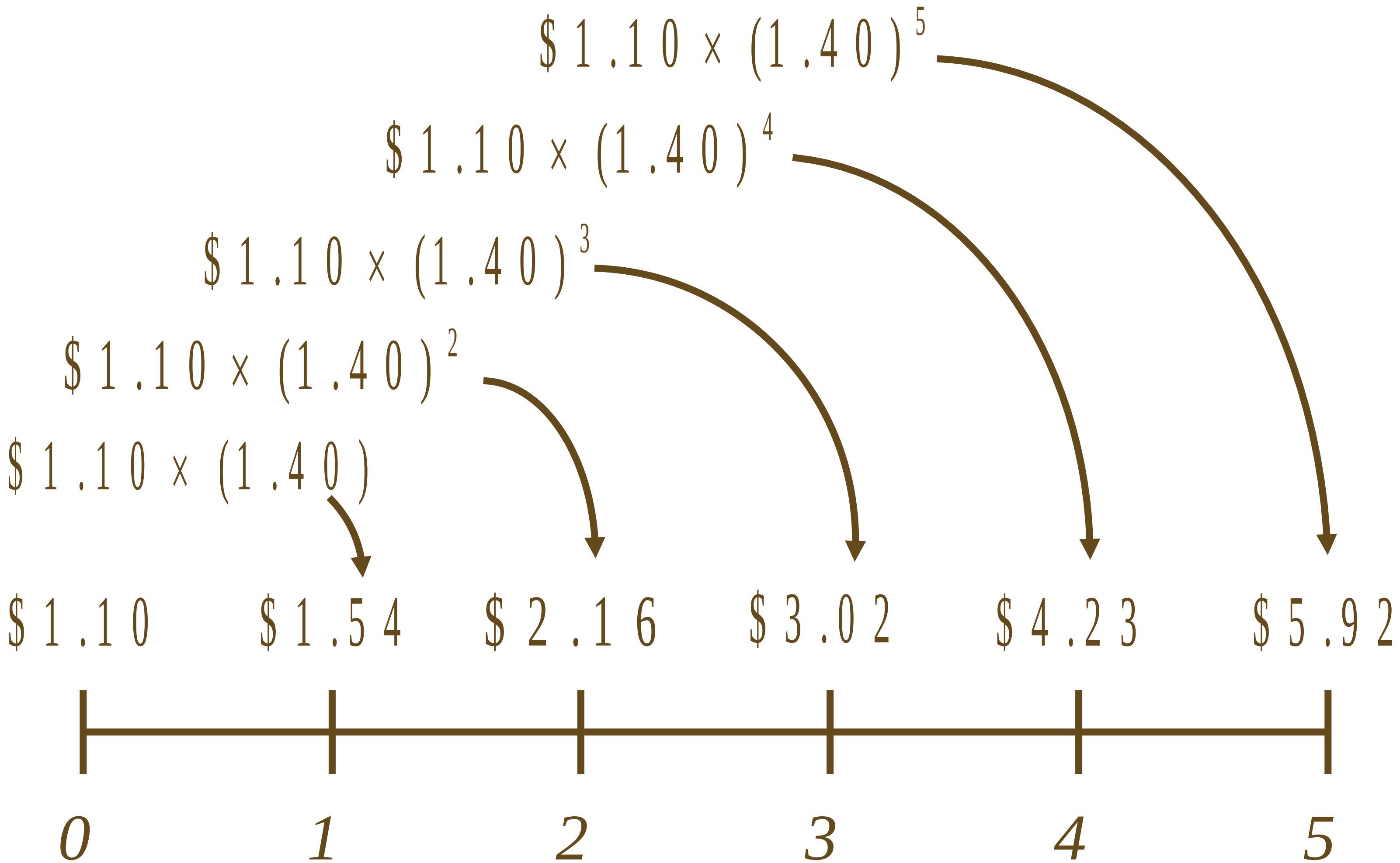
Future Value and Compounding

- Notice that the dividend in year five, \$5.92, is considerably higher than the sum of the original dividend plus five increases of 40-percent on the original \$1.10 dividend:

$$\$5.92 > \$1.10 + 5\times[\$1.10\times.40] = \$3.30$$

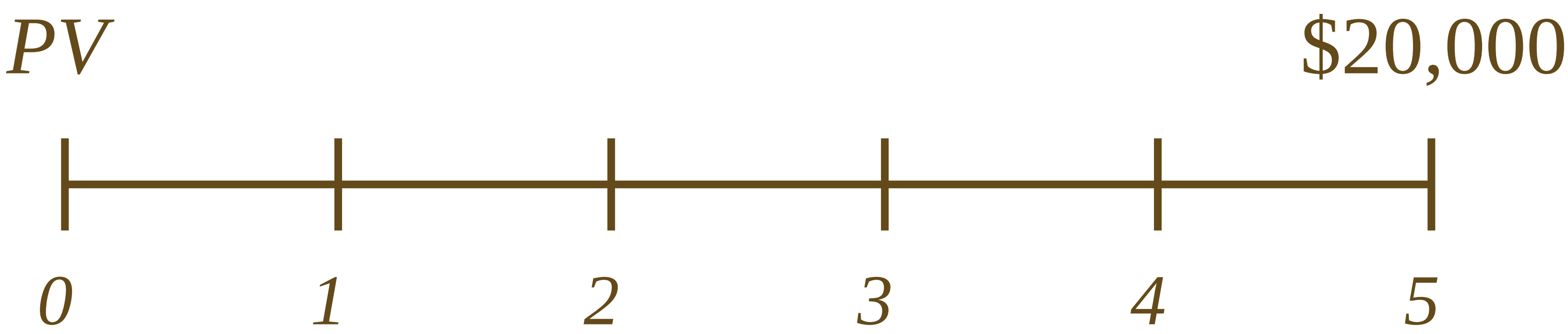
This is due to *compounding*.

