Chapter 11

Return and Risk: The Capital Asset Pricing Model (CAPM)

收益与风险：资本资产定价模型

Key Concepts and Skills

- Know how to calculate expected returns
- Know how to calculate covariances, correlations, and betas
- Understand the impact of diversification
- Understand the systematic risk principle
- Understand the security market line
- Understand the risk-return tradeoff
- Be able to use the Capital Asset Pricing Model

11.1 Individual Securities

- The characteristics of individual securities that are of interest are the:
  - Expected Return （期望收益）
  - Variance and Standard Deviation
  - Covariance and Correlation (to another security or index)

11.2 Expected Return, Variance, and Covariance

Consider the following two risky asset world. There is a 1/3 chance of each state of the economy, and the only assets are a stock fund and a bond fund.

		Rate of Return	
Scenario	***Probability***	***Stock Fund***	***Bond Fund***
Recession	33.3%	-7%	17%
Normal	33.3%	12%	7%
Boom	33.3%	28%	-3%

Expected Returns

- The Expected return, E(R), is a weighted average of outcomes in different states

$$E(R) = \sum_{i=1}^{n} p_i R_i$$

Expected Return

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$E(r_S) = \frac{1}{3} \times (-7\%) + \frac{1}{3} \times (12\%) + \frac{1}{3} \times (28\%)$$

Variance

- Variance, σ^2, is measured different from before
- It is now the assets weighted average squared deviation from the expected return
- As before, the larger the variance, the wider the spread of possible returns

Variance

$$\sigma^2 = \sum_{i=1}^{n} p_i (R_i - E(R))^2$$

Variance

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(-7\% - 11\%)^2 = .0324$$

Variance

	Stock Fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$.0205 = \frac{1}{3}(.0324 + .0001 + .0289)$$

Standard Deviation

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
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Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$14.3\% = \sqrt{0.0205}$$

11.3 The Return and Risk for Portfolios

	Stock Fund		Bond Fund	
Scenario	***Rate of Return***	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

Note that stocks have a higher expected return than bonds and higher risk. Let us turn now to the risk-return tradeoff of a portfolio that is 50% invested in bonds and 50% invested in stocks.

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S$$

$$5\% = 50\% \times (-7\%) + 50\% \times (17\%)$$

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
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Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The *expected* rate of return on the portfolio is a weighted average of the *expected* returns on the securities in the portfolio.

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

$$9\% = 50\% \times (11\%) + 50\% \times (7\%)$$

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
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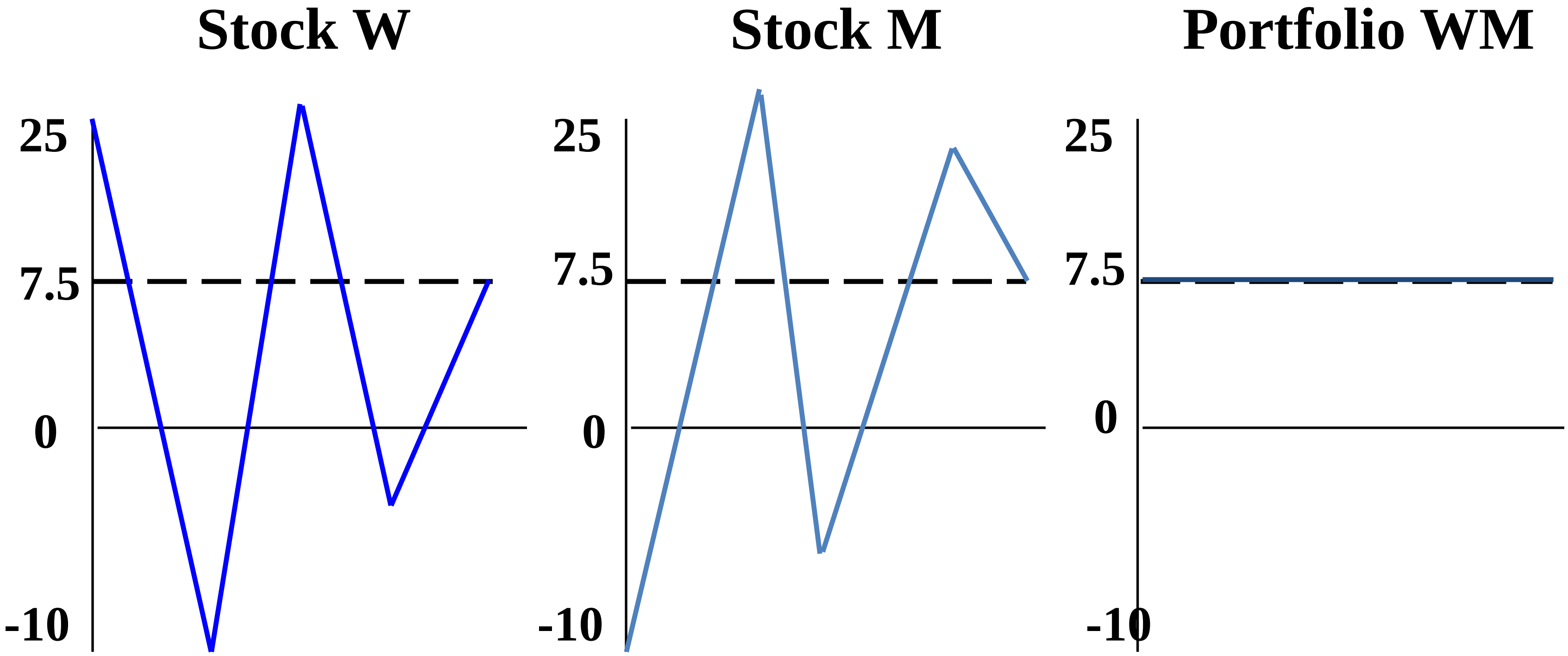
The variance of a portfolio is **NOT** the weighted average of the variances of the individual assets

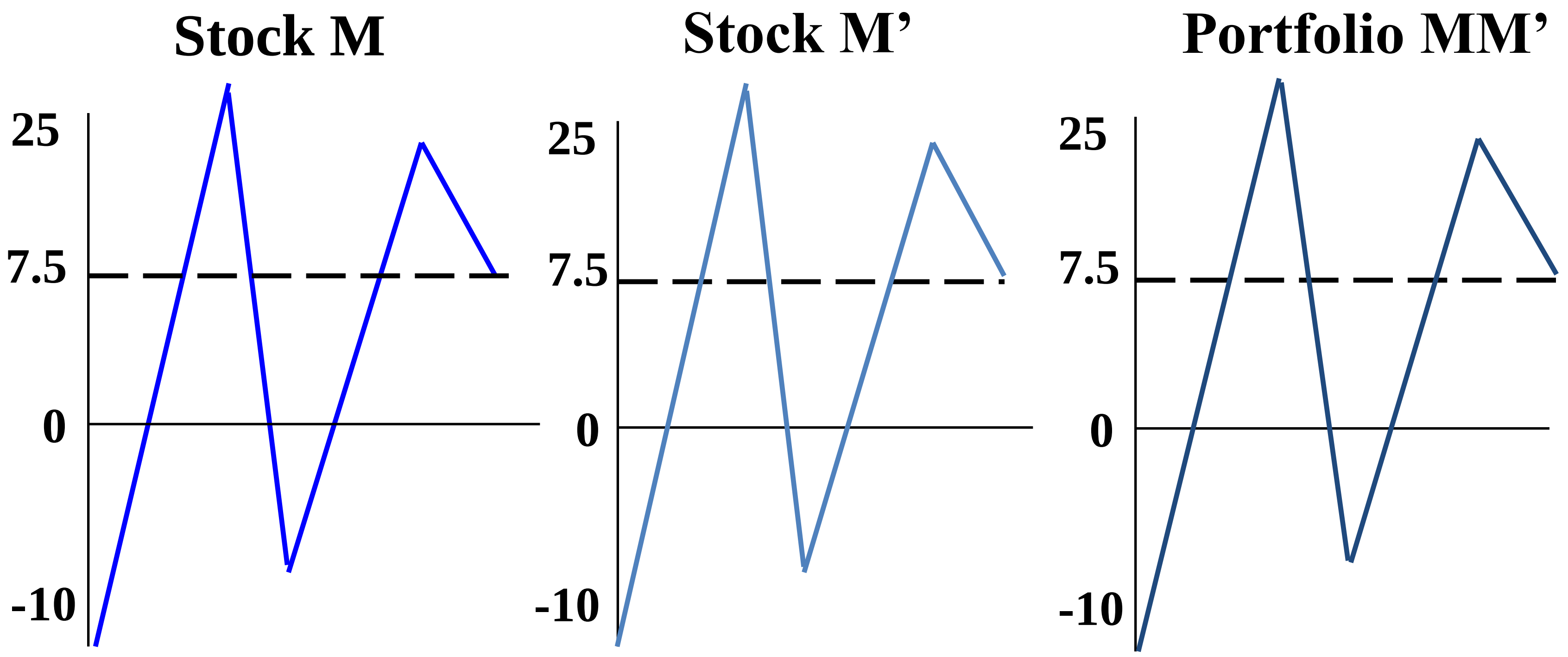
The Volatility of a Two-Stock Portfolio

- Combining Risks
  - By combining stocks into a portfolio, we reduce risk through diversification.
  - The amount of risk that is eliminated in a portfolio depends on the degree to which the stocks face common risks and their prices move together.
  - For example, when the airline stocks performed well, the oil stock tended to do poorly, and when the airlines did poorly, the oil stock tended to do well. The expenditure on fuel accounts for a considerable proportion of airline companies' gross costs