Future Value and Compounding



Present Value and Discounting

- How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



$$\$9{,}943.53 = \frac{\$20{,}000}{(1.15)^5}$$

Finding the Number of Periods

If we deposit $5,000 today in an account paying 10%, how long does it take to grow to $10,000?

$$FV = C_0 \times (1 + r)^T \qquad \$10{,}000 = \$5{,}000 \times (1.10)^T$$

$$(1.10)^T = \frac{\$10{,}000}{\$5{,}000} = 2$$

$$\ln(1.10)^T = \ln(2)$$

$$\boxed{t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + r)}}$$

$$T = \frac{\ln(2)}{\ln(1.10)} = \frac{0.6931}{0.0953} = 7.27 \text{ years}$$

What Rate Is Enough?

Assume the total cost of a college education will be $50,000 when your child enters college in 12 years. You have $5,000 to invest today. What rate of interest must you earn on your investment to cover the cost of your child's education? About 21.15%.

$$FV = C_0 \times (1 + r)^T \qquad \$50,000 = \$5,000 \times (1 + r)^{12}$$

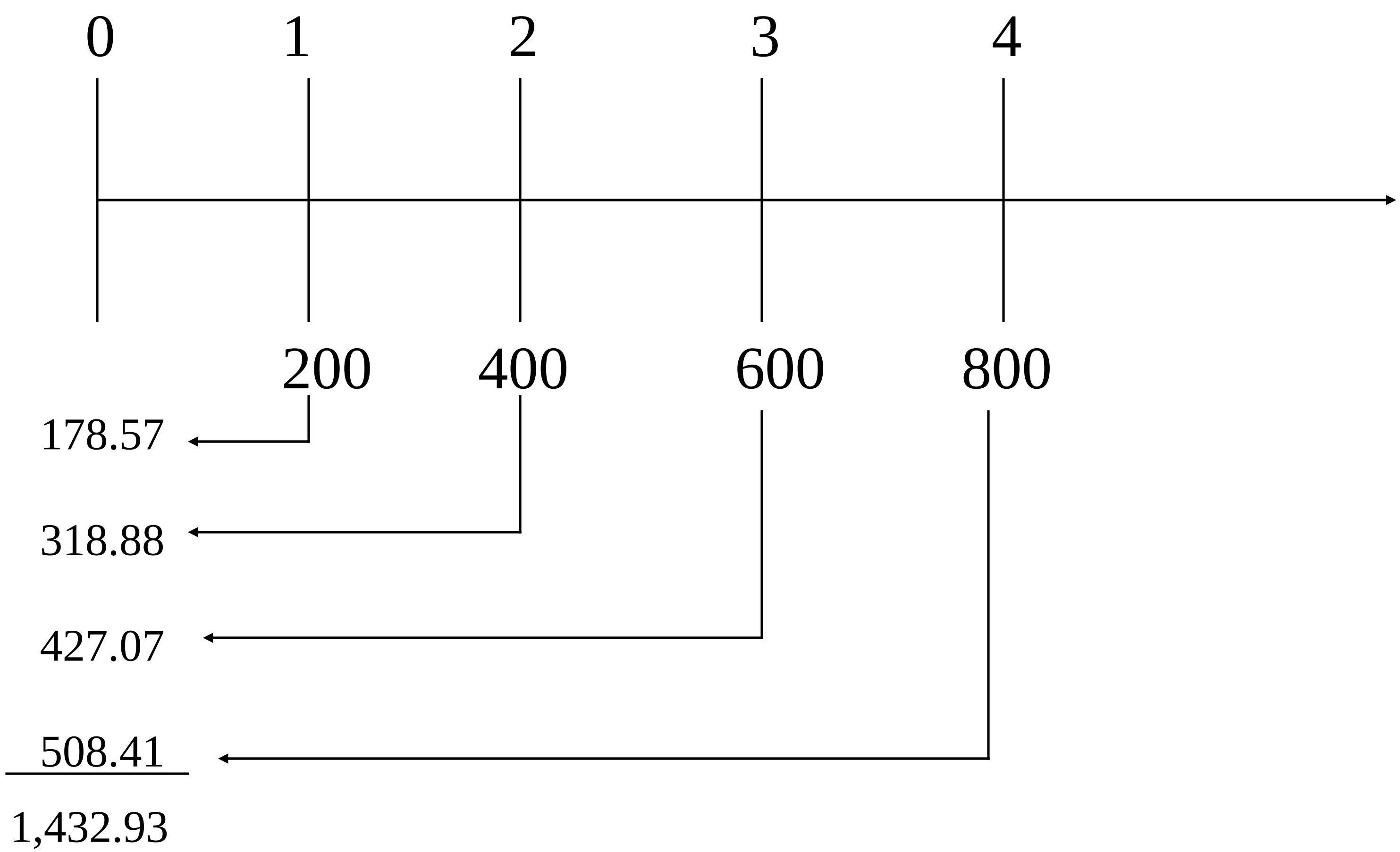
$$(1 + r)^{12} = \frac{\$50,000}{\$5,000} = 10 \qquad (1 + r) = 10^{1/12}$$

$$r = 10^{1/12} - 1 = 1.2115 - 1 = .2115$$

Multiple Cash Flows

- Consider an investment that pays $200 one year from now, with cash flows increasing by $200 per year through year 4. If the interest rate is 12%, what is the present value of this stream of cash flows?
- If the issuer offers this investment for $1,500, should you purchase it?

Multiple Cash Flows



Present Value < Cost → Do Not Purchase

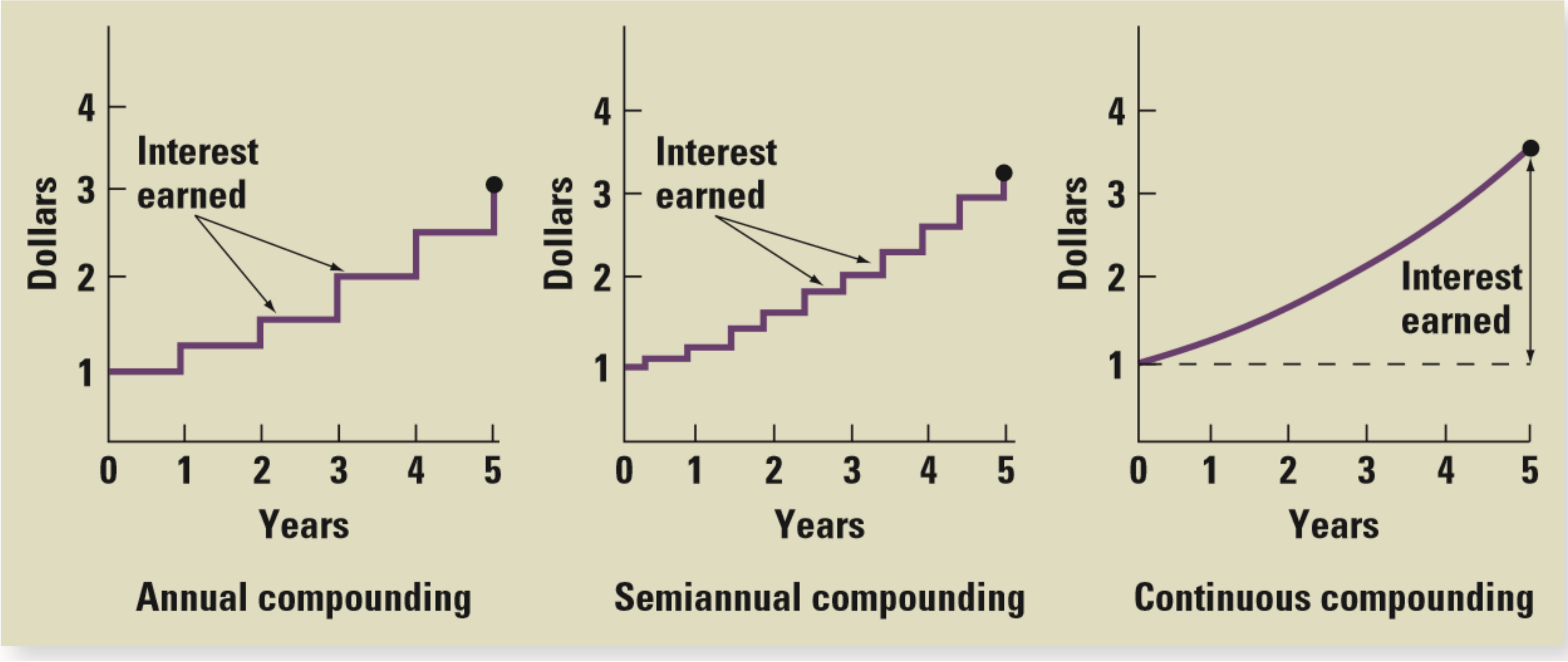
Compounding Periods

- The frequency of compounding affects both the future and present values of cash flows
- The Annual Percentage Rate (APR) is calculated as: the rate, for a payment period, multiplied by the number of payment periods in a year.
- Compounding an investment *m* times a year for *T* years provides for future value of wealth:

$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{m \times T}$$

Annual, Semiannual, and Continuous Compounding

- Continuous compounding has both the smoothest curve and the highest ending value of all