Returns distribution for two perfectly negatively correlated stocks



Returns distribution for two perfectly positively correlated stocks



Computing a Portfolio's Variance and Volatility

- For a two security portfolio:

$$\begin{aligned} Var(R_P) &= Cov(R_P, R_P) \\ &= Cov(x_1 R_1 + x_2 R_2, x_1 R_1 + x_2 R_2) \\ &= x_1 x_1 Cov(R_1, R_1) + x_1 x_2 Cov(R_1, R_2) + x_2 x_1 Cov(R_2, R_1) + x_2 x_2 Cov(R_2, R_2) \end{aligned}$$

- The Variance of a Two-Stock Portfolio

$$Var(R_P) = x_1^2 Var(R_1) + x_2^2 Var(R_2) + 2 x_1 x_2 Cov(R_1, R_2)$$

Determining Covariance and Correlation

- Covariance
  - The expected product of the deviations of two returns from their means
  - Covariance between Returns R_i and R_j

$$Cov(R_i, R_j) = E[(R_i - E[R_i])(R_j - E[R_j])]$$

  - Estimate of the Covariance from Historical Data

$$Cov(R_i, R_j) = \frac{1}{T-1}\sum_t (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)$$

    - If the covariance is positive, the two returns tend to move together.
    - If the covariance is negative, the two returns tend to move in opposite directions.

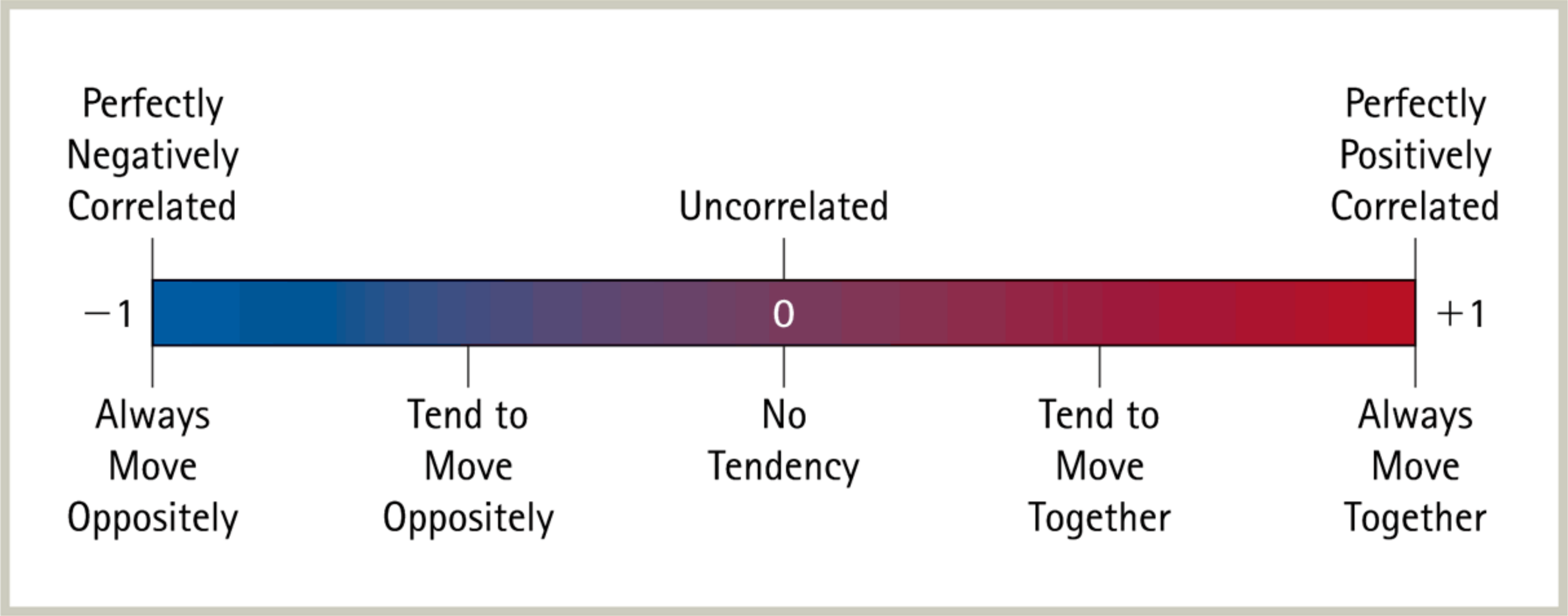
Determining Covariance and Correlation (cont'd)

- Correlation
  - A measure of the common risk shared by stocks that does not depend on their volatility

$$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{SD(R_i)\, SD(R_j)}$$

- The correlation between two stocks will always be between –1 and +1.

Correlation



Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
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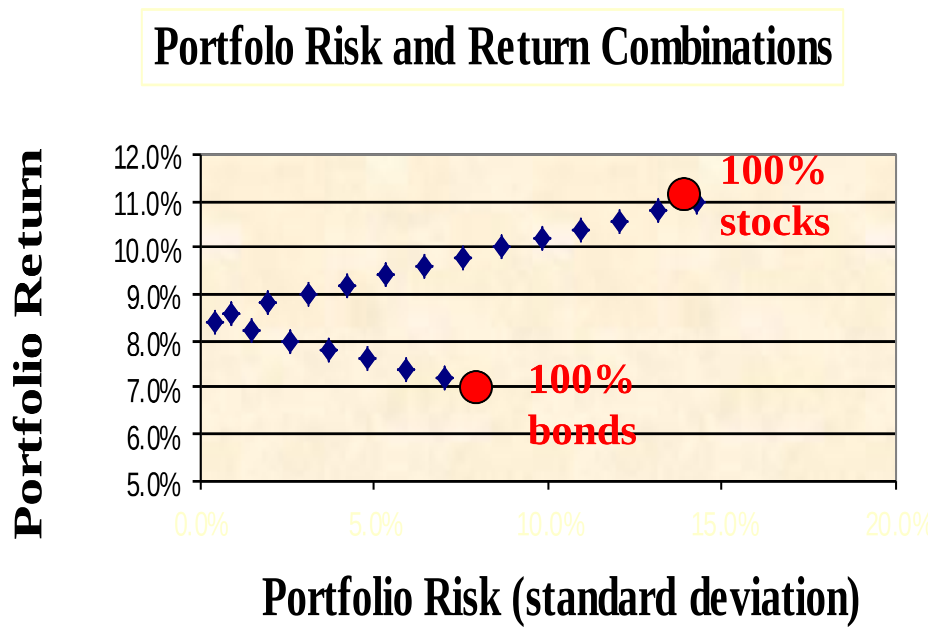
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

Observe the decrease in risk that diversification offers.

An equally weighted portfolio (50% in stocks and 50% in bonds) has less risk than either stocks or bonds held in isolation.