Compounding Periods

- For example, if you invest \$50 for 3 years at 12% compounded semi-annually, your investment will grow to

$$FV = \$50 \times \left(1 + \frac{.12}{2}\right)^{2 \times 3} = \$50 \times (1.06)^{6} = \$70.93$$

- Annual percentage rate (APR) is the annual interest rate without consideration of compounding

Effective Annual Rates of Interest 实际年利率

A reasonable question to ask in the above example is "what is the effective *annual* rate of interest on that investment?"

$$FV = \$50 \times (1 + \frac{.12}{2})^{2\times 3} = \$50 \times (1.06)^6 = \$70.93$$

The Effective Annual Rate (EAR) of interest is the annual rate that would give us the same end-of-investment wealth after 3 years:

$$\$50 \times (1 + EAR)^3 = \$70.93$$

Effective Annual Rates of Interest

$$FV = \$50 \times (1 + EAR)^3 = \$70.93$$

$$(1 + EAR)^3 = \frac{\$70.93}{\$50}$$

$$EAR = \left(\frac{\$70.93}{\$50}\right)^{1/3} - 1 = .1236$$

- So, investing at 12.36% compounded annually is the same as investing at 12% compounded semi-annually

Effective Annual Rates of Interest

- Find the Effective Annual Rate (EAR) of an 18% APR loan that is compounded monthly.
- What we have is a loan with a monthly interest rate rate of 1½%.
- This is equivalent to a loan with an annual interest rate of 19.56%.

$$\left(1+\frac{r}{m}\right)^m = \left(1+\frac{.18}{12}\right)^{12} = (1.015)^{12} = 1.1956$$

Continuous Compounding

- The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = C_0 \times e^{rT}$$

Where

C_0 is cash flow at date 0,

r is the stated annual interest rate,

T is the number of years, and

e is a transcendental number approximately equal to 2.718. e^x is a key on your calculator.

Effective Annual Rate Based on Frequency of Compounding

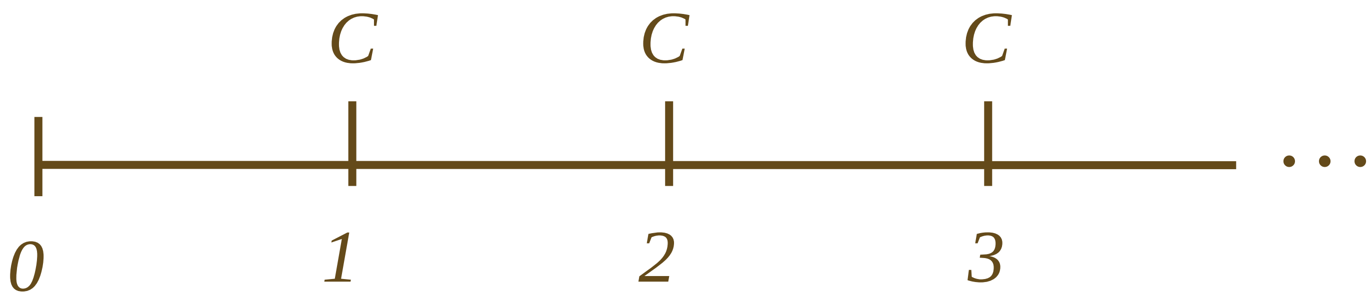
Nominal Rate	Semi-Annual	Quarterly	Monthly	Daily	Continuous
1%	1.003%	1.004%	1.005%	1.005%	1.005%
5%	5.063%	5.095%	5.116%	5.127%	5.127%
10%	10.250%	10.381%	10.471%	10.516%	10.517%
15%	15.563%	15.865%	16.075%	16.180%	16.183%
20%	21.000%	21.551%	21.939%	22.134%	22.140%
30%	32.250%	33.547%	34.489%	34.969%	34.986%
40%	44.000%	46.410%	48.213%	49.150%	49.182%
50%	56.250%	60.181%	63.209%	64.816%	64.872%

Simplifications

- Perpetuity 永续年金
  - A constant stream of cash flows that occurs at regular intervals and **lasts forever**
- Growing perpetuity 增长型永续年金
  - A stream of cash flows that occurs at regular intervals and grows at a constant rate forever
- Annuity 年金
  - A stream of constant cash flows that occurs at regular intervals and **lasts for a fixed number of periods**
- Growing annuity增长型年金
  - A stream of cash flows that occurs at regular intervals and grows at a constant rate for a fixed number of periods

Perpetuity

A constant stream of cash flows that lasts forever

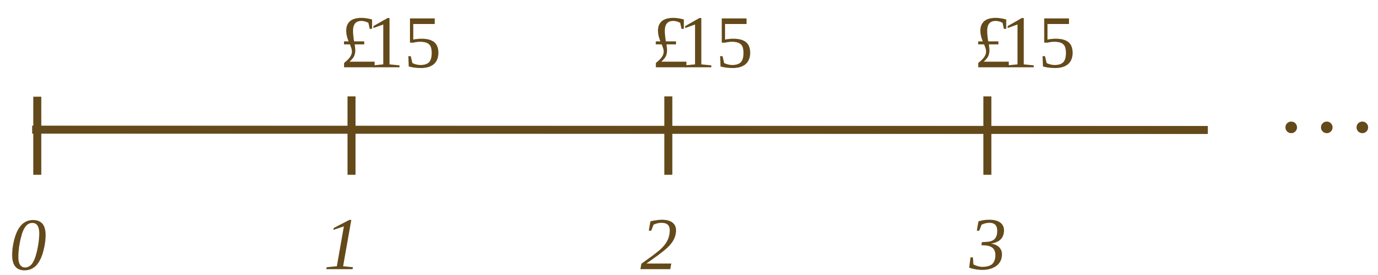


A Geometric Sequence

Perpetuity: Example

What is the value of a British consol that promises to pay £15 every year for ever?