11.4 The Efficient Set for Two Assets

% in stocks	*Risk*	*Return*
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50.00%	**3.08%**	**9.00%**
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



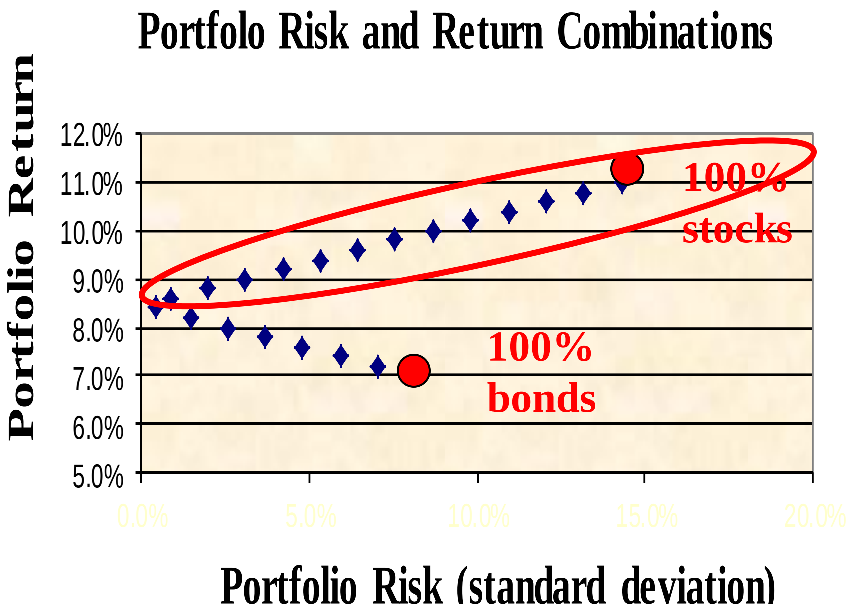
We can consider other portfolio weights besides 50% in stocks and 50% in bonds.

Opportunity set （机会集）/feasible set （可行集）

The possible expected return and standard deviation pairs of all portfolios that can be constructed from a given set of assets.

The Efficient Set for Two Assets

% in stocks	*Risk*	*Return*
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50%	3.1%	9.0%
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



Note that some portfolios are “better” than others. They have higher returns for the same level of risk or less.

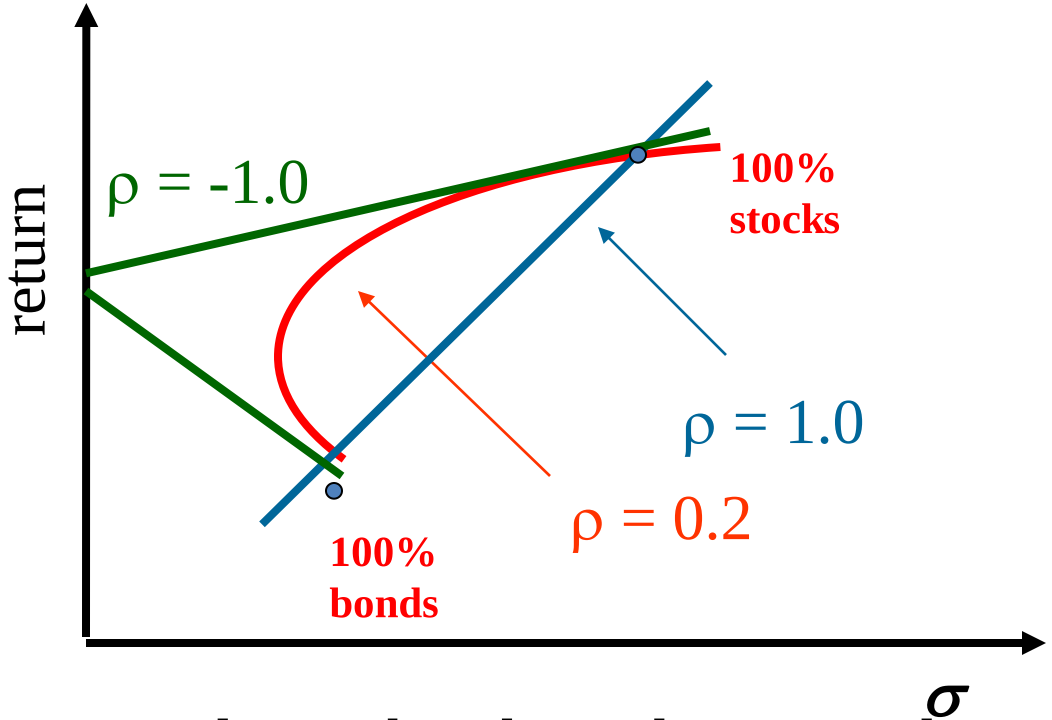
Efficient Set 有效集 /Efficient frontier 有效边界

The efficient frontier is the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return.

The Effect of Correlation

- Correlation has no effect on the expected return of a portfolio. However, the volatility of the portfolio will differ depending on the correlation.
- The lower the correlation, the lower the volatility we can obtain. As the correlation decreases, the volatility of the portfolio falls.

Portfolios with Various Correlations



- Relationship depends on correlation coefficient

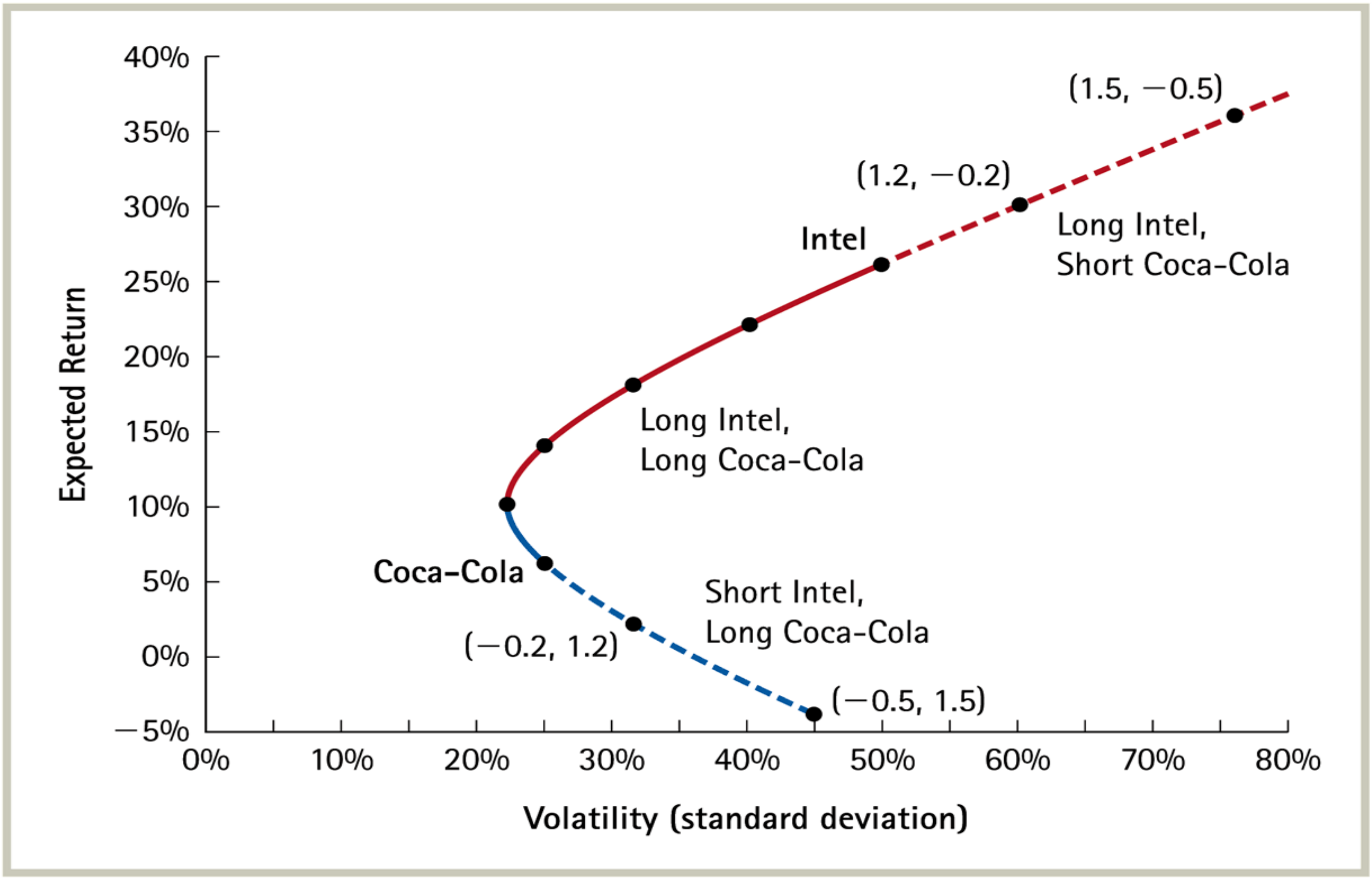
$$-1.0 \leq r \leq +1.0$$

- If r = +1.0, no risk reduction is possible
- If r = –1.0, complete risk reduction is possible