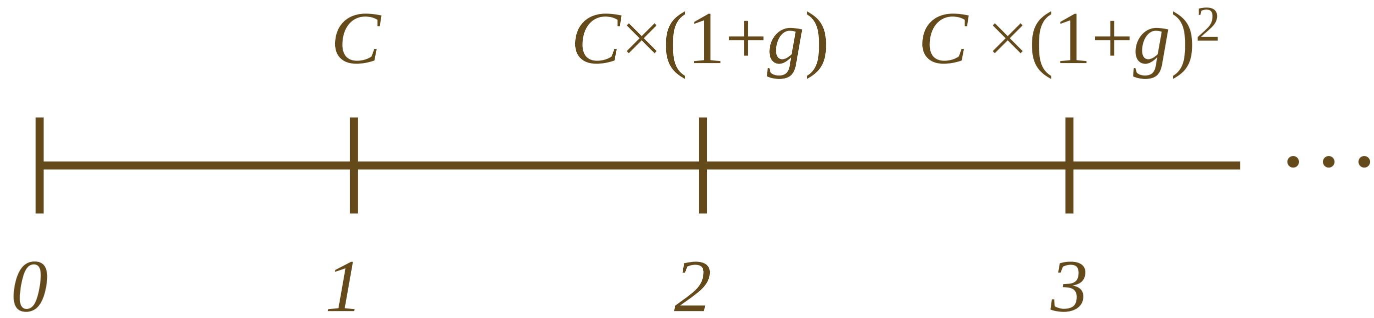
The interest rate is 10-percent.



$$PV = \frac{£15}{.10} = £150$$

Growing Perpetuity

A growing stream of cash flows that lasts forever



$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

Growing Perpetuity: Example

The expected dividend next year is \$1.30, and dividends are expected to grow at 5% forever.

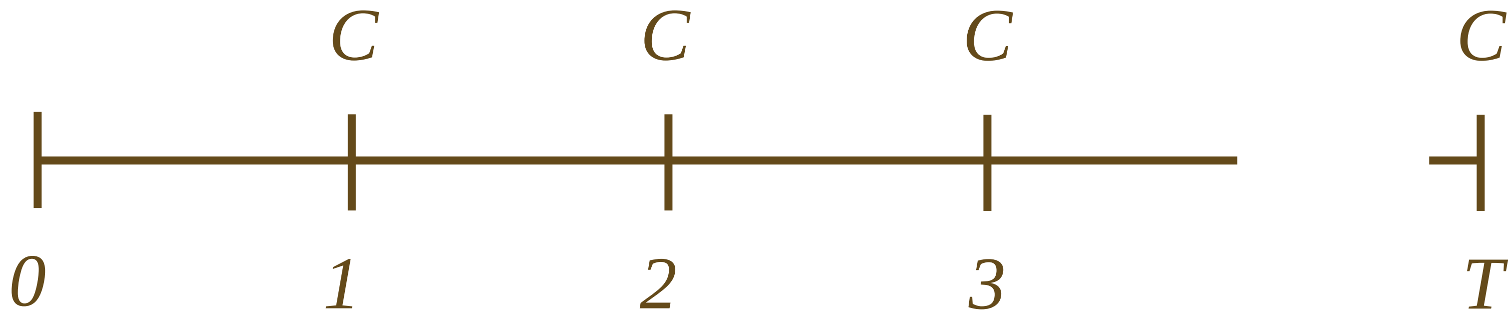
If the discount rate is 10%, what is the value of this promised dividend stream?

$$\begin{array}{cccc} & \$1.30 & \$1.30\times(1.05) & \$1.30\times(1.05)^2 \quad \cdots \\ 0 & 1 & 2 & 3 \end{array}$$