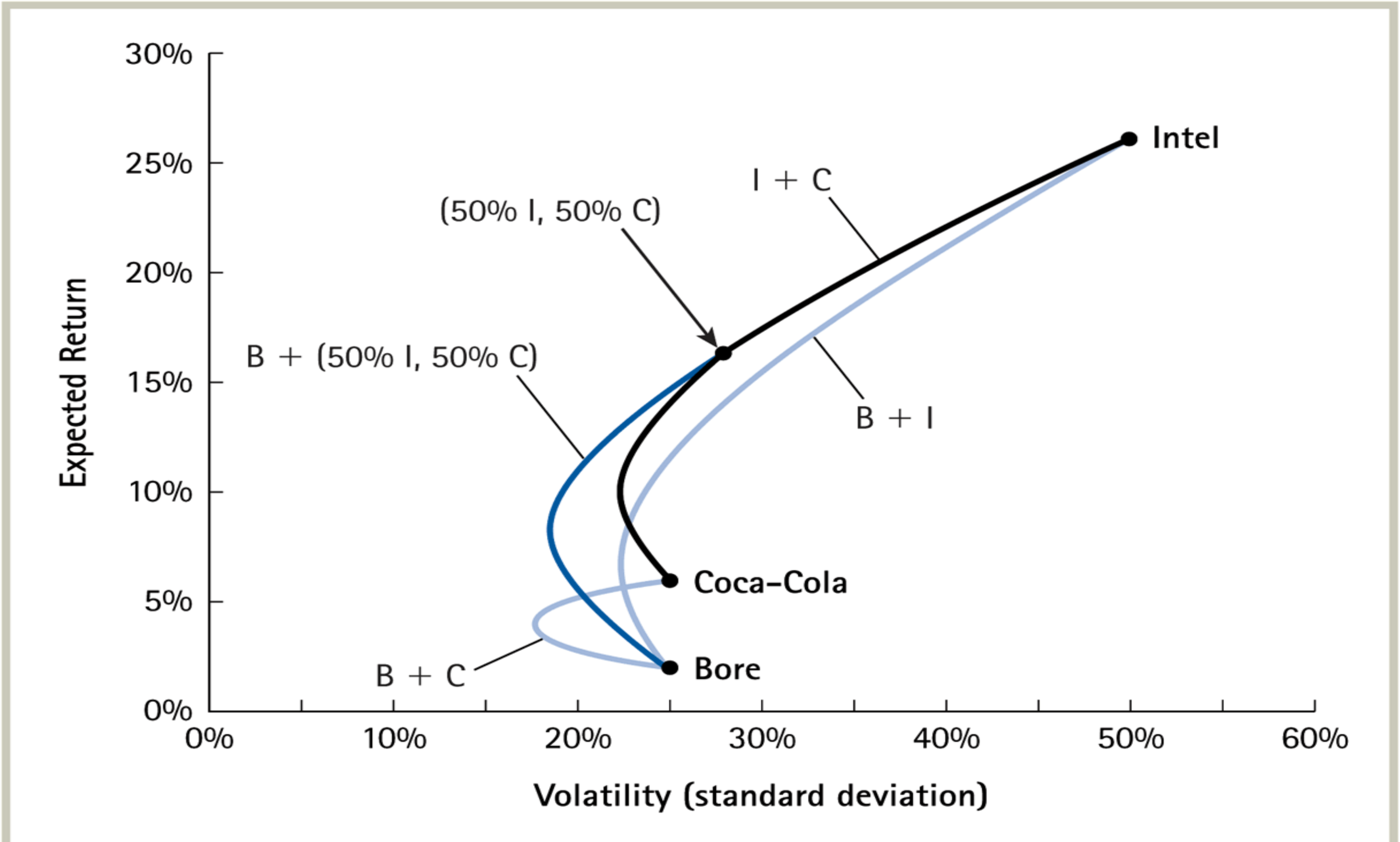
Short Sales 卖空

- Long Position
  - A positive investment in a security
- Short Position
  - A negative investment in a security
  - In a short sale, you sell a stock that you do not own and then buy that stock back in the future.
  - Short selling is an advantageous strategy if you expect a stock price to decline in the future.

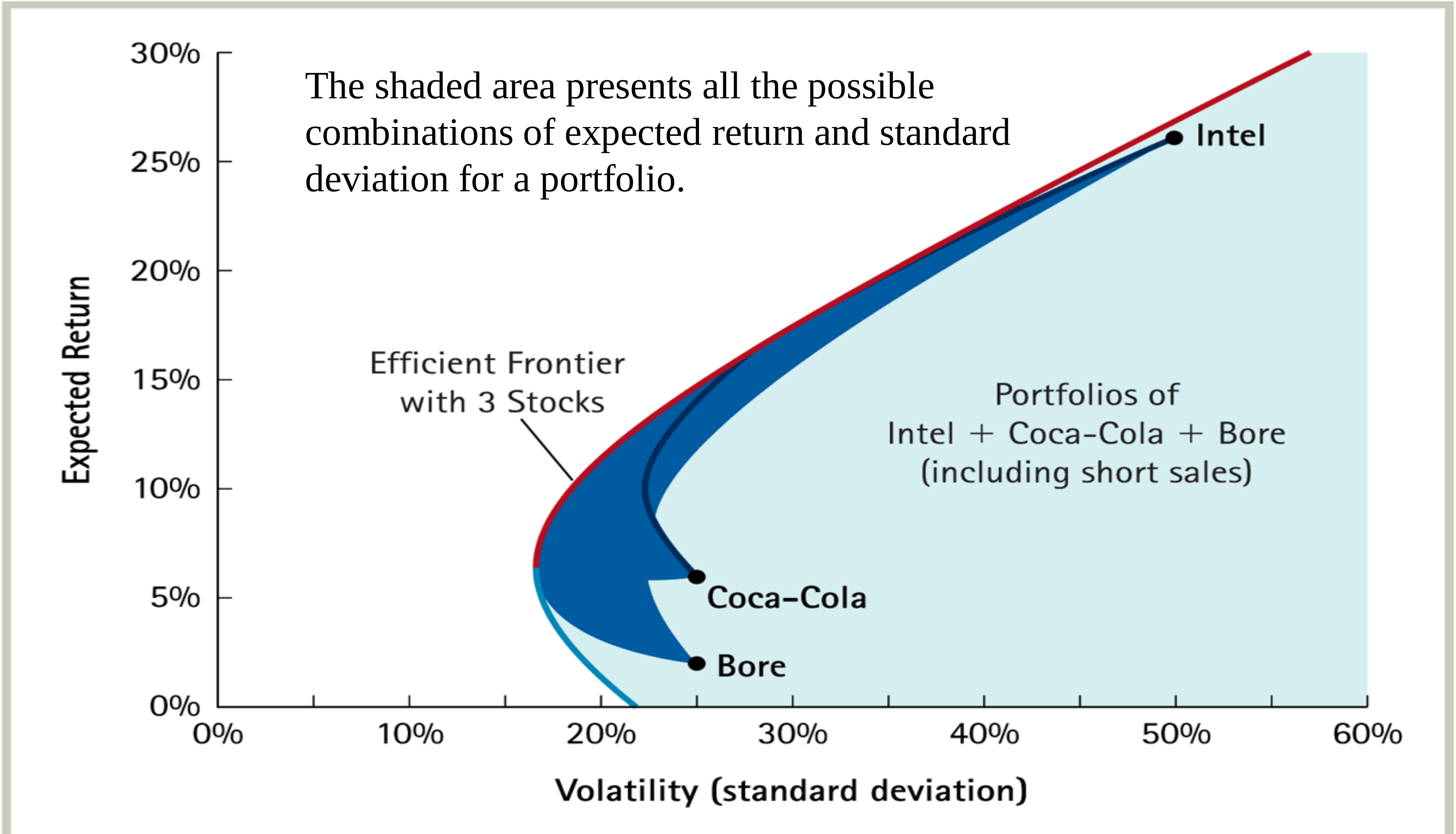
Portfolios of Intel and Coca-Cola Allowing for Short Sales



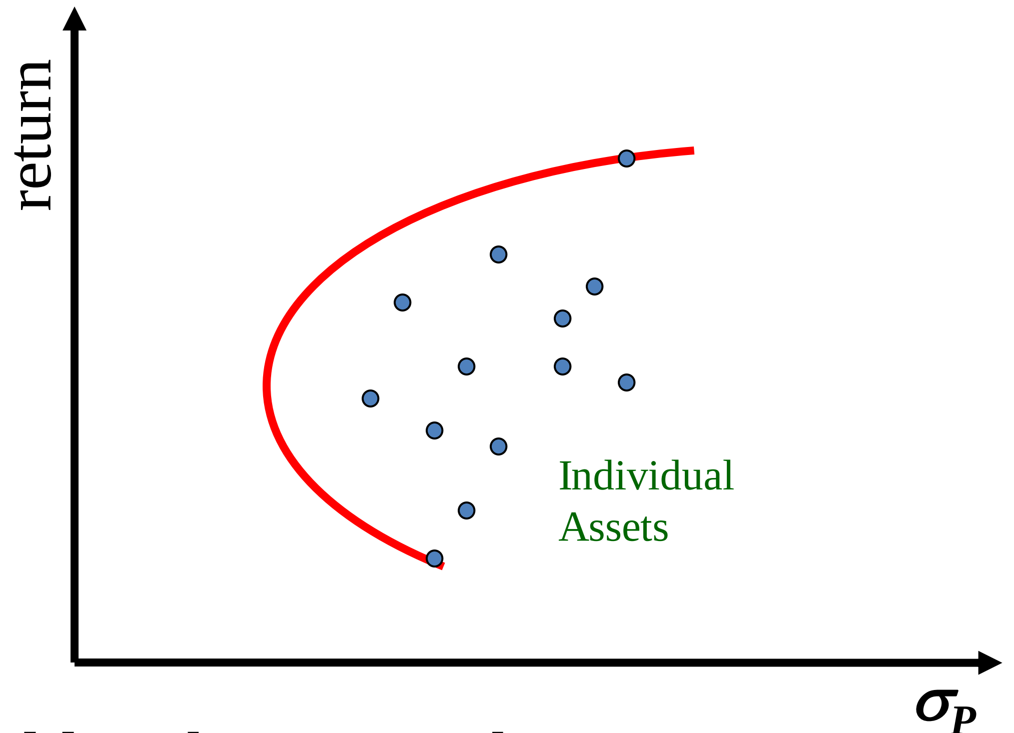
Expected Return and Volatility for Selected Portfolios of Intel, Coca-Cola, and Bore Industries Stocks



The Volatility and Expected Return for All Portfolios of Intel, Coca-Cola, and Bore Stock

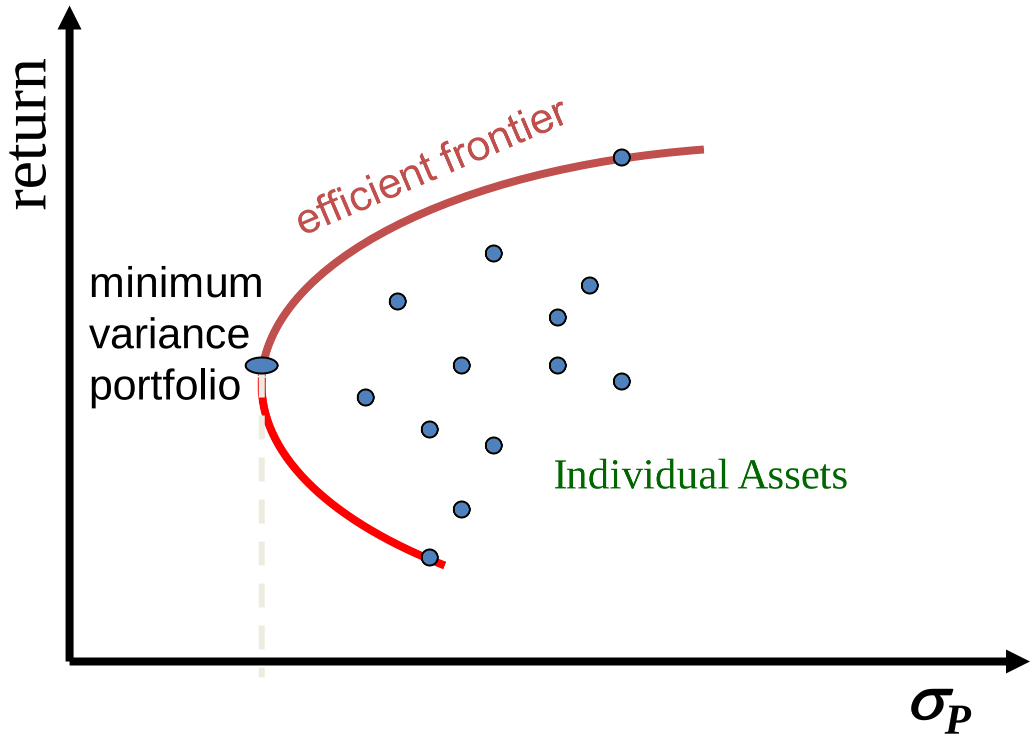


11.5 The Efficient Set for Many Securities



Consider a world with many risky assets; we can still identify the *opportunity set* of risk-return combinations of various portfolios.