$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

Annuity

A constant stream of cash flows with a fixed maturity

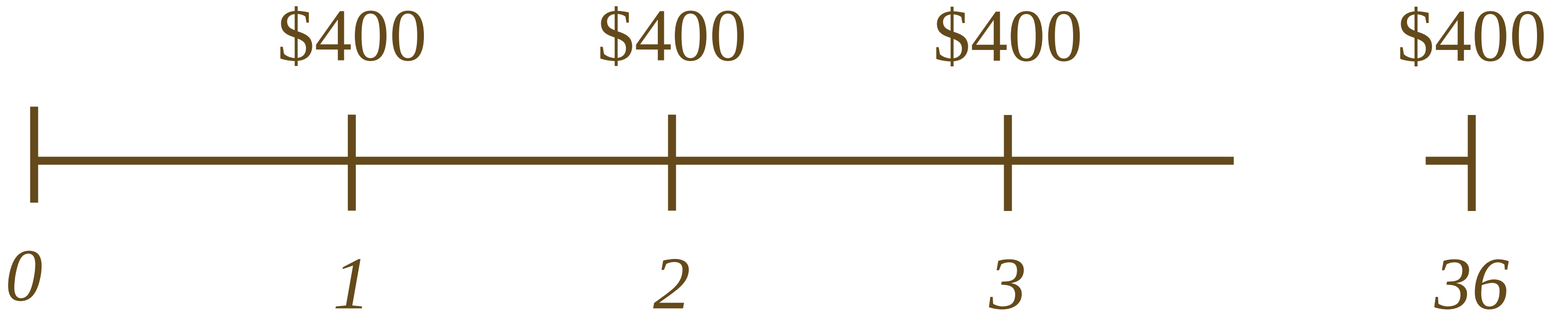


$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Annuity: Example

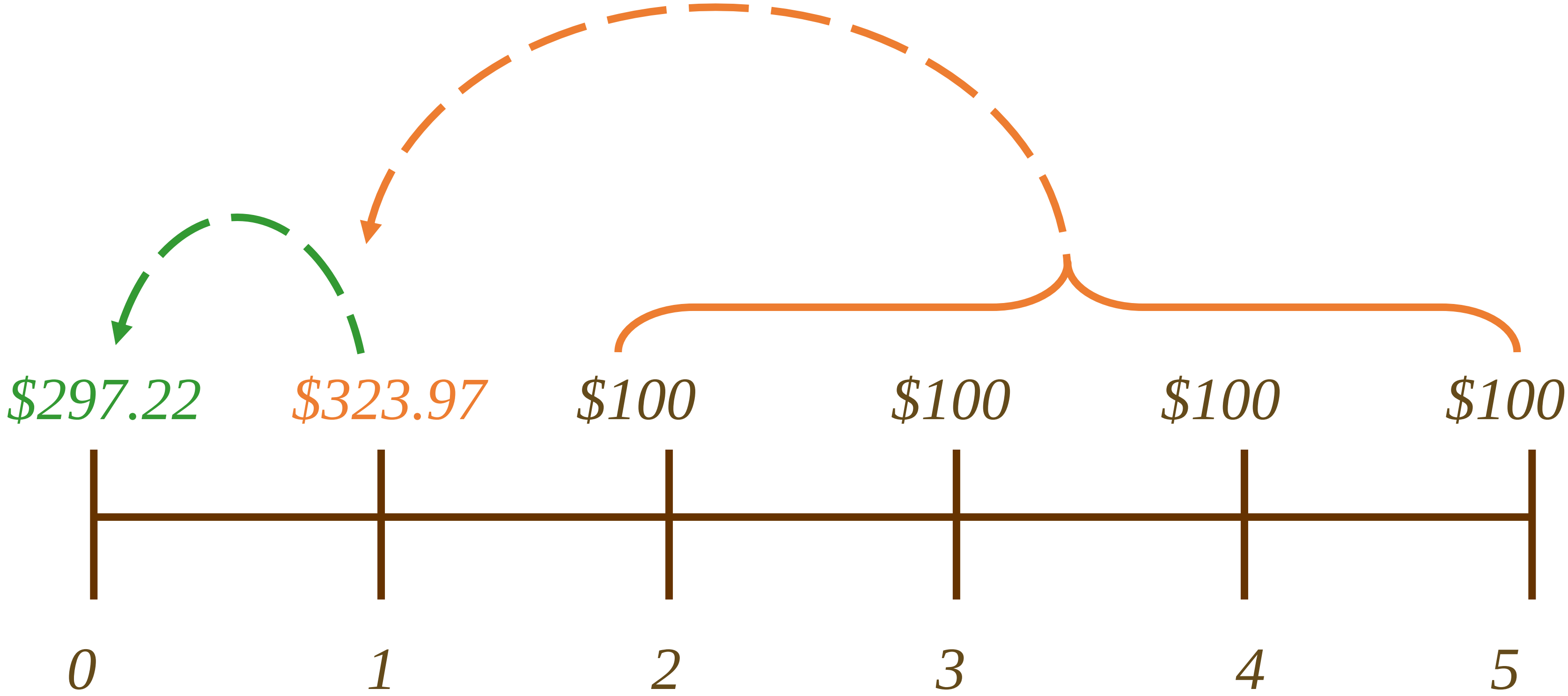
If you can afford a $400 monthly car payment, how much car can you afford if interest rates are 7% on 36-month loans?



$$PV = \frac{\$400}{.07/12}\left[1 - \frac{1}{(1+.07/12)^{36}}\right] = \$12,954.59$$

What is the present value of a four-year annuity of \$100 per year that makes its first payment two years from today if the discount rate is 9%?