The Efficient Set for Many Securities

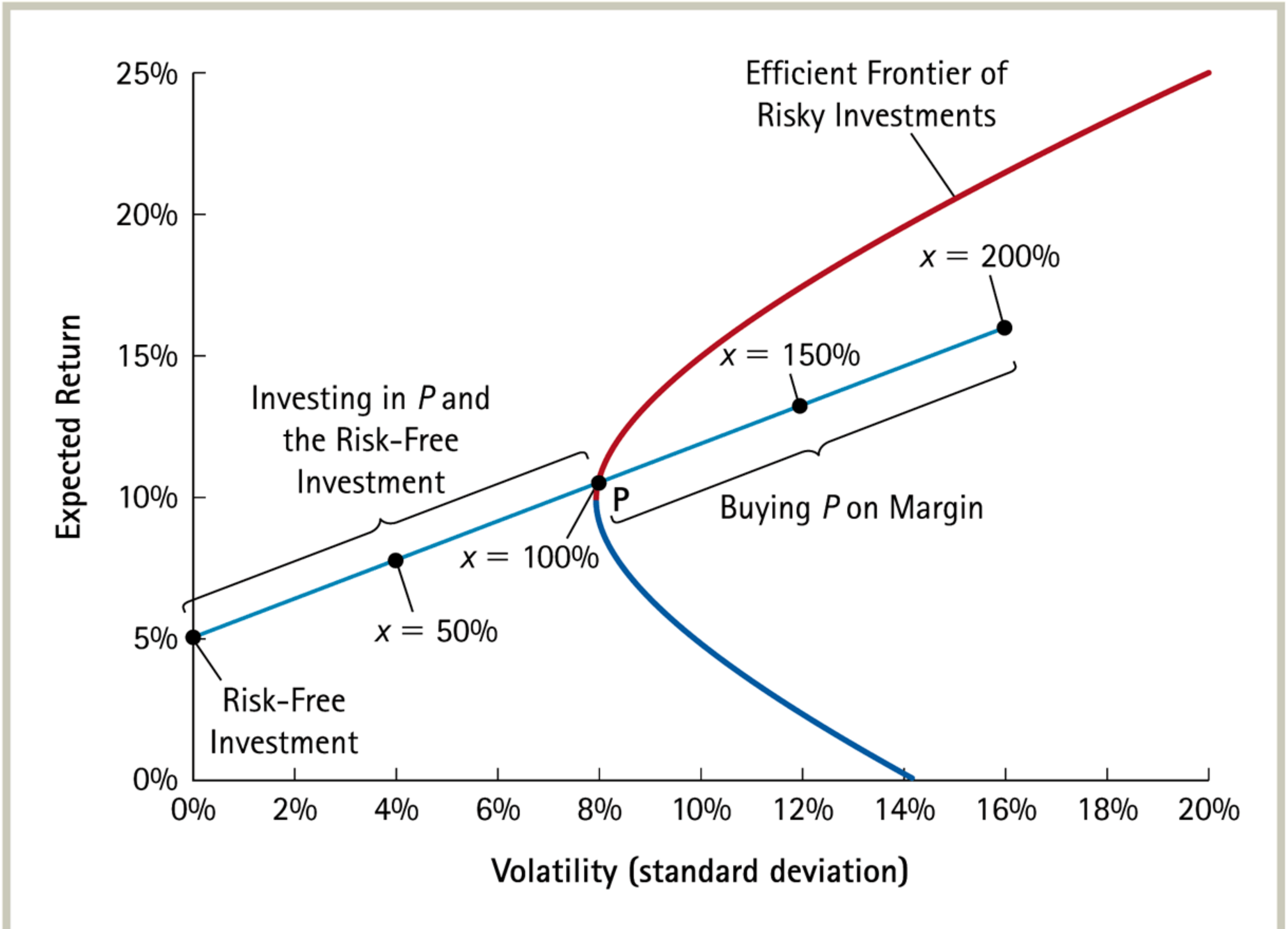


The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

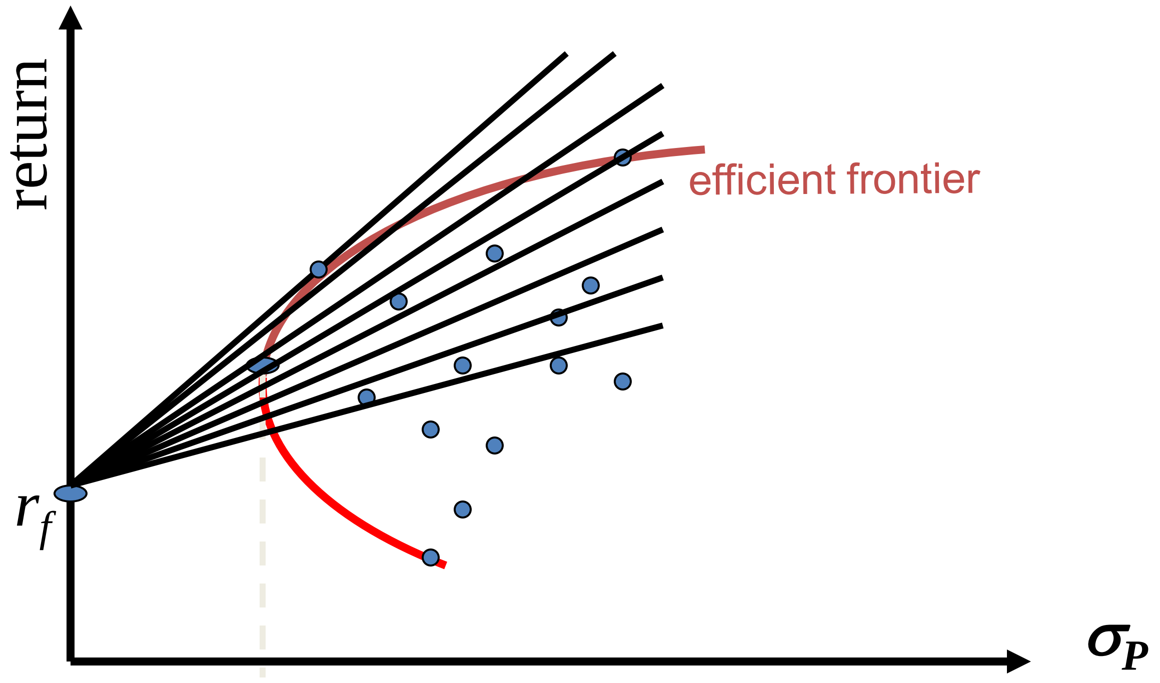
Risk-Free Saving and Borrowing

- Risk can also be reduced by investing a portion of a portfolio in a risk-free investment, like T-Bills. However, doing so will likely reduce the expected return.
- On the other hand, an aggressive investor who is seeking high expected returns might decide to borrow money to invest even more in the stock market.

The Risk–Return Combinations from Combining a Risk-Free Investment and a Risky Portfolio

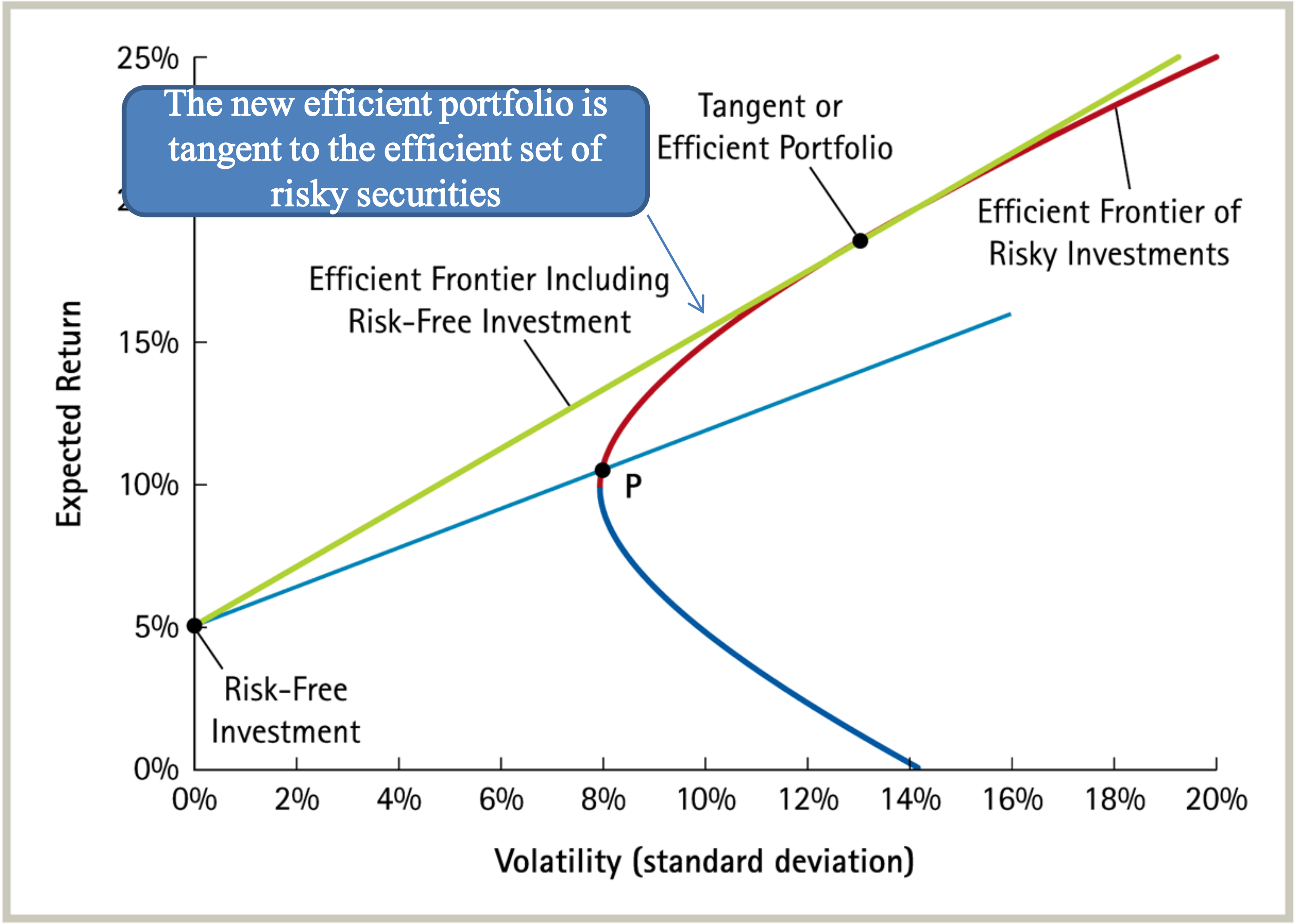


Riskless Borrowing and Lending



With a risk-free asset available and the efficient frontier identified, we choose the capital allocation line with the steepest slope.

The Tangent or Efficient Portfolio



Assumptions

- Assumption 1
  - Investors can buy and sell all securities at competitive market prices (without incurring taxes or transactions costs) and can borrow and lend at the risk-free interest rate.
- Assumption 2
  - Investors hold only efficient portfolios of traded securities—portfolios that yield the maximum expected return for a given level of volatility.
- Assumption 3
  - Investors have **homogeneous expectations** regarding the volatilities, correlations, and expected returns of securities.

Security Demand Must Equal Supply

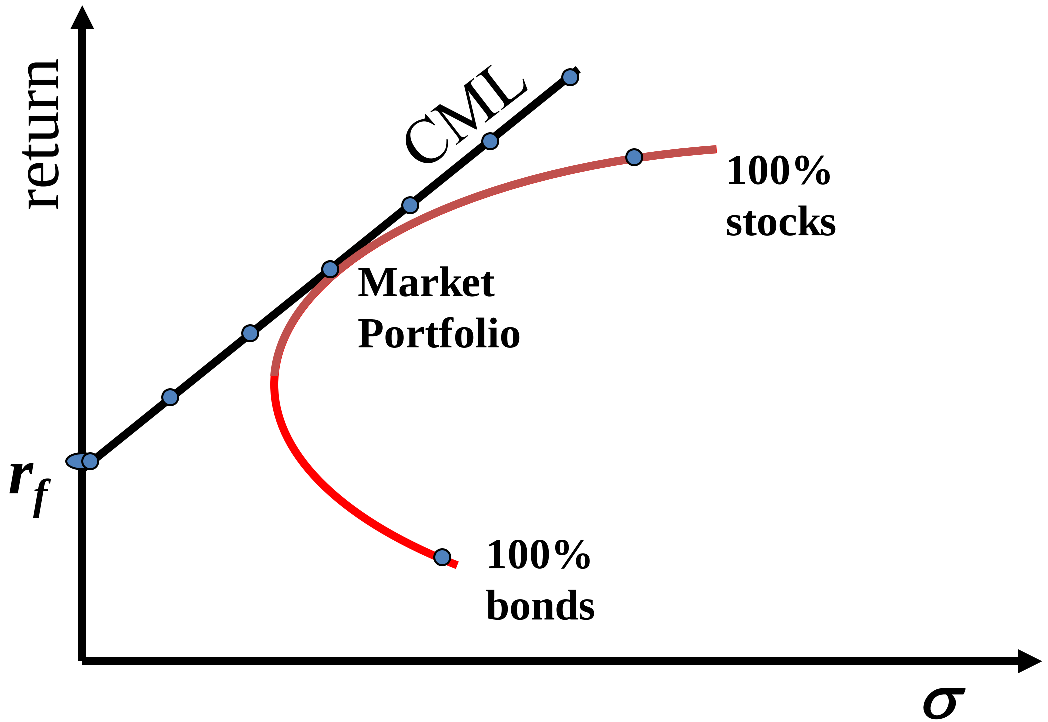
- Given homogeneous expectations, all investors will demand the same efficient portfolio of risky securities.
- The combined portfolio of risky securities of all investors must equal the efficient portfolio.
- Thus, if all investors demand the efficient portfolio, and the supply of securities is the market portfolio, the demand for market portfolio must equal the supply of the market portfolio.

The Capital Market Line (CML)

- In equilibrium, the market portfolio is the tangency portfolio.
- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio:

$$E(R_P) = R_f + \frac{[E(R_M) - R_f]}{\sigma_M}\sigma_P$$

Market Equilibrium



–Where the investor chooses along the Capital Market Line（资本市场线） depends on her risk tolerance. The big point is that all investors have the same CML.

Risk: Systematic and Unsystematic

- A *systematic risk* is any risk that affects a large number of assets, each to a greater or lesser degree.
- An *unsystematic risk* is a risk that specifically affects a single asset or small group of assets.
- Unsystematic risk can be diversified away.
- Examples of systematic risk include uncertainty about general economic conditions, such as GNP, interest rates or inflation.
- On the other hand, announcements specific to a single company are examples of unsystematic risk.

Breaking down sources of risk

Total risk = Systematic risk + Firm Specific Risk

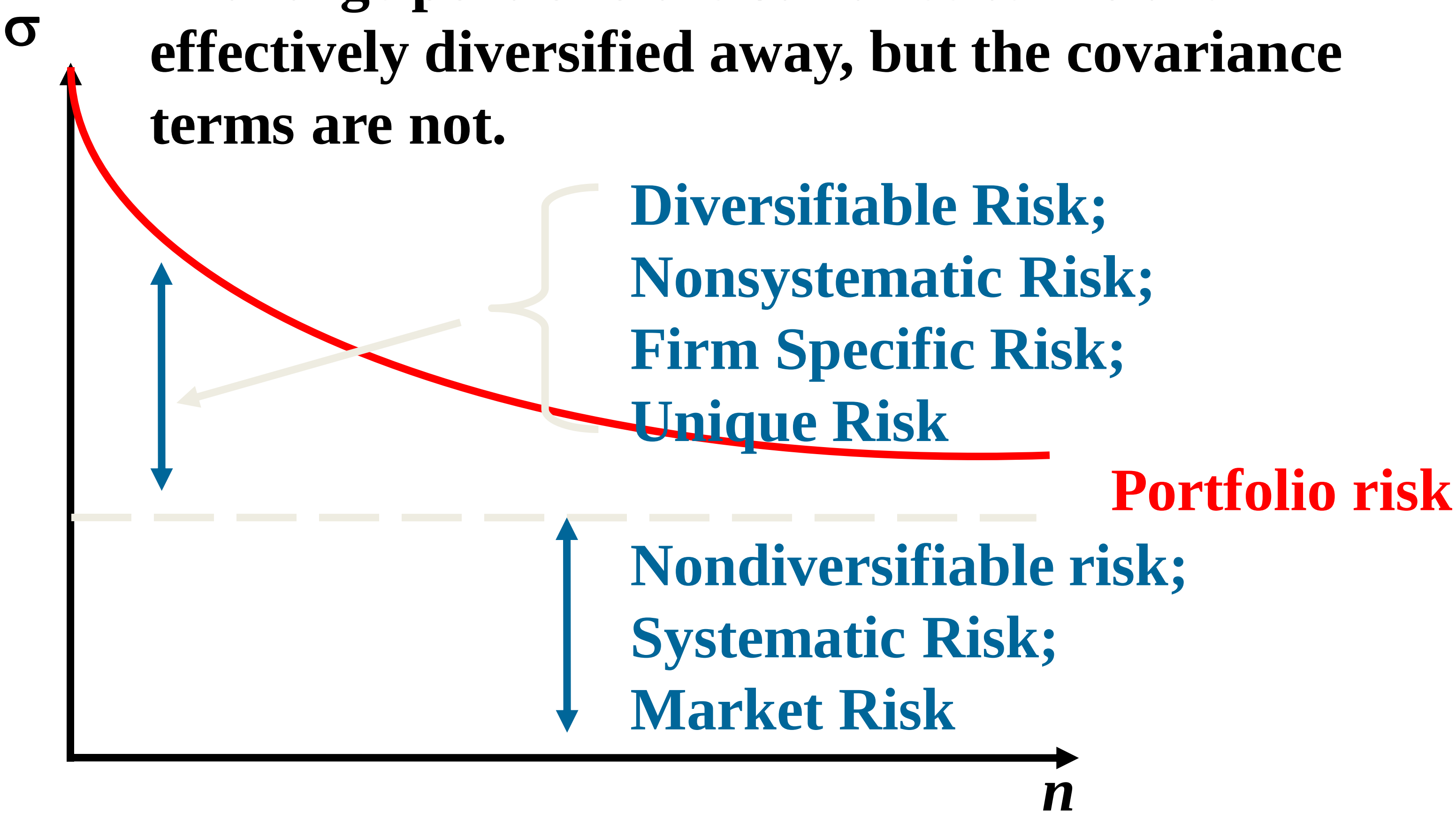
- Systematic risk – portion of a security's stand-alone risk that cannot be eliminated through diversification. Measured by beta.
- Firm Specific Risk – portion of a security's stand-alone risk that can be eliminated through proper diversification.

Diversification and Portfolio Risk

- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns.
- This reduction in risk arises because worse than expected returns from one asset are offset by better than expected returns from another.
- However, there is a minimum level of risk that cannot be diversified away, and that is the systematic portion.

Portfolio Risk and Number of Stocks

In a large portfolio the variance terms are effectively diversified away, but the covariance terms are not.



Measuring Systematic Risk (cont'd)

- Beta (β)
  - The expected percent change in the excess return of a security for a 1% change in the excess return of the market portfolio.
    - Beta differs from volatility. Volatility measures total risk (systematic plus unsystematic risk), while beta is a measure of only systematic risk.
  - Measures a stock's market risk, and shows a stock's volatility relative to the market.
  - Indicates how risky a stock is if the stock is held in a well-diversified portfolio.

Measuring Systematic Risk (cont'd)

- Beta (β)
  - A security's beta is related to how sensitive its underlying revenues and cash flows are to general economic conditions.