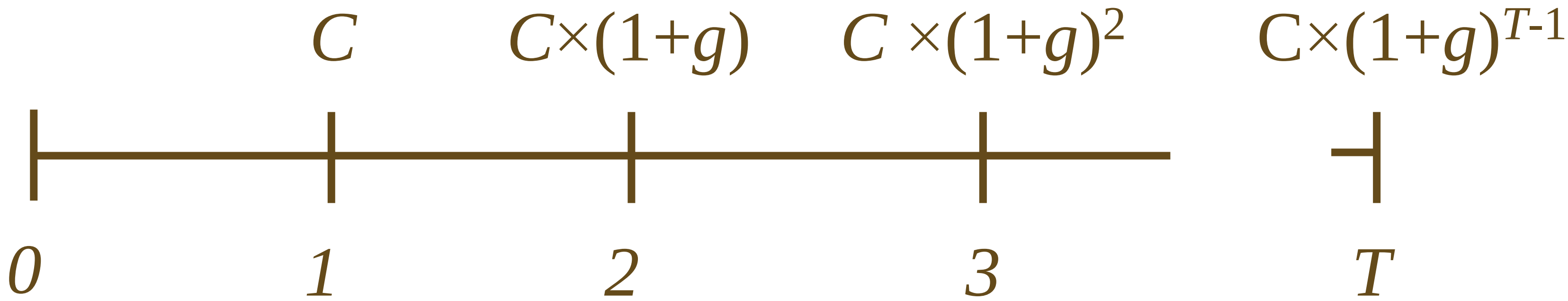
$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$



$$PV_0 = \frac{\$327.97}{1.09} = \$297.22$$

Growing Annuity

A growing stream of cash flows with a fixed maturity

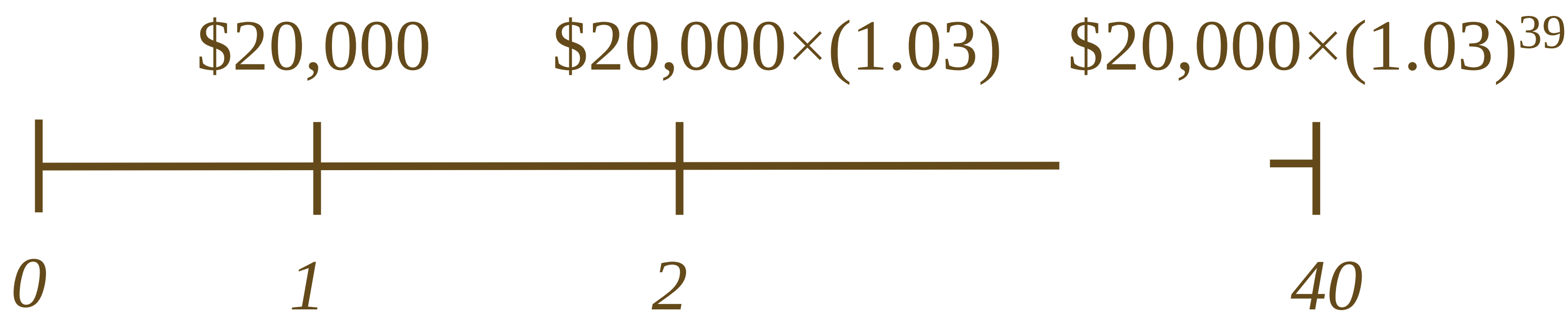


$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \cdots + \frac{C \times (1+g)^{T-1}}{(1+r)^T}$$

$$PV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{(1+r)}\right)^T\right]$$

Growing Annuity: Example

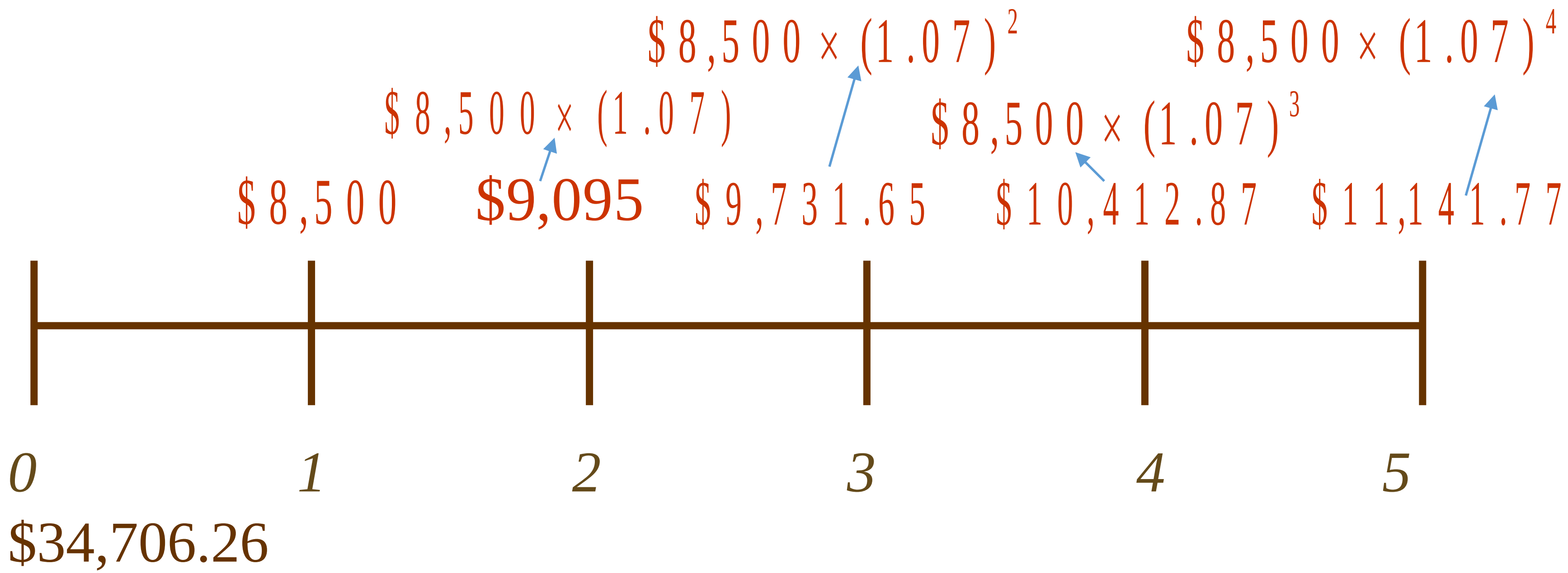
A defined-benefit retirement plan offers to pay \$20,000 per year for 40 years and increase the annual payment by 3% each year. What is the present value at retirement if the discount rate is 10%?



$$PV = \frac{\$20{,}000}{.10 - .03}\left[1 - \left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

Growing Annuity: Example

You are evaluating an income generating property. Net rent is received at the end of each year. The first year's rent is expected to be $8,500, and rent is expected to increase 7% each year. What is the present value of the estimated income stream over the first 5 years if the discount rate is 12%?



Loan Amortization 分期偿还贷款

- Pure Discount Loans are the simplest form of loan. The borrower receives money today and repays a single lump sum (principal and interest) at a future time.
- Interest-Only Loans require an interest payment each period, with full principal due at maturity.
- Amortized Loans require repayment of principal over time, in addition to required interest.

Pure Discount Loans

- Treasury bills are excellent examples of pure discount loans. The principal amount is repaid at some future date, without any periodic interest payments.
- If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?

  - PV = 10,000 / 1.07 = 9,345.79

Interest-Only Loan

- Consider a 5-year, interest-only loan with a 7% interest rate. The principal amount is $10,000. Interest is paid annually.
  - What would the stream of cash flows be?
    - Years 1 – 4: Interest payments of .07(10,000) = 700
    - Year 5: Interest + principal = 10,700
- This cash flow stream is similar to the cash flows on corporate bonds, and we will talk about them in greater detail later.

Amortized Loan with **Fixed Principal Payment (等额本金还款）**

- Consider a $50,000, 10 year loan at 8% interest. The loan agreement requires the firm to pay $5,000 in principal each year plus interest for that year.

Year	Beginning Balance	Interest Payment	Principal Payment	Total Payment	Ending Balance
1	50,000	4,000	5,000	9,000	45,000
2	45,000	3,600	5,000	8,600	40,000
3	40,000	3,200	5,000	8,200	35,000
4	35,000	2,800	5,000	7,800	30,000
5	30,000	2,400	5,000	7,400	25,000
6	25,000	2,000	5,000	7,000	20,000
7	20,000	1,600	5,000	6,600	15,000
8	15,000	1,200	5,000	6,200	10,000
9	10,000	800	5,000	5,800	5,000
10	5,000	400	5,000	5,400	0

Amortized Loan with **Fixed Payment (等额本息还款）**

- Each payment covers the interest expense plus reduces principal
- Consider a 4 year loan with annual payments. The interest rate is 8%, and the principal amount is $5,000.

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	5,000.00	1,509.60	400.00	1,109.60	3,890.40
2	3,890.40	1,509.60	311.23	1,198.37	2,692.03
3	2,692.03	1,509.60	215.36	1,294.24	1,397.79
4	1,397.79	1,509.60	111.82	1,397.78	0.01
Totals		6,038.40	1,038.41	4,999.99	

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right]$$

Buying a House

- You are ready to buy a house and you have $20,000 for a down payment and closing costs.
- Closing costs are estimated to be 4% of the loan value. Closing costs are fees associated with your home purchase that are paid at the closing of a real estate transaction.
- You have an annual salary of $36,000.
- The bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income.
- The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan.
- How much money will the bank loan you?
- How much can you offer for the house?

Buying a House - Continued

- Bank loan
  - Monthly income = 36,000 / 12 = 3,000
  - Maximum payment = .28(3,000) = 840
    - T=30*12=360
    - R=0.5%
    - PMT=840

    PV = 140,105

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

- Total Price
  - Closing costs = .04(140,105) = 5,604
  - Down payment = 20,000 – 5604 = 14,396
  - Total Price = 140,105 + 14,396 = **154,501**

0% APR vs. $1000 Rebate

Quick Quiz

- How is the future value of a single cash flow computed?
- How is the present value of a series of cash flows computed.
- What is the Net Present Value of an investment?
- What is an EAR, and how is it computed?
- What is a perpetuity? An annuity?