Measuring Systematic Risk

$$\beta_i = Cov(R_i, R_M)/Var(R_M)$$

- A beta of 1 implies the asset has the same systematic risk as the overall market
- A beta < 1 implies the asset has less systematic risk than the overall market
- A beta > 1 implies the asset has more systematic risk than the overall market

Estimating the Risk Premium

- Market Risk Premium
  - The market risk premium is the reward investors expect to earn for holding a portfolio with a beta of 1.

$$\text{Market Risk Premium} = E\left[R_{Mkt}\right] - r_f$$

Market Equilibrium

In equilibrium, all assets and portfolios must have the same reward-to-risk ratio, and they all must equal the reward-to-risk ratio for the market

$$\frac{E(R_i) - R_f}{\beta_i} = \frac{E(R_M - R_f)}{\beta_M} = \frac{E(R_M - R_f)}{1}$$

Estimating the Risk Premium (cont'd)

- Estimating a Traded Security's Expected Return from Its Beta

$$E[R] = \text{Risk-Free Interest Rate} + \text{Risk Premium}$$

$$= r_f + \beta \times (E[R_{Mkt}] - r_f)$$

Capital Asset Pricing Model (CAPM)

- This can be rewritten as the capital asset pricing model:

$$\mathbf{E(R_i) = R_f + \beta_i(E(R_M) - R_f)}$$

- If we know an asset's systematic risk, we can use the CAPM to determine its expected return

Example

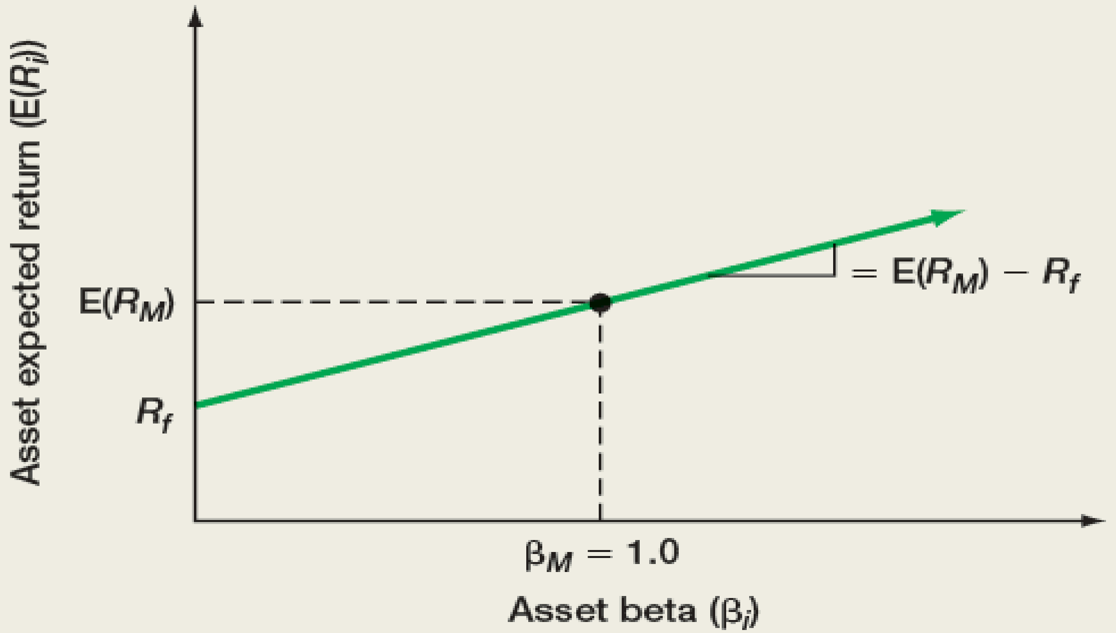
- Problem
  - Assume the economy has a 60% chance the market return will 15% next year and a 40% chance the market return will be 5% next year.
  - Assume the risk-free rate is 6%.
  - **If Microsoft's beta is 1.18, what is its expected return next year?**

Example

- Solution
  - $E[R_{Mkt}] = (60\% \times 15\%) + (40\% \times 5\%) = 11\%$
  - $E[R] = r_f + \beta \times (E[R_{Mkt}] - r_f)$
  - $E[R] = 6\% + 1.18 \times (11\% - 6\%)$
  - $E[R] = 6\% + 5.9\% = 11.9\%$

Security Market Line (SML)

- A positively sloped straight line displaying the relationship between expected return and beta

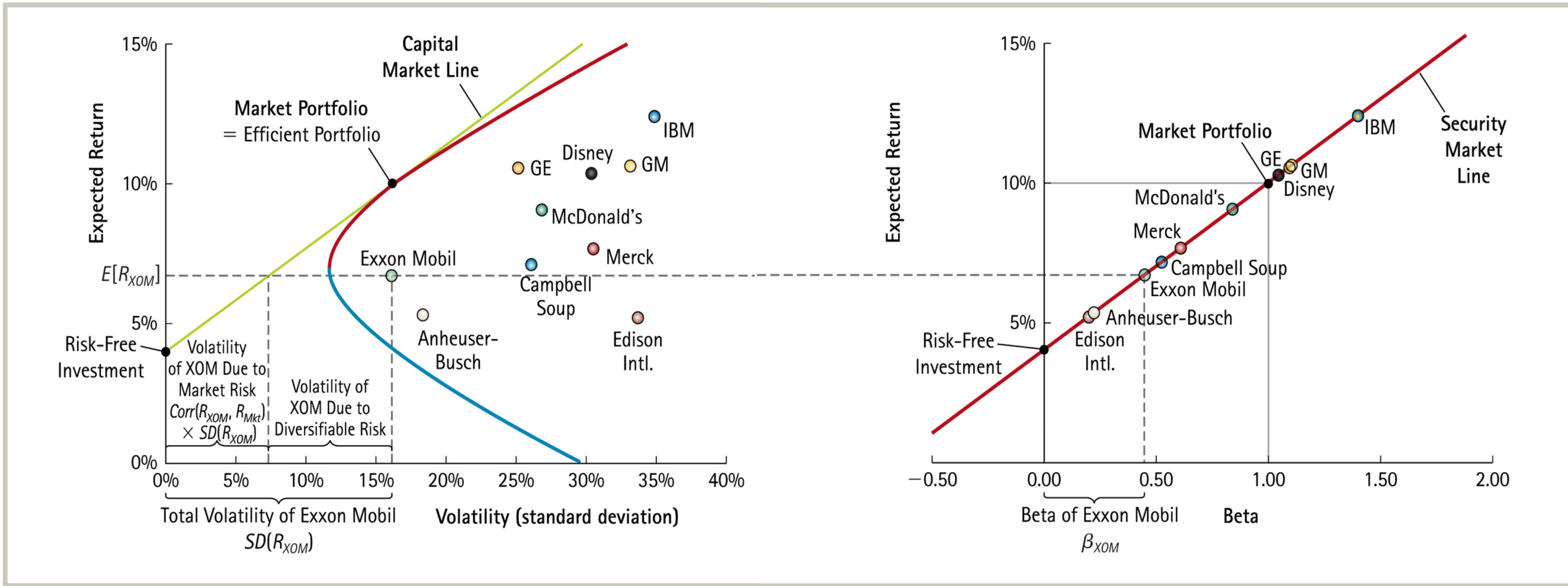


The slope of the security market line is equal to the market risk premium—that is, the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

which is the capital asset pricing model (CAPM).

The Capital Market Line and the Security Market Line



(a) The CML depicts portfolios combining the risk-free investment and the efficient portfolio, and shows the highest expected return that can be attained for each level of volatility. According to the CAPM, the market portfolio is on the CML and all other stocks and portfolios contain diversifiable risk and lie to the right of the CML, as illustrated for Exxon Mobil (XOM).

(b) The SML shows the required return for each security as a function of its beta with the market. According to the CAPM, the market portfolio is efficient, which is equivalent to the required return equaling the expected return for every security. According to the CAPM, all stocks and portfolios should lie on the SML.

The Security Market Line (cont'd)

- The beta of a portfolio is the weighted average beta of the securities in the portfolio.

$$\beta_P = \frac{Cov(R_P, R_{Mkt})}{Var(R_{Mkt})} = \frac{Cov\left(\sum_i x_i R_i, R_{Mkt}\right)}{Var(R_{Mkt})} = \sum_i x_i \frac{Cov(R_i, R_{Mkt})}{Var(R_{Mkt})} = \sum_i x_i \beta_i$$

Example

- Problem
  - Suppose the stock of the 3M Company (MMM) has a beta of 0.69 and the beta of Hewlett-Packard Co. (HPQ) stock is 1.77.
  - Assume the risk-free interest rate is 5% and the expected return of the market portfolio is 12%.
  - What is the expected return of a portfolio of 40% of 3M stock and 60% Hewlett-Packard stock, according to the CAPM?

Example

- Solution

$$\beta_P = \sum_i x_i \beta_i = (.40)(0.69) + (.60)(1.77) = 1.338$$

$$E[R_i] = 5\% + 1.338(12\% - 5\%) = 14.37\%$